Origin of Mass of the Higgs Boson

Christopher T. Hill
Fermilab

University of Toronto, April 28, 2015

A fundamental Higgs Mechanism was supposed to explain the origin of electroweak mass

A fundamental Higgs Mechanism was supposed to explain the origin of electroweak mass

We now know that a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z

A fundamental Higgs Mechanism was supposed to explain the origin of electroweak mass

We now know that a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z

But, the Higgs Boson does NOT explain the origin of the electroweak mass-scale

A fundamental Higgs Mechanism was supposed to explain the origin of electroweak mass

We now know that a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z

The Higgs Boson does NOT explain the origin of the electroweak mass-scale

i.e., what is the origin of the Higgs Boson mass itself?

A fundamental Higgs Mechanism was supposed
to explain the origin of electroweak mass

We now know that a fundamental Higgs Boson
exists and explains the masses of quarks, leptons, W and Z

The Higgs Boson does NOT
explain the origin of the electroweak mass-scale

i.e., what is the origin
of the Higgs Boson mass itself?

This is either very sobering, or it
presents theoretical opportunities

Nature *may* be telling us something very profound:

Nature *may* be telling us something very profound:

Point-like spin-0 bosons may be natural afterall !

Nature *may* be telling us something very profound:

Isolated, point-like spin-0 bosons may be natural afterall !

If true, it's a serious challenge to our understanding of naturalness

Nature *may* be telling us something very profound:

Isolated, point-like spin-0 bosons may be natural afterall !

What is the custodial symmetry?

The world of masslessness features a symmetry:

Scale Invariance

The world of masslessness features a symmetry:

Scale Invariance

Scale Invariance is (almost) always broken by quantum effects

Feynman Loops $\propto$ h

Scale Symmetry in QCD
is broken by quantum loops
and this gives rise to:

Scale Symmetry in QCD
is broken by quantum loops
and this gives rise to:

The Origin of the Nucleon Mass
(aka, most of the visible mass in
The Universe)

Gell-Mann and Low:

$$\frac{dg}{d\ln\mu} = \beta(g)$$

Gell-Mann and Low:

$$\frac{dg}{d\ln\mu} = \beta(g)$$

Gross, Politzer and Wilczek (1973):

$$\beta(g) = \beta_0 g^3 \qquad \text{where} \qquad \beta_0 = -\frac{\hbar}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)$$

Gell-Mann and Low:

$$\frac{dg}{d\ln\mu} = \beta(g)$$

Gross, Politzer and Wilczek (1973):

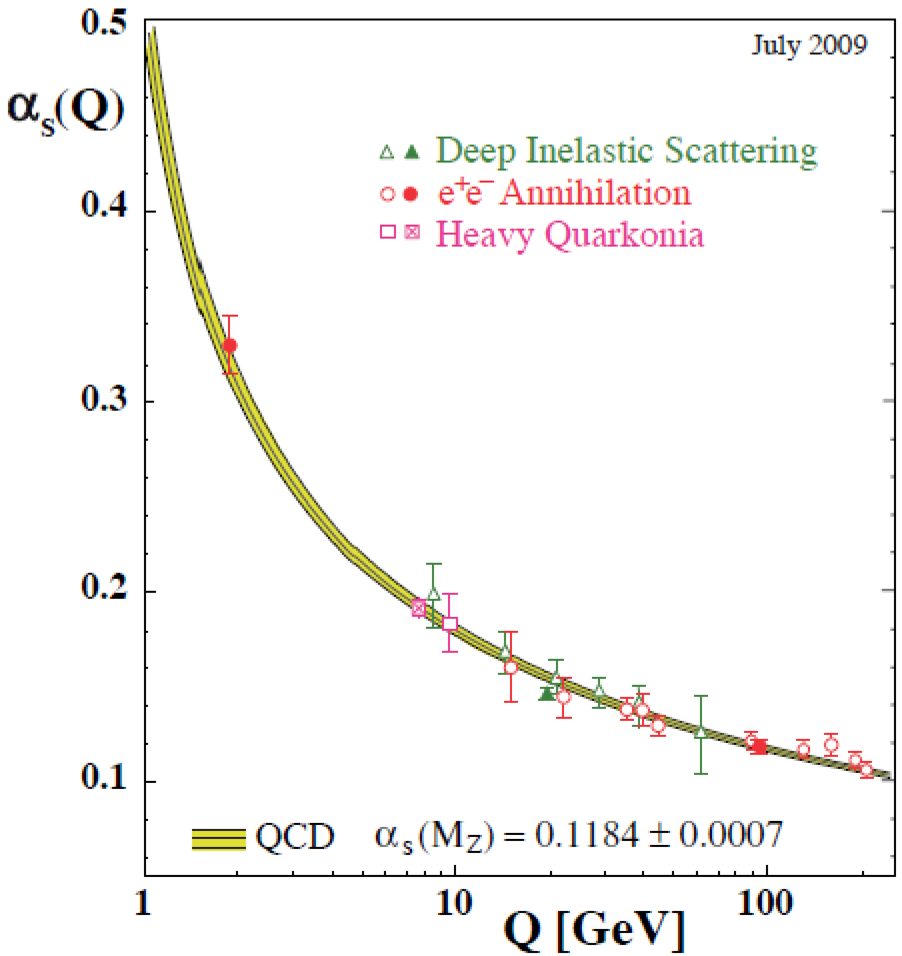
$$\beta(g) = \beta_0 g^3 \quad \text{where} \quad \beta_0 = -\frac{\hbar}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)$$

"running coupling constant"

$$\alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{|\beta_0|\ln(k^2/\Lambda^2)}$$

$\Lambda = 200$ MeV

S. Burby and C. Maxwell
arXiv:hep-ph/0011203



QCD running coupling constant

$$\alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{|\beta_0| \ln(k^2/\Lambda^2)}$$

A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of
Scale symmetry

$$S_\mu = x^\nu T_{\mu\nu}$$

A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of Scale symmetry $$S_\mu = x^\nu T_{\mu\nu}$$

Current divergence $$\partial_\mu S^\mu = T^\mu_\mu$$

A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of Scale symmetry $S_\mu = x^\nu T_{\mu\nu}$

Current divergence $\partial_\mu S^\mu = T^\mu_\mu$

Yang-Mills Stress Tensor $T_{\mu\nu} = \mathrm{Tr}(G_{\mu\rho} G^\rho_\nu) - \frac{1}{4} g_{\mu\nu} \, \mathrm{Tr}(G_{\rho\sigma} G^{\rho\sigma})$

A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of Scale symmetry $$S_\mu = x^\nu T_{\mu\nu}$$

Current divergence $$\partial_\mu S^\mu = T^\mu_\mu$$

Yang-Mills Stress Tensor $$T_{\mu\nu} = \mathrm{Tr}(G_{\mu\rho} G^\rho_\nu) - \frac{1}{4} g_{\mu\nu}\, \mathrm{Tr}(G_{\rho\sigma} G^{\rho\sigma})$$

Compute: $$\partial_\mu S^\mu = T^\mu_\mu = \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) - \frac{4}{4} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) = 0$$

A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of Scale symmetry $$S_\mu = x^\nu T_{\mu\nu}$$

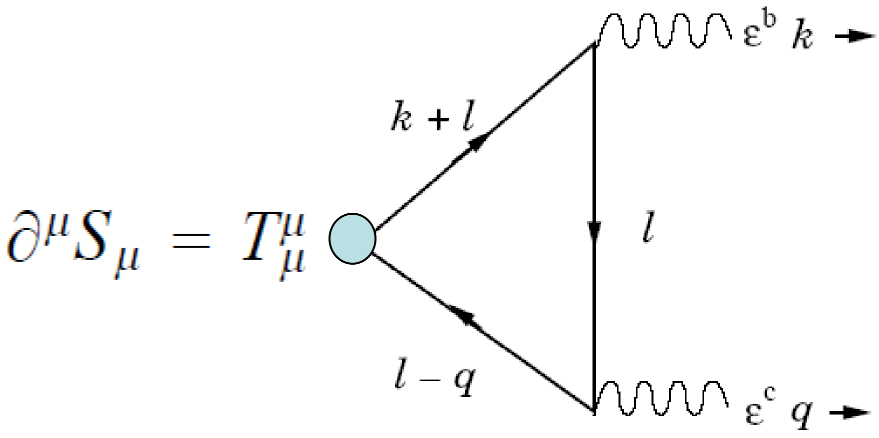
Current divergence $$\partial_\mu S^\mu = T^\mu_\mu$$

Yang-Mills Stress Tensor $$T_{\mu\nu} = \mathrm{Tr}(G_{\mu\rho} G^\rho_\nu) - \frac{1}{4} g_{\mu\nu} \mathrm{Tr}(G_{\rho\sigma} G^{\rho\sigma})$$

Compute: $$\partial_\mu S^\mu = T^\mu_\mu = \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) - \frac{4}{4} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) = 0$$

QCD is scale invariant!!!???

Resolution: The Scale Anomaly



Resolution: The Scale Anomaly

$$\partial_\mu S^\mu = \frac{\beta(g)}{g} \,\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} = \mathcal{O}(\hbar)$$

Origin of Mass in QCD
= Quantum Mechanics

See Murraypalooza talk:
Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill . **hep-th/0510177**

't Hooft Naturalness:

"Small ratios of physical parameters are controlled by symmetries. In the limit that a ratio goes to zero, there is enhanced symmetry ("custodial symmetry")."

$$\frac{\Lambda}{M} = \exp\left(-\frac{8\pi^2}{|b_0| g^2(M)}\right) \qquad b_0 \propto \hbar.$$

't Hooft Naturalness:

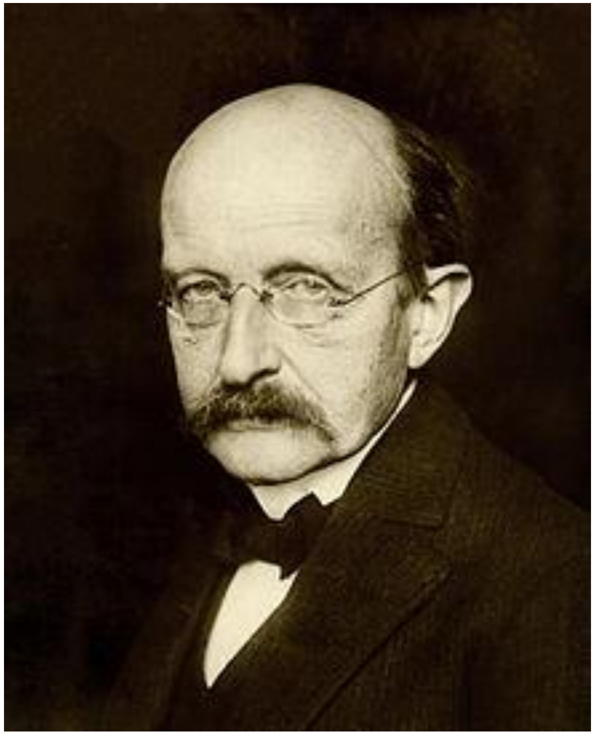
Small ratios of physical parameters are controlled by symmetries. In the limit that a ratio goes to zero, there is enhanced symmetry ("custodial symmetry").

$$\frac{\Lambda}{M} = \exp\left(-\frac{8\pi^2}{|b_0|g^2(M)}\right) \qquad b_0 \propto \hbar.$$

$\Rightarrow$ 0 $\qquad\qquad$ $\hbar \Rightarrow 0$

Classical Scale Invariance
is the "Custodial Symmetry" of Λ_{QCD}

Conjecture:



Max Planck

All mass is a quantum phenomenon.

Murraypalooza talk:
Christopher T. Hill . hep-th/0510177

Many theories were proposed to imitate QCD for the electroweak scale.

Many theories were proposed to imitate QCD for the electroweak scale.

All of these featured "strong dynamics" and classical scale invariance as the custodial symmetry of $v_{Weak} \ll M_{Gut,\ Planck}$

Many theories were proposed to imitate QCD for the electroweak scale.

All of these featured "strong dynamics" and classical scale invariance as the custodial symmetry of $v_{Weak} \ll M_{Gut,\,Planck}$

(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
(10)....

Many theories were proposed to imitate QCD for the electroweak scale.

All of these featured "strong dynamics" and classical scale invariance as the custodial symmetry of $v_{Weak} \ll M_{Gut,\, Planck}$

(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
(10)....

Many theories were proposed to imitate QCD for the electroweak scale.

All of these featured "strong dynamics" and classical scale invariance as the custodial symmetry of $v_{Weak} \ll M_{Gut,\,Planck}$

(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
(10)....

Mass extinction of theories on July 4th 2012

Susy is still alive.

Susy is still alive?

If so, where is it?

How much fine tuning should we tolerate?

