The Quantum Critical Higgs

John Terning

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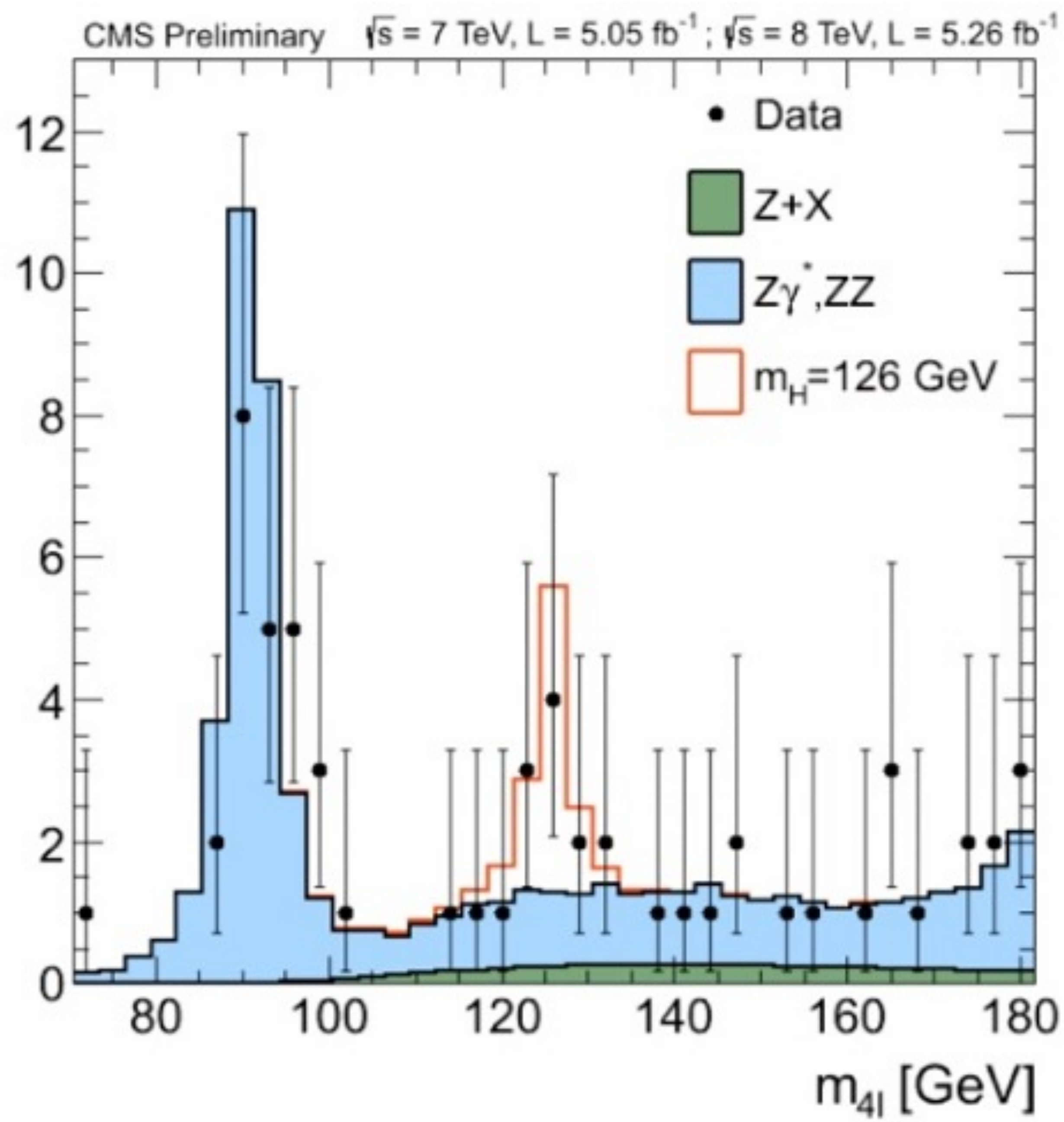
Jay Hubisz, Seung J. Lee, Javi Serra

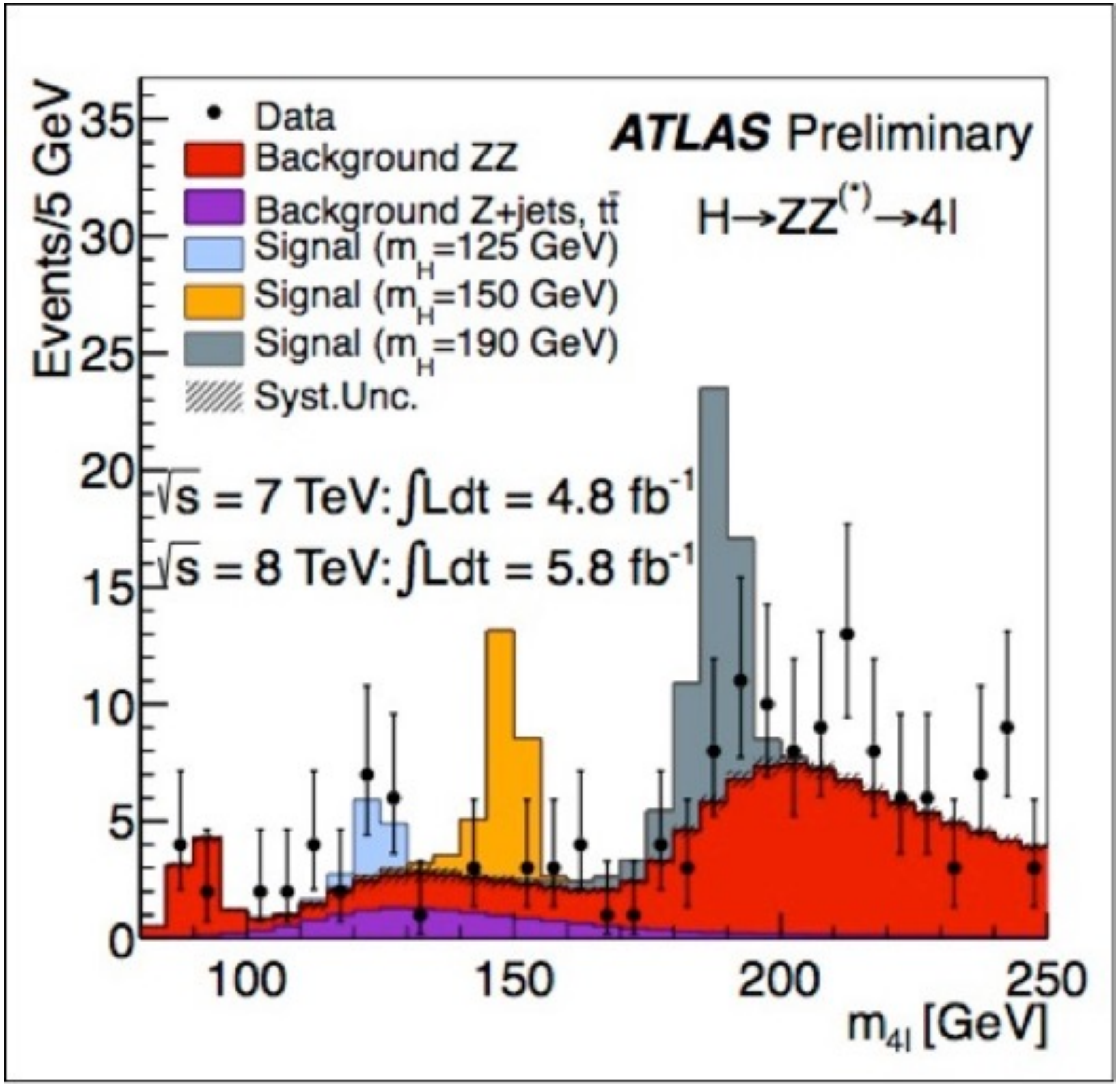
hep-ph/1511.????

Outline

* Motivation
* AdS/CFT/unparticle correspondence
* effective action for quantum critical Higgs
* quasi-local gauge interactions
* LHC measurements

Higgs-like Resonance



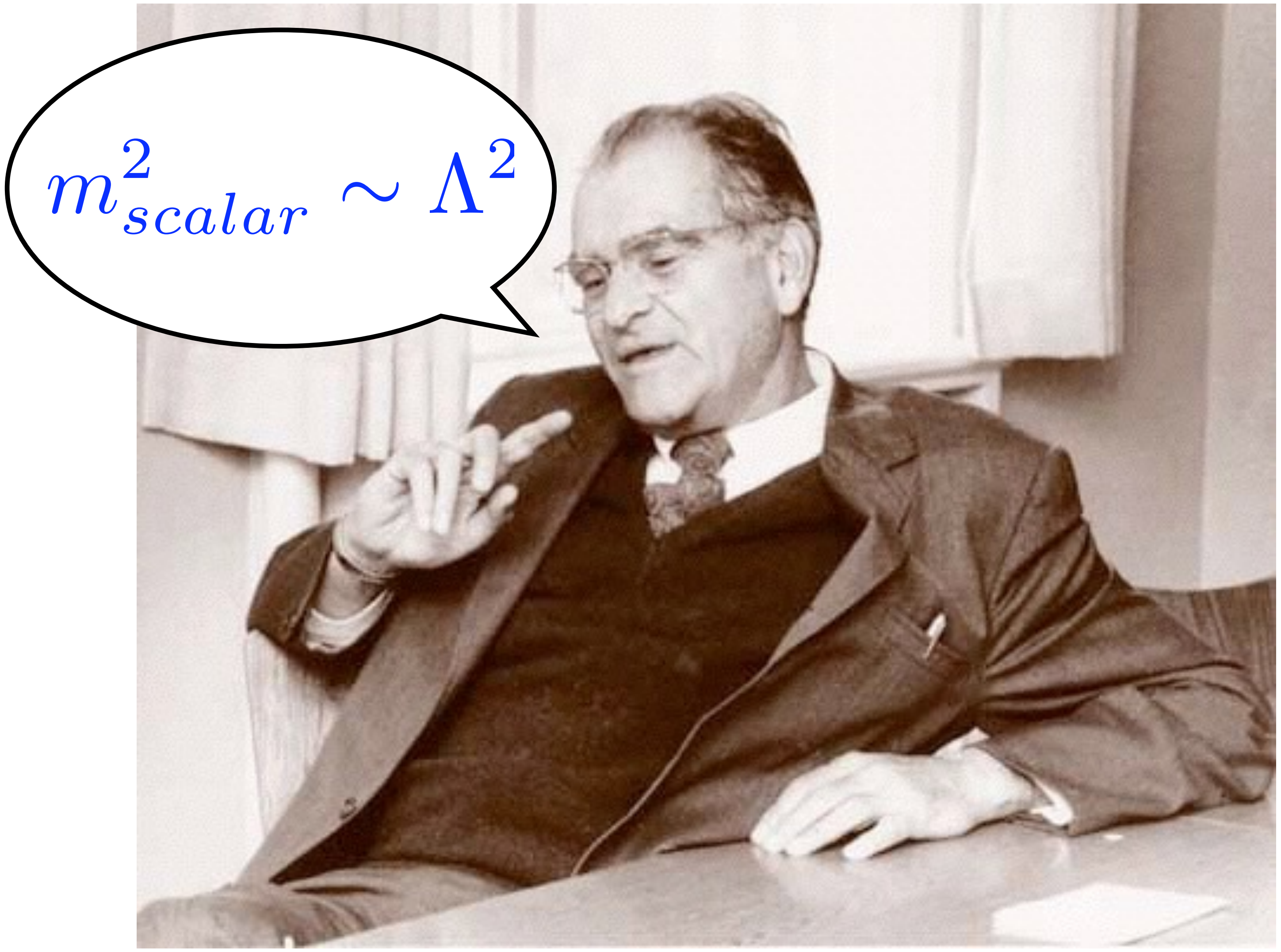


finally something really new!

What's the problem?

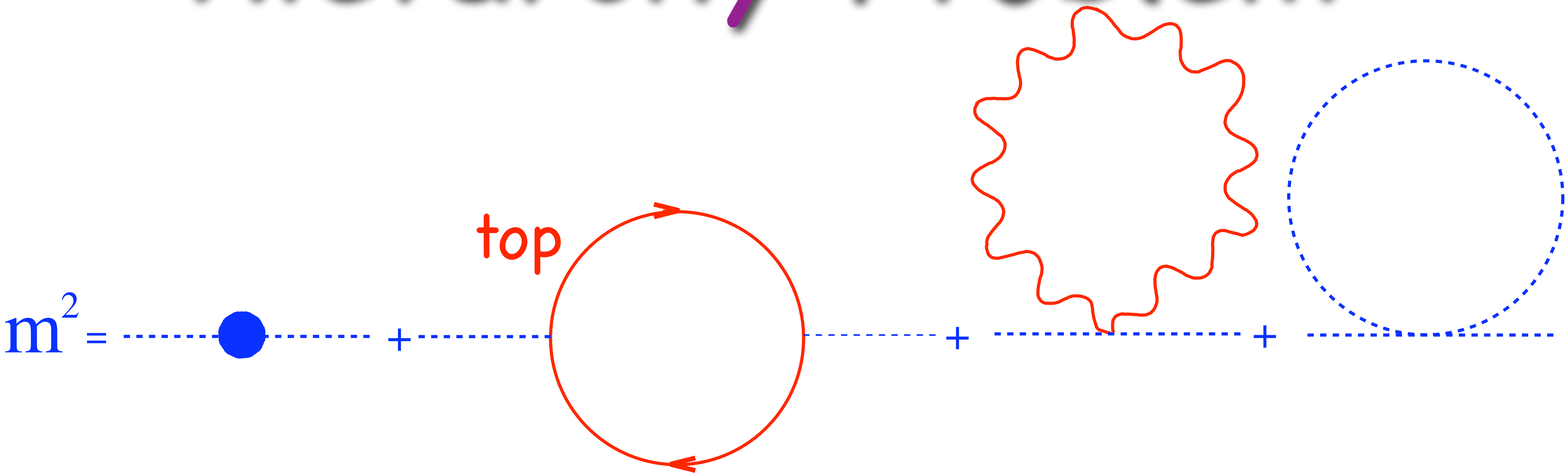
Weisskopf Phys. Rev.56 (1939) 72

What's the problem?

