Higgsing the stringy higher spin symmetry

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Stringy theory on AdS_3 at tensionless point

In the context of the AdS_3/CFT_2 correspondence, the symmetric product orbifold CFT of the D1-D5 system is dual to string theory on $AdS_3 \times S^3 \times \mathbb{T}^4$ at the tensionless point.

[Gaberdiel & Gopakumar, '14]

The symmetric orbifold CFT has an infinite tower of massless conserved higher spin (HS) currents, a closed subsector of which are dual to the HS fields of the Vasiliev theory.

This work: we consider deformation of the symmetric orbifold CFT which corresponds to switching on the string tension and study the behaviour of symmetry generators of the theory.

Outline

- Symmetric orbifold CFT and the stringy symmetries
- Higgsing stringy symmetries
- Results







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It flows in IR to a CFT described by a sigma model whose target space is a resolution of symmetric product orbifold

[Vafa, '95]

$$Sym_{N+1}(\mathbb{T}^4) = (\mathbb{T}^4)^{N+1}/S_{N+1}, \qquad (N+1 = N_1N_5).$$



$\mathsf{AdS}_3/\mathsf{CFT}_2$

 $\begin{array}{ll} \mbox{String theory on } AdS_3 \times S^3 \times \mathbb{T}^4 \mbox{ is dual to symmetric product} \\ \mbox{orbifold CFT.} & \mbox{[Maldacena, '97]} \end{array}$

Free orbifold point is the analogue of free Yang Mills theory for the case of D3 branes.

Symmetric product orbifold CFT

- ▶ 2d $\mathcal{N} = (4, 4)$ SCFT with $SO(4)_R \cong SU(2)_L \times SU(2)_R$ *R*-symmetry.
- Generators of left-moving superconformal algebra: L_n, G_r^α, and J_n^l (similar for right-moving generators).
- ► At the orbifold point, we have a free CFT of 2(N+1) complex bosons and 2(N+1) complex fermions and their conjugates:

 $\partial \phi_a^i, \ \partial \bar{\phi}_a^i, \ \psi_a^i, \ \bar{\psi}_a^i, \quad i \in \{1, 2\}, \ a \in \{1, \cdots, N+1\},$

plus right-moving counterparts.

• S_{N+1} acts by permuting N+1 copies of \mathbb{T}^4 .

Twisted sector

- ► The orbifold group, S_{N+1}, acts by permuting the N + 1 copies (labelled by a). It defines states belonging to a new sector of the Hilbert space: the *twisted* sector.
- ► Twist operators σ_(12...n) define new boundary conditions of the fundamental fields ∂φⁱ_a and ψⁱ_a and their conjugates:

Higher spin holography

Non-supersymmetric version:

in the large-N 't Hooft limit, where $\lambda = \frac{N}{N+k}$. [Gaberdiel & Gopakumar, '10]

 $\mathcal{W}_{N,k}$ minimal model series are described in terms of cosets

 $\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}.$

Agreement in the 't Hooft limit:

symmetries, [Gaberdiel & Hartman, '11; Gaberdiel & Gopakumar, '13]
 spectra, [Gaberdiel, Hartman, Gopakumar, Raju, '11]
 correlation functions. [Chang & Yin, '11; Ammon, Kraus, Perlmutter, '11]

Higher spin holography

Supersymmetric version:

The Wolf space coset CFTs are described by cosets

$$\frac{\mathfrak{su}(N+2)_{k+N+2}^{(1)}}{\mathfrak{su}(N)_{k+N+2}^{(1)}} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(N+4)_1}{\mathfrak{su}(N)_{k+2}}$$

[Sevrin et. al., '88; Spindel et. al., '88; Van Proeyen, '89]

- ▶ Both theories have large N = 4 symmetry: this is the symmetry algebra of the CFT dual to string theory on AdS₃ × S³ × S³ × S¹ (with two su(2) affine algebras).
- The symmetries and spectra of the two theories agree in the large *N*, *k* 't Hooft limit, $\lambda = \frac{N+1}{N+k+2}$.