Weak Scale SUSY was seriously challenged before the LHC turned on (e.g. EDM's)

MSSM now copes with severe direct limits; Some nMSSM models survive

If SUSY is the custodial symmetry we may see it in LHC RUN-II

Why EDM's are so powerful:

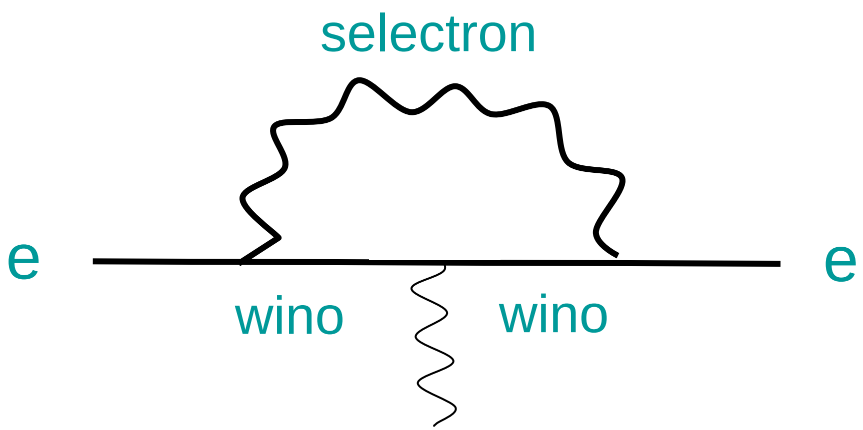
$$\frac{e\, m_e}{\Lambda^2}\, \overline{\Psi}\, \gamma_5 \sigma_{\mu\nu} F^{\mu\nu}\, \Psi$$

$$d_e = \frac{e\, m_e}{\Lambda^2} = 0.2 \times 10^{-16}\ (e\text{-cm}) \times \frac{(m_e/\text{MeV})}{(\Lambda/\text{GeV})^2}$$

Current limit: $d_e < 10^{-27}$ e-cm

$\Lambda > 1.4 \times 10^5$ GeV

Are EDM's telling us something about SUSY?:



$$\frac{e\, m_e}{\Lambda^2}\, \overline{\Psi}\, \gamma_5\, \sigma_{\mu\nu} F^{\mu\nu}\, \Psi$$

$$1/(\Lambda)^2 = (\alpha \sin(\gamma) / 4\pi \sin^2\theta)\, (1/M_{selectron})^2$$

$$M_{selectron} > 6.8 \times 10^3 \text{ GeV } (\sin(\gamma))^{1/2}$$

Future limit: $d_e < 10^{-29}$ e-cm -- 10^{-32} e-cm ?

Can a perturbatively
light Higgs Boson mass
come from quantum mechanics?

Bardeen: Classical Scale Invariance
could be the custodial symmetry
of a fundamental, perturbatively
light Higgs Boson in SU(3)xSU(2)xU(1).

The only manifestations of
Classical Scale Invariance breaking by
quantum loops are d = 4 scale anomalies.

On naturalness in the standard model.
William A. Bardeen
FERMILAB-CONF-95-391-T, Aug 1995. 5pp.

Bardeen: Classical Scale Invariance
could be the custodial symmetry
of a fundamental, perturbatively
light Higgs Boson in SU(3)xSU(2)xU(1).

The only manifestations of
Classical Scale Invariance breaking by
quantum loops are d = 4 scale anomalies.
There is no meaning to the "quadratic divergence"
as a source of scale breaking.

On naturalness in the standard model.
William A. Bardeen
FERMILAB-CONF-95-391-T, Aug 1995. 5pp.

The only manifestations of
Classical Scale Invariance breaking by
quantum loops are d = 4 scale anomalies.

There are possible additive effects from higher mass scales: $\delta m_H^2 = \alpha^p M_{GUT}^2 + \alpha^q M_{Planck}^2$.

But the existence of the low mass Higgs
may be telling us that such effects are absent!

Something seems to be missing in our understanding
of scale symmetry and fine-tuning.

The only manifestations of
Classical Scale Invariance breaking by
quantum loops are d = 4 scale anomalies.

There are possible additive effects from higher mass scales: $\delta m_H^2 = \alpha^p M_{GUT}^2 + \alpha^q M_{Planck}^2$.

But the existence of the low mass Higgs
may be telling us that such effects are absent!

Something seems to be missing in our understanding
of scale symmetry and fine-tuning.

This is profoundly important in sculpting
our view of the physical world!

Treat this as a
phenomenological question:

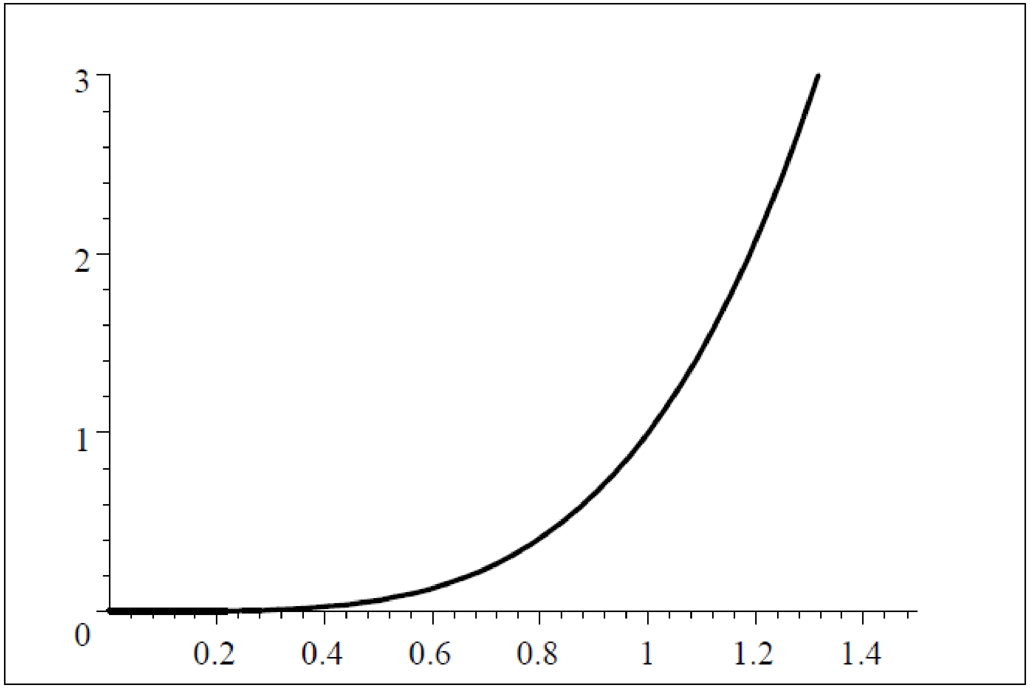
Is the Higgs Potential Generated by
Infra-red (scale-breaking)
Quantum Loop Effects?

i.e., is the Higgs potential a
Coleman-Weinberg Potential?

Start with the Classically Scale Invariant Higgs Potential

$$\frac{\lambda}{2}|H|^4$$

$\langle H \rangle = v$

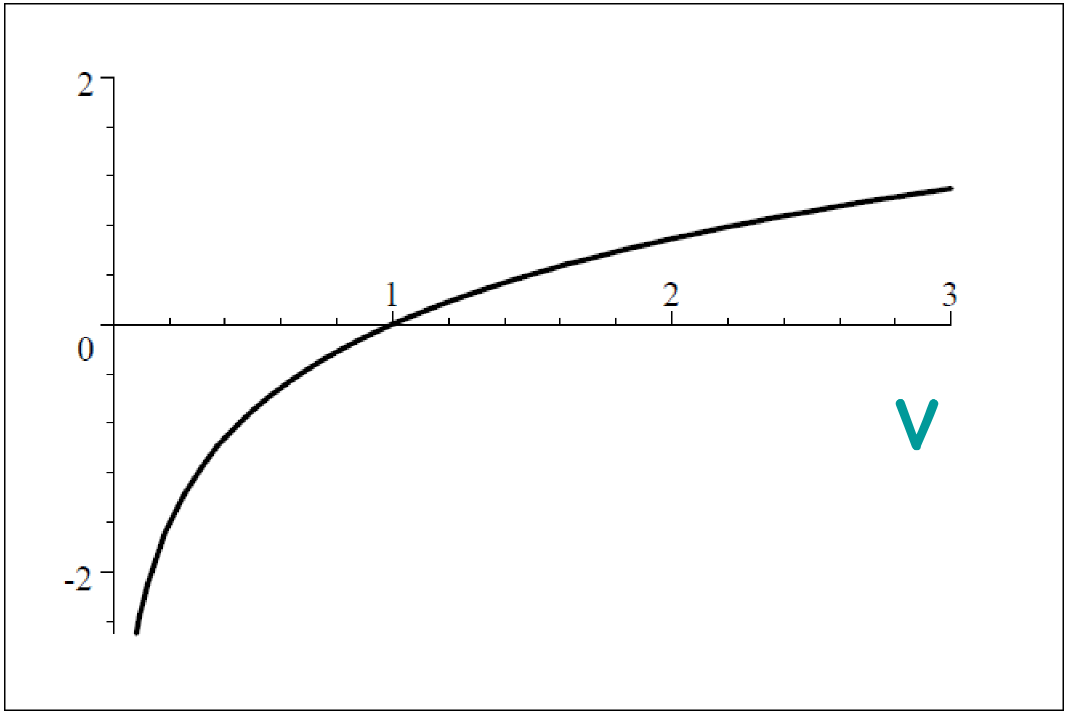


v

Scale Invariance -> Quartic Potential -> No VEV

Quantum loops generate a logarithmic "running" of the quartic coupling

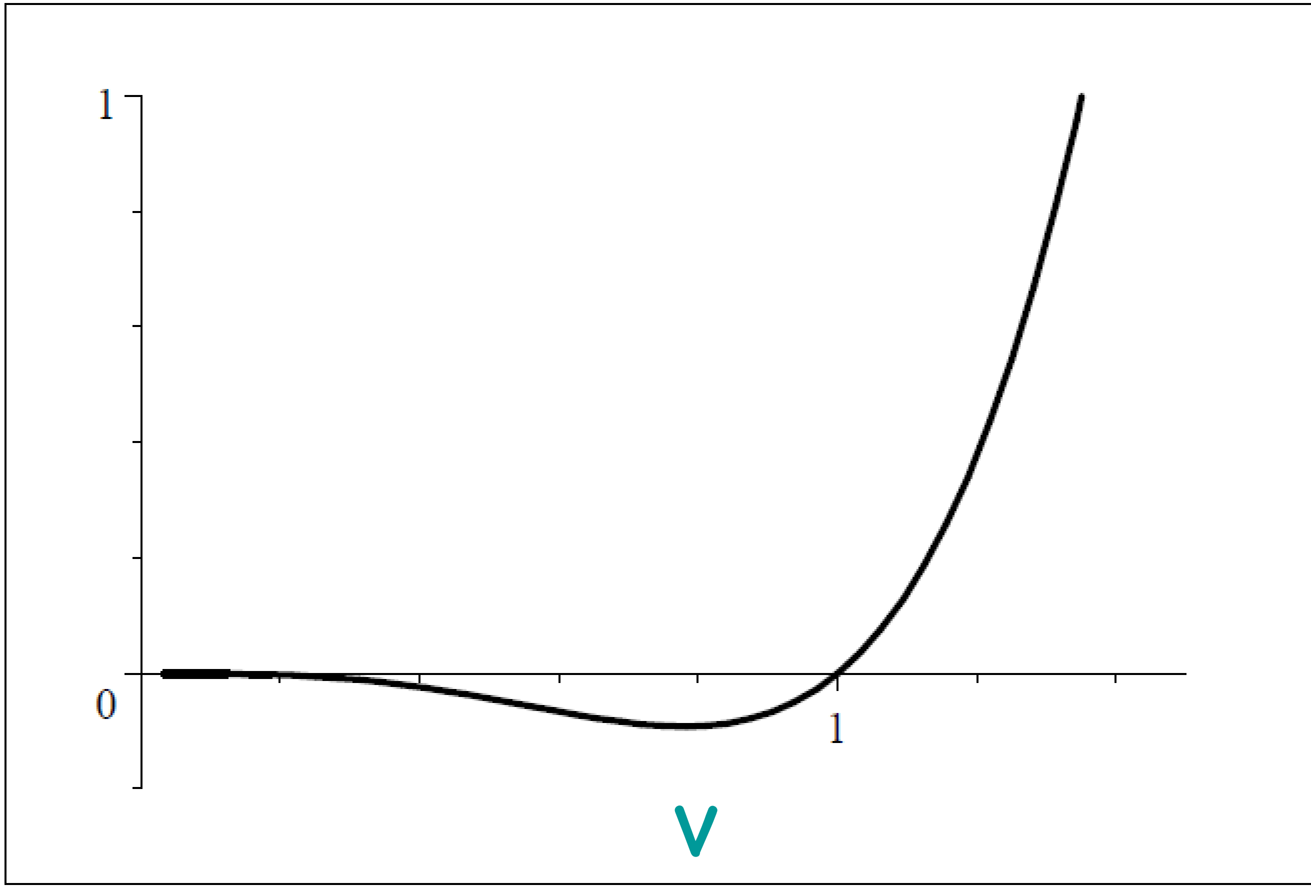
$$\lambda(v) \propto \hbar \beta \log(v/M)$$



Nature chooses a particular trajectory determined by dimensionless cc's.

