Renormalization of Subleading Dijet Operators in Soft-Collinear Effective Theory

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Work completed with Simon Freedman:
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Physics
UNIVERSITY OF TORONTO

THEP Seminar
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Outline

Punchline: We calculated the anomalous dimensions of subleading dijet operators in Soft-Collinear Effective Theory – a step towards improving theoretical precision in the thrust distribution, allowing for an improved determination of $\alpha_s(M_Z)$

The Thrust Observable

Factorization/Resummation in General

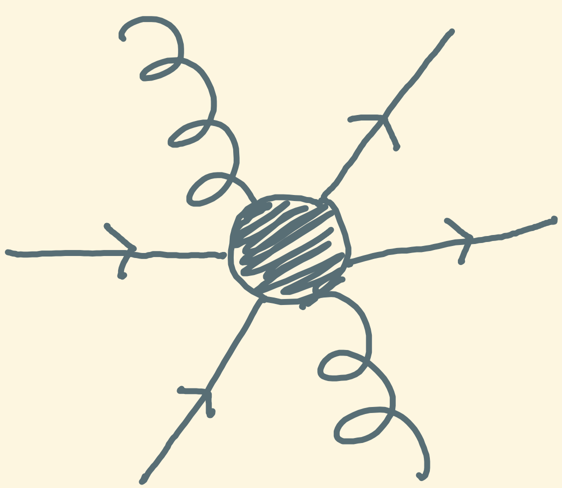
Soft-Collinear Effective Theory (SCET)

Applying SCET to Thrust

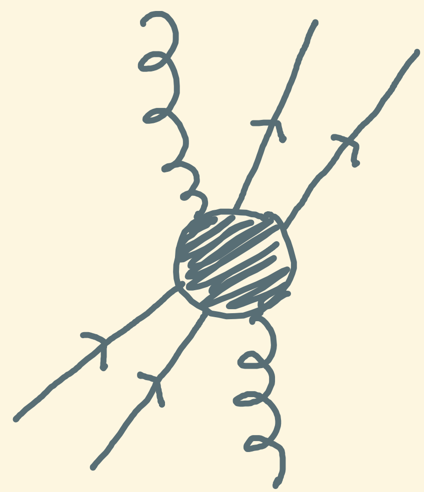
Results

Thrust

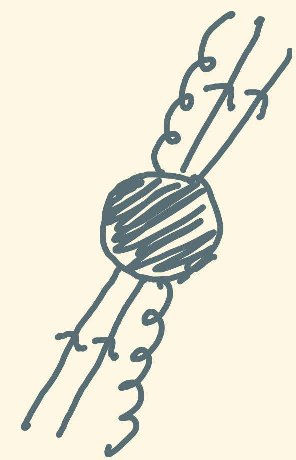
Thrust, τ, parametrizes the geometry of the energy-momentum flow of an event:



