# The Future is Stochastic

EFT for super-Hubble modes
&
Resumming IR inflationary behaviour

CPB, Holman, Tasinato, Williams

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# The Punchline

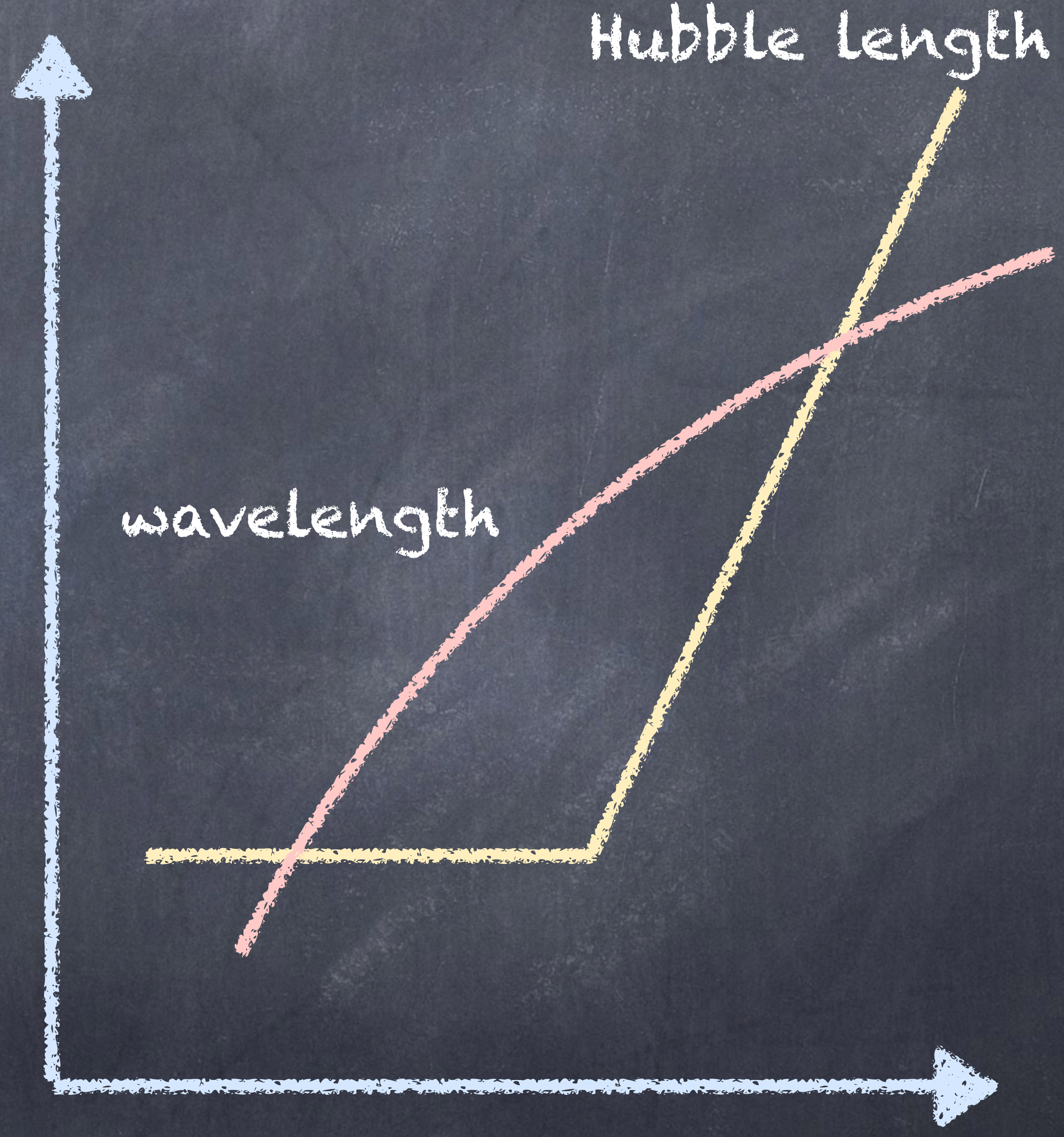
- EFT for super-Hubble modes
- IR problems for inflationary calcs
- Quantum optics
- Stochastic inflation
- Schrodinger's cosmologist
- Information loss in BHs?

# Contents

- Motivation
- Open EFTs
  - Secular evolution
  - Stochastic inflation
- Decoherence

# Motivation

- Extra-Hubble modes are key to success of inflationary predictions
  - What is their EFT?
- What quantifies theoretical error?



