# Higgsing the stringy higher spin symmetry 

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## Stringy theory on $\mathrm{AdS}_{3}$ at tensionless point

In the context of the $A d S_{3} / \mathrm{CFT}_{2}$ correspondence, the symmetric product orbifold CFT of the D1-D5 system is dual to string theory on $A d S_{3} \times S^{3} \times \mathbb{T}^{4}$ at the tensionless point.
[Gaberdiel \& Gopakumar, '14]

The symmetric orbifold CFT has an infinite tower of massless conserved higher spin (HS) currents, a closed subsector of which are dual to the HS fields of the Vasiliev theory.

This work: we consider deformation of the symmetric orbifold CFT which corresponds to switching on the string tension and study the behaviour of symmetry generators of the theory.

## Outline

- Symmetric orbifold CFT and the stringy symmetries
- Higgsing stringy symmetries
- Results
- Summary

D1-D5 system

\[

\]

where $\mathcal{M}$ is $\mathbb{T}^{4}$ or $K 3$.

## D1-D5 system

$$
\begin{array}{lcccccccccc} 
& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
N_{1} \text { D1 } & - & - & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & . \\
N_{5} \text { D5 } & - & - & - & - & - & - & \cdot & \cdot & \cdot & .
\end{array}
$$

$$
\text { where } \mathcal{M} \text { is } \mathbb{T}^{4} \text { or } K 3 . \quad S^{1} \quad \mathcal{M}
$$

In the limit where size of $\mathbb{T}^{4} \ll$ size of $S^{1}$, worldvolume gauge theory of D branes is a 2d field theory that lives on $S^{1}$.

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It flows in IR to a CFT described by a sigma model whose target space is a resolution of symmetric product orbifold
[Vafa, '95]

$$
\operatorname{Sym}_{N+1}\left(\mathbb{T}^{4}\right)=\left(\mathbb{T}^{4}\right)^{N+1} / S_{N+1}, \quad\left(N+1=N_{1} N_{5}\right) .
$$

## D1-D5 system

where $\mathcal{M}$ is $\mathbb{T}^{4}$ or $K 3$.

$$
\begin{array}{lllllllllll} 
& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
N_{1} \text { D1 } & - & - & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_{5} \text { D5 } & - & - & - & - & - & - & \cdot & \cdot & \cdot & . \\
K 3 & & S^{1} & \mathcal{M} & & & & &
\end{array}
$$

$\mathrm{AdS}_{3} / \mathrm{CFT}_{2}$
String theory on $A d S_{3} \times S^{3} \times \mathbb{T}^{4}$ is dual to symmetric product orbifold CFT.
[Maldacena, '97]

Free orbifold point is the analogue of free Yang Mills theory for the case of D3 branes.

## Symmetric product orbifold CFT

- $2 \mathrm{~d} \mathcal{N}=(4,4)$ SCFT with $S O(4)_{R} \cong S U(2)_{L} \times S U(2)_{R}$ $R$-symmetry.
- Generators of left-moving superconformal algebra: $L_{n}, G_{r}^{\alpha}$, and $J_{n}^{\prime}$ (similar for right-moving generators).
- At the orbifold point, we have a free CFT of $2(N+1)$ complex bosons and $2(N+1)$ complex fermions and their conjugates:

$$
\partial \phi_{a}^{i}, \partial \bar{\phi}_{a}^{i}, \quad \psi_{a}^{i}, \bar{\psi}_{a}^{i}, \quad i \in\{1,2\}, \quad a \in\{1, \cdots, N+1\},
$$

plus right-moving counterparts.

- $S_{N+1}$ acts by permuting $N+1$ copies of $\mathbb{T}^{4}$.


## Twisted sector

- The orbifold group, $S_{N+1}$, acts by permuting the $N+1$ copies (labelled by a). It defines states belonging to a new sector of the Hilbert space: the twisted sector.
- Twist operators $\sigma_{(12 \cdots n)}$ define new boundary conditions of the fundamental fields $\partial \phi_{a}^{i}$ and $\psi_{a}^{i}$ and their conjugates:

$$
\begin{aligned}
& \partial \phi_{1}^{i} \rightarrow \partial \phi_{2}^{i} \rightarrow \partial \phi_{3}^{i} \rightarrow \cdots \rightarrow \partial \phi_{n}^{i} \rightarrow \partial \phi_{1}^{i} \\
& \psi_{1}^{i} \rightarrow \psi_{2}^{i} \rightarrow \psi_{3}^{i} \rightarrow \cdots \rightarrow \psi_{n}^{i} \rightarrow \psi_{1}^{i}
\end{aligned}
$$