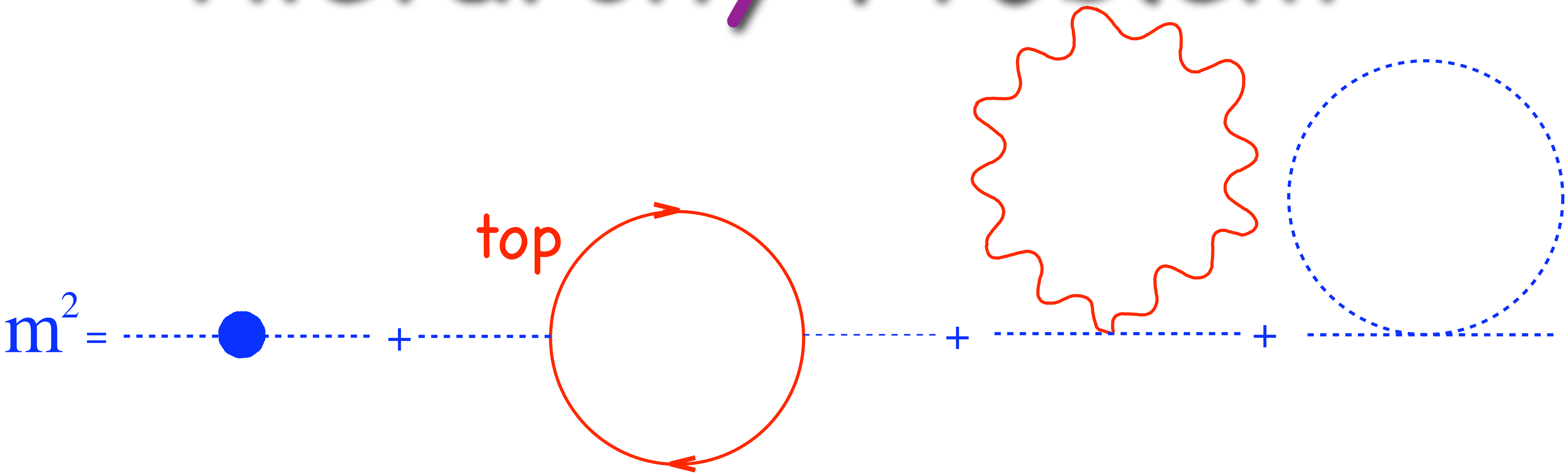


Weisskopf Phys. Rev.56 (1939) 72

Hierarchy Problem

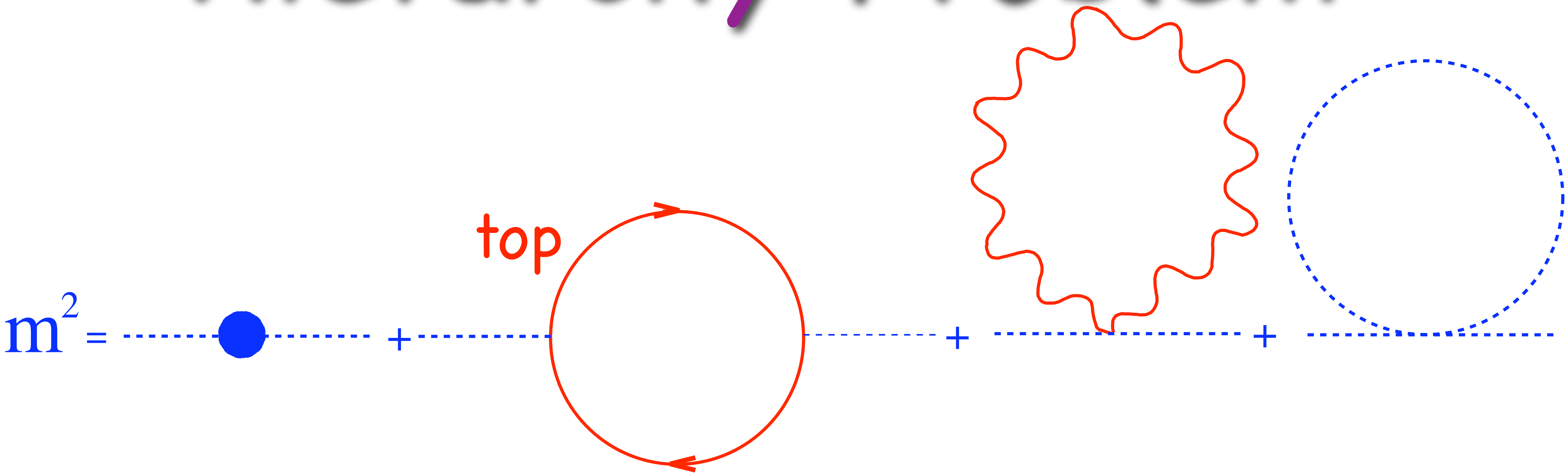


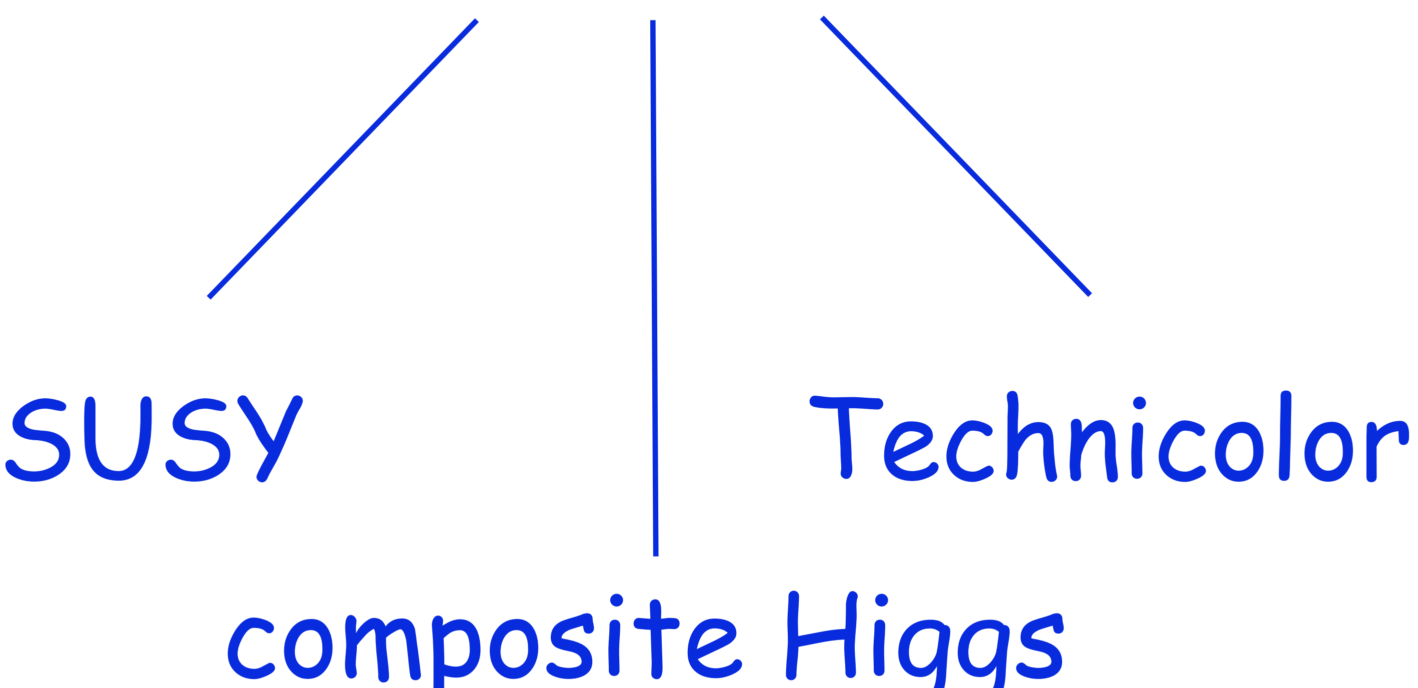
Hierarchy Problem



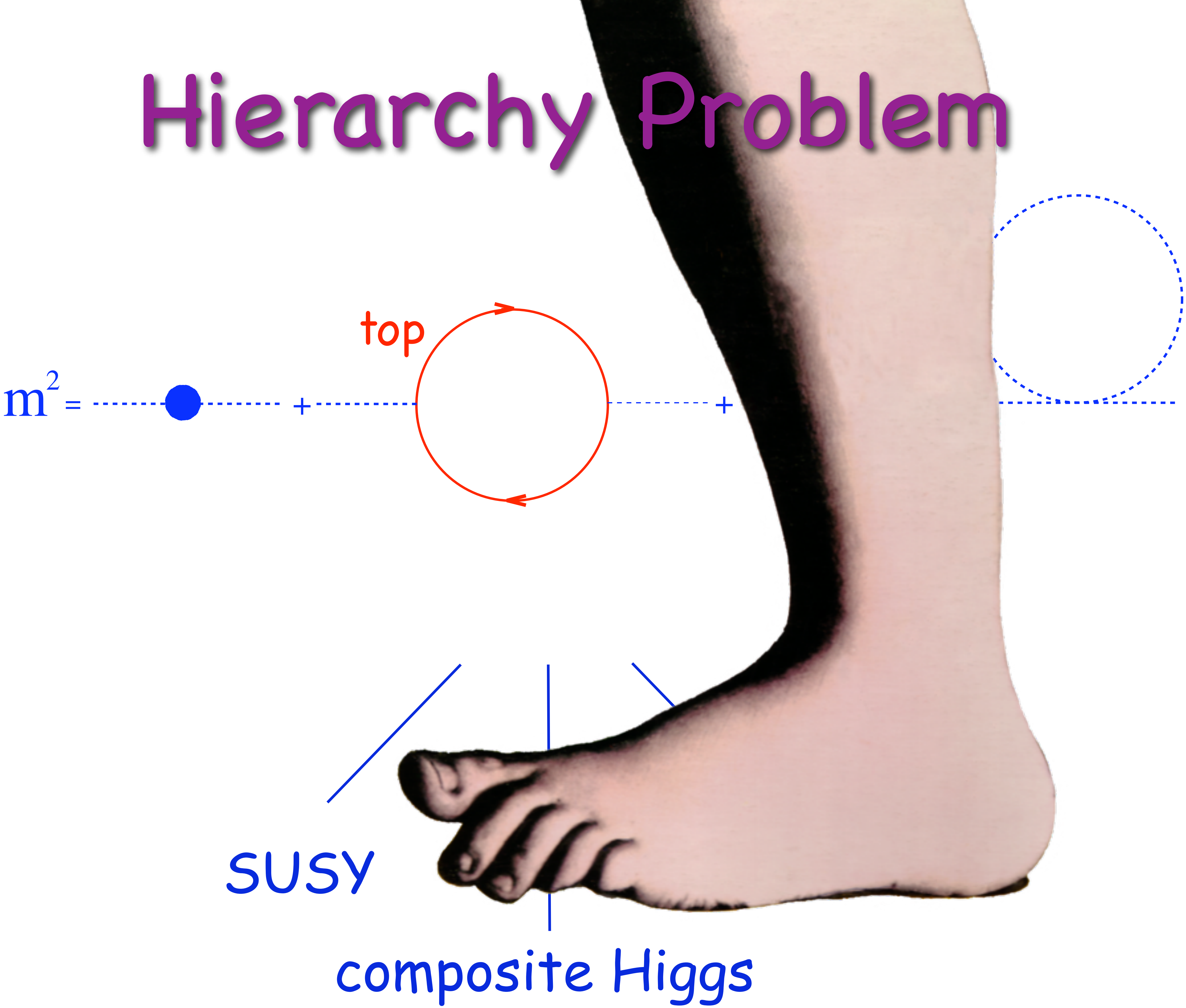
$$\left(\frac{125}{\sqrt{2}}\right)^2 = \begin{array}{l} 16419971512763993607881093447038089115 \\ -19402031160008016677277886179991476752 \\ +2441281099066559954943818225739637142 \\ +540778548177463114452974507213751495 \end{array}$$