# Motivation

- Late-time and IR effects make finding an EFT even more important

$$\mathcal{L} = -\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4!}\,\phi^4\right]$$

$$\langle\phi^{2n}\rangle = (2n-1)!!\left(\frac{H^2}{4\pi^2}\,\ln a\right)^n\left[1 - \frac{n(n+1)}{2}\left(\frac{\lambda}{36\pi^2}\right)\ln^2 a + \cdots\right]$$

Tsamis & Woodard

# Motivation

- Secular effects have their root in a general issue

$$\langle \phi^{2n} \rangle = (2n-1)!! \left( \frac{H^2}{4\pi^2} \ln a \right)^n \left[ 1 - \frac{n(n+1)}{2} \left( \frac{\lambda}{36\pi^2} \right) \ln^2 a + \cdots \right]$$

- Perturbative methods generically fail at late times

$$U(t) = \exp\left[ -i(H_0 + H_{\text{int}})t \right]$$

# Motivation

- Normally IR divergences cancel for physical quantities (a la Bloch-Nordsieck)
- This appears to be true as well for single-field inflationary models with IR effects gauge artefacts
- General statement (multiple scalars, other massless fields, tensor modes, etc) not known

Senatore, Zaldarriaga

# Motivation

- Normally IR divergences cancel for physical quantities (a la Bloch-Nordsieck)
- Often large logarithms survive IR cancellation, with IR scale that is system-dependent (not universal)

$$A\ln\left(\frac{\mu}{\omega}\right)+A\ln\left(\frac{m}{\mu}\right)=A\ln\left(\frac{m}{\omega}\right)$$

# Motivation

- Usually define EFT in terms of effective action (or hamiltonian), but none* has emerged for inflation

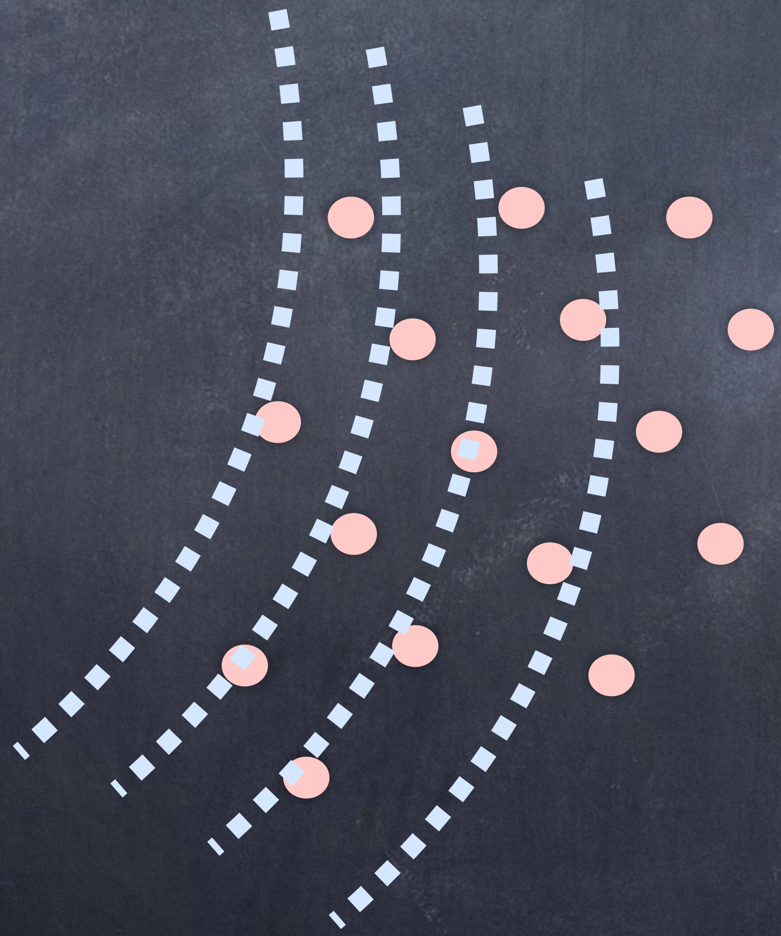
$$\langle \mathcal{O}_1(\ell) \cdots \mathcal{O}_n(\ell) \rangle = \int \mathcal{D}\ell \, \mathcal{D}h \, e^{iS(\ell,h)} \, \mathcal{O}_1(\ell) \cdots \mathcal{O}_n(\ell)$$

$$= \int \mathcal{D}\ell \, e^{iS_{\rm eff}(\ell)} \, \mathcal{O}_1(\ell) \cdots \mathcal{O}_n(\ell)$$

$$e^{iS_{\rm eff}(\ell)} = \int \mathcal{D}h \, e^{iS(\ell,h)}$$

# Open EFTs

- Better analogy for inflation is effective description of a particle moving through a medium



