

# Unsafe but Calculable

Jesse Thaler



University of Toronto, THEP Seminar — December 8, 2015

		IRC Safe	IRC Unsafe
Lore:	Calculable in pQCD?	✓	✗
	Controlled $\Lambda_{\text{QCD}}$ Effects?	✓	✗

		IRC Safe	IRC Unsafe
New Lore:	Calculable in pQCD?	✓	✓*
	Controlled $\Lambda_{\text{QCD}}$ Effects?	✓*	✓*

## Sudakov Safety

[Andrew Larkoski, JDT, 1307.1699, 1406.7011]

[Andrew Larkoski, Simone Marzani, Gregory Soyez, JDT, 1402.2657]

[Andrew Larkoski, Simone Marzani, JDT, 1502.01719]

## Offline: Generalized Fragmentation Functions

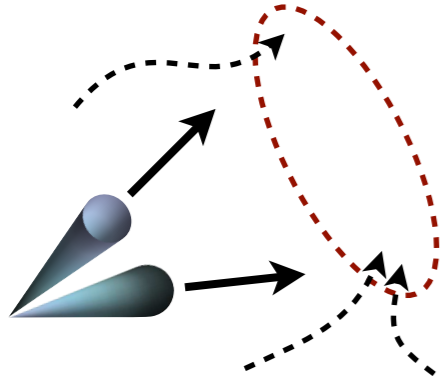
[Hsi-Ming Chang, Massimiliano Procura, JDT, Wouter Waalewijn, 1303.6637, 1306.6630]

[Andrew Larkoski, JDT, Wouter Waalewijn, 1408.3122]

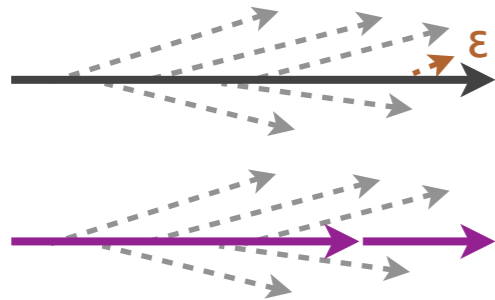
*All observables are calculable,  
but some observables are  
more calculable than others.*

≈ George Orwell, *Animal Farm*

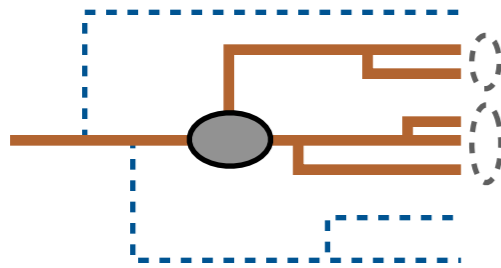
# Outline



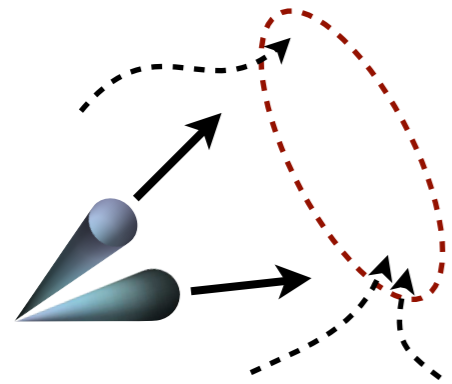
Inspiration from Jet Substructure



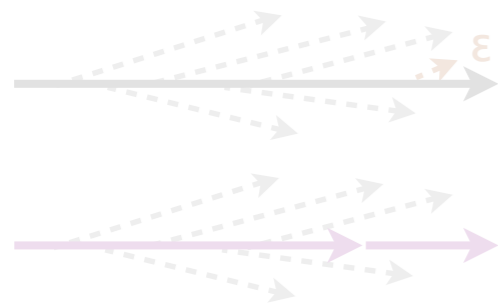
From IRC Safe to Sudakov Safe



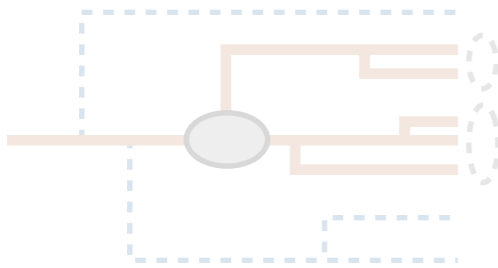
Probing the Core of QCD



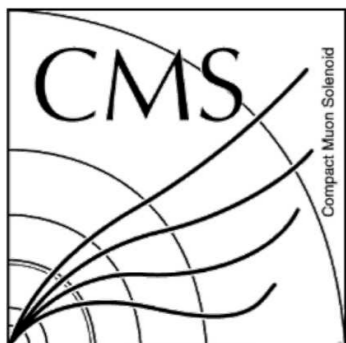
## Inspiration from Jet Substructure



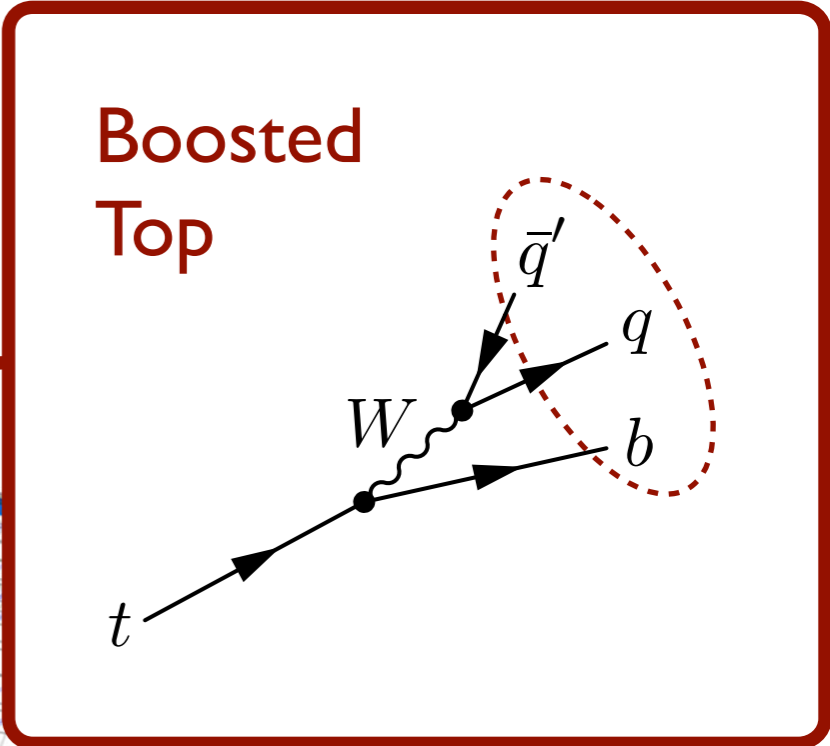
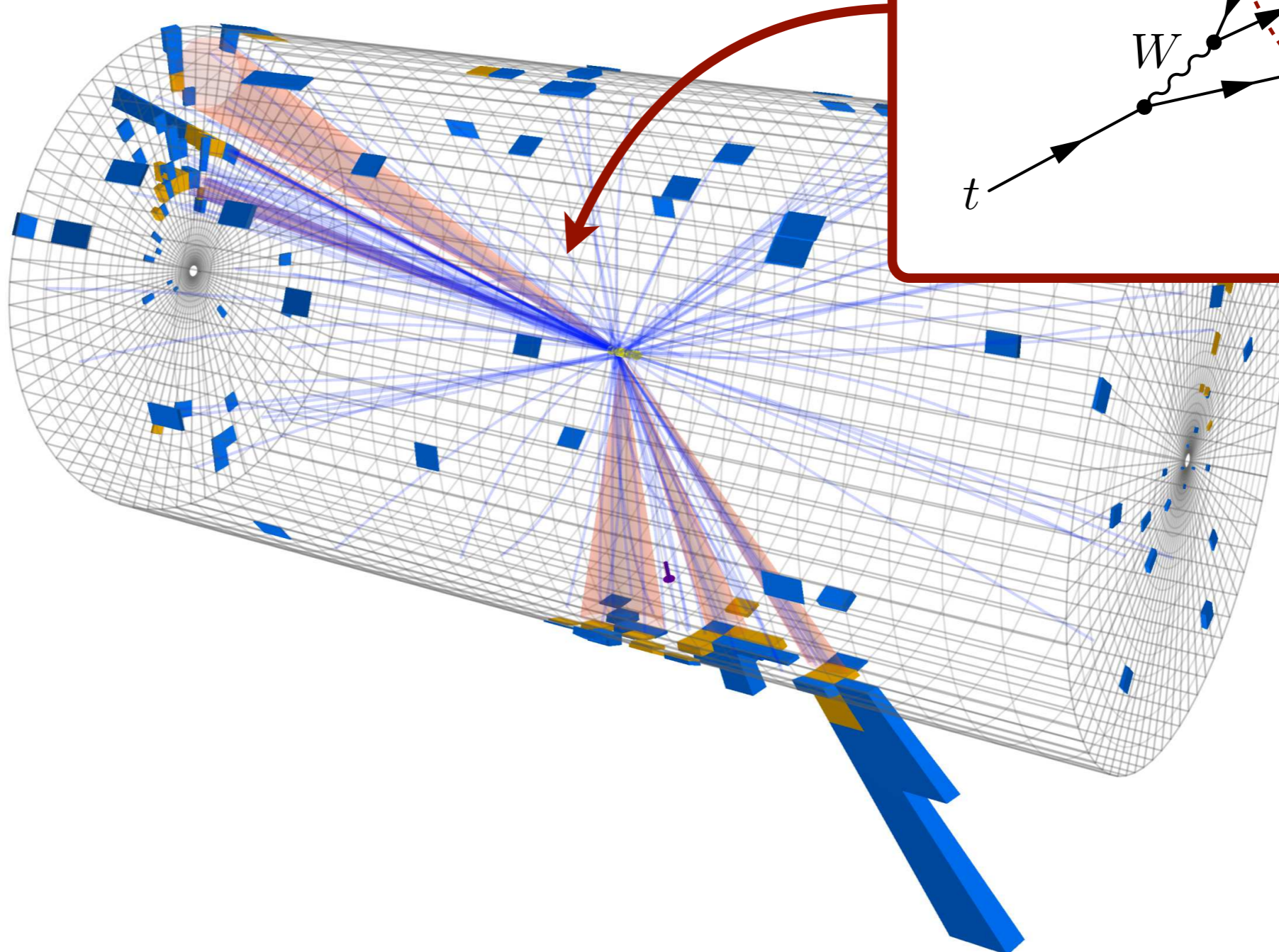
## From IRC Safe to Sudakov Safe



## Probing the Core of QCD



CMS Experiment at LHC, CERN  
Data recorded: Sun Jul 12 07:25:11 2015 CEST  
Run/Event: 251562 / 111132974  
Lumi section: 122  
Orbit/Crossing: 31722792 / 2253

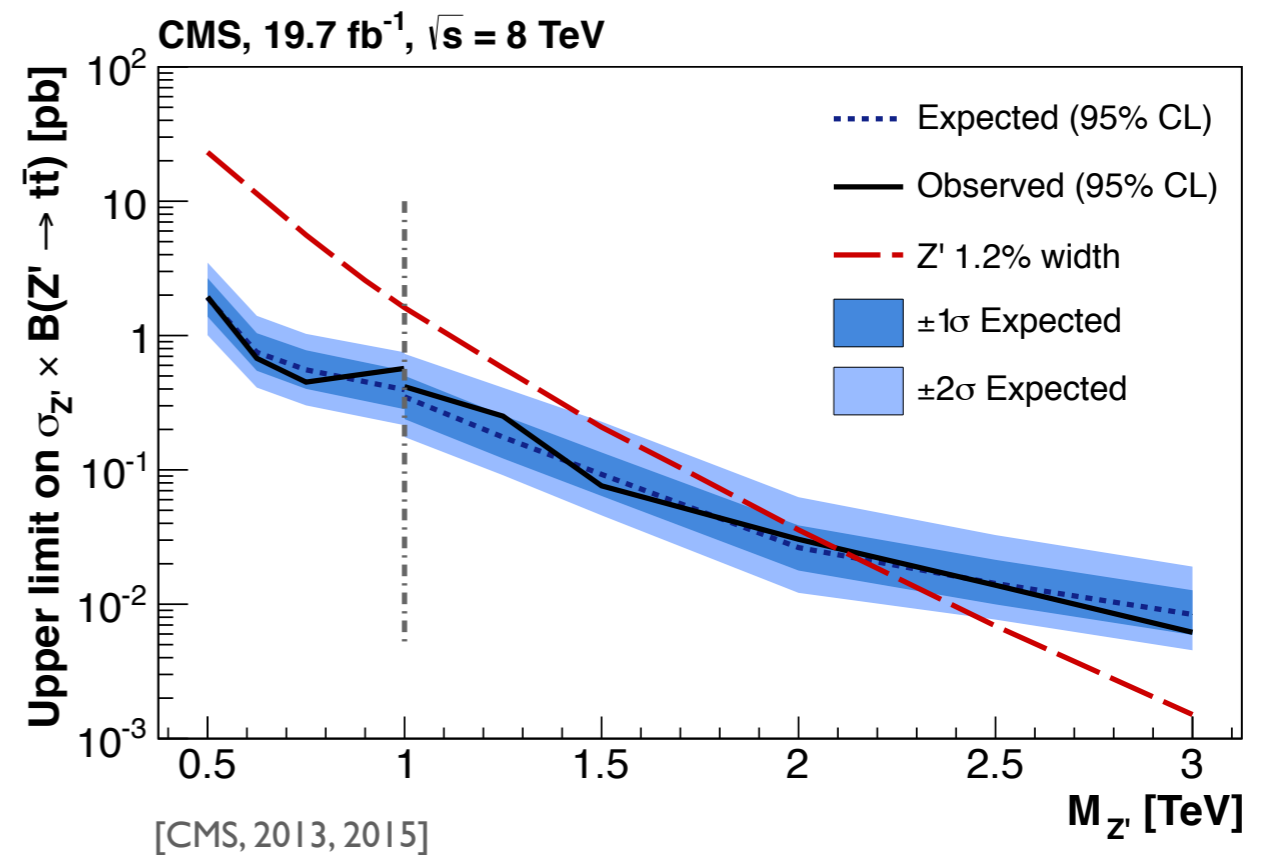
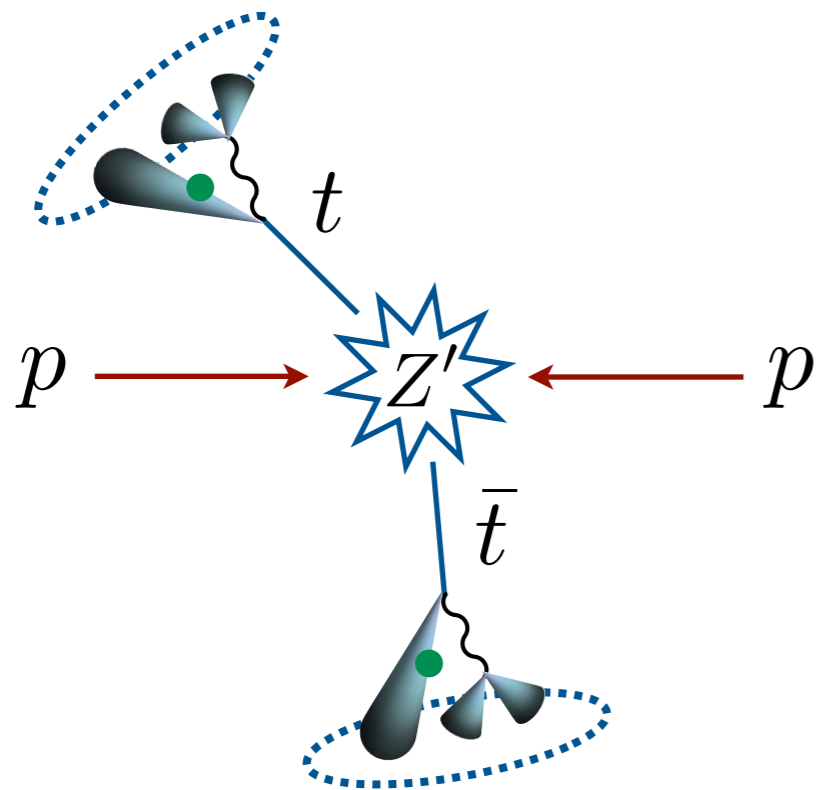


[CMS 2011, 2013, 2015]

[using Kaplan, Rehermann, Schwartz, Tweedie, 2008; using Ellis, Vermilion, Walsh, 2009]

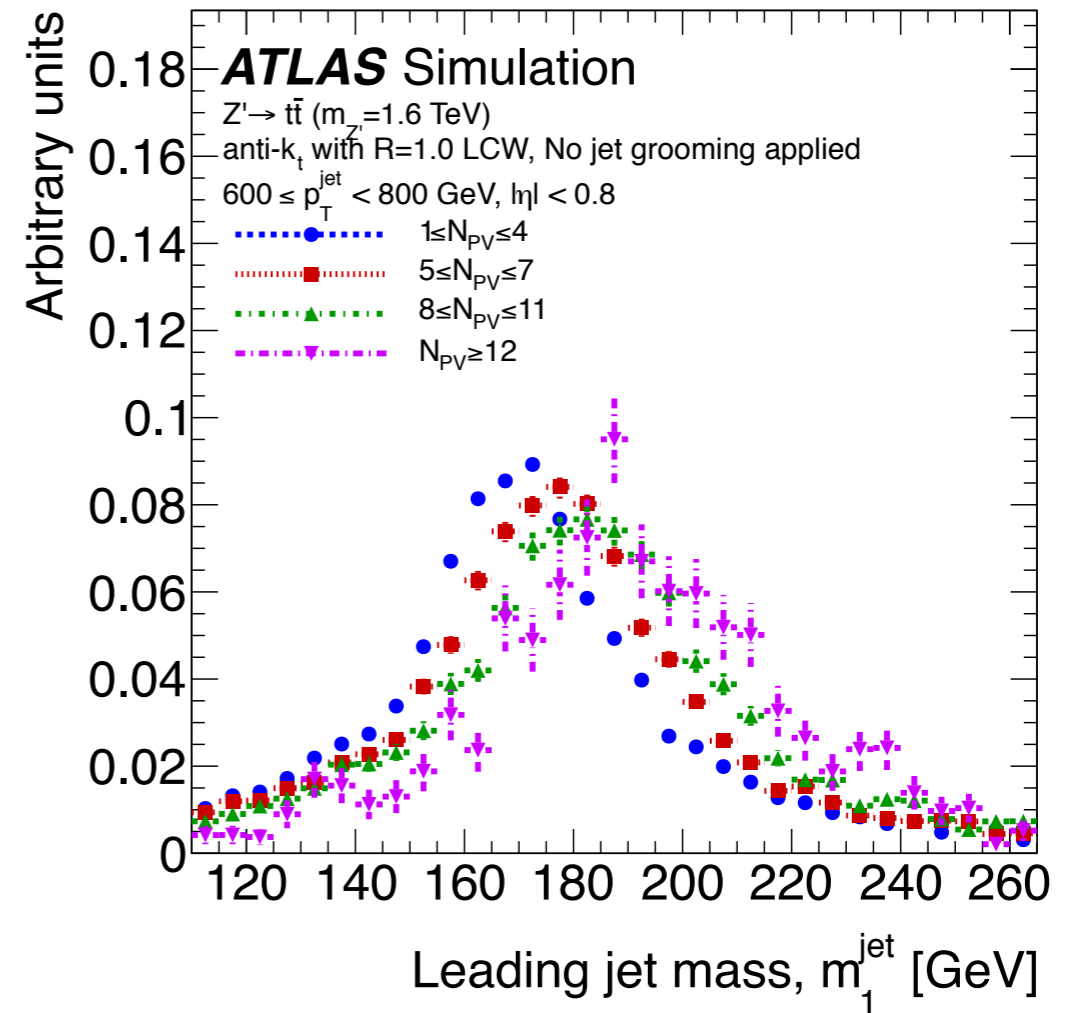
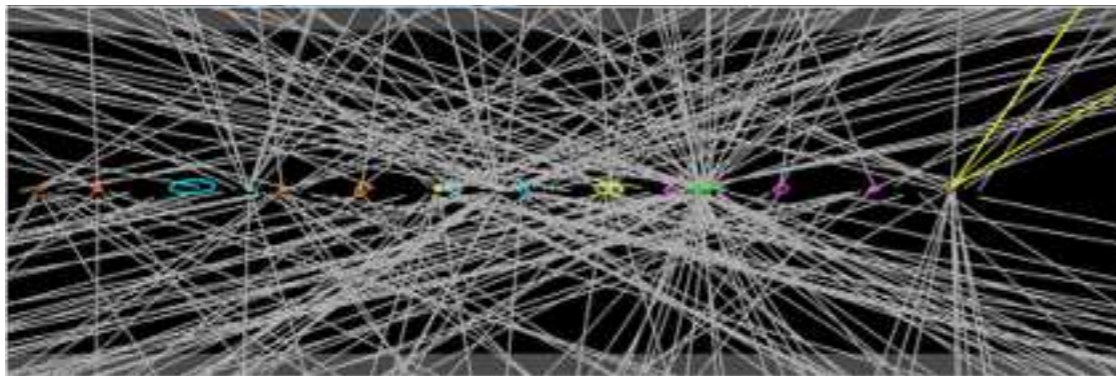
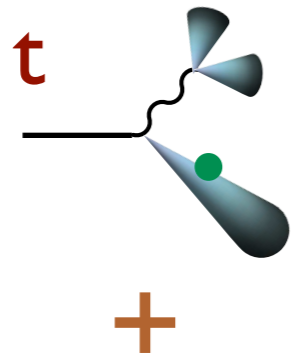
# High Energy $\Rightarrow$ Boosted Regime