$\tau \sim 0.5$



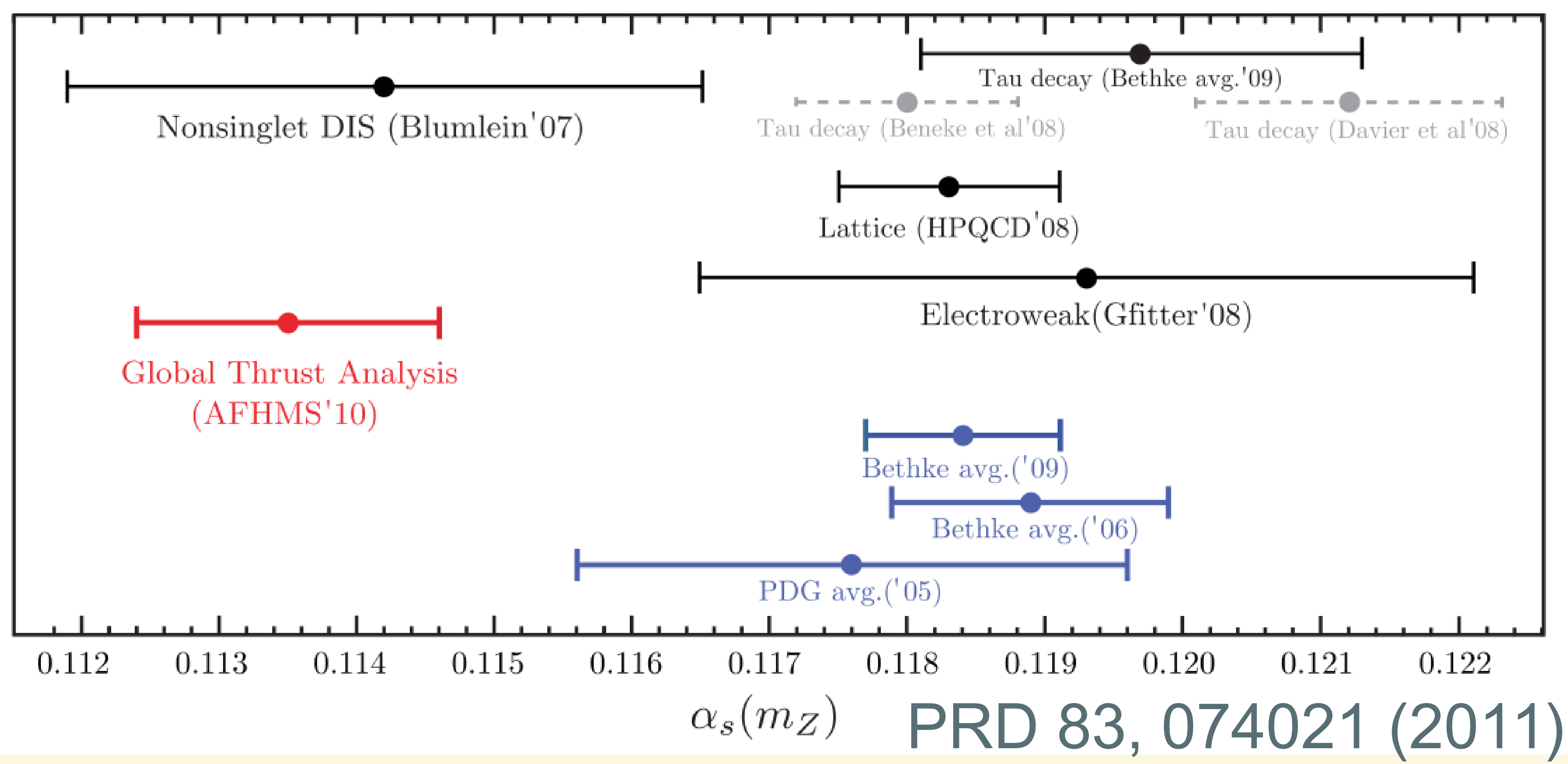
$0.5 \gtrsim \tau \gtrsim 0$



$\tau \sim 0$

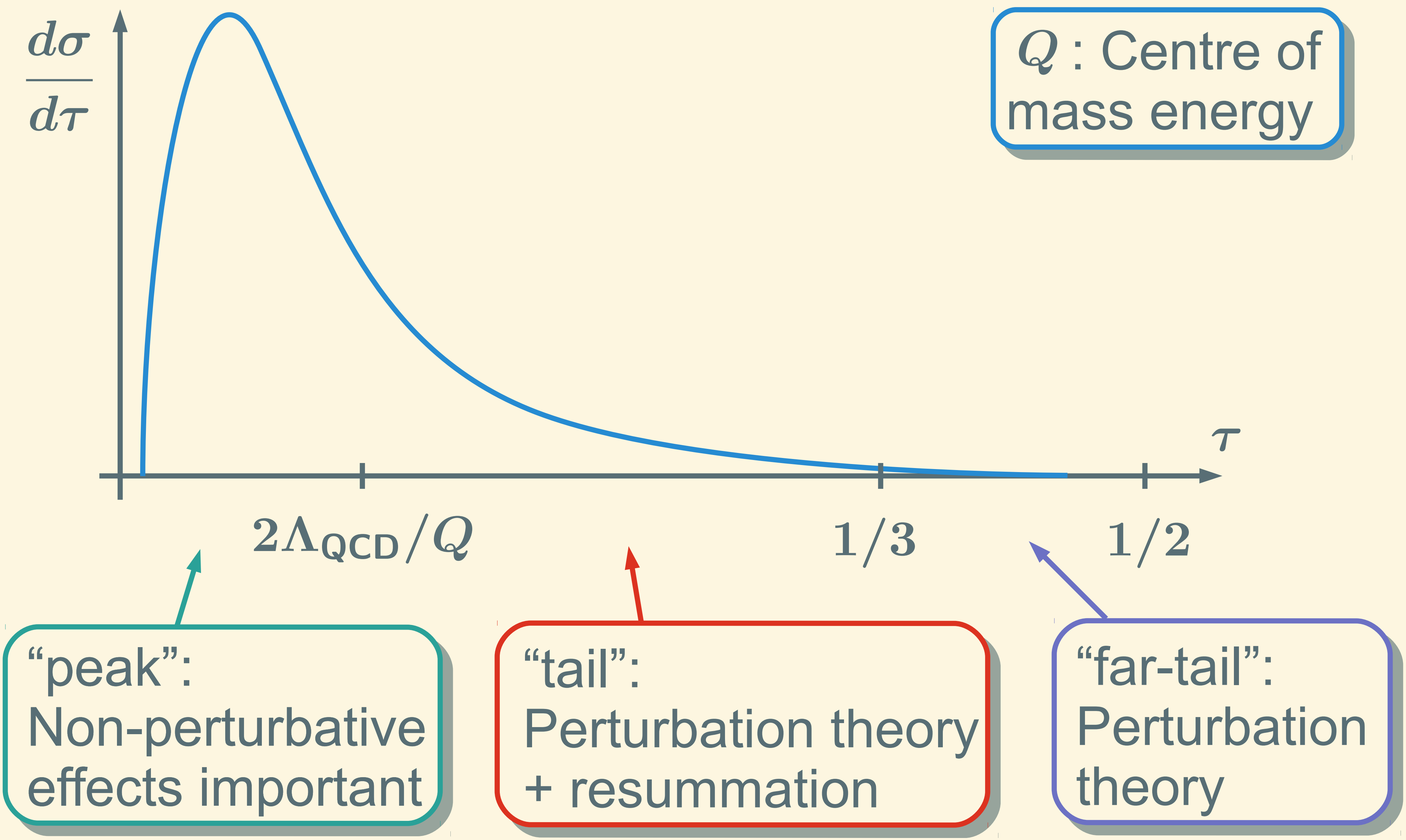
Thrust

By fitting thrust distributions to LEP data, we can make precise determinations of $\alpha_s(M_Z)$