Hierarchy Problem





Hierarchy Problem



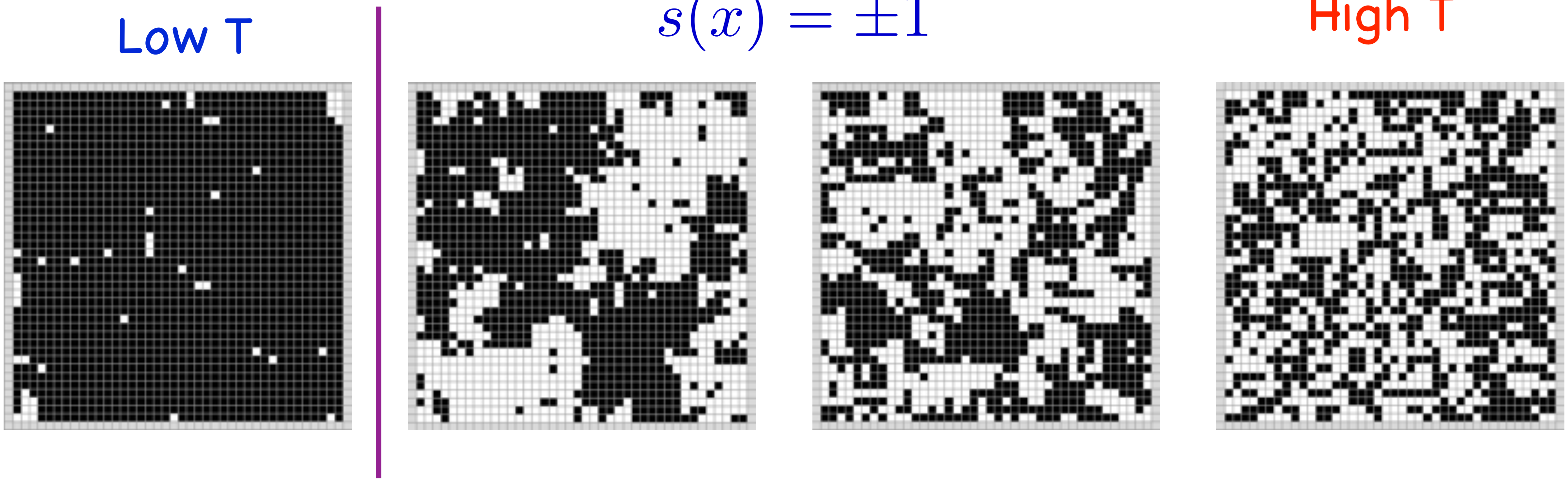
Ising Model

$$H = -J \sum s(x)s(x+n)$$

$$s(x) = \pm 1$$

Low T

High T

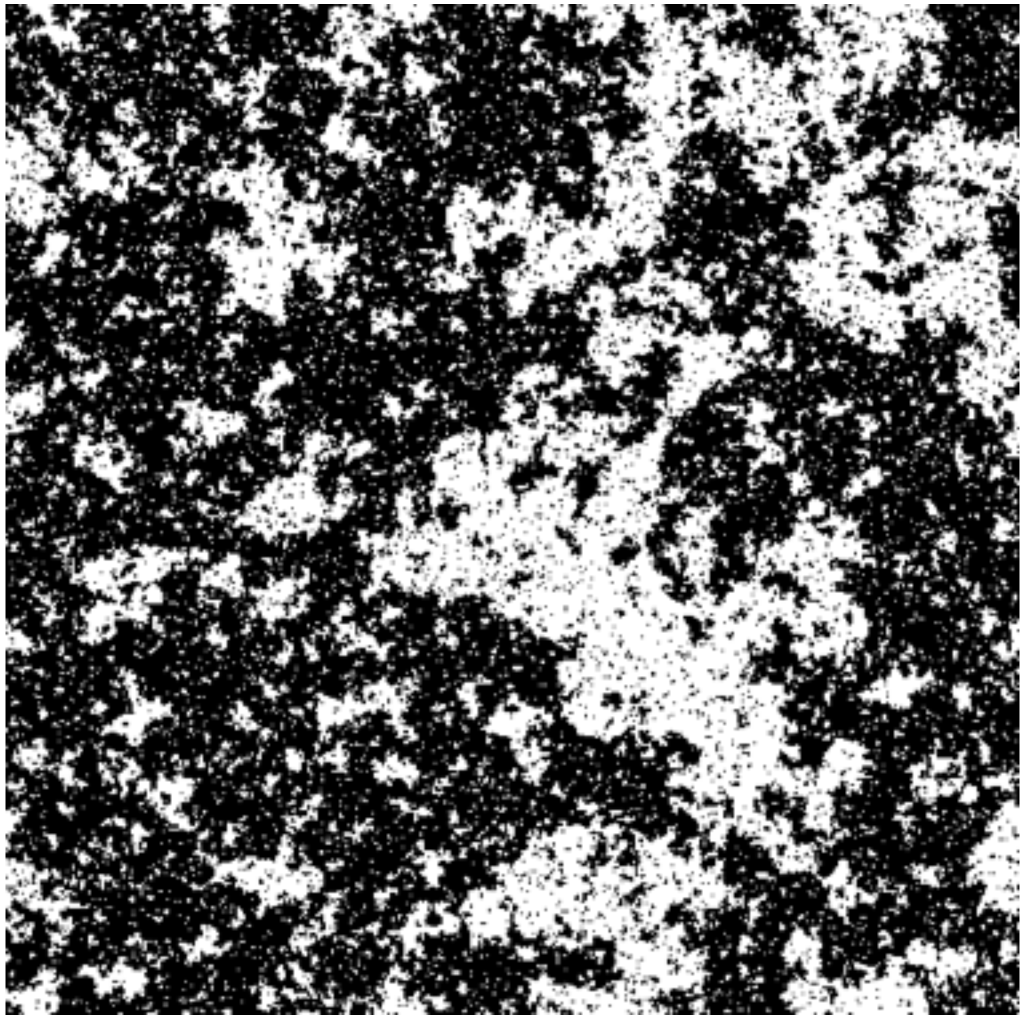


T_c

$$\langle s(0)s(x) \rangle = e^{-|x|/\xi}$$

at $T=T_c$ $\quad \xi \to \infty$

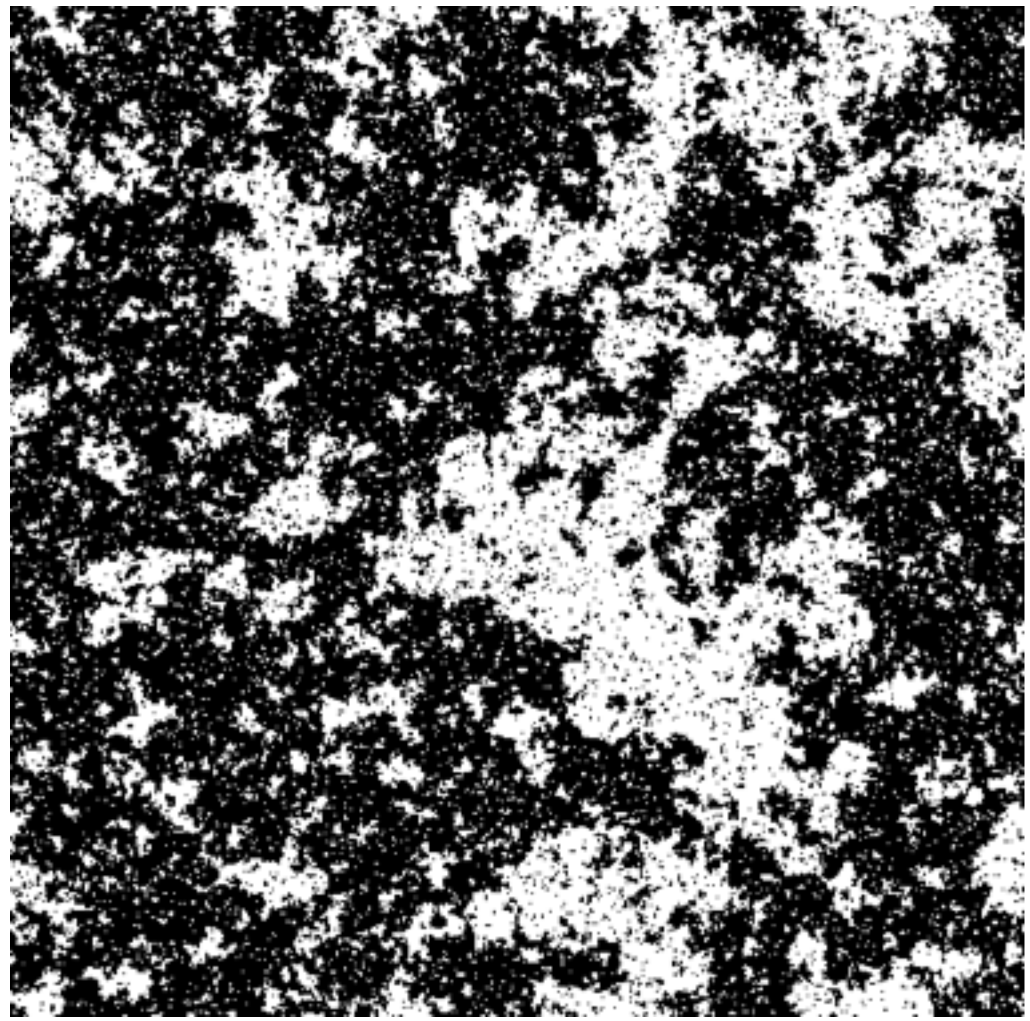
Critical Ising Model is Scale Invariant



http://bit.ly/2Dcrit

at $T=T_c$ $\quad \langle s(0)s(x)\rangle \propto \dfrac{1}{|x|^{2\Delta-1}}$

Critical Ising Model is Scale Invariant

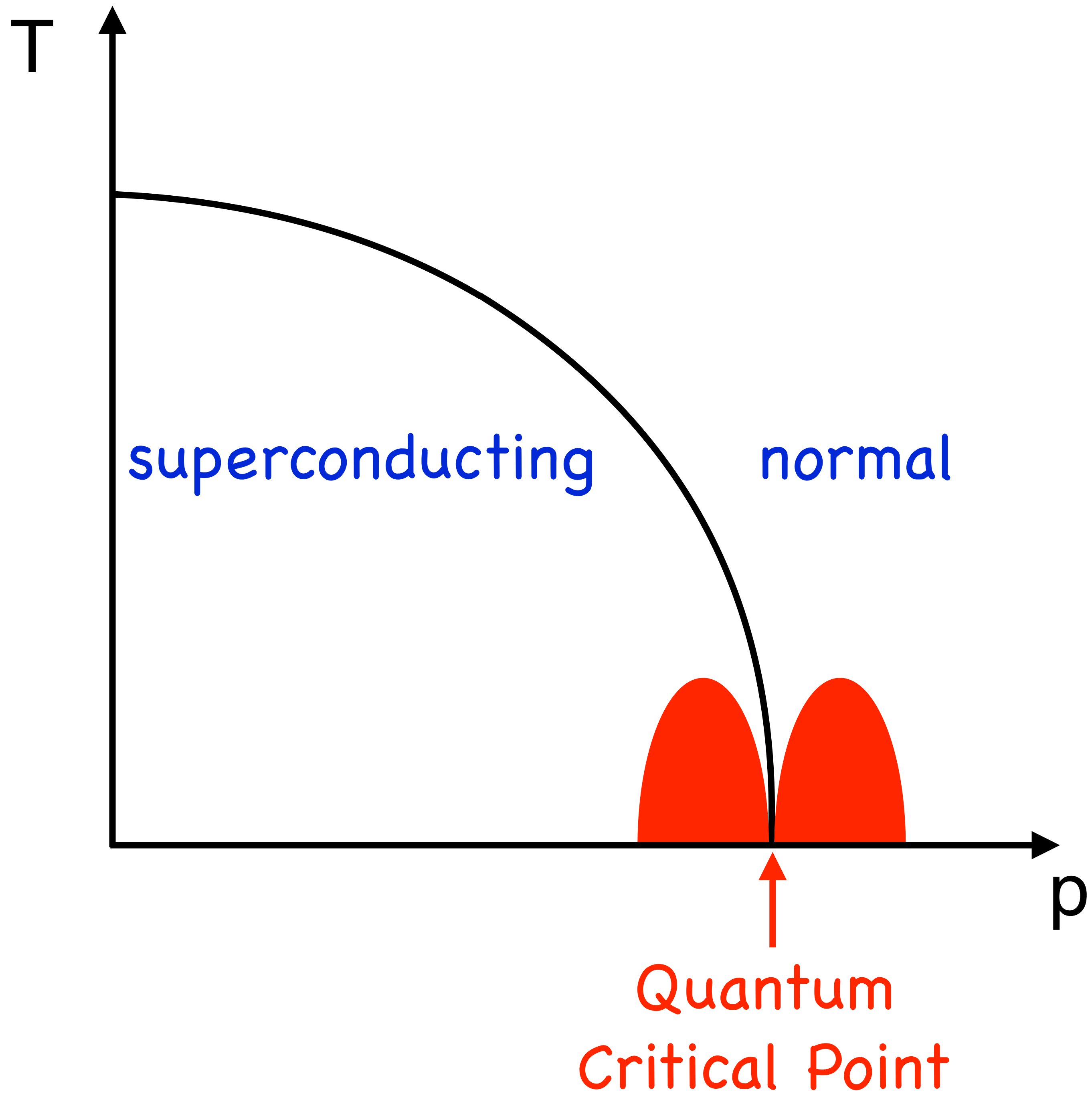


http://bit.ly/2Dcrit

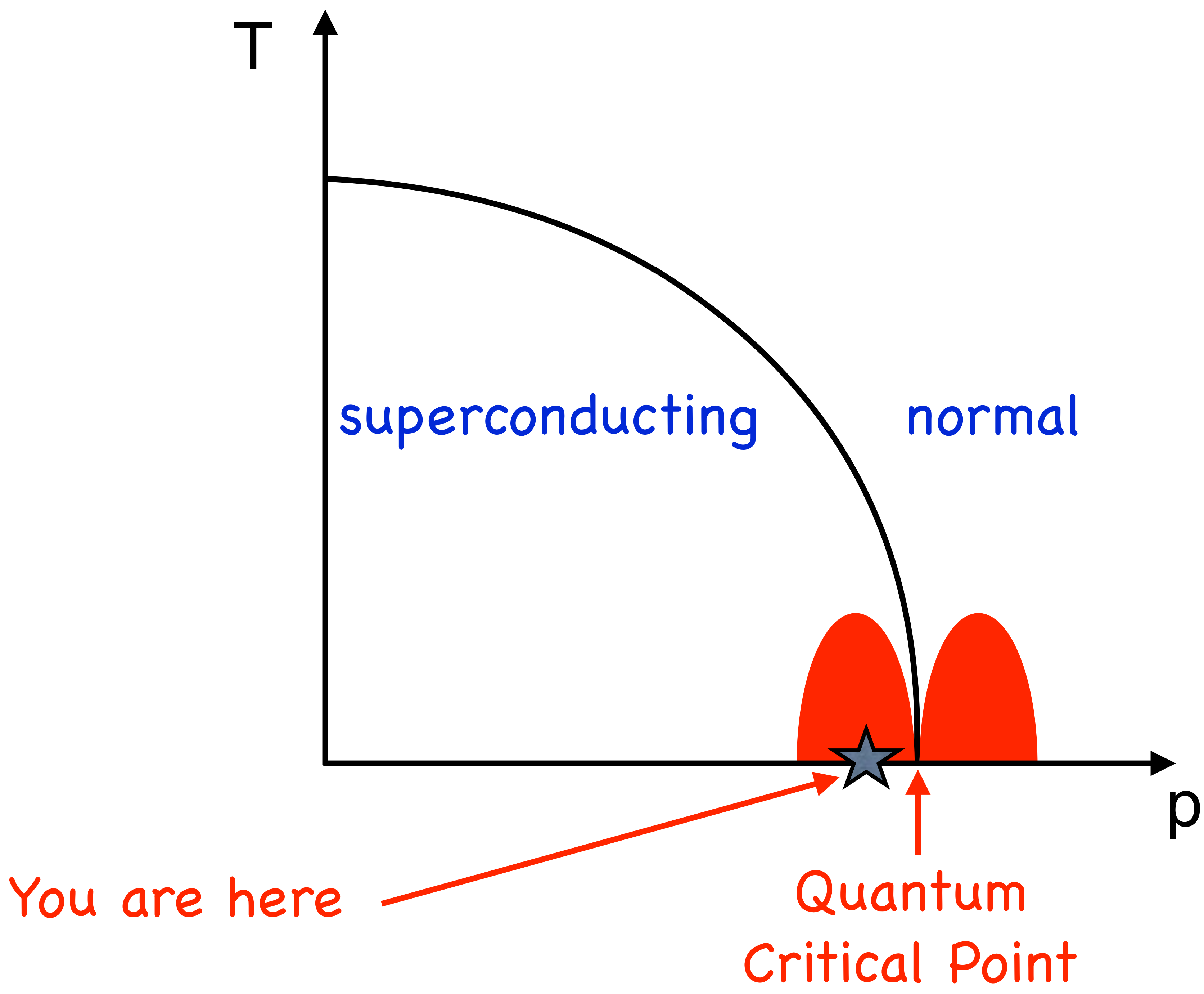
at T=T$_c$ $\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}} = \int d^3p \, \frac{e^{ip\cdot x}}{|p|^{4-2\Delta}}$