Higher spin embedding

- Symmetry algebra of string theory on AdS₃ × S³ × T⁴ is small N = 4: take the limit where N = 4 contracts to N = 4.
- ► One su(2) becomes the R-symmetry algebra of N = 4 theory and the other one becomes a global symmetry.
- In this limit, the coset CFT reduces to a U(N) group orbifold (a continuous orbifold):

 $(\mathbb{T}^4)^{N+1}/U(N).$

Embed continuous orbifold into the symmetric group orbifold:

 $S_{N+1} \subset U(N).$

Higher spin embedding

The perturbative part of the HS dual coset CFT forms a closed subsector of the symmetric orbifold CFT.

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[Gaberdiel & Gopakumar, '14]
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- ► All states of the symmetric orbifold CFT (dual to string theory) are organised in terms of representations of the HS W^(N=4)_∞ symmetry of the continuous orbifold.
- The chiral algebra of symmetric orbifold CFT is written as

$$Z_{\mathsf{vac},\mathsf{stringy}}(q,y) = \sum_{\Lambda} \mathit{n}(\Lambda) \ \chi_{(0;\Lambda)}(q,y).$$

Original $\mathcal{W}_\infty^{\,\mathcal{N}=4}$ algebra

$$(\mathcal{N} = 4) \oplus \bigoplus_{s=1}^{\infty} \mathbb{R}^{(s)}, \qquad \mathbb{R}^{(s)} : \begin{array}{c} s : & (\mathbf{1}, \mathbf{1}) \\ s + \frac{1}{2} : & (\mathbf{2}, \mathbf{2}) \\ s + 1 : & (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}). \\ s + \frac{3}{2} : & (\mathbf{2}, \mathbf{2}) \\ s + 2 : & (\mathbf{1}, \mathbf{1}) \end{array}$$

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Free field realisation of HS fields dual to Vasiliev theory is in terms of neutral bilinears:

$$\sum_{a=1}^{N+1} P_a^1 P_a^2, \qquad P_a^1 \in \{\partial^{\#} \phi^i, \partial^{\#} \psi^i\}, \quad P_a^2 \in \{\partial^{\#} \bar{\phi}^i, \partial^{\#} \bar{\psi}^i\}.$$

Stringy HS fields

HS fields of symmetric orbifold theory come from the untwisted sector of orbifold. Their single particle symmetry generators are:

$$\sum_{a=1}^{N+1} P_a^1 \cdots P_a^m,$$

where P_a^j is one of the 4 bosons/fermions or their derivatives in the $a^{\rm th}$ copy.

They fall into additional $\mathcal{W}_\infty^{\mathcal{N}=4}$ representations: hugely extend coset $\mathcal W$ algebra

$$\mathcal{W}_{\infty}^{\mathcal{N}=4} \oplus \bigoplus_{n,\bar{n}} (0; [n, 0, \cdots, 0, \bar{n}]), \qquad m = n + \bar{n}.$$

Stringy HS fields



[Gaberdiel & Gopakumar, '15]

Example: cubic generators (m = 3)

 $P_a^1, P_a^2, P_a^3 \in \{\partial^{\#}\phi^i, \partial^{\#}\psi^i\} \quad \text{or} \quad P_a^1, P_a^2, P_a^3 \in \{\partial^{\#}\bar{\phi}^i, \partial^{\#}\bar{\psi}^i\},$

lie in the multiplets

$$(0; [3, 0, \cdots, 0, 0]), (0; [0, 0, \cdots, 0, 3]) : \bigoplus_{s=2}^{\infty} n(s) \left[R^{(s)}(\mathbf{2}, \mathbf{1}) \oplus R^{(s+3/2)}(\mathbf{1}, \mathbf{2}) \right] ,$$

where
$$\frac{q^2}{(1-q^2)(1-q^3)} = \sum_{s=2}^{\infty} n(s)q^s$$
, and

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- As string tension is switched on, HS symmetries are broken. Expect that Regge trajectories emerge: Vasiliev fields fall into the leading trajectory. Higher trajectories correspond to additional HS fields which become massless at tensionless point.

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- As string tension is switched on, HS symmetries are broken. Expect that Regge trajectories emerge: Vasiliev fields fall into the leading trajectory. Higher trajectories correspond to additional HS fields which become massless at tensionless point.
- We examine this picture by switching on string tension and studying behaviour of symmetry generators of symmetric orbifold CFT.