$$\Delta R \simeq \frac{2m_{\text{top}}}{p_T}$$



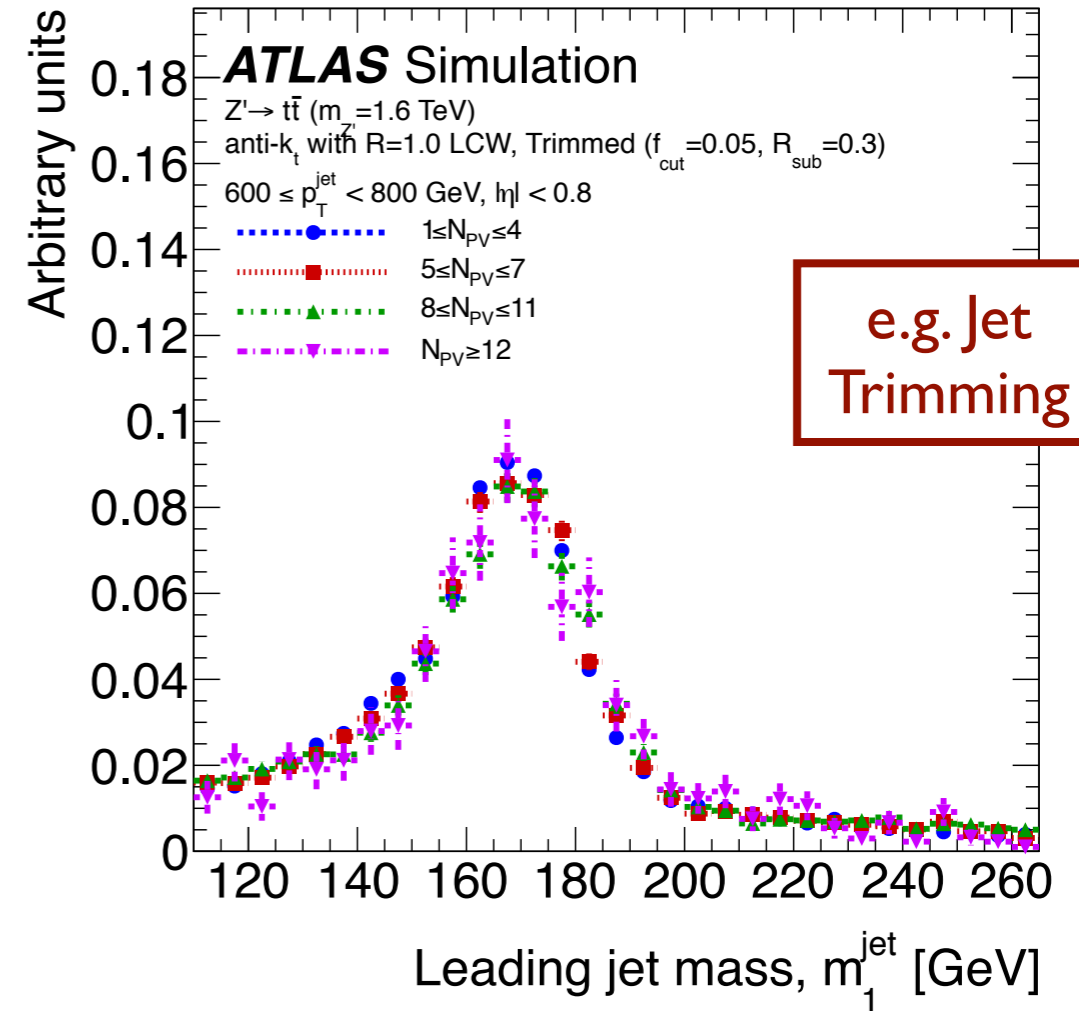
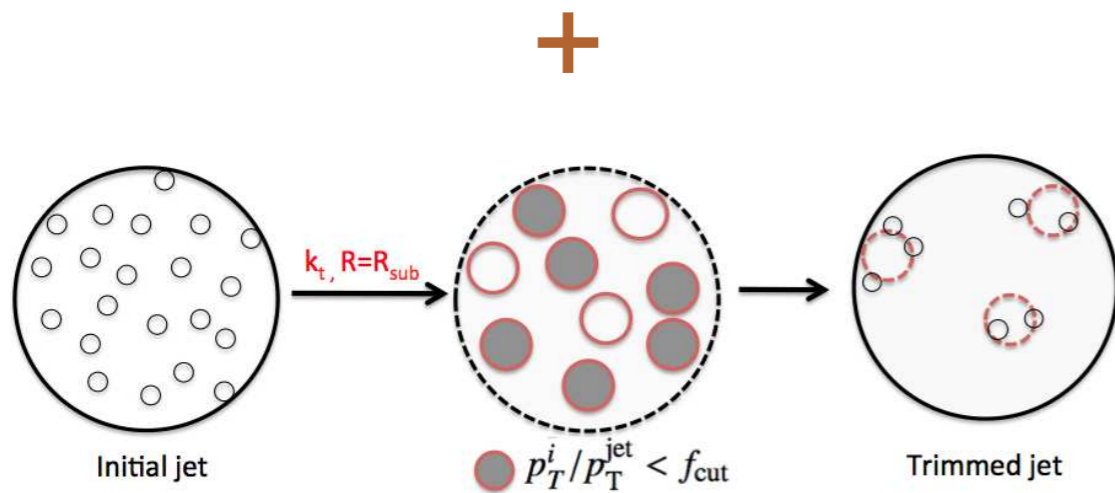
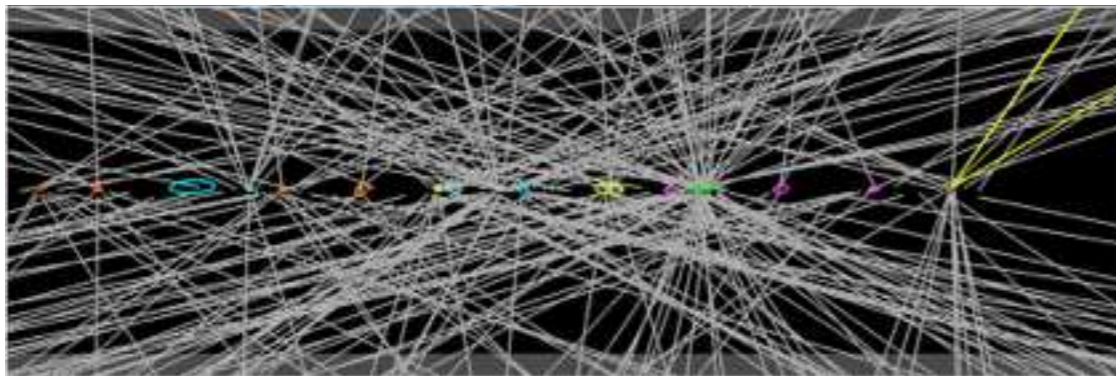
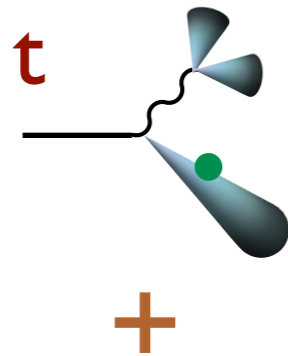


# High Luminosity $\Rightarrow$ Pileup

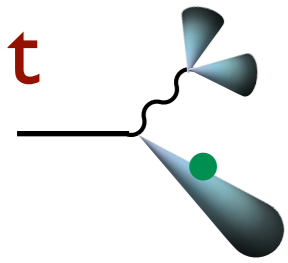


[ATLAS, 2012]

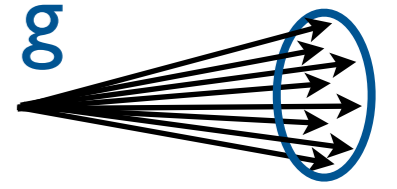
# High Luminosity $\Rightarrow$ Pileup



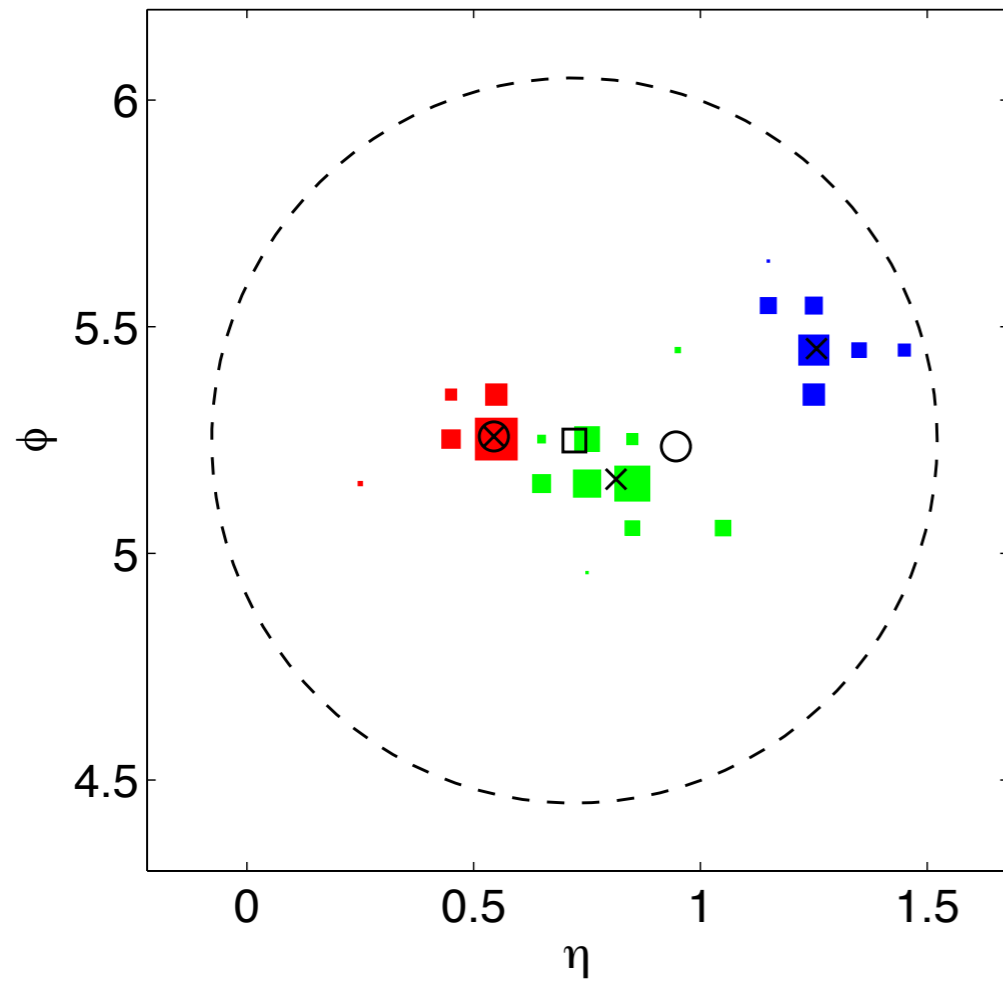
[ATLAS, 2012; using Krohn, JDT, Wang, 2009]



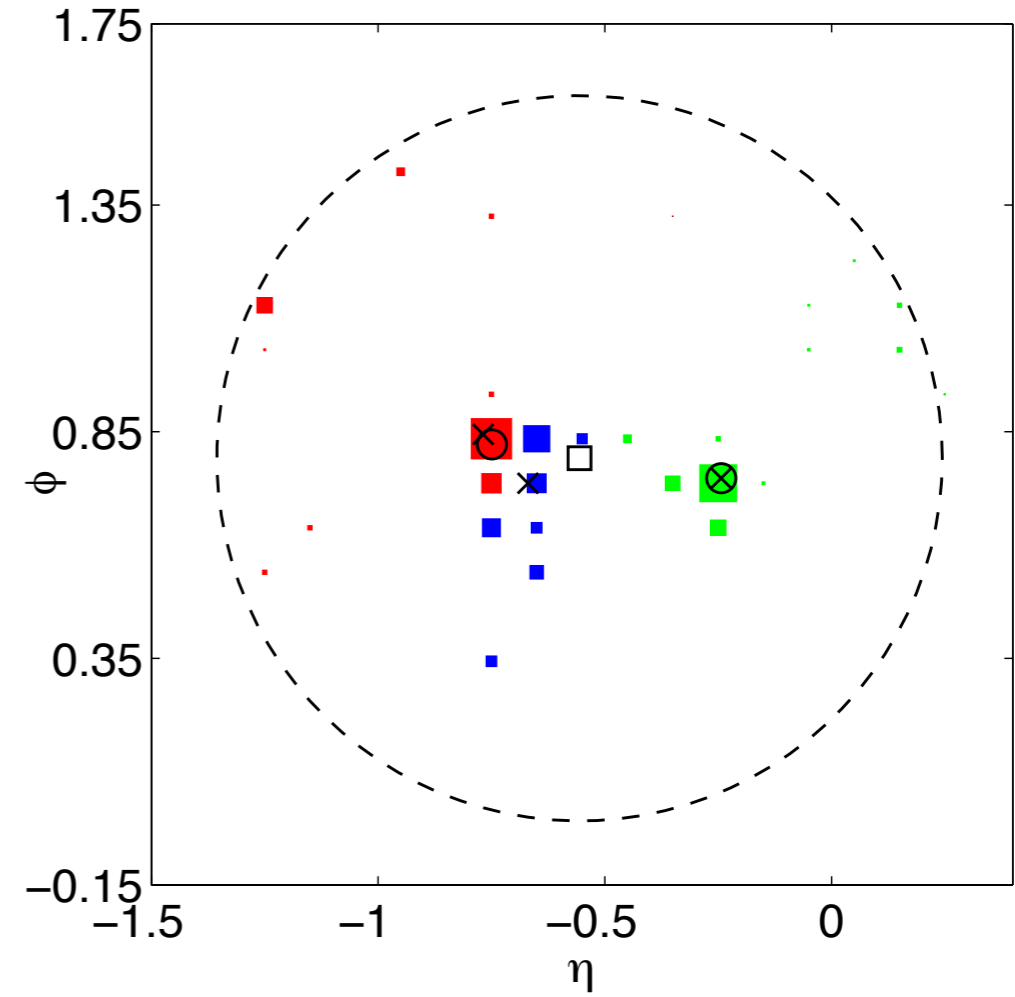
# N-Prong vs. I-Prong



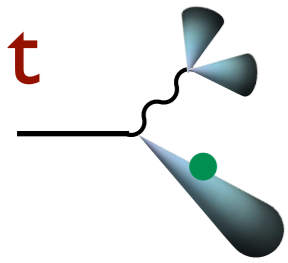
Boosted Top Jet,  $R = 0.8$



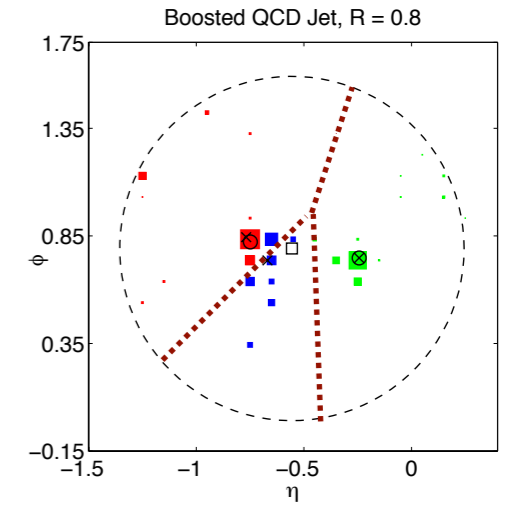
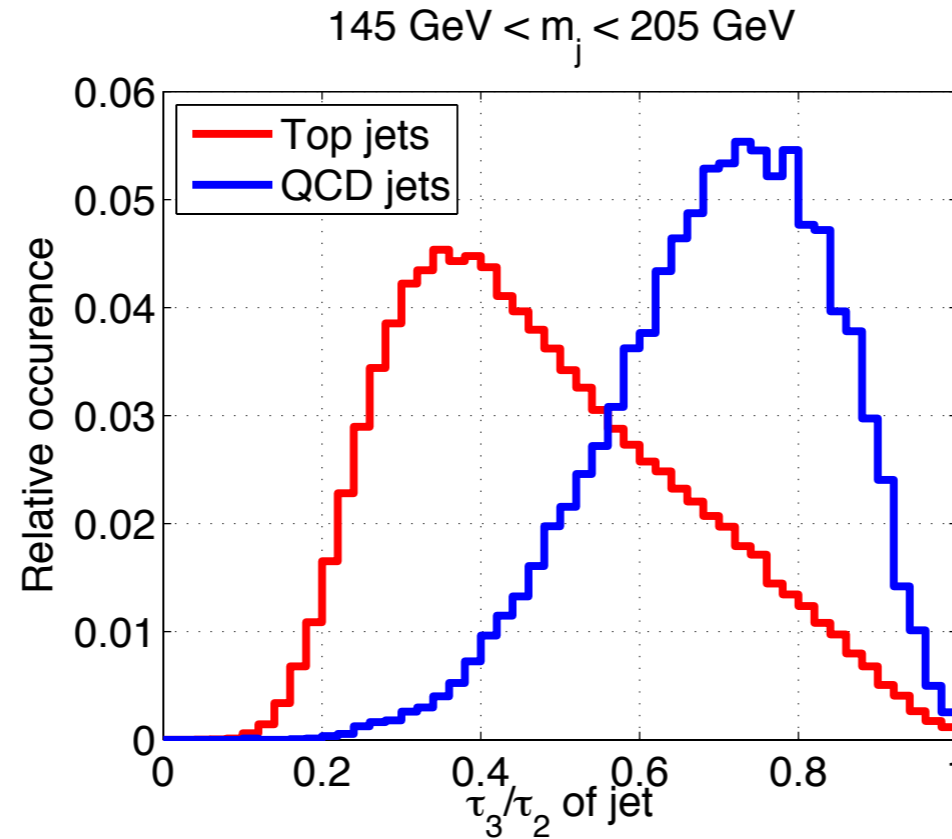
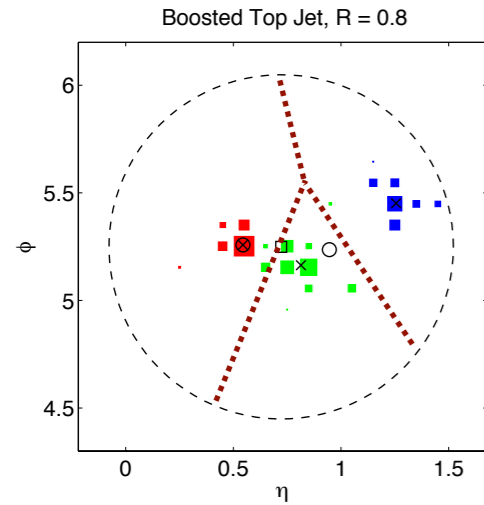
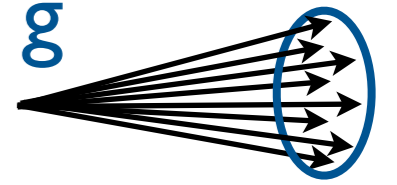
Boosted QCD Jet,  $R = 0.8$



(Both jets have  $m \approx 170$  GeV)



# N-Prong vs. 1-Prong



## N-subjettiness

$$\tau_N = \sum_k p_{T,k} \min \{ \Delta R_{k,1}, \Delta R_{k,2}, \dots, \Delta R_{k,N} \}^\beta$$