## Higher spin holography

Non-supersymmetric version:

Vasiliev HS theory on $\mathrm{AdS}_{3}$ along with a complex scalar

minimal model CFTs
in the large- N 't Hooft limit, where $\lambda=\frac{N}{N+k}$.
[Gaberdiel \& Gopakumar, '10]
$\mathcal{W}_{N, k}$ minimal model series are described in terms of cosets

$$
\frac{\mathfrak{s u}(N)_{k} \oplus \mathfrak{s u}(N)_{1}}{\mathfrak{s u}(N)_{k+1}}
$$

Agreement in the 't Hooft limit:

- symmetries,
- spectra,
- correlation functions.
[Gaberdiel \& Hartman, '11; Gaberdiel \& Gopakumar, '13]
[Gaberdiel, Hartman, Gopakumar, Raju, '11]
[Chang \& Yin, '11; Ammon, Kraus, Perlmutter, '11]


## Higher spin holography

Supersymmetric version:
Vasiliev HS theory on $\mathrm{AdS}_{3}$ a class 2d coset CFTs
[Gaberdiel \& Gopakumar, '13]
The Wolf space coset CFTs are described by cosets

$$
\frac{\mathfrak{s u}(N+2)_{k+N+2}^{(1)}}{\mathfrak{s u}(N)_{k+N+2}^{(1)}} \cong \frac{\mathfrak{s u}(N+2)_{k} \oplus \mathfrak{s o}(N+4)_{1}}{\mathfrak{s u}(N)_{k+2}}
$$

[Sevrin et. al., '88; Spindel et. al., '88; Van Proeyen, '89]

- Both theories have large $\mathcal{N}=4$ symmetry: this is the symmetry algebra of the CFT dual to string theory on $A d S_{3} \times S^{3} \times S^{3} \times S^{1}$ (with two $\mathfrak{s u}(2)$ affine algebras).
- The symmetries and spectra of the two theories agree in the large $N$, $k$ 't Hooft limit, $\lambda=\frac{N+1}{N+k+2}$.


## Higher spin embedding

- Symmetry algebra of string theory on $A d S_{3} \times S^{3} \times \mathbb{T}^{4}$ is small $\mathcal{N}=4$ : take the limit where $\mathcal{N}=4$ contracts to $\mathcal{N}=4$.
- One $\mathfrak{s u}(2)$ becomes the R-symmetry algebra of $\mathcal{N}=4$ theory and the other one becomes a global symmetry.
- In this limit, the coset CFT reduces to a $U(N)$ group orbifold (a continuous orbifold):

$$
\left(\mathbb{T}^{4}\right)^{N+1} / U(N) .
$$

- Embed continuous orbifold into the symmetric group orbifold:

$$
S_{N+1} \subset U(N)
$$

## Higher spin embedding

- The perturbative part of the HS dual coset CFT forms a closed subsector of the symmetric orbifold CFT.
[Gaberdiel \& Gopakumar, '14]
- All states of the symmetric orbifold CFT (dual to string theory) are organised in terms of representations of the $\mathrm{HS} \mathcal{W}_{\infty}^{(\mathcal{N}=4)}$ symmetry of the continuous orbifold.
- The chiral algebra of symmetric orbifold CFT is written as

$$
Z_{\text {vac, stringy }}(q, y)=\sum_{\Lambda} n(\Lambda) \chi_{(0 ; \Lambda)}(q, y)
$$

Original $\mathcal{W}_{\infty}^{\mathcal{N}=4}$ algebra

$$
\begin{array}{cccc} 
& & s: & (\mathbf{1}, \mathbf{1}) \\
(\mathcal{N}=4) \oplus \bigoplus_{s=1}^{\infty} R^{(s)}, & & R^{(s)}: & s+\frac{1}{2}: \\
s+1: & (\mathbf{2}, \mathbf{2}) \\
& & s+\frac{3}{2}: & (\mathbf{3}, \mathbf{1}) \oplus(\mathbf{1}, \mathbf{3}) . \\
& s+2: & (\mathbf{2}, \mathbf{2}) \\
& & (\mathbf{1}, \mathbf{1})
\end{array}
$$

Free field realisation of HS fields dual to Vasiliev theory is in terms of neutral bilinears:

$$
\sum_{a=1}^{N+1} p_{a}^{1} p_{a}^{2}, \quad P_{a}^{1} \in\left\{\partial^{\#} \phi^{\prime}, \partial^{\#} \psi^{\prime}\right\}, \quad P_{a}^{2} \in\left\{\partial^{\#} \bar{\phi}^{-}, \partial \partial^{*} \bar{\psi}^{\top}\right\} .
$$

## Stringy HS fields

HS fields of symmetric orbifold theory come from the untwisted sector of orbifold. Their single particle symmetry generators are:

$$
\sum_{a=1}^{N+1} P_{a}^{1} \cdots P_{a}^{m}
$$

where $P_{a}^{j}$ is one of the 4 bosons/fermions or their derivatives in the $a^{\text {th }}$ copy.