 Switching on tension corresponds to deforming CFT away from orbifold point by an exactly marginal operator Φ, which belongs to twist-2 sector.



• Φ has $(h, \bar{h}) = (1, 1)$, and transforms as (1, 1) under the $SU(2)_L \times SU(2)_R$ *R*-symmetry and the global SO(4). It is the super-descendant of BPS ground state: $\propto G_{-1/2}\tilde{G}_{-1/2}|\Psi_2\rangle$

Symmetries broken?

First order deformation analysis: criterion for spin s field $W^{(s)}$ of the chiral algebra to remain chiral under deformation by Φ

[Cardy, '90; Fredenhagen, Gaberdiel, Keller, '07;

Gaberdiel, Jin, Li, '13]

$$\mathcal{N}(W^{(s)}) \equiv \sum_{l=0}^{\lfloor s+h_{\Phi}\rfloor - 1} \frac{(-1)^{l}}{l!} (L_{-1})^{l} W^{(s)}_{-s+1+l} \Phi = 0,$$

where

$$\partial_{\bar{z}} W^{(s)}(z,\bar{z}) = g \,\pi \,\mathcal{N}(W^{(s)}).$$

 $\mathcal{N}=4$ superconformal algebra is preserved, while HS currents are not conserved: gigantic symmetry algebra is broken down to the $\mathcal{N}=4$ SCA.

Conformal perturbation theory

Compute relevant anomalous dimensions quantitatively and determine masses of the corresponding fields.

Consider adding a small perturbation to the action of free CFT. The normalised perturbed 2pf is:

$$\left\langle W^{(s)i}(z_1)W^{(s)j}(z_2)\right\rangle_{\Phi} = \frac{\left\langle W^{(s)i}(z_1)W^{(s)j}(z_2)e^{\delta S}\right\rangle}{\left\langle e^{\delta S}\right\rangle}, \quad \delta S = g\int d^2w \,\Phi(w,\bar{w}) \,.$$

Upon expanding in powers of g, we have

$$\begin{split} \left\langle W^{(s)i}(z_1)W^{(s)j}(z_2) \right\rangle_{\Phi} &- \left\langle W^{(s)i}(z_1)W^{(s)j}(z_2) \right\rangle = \\ \frac{g^2}{2} \left(\int d^2 w_1 \, d^2 w_2 \left\langle W^{(s)i}(z_1) \, W^{(s)j}(z_2) \, \Phi(w_1, \bar{w}_1) \, \Phi(w_2, \bar{w}_2) \right\rangle \right. \\ &\left. - \int d^2 w_1 \, d^2 w_2 \left\langle W^{(s)i}(z_1) \, W^{(s)j}(z_2) \right\rangle \left\langle \Phi(w_1, \bar{w}_1) \, \Phi(w_2, \bar{w}_2) \right\rangle \right) + \mathcal{O}(g^3) \; . \end{split}$$

Anomalous dimensions

2pf of quasiprimary operators is of the form

$$\left\langle {\mathcal W}^{(s)i}(z_1) {\mathcal W}^{(s)j}(z_1)
ight
angle_{\Phi} \sim rac{c^{ij}}{(z_1-z_2)^{2(s+\gamma^{ij})} \; (ar z_1-ar z_2)^{2ar \gamma^{ij}}} \; ,$$

where for small γ^{ij} reads

$$pprox rac{c^{ij}}{(z_1-z_2)^{2s}} \left(1\!-\!2\gamma^{ij}\ln(z_1\!-\!z_2)\!-\!2ar\gamma^{ij}\ln(ar z_1\!-\!ar z_2)\!+\!\cdots
ight).$$

Read coefficient of the log term in perturbed 2pf.

Anomalous dimensions

To first order, γ_{ij} is given by 3 point function

$$\left\langle W^{(s)i}(z_1) \Phi(w_1, \bar{w}_1) W^{(s)j}(z_2) \right\rangle$$

which vanishes: Φ has $h_{\Phi} = \bar{h}_{\Phi} = 1$ while *W*'s have $\bar{h}_W = 0$.