$\uparrow$  transverse momentum  
 $\uparrow$  distance to subjet core  
 $\uparrow$  angular exponent

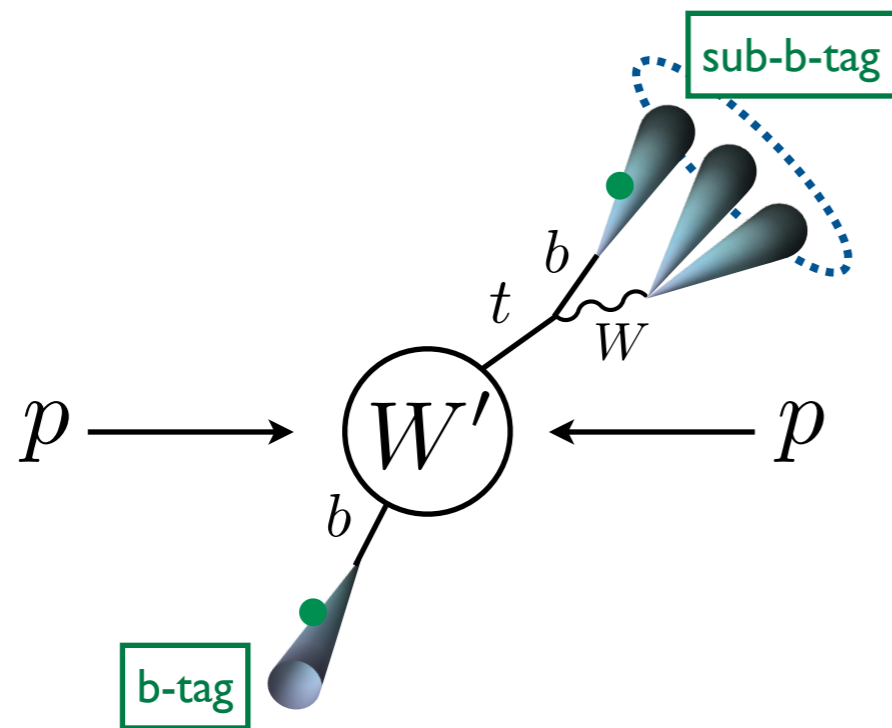
$\tau_3/\tau_2$ :	$t \rightarrow bW$
$\tau_2/\tau_1$ :	$H \rightarrow b\bar{b}$
	$Z \rightarrow q\bar{q}$
	$W \rightarrow q\bar{q}'$

[JDT, Van Tilburg, 2010, 2011]

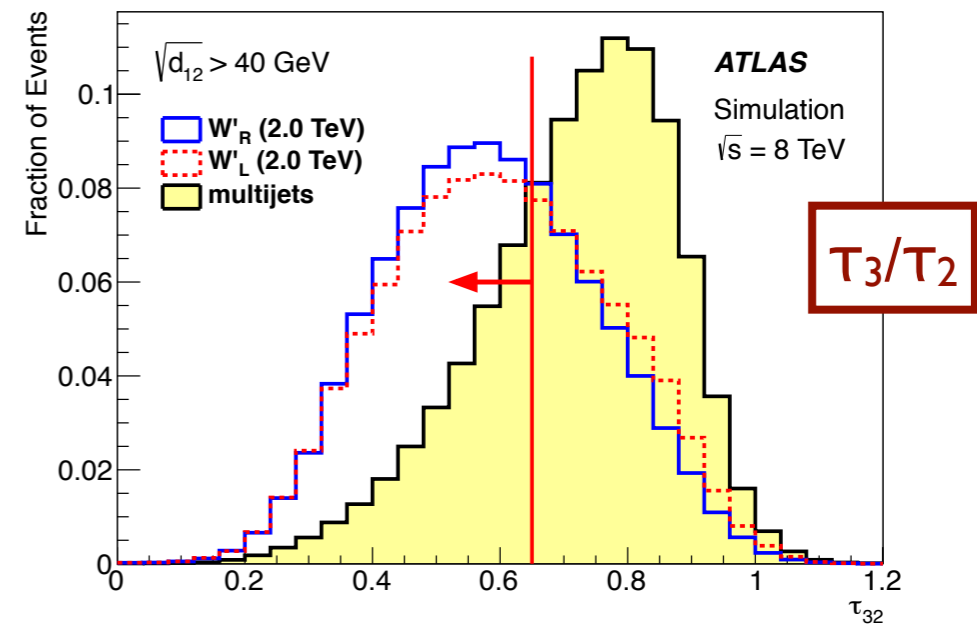
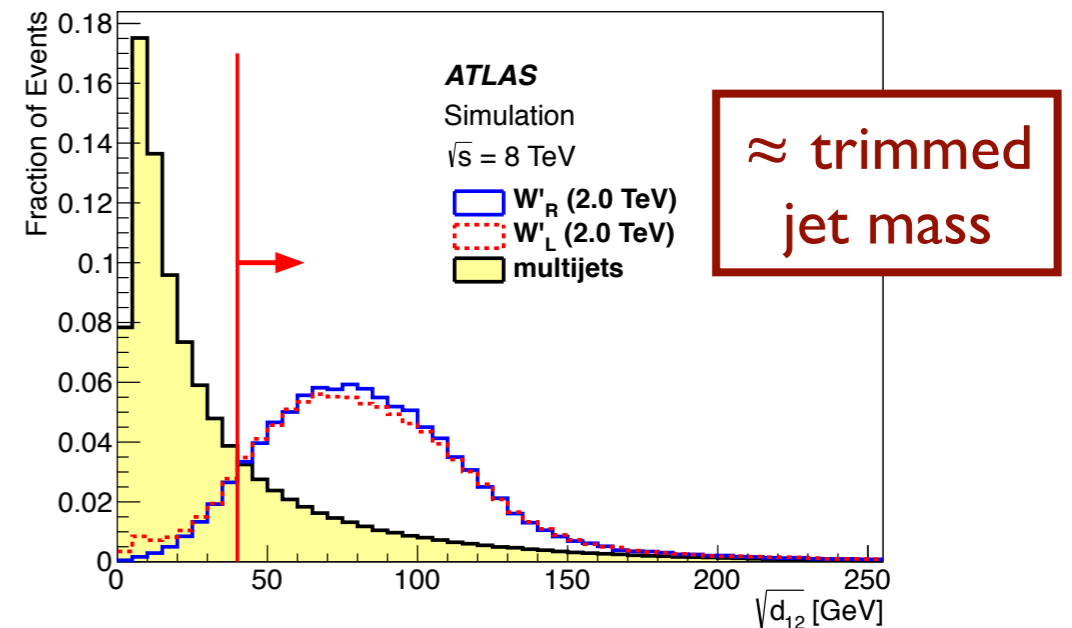
[see also Stewart, Tackmann, Waalewijn, 2010; Larkoski, Mout, Neill, 2014]

# ATLAS Search for Heavy W Bosons

*Trimming + B-tagging*  
+ *N-subjettiness*



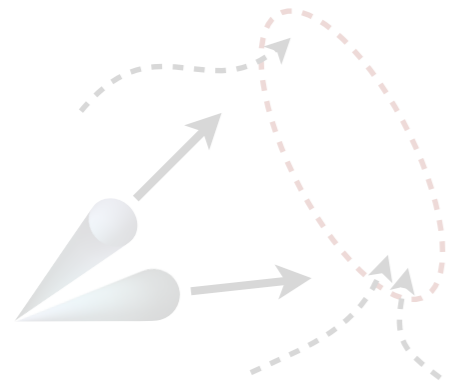
Similar techniques used for  
ATLAS diboson excess



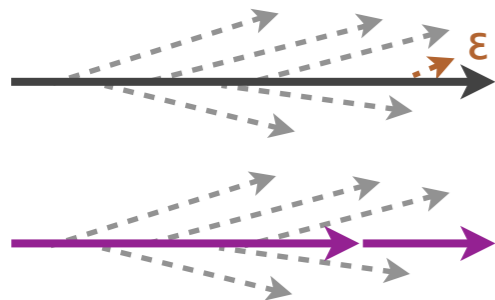
[ATLAS, 2014]

Rest of this talk:

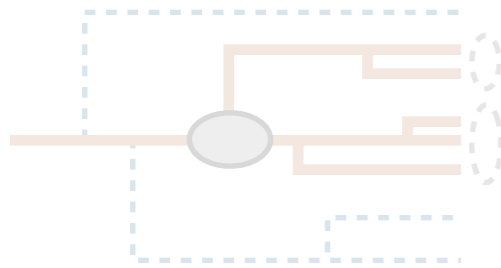
*Jet substructure as motivation to  
delve into subtleties of QCD*



Inspiration from Jet Substructure



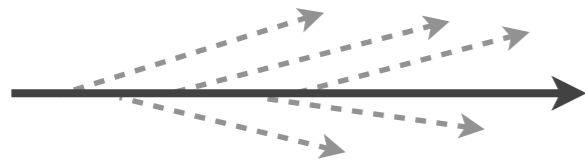
From IRC Safe to Sudakov Safe



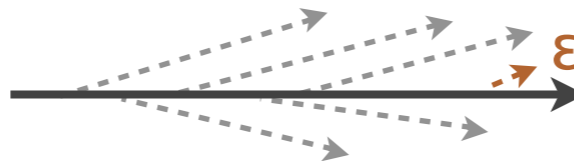
Probing the Core of QCD

# Infrared/Collinear Safety

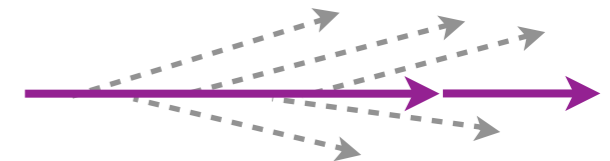
Original Jet



Infrared



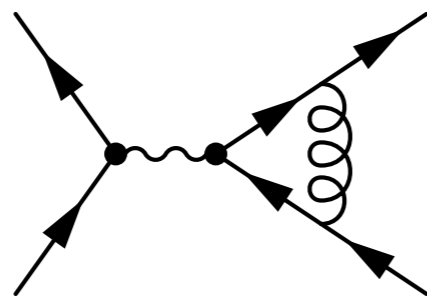
Collinear



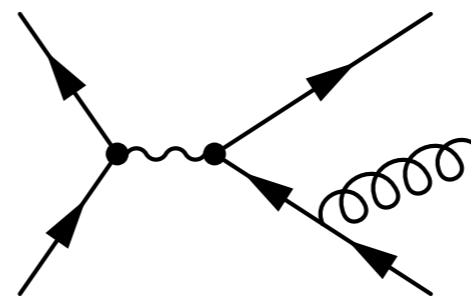
IRC Safe Observable: Insensitive to **IR** or **C** emissions

Formally:

Virtual Diagrams



$\Sigma$  Real Emissions



IRC divergences cancel order-by-order in  $\alpha_s$



# Examples from Jet Substructure

Jet  $p_T$ :  $\sum_{i \in \text{jet}} p_{T,i}$  **IRC Safe**

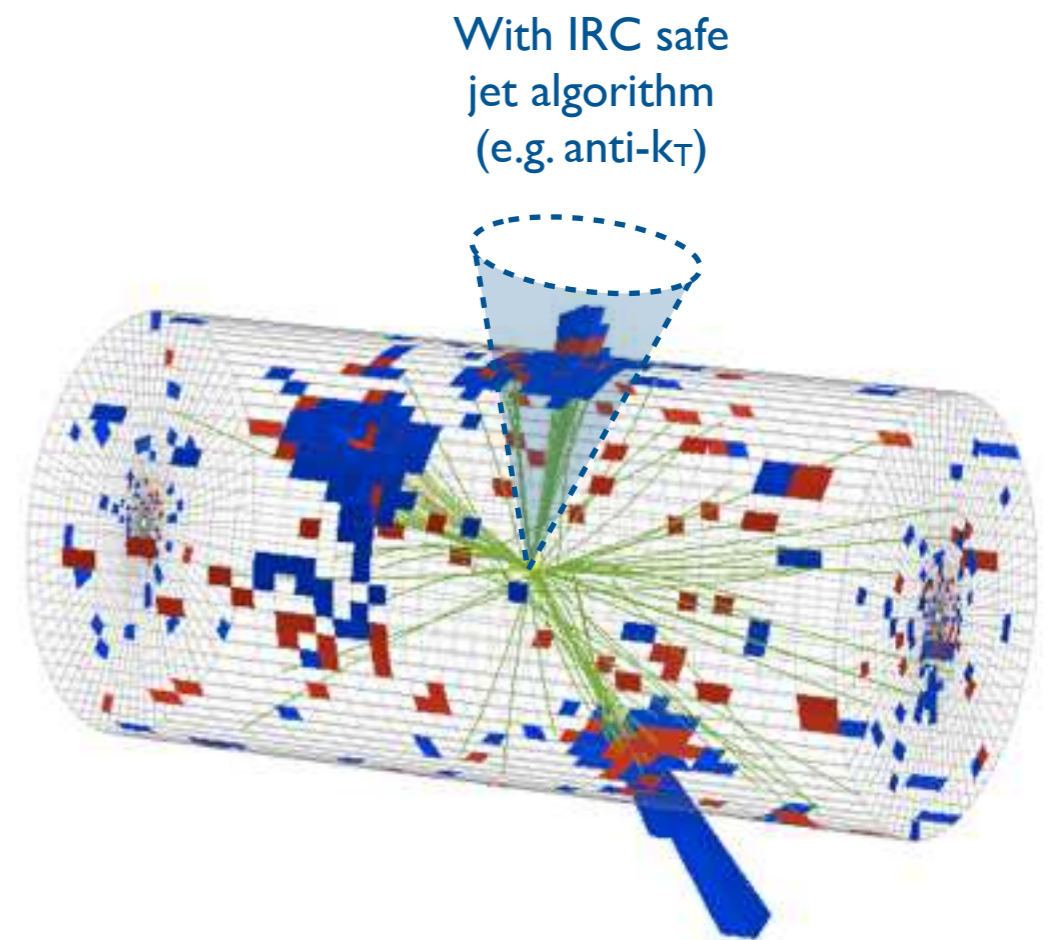
$p_T^D$ :  $\sum_{i \in \text{jet}} \frac{p_{T,i}^2}{p_{T,\text{jet}}^2}$  **IR Safe**  
**C Unsafe**  
 [CMS HIG-11-027]

Multiplicity:  $\sum_{i \in \text{jet}} 1$  **IRC Unsafe**

Jet Mass:  $\sum_{i,j \in \text{jet}} p_i \cdot p_j$  **IRC Safe**

N-subjettiness:  $\sum_{i \in \text{jet}} p_{T,i} \min \{ \Delta R_{i,1}, \Delta R_{i,2}, \dots, \Delta R_{i,N} \}^\beta$  **IRC Safe**

[JDT, Van Tilburg, 1011.2268, 108.2701]



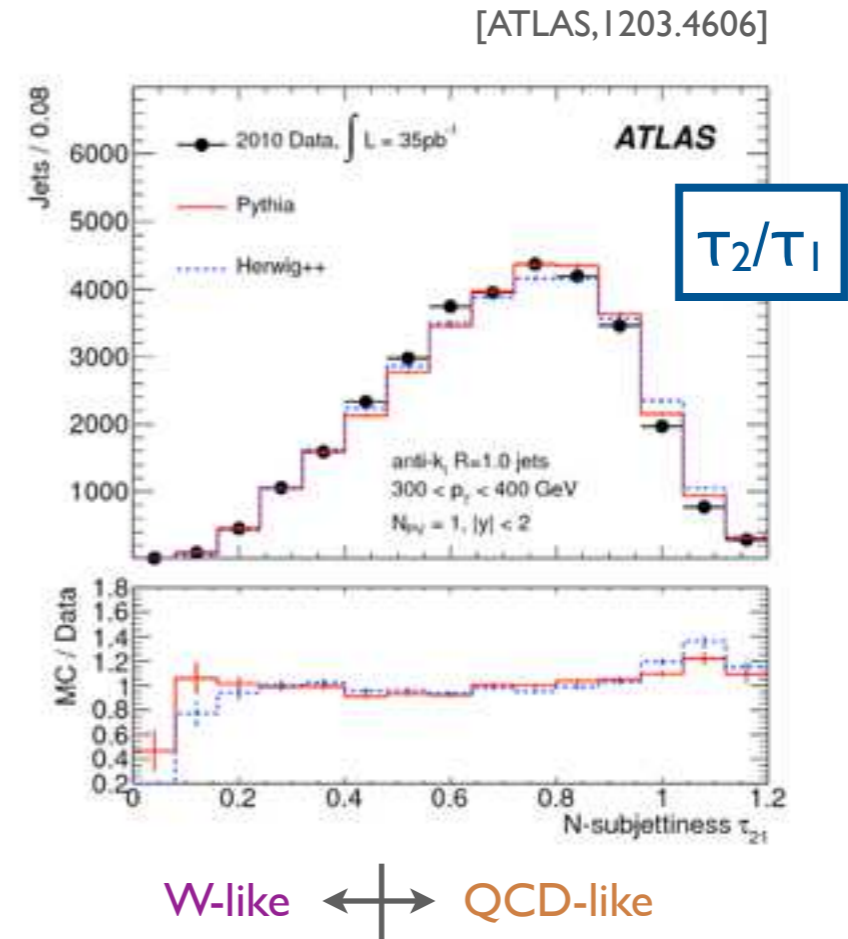
# Ratio Observables?

IRC Safe  $\Rightarrow$  Useful Ratio

$$\tau_N \Rightarrow \frac{\tau_N}{\tau_{N-1}}$$

Ubiquitous in jet substructure  
(esp. N-subjettiness)