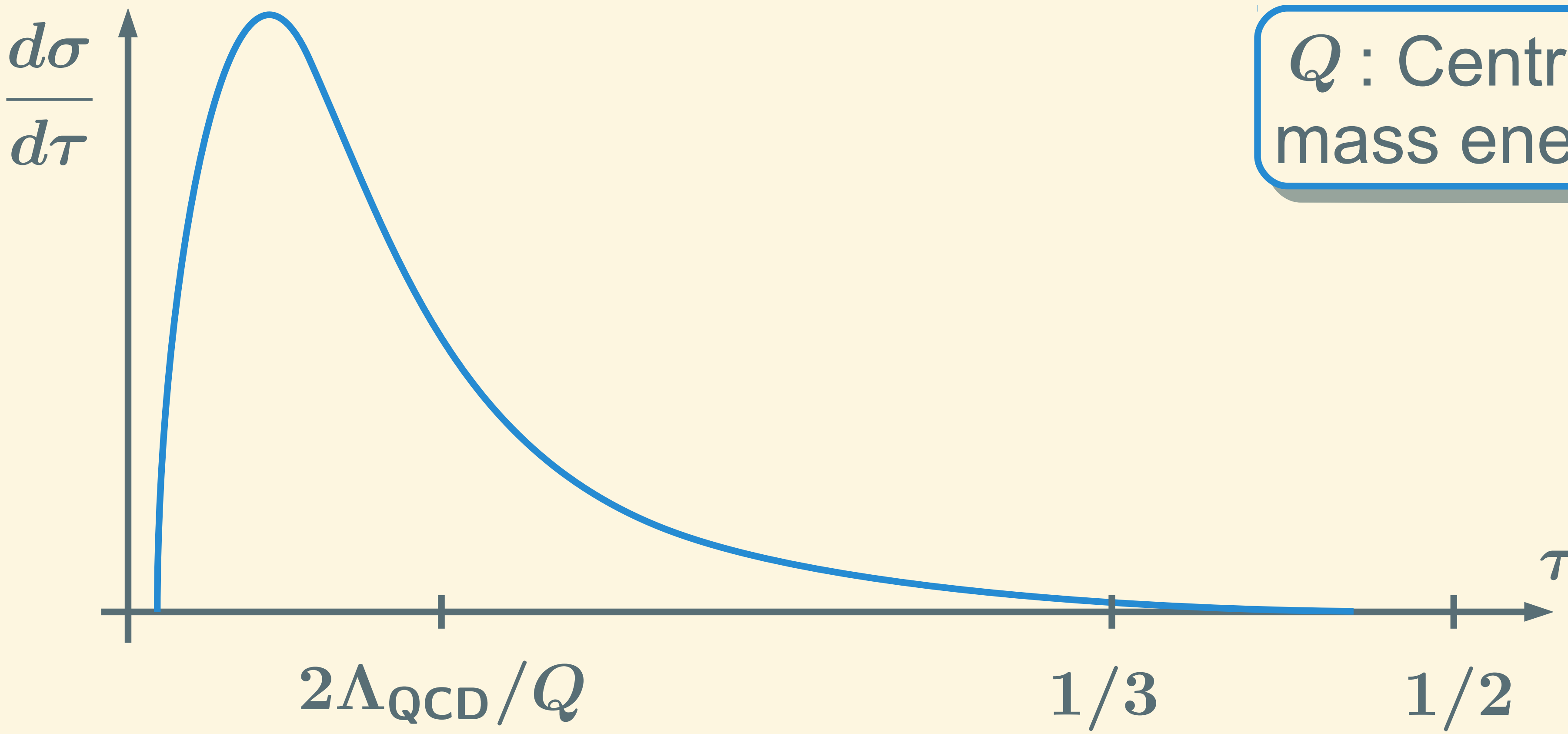
PRD 83, 074021 (2011)

Thrust



$\frac{d\sigma}{d\tau}$
τ
$2\Lambda_{\text{QCD}}/Q$
$1/3$
$1/2$
Q : Centre of mass energy
“peak”: Non-perturbative effects important
“tail”: Perturbation theory + resummation
“far-tail”: Perturbation theory