Result: "Coleman-Weinberg Potential"

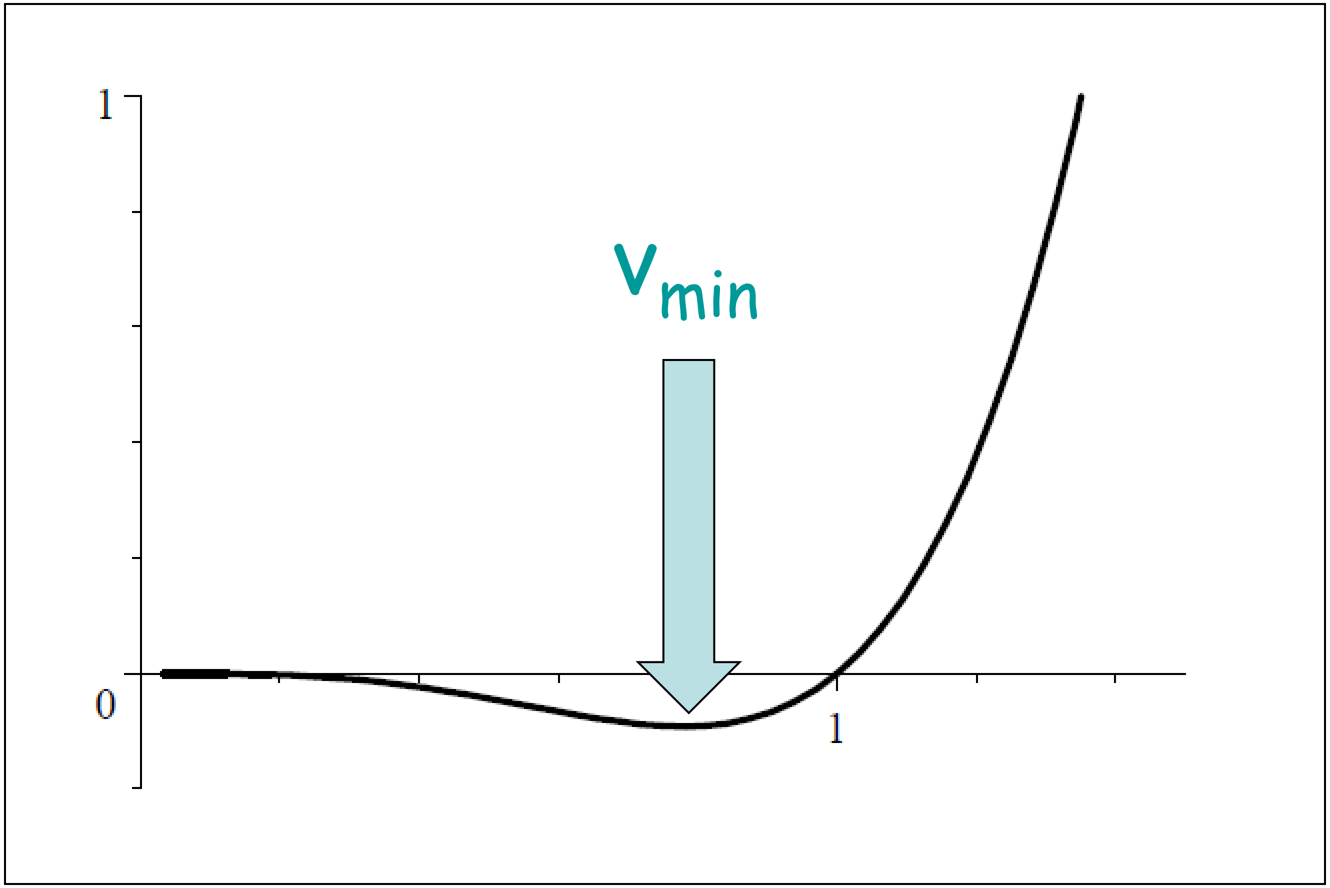
$$\frac{\tilde{\lambda}(v)}{2} \times v^4$$



Potential Minimum arises from running of λ
i.e. Quantum Mechanics

Result: “Coleman-Weinberg Potential

$$\frac{\tilde{\lambda}(v)}{2} \times v^4$$



Require: $\beta > 0$ $\lambda < 0$ for a minimum, positive curvature

Boson Mass is determined by curvature at minimum

An Improved Coleman-Weinberg Potential

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

An Improved Coleman-Weinberg Potential

$$S=\int d^4x\,\mathcal{L}=\int d^4x\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right)$$

Improved Stress tensor:

Callan, Coleman, Jackiw

$$\widetilde{T}_{\mu\nu}\ =\ T_{\mu\nu}+Q_{\mu\nu}$$

$$=\frac{2}{3}\partial_\mu\phi\partial_\nu\phi-\frac{1}{6}\eta_{\mu\nu}\partial_\rho\phi\partial^\rho\phi-\frac{1}{3}\phi\partial_\mu\partial_\upsilon\phi\ \ +\frac{1}{3}\eta_{\mu\nu}\phi\partial^2\phi+\eta_{\mu\nu}V(\phi)$$

Trace of improved stress tensor:

$$\widetilde{T}^{\mu}_{\mu} = \phi\partial^2\phi + 4V(\phi) = -\phi\frac{\delta}{\delta\phi}V(\phi) + 4V(\phi)$$

Traceless for a classical scale invariant theory:

$$V(\phi) = \frac{\lambda}{4}\phi^4, \quad \Longrightarrow \quad \widetilde{T}^{\mu}_{\mu} = 0$$

Conserved scale current

Trace of improved stress tensor:

$$\widetilde{T}^{\mu}_{\mu} = \phi \partial^2 \phi + 4V(\phi) = -\phi \frac{\delta}{\delta \phi} V(\phi) + 4V(\phi)$$

Traceless for a classical scale invariant theory:

$$V(\phi) = \frac{\lambda}{4}\phi^4, \quad \Rightarrow \quad \widetilde{T}^{\mu}_{\mu} = 0$$

Conserved scale current

Running coupling constant: $\frac{\delta}{\delta \phi}\lambda(\phi) = \beta(\lambda)$

$$\Rightarrow \quad \widetilde{T}^{\mu}_{\mu} = -\frac{\beta(\lambda)}{\lambda} V(\phi)$$

Trace Anomaly associated with running coupling

Improved Coleman-Weinberg Potential is the solution to the equations:

$$\widetilde{T}^{\mu}_{\mu} = -\frac{\beta(\lambda)}{\lambda} V(\phi)$$

$$\phi \frac{\delta}{\delta\phi} V(\phi) - 4V(\phi) = \frac{\beta}{\lambda} V(\phi)$$

$$\frac{d\lambda(\mu)}{d \ln \mu} = \beta(\lambda)$$

Improved Coleman-Weinberg Potential is the solution to the equations:

$$\phi\frac{\delta}{\delta\phi}V(\phi) - 4V(\phi) = \frac{\beta}{\lambda}V(\phi)$$

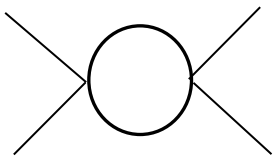
$$\frac{d\lambda(\mu)}{d\ln\mu} = \beta(\lambda)$$

The solution is: $V(\phi) = \frac{1}{2}\lambda(\phi)\phi^4$

CTH arXiv:1401.4185 [hep-ph].
Phys Rev D.89. 073003.

Example: ϕ^4 Field theory

$$\frac{d\lambda}{d\ln(\phi)} = \beta(\lambda) = \frac{9\lambda^2}{32\pi^2}$$

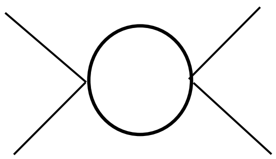


$$V_{RG} = \frac{\lambda}{4}\phi^4 + \hbar\frac{9\lambda^2}{32\pi^2}\phi^4 \ln(\phi/M) \qquad = \hbar\frac{m_h^4}{32\pi^2}(\phi/v)^4 \ln(\phi/\tilde{M})$$

agrees with CW original result log (path Integral)

Example: ϕ^4 Field theory

$$\frac{d\lambda}{d\ln(\phi)} = \beta(\lambda) = \frac{9\lambda^2}{32\pi^2}$$



$$V_{RG} = \frac{\lambda}{4}\phi^4 + \hbar\frac{9\lambda^2}{32\pi^2}\phi^4\ln(\phi/M) \qquad = \hbar\frac{m_h^4}{32\pi^2}(\phi/v)^4\ln(\phi/\tilde{M})$$

agrees with CW original result log (path Integral)

Example: Scalar Electrodynamics

$$V(\phi) = \frac{\lambda_0}{2}|\phi|^4 + \frac{1}{16\pi^2}\left(5\lambda^2 - 6\lambda e^2 + 6e^4\right)|\phi|^4\ln\left(\frac{|\phi|}{M}\right)$$

$$\phi_c^2 = 2|\phi|^2 \qquad \text{and} \qquad \frac{\lambda_{CW}}{4!}\phi_c^4 = \frac{\lambda_0}{2}|\phi|^4$$

$$V(\phi'_c) = \frac{\lambda_{CW}}{4!}\phi_c'^4 + \left(\frac{5\lambda_{CW}^2}{1152\pi^2} + \frac{3e^4}{64\pi^2}\right)\phi_c'^4\ln\left(\frac{\phi_c^2}{M'^2}\right) \qquad (C.6)$$

agrees with CW original result BUT with canonical normalization

The Renormalization Group generates the Coleman Weinberg potential

Expand about minimum:

$$V_{CW}(h) \;=\; \frac{1}{2}\lambda(v+h/\sqrt{2})(v+h/\sqrt{2})^4$$

$$\left.\frac{dV}{dh}\right|_{h=0} = \sqrt{2}v^3\left(\lambda + \frac{1}{4}\beta\right) = 0$$

$$\beta_1(\lambda_i(v)) = -4\lambda_1(v)$$ at minimum

$$\frac{d^2V}{dh^2} = m_h^2 = \left(3\lambda + \frac{7}{4}\beta\right)v^2$$

$$m_h^2 = -4\lambda v^2 = \beta v^2 > 0$$

The Renormalization Group generates the Coleman Weinberg potential

Expand about minimum:

$$V_{CW}(h) \;=\; \frac{1}{2}\lambda(v + h/\sqrt{2})(v + h/\sqrt{2})^4$$

$$\left.\frac{dV}{dh}\right|_{h=0} = \sqrt{2}v^3\left(\lambda + \frac{1}{4}\beta\right) = 0$$

$$\beta_1(\lambda_i(v)) = -4\lambda_1(v)$$ at minimum

$$\frac{d^2V}{dh^2} = m_h^2 = \left(3\lambda + \frac{7}{4}\beta\right)v^2$$

$$m_h^2 = -4\lambda v^2 = \beta v^2 > 0$$

We label all relevant coupling constants that enter in any order of the loop diagrams for the running of λ (*e.g.*, g_{top}, g_2, g_{QCD}, etc.) as λ_i. We denote the scalar quartic (Higgs) coupling as $\lambda \equiv \lambda_1$ with β-function $\beta_1(\lambda_i)$. Each λ_i has its own β_i:

$$\frac{d\lambda_i}{d\ln(\mu)} = \beta_i(\lambda_j)$$

$$\begin{aligned}
v\lambda_1'(v) &= \beta_1 \\
v^2\lambda_1''(v) &= \beta_j\frac{\partial\beta_1}{\partial\lambda_j} - \beta_1 \\
v^3\lambda_1'''(v) &= \beta_i\beta_j\frac{\partial^2\beta_1}{\partial\lambda_i\partial\lambda_j} + \beta_j\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta_1}{\partial\lambda_i} \\
&\quad -3\beta_j\frac{\partial\beta_i}{\partial\lambda_i} + 2\beta_1 \\
v^4\lambda_1''''(v) &= \beta_i\beta_j\beta_k\frac{\partial^3\beta_1}{\partial\lambda_i\partial\lambda_j\partial\lambda_k} + \beta_k\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta_1}{\partial\lambda_i} \\
&\quad +\beta_k\beta_j\frac{\partial^2\beta_i}{\partial\lambda_j\partial\lambda_k}\frac{\partial\beta_1}{\partial\lambda_i} + 3\beta_k\beta_j\frac{\partial\beta_i}{\partial\lambda_k}\frac{\partial^2\beta_1}{\partial\lambda_i\partial\lambda_j} \\
&\quad -6\beta_i\beta_j\frac{\partial^2\beta_1}{\partial\lambda_i\partial\lambda_j} - 6\beta_k\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial\beta_1}{\partial\lambda_j} \\
&\quad +11\beta_i\frac{\partial\beta_1}{\partial\lambda_i} - 6\beta_1 \qquad (21)
\end{aligned}$$