Leading correction to the 2pf appears at second order:

$$\gamma^{ij} = g^2 \pi^2 \left\langle \mathcal{N}(W^{(s)i}) \ \mathcal{N}(W^{(s)j}) \right\rangle,$$

$$\mathcal{N}(W^{(s)}) \equiv \sum_{l=0}^{\lfloor s+h_{\Phi} \rfloor - 1} \frac{(-1)^{l}}{l!} (L_{-1})^{l} W^{(s)}_{-s+1+l} \Phi$$

Operator mixing

In general, matrix γ_{ij} is not diagonal: need to diagonalise it to extract anomalous dimensions.

- ▶ In general, fields within each family, $m = 2, 3, \cdots$, mix (multiplicities n(s) > 1).
- There is also mixing present between fields from different families.



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Vasiliev HS fields:

$$W^{(s)} = \sum_{q=0}^{s-2} (-1)^q \binom{s-1}{q} \binom{s-1}{q+1} \partial^{s-1-q} \overline{\phi}^1 \partial^{q+1} \phi^2,$$

$$\gamma^{ij} = g^2 \pi^2 \left\langle \mathcal{N}(W^{(s)i}) \ \mathcal{N}(W^{(s)j}) \right\rangle.$$

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The diagonal elements γ^{ii} can be computed analytically and in closed form: $r^2 - 2 \sum_{i=1}^{s} (-1)^{s-p} (2^{s}) P(r, p)$

$$\gamma^{(s)} = \frac{g^2 \pi^2 \sum_{p=0}^{s} (-1)^{s-p} {2s \choose s-p} P_2(s,p)}{(N+1) E_2(s)},$$

where

$$\begin{split} E_2(s) &= \sum_{q=0}^{s-1} \sum_{p=0}^{s-1} (-1)^{s+1+p+q} {s \choose q} {s \choose q+1} {s \choose p} {s \choose p+1} \\ &\times ((-2)_{(q)} (-2-q)_{(s-p-1)} (-2)_{(s-q-1)} (q-s-1)_{(p)}) , \\ P_2(s,p) &= \sum_{n=3/2}^{p-3/2} n(p-n) f(s,p,n) f(s,-p,n-p) \\ &+ \frac{3}{2} (-1)^{s+1} \Theta(p-2) f(s,p,1/2) f(s,-p,-1/2) (p-1/2) \\ &+ \frac{1}{2} \delta_{p,1} f(s,1,1/2) f(s,-1,-1/2) , \end{split}$$

$$f(s, p, n) = \sum_{q=0}^{s-1} (-1)^q {\binom{s}{q}} {\binom{s}{q+1}} (-1-p+n)_{(s-q-1)} (-1-n)_{(q)}.$$

Regge trajectories

- Vasiliev HS generators correspond to the leading Regge trajectory (blue diamonds); have lowest masses for a given spin.
- Cubic generators describe the first sub-leading Regge trajectory (brown circles).



Regge trajectories

- Diagonalisation of complete mixing matrix becomes complicated as spin increases: we have solved it completely for low-lying fields (X's).
- For cubic generators, we perform partial diagonalisation at larger spin where we only diagonalise γ_{ij} among the fields of m = 3.



Regge trajectories

► Diagonal entries of Regge trajectories behave as γ^(s) ≅ a log s at large spin, with dispersion relation E(s) ≅ s + a log s. This suggests that symmetric orbifold CFT is dual to an AdS₃ background with pure RR flux. [Loewy, Oz, '03; David, Sadhukhan, '14]



Summary:

- Computed anomalous dimensions of the HS generators of symmetric orbifold CFT as the string tension is switched on.
- ► HS fields of original W^(N=4)_∞ algebra form a decoupled subsector at tensionless point. As tension is switched on, they couple with stringy symmetry generators.

Future directions:

- Solve for exact anomalous dimensions for higher spins and determine shape of dispersion relations.
- Derive anomalous diemensions for symmetric product orbifold of K3.
 [Baggio, Gaberdiel, and Peng, '15]
- ► Compute the anomalous dimensions from the dual AdS viewpoint.