critical exponent

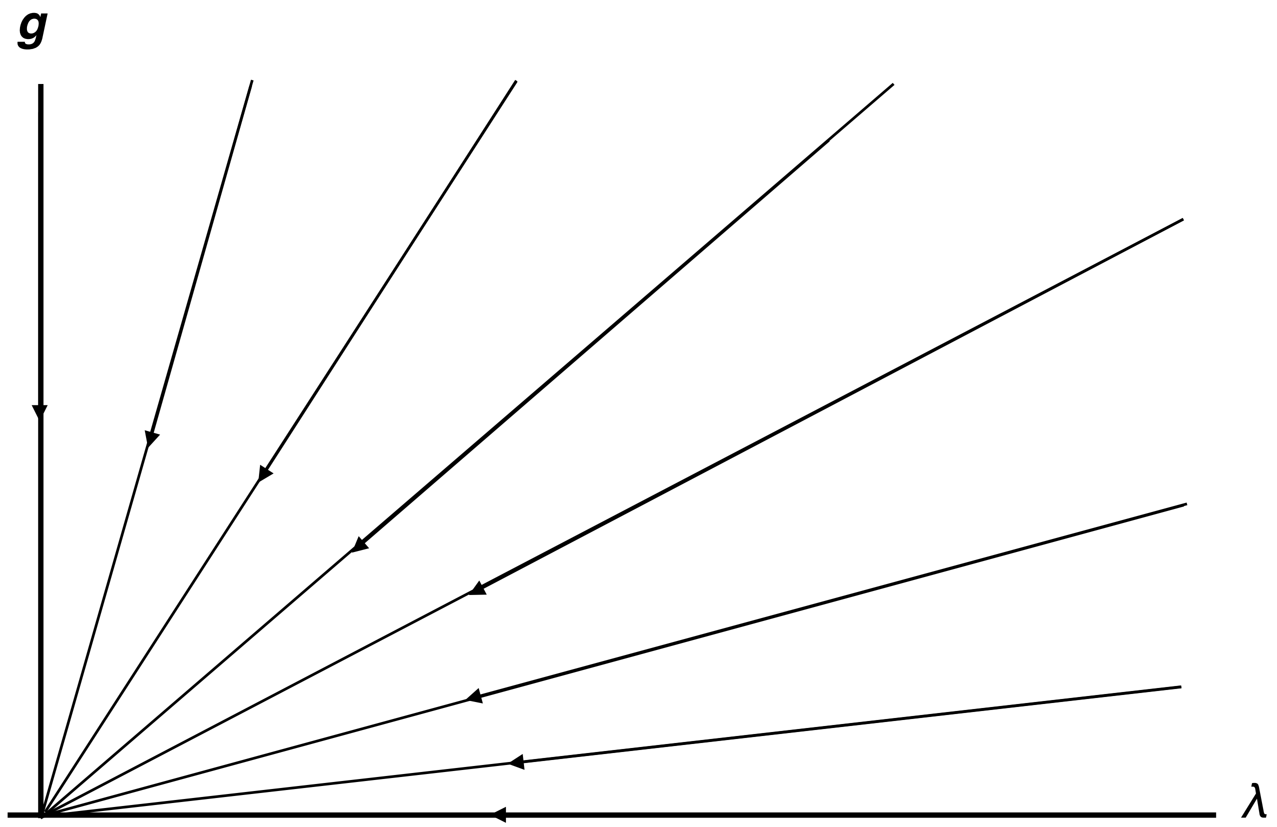
Quantum Phase Transition

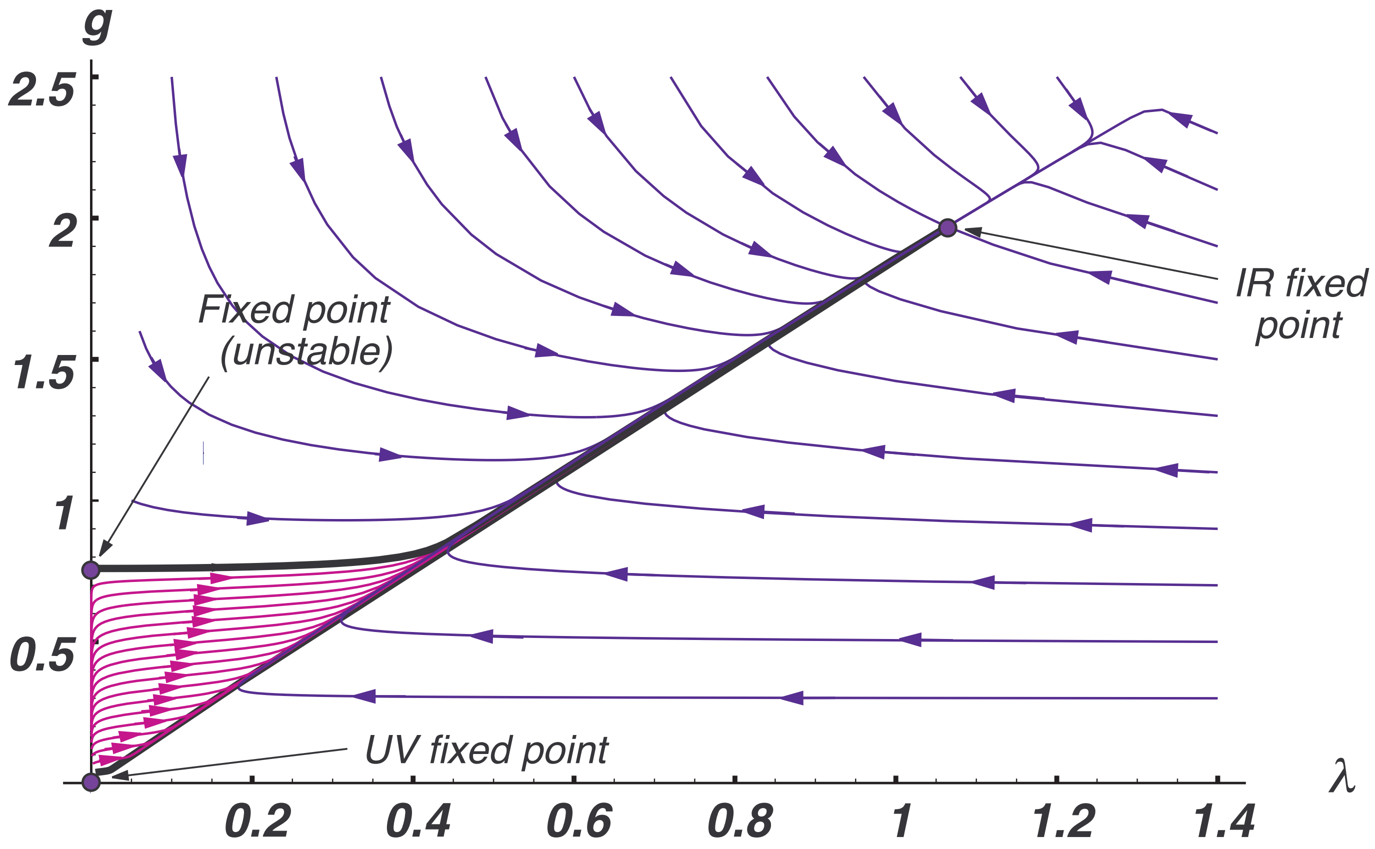


Quantum Phase Transition



Quantum Critical Point





arbitrarily long RG flow

AdS/CFT

$$\langle e^{\int d^4x\, \phi_0(x)\mathcal{O}(x)} \rangle_{\rm CFT} \approx e^{-S_{\rm 5Dgrav}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2}\left(dx^2 - dz^2\right)$$

$\mathcal{O} \subset$ CFT $\leftrightarrow \phi$ AdS$_5$ field, $\phi_0(x)$ is boundary value

AdS/CFT

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)$$

$$z > \epsilon$$

$$S_{bulk} = \frac{1}{2}\int d^4x\, dz \sqrt{g}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + m^2\phi^2)$$

$$\phi(p,z) = az^2 J_\nu(pz) + bz^2 J_{-\nu}(pz)$$

$$\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2 R^2}$$

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\,\phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\,dz\,\partial_z\left(\frac{R^3}{z^3}\phi\partial_z\phi\right)$$

surface term

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\, \phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3} \phi \partial_z \phi \right)$$

surface term

$$S = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \phi_0(-p)\phi_0(p) K(p)$$

$$K(p) = (2-\nu)\epsilon^{-2\nu} + b\, p^{2\nu} + c\, p^2 \epsilon^{2-2\nu} + \ldots$$

$$K(p) = G(p) \qquad \Delta = 2 + \nu$$

unparticle propagator

$$G(p) \equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle$$

unparticle propagator

$$G(p) \equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle$$

$$= \frac{A_d}{2\pi}\int_0^\infty (M^2)^{\Delta - 2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2$$

spectral density

unparticle propagator

$$
\begin{aligned}
G(p) &\equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle \\
&= \frac{A_d}{2\pi}\int_0^\infty (M^2)^{\Delta-2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2 \\
&= i\frac{A_d}{2} \frac{\left(-p^2 - i\epsilon\right)^{\Delta-2}}{\sin d\pi}
\end{aligned}
$$

spectral density

$$
A_d = \frac{16\pi^{5/2}}{(2\pi)^{2\Delta}} \frac{\Gamma(\Delta + 1/2)}{\Gamma(\Delta - 1)\Gamma(2\Delta)}
$$

Legendre Transform

$$\Delta = 2 - \nu$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)K\phi_0(p) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(p)A(p)$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

A is the source

$K(p)^{-1} = G(p)$ ϕ_0 is the field

Klebanov, Witten hep-th/9905104

Legendre Transform

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$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

$$\langle \mathcal{O}(p')\mathcal{O}(p)\rangle \quad \propto \quad \frac{\delta^2 S'}{\delta A(p')\,\delta A(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4}\,(p^2)^{\Delta-2}$$

Klebanov, Witten hep-th/9905104

AdS/CFT/Un Dictionary

“Georgi” “string theorist”

AdS/CFT/Un Dictionary

"Georgi" "string theorist"