They fall into additional $\mathcal{W}_{\infty}^{\mathcal{N}}=4$ representations: hugely extend coset $\mathcal{W}$ algebra

$$
\mathcal{W}_{\infty}^{\mathcal{N}=4} \oplus \bigoplus_{n, \bar{n}}(0 ;[n, 0, \cdots, 0, \bar{n}]), \quad m=n+\bar{n}
$$

Stringy HS fields

[Gaberdiel \& Gopakumar, '15]

Example: cubic generators $(m=3)$
$P_{a}^{1}, P_{a}^{2}, P_{a}^{3} \in\left\{\partial^{\#} \phi^{i}, \partial^{\#} \psi^{i}\right\} \quad$ or $\quad P_{a}^{1}, P_{a}^{2}, P_{a}^{3} \in\left\{\partial^{\#} \bar{\phi}^{i}, \partial^{\#} \bar{\psi}^{i}\right\}$,
lie in the multiplets

$$
\begin{aligned}
& (0 ;[3,0, \cdots, 0,0]),(0 ;[0,0, \cdots, 0,3]): \\
& \bigoplus_{s=2}^{\infty} n(s)\left[R^{(s)}(\mathbf{2}, \mathbf{1}) \oplus R^{(s+3 / 2)}(\mathbf{1}, \mathbf{2})\right],
\end{aligned}
$$

where $\frac{q^{2}}{\left(1-q^{2}\right)\left(1-q^{3}\right)}=\sum_{s=2}^{\infty} n(s) q^{s}$, and

|  | $s:$ | $(\mathbf{2}, \mathbf{1})$ | $s:$ | $(\mathbf{1}, \mathbf{2})$ |
| :---: | :---: | :---: | :---: | :---: |
| $R^{(s)}(\mathbf{2}, \mathbf{1}):$ | $s+\frac{1}{2}:$ | $(\mathbf{3}, \mathbf{2 )} \oplus(\mathbf{1}, \mathbf{2})$ |  | $s+\frac{1}{2}:$ |
|  | $s+1:$ | $(\mathbf{4}, \mathbf{1}) \oplus(\mathbf{2}, \mathbf{1}) \oplus(\mathbf{2}, \mathbf{3})$, | $R^{(s)}(\mathbf{1}, \mathbf{2}):$ | $s+\mathbf{3}) \oplus(\mathbf{2}, \mathbf{1})$ |
|  | $s+\frac{3}{2}:$ | $(\mathbf{3}, \mathbf{2}) \oplus(\mathbf{1}, \mathbf{2})$ | $(\mathbf{1}, \mathbf{4}) \oplus(\mathbf{1}, \mathbf{2}) \oplus(\mathbf{3}, \mathbf{2})$. |  |
|  | $s+2:$ | $(\mathbf{2}, \mathbf{1})$ | $s+\frac{3}{2}:$ | $(\mathbf{2 , 3}) \oplus(\mathbf{2}, \mathbf{1})$ |
|  |  |  | $s+2:$ | $(\mathbf{1}, \mathbf{2})$ |

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## Higgsing of stringy symmetries

- Showing that CFT dual of HS theory forms a closed subsector of symmetric orbifold CFT provides convincing evidence for the claim that orbifold point corresponds to tensionless limit of string thoery.


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- As string tension is switched on, HS symmetries are broken. Expect that Regge trajectories emerge: Vasiliev fields fall into the leading trajectory. Higher trajectories correspond to additional HS fields which become massless at tensionless point.


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- At the tensionless point, the symmetry algebra is much bigger than $\mathcal{N}=4$ superconformal algebra + algebra of Vasiliev HS theory.
- As string tension is switched on, HS symmetries are broken. Expect that Regge trajectories emerge: Vasiliev fields fall into the leading trajectory. Higher trajectories correspond to additional HS fields which become massless at tensionless point.
- We examine this picture by switching on string tension and studying behaviour of symmetry generators of symmetric orbifold CFT.