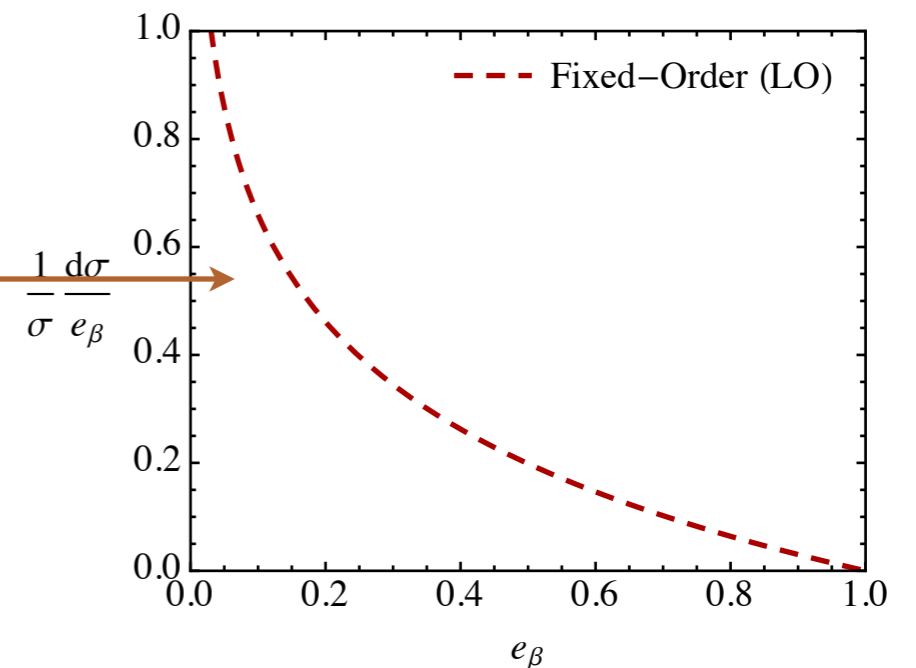
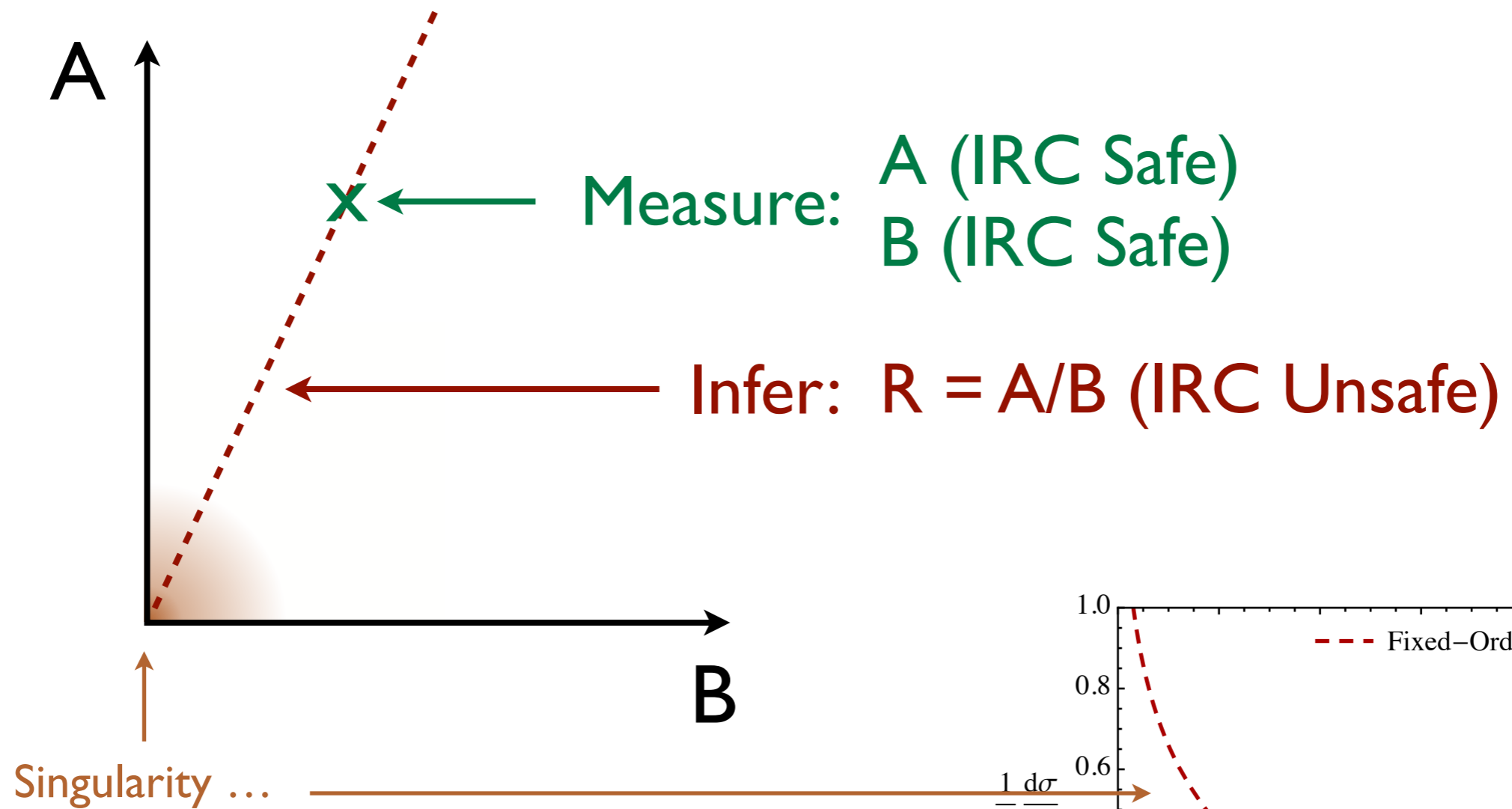
$$\frac{\text{IRC Safe Numerator}}{\text{IRC Safe Denominator}} = \text{IRC Unsafe Ratio}$$



[Soyez, Salam, Kim, Dutta, Cacciari, 1211.2811]

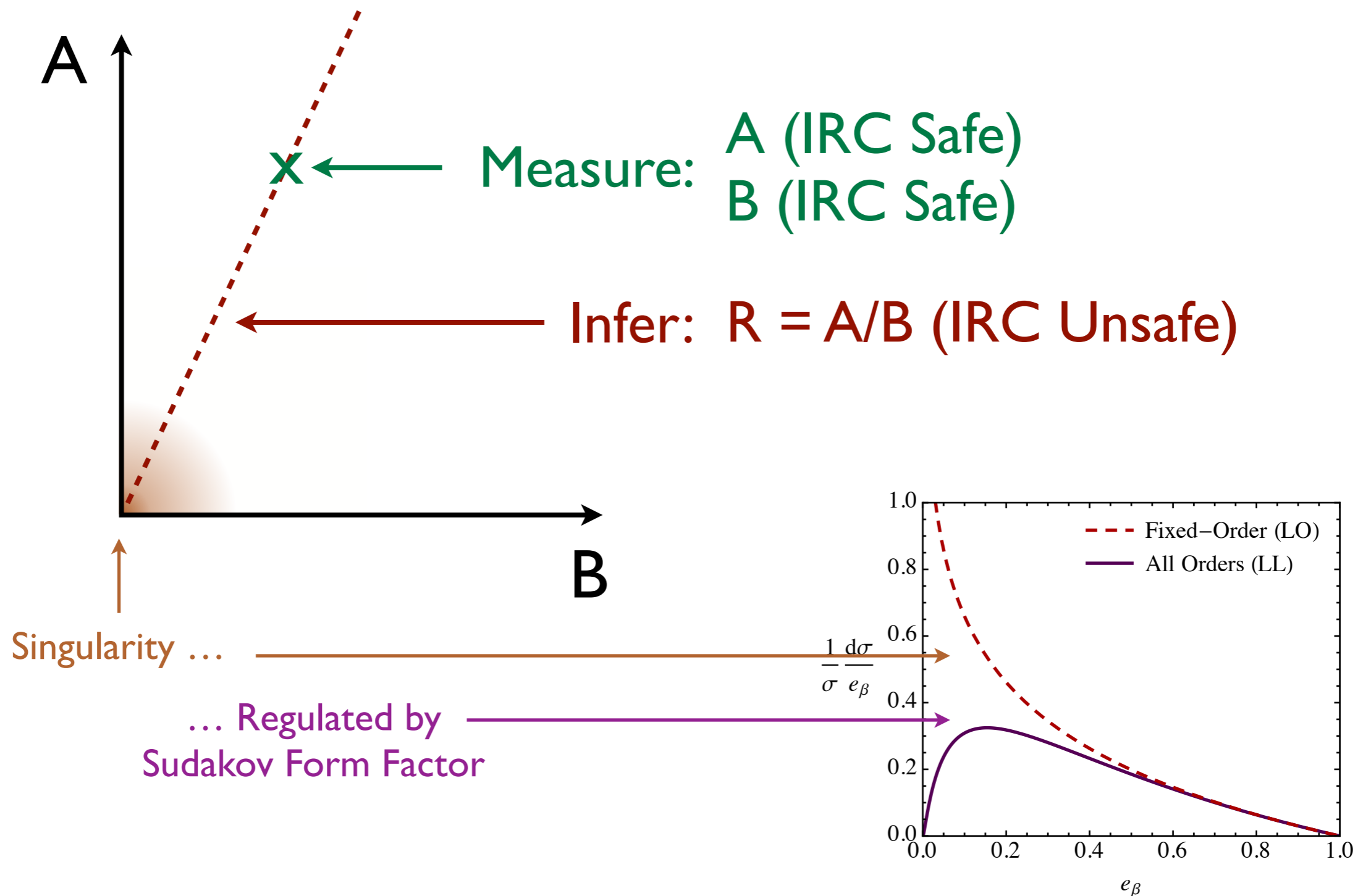
# WHAT?!

*Safe/Safe = Unsafe?!*



# WHAT?!

*Safe/Safe = Unsafe?!*

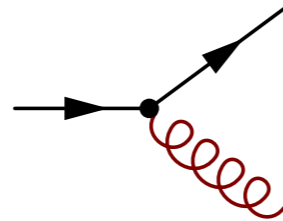


# The Key Realization

*Generalization in backup*

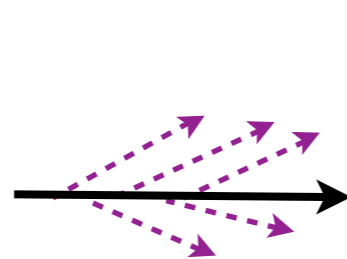
$$\frac{d\sigma}{dr} = \int de_\alpha de_\beta \frac{d^2\sigma}{de_\alpha de_\beta} \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$

↑  
**IRC Unsafe**  
Infinity at  $O(\alpha_s)$



↑  
**IRC Safe**  
“I can simultaneously measure  $e_\alpha$  and  $e_\beta$ ”

↑  
**“Sudakov Safe”**  
Non-analytic in  $\alpha_s$



[Larkoski, JDT, 1307.1699]

↑  
**Joint Resummable**  
“I can find a Sudakov form factor that resums large logarithms in  $e_\alpha$  and  $e_\beta$  to *all orders* in  $\alpha_s$  (e.g. a parton shower)”

# Sudakov Safety in Action

Ratios of angularities ( $I$ -subjettiness)

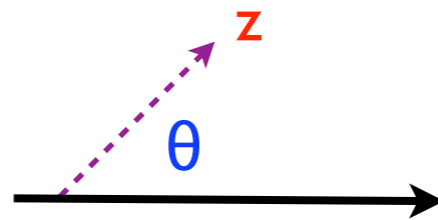
$$e_\beta \simeq \sum_{i \in \text{jet}} z_i (\theta_i)^\beta \quad \text{IRC Safe}$$

↑ ↑  
momentum fraction     angle to axis

IR Limit     C Limit  
 $z \rightarrow 0$       $\theta \rightarrow 0$

Single emission:

Order  $\alpha_s$  (LO)



$$r = \frac{e_\alpha}{e_\beta} = \theta^{\alpha-\beta}$$

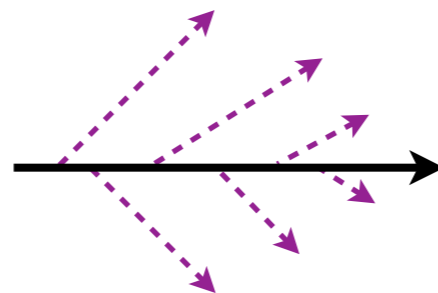
IRC Unsafe

“Sudakov Safe”

[Larkoski, JDT, 1307.1699]

Many emissions:

All orders in  $\alpha_s$  (LL)

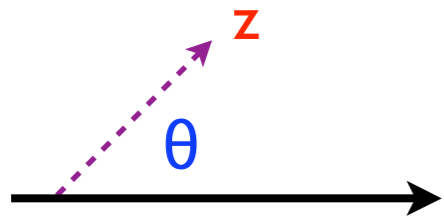


[Berger, Kucs, Sterman, 2003; Ellis, Vermilion, Walsh, Hornig, Lee, 2010]  
 [Recoil-free Versions: Larkoski, Salam, JDT, 1305.0007; Larkoski, Neill, JDT, 1401.2158]

# Turning the Crank

$$e_\beta \simeq \sum_{i \in \text{jet}} z_i (\theta_i)^\beta$$

$$\frac{d\sigma}{dr} = \int de_\alpha de_\beta \frac{d^2\sigma}{de_\alpha de_\beta} \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$



$$\frac{d^2\sigma^{\text{FO}}}{de_\alpha de_\beta} \simeq \frac{2\alpha_s}{\pi} \frac{C_F}{\alpha - \beta} \frac{1}{e_\alpha e_\beta}$$

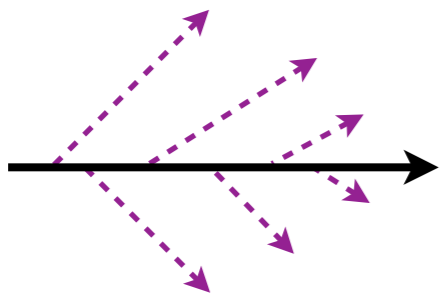
Single emission:  
Order  $\alpha_s$  (LO)

$$\frac{d\sigma^{\text{FO}}}{dr} \Rightarrow \text{Not integrable! (IRC Unsafe)}$$

# Turning the Crank

$$e_\beta \simeq \sum_{i \in \text{jet}} z_i (\theta_i)^\beta$$

$$\frac{d\sigma}{dr} = \int de_\alpha de_\beta \frac{d^2\sigma}{de_\alpha de_\beta} \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$



Many emissions:  
All orders in  $\alpha_s$  (LL)

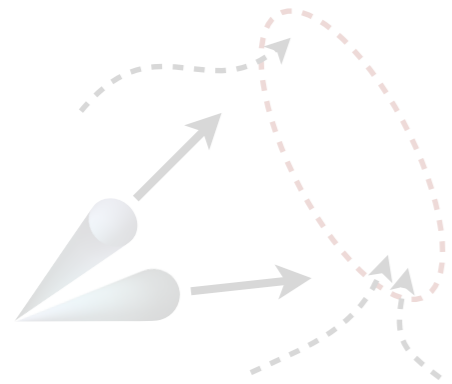
$$\frac{d^2\sigma^{\text{LL}}}{de_\alpha de_\beta} \simeq \frac{2\alpha_s}{\pi} \frac{C_F}{\alpha - \beta} \frac{1}{e_\alpha e_\beta} \overbrace{e^{-\frac{\alpha_s}{\pi} \frac{C_F}{\beta} \log^2 e_\beta}}^{\text{Sudakov form factor}}$$

$$\frac{d\sigma^{\text{LL}}}{dr} \simeq \sqrt{\alpha_s} \frac{\sqrt{C_F \beta}}{\alpha - \beta} \frac{1}{r} e^{-\frac{\alpha_s}{\pi} \frac{C_F}{\alpha - \beta} \log^2 r}$$

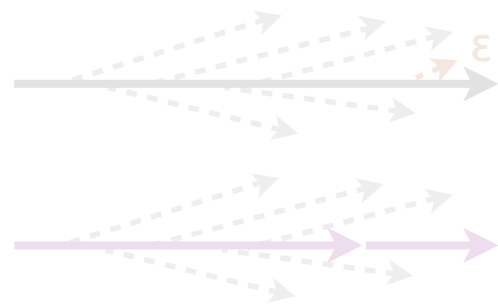
Unsafe... but Calculable (Sudakov Safe)

[Larkoski, JDT, 1307.1699]

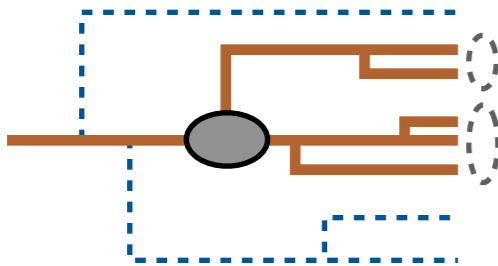




Inspiration from Jet Substructure



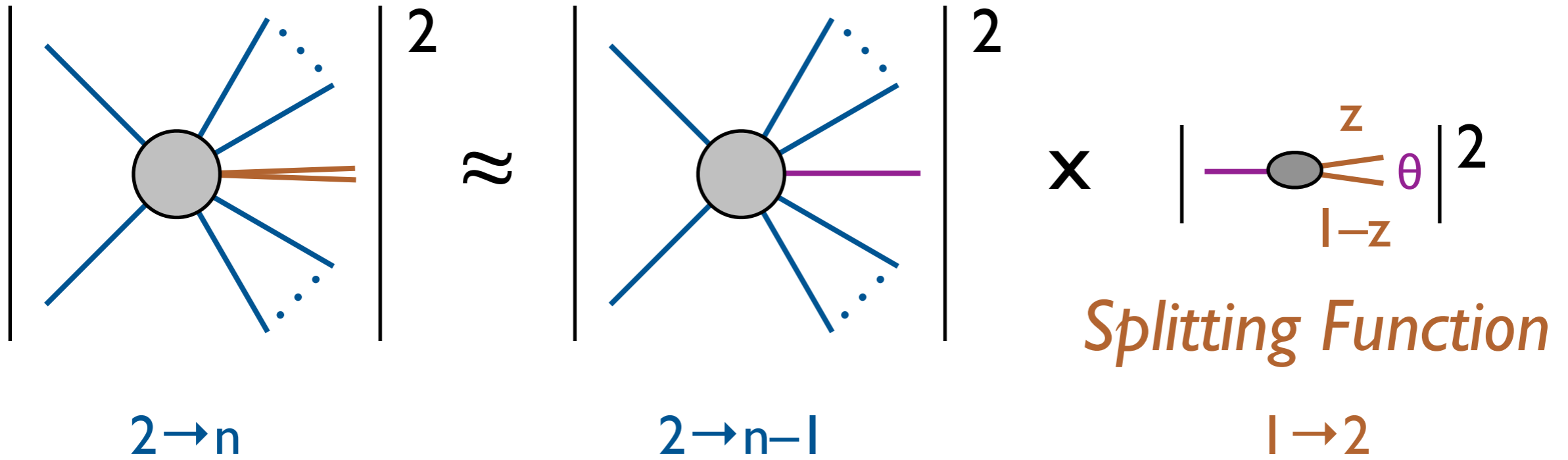
From IRC Safe to Sudakov Safe



Probing the Core of QCD

# Textbook QCD

*Universal collinear limit*

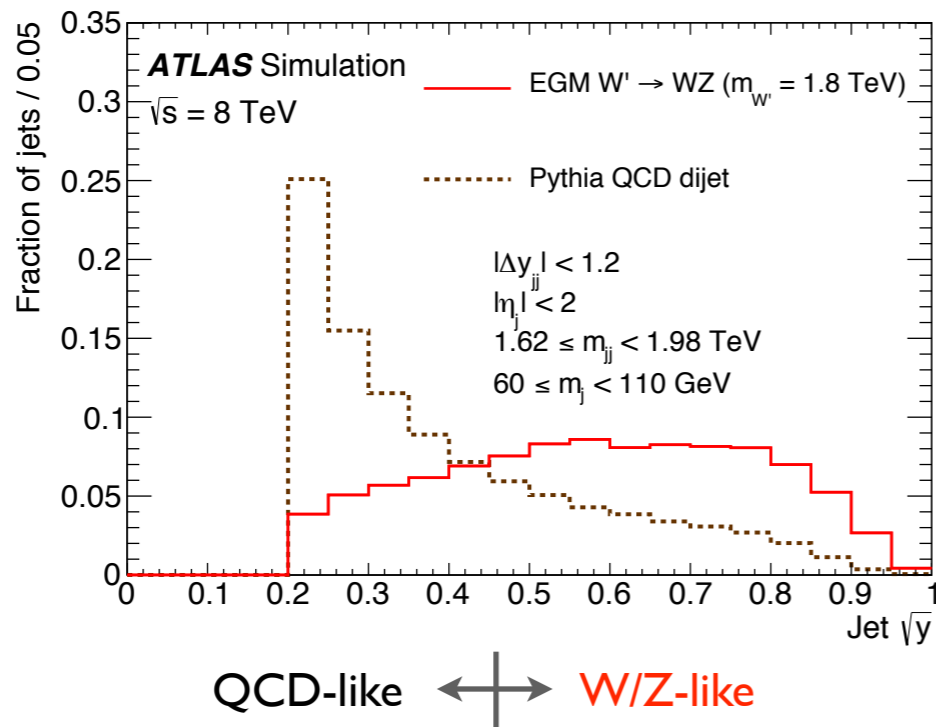


$$d\sigma_{2 \rightarrow n} = d\sigma_{2 \rightarrow n-1} dP_{i \rightarrow jk}$$

$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear singularity}} \underbrace{\frac{dz}{z}}_{\text{Soft singularity}}$$

# The Core of QCD

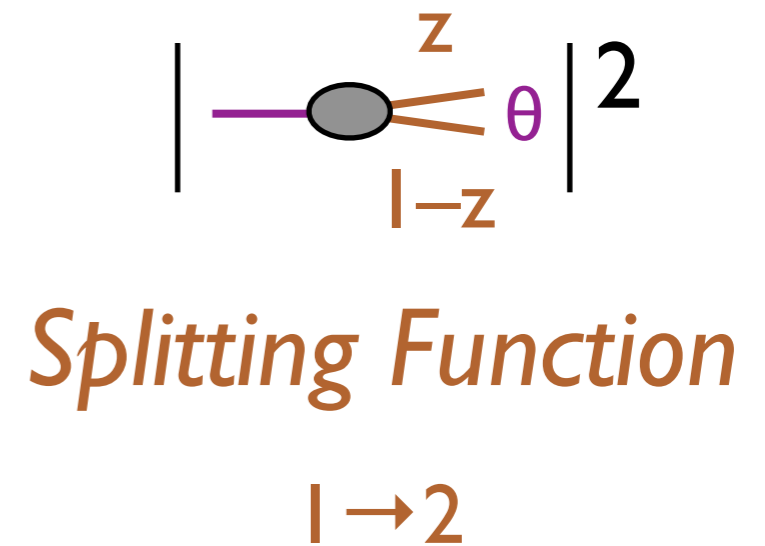
Basis for parton shower MC generators,  
PDF evolution, NLO subtractions,  
 $k_t$  clustering, jet substructure studies...