Information can be exchanged so Hamiltonian description need not exist

# Open EFTs

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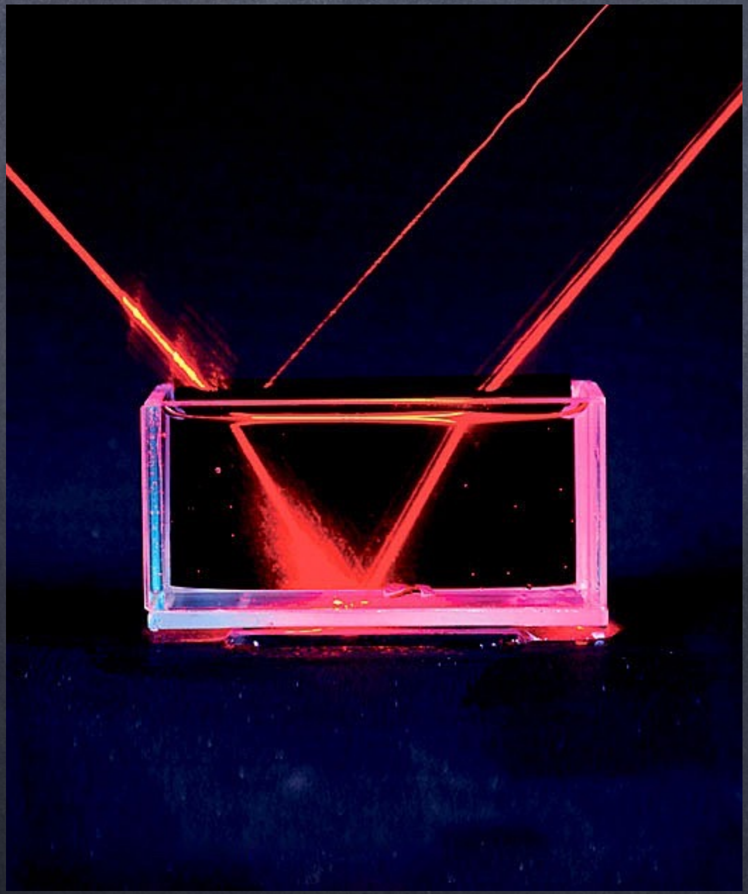


Information can be exchanged so Hamiltonian description need not exist

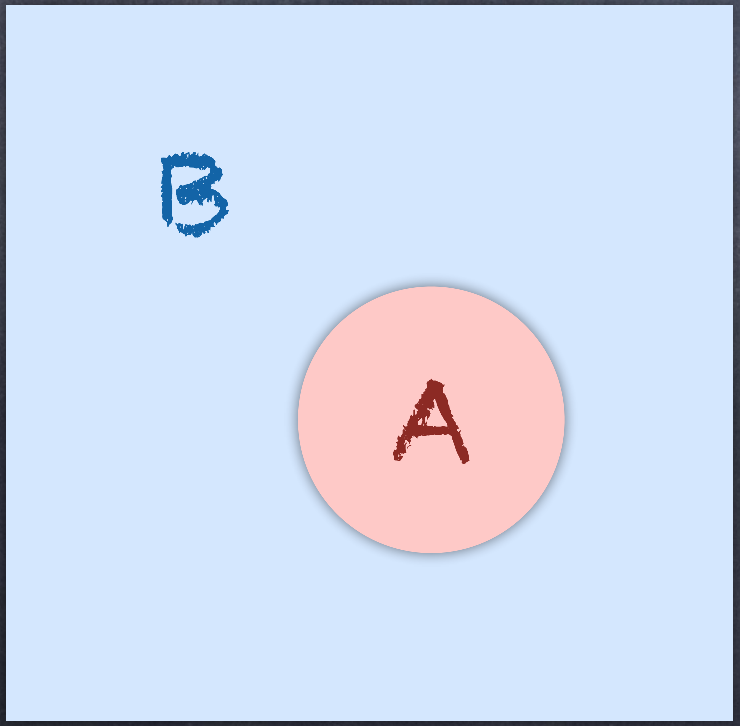
Naive perturbation theory fails at late times