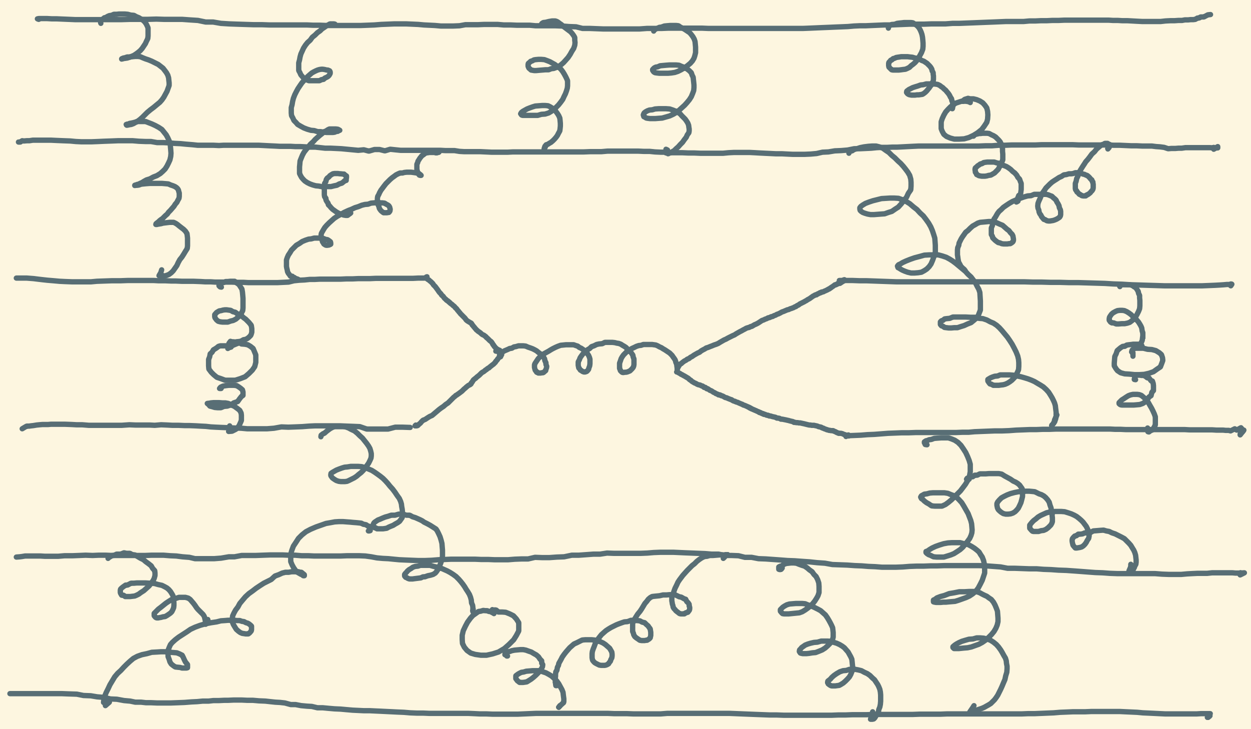
Thrust



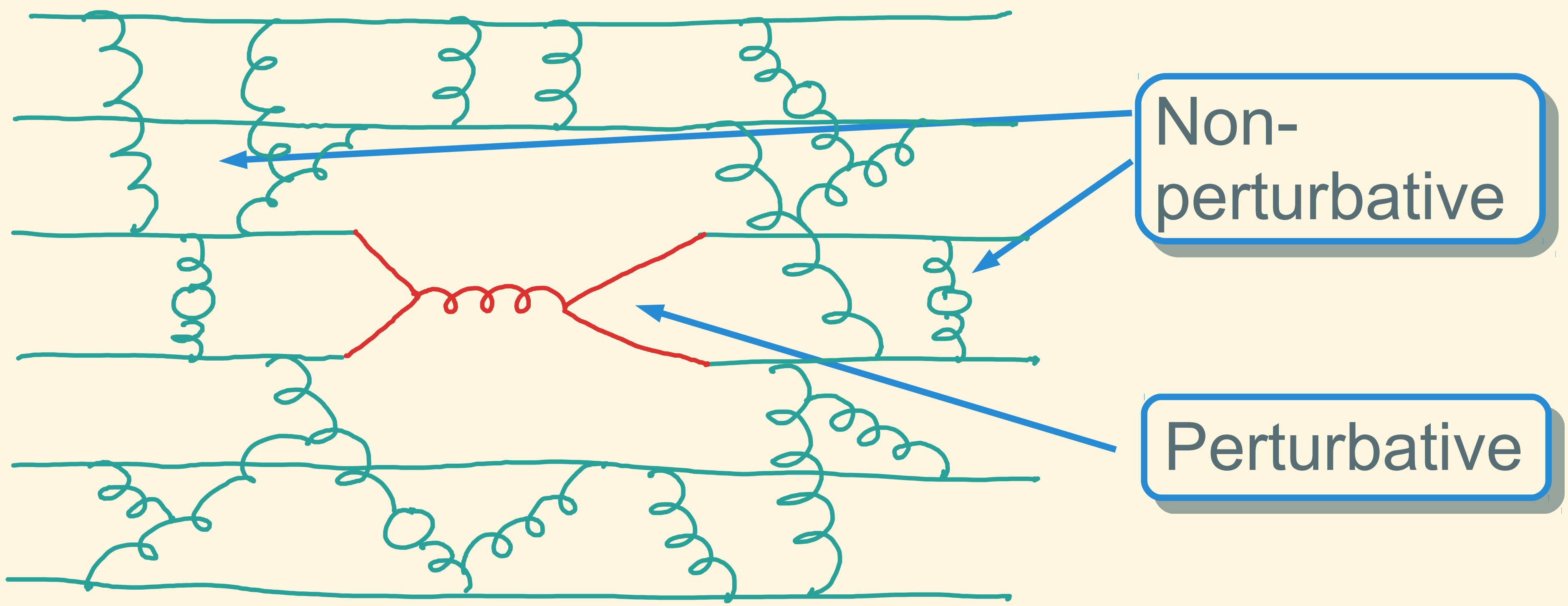
Q : Centre of mass energy

$$\frac{d\sigma}{d\tau} = \frac{d\sigma}{d\tau}^{(0)} + \tau \frac{d\sigma}{d\tau}^{(1)} + \cdots$$