## Higgsing of stringy symmetries

- Switching on tension corresponds to deforming CFT away from orbifold point by an exactly marginal operator $\Phi$, which belongs to twist-2 sector.

- $\Phi$ has $(h, \bar{h})=(1,1)$, and transforms as $(1,1)$ under the $S U(2)_{L} \times S U(2)_{R} R$-symmetry and the global $S O(4)$. It is the super-descendant of BPS ground state: $\propto G_{-1 / 2} \tilde{G}_{-1 / 2}\left|\Psi_{2}\right\rangle$


## Symmetries broken?

First order deformation analysis: criterion for spin $s$ field $W^{(s)}$ of the chiral algebra to remain chiral under deformation by $\Phi$
[Cardy, '90; Fredenhagen, Gaberdiel, Keller, '07; Gaberdiel, Jin, Li, '13]

$$
\mathcal{N}\left(W^{(s)}\right) \equiv \sum_{l=0}^{\left\lfloor s+h_{\Phi}\right\rfloor-1} \frac{(-1)^{\prime}}{l!}\left(L_{-1}\right)^{l} W_{-s+1+l}^{(s)} \Phi=0
$$

where

$$
\partial_{\bar{z}} W^{(s)}(z, \bar{z})=g \pi \mathcal{N}\left(W^{(s)}\right)
$$

$\mathcal{N}=4$ superconformal algebra is preserved, while HS currents are not conserved: gigantic symmetry algebra is broken down to the $\mathcal{N}=4$ SCA.

## Conformal perturbation theory

Compute relevant anomalous dimensions quantitatively and determine masses of the corresponding fields.

Consider adding a small perturbation to the action of free CFT. The normalised perturbed 2pf is:
$\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right)\right\rangle_{\Phi}=\frac{\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right) e^{\delta S}\right\rangle}{\left\langle e^{\delta S}\right\rangle}, \quad \delta S=g \int d^{2} w \Phi(w, \bar{w})$.
Upon expanding in powers of $g$, we have

$$
\begin{aligned}
& \left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right)\right\rangle_{\Phi}-\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right)\right\rangle= \\
& \frac{g^{2}}{2}\left(\int d^{2} w_{1} d^{2} w_{2}\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right) \Phi\left(w_{1}, \bar{w}_{1}\right) \Phi\left(w_{2}, \bar{w}_{2}\right)\right\rangle\right. \\
& \left.\quad-\int d^{2} w_{1} d^{2} w_{2}\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{2}\right)\right\rangle\left\langle\Phi\left(w_{1}, \bar{w}_{1}\right) \Phi\left(w_{2}, \bar{w}_{2}\right)\right\rangle\right)+\mathcal{O}\left(g^{3}\right) .
\end{aligned}
$$

## Anomalous dimensions

2 pf of quasiprimary operators is of the form
$\left\langle W^{(s) i}\left(z_{1}\right) W^{(s) j}\left(z_{1}\right)\right\rangle_{\Phi} \sim \frac{c^{i j}}{\left(z_{1}-z_{2}\right)^{2\left(s+\gamma^{i j}\right)}\left(\bar{z}_{1}-\bar{z}_{2}\right)^{2 \bar{\gamma}^{i j}}}$,
where for small $\gamma^{i j}$ reads

$$
\approx \frac{c^{i j}}{\left(z_{1}-z_{2}\right)^{2 s}}\left(1-2 \gamma^{i j} \ln \left(z_{1}-z_{2}\right)-2 \bar{\gamma}^{i j} \ln \left(\bar{z}_{1}-\bar{z}_{2}\right)+\cdots\right) .
$$

Read coefficient of the log term in perturbed 2pf.

## Anomalous dimensions

To first order, $\gamma_{i j}$ is given by 3 point function

$$
\left\langle W^{(s) i}\left(z_{1}\right) \Phi\left(w_{1}, \bar{w}_{1}\right) W^{(s) j}\left(z_{2}\right)\right\rangle
$$

which vanishes: $\Phi$ has $h_{\Phi}=\bar{h}_{\Phi}=1$ while $W$ 's have $\bar{h}_{W}=0$.