moose

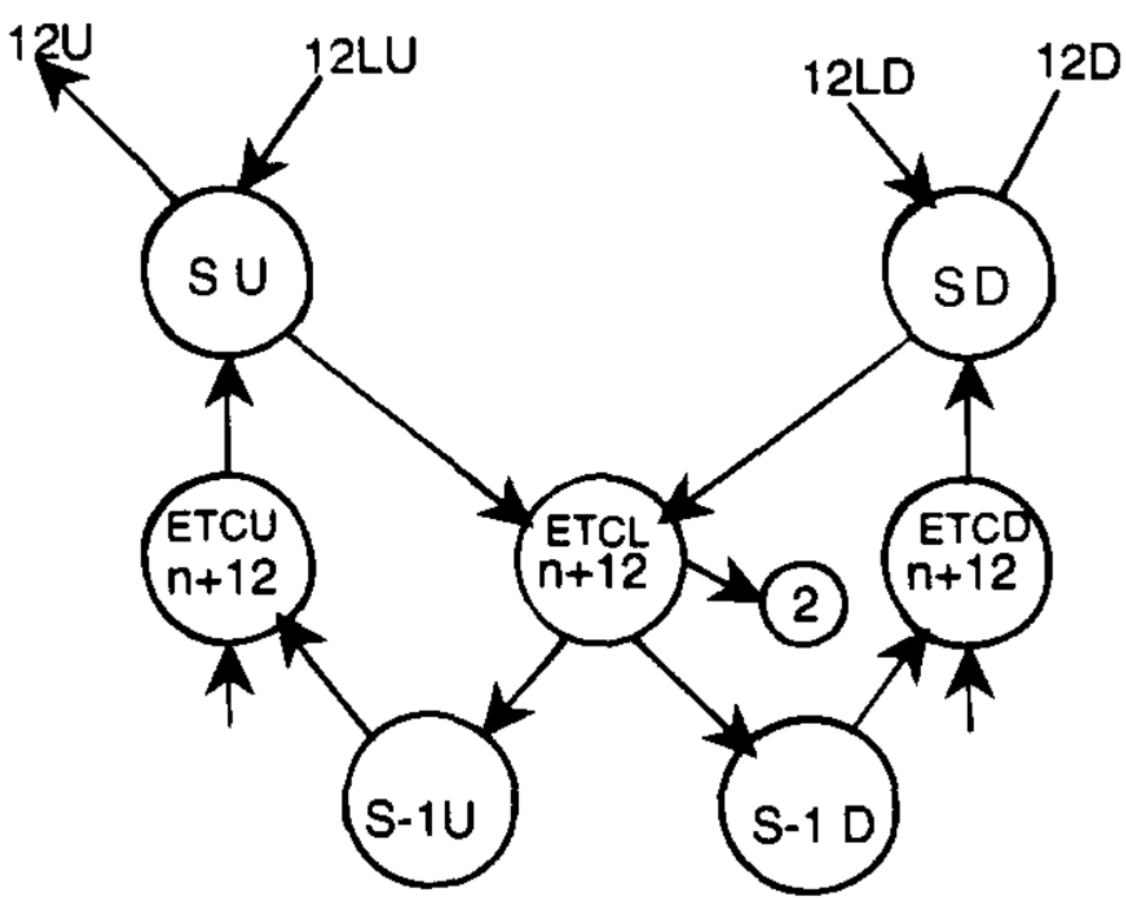
AdS/CFT/Un Dictionary

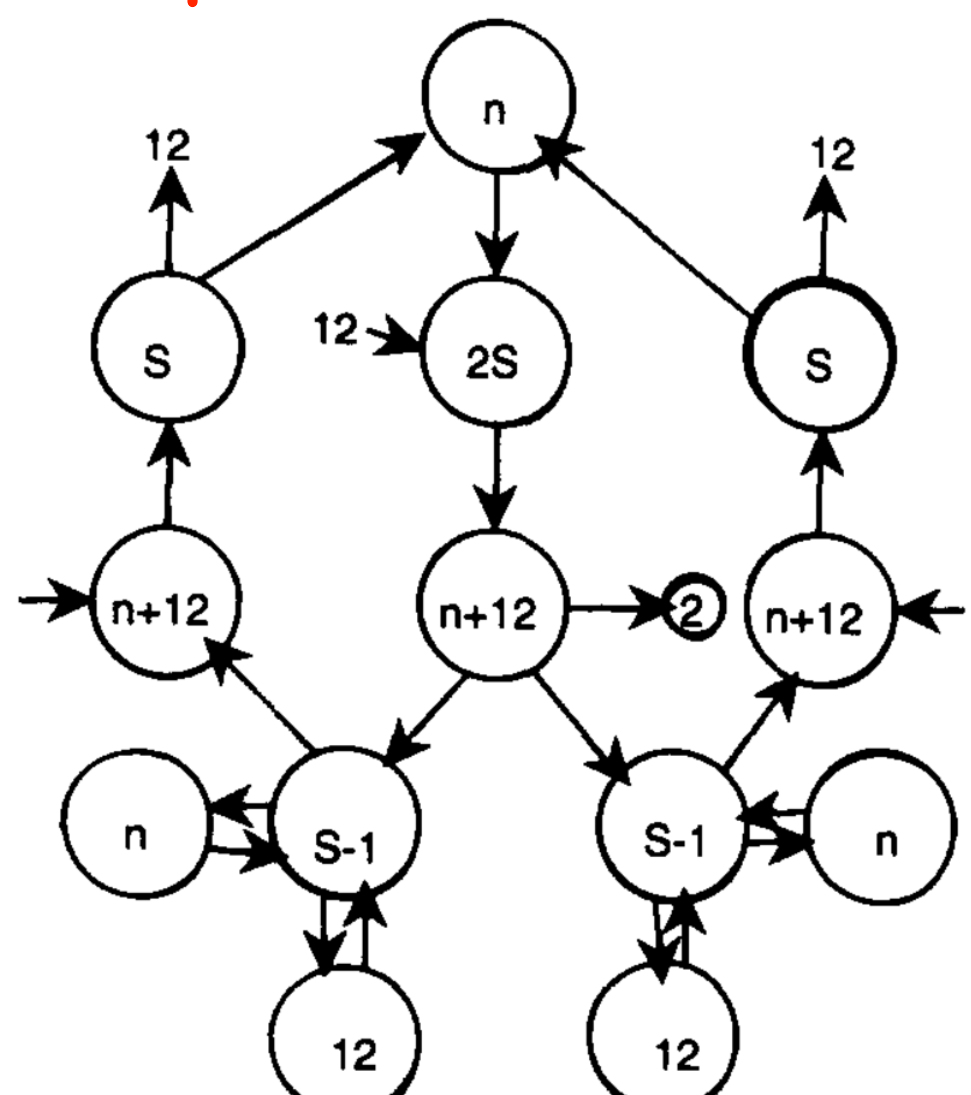
"Georgi"	"string theorist"
moose	quiver model

AdS/CFT/Un Dictionary

"Georgi" — moose

"string theorist" — quiver model





AdS/CFT/Un Dictionary

"Georgi"	"string theorist"
moose	quiver model
	double trace perturbation

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AdS/CFT/Un Dictionary

"Georgi"	"string theorist"
moose	quiver model
mass term	double trace perturbation
unparticle	state created by a CFT operator
unparticle action	Legendre transform of a holographic boundary action

Why (broken) CFT's are Interesting

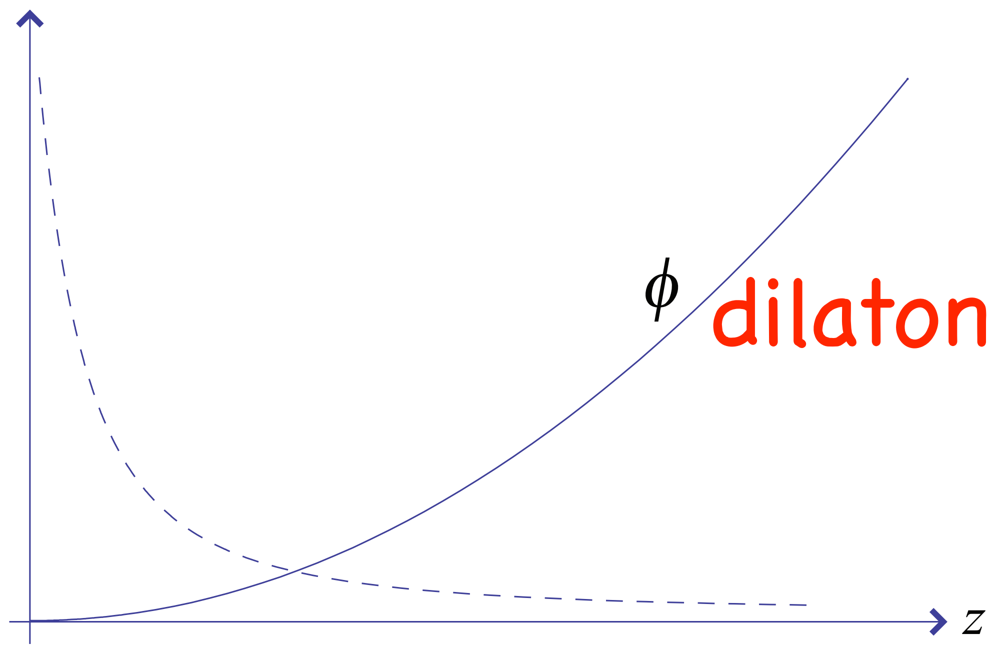
pure unparticles are equivalent to RS2

IR cutoff at TeV turns RS2 into RS1

IR brane cutoff is one type of scale breaking

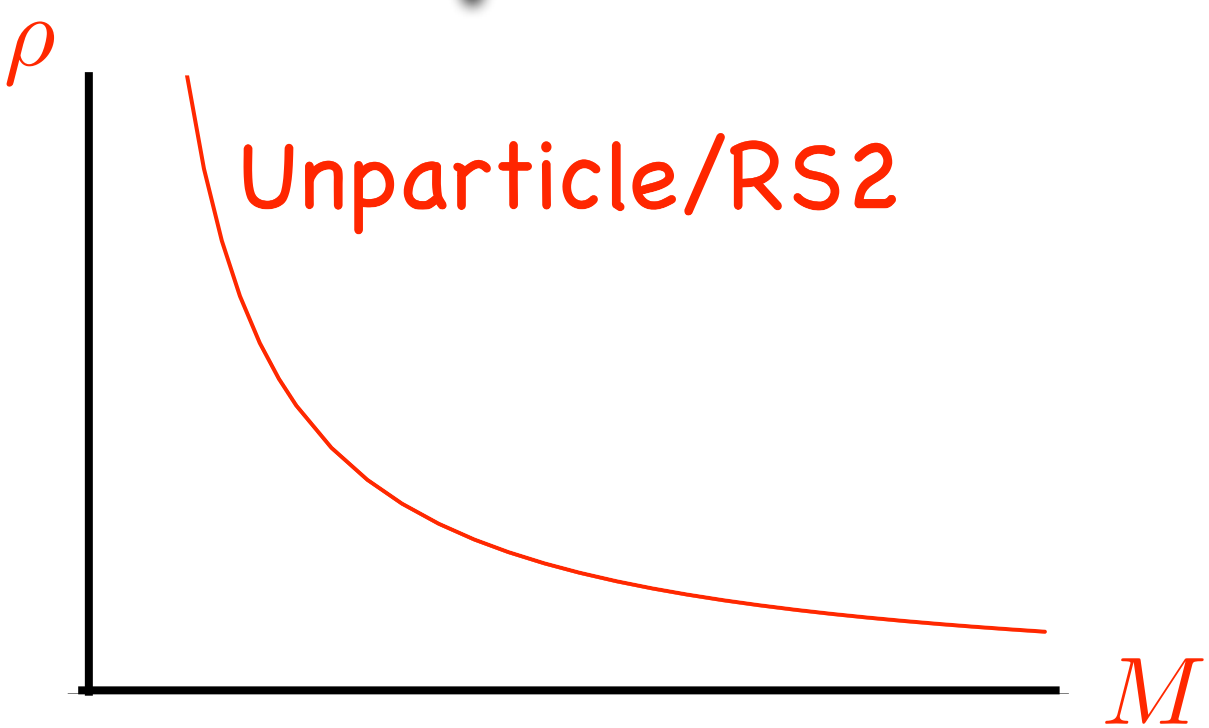
a new type of IR cutoff will lead to new LHC phenomenology