Can nevertheless simplify when there is a hierarchy of scales

# Open EFTs

- Open EFTs: consider the evolution of a subset A of a larger system B

  eg: light in glass or neutrinos in Sun




$$\rho_A = \mathrm{Tr}_B \, \rho$$

$$\frac{\partial \rho}{\partial t} = -i \left[ \rho, H_{\rm int} \right]$$

# Open EFTs

- Open EFTs: direct perturbative evolution shows state generically depends on entire entanglement history

$$\rho(t) = \rho_0 - i \int_0^t \mathrm{d}\tau \Big[H_{\rm int}(\tau), \rho_0\Big] + (-i)^2 \int_0^t \mathrm{d}\tau \int_0^\tau \mathrm{d}\tilde{\tau} \Big[H_{\rm int}(\tilde{\tau}), \Big[H_{\rm int}(\tau), \rho_0\Big]\Big] + \cdots$$

but can simplify if correlations die out over time

$$\langle H_{\rm int}(t) H_{\rm int}(t+\tau)\rangle_B \to 0 \qquad \text{for} \quad \tau \gg t_c$$

provided tc << tp not so large that perturbation theory fails

# Open EFTs

- Open EFTs: can then define 'coarse-grained' time evolution for state of system A

$$\frac{D\rho_A}{dt} = \frac{\rho_A(t+\Delta t) - \rho_A(t)}{\Delta t}$$

$$= -i\Big[\rho_A, \overline{H}_{\rm int}\Big] + \int {\rm d}\tau {\rm Tr}_B \Big[\rho(t), \Big[H_{\rm int}(t), H_{\rm int}(\tau)\Big]\Big] + \cdots$$

$$= F(\rho_A, \rho_B, H_{\rm int})$$

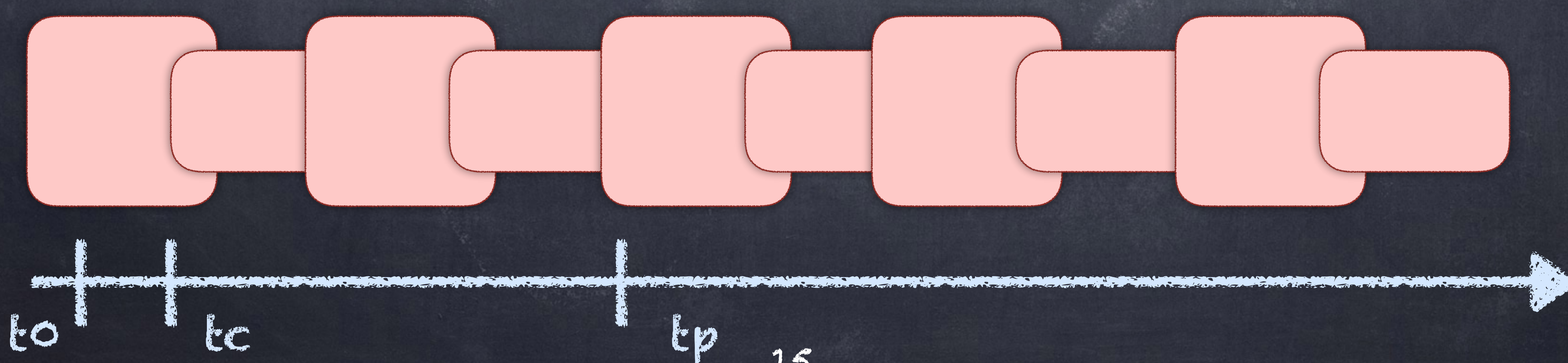
- can compute F perturbatively if $\Delta t \ll t_p$
- F depends only on instantaneous $\rho A$ and $\rho B$ if $\Delta t \gg t_c$.