The Coleman-Weinberg Potential is completely determined by β-functions:

$$
\begin{aligned}
V_{CW}(h) \;=\; & -\frac{1}{8}\beta_1 v^4 + \frac{1}{2}v^2h^2\left(\beta_1 + \frac{1}{4}\beta_j\frac{\partial\beta_1}{\partial\lambda_j}\right) \\
& + \frac{5}{6\sqrt{2}}vh^3\left(\beta_1 + \frac{9}{20}\beta_i\frac{\partial\beta_1}{\partial\lambda_i} + \frac{1}{20}\beta_j\beta_i\frac{\partial^2\beta_1}{\partial\lambda_j\partial\lambda_i}\right. \\
& \quad \left. + \frac{1}{20}\beta_j\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta_1}{\partial\lambda_i}\right) \\
& + \frac{11}{48}h^4\left(\beta_1 + \frac{35}{44}\beta_i\frac{\partial\beta_1}{\partial\lambda_i} + \frac{5}{22}\beta_j\beta_i\frac{d^2\beta_1}{\partial\lambda_j\partial\lambda_i}\right. \\
& \quad + \frac{5}{22}\beta_j\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta_1}{\partial\lambda_i} + \frac{1}{44}\beta_k\beta_j\beta_i\frac{d^3\beta_1}{\partial\lambda_k\partial\lambda_j\partial\lambda_i} \\
& \quad + \frac{1}{44}\beta_k\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta_1}{\partial\lambda_i} + \frac{1}{44}\beta_j\beta_i\frac{d^2\beta_i}{\partial\lambda_j\partial\lambda_i}\frac{\partial\beta_1}{\partial\lambda_i} \\
& \quad \left. + \frac{3}{44}\beta_j\beta_k\frac{\partial\beta_i}{\partial\lambda_k}\frac{d^2\beta_1}{\partial\lambda_j\partial\lambda_i}\right)
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{h^5}{40\sqrt{2}v}\left(\beta + \frac{25}{12}\beta_i\frac{d\beta}{d\lambda_i} + \frac{35}{24}\beta_j\beta_i\frac{d^2\beta}{d\lambda_j d\lambda_i}\right. \\
& + \frac{35}{24}\beta_j\frac{d\beta_i}{d\lambda_j}\frac{d\beta}{d\lambda_i} + \frac{5}{12}\beta_k\beta_j\beta_i\frac{d^3\beta}{d\lambda_k d\lambda_j d\lambda_i} \\
& + \frac{5}{12}\beta_k\frac{d\beta_j}{d\lambda_k}\frac{d\beta_i}{d\lambda_j}\frac{d\beta}{d\lambda_i} + \frac{5}{12}\beta_j\beta_i\frac{d^2\beta_i}{d\lambda_j d\lambda_i}\frac{d\beta}{d\lambda_i} \\
& + \frac{5}{4}\beta_j\beta_k\frac{d\beta_i}{d\lambda_k}\frac{d^2\beta}{d\lambda_j d\lambda_i} + \frac{1}{24}\beta_i\beta_j\beta_k\beta_\ell\frac{\partial^4\beta}{\partial\lambda_i\partial\lambda_j\partial\lambda_k\partial\lambda_\ell} \\
& + \frac{1}{24}\beta_\ell\frac{\partial\beta_k}{\partial\lambda_\ell}\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta}{\partial\lambda_i} + \frac{1}{4}\beta_i\beta_j\beta_k\frac{\partial\beta_\ell}{\partial\lambda_k}\frac{\partial^3\beta}{\partial\lambda_i\partial\lambda_j\partial\lambda_\ell} \\
& + \frac{1}{8}\beta_\ell\beta_k\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial^2\beta_i}{\partial\lambda_j\partial\lambda_\ell}\frac{\partial\beta}{\partial\lambda_i} + \frac{1}{6}\beta_i\beta_\ell\frac{\partial\beta_k}{\partial\lambda_\ell}\frac{\partial\beta_j}{\partial\lambda_k}\frac{\partial^2\beta}{\partial\lambda_j\partial\lambda_i} \\
& + \frac{1}{6}\beta_i\beta_j\beta_\ell\frac{\partial^2\beta_k}{\partial\lambda_\ell\partial\lambda_j}\frac{\partial^2\beta}{\partial\lambda_k\partial\lambda_i} + \frac{1}{24}\beta_\ell\beta_k\beta_j\frac{\partial^3\beta_i}{\partial\lambda_j\partial\lambda_k\partial\lambda_\ell}\frac{\partial\beta}{\partial\lambda_i} \\
& \left. + \frac{1}{8}\beta_\ell\beta_i\frac{\partial\beta_k}{\partial\lambda_\ell}\frac{\partial^2\beta}{\partial\lambda_k\partial\lambda_j}\frac{\partial\beta_j}{\partial\lambda_i} + \frac{1}{24}\beta_\ell\beta_k\frac{\partial^2\beta_j}{\partial\lambda_\ell\lambda_k}\frac{\partial\beta_i}{\partial\lambda_j}\frac{\partial\beta}{\partial\lambda_i}\right) \\
&)(h^6).
\end{aligned}
$$

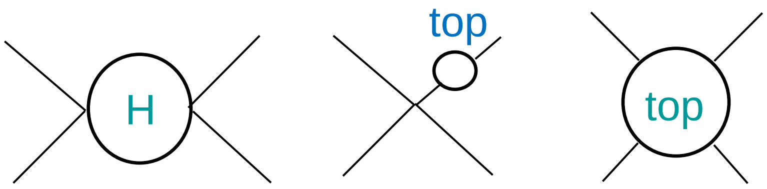
CTH arXiv:1401.4185 [hep-ph].
Phys Rev D.89. 073003.

Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2}(\lambda^2 + \lambda g^2 - g^4) = \beta$$

g : top Yukawa cc



(I am ignoring EW contributions
for simplicity of discussion)

Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2}(\lambda^2 + \lambda g^2 - g^4) = \beta$$

approximate physical values:

Higgs quartic cc: $\lambda = 1/4$

Top Yukawa cc: $g = 1$

$\Rightarrow \beta = -5.2244 \times 10^{-2}$

β is small and negative in standard model

No solution !

Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2}(\lambda^2 + \lambda g^2 - g^4) = \beta$$

approximate physical values:

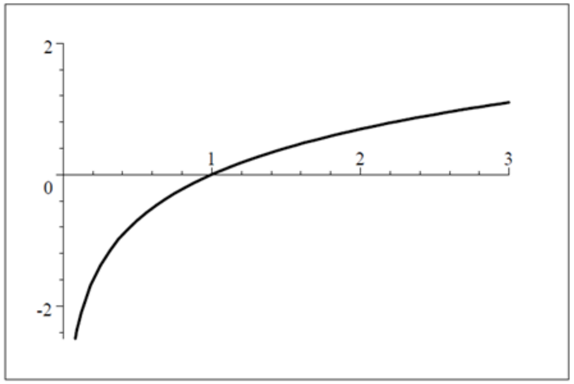
Higgs quartic cc: $\lambda = 1/4$

Top Yukawa cc: $g = 1$

$\Rightarrow \beta = -5.2244 \times 10^{-2}$

β is small and negative in standard model

We require positive β to have a Coleman-Weinberg potential



Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2}(\lambda^2 + \lambda g^2 - g^4) = \beta$$

approximate physical values:

Higgs quartic cc: $\lambda = 1/4$

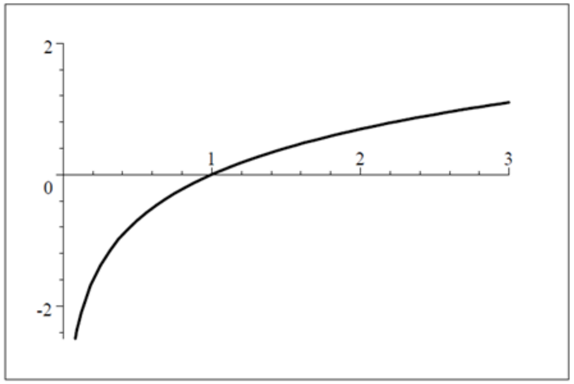
Top Yukawa cc: $g = 1$

$\Rightarrow$ $\beta = -5.2244 \times 10^{-2}$

β is small and negative in standard model

We require positive β to have a Coleman-Weinberg potential

Requires New Bosonic physics beyond the standard model



Higgs Quartic coupling $\beta(\lambda)$

Introduce a new field: S

Higgs-Portal Interaction $\lambda' |H|^2 |S|^2$

Higgs Quartic coupling $\beta(\lambda)$

Introduce a new field: S

Higgs-Portal Interaction $\lambda' |H|^2 |S|^2$

Two possibilities:

(1) Modifies RG equation to make $\beta > 0$:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2}(\lambda^2 + \lambda g^2 - g^4 + c\,\lambda'^2)$$

(2) S develops its own CW potential, and VEV $\langle S \rangle = V'$ and Higgs gets mass, $\lambda' V'$

Simplest hypotheses:

S may be:

(1)A singlet field with or without VEV

e.g., Ultra-weak sector, Higgs boson mass, and the dilaton
Kyle Allison, Christopher T. Hill, Graham G. Ross. : **arXiv:1404.6268 [hep-ph]**

(2) A new doublet NOT coupled to
SU(2) x U(1) (inert) w or wo VEV

Hambye and Strumia Phys.Rev. D88 (2013) 055022

} S sector is Dark Matter

Simplest hypotheses:

S may be:

(1)A singlet field with or without VEV
e.g., Ultra-weak sector, Higgs boson mass, and the dilaton
Kyle Allison, Christopher T. Hill, Graham G. Ross. : **arXiv:1404.6268 [hep-ph]**

(2) A new doublet NOT coupled to
SU(2) x U(1) (inert) w or wo VEV
Hambye and Strumia Phys.Rev. D88 (2013) 055022

} S sector is Dark Matter

(3) New doublet COUPLED to
SU(2)xU(1) with no VEV (dormant)

Is the Higgs Boson Associated with
Coleman-Weinberg Dynamical Symmetry Breaking?
CTH, arXiv:1401.4185 [hep-ph]. Phys Rev D.89.073003.

} S sector is visible

Simplest hypotheses
S may be:

A new doublet NOT coupled to SU(2) x U(1) (inert) w or wo VEV

e.g., Hambye and Strumia Phys.Rev. D88 (2013) 055022;
S. Iso, and Y. Orikasa, PTEP (2013) 023B08; …
"Ultra-weak sector, Higgs boson mass, and the dilaton,"
K. Allison, C. T. Hill, G. G. Ross. arXiv:1404.6268 [hep-ph]
Light Dark Matter, Naturalness, and the Radiative Origin of the Electroweak Scale, W. Altmannshofer, W. Bardeen, M Bauer, M. Carena, J. Lykken e-Print: arXiv:1408.3429 [hep-ph] …

Many, many papers on this approach!

S sector is Dark Matter

A New doublet COUPLED to SU(2)xU(1) with no VEV (dormant)

e.g., Is the Higgs Boson Associated with Coleman-Weinberg Dynamical Symmetry Breaking?
CTH, arXiv:1401.4185 [hep-ph]. Phys Rev D.89.073003….

S sector is visible at LHC

Massless two doublet potential

$$V(H_1,H_2) = \frac{\lambda_1}{2}|H_1|^4 + \frac{\lambda_2}{2}|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 e^{i\theta} + h.c.\right]$$

Two doublet RG equations

$$16\pi^2\frac{d\lambda_1(\mu)}{d\ln(\mu)} = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_1(3g_2^2+g_1^2) + \frac{3}{2}g_2^4 + \frac{3}{4}(g_1^2+g_2^2)^2 + 12\lambda_1 g_t^2 - 12 g_t^4$$

$$16\pi^2\frac{d\lambda_2(\mu)}{d\ln(\mu)} = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_2(3g_2^2+g_1^2) + \frac{3}{2}g_2^4 + \frac{3}{4}(g_1^2+g_2^2)^2 + 12\lambda_2 g_b^2 - 12 g_b^4$$

$$16\pi^2\frac{d\lambda_3(\mu)}{d\ln(\mu)} = (\lambda_1+\lambda_2)(6\lambda_3+2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_3(3g_2^2+g_1^2) + \frac{9}{4}g_2^4 + \frac{3}{4}g_1^4 - \frac{3}{2}g_1^2 g_2^2 + 6\lambda_3(g_t^2+g_b^2) - 12 g_t^2 g_b^2$$

$$16\pi^2\frac{d\lambda_4(\mu)}{d\ln(\mu)} = 2(\lambda_1+\lambda_2)\lambda_4 + 4(2\lambda_3+\lambda_4)\lambda_4 + 8\lambda_5^2 - 3\lambda_4(3g_2^2+g_1^2) + 3g_1^2 g_2^2 - 12 g_t^2 g_b^2$$

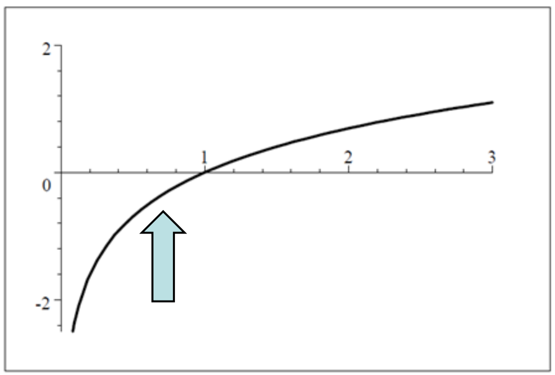
$$16\pi^2\frac{d\lambda_5(\mu)}{d\ln(\mu)} = \lambda_5[2(\lambda_1+\lambda_2) + 8\lambda_3 + 12\lambda_4 - 3(3g_2^2+g_1^2) + 2(g_t^2+g_b^2)]$$