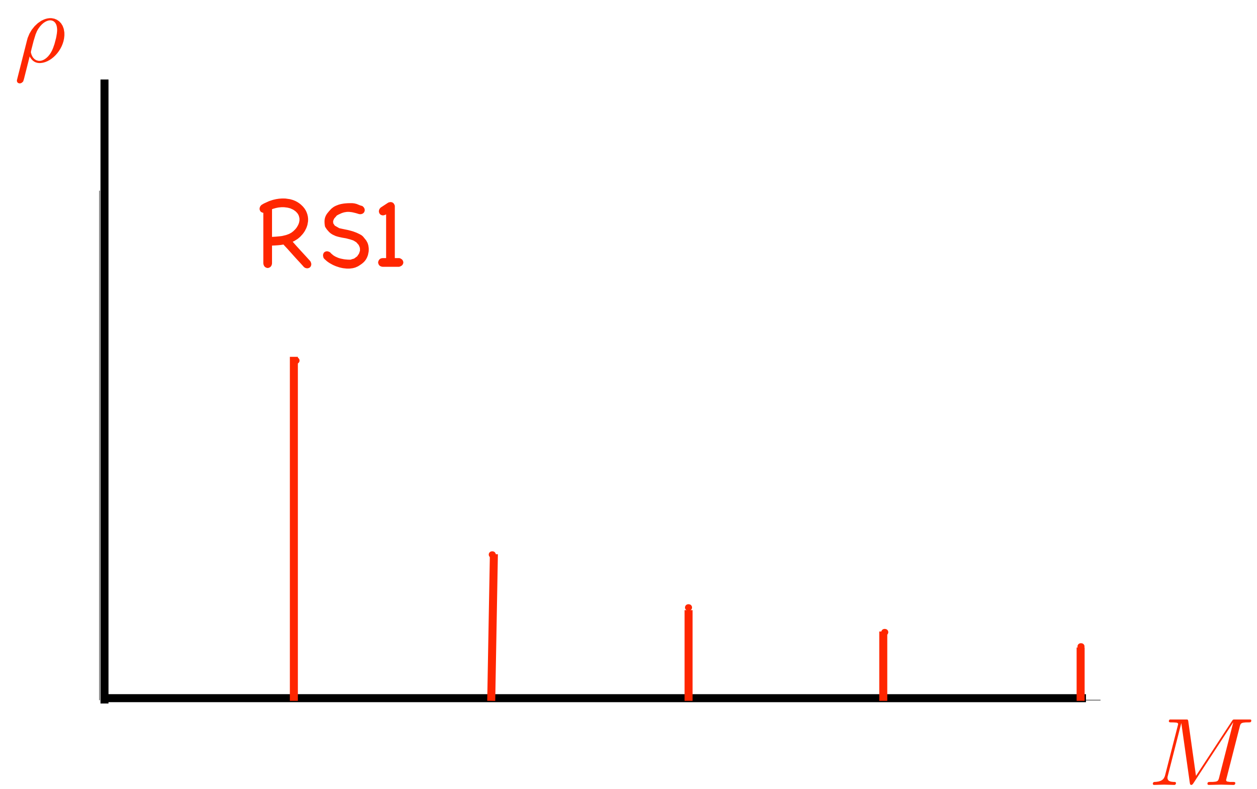
Soft-Wall



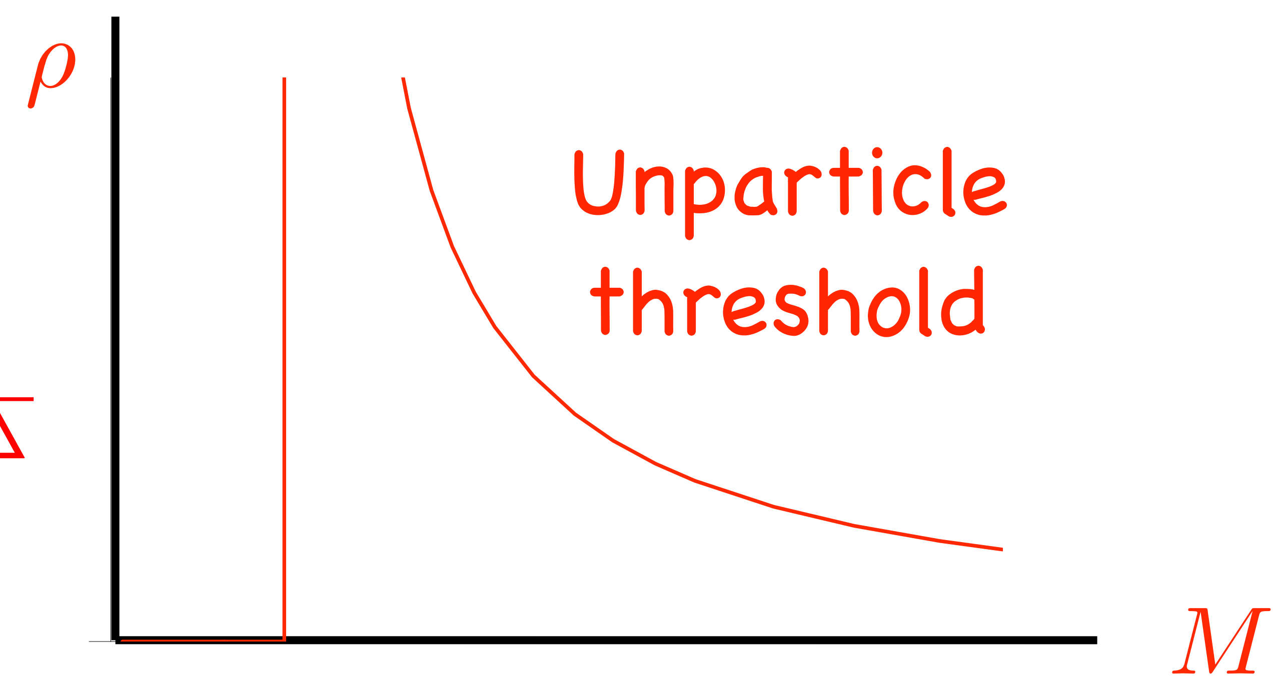
Karch, Katz, Son, Stephanov hep-ph/0602229
Gherghetta, Batell hep-th/0801.4383

Spectral Densities





$$G(p) \quad = \quad -\frac{i}{(\mu^2 - p^2 - i\epsilon)^{2-\Delta}}$$



Effective Action

$$S = \int \frac{d^4p}{(2\pi)^4}\, \mathcal{H}^\dagger(p) \left[\mu^2 - p^2\right]^{2-\Delta} \mathcal{H}(p)$$

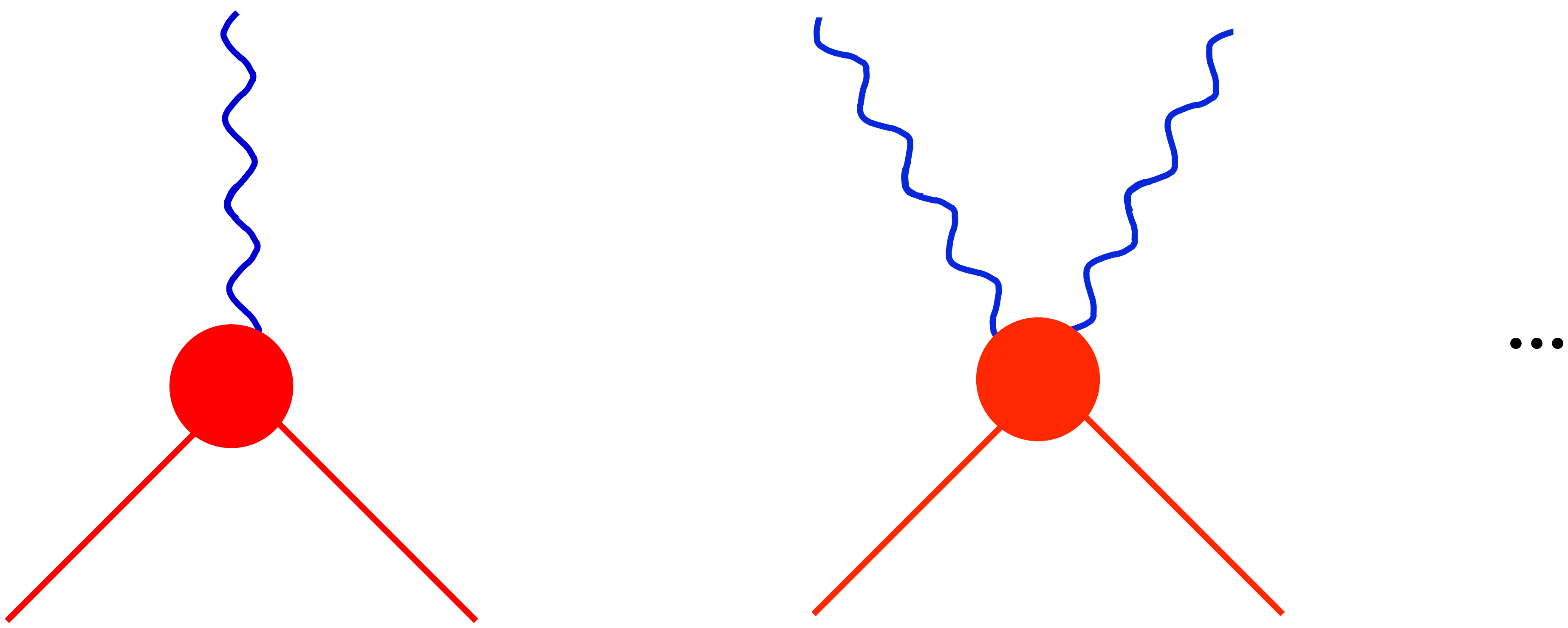
$$S = \int d^4x d^4y\, \mathcal{H}^\dagger(x) F(x-y) \mathcal{H}(y)$$

$$F(x-y) = \left[\partial^2 - \mu^2\right]^{2-\Delta} \delta(x-y)$$

Minimal Gauge Coupling

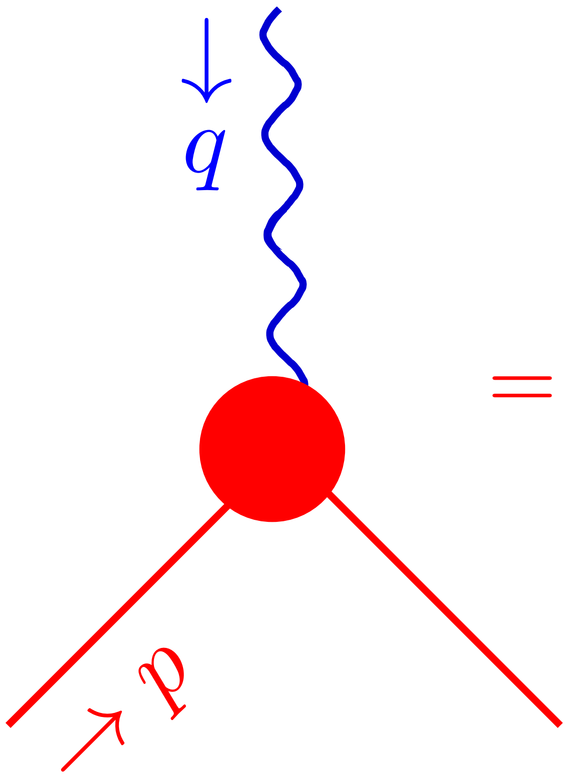
$$F(x-y) \rightarrow F(x-y)W(x,y)$$

$$W(x,y) = P\exp\left[-igT^a \int_x^y A_\mu^a dw^\mu\right]$$



cf Mandelstam Ann Phys **19** (1962) 1

Gauge Vertex



$$= \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

Ward-Takahashi Identity

$$ig\Gamma^{a\alpha}(p,q) \;=\; \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

$$iq_\mu\Gamma^{a\mu} = G^{-1}(p+q)T^a - T^a G^{-1}(p)$$

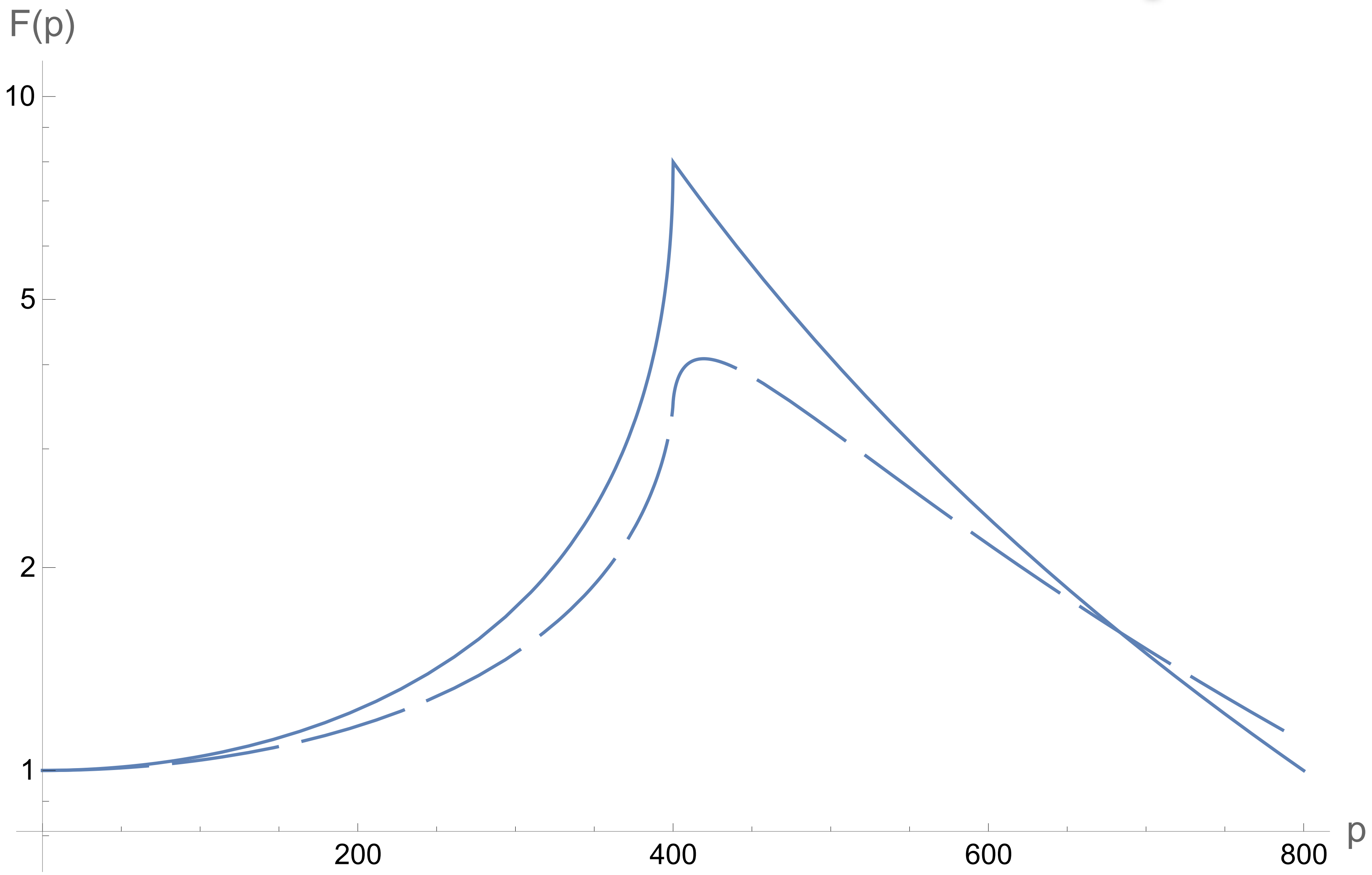
Higher Dimension Operator

$$H^\dagger F^a_{\alpha\beta} F^{b\alpha\beta} H$$

$$\mathcal{M} = \{T^a, T^b\} \left(g^{\alpha\beta} p_1 \cdot p_2 - p_1^\beta p_2^\alpha \right) F^{ab}_{VVh} \ ,$$

$$F^{ab}_{VVh} \propto \tilde{v}^\Delta g_5^2 \int_R^\infty dz \, z^3 \, \frac{K_{2-\Delta}(\sqrt{\mu^2 - (p_1+p_2)^2} \, z) K_{2-\Delta}(\mu \, z)}{K_{2-\Delta}(\sqrt{\mu^2 - (p_1+p_2)^2} \, R) K_{2-\Delta}(\mu \, R)} \ ,$$