Factorization



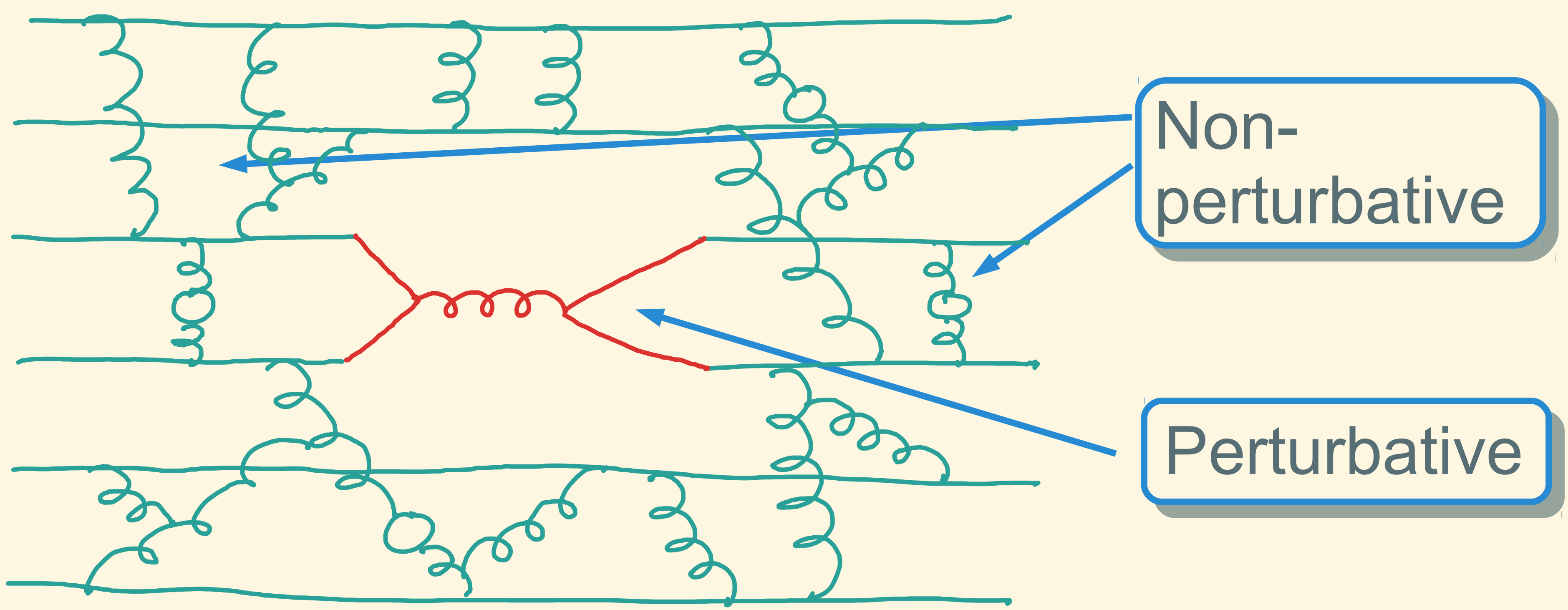
Factorization



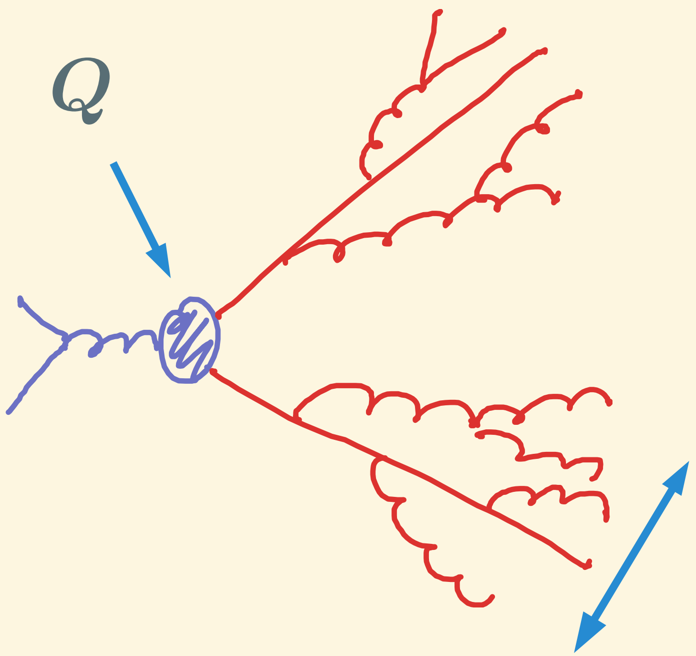
Factorization

Factorization Theorem:

$$d\sigma = \sum_{ij} d\sigma_{ij}^{\text{part}} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$ (inclusive)



Factorization



An event with thrust τ

Typical relative momentum of particles within a jet:

$$p_\perp \sim \sqrt{\tau} Q$$

$$\log \frac{\sqrt{\tau} Q}{Q} = \log(\tau)/2$$

Resummation

$$R = \int_0^\tau d\tau' \frac{d\sigma}{d\tau'}$$

$$L = \log(\tau)$$

$$\begin{aligned} R = 1 &+ \alpha_s \left(R_{12}L^2 + R_{11}L + R_{10} \right) \\ &+ \alpha_s^2 \left(R_{24}L^4 + R_{23}L^3 + R_{22}L^2 + R_{21}L + R_{20} \right) \\ &+ \alpha_s^3 \left(R_{36}L^6 + R_{35}L^5 + R_{34}L^4 + R_{33}L^3 + \cdots \right) \\ &\quad\ \ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \end{aligned}$$

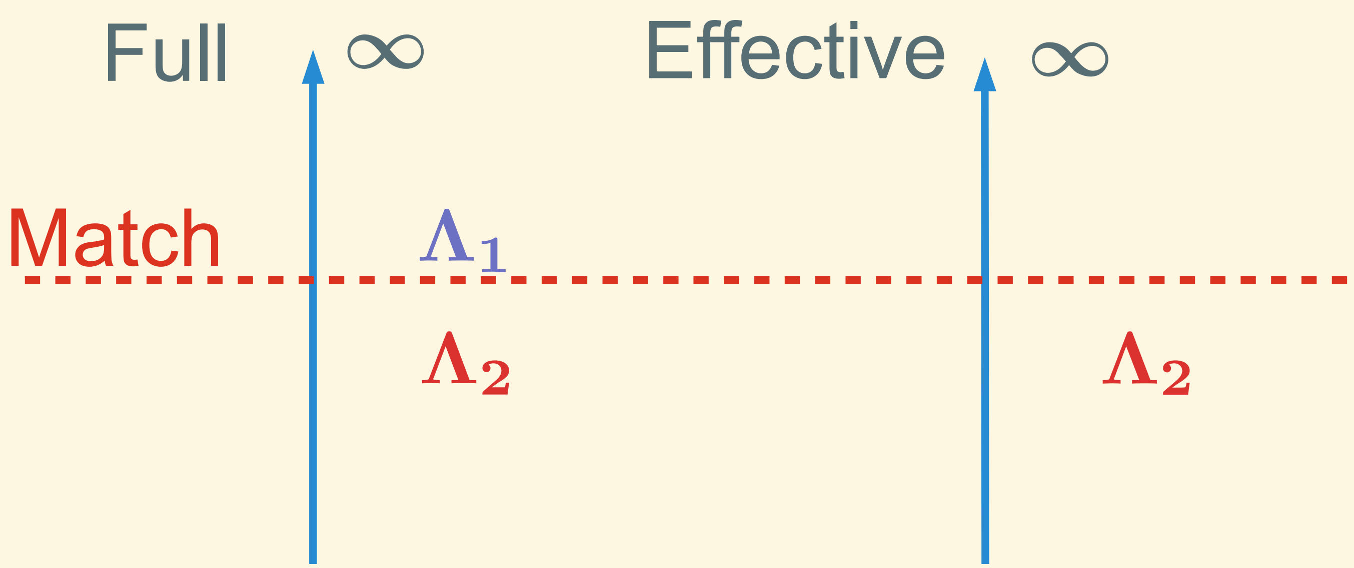
$$\alpha_s L^2 \sim 1 \longrightarrow R = C(\alpha_s) e^{E(\alpha_s, L)}$$

Effective Field Theory

Consider a toy theory with two scales $\Lambda_1 > \Lambda_2$

$$\langle O(\mu)\rangle \supset \log(\mu/\Lambda_1), \log(\mu/\Lambda_2)$$

Construct a low-energy effective theory that does not depend on Λ_1, but get's the IR physics right.