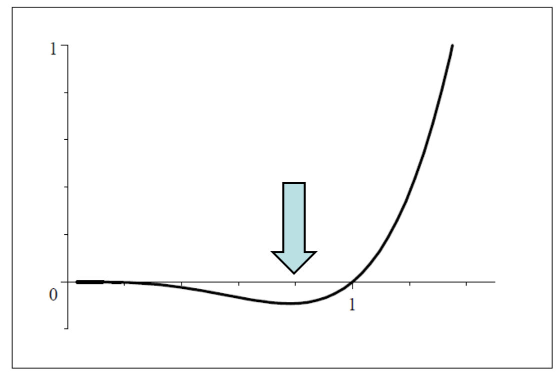
CTH, C N Leung, S Rao
NPB262 (1985) 517

The observed Higgs boson mass implies:

$$m_h^2 = -4\lambda v^2 = \beta v^2 > 0 \quad \Rightarrow \quad \beta = \left(\frac{126}{175}\right)^2 = 0.5184$$

Note that λ is negative: $\lambda = -(0.25)(0.5184) = -0.1296$





Can now solve for λ_3: $\beta = \frac{1}{16\pi^2}(12\lambda^2 + 12\lambda g^2 - 12g^4 + 4\lambda_3^2)$

$g = g_{top} \approx 1$

Solution is: $\lambda_3 = 4.8789$

Mass of New Doublet: $\sqrt{4.8789} \times (175) = 386.54$ GeV

M^2 is determined ⇨ heavy "dormant" Higgs doublet

No VeV but coupled to SU(2) xU(1):

"Dormant" Higgs Doublet (vs. "Inert")

Production, mass, and decay details are model dependent

If Dormant Higgs couples to SU(2) x U(1) but not fermions

Parity $H_2 \to -H_2$ implies stabity: $H_2^+ \to H_2^0 + (e^+\nu)$ if $M^+ > M^0$

Then H_2^0 is stable dark matter WIMP

The Dormant Doublet is pair produced above threshold near $2M_H \approx 800$ GeV

CalcHEP estimates

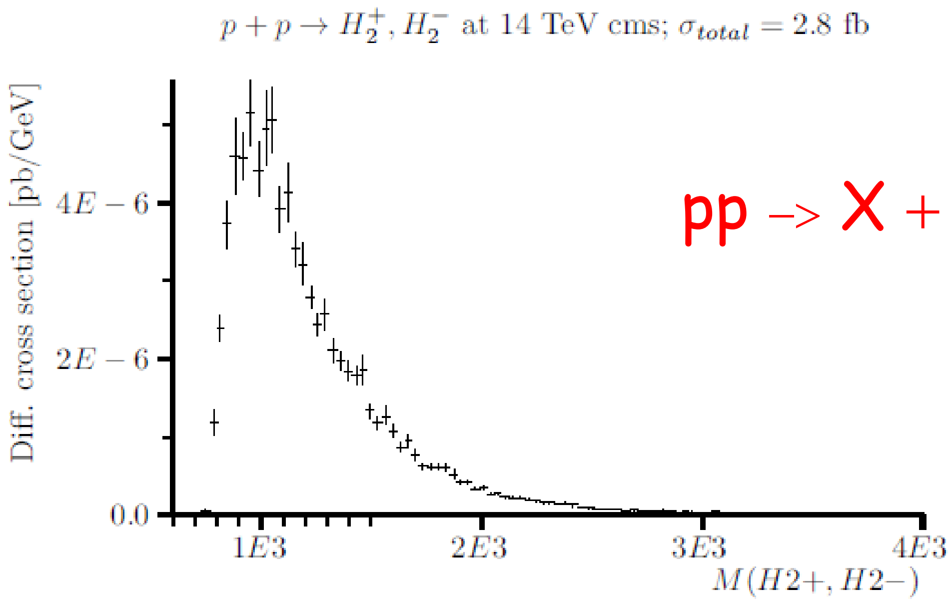


FIG. 1: H^+H^- production at LHC.

$pp \to X + (\gamma^*, Z^*, W^*, h^*) \to X + H\,H^*$

$pp \to H^0 H^0$ $\quad \sigma = 1.4$ fb

$pp \to H^+ H^-$ $\quad \sigma = 2.8$ fb

$pp \to H^+ H^0$ $\quad \sigma = 0.9$ fb

$\Gamma_{H^0 \to bb} = 45$ GeV

$\Gamma_{H+ \to tb} = 14$ GeV

Assume $g_b' = 1$

Maybe Run II?

TABLE I: Predicted decay widths and production cross-sections for the dormant Higgs bosons. We used CalcHep, and production runs CTEQ61 proton structure functions, 1.64×10^5 calls. All cross-sections are evaluated at 14 TeV cms energy with the mass of H_2 doublet set to 380 GeV/c^2. Model dependent processes have rates or cross-sections that are indicated as $\propto (g_b')^2$.

Process	value	comments
$\Gamma(H^+ \to t + \bar{b}) = \Gamma(H^- \to b + \bar{t})$	$14.5\ (g_b')^2 \pm 5 \times 10^{-5}\%$ GeV	
$\Gamma(H^0 \to b + \bar{b}) = \Gamma(A^0 \to b + \bar{b})$	$5.67\ (g_b')^2 \pm 5 \times 10^{-5}\%$ GeV	
$\Gamma(H^0 \to 2h, 3h) = \Gamma(A^0 \to 2h, 3h)$		absent in model
$pp \to (\gamma, Z^0) \to H^+H^-$	$\sigma_t = 1.4$ fb	
$pp \to (\gamma, Z) \to H^0H^0$		absent in model
$pp \to (\gamma, Z) \to A^0H^0$	$\sigma_t = 1.3$ fb	
$pp \to (\gamma, Z) \to A^0A^0$		absent in model
$pp(gg) \to h \to H^0H^0$ or A^0A^0	$\sigma_t = 1.7 \times 10^{-5}$ fb	
$pp \to W^+ \to H^0H^+$	$\sigma_t = 1.8$ fb	
$pp \to W^+ \to A^0H^+$	$\sigma_t = 1.8$ fb	
$pp \to W^- \to H^0H^-$	$\sigma_t = 0.74$ fb	
$pp \to W^- \to A^0H^-$	$\sigma_t = 0.74$ fb	
$pp \to b + \bar{b} + H^0$ or A^0	$\sigma_t = 1.8\ (g_b')^2$ pb $\pm 2.4\%$	No p_T cuts
	$\sigma_t = 67\ (g_b')^2$ fb $\pm 5\%$	$p_T(b)$ and $p_T(\bar{b}) > 50$ GeV
	$\sigma_t = 9.6\ (g_b')^2$ fb $\pm 3.5\%$	$p_T(b)$ and $p_T(\bar{b}) > 100$ GeV
$pp \to t + \bar{b} + (H^-)$	$\sigma_t = 220\ (g_b')^2$ fb	No cuts
	$\sigma_t = 44\ (g_b')^2$ fb	$p_T(t)$, $p_T(\bar{b}) > 50$ GeV
	$\sigma_t = 14\ (g_b')^2$ fb	$p_T(t)$, $p_T(\bar{b}) > 100$ GeV
$pp \to \bar{t} + b + (H^+)$	$\sigma_t = 270\ (g_b')^2$ fb	No cuts
	$\sigma_t = 46\ (g_b')^2$ fb $p_T(\bar{t})$	$p_T(b) > 50$ GeV
	$\sigma_t = 14\ (g_b')^2$ fb $p_T(\bar{t})$	$p_T(b) > 100$ GeV

CTH, arXiv:1401.4185 [hep-ph].
Phys Rev D.89.073003.

The trilinear, quartic and quintic Higgs couplings will be significantly different than in SM case

$$V_{CW}(H) = \frac{1}{2}m_h^2h^2 + \frac{5}{6\sqrt{2}v}h^3\left(\beta_1 + \frac{9}{20}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \frac{11}{48v^2}h^4\left(\beta_1 + \frac{35}{44}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \frac{1}{40\sqrt{2}v}h^5\left(\beta_1 + \frac{25}{12}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \ldots$$

$$\text{trilinear} = \frac{5}{3}\left(1 + \frac{v^2}{5m_h^2}\frac{\lambda_3^3}{8\pi^4}\right) \approx 1.75$$

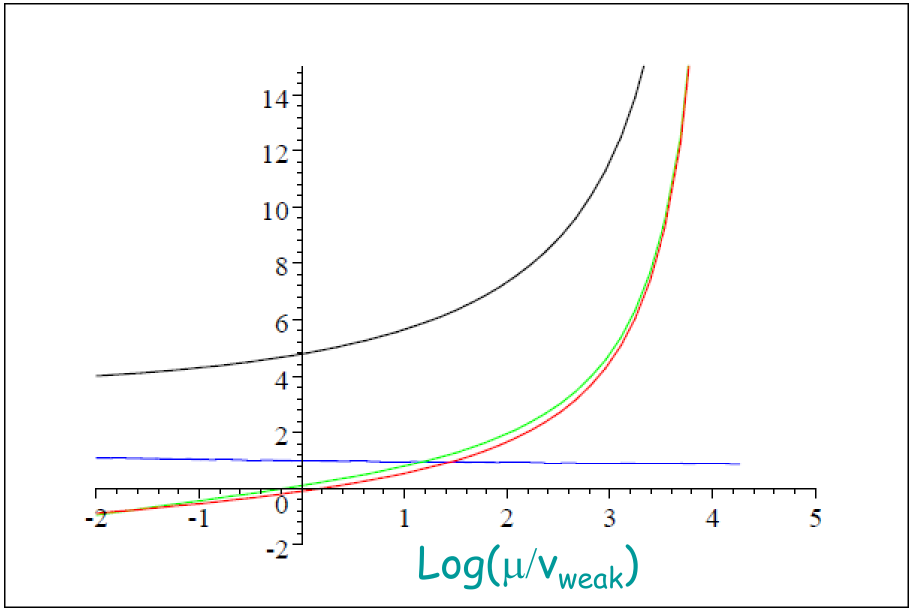
$$\text{quadrilinear} = \frac{11}{3}\left(1 + \frac{35v^2}{44m_h^2}\frac{\lambda_3^3}{8\pi^4}\right) \approx 4.43$$

$$\text{quintic} = \frac{3}{5}\left(\frac{\beta_1}{\hat{\beta}} + \frac{25}{12\hat{\beta}}\frac{\lambda_3^3}{6\pi^4}\right) \approx -8.87$$

This may be doable at LHC !

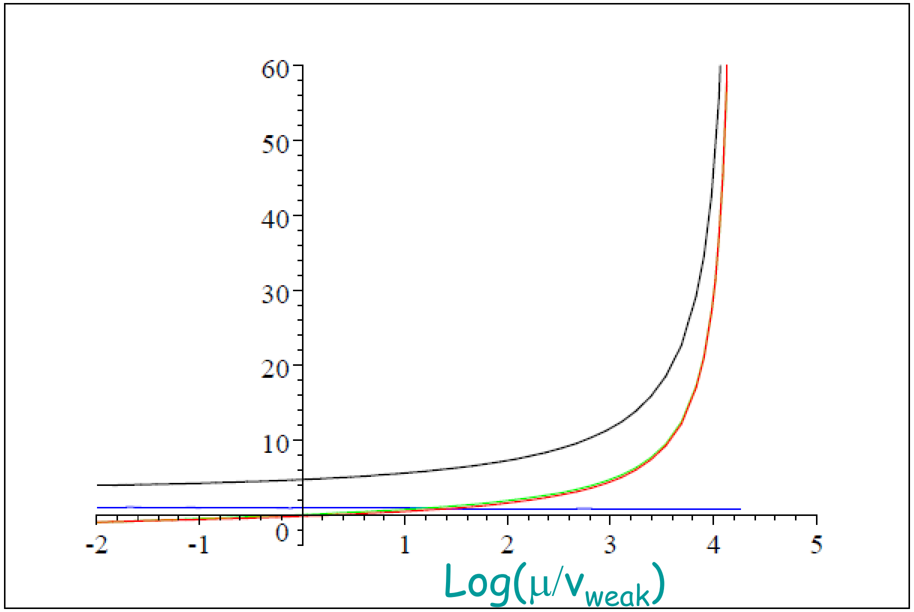
Problem:
UV Landau Pole
implying strong scale

λ_3(175 GeV) = 4.79 (black)
λ_1(175 GeV) = -0.1 (red)
λ_2(175 GeV) = 0.1 (green)
g_{top} = 1 (blue)
$\lambda_4 = \lambda_5 = 0$



Landau Pole = 10 – 100 TeV

Landau Pole ->
Composite H_2
New Strong Dynamics



e.g. Higgs mass from compositeness at a multi-TeV scale,
Hsin-Chia Cheng Bogdan Dobrescu, Jiayin Gu
e-Print: arXiv:1311.5928

Hambye-Strumia Dark Matter Portal Model
[hep-ph]1306.2329

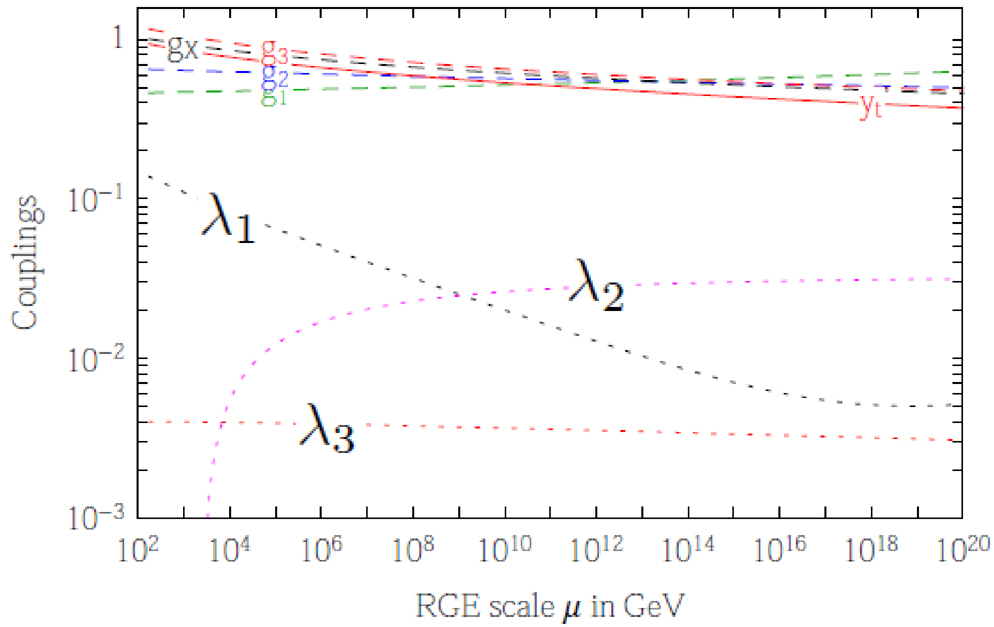
S develops a Coleman-Weinberg potential and VEV v_2

λ_3 is negative and gives the Higgs boson mass $-m^2 = \lambda_3 |S|^2$

The model does not require large quartic cc's, has sensible UV behavior

H_2 and associated gauge fields become viable dark matter

But, hard to detect !