Higher Dimension Operator



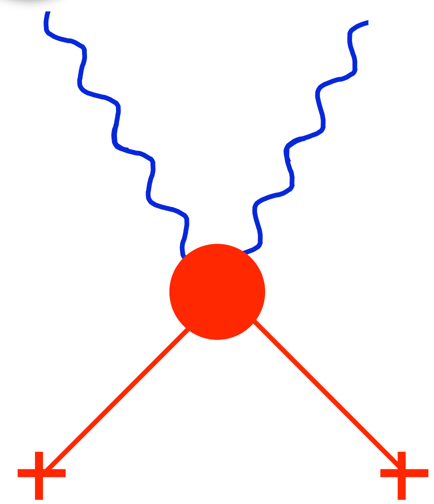
QC Higgs Model

$$G = \frac{-i}{(\mu^2 - p^2)^{2-\Delta} + m^{4-2\Delta}}$$

minimal parameterization requires
two mass scales: pole and cut threshold

approach the SM in two limits: $\Delta \to 1$ or $\mu \to \infty$

QC Higgs and M_W

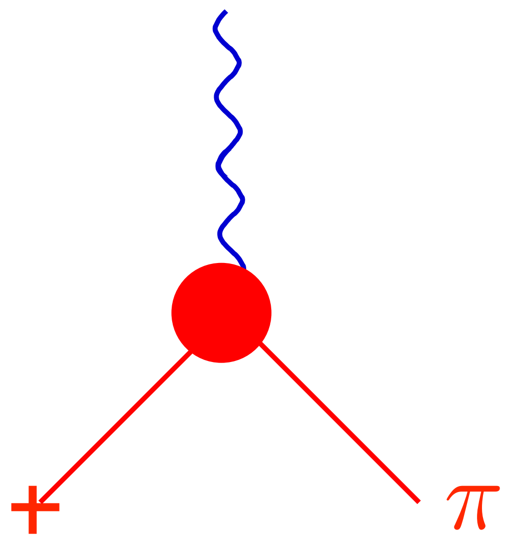


$$-g^2 A^a_\alpha A^b_\beta \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \left\{ \; g^{\alpha\beta} (\Delta - 2) \mu^{2-2\Delta} \right.$$

$$\left. - \frac{q^\alpha q^\beta}{q^2} \left[(\Delta - 2) \mu^{2-2\Delta} - \frac{\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}}{q^2} \right] \right\}$$

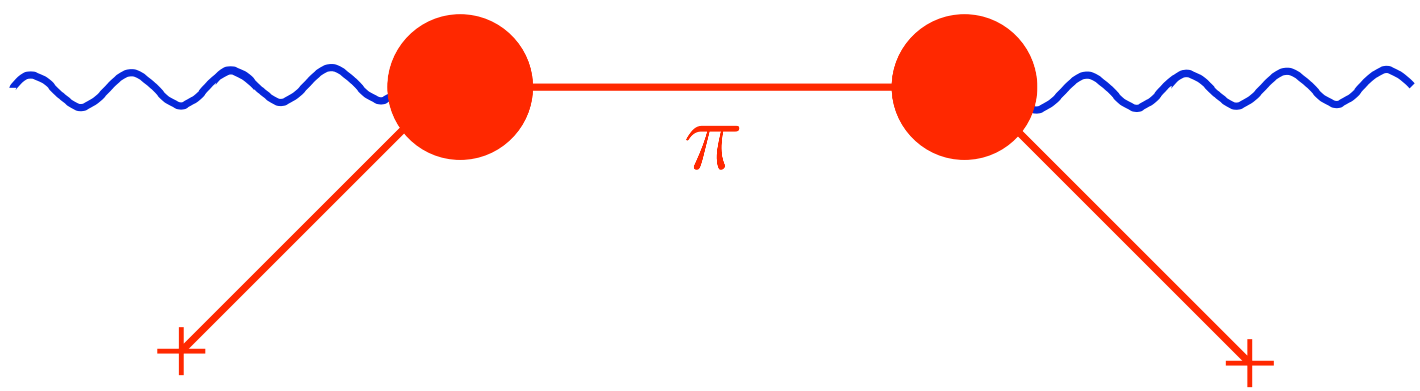
$$M_W^2 \quad = \quad \frac{g^2 (2 - \Delta) \mu^{2-2\Delta} v^{2\Delta}}{4}$$

GB mixing



$$g\left(\langle\mathcal{H}^\dagger\rangle A^a_\alpha T^a \Pi - \Pi^\dagger A^a_\alpha T^a \langle\mathcal{H}\rangle\right)\left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}\right] q^\alpha / q^2$$

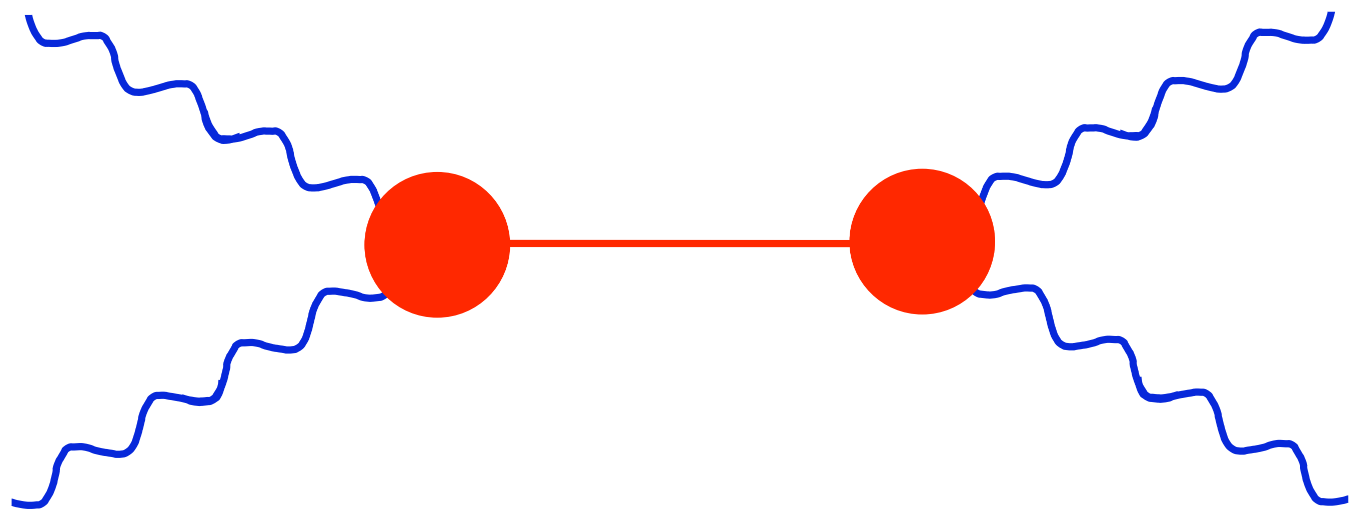
Gauge invariance is maintained



$$\Pi^{ab\alpha\beta}(q) = -g^2 \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \frac{q^\alpha q^\beta}{q^4} \times \left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta} \right]^2 G_{GB}(q)$$

$$G_{GB}(q) = -\frac{i}{\left(\mu^2 - q^2 - i\epsilon\right)^{2-\Delta} - \mu^{4-2\Delta}}$$

WW Scattering

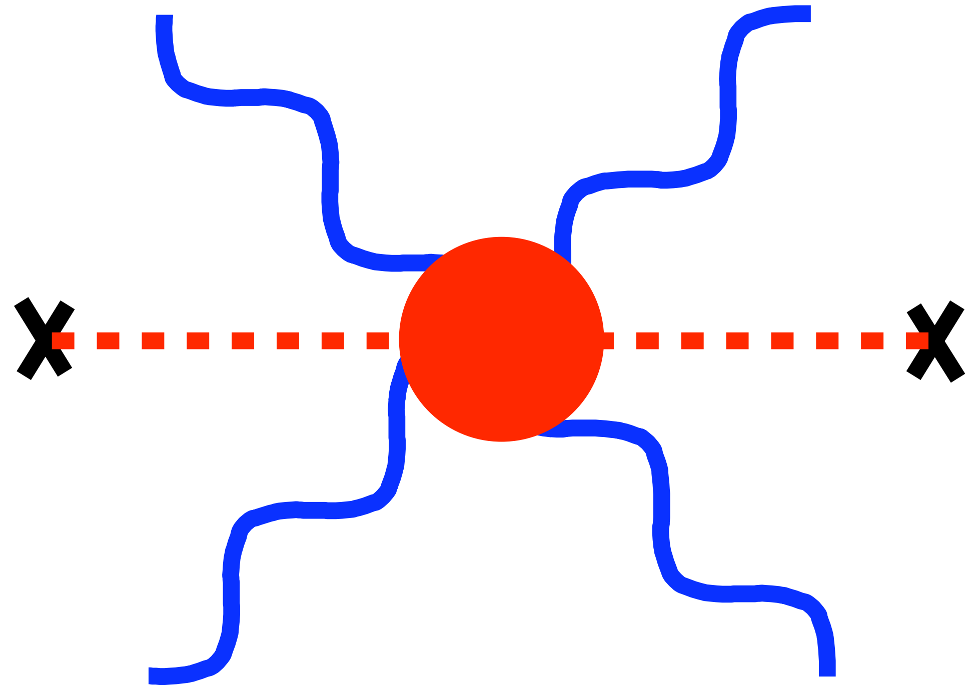


at large s

$$\mathcal{M}_h = -i\frac{g^4}{4M_W^2(2-\Delta)\mu^{2-2\Delta}}(-s)^{2-\Delta}$$

QC Higgs exchange is insufficient
to unitarize WW scattering

WW Scattering

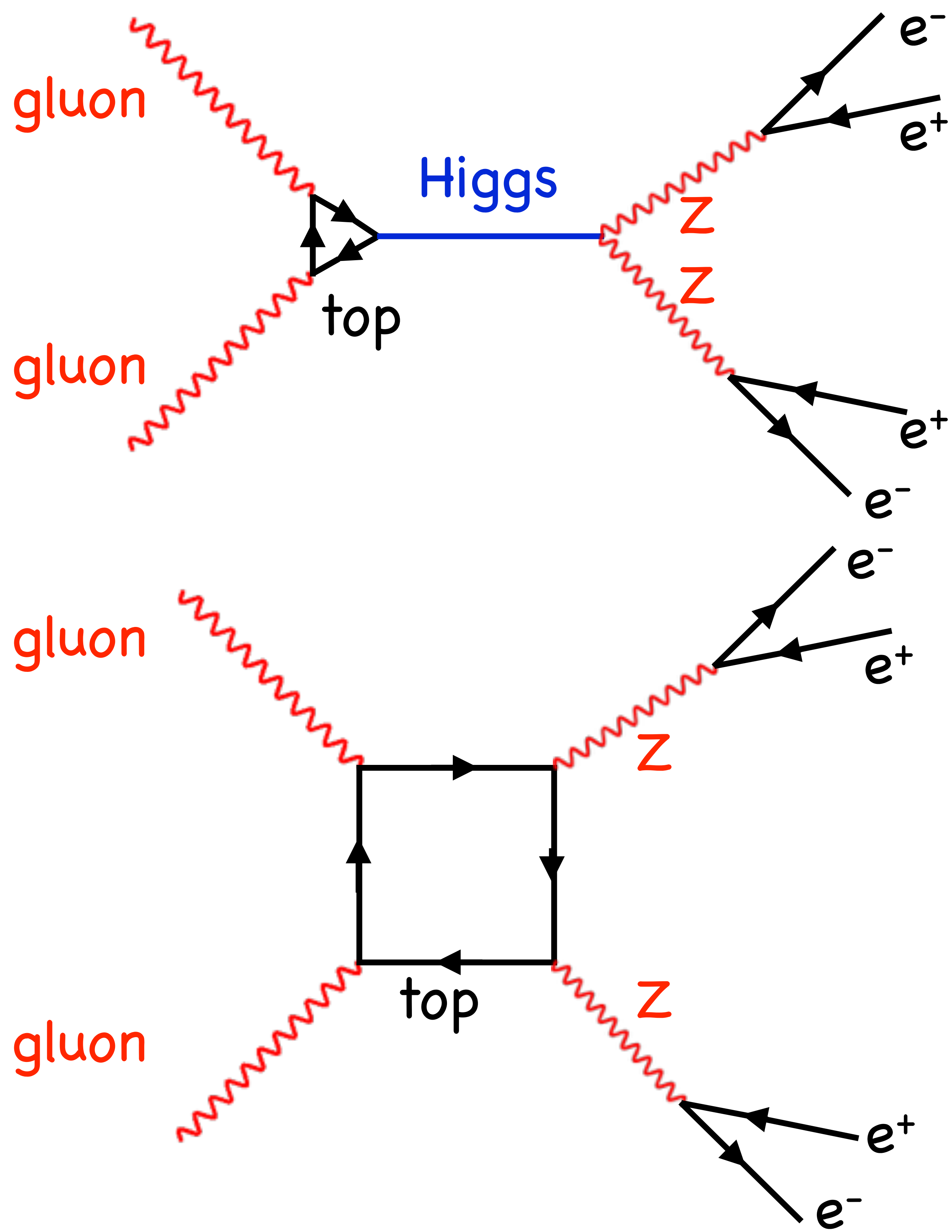


$$\mathcal{M}_{hh} = -i\frac{g^2}{4M_W^2}\left[s + \frac{(-s)^{2-\Delta}}{(2-\Delta)\mu^{2-2\Delta}}\right]$$