Leading correction to the 2 pf appears at second order:

$$
\begin{gathered}
\gamma^{i j}=g^{2} \pi^{2}\left\langle\mathcal{N}\left(W^{(s) i}\right) \mathcal{N}\left(W^{(s) j}\right)\right\rangle, \\
\mathcal{N}\left(W^{(s)}\right) \equiv \sum_{l=0}^{\left\lfloor s+h_{\phi}\right\rfloor-1} \frac{(-1)^{\prime}}{!!}\left(L_{-1}\right)^{\prime} W_{-s+1+1}^{(s)} \Phi .
\end{gathered}
$$

## Operator mixing

In general, matrix $\gamma_{i j}$ is not diagonal: need to diagonalise it to extract anomalous dimensions.

- In general, fields within each family, $m=2,3, \cdots$, mix (multiplicities $n(s)>1)$.
- There is also mixing present between fields from different families.



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Vasiliev HS fields:

$$
\begin{gathered}
W^{(s)}=\sum_{q=0}^{s-2}(-1)^{q}\binom{s-1}{q}\binom{s-1}{q+1} \partial^{s-1-q} \bar{\phi}^{1} \partial^{q+1} \phi^{2} \\
\gamma^{i j}=g^{2} \pi^{2}\left\langle\mathcal{N}\left(W^{(s) i}\right) \mathcal{N}\left(W^{(s) j}\right)\right\rangle .
\end{gathered}
$$

## Vasiliev HS fields:

$$
W^{(s)}=\sum_{q=0}^{s-2}(-1)^{q}\binom{s-1}{q}\binom{s-1}{q+1} \partial^{s-1-q} \bar{\phi}^{1} \partial^{q+1} \phi^{2} .
$$

The diagonal elements $\gamma^{i i}$ can be computed analytically and in closed form:

$$
\gamma^{(s)}=\frac{g^{2} \pi^{2} \sum_{p=0}^{s}(-1)^{s-p}\binom{2 s}{s-p} P_{2}(s, p)}{(N+1) E_{2}(s)}
$$

where

$$
\begin{aligned}
& \begin{aligned}
E_{2}(s) & =\sum_{q=0}^{s-1} \sum_{p=0}^{s-1}(-1)^{s+1+p+q}\binom{s}{q}\binom{s}{q+1}\binom{s}{p}\binom{s}{p+1} \\
& \times\left((-2)_{(q)}(-2-q)_{(s-p-1)}(-2)_{(s-q-1)}(q-s-1)_{(p)}\right)
\end{aligned} \\
& \begin{aligned}
P_{2}(s, p) & =\sum_{n=3 / 2}^{p-3 / 2} n(p-n) f(s, p, n) f(s,-p, n-p)
\end{aligned} \\
& \quad+\frac{3}{2}(-1)^{s+1} \Theta(p-2) f(s, p, 1 / 2) f(s,-p,-1 / 2)(p-1 / 2) \\
& \quad+\frac{1}{2} \delta_{p, 1} f(s, 1,1 / 2) f(s,-1,-1 / 2)
\end{aligned} \quad \begin{aligned}
& f(s, p, n)=\sum_{q=0}^{s-1}(-1)^{q}\binom{s}{q}\binom{s}{q+1}(-1-p+n)_{(s-q-1)}(-1-n)_{(q)} .
\end{aligned}
$$

Regge trajectories

- Vasiliev HS generators correspond to the leading Regge trajectory (blue diamonds); have lowest masses for a given spin.
- Cubic generators describe the first sub-leading Regge trajectory (brown circles).



## Regge trajectories

- Diagonalisation of complete mixing matrix becomes complicated as spin increases: we have solved it completely for low-lying fields ( $X$ 's).
- For cubic generators, we perform partial diagonalisation at larger spin where we only diagonalise $\gamma_{i j}$ among the fields of $m=3$.



## Regge trajectories

- Diagonal entries of Regge trajectories behave as $\gamma^{(s)} \cong a \log s$ at large spin, with dispersion relation $E(s) \cong s+a \log s$. This suggests that symmetric orbifold CFT is dual to an $\mathrm{AdS}_{3}$ background with pure RR flux.
[Loewy, Oz, '03; David, Sadhukhan, '14]



## Summary:

- Computed anomalous dimensions of the HS generators of symmetric orbifold CFT as the string tension is switched on.
- HS fields of original $\mathcal{W}_{\infty}^{(\mathcal{N}=4)}$ algebra form a decoupled subsector at tensionless point. As tension is switched on, they couple with stringy symmetry generators.


## Future directions:

- Solve for exact anomalous dimensions for higher spins and determine shape of dispersion relations.
- Derive anomalous diemensions for symmetric product orbifold of K3.
[Baggio, Gaberdiel, and Peng, '15]
- Compute the anomalous dimensions from the dual AdS viewpoint.