Ultra-Weak Sector, dilaton, axion,

K. Allison, CTH, G. G. Ross, PL B738 191 (2015), NP B891, 613 (205)

$$V(H,\sigma) = \frac{\lambda}{2}(H^\dagger H)^2 + \frac{\zeta_1}{2}\sigma^2 H^\dagger H + \frac{\zeta_2}{4}\sigma^4 \qquad V(\sigma,\phi_i,\lambda_i,\zeta_i) = V_1(\phi_i,\lambda_i) + V_2(\sigma_i,\phi_i,\zeta_i)$$

σ is a complex singlet

Here the full potential decomposes into components V_1 and V_2 where $\frac{\delta}{\delta\sigma_i}V_1 = \frac{\delta}{\delta\zeta_i}V_1 = 0$, and $\frac{\delta}{\delta\lambda_i}V_2 = 0$.

The ζ_i are technically naturally small: shift symmetry

$$\begin{aligned}
\beta_\lambda &= \frac{d\lambda(\mu)}{d\ln(\mu)} = \frac{1}{16\pi^2}\Big(12\lambda^2 - 3\lambda(3g_2^2+g_1^2) \\
&\quad + \frac{3}{4}(g_1^2+g_2^2)^2 + \frac{3}{2}g_2^4 + 12\lambda g_t^2 - 12g_t^4 + \zeta_1^2\Big)\,, \\
\beta_1 &= \frac{d\zeta_1(\mu)}{d\ln(\mu)} = \frac{1}{16\pi^2}\Big(6\zeta_1\zeta_2 + 6\zeta_1\lambda + 4\zeta_1^2 \\
&\quad - \frac{3}{2}\zeta_1(3g_2^2+g_1^2) + 6\zeta_1 g_t^2\Big)\,, \\
\beta_2 &= \frac{d\zeta_2(\mu)}{d\ln(\mu)} = \frac{1}{16\pi^2}\left(18\zeta_2^2 + 2\zeta_1^2\right).
\end{aligned}$$

$\langle\sigma\rangle = f$ can be very large, e.g. GUT scale

Ultra-weak sector, dilaton, axion,

$$f \gtrsim 10^{10}\ \text{GeV}.$$

Incorporates the axion,
GUT scale breaking, f,
yields the Higgs boson mass

$m_{axion} \sim \Lambda^2{}_{QCD}/f$

$m_{Dilaton} \sim m^2{}_{Higgs}/f$

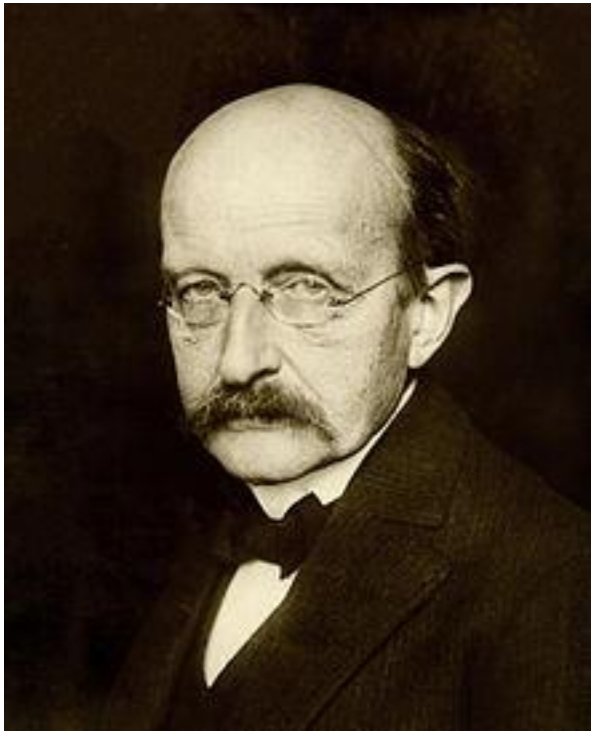
$$m_\sigma \approx 0.179 \left(\frac{10^{10}\ \text{GeV}}{f}\right)\ \text{keV}. \qquad (20)$$

The model therefore predicts a low mass 0^+ particle for $f \gtrsim 10^{10}$ GeV.

The ζ_i are technically naturally small.
Extend to include right-handed neutrinos;

(σ, ν_R) can form an N = 1 SUSY multiplet;
SUSY broken by ζ_i allows tiny neutrino Dirac masses.

A Conjecture:



Max Planck

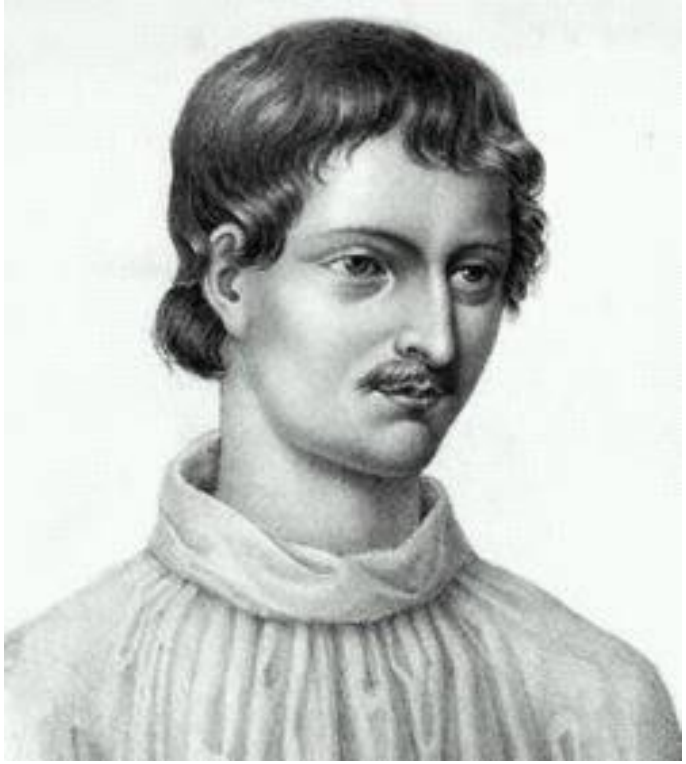
All mass is a quantum phenomenon.

$\hbar \rightarrow 0 \rightarrow$ Classical scale symmetry

Conjecture on the physical implications of the scale anomaly:
M. Gell-Mann 75th birthday talk: C. T. Hill hep-th/0510177

Musings:
What if it's true?

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The $\hbar \to 0$ limit of nature is exactly scale invariant.

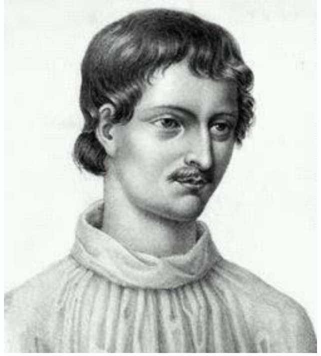


(a heretic)

“Predictions” of the Conjecture:

We live in D=4!

$$T^{\mu}_{\mu} = \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} - \frac{D}{4}\, \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$



Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions!

“Predictions” of the Conjecture:

We live in D=4!

$$T^{\mu}_{\mu} = \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} - \frac{D}{4} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions!

Does the Planck Mass Come From Quantum Mechanics?

Can String Theory be an effective theory?

... or Weyl Gravity? (A-gravity?)

Weyl Gravity is Renormalizable!

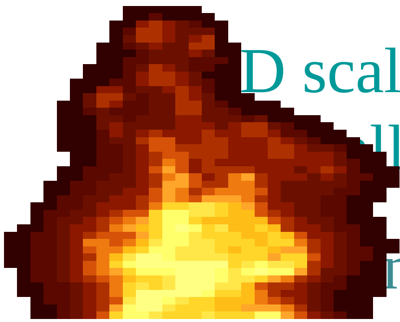
Weyl Gravity is QCD-like:

$$\frac{1}{h^2}\sqrt{-g}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)$$

"Predictions" of the Conjecture:

We live in D=4! $$T^{\mu}_{\mu} = \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \frac{D}{4}\,\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$$

Cosmological constant is zero in classical limit



D scale is generated in this way; Hierarchy
lly generated

n the Weak Interactions!

⇨ String Theory RULED OUT (classical string scale)

⇨ Weyl Gravity?

Weyl Gravity is Renormalizeab =4!

Weyl Gravity in D=4 is QCD $_{\nu}R^{\mu\nu} - \frac{1}{3}R^2)$

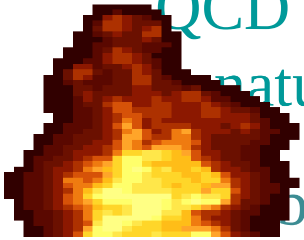
⇨ The Planck Mass Comes From Quantum Mechanics!

See: **Conjecture on the physical implications of the scale anomaly.**
Christopher T. Hill (Fermilab) . **hep-th/0510177** (and refs.therein)

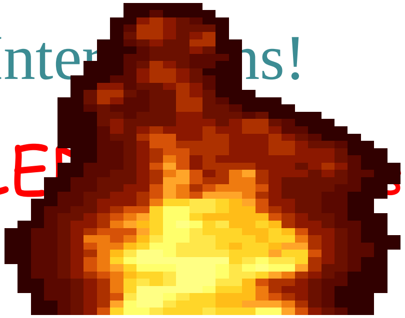
“Predictions” of the Conjecture:

We live in D=4! $$T^\mu_\mu = \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} - \frac{D}{4}\, \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$

Cosmological constant is zero in classical limit

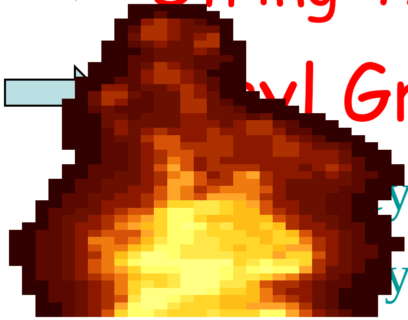


QCD scale is generated in this way; Hierarchy naturally generated



...ble in the Weak Inter...ns!

⇨ String Theory RULE... ...ical string scale)



⇨ ...yl Gravity:

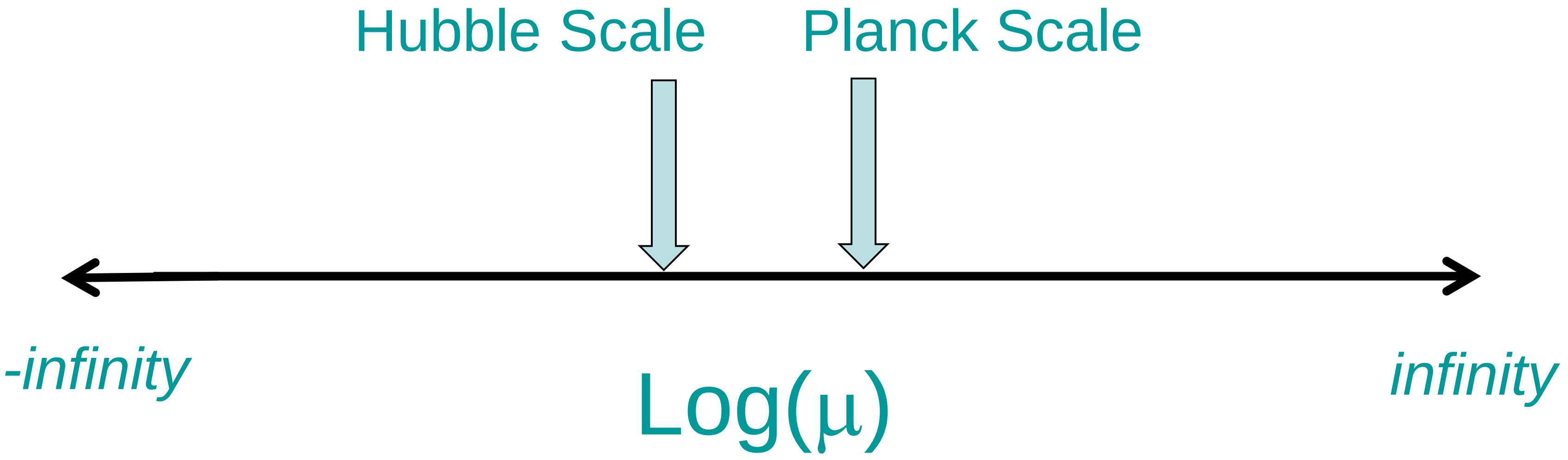
...yl Gravity is Renormalizeable! Predicts D=4!