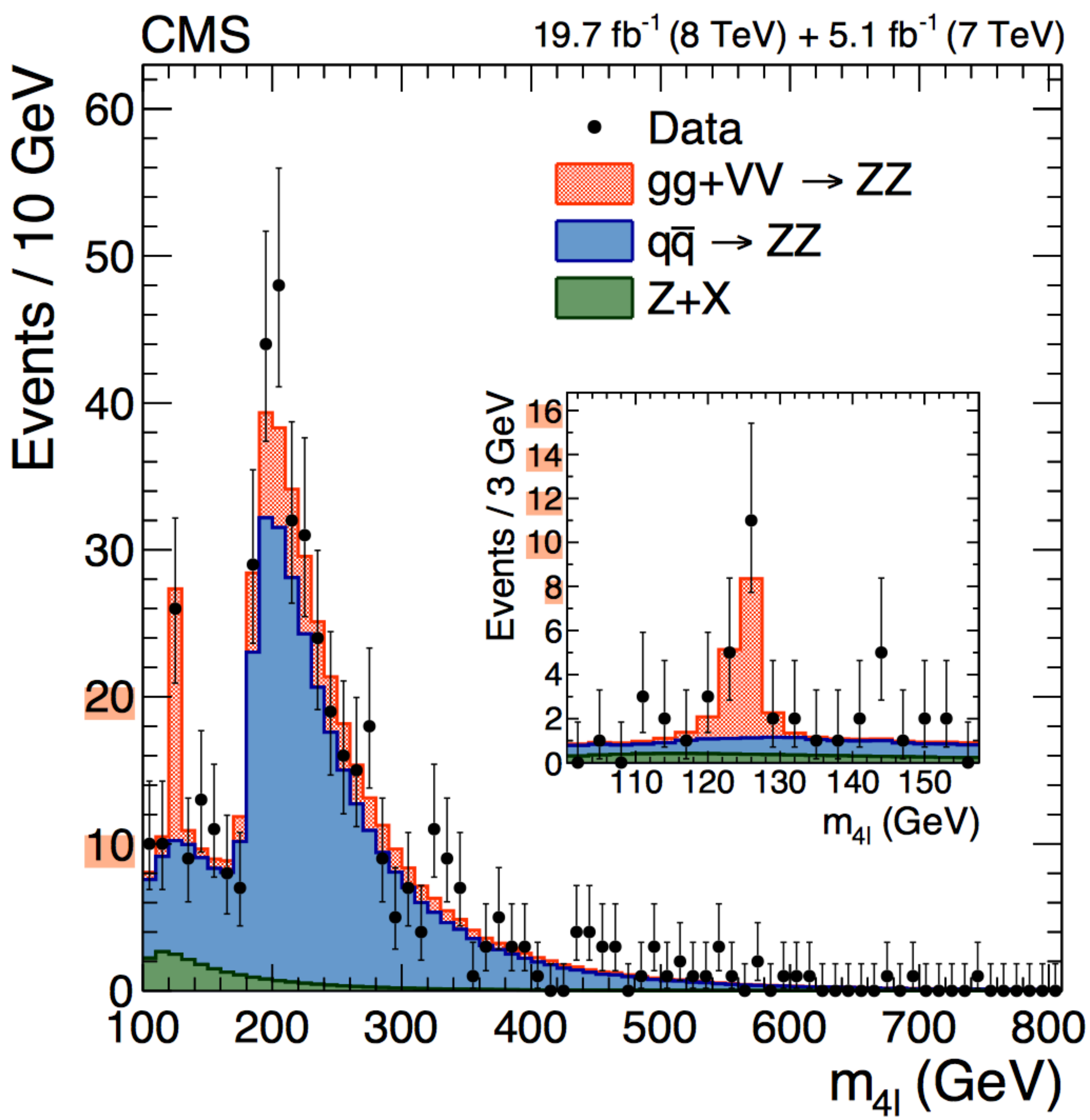
QC Higgs 6 point vertex does unitarize WW scattering

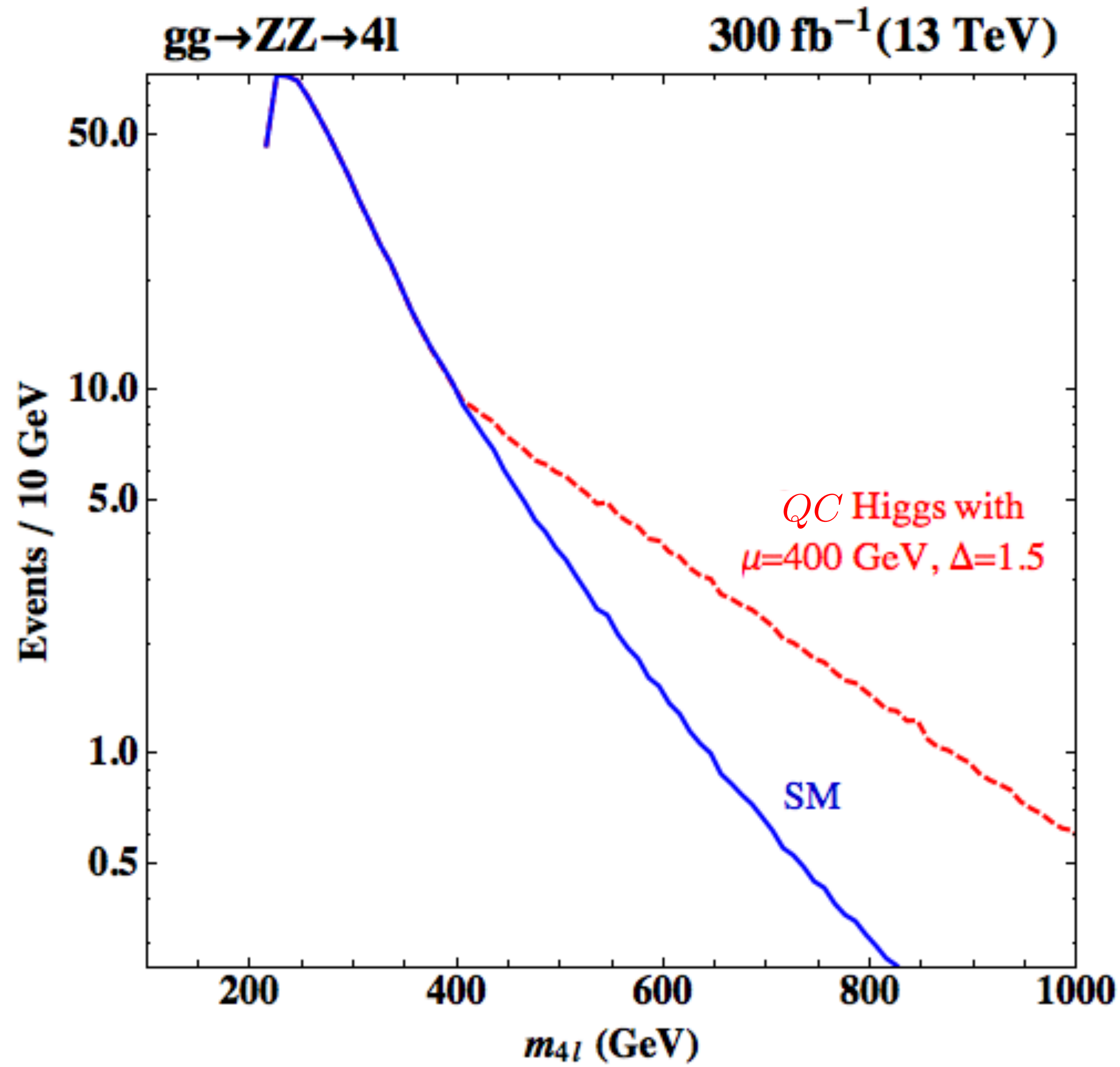
Stancato JT, hep-ph/0807.3961

LHC Interference



LHC Experiment





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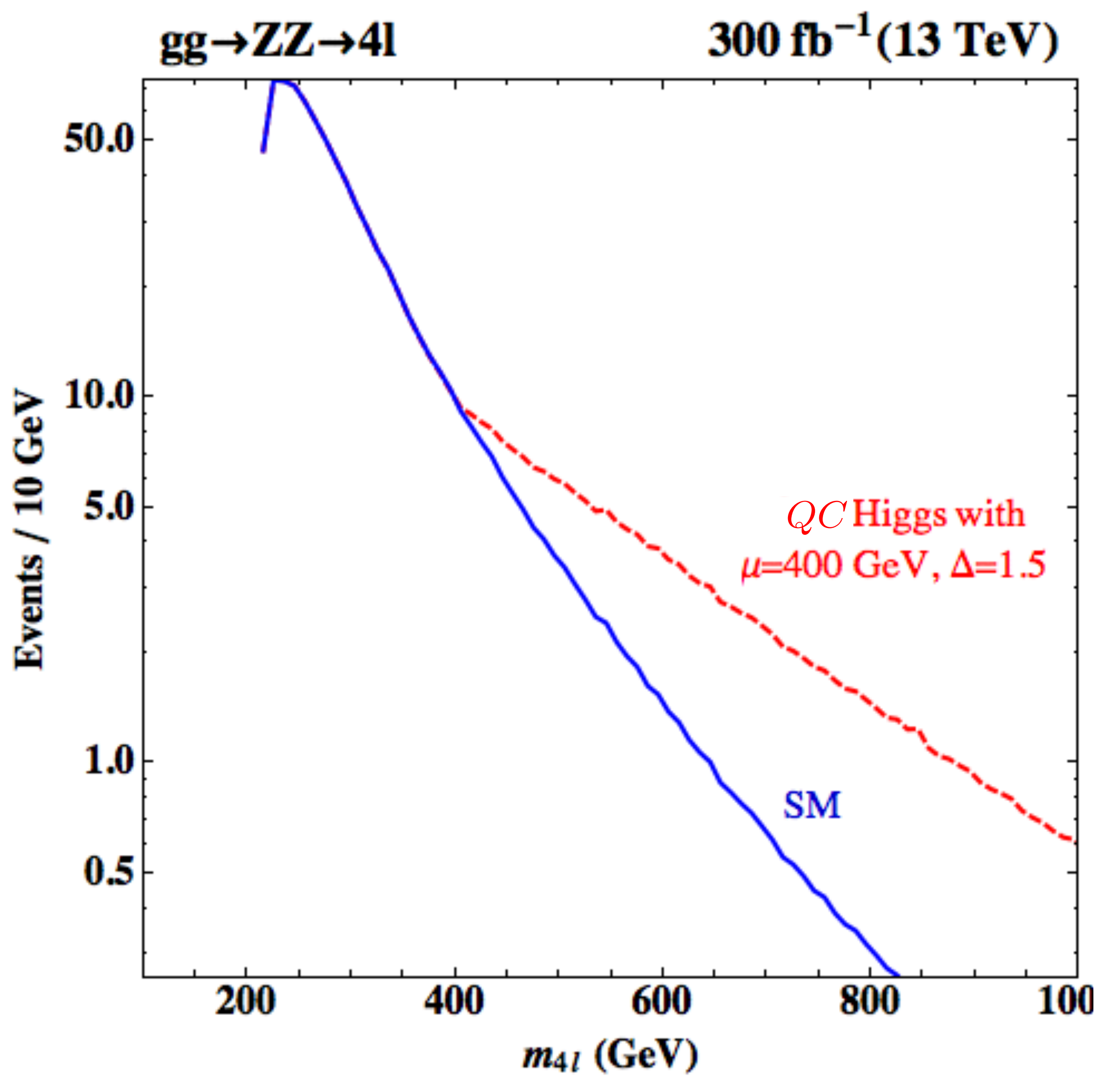
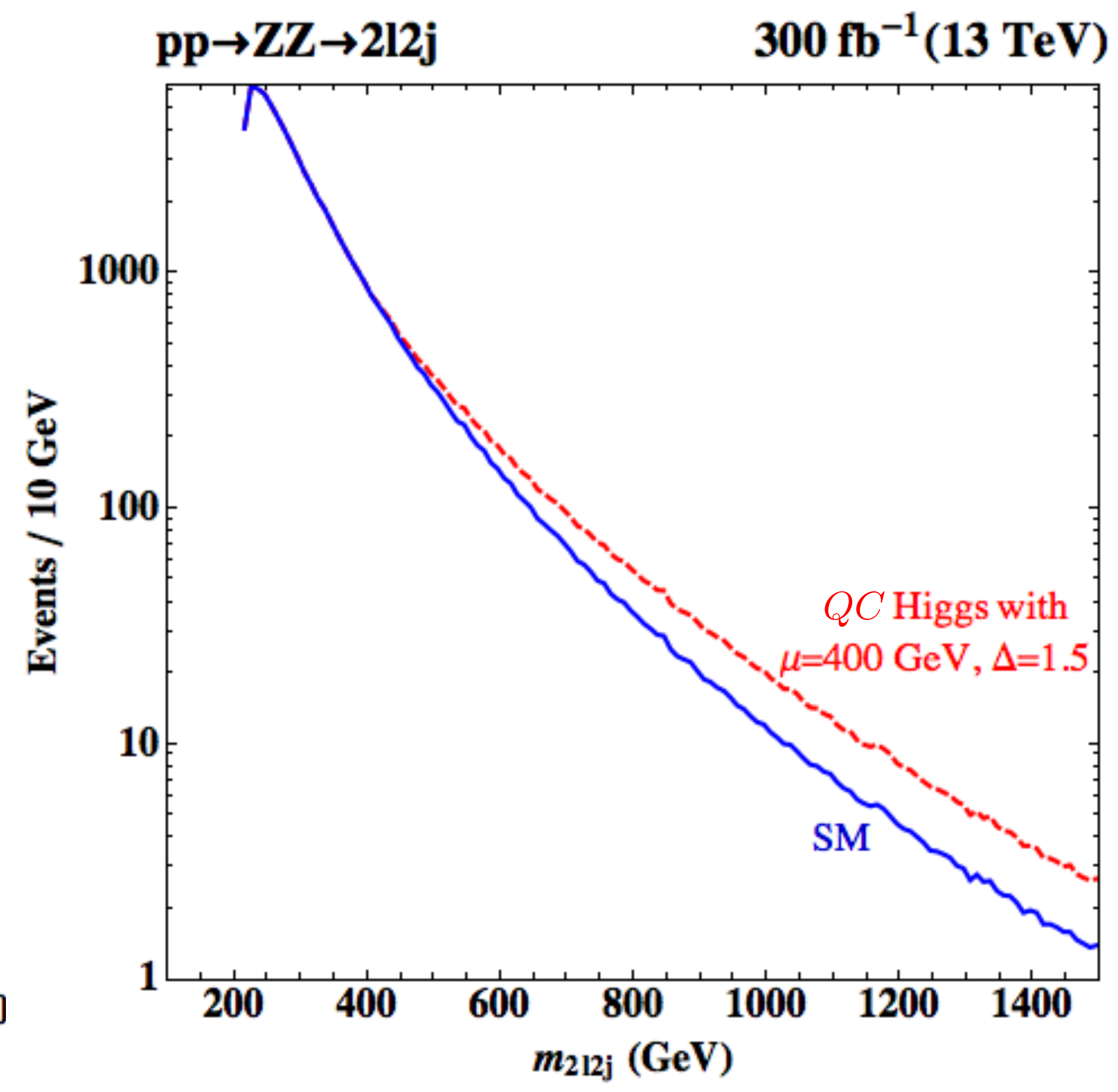
LHC Experiment





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Conclusions

The Electroweak Phase Transition is close to a Quantum Critical Point

The LHC can test whether the Higgs has a non-trivial critical exponent