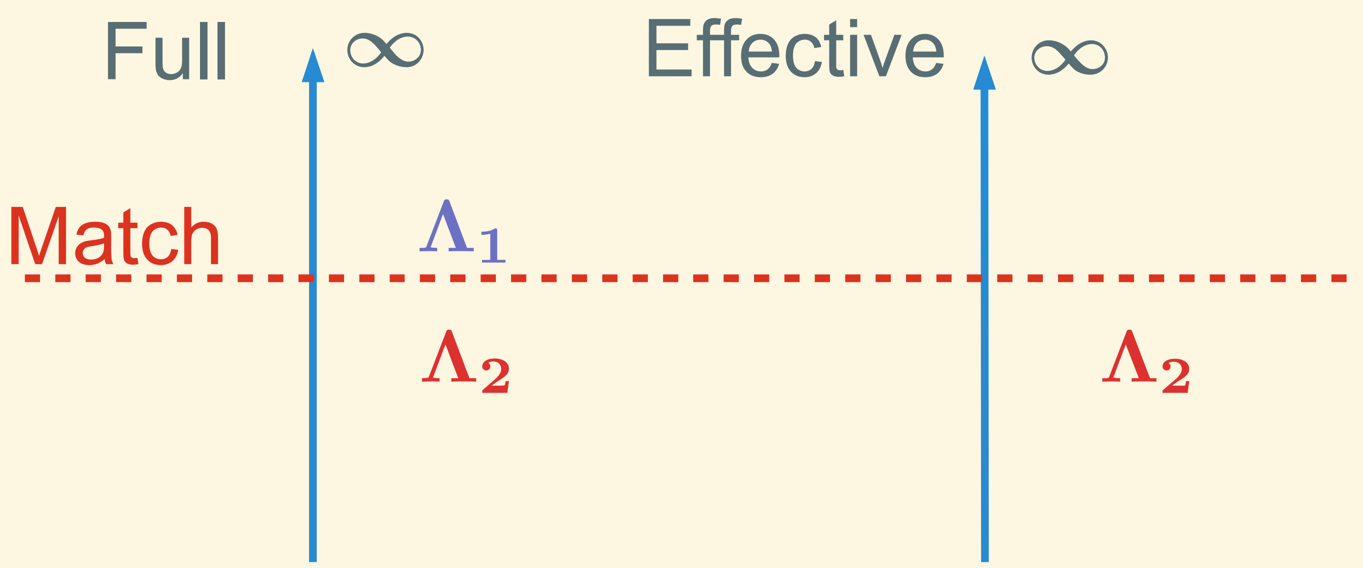


Effective Field Theory

$$\langle O(\mu)\rangle\Big|_{\mu=\Lambda_1} = C(\mu)O_{\text{Eff}}(\mu)\Big|_{\mu=\Lambda_1} + \mathcal{O}\left(\frac{\Lambda_2}{\Lambda_1}\right)$$

$$C(\mu) \supset \log(\mu/\Lambda_1)$$

$$\langle O_{\text{Eff}}(\mu)\rangle \supset \log(\mu/\Lambda_2)$$



Effective Field Theory

$$\langle O(\mu)\rangle\Big|_{\mu=\Lambda_1} = C(\mu)O_{\text{Eff}}(\mu)\Big|_{\mu=\Lambda_1} + \mathcal{O}\left(\frac{\Lambda_2}{\Lambda_1}\right)$$

$$C(\mu) \supset \log(\mu/\Lambda_1)$$

$$\langle O_{\text{Eff}}(\mu)\rangle \supset \log(\mu/\Lambda_2)$$

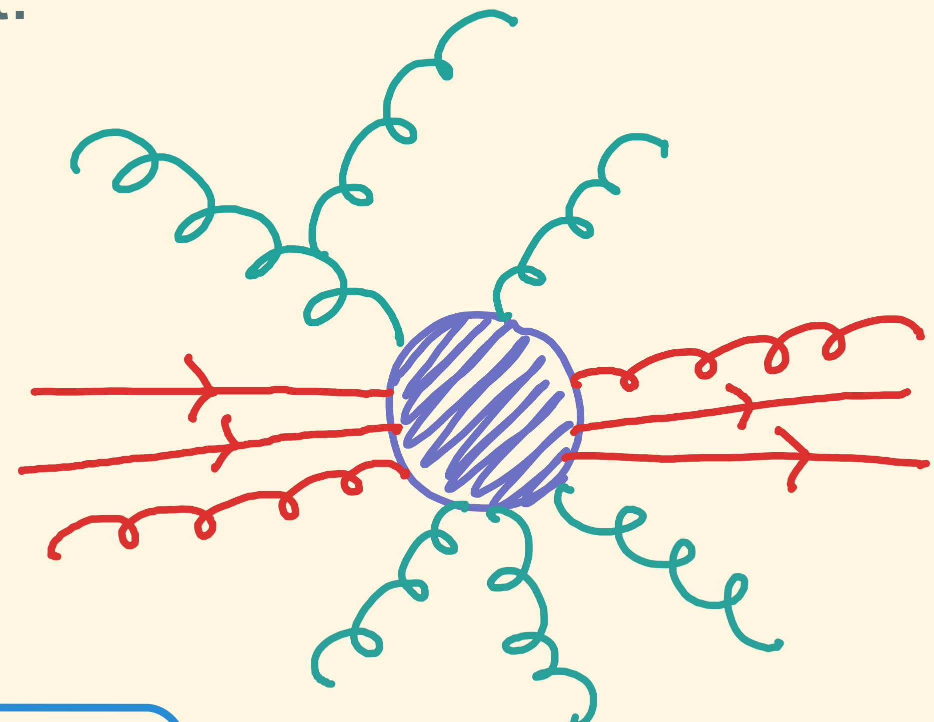
By solving the renormalization group equation for C_1:

$$C_1(\mu)\langle j_{\text{Eff}}(\mu)\rangle = C_1(\Lambda_1)U_1(\Lambda_1,\mu)\langle j_{\text{Eff}}(\mu)\rangle$$