...yl Gravity in D=4 is QCD-like: $\frac{1}{h^2}\sqrt{-g}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)$

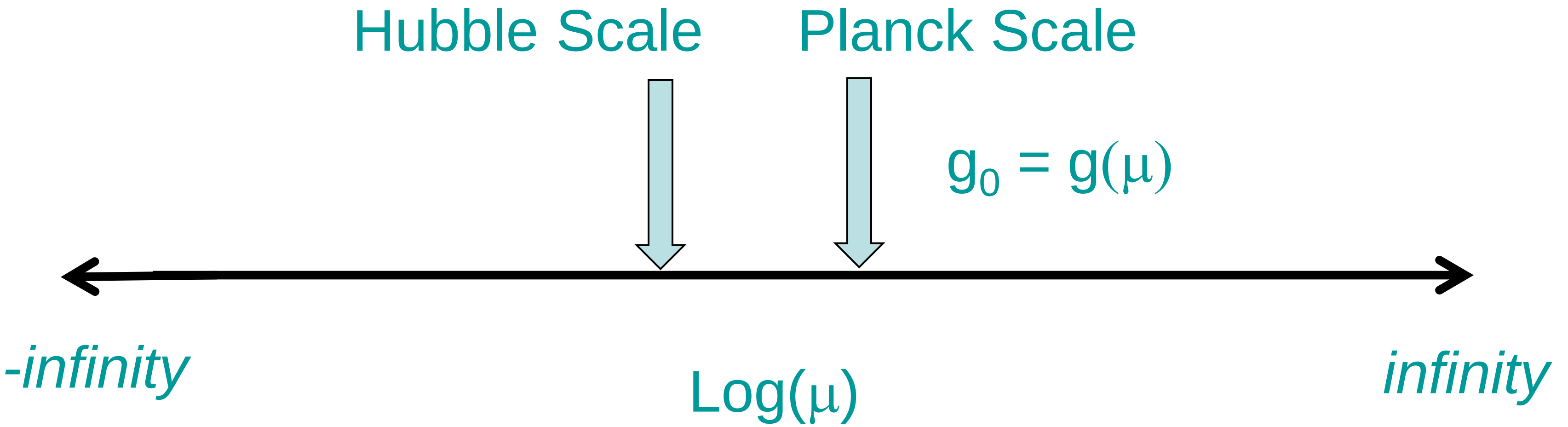
⇨ The Planck Mass Comes From Quantum Mech...ics!

⇨ We Live in a Scaloplex !!!

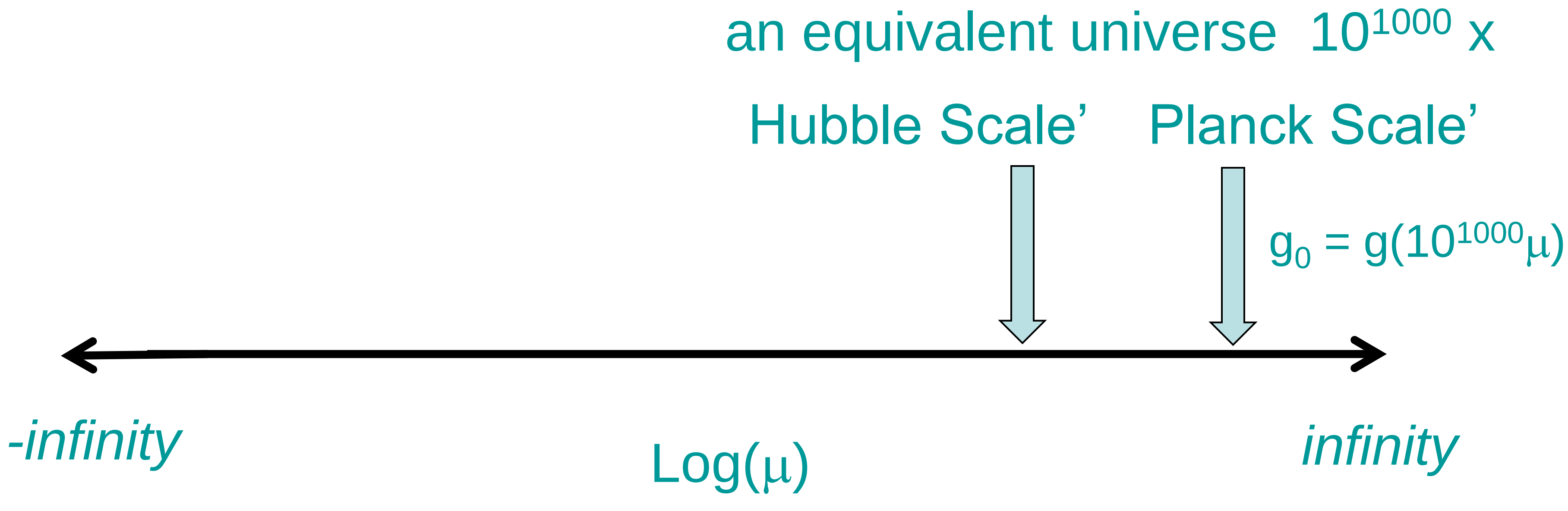
The "Scaloplex" (scale continuum) infinite, uniform, and classically isotropic



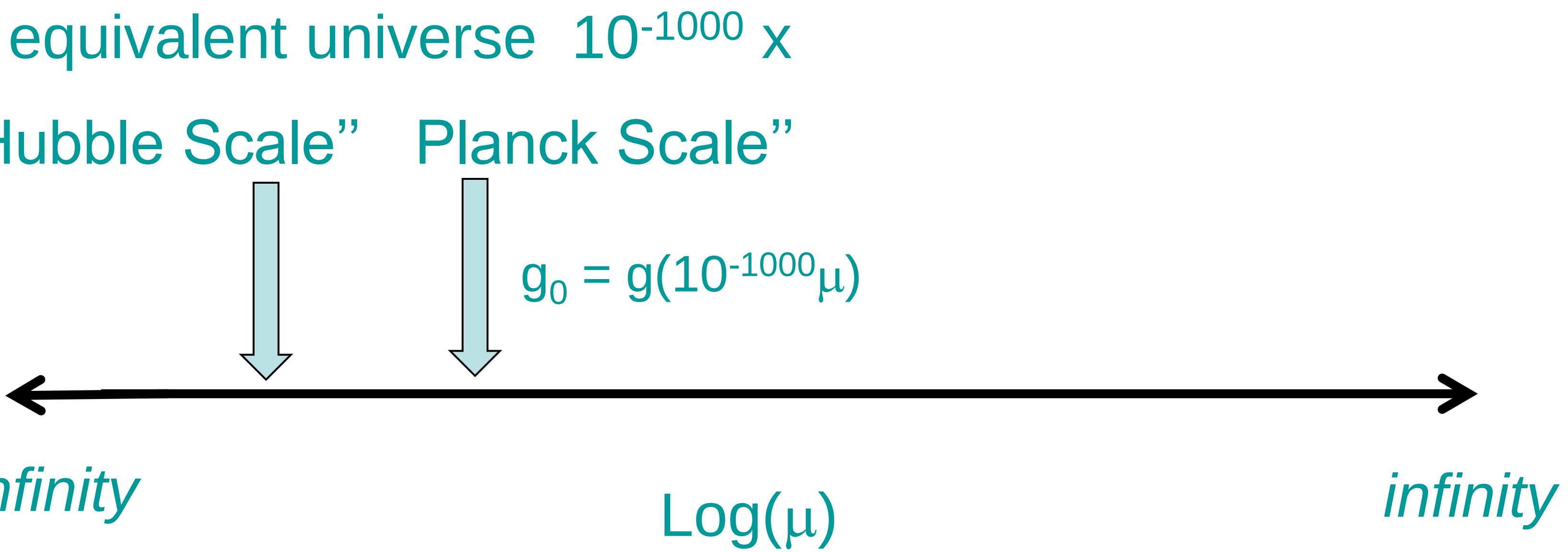
Physics is determined by local values of dimensionless coupling constants



Physics is determined by local values of dimensionless coupling constants



Physics is determined by local values of dimensionless coupling constants



Lack of additive scales:
Is the principle of scale recovery actually a "Principle of Locality" in Scale?

Physical Mass Scales, generated by e.g. Coleman-Weinberg or QCD-like mechanisms, are Local in scale, and do not add to scales far away in the scaloplex

E.g, "shining" with Yukawa suppression in extra dimensional models.

Does Coleman-Weinberg mechanism provide immunity from additive scales?

Conjecture on a solution to the Unitarity Problem of Weyl Gravity

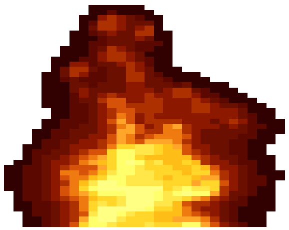
CTH, P. Agrawal

M_{Planck} arises via QCD-like mechanism.

Theory becomes Euclidean for $\mu > M_{Planck}$ (infinite temperature or instanton dominated)

Time is emergent for $\mu \ll M_{Planck}$

Passage through the Planck Scale



D=4 quantum theory T = 0 → D=5 quantum theory T = 0 → D=5 T -> μ theory → D=5 T -> infinity theory -> D=4 Euclidean Theory

Log(μ)

Time emerges
$T < M_{Planck}$

Euclidean D=4

Hawking-Hartle Boundary Condition?

I think this is a profoundly important scientific question:

Is the Higgs potential Coleman-Weinberg?

- Examined a "maximally visible" scheme
- Dormant Higgs Boson from std 2-doublet scheme $M \approx 400$ GeV
- May be observable, LHC run II, III?
- Higgs trilinear and quartic couplngs non-standard
- UV problem -> new strong scale $<$ 100 TeV
- or New bosons may be dark matter

Perhaps we live in a world where all
Mass comes from quantum effects
No classical mass input parameters.

Conclusions:

An important answerable scientific question:

Is the Higgs potential Coleman-Weinberg?

- We examined a "maximally visible" scheme
- Dormant Higgs Boson from std 2-doublet scheme

$M \approx 386$ GeV

- May be observable, LHC run II, III?
- Higgs trilinear … couplings non-standard

or New bosons may be dark matter

Perhaps we live in a world where all
mass comes from quantum effects
No classical mass input parameters.

Everyone is still missing the
solution to the scale recovery problem!

End

Why do couplings run with field VEV?

$$S = \int d^4x\, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

Equation of motion

$$\partial^\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = \partial^2 \phi + V'(\phi) = 0 \qquad V'(\phi) = \frac{\delta}{\delta \phi} V(\phi)$$

$$x^{\mu\prime} = x^\mu - \zeta^\mu, \qquad \phi'(x') = \phi(x)$$

$$\begin{aligned} \delta dx^\mu &= -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu)\, dx^\lambda \\ \delta \partial_\mu &= (\partial^\nu \zeta_\mu) \partial_\nu \\ \delta d^4x &= -(\partial_\mu \zeta^\mu) d^4x \end{aligned}$$

$$\begin{aligned} \delta S = \int d^4x [&-\frac{1}{2} (\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi \\ &+ (\partial_\mu \zeta^\mu) V(\phi)] \equiv -\frac{1}{2} \int d^4x \, [(\partial_\mu \zeta_\nu)\, T^{\mu\nu}] \end{aligned}$$

Why do couplings run with field VEV?

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

Equation of motion

$$\partial^\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = \partial^2 \phi + V'(\phi) = 0 \qquad V'(\phi) = \frac{\delta}{\delta \phi} V(\phi)$$

Infinitesimal translation (diffeomorphism)

$$x^{\mu\prime} = x^\mu - \zeta^\mu,$$

$$\phi'(x') = \phi(x)$$

$$\left\{ \begin{array}{rcl} \delta dx^\mu & = & -d\zeta^\mu(x) = -\left(\partial_\lambda \zeta^\mu\right) dx^\lambda \\ \delta \partial_\mu & = & (\partial^\nu \zeta_\mu) \partial_\nu \\ \delta d^4x & = & -(\partial_\mu \zeta^\mu) d^4x \end{array} \right.$$

Why do couplings run with field VEV?