[ATLAS, 2015]

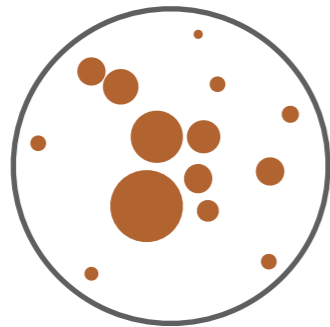
Measurable? Calculable?

↳ IRC Unsafe

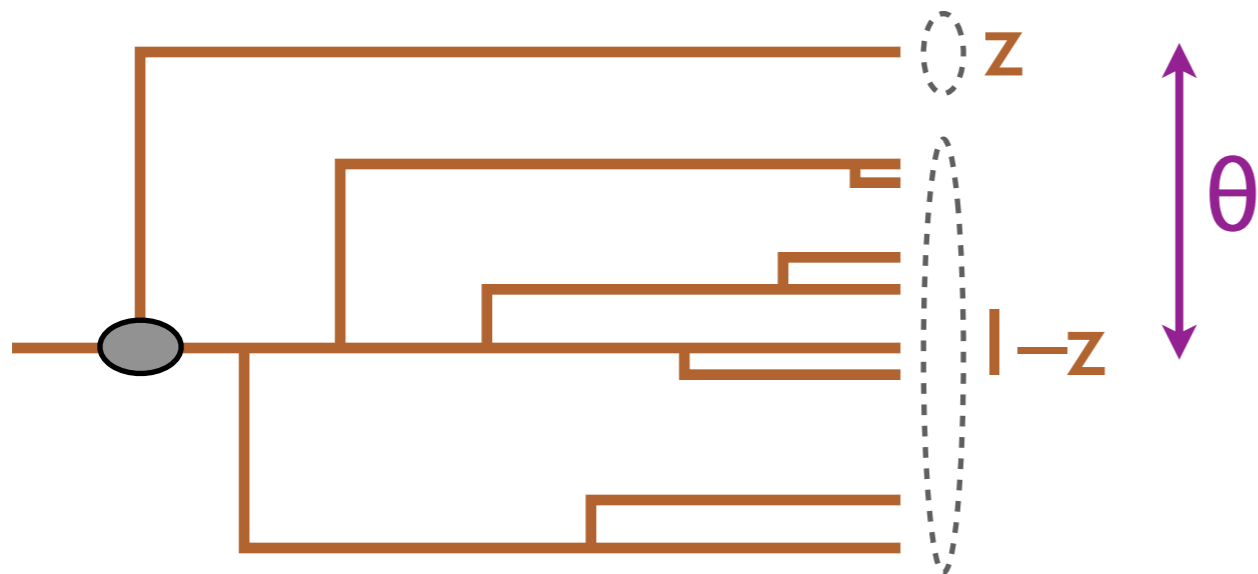


$$\frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear singularity}} \underbrace{\frac{dz}{z}}_{\text{Soft singularity}}$$

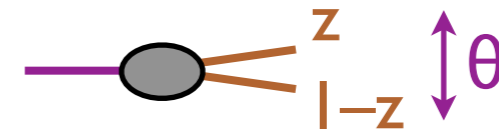
# Measure Universal Singularity?



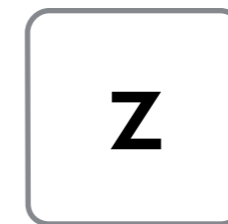
Angular-ordered tree...



...gives splitting function?

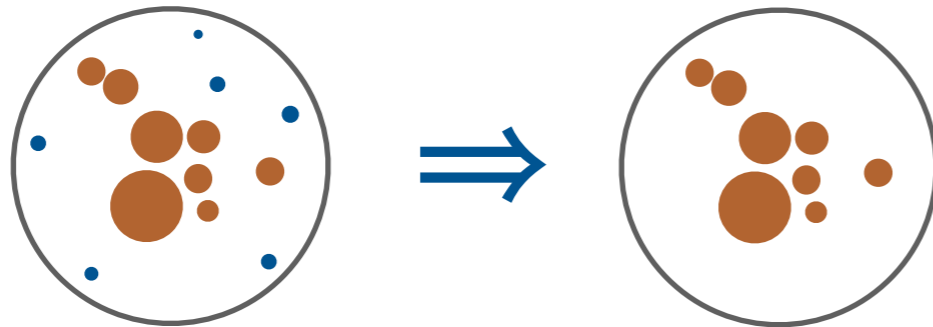


$$\frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



IRC Unsafe

# Measure Universal Singularity?

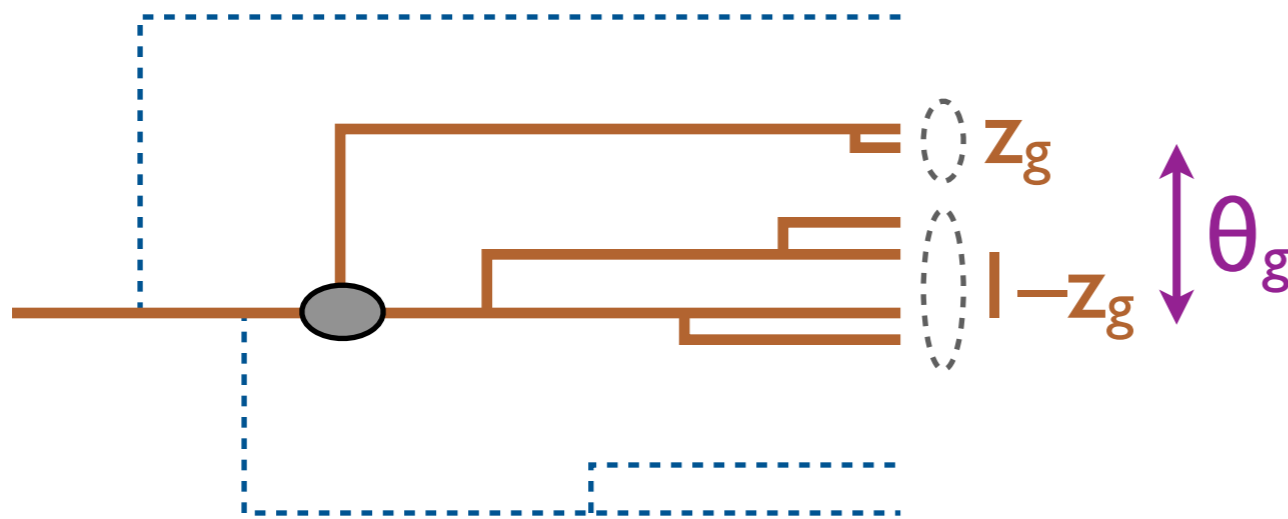


## Soft Drop

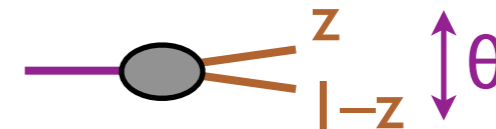
$$z > z_{\text{cut}} \theta^\beta$$

↑ energy threshold      ↑ angular exponent

Groomed angular-ordered tree...



...gives splitting function?



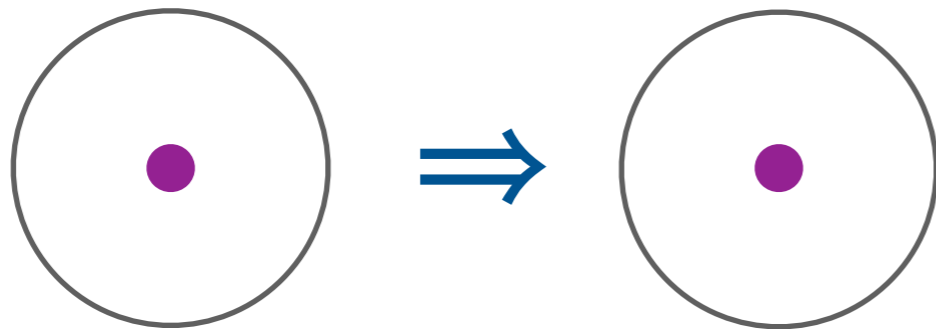
$$\frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

**$z_g$**

IR Safe  
C Unsafe ( $\beta \geq 0$ )

[Larkoski, Marzani, Soyez, JDT, 1402.2657]  
[see also Butterworth, Davison, Rubin, Salam, 0802.2470; Dasgupta, Fregoso, Marzani, Salam, 1307.0007]

# Measure Universal Singularity?



## Soft Drop

$$z > z_{\text{cut}} \theta^\beta$$

↑ energy threshold      ↑ angular exponent

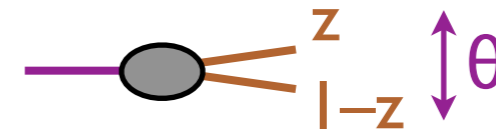
One prong jet...



vs.



...gives splitting function?

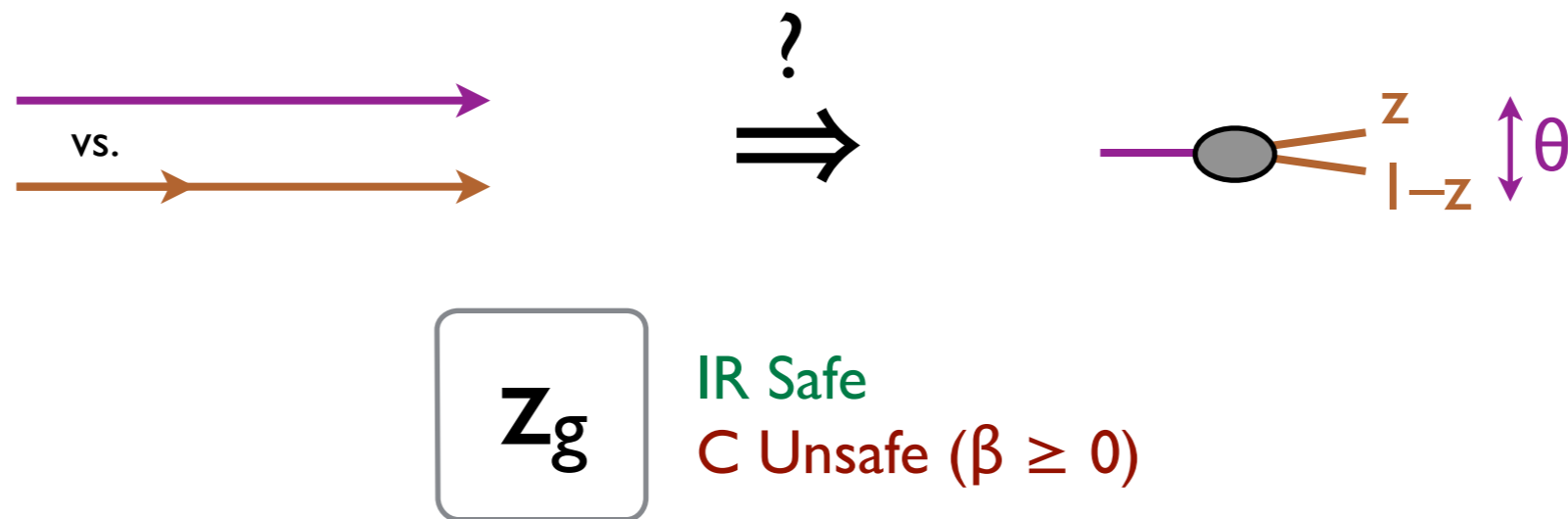


$$\frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

$z_g$

IR Safe  
C Unsafe ( $\beta \geq 0$ )

[Larkoski, Marzani, Soyez, JDT, 1402.2657]  
[see also Butterworth, Davison, Rubin, Salam, 0802.2470; Dasgupta, Fregoso, Marzani, Salam, 1307.0007]

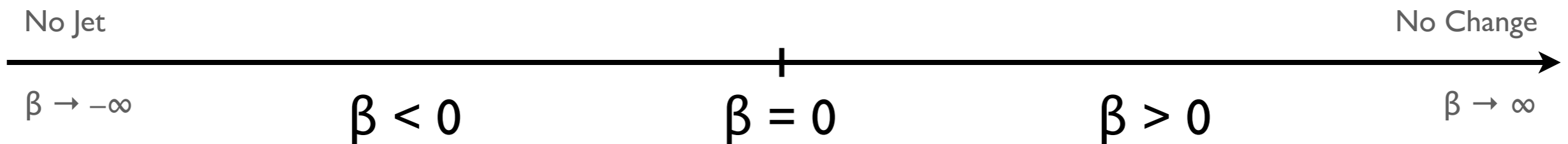
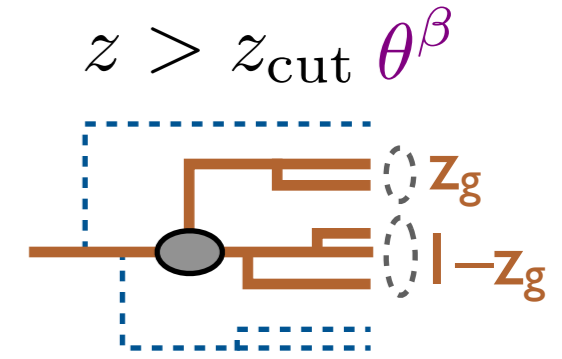
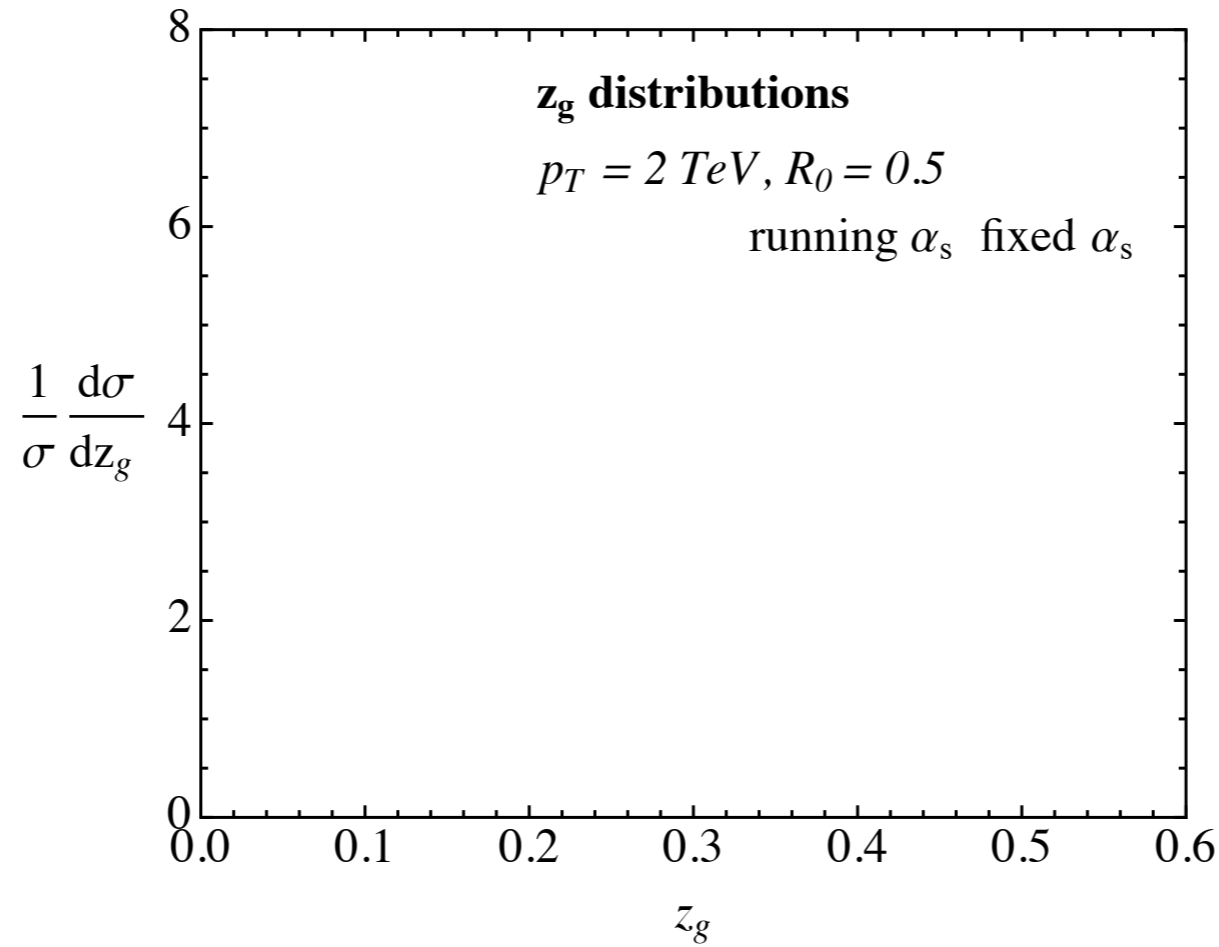


*How to calculate from first principles?*

## Exploit Sudakov Safety

(see backup for two additional approaches)

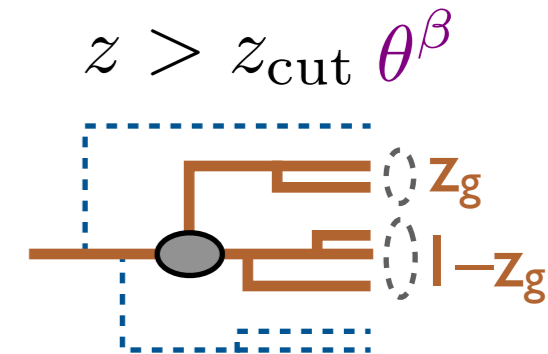
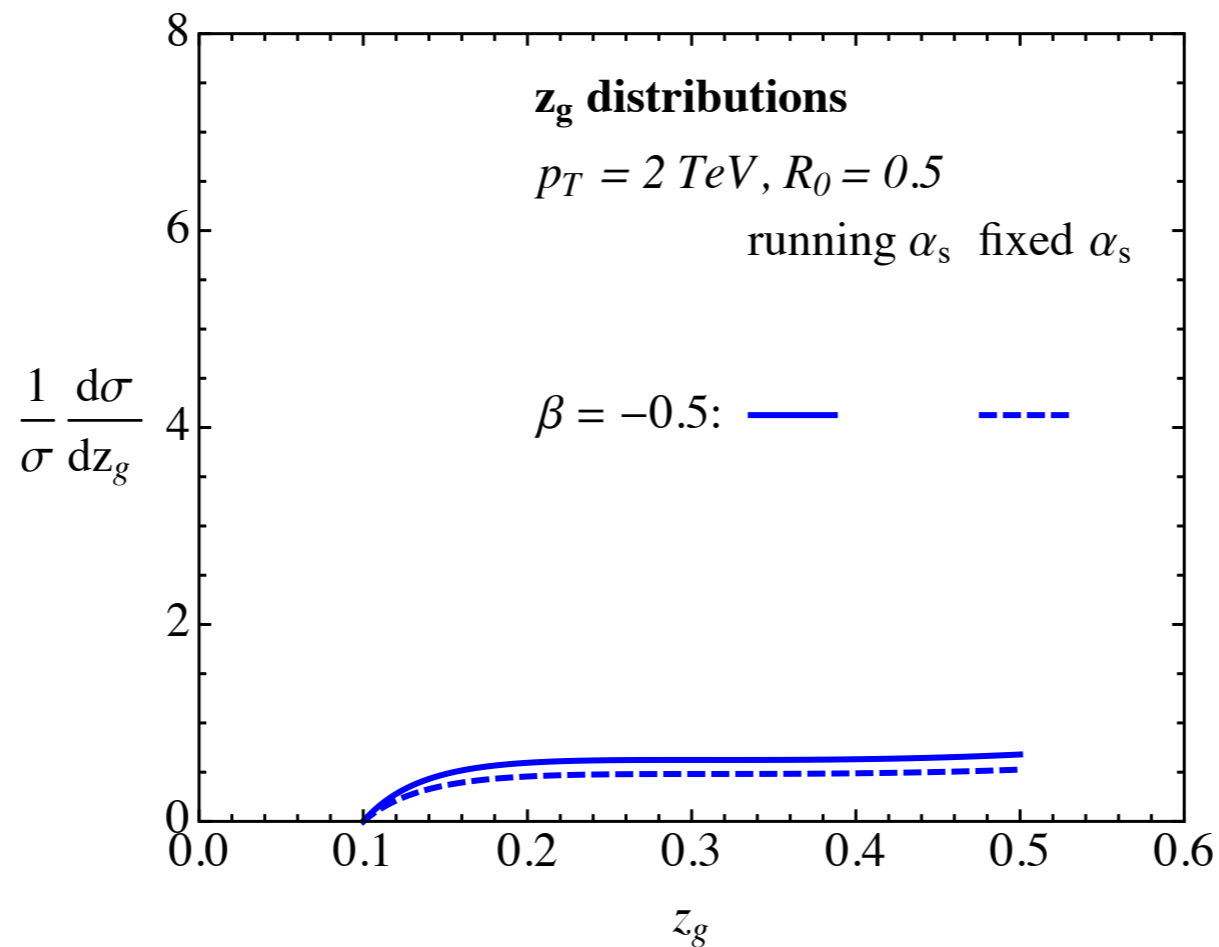
# First-Principles QCD