# Open EFTs

- **The point:** if you can integrate the coarse-grained evolution equation

$$\frac{\partial \rho_A}{\partial t} = F(\rho_A, \rho_B, H_{\rm int})$$

can trust the solution even for $t \gg t_p$ provided there are overlapping regions each of which satisfies $t_c \ll \Delta t \ll t_p$



# Stochastic Inflation

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- For super-Hubble modes in cosmology: sector A consists of modes with $k/a \ll H$ and it is crossing of modes through Hubble scale that makes Open EFT the more useful framework
- Stochastic inflation corresponds to dropping all interactions at horizon exit except those with the background spacetime
  - Useful to compute $\langle \phi \mid \rho_A \mid \phi' \rangle$ for $\phi$ position-space field coarse grained over Hubble scale

# Stochastic Inflation

Starobinsky

- For gaussian system Schrodinger equation implies diagonal elements P(ϕ) = < ϕ | ρA | ϕ > satisfy Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left[N\frac{\partial P}{\partial \phi} + \frac{\partial}{\partial \phi}\Big(FP\Big)\right]$$

with N, F functions of time. In slow roll N and F instead can be given as functions of ϕ by

$$N = \frac{H^3}{8\pi^2} + \cdots \qquad \text{and} \qquad F = \frac{m^2\phi}{3H} + \cdots$$

# Stochastic Inflation

Starobinsky

- When mass is a function of $\phi$ then it slowly drifts with time, changing leading behaviour to

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left[N\frac{\partial P}{\partial \phi} + \frac{\partial}{\partial \phi}\Big(FP\Big)\right]$$

with coefficients N and F given by

$$N = \frac{H^3}{8\pi^2} + \cdots \qquad \text{and} \qquad F = \frac{V'(\phi)}{3H} + \cdots$$

# Stochastic Inflation

- For V = λ ϕ^4 this FP equation is known to capture the IR singular part of the field theory

Tsamis & Woodard

$$\frac{\partial P}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2 P}{\partial \phi^2} + \frac{\lambda}{18H}\frac{\partial}{\partial \phi}\left(\phi^3 P\right)$$

so late-time is described by static solution, dP/dt = 0:

Starobinsky

$$P \propto \exp\left[-\frac{8\pi^2 V}{3H^4}\right]$$

# Stochastic Inflation

- Derivation as leading part of Master equation shows why FP equation should generically resum the late-time behaviour in a general system
- Can check whether it gives results that are IR safe:
  - Expect late-time $P(\phi)$ to be finite because this appears in the expression for any observable

# Stochastic Inflation

- Check using: massive field in power-law inflation with time-dependent speed of sound
  - gives IR singularities when regarded as perturbations to massless field in dS
  - can be solved exactly to see what really happens, and compare with FP eq

# Stochastic Inflation

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- For this case can compute N and F as functions of m/H and ε = -(dH/dt)/H^2

$$\langle \dot{\phi} \rangle = C H \langle \phi \rangle$$

$$C = \frac{3-\epsilon}{2}\left[1 - \sqrt{1 - \frac{4m^2}{(3-\epsilon)H^2}}\right]$$

so

$$F = C H \phi \approx \frac{m^2 \phi}{(3-\epsilon)H} + \cdots$$

# Stochastic Inflation

CPB, Holman & Tasinato

- Similarly N is found from variance

$$\frac{1}{2}\partial_t\langle(\phi-\langle\phi\rangle)^2\rangle = \langle N\rangle + (\langle\phi F\rangle - \langle\phi\rangle\langle F\rangle)$$

- Although rate of change of variance is IR singular, N and F(ϕ) are not; guaranteeing IR finite P(ϕ) at late times

$$N = \frac{H^3}{8\pi^2}\mathcal{F}(\nu) \approx \frac{H^3}{8\pi^2}\left[1 + k(3-2\nu) + \cdots\right]$$

$$\nu = \frac{3-\epsilon}{2(1-\epsilon)}\sqrt{1 - \frac{4m^2}{(3-\epsilon)^2 H^2}}$$

# Decoherence

- Interesting spin-off: can also compute evolution of off-diagonal terms of density matrix and when doing so find:
  - Off-diagonal terms are gaussian distributed with variance that tends to zero like 1/a^3
  - Squeezing during inflation ensures the resulting density matrix always becomes diagonal in the field basis
  - Provides an explanation of why quantum fluctuations rapidly go classical when outside the Hubble scale.

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# Conclusions

- IR and secular issues in cosmology are a special case of the general problem of perturbative failure at late times
  - Similar statement for BH information loss?
- Solution and late-time resummation is provided by same methods as are useful elsewhere
  - Master-equation methods for reduced density matrix
- For inflation find Stochastic inflation as leading description, and this explains evidence for stochastic inflation resumming IR divergences in simple examples

# Open Issues

- What is the stochastic formulation for gravity and the inflaton sector?
- What can be said about eternal inflation and the late-time regime?
- Can BH information loss be understood within the same kind of framework?
- ...