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

Equation of motion

$$\partial^\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = \partial^2 \phi + V'(\phi) = 0 \qquad V'(\phi) = \frac{\delta}{\delta \phi} V(\phi)$$

Infinitesimal translation (diffeomorphism)

$$x^{\mu\prime} = x^\mu - \zeta^\mu,$$

$$\phi'(x') = \phi(x)$$

$$\left\{ \begin{array}{lcl} \delta dx^\mu & = & -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu)\, dx^\lambda \\ \delta \partial_\mu & = & (\partial^\nu \zeta_\mu) \partial_\nu \\ \delta d^4x & = & -(\partial_\mu \zeta^\mu) d^4x \end{array} \right.$$

$$\delta S = \int d^4x [-\frac{1}{2}(\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi + (\partial_\mu \zeta^\mu) V(\phi)] \equiv -\frac{1}{2} \int d^4x \, [(\partial_\mu \zeta_\nu)\, T^{\mu\nu}]$$

Stress tensor:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \eta_{\mu\nu} \left(\frac{1}{2} \partial_\rho \phi \partial^\rho \phi - V(\phi) \right)$$

$$\partial^\mu T_{\mu\nu} = \partial^2 \phi \partial_\nu \phi + \partial_\mu \phi \partial^\mu \partial_\nu \phi - \partial_\nu \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

Equation of motion:

$$= \partial_\nu \phi \left(\partial^2 \phi + V'(\phi) \right)$$

can choose $\zeta^\mu = -\epsilon x^\mu$

$$\delta S = \frac{1}{2} \int d^4 x \left[\left(\partial_\mu \epsilon x_\nu \right) T^{\mu\nu} \right]$$

Scale Current:

$$\frac{\delta S}{\partial_\mu \epsilon} \equiv S^\mu = x_\nu T^{\mu\nu} \qquad \partial_\mu S^\mu = T^\mu_\mu$$

Scale Current not conserved with canonical stress tensor:

$$\partial_\mu S^\mu = T^\mu_\mu \qquad T^\mu_\mu = -\partial_\rho \phi \partial^\rho \phi + 4V(\phi)$$

The "Improved Stress Tensor"

$$S \to S + S_2 \qquad S_2 = \xi \int d^4x \; \partial^2 \phi^2$$

$$\delta S_2 = \xi \int d^4x[-(\partial_\mu \zeta^\mu)\partial^2\phi^2 + \partial^\mu((\partial^\nu \zeta_\mu)\partial_\nu \phi^2 \quad \equiv \int d^4x(\partial_\mu \zeta_\nu)[Q^{\mu\nu}$$

$$Q_{\mu\nu} = \xi(\partial_\mu \partial_\upsilon \phi^2 - \eta_{\mu\nu}\partial^2\phi^2) \qquad \widetilde{T}_{\mu\nu} = T_{\mu\nu} + Q_{\mu\nu} \qquad \xi = 1/6$$

$$= \frac{2}{3}\partial_\mu \phi \partial_\nu \phi - \frac{1}{6}\eta_{\mu\nu}\partial_\rho \phi \partial^\rho \phi - \frac{1}{3}\phi \partial_\mu \partial_\upsilon \phi \quad + \frac{1}{3}\eta_{\mu\nu}\phi \partial^2 \phi + \eta_{\mu\nu} V(\phi)$$

Trace of improved stress tensor

$$\widetilde{T}^{\mu}_{\mu} = \phi \partial^2 \phi + 4V(\phi) = -\phi \frac{\delta}{\delta\phi} V(\phi) + 4V(\phi)$$

Traceless for a scale invariant theory

$$\phi \frac{\delta}{\delta\phi} V(\phi) = DV(\phi)$$

D = 4 →

$$V(\phi) = \frac{\lambda}{4}\phi^4,$$

In general, $D = 4 + \gamma$

$$\frac{\delta}{\delta\phi}\lambda(\phi) = \beta(\lambda)$$

$$\widetilde{T}^{\mu}_{\mu} = -\frac{\beta(\lambda)}{\lambda} V(\phi)$$

Trace anomaly associate with running coupling

A Canonical Picture of Scale Breaking

Stress tensor:

$$T_{\mu\nu} \;=\; \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V(\phi)\right)$$

Scale Current:

$$\frac{\delta S}{\partial_\mu \epsilon} \equiv S^\mu = x_\nu T^{\mu\nu} \qquad \partial_\mu S^\mu = T^\mu_\mu$$

Scale Current not conserved with canonical stress tensor:

$$\partial_\mu S^\mu = T^\mu_\mu \qquad T^\mu_\mu = -\partial_\rho\phi\partial^\rho\phi + 4V(\phi)$$

Derivation of stress tensor: (diffeomorphism, constant metric)

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$x^{\mu\prime} = x^\mu - \zeta^\mu,$$

$$\phi'(x') = \phi(x)$$

$$\left\{ \begin{array}{rcl} \delta dx^\mu & = & -d\zeta^\mu(x) = -\left(\partial_\lambda \zeta^\mu\right) dx^\lambda \\ \delta \partial_\mu & = & (\partial^\nu \zeta_\mu) \partial_\nu \\ \delta d^4x & = & -(\partial_\mu \zeta^\mu) d^4x \end{array} \right.$$

$$\begin{aligned} \delta S &= \int d^4x [-\frac{1}{2}(\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi \\ &\quad +(\partial_\mu \zeta^\mu) V(\phi)] \end{aligned}$$

$$\equiv -\frac{1}{2} \int d^4x \left[\left(\partial_\mu \zeta_\nu \right) T^{\mu\nu} \right]$$

Derivation of stress tensor: (diffeomorphism, constant metric)

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$x^{\mu\prime} = x^\mu - \zeta^\mu ,$$

$$\phi'(x') = \phi(x)$$

$$\left\{ \begin{array}{rcl} \delta dx^\mu & = & -d\zeta^\mu(x) = -\left(\partial_\lambda \zeta^\mu\right) dx^\lambda \\ \delta \partial_\mu & = & (\partial^\nu \zeta_\mu) \partial_\nu \\ \delta d^4x & = & -(\partial_\mu \zeta^\mu) d^4x \end{array} \right.$$

$$\begin{aligned} \delta S &= \int d^4x [-\frac{1}{2}(\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi \\ &\quad + (\partial_\mu \zeta^\mu) V(\phi)] \end{aligned}$$

$$\equiv -\frac{1}{2} \int d^4x \left[(\partial_\mu \zeta_\nu) T^{\mu\nu} \right]$$

Derivation of the "Improved Stress Tensor"

$$S \to S + S_2 \qquad S_2 = \xi \int d^4x \, \partial^2 \phi^2$$

$$\delta S_2 \;=\; \xi \int d^4x [-(\partial_\mu \zeta^\mu)\partial^2\phi^2 + \partial^\mu((\partial^\nu \zeta_\mu)\partial_\nu \phi^2)$$

$$\equiv \int d^4x (\partial_\mu \zeta_\nu)[Q^{\mu\nu}]$$

$$Q_{\mu\nu} \;=\; \xi(\partial_\mu \partial_\upsilon \phi^2 - \eta_{\mu\nu}\partial^2\phi^2) \qquad \widetilde{T}_{\mu\nu} \;=\; T_{\mu\nu} + Q_{\mu\nu} \qquad \xi{=}1/6$$

$$= \frac{2}{3}\partial_\mu \phi \partial_\nu \phi - \frac{1}{6}\eta_{\mu\nu}\partial_\rho \phi \partial^\rho \phi - \frac{1}{3}\phi \partial_\mu \partial_\upsilon \phi \quad + \frac{1}{3}\eta_{\mu\nu}\phi \partial^2 \phi + \eta_{\mu\nu} V(\phi)$$

Classical Standard Model Higgs Potential

$$m_{Higgs}^2 \approx \frac{1}{2} m_{top}^2 \qquad m_{top} \approx v_{weak}$$

$$v_{weak} \approx 175 \text{ GeV}$$

$$\mathcal{L} = \mathcal{L}_{kinetic} + g_t \overline{\psi}_L t_R H + h.c. - \frac{\lambda}{2} \left(H^\dagger H - v_{weak}^2 \right)^2$$

$$g_t \approx 1, \qquad \lambda \approx \frac{1}{4}$$

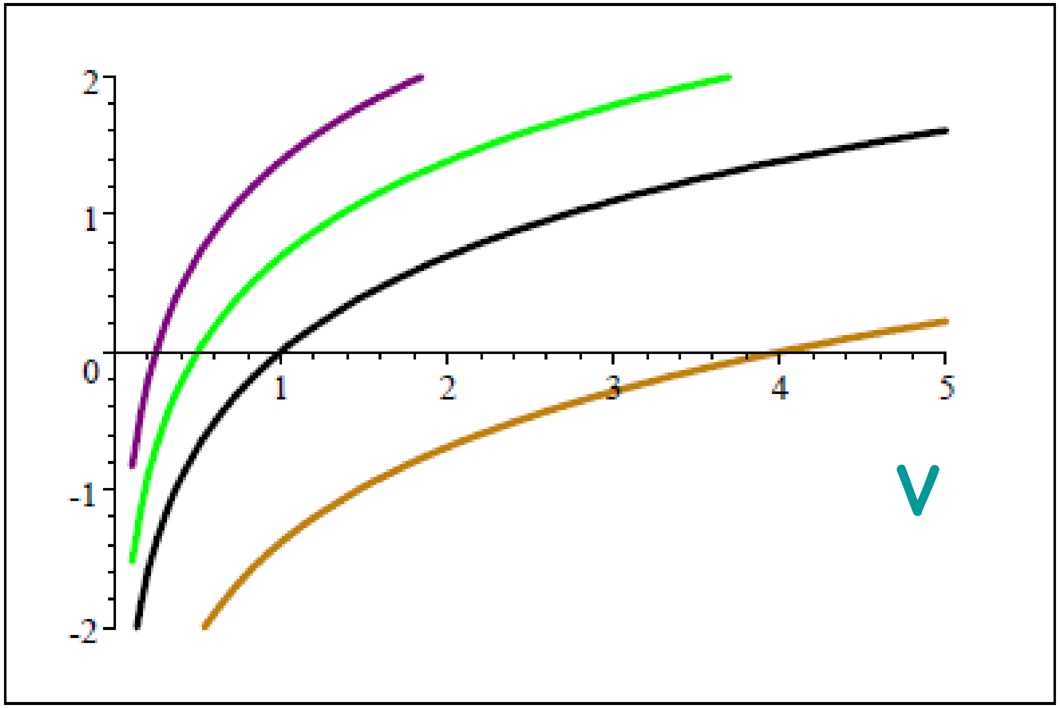
It is possible that we need
only the strongest coupled SUSY
partners to the Higgs Boson to be nearby in mass

e.g., "Natural SUSY" : A Light Stop

The More minimal supersymmetric standard model
A, G. Cohen , D.B. Kaplan, A.E. Nelson Phys.Lett. B388 (1996) 588-598

Quantum loops generate a logarithmic “running” of the quartic coupling

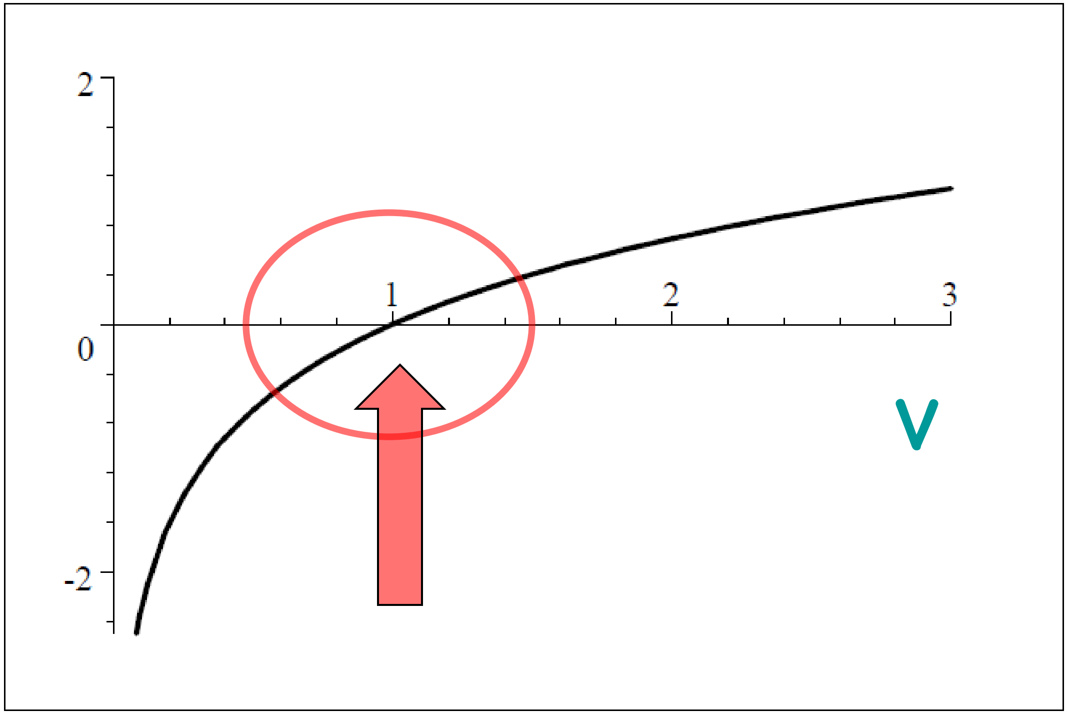
$\lambda(v) \propto \hbar\,\beta \log(v/M)$



running couplings have many possible trajectories, each parameterized by some M

Quantum loops can generate a logarithmic "running" of the quartic coupling

$\lambda(v) \propto \hbar\,\beta \log(v/M)$



this is the relevant behavior

$\tilde{\lambda}$ passing from < 0 to > 0 requires $\beta > 0$