[Larkoski, Marzani, JDT, 2015; using calculational techniques in Dasgupta, Fregoso, Marzani, Salam, 2013; Larkoski, JDT, 2013]



# First-Principles QCD



$$\simeq \frac{2\alpha_s C_i}{\pi|\beta|} \frac{1}{z_g} \log \frac{z_g}{z_{\text{cut}}}$$

No Jet

No Change

$\beta \rightarrow -\infty$

$\beta < 0$

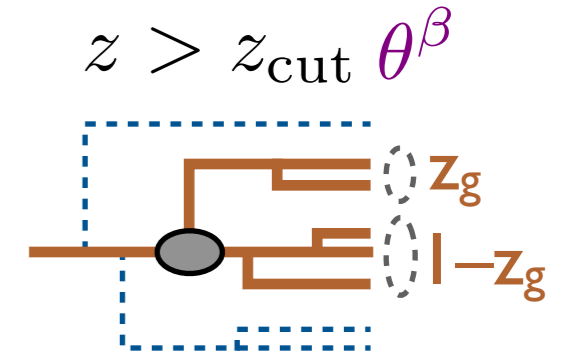
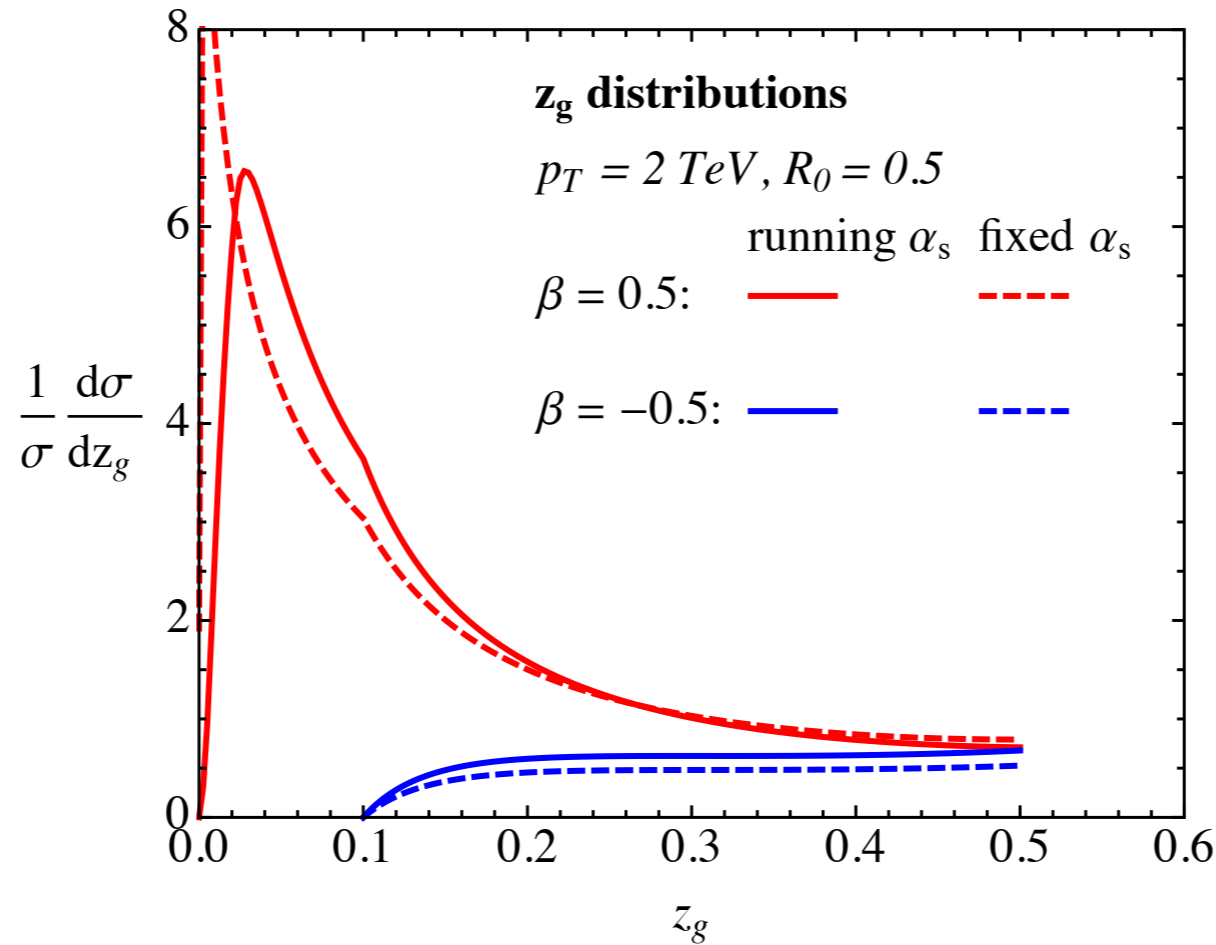
$\beta = 0$

$\beta > 0$

$\beta \rightarrow \infty$

[Larkoski, Marzani, JDT, 2015; using calculational techniques in Dasgupta, Fregoso, Marzani, Salam, 2013; Larkoski, JDT, 2013]

# First-Principles QCD



$$\approx \frac{2\alpha_s C_i}{\pi|\beta|} \frac{1}{z_g} \log \frac{z_g}{z_{\text{cut}}}$$

└──────────────────┘
└──────────────────┘

No Jet No Change

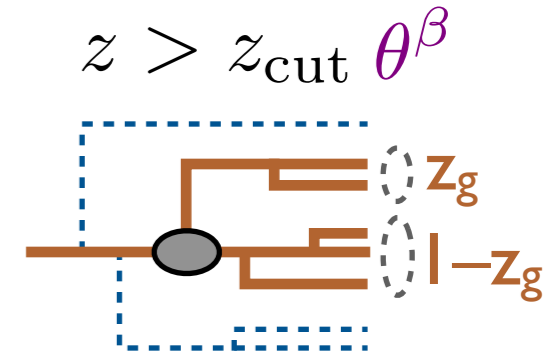
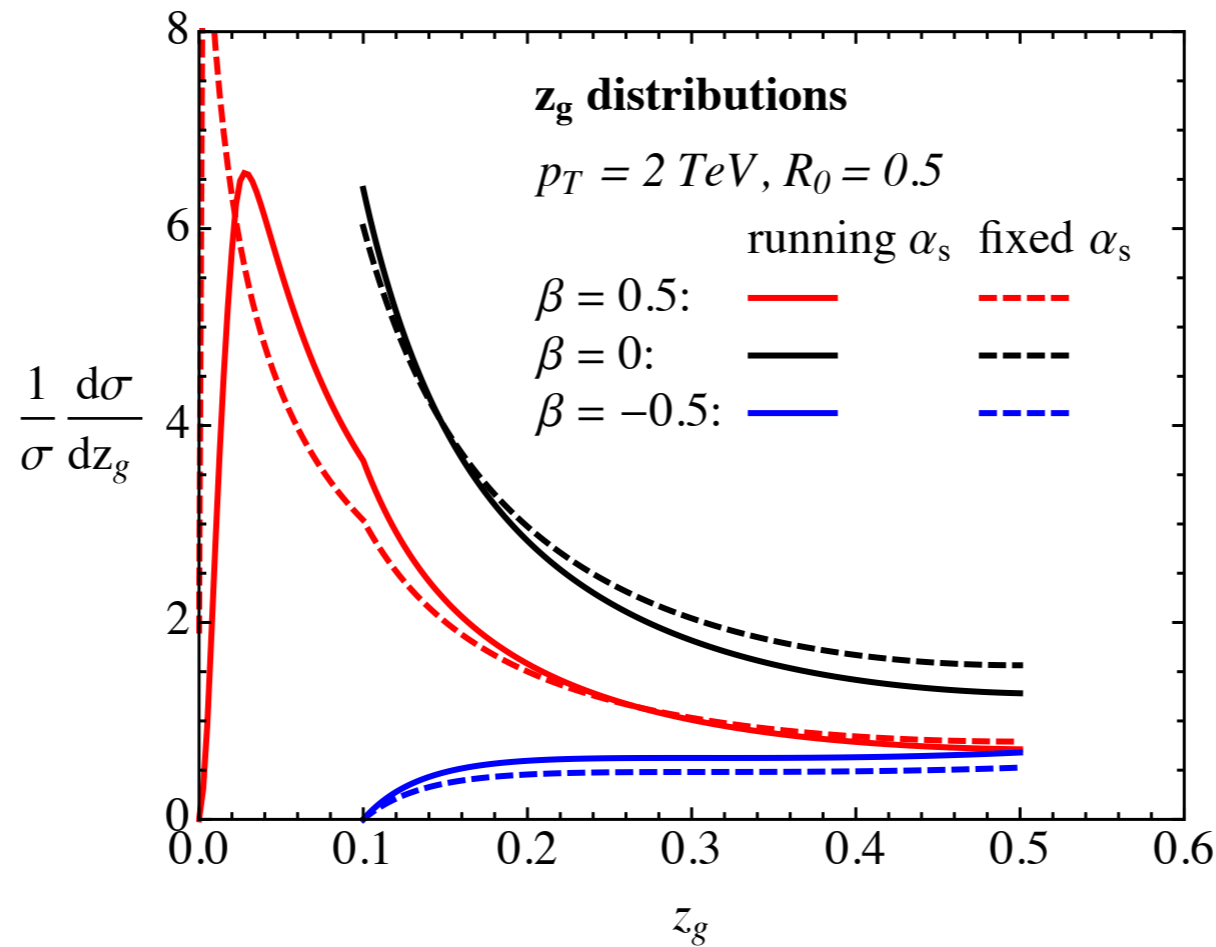
$\beta \rightarrow -\infty$        $\beta < 0$        $\beta = 0$        $\beta > 0$        $\beta \rightarrow \infty$

$$\approx \sqrt{\frac{\alpha_s C_i}{\beta}} \frac{1}{z_g}$$

Non-analytic because of Sudakov safety

[Larkoski, Marzani, JDT, 2015; using calculational techniques in Dasgupta, Fregoso, Marzani, Salam, 2013; Larkoski, JDT, 2013]

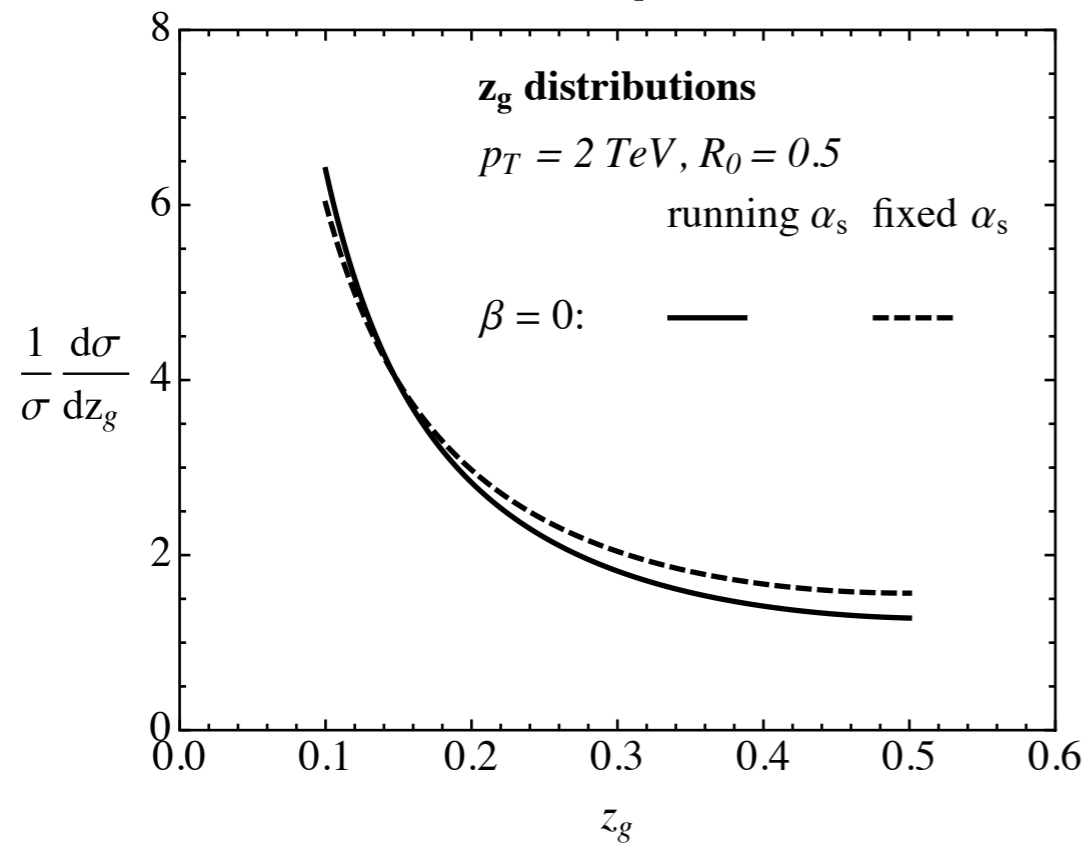
# First-Principles QCD



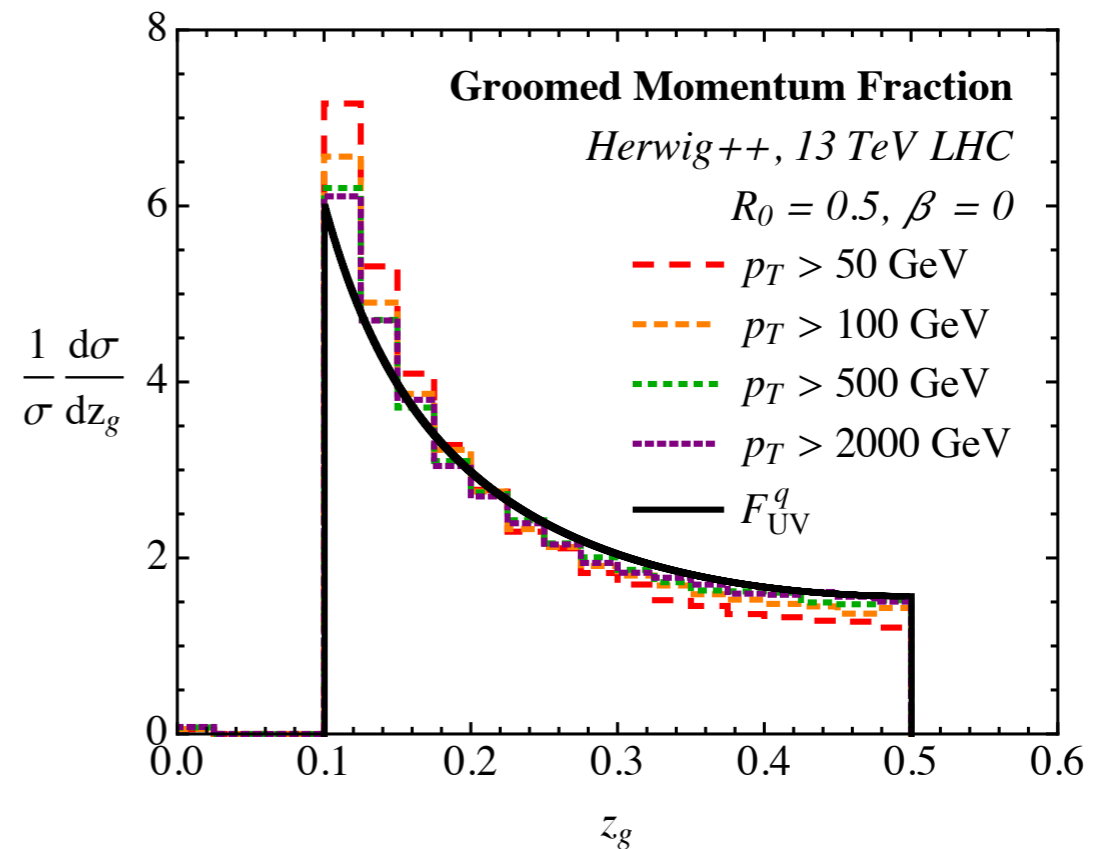
$$\begin{aligned}
 &\simeq \frac{2\alpha_s C_i}{\pi|\beta|} \frac{1}{z_g} \log \frac{z_g}{z_{\text{cut}}} && \simeq \frac{1}{z_g} (!) && \simeq \sqrt{\frac{\alpha_s C_i}{\beta}} \frac{1}{z_g} && \text{Non-analytic because of Sudakov safety} \\
 &\downarrow && \downarrow && \downarrow && \\
 \text{No Jet} &&& && && \text{No Change} \\
 \beta \rightarrow -\infty && \beta < 0 && \beta = 0 && \beta > 0 && \beta \rightarrow \infty
 \end{aligned}$$

[Larkoski, Marzani, JDT, 2015; using calculational techniques in Dasgupta, Fregoso, Marzani, Salam, 2013; Larkoski, JDT, 2013]

# First-Principles QCD



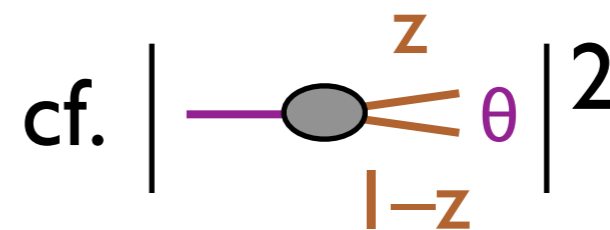
# Simulated LHC Data



**Core Feature of QCD:**  $\simeq \frac{1}{z_g}$

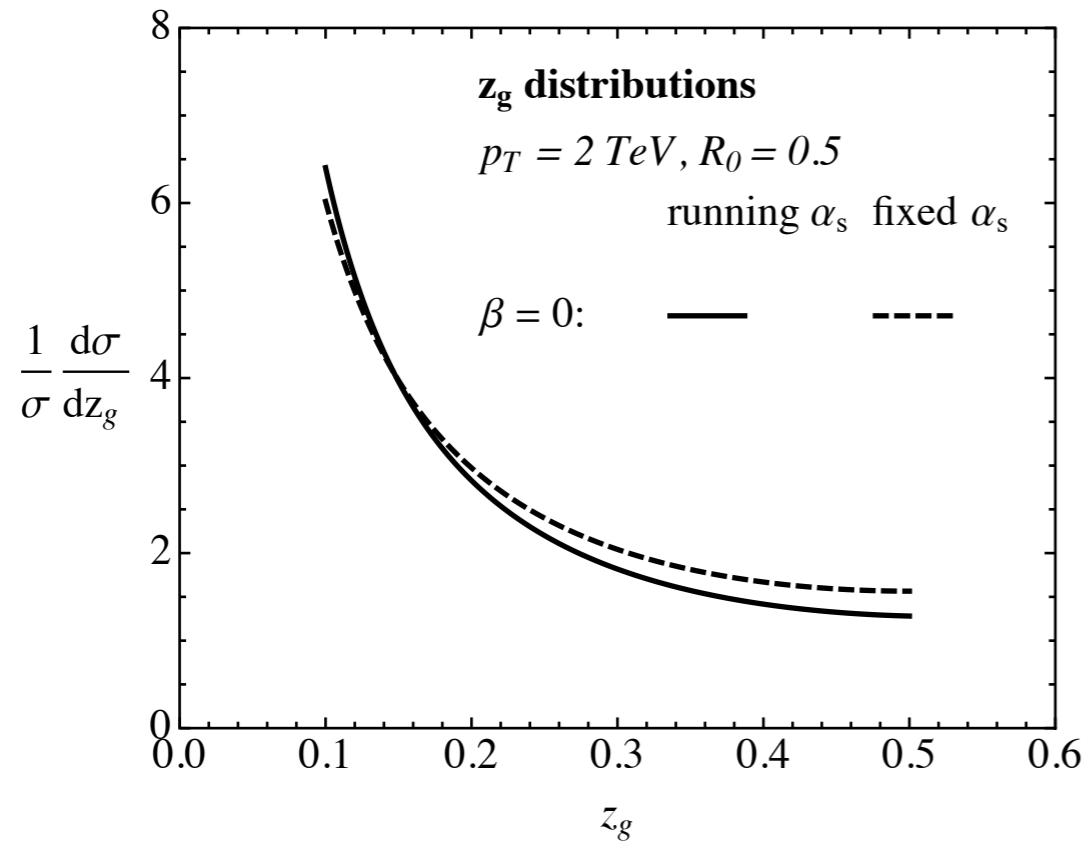
$\simeq$  independent of  $\alpha_s$  (!)  
 $\simeq$  independent of jet energy/radius  
 $\simeq$  same for quarks/gluons

$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

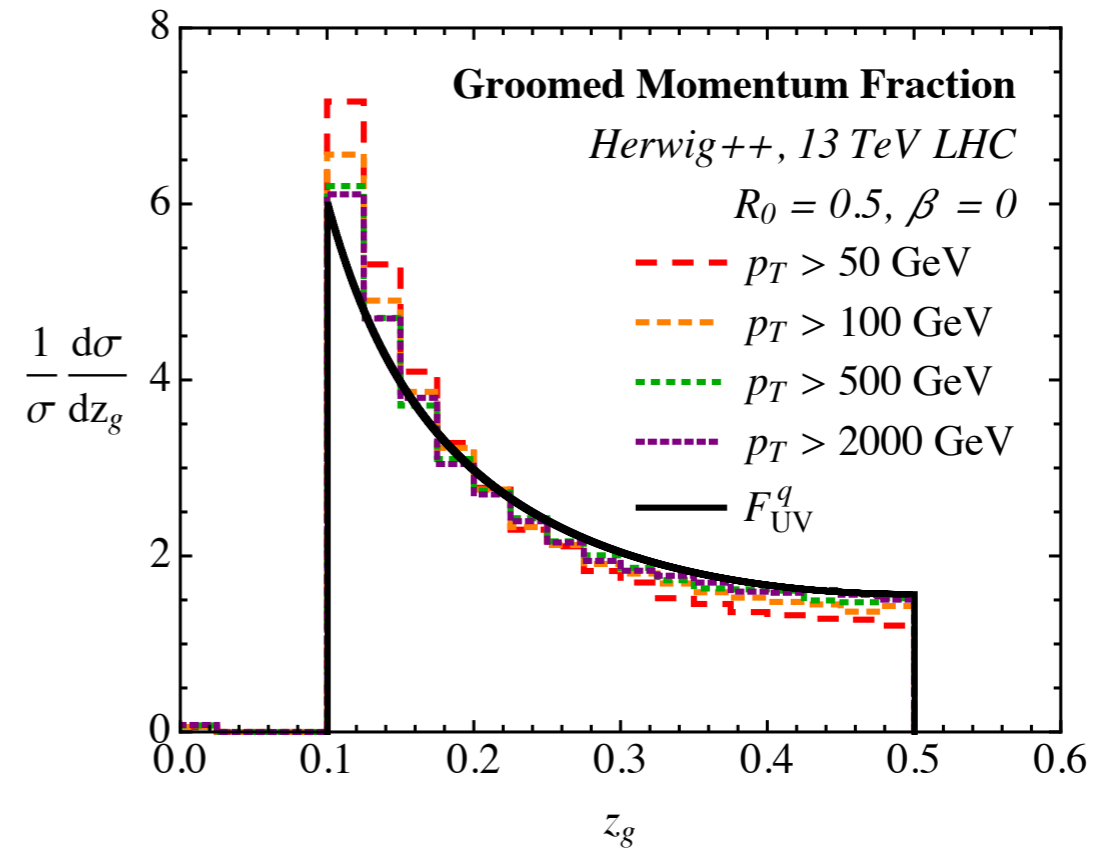


[Larkoski, Marzani, JDT, 2015; using Larkoski, JDT, 2013]

# First-Principles QCD

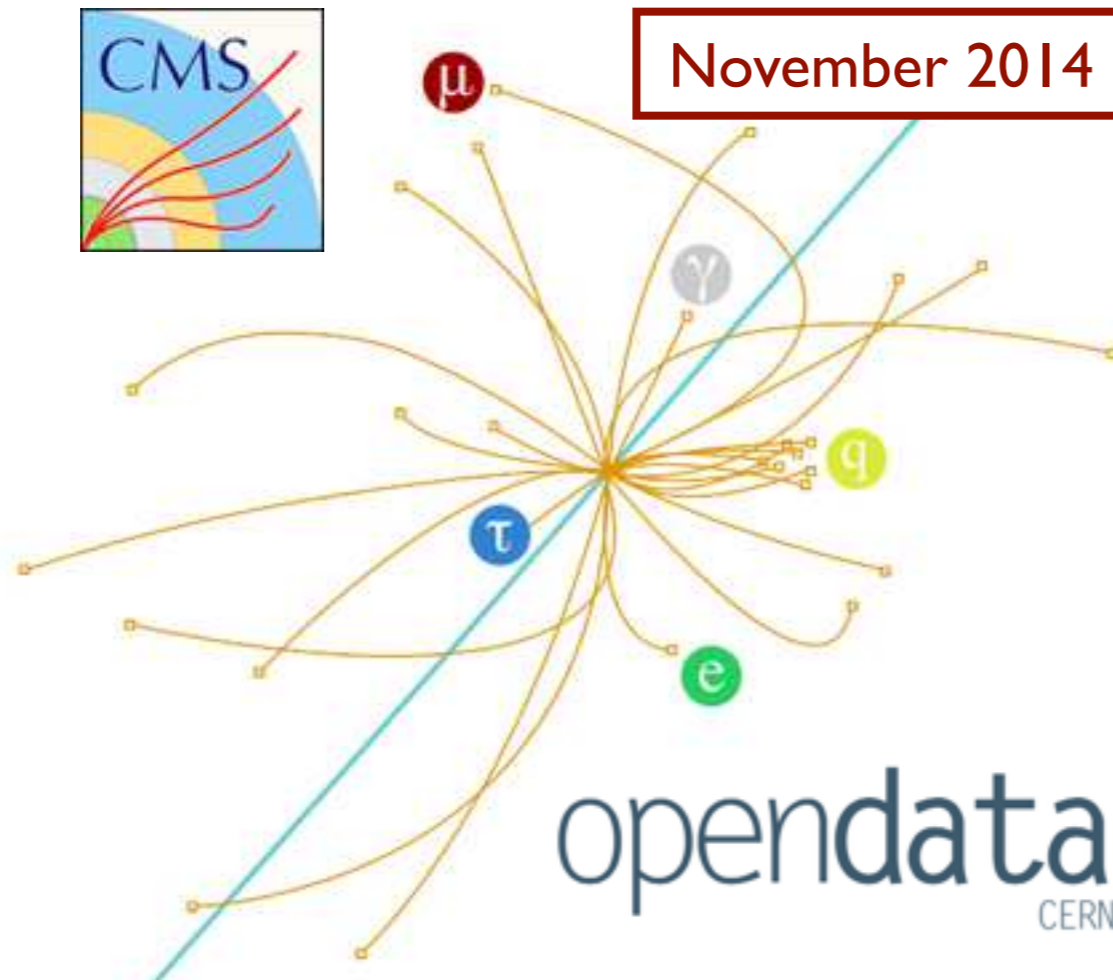


# Simulated LHC Data



*Actual  
LHC Data?*

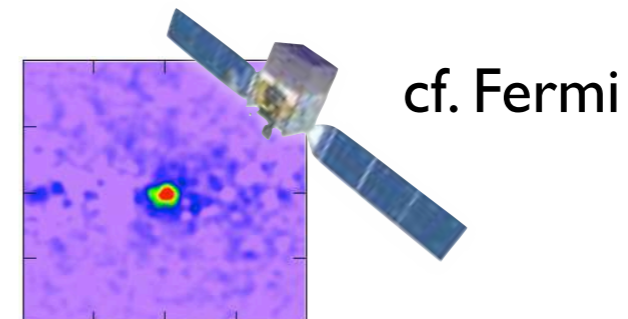
# The Future is Open



CMS 2010:

Unique data set  
with very low pileup

*Accelerating science  
through public data*

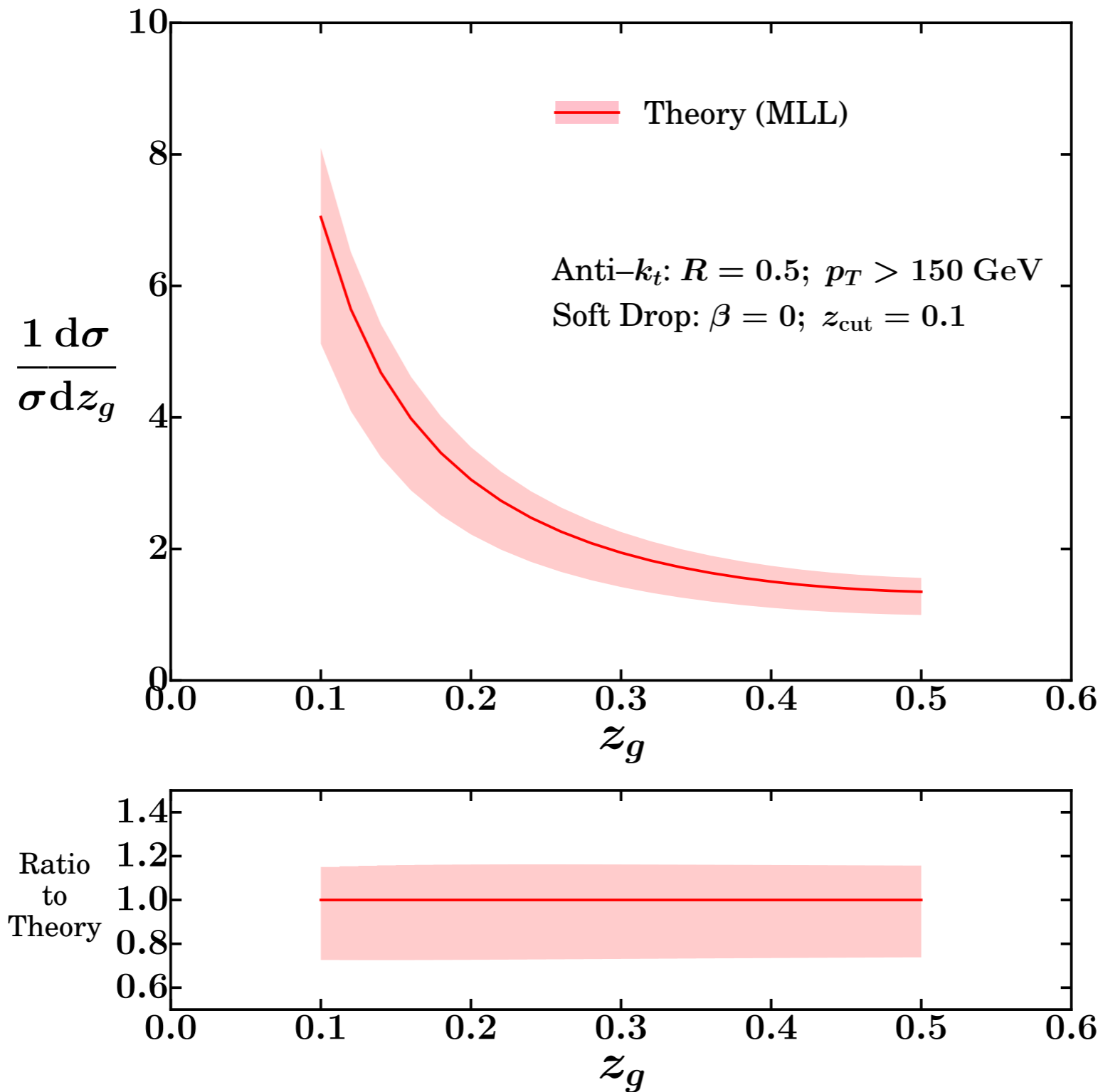


# Theory Calculation

Andrew Larkoski



Simone Marzani



# Simulated LHC Data

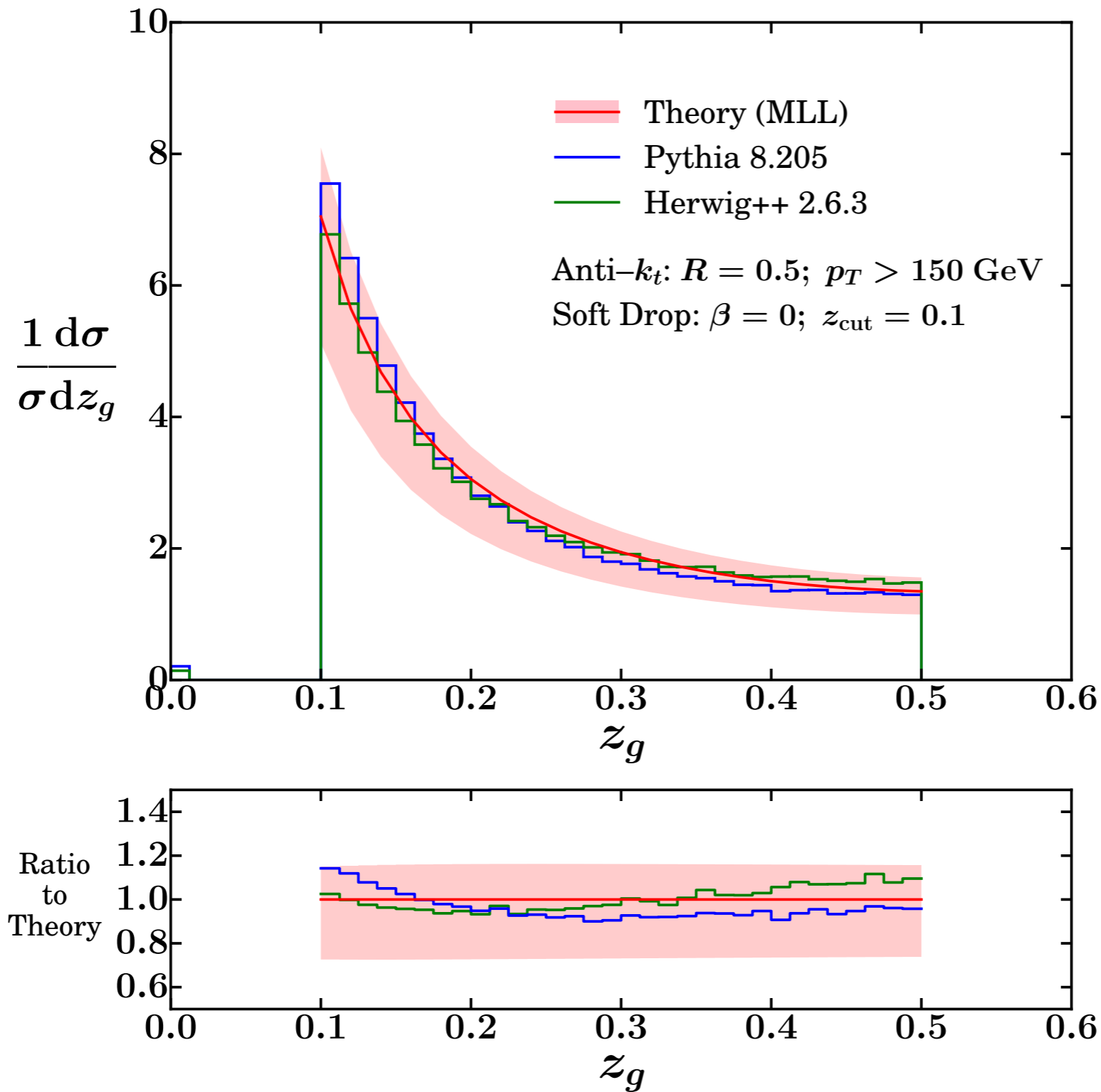
Andrew Larkoski



Simone Marzani

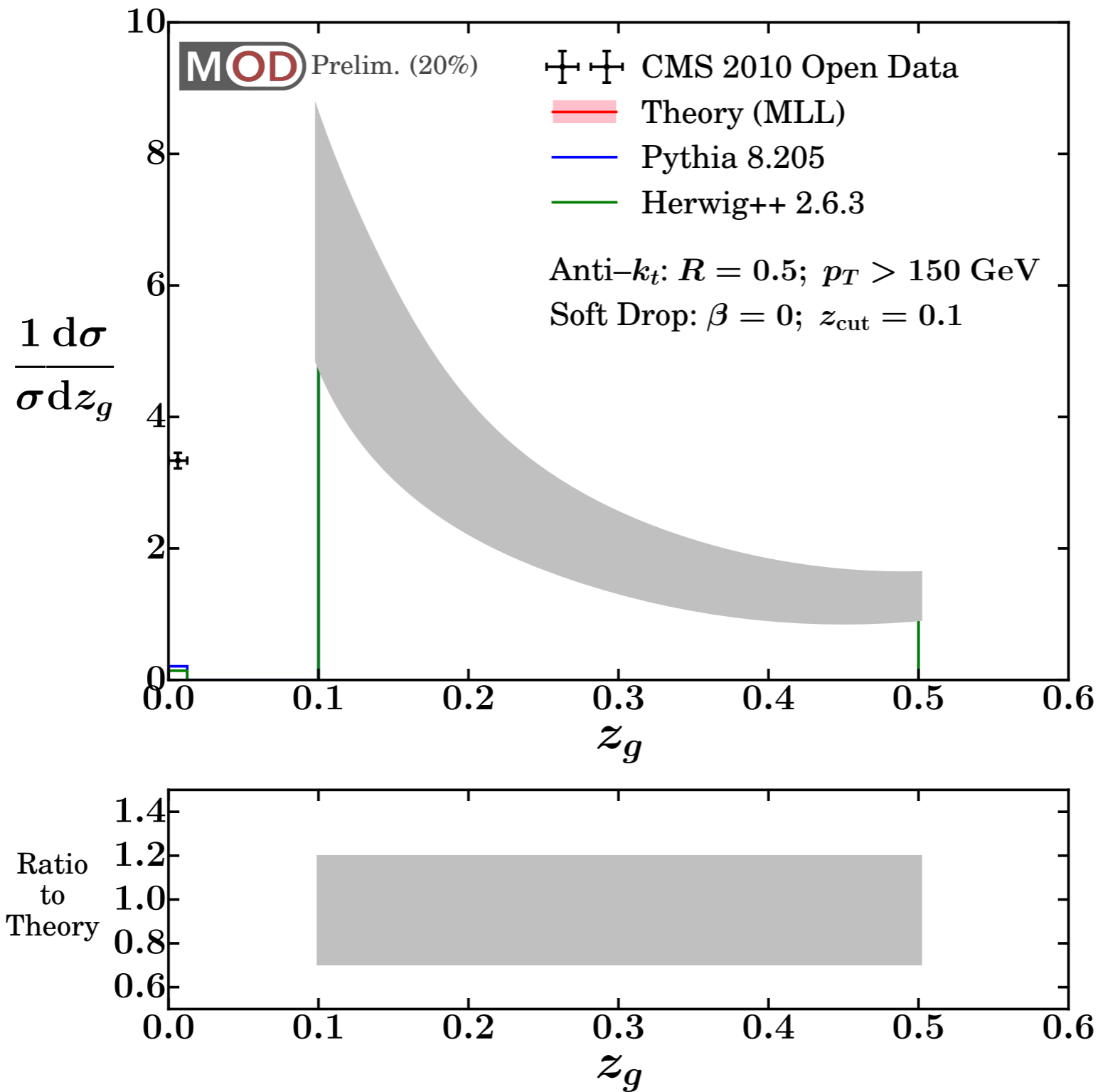


Alexis Romero





# CMS Open Data



Andrew Larkoski



Simone Marzani



Alexis Romero



Aashish Tripathy



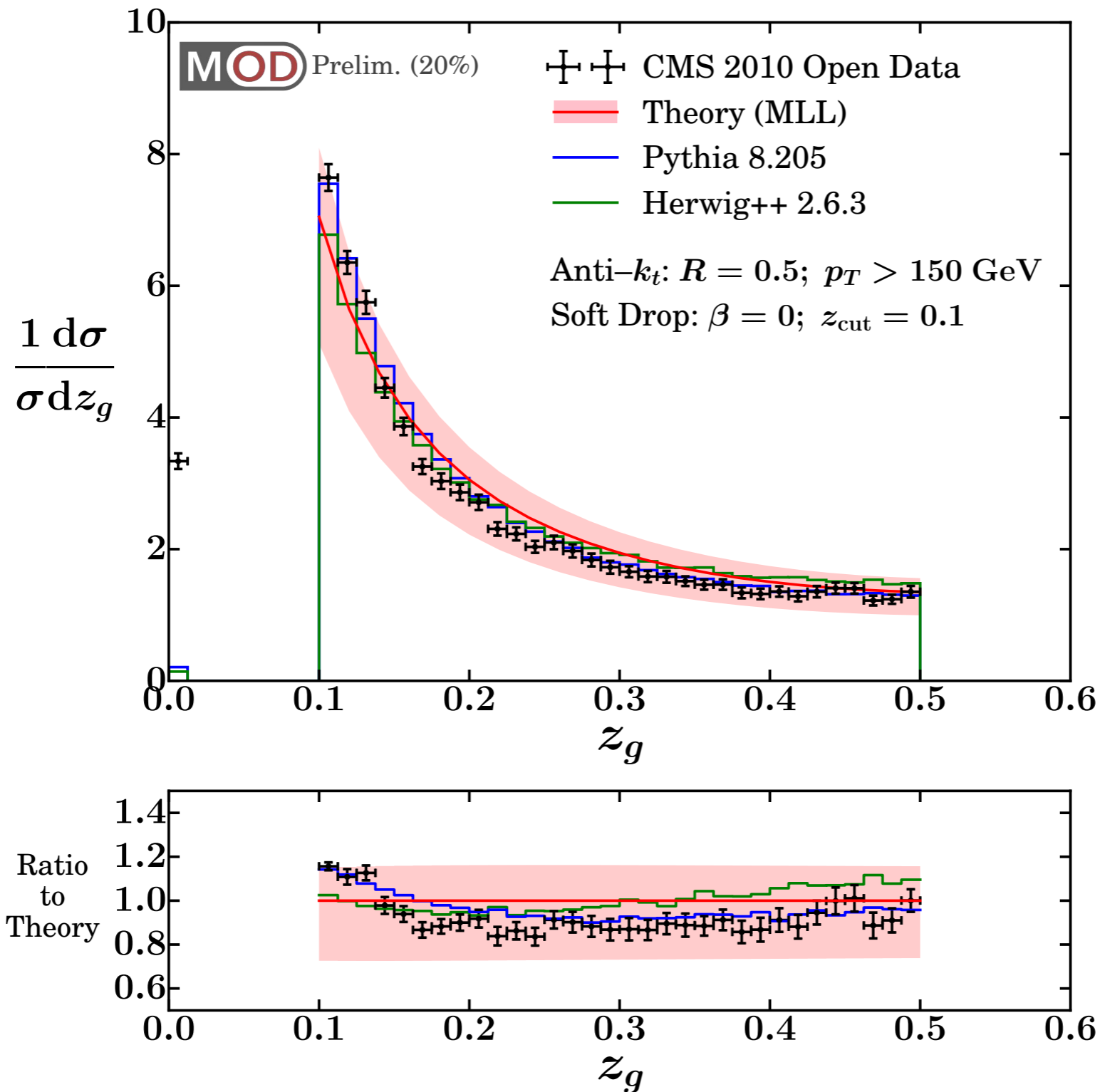
Wei Xue



CMS advice from Sal Rappoccio



# CMS Open Data



Andrew Larkoski



Simone Marzani



Alexis Romero



Aashish Tripathy



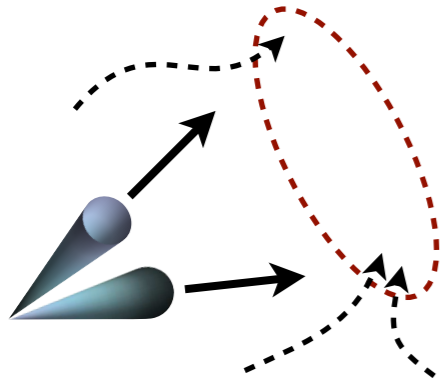
Wei Xue



CMS advice from Sal Rappoccio

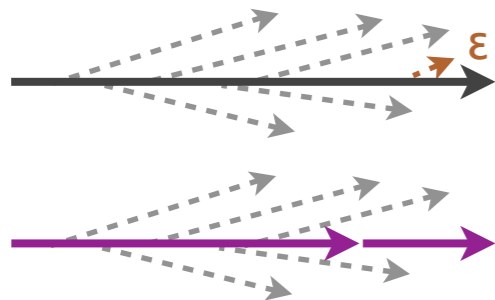


# Summary



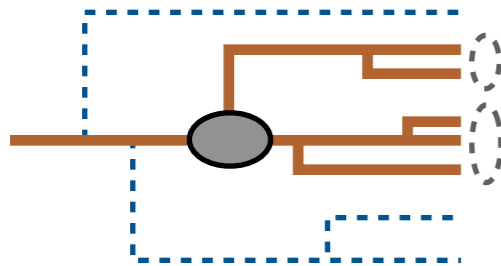
## Inspiration from Jet Substructure

*Exceptional LHC performance + (B)SM physics*



## From IRC Safe to Sudakov Safe

*All orders in  $\alpha_s$  yields new insights into QFT*



## Probing the Core of QCD

*Measuring the universal singularity structure of gauge theories*

*All IRC safe observables are alike;  
each IRC unsafe observable  
is unsafe in its own way.*

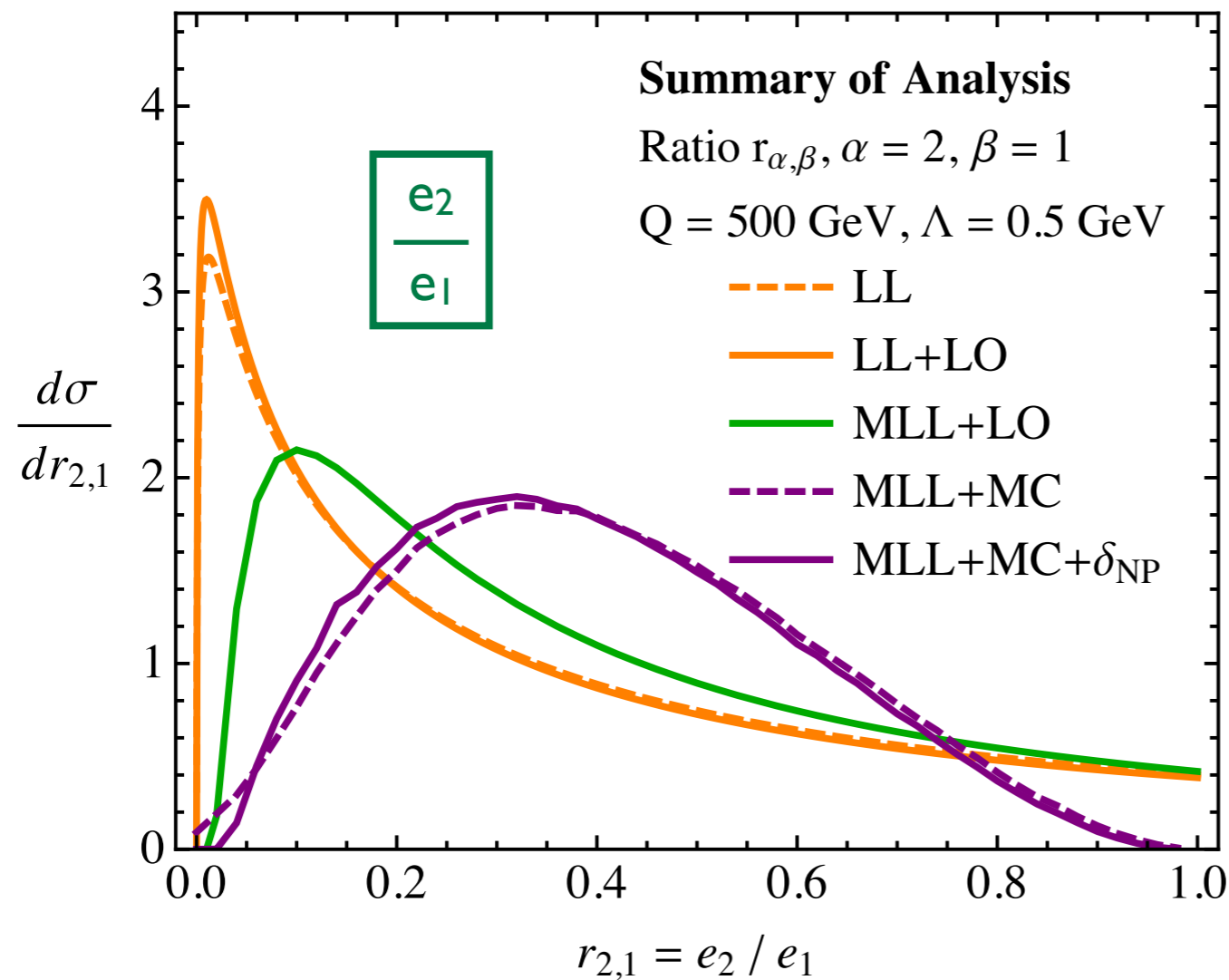
*≈ Leo Tolstoy, Anna Karenina*

# *Backup Slides*

*IRC safe observables, useful, measurable,  
and calculable...seemed to unite some of the  
best blessings of perturbation theory; and  
have existed nearly forty years in the world  
with very little to distress or vex them.*

*≈ Jane Austen, Emma*

# Systematically Improvable



Minimal set of effects for comparison to LHC data:

- = Sudakov factor
- + tree-level matching
- + running  $\alpha_s$
- + multiple emissions
- + hadronization correction

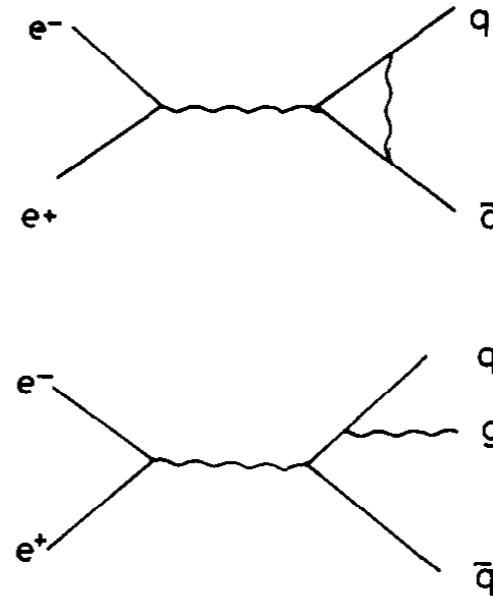
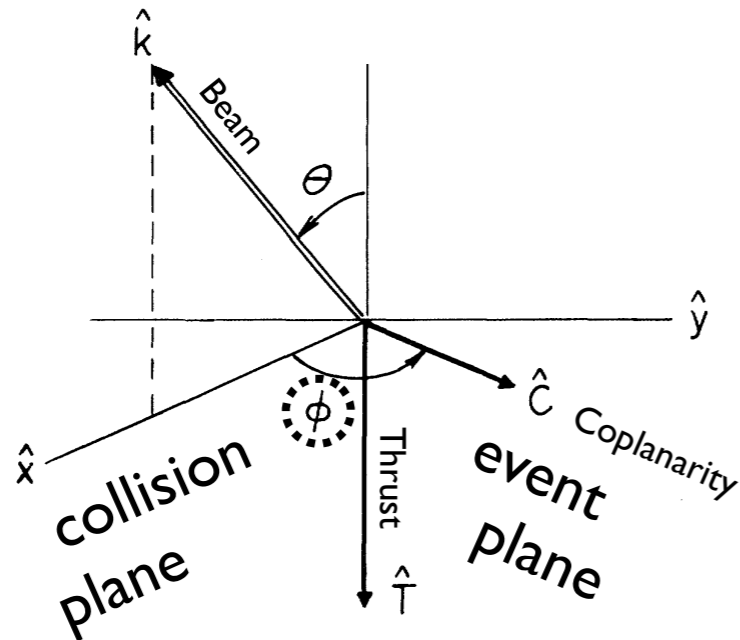
[Larkoski, JDT, 1307.1699]

**NLL:** [Larkoski, Mout, Neill, 1401.4458]

[Procura, Waalewijn, Zeune, 1410.6483]

Predictions for jet substructure from first-principles QCD

# 0. Learn from Our Elders



$\varphi$  ambiguous

$\varphi$  well-defined

Me: “ $\varphi$  is IRC unsafe”

My Elder: “We explicitly calculated  $d\sigma/d\varphi$  in 1978”

$$\frac{2\pi}{\sigma_0} \frac{d\sigma}{d\varphi} = 1 + O(\alpha_s(Q^2)) + \frac{\alpha_s(Q^2)}{\pi} \left( \frac{16}{3} \ln \frac{3}{2} - 2 \right) \cos 2\varphi$$

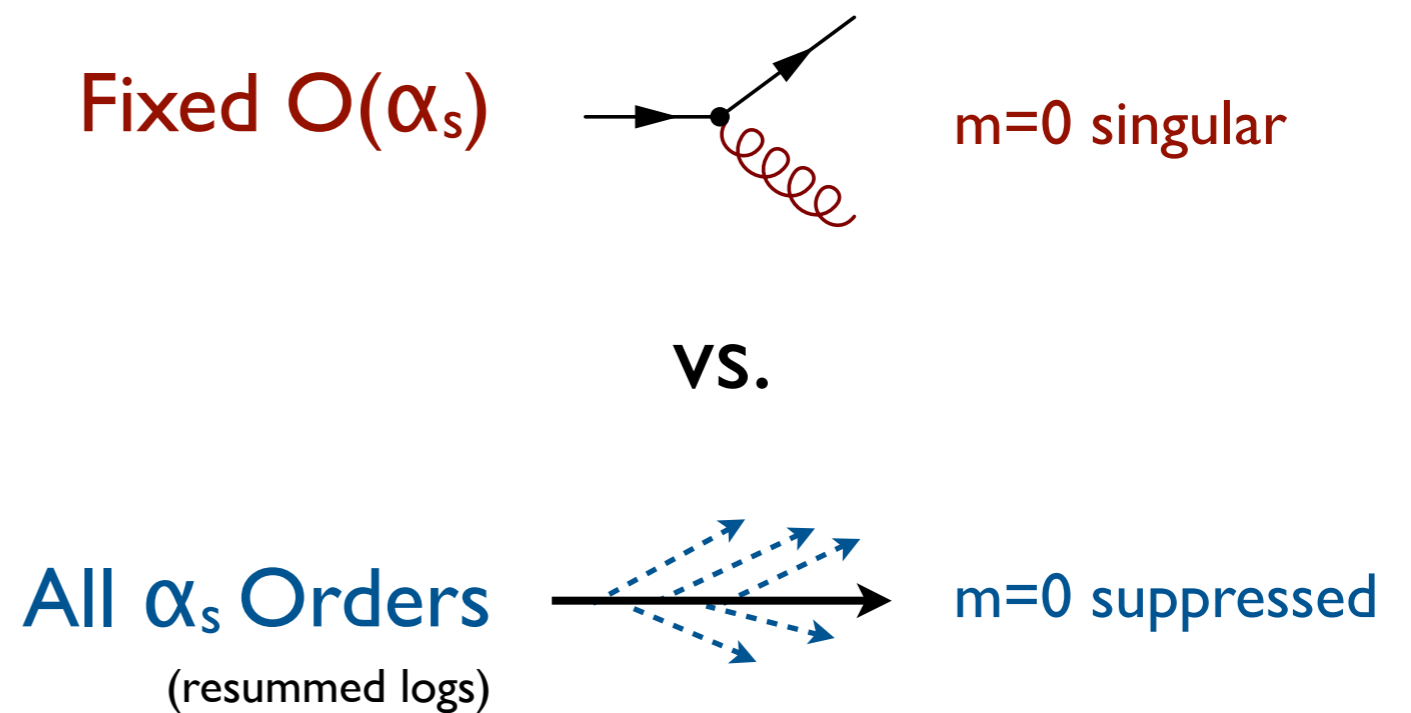
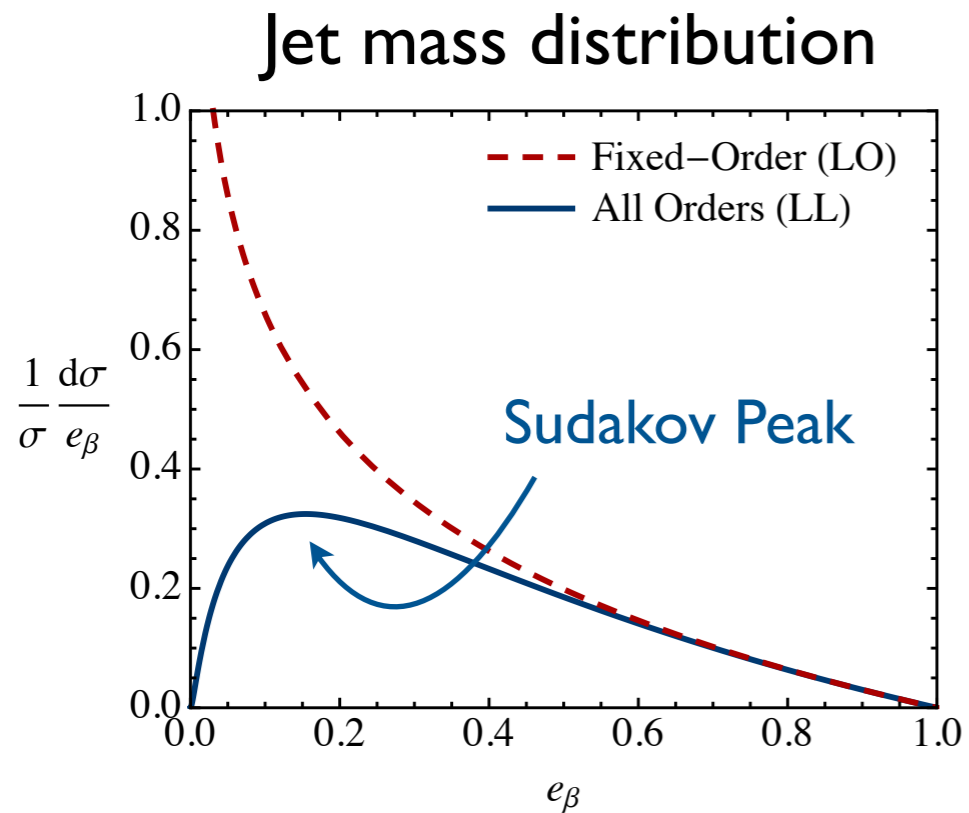
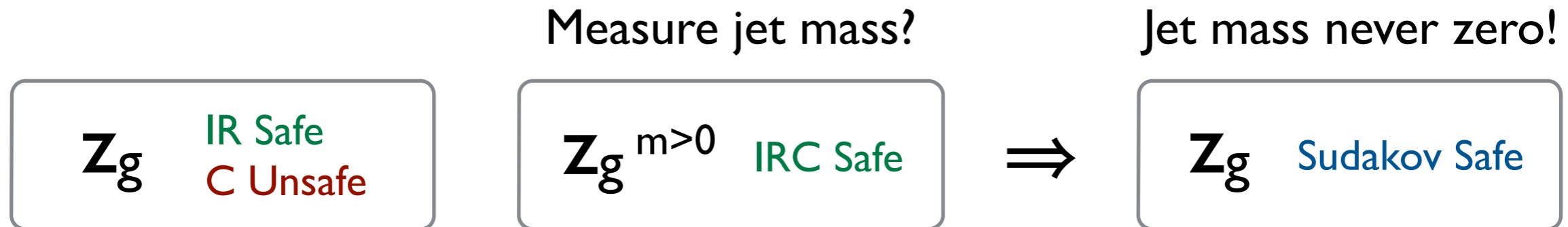
↑ Born cross section despite ambiguity (!)

Lesson: Use IRC limit to resolve ambiguities

[Pi, Jaffe, Low, 1978;  
Kramer, Schierholz, Willrodt, 1978]



# I. Use Sudakov Form Factors



# I. Use Sudakov Form Factors

**Unsafe**  
↓  
**Want:**  $p(u) = \frac{1}{\sigma} \frac{d\sigma}{du}$

**Calculable...**  
↓  
**Need:**  $p(u|s) = \frac{p(u, s)}{p(s)}$   
↑  
...with Safe companion

**Sudakov Safe**  
↓  
**Insight:**  $p(u) = \int ds p(s) p(u|s)$

↑  
**Sudakov form factor**  
(all orders in  $\alpha_s$ )

↑  
**Perturbative**  
(fixed order in  $\alpha_s$ )

Suppresses isolated singularities...

...at each perturbative order

# 2. Use Fragmentation Functions

$z_g$  IR Safe  
C Unsafe

Absorb singularities into universal function (just like PDFs!)

$$\frac{d\sigma}{dz_g} \simeq F(z_g) - \frac{1}{2\epsilon} \frac{\alpha_s C}{\pi} F(z_g) + \frac{\alpha_s C}{\pi} \int \frac{d\theta}{\theta} P(z_g)$$