Soft-Collinear Effective Theory

Relevant scales that must be factorized in order to sum all large logs for thrust:

$$Q, \quad \sqrt{\tau}Q, \quad \tau Q$$

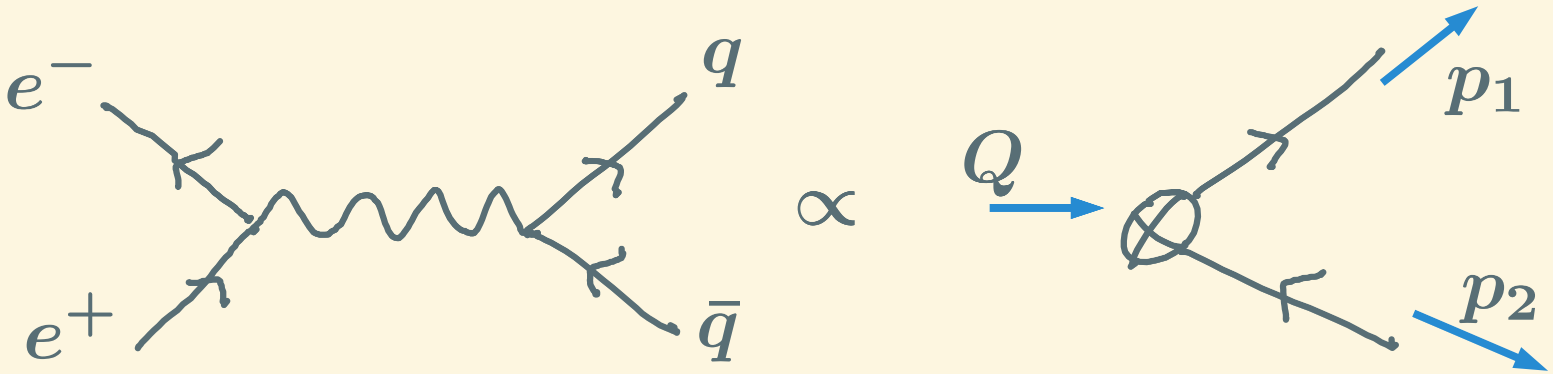


Typical pattern of scales for observables with double-logs.

The correct EFT is Soft-Collinear Effective Theory

Soft-Collinear Effective Theory

Consider the thrust distribution for e^+e^- collisions:



$\vec{p_1} \,||\, \vec{n}$

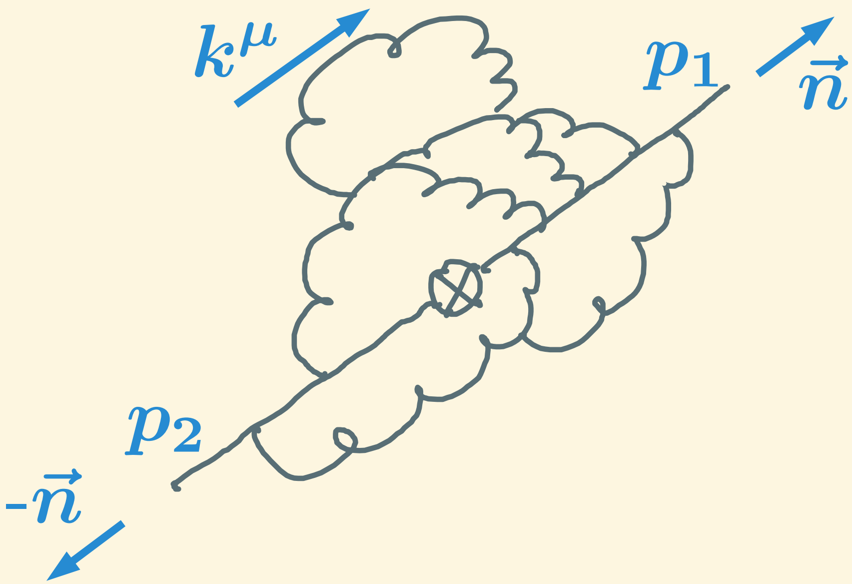
$\vec{p_2} \,||\, \text{-}\vec{n}$

$n^\mu = (1, \vec{n})$

$\bar{n}^\mu = (1, \text{-}\vec{n})$

Soft-Collinear Effective Theory

Consider a generic feynman diagram:



$\vec{p_1} \,||\, \vec{n}$

$\vec{p_2} \,||\, -\vec{n}$

$n^\mu = (1, \vec{n})$

$\bar{n}^\mu = (1, -\vec{n})$

Need to reproduce IR divergences:

In massless gauge theory:

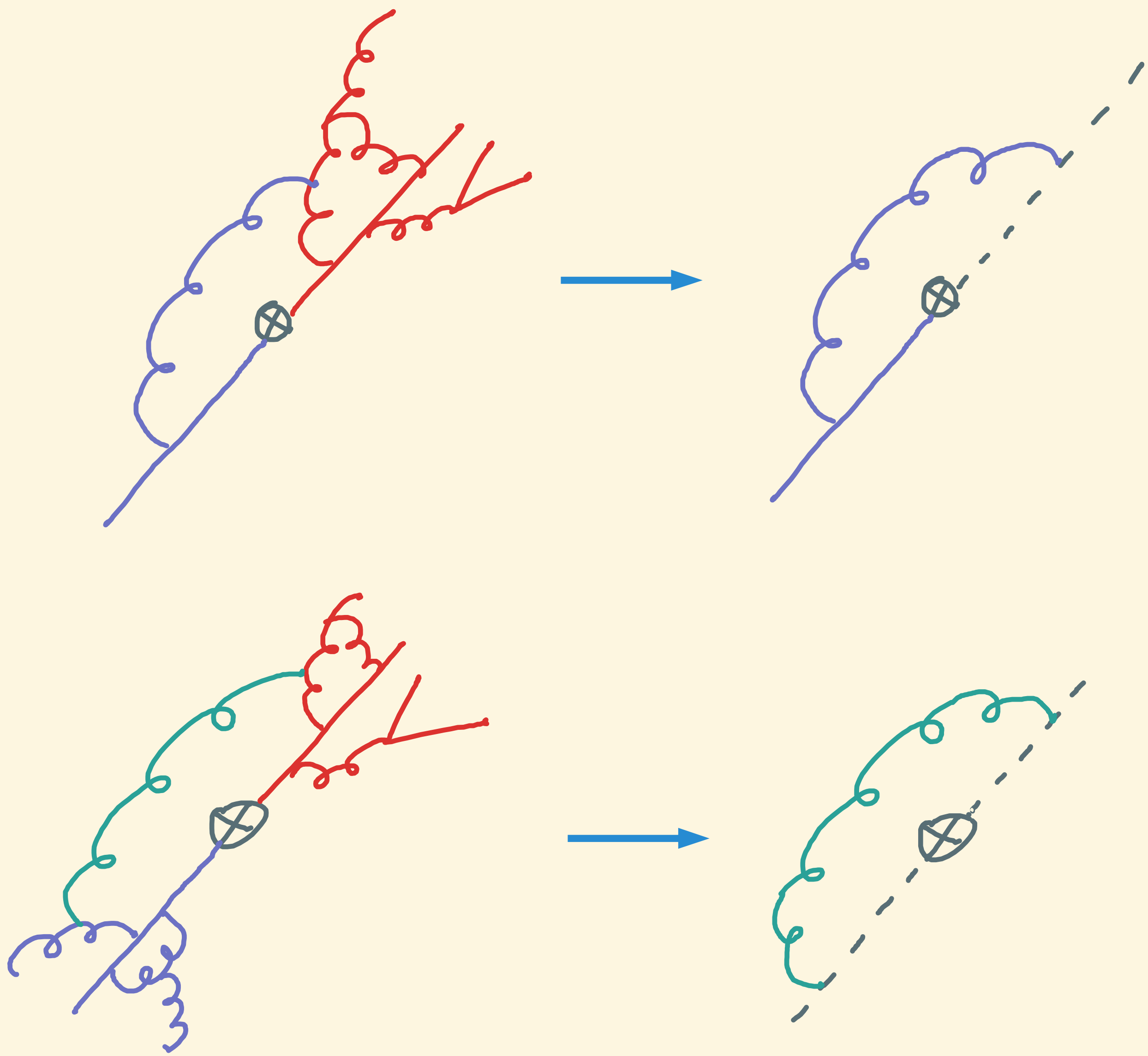
$k^\mu \to 0$ (soft)

$k^\mu \,||\, p_1^\mu$ (n-collinear)

$k^\mu \,||\, p_2^\mu$ ($\bar{n}$-collinear)

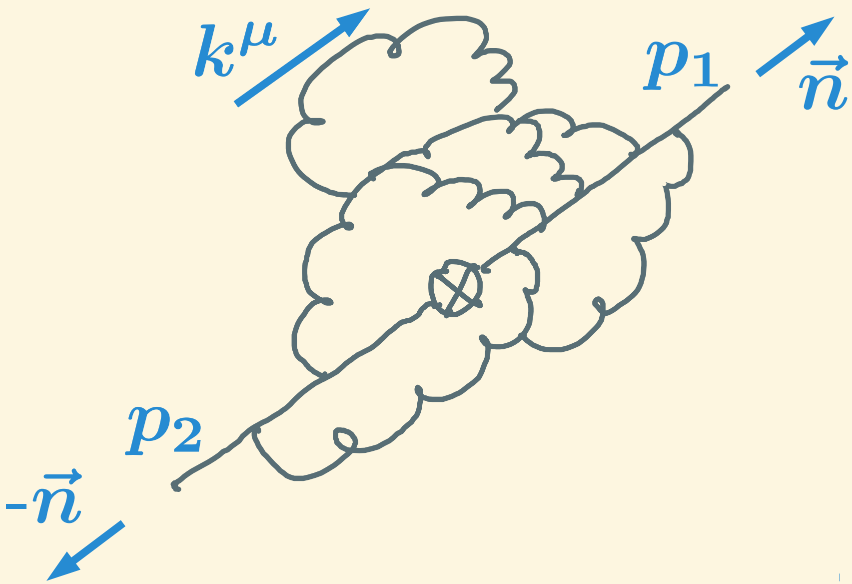
Soft-Collinear Effective Theory

Physical picture:



Soft-Collinear Effective Theory

Consider a generic feynman diagram:



$k^\mu \to 0$	(soft)
$k^\mu \,\|\, p_1^\mu$	(n-collinear)
$k^\mu \,\|\, p_2^\mu$	($\bar{n}$-collinear)

$\vec{p_1} \,||\, \vec{n}$

$\vec{p_2} \,||\, -\vec{n}$

$n^\mu = (1, \vec{n})$

$\bar{n}^\mu = (1, -\vec{n})$

$k \cdot p_1 \sim \lambda^2 Q$

$k \cdot p_2 \sim Q$

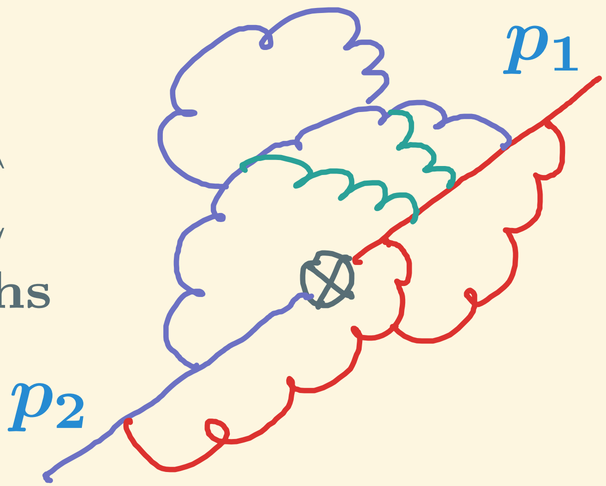
$k \cdot p_2 \sim \lambda^2 Q$

$k \cdot p_1 \sim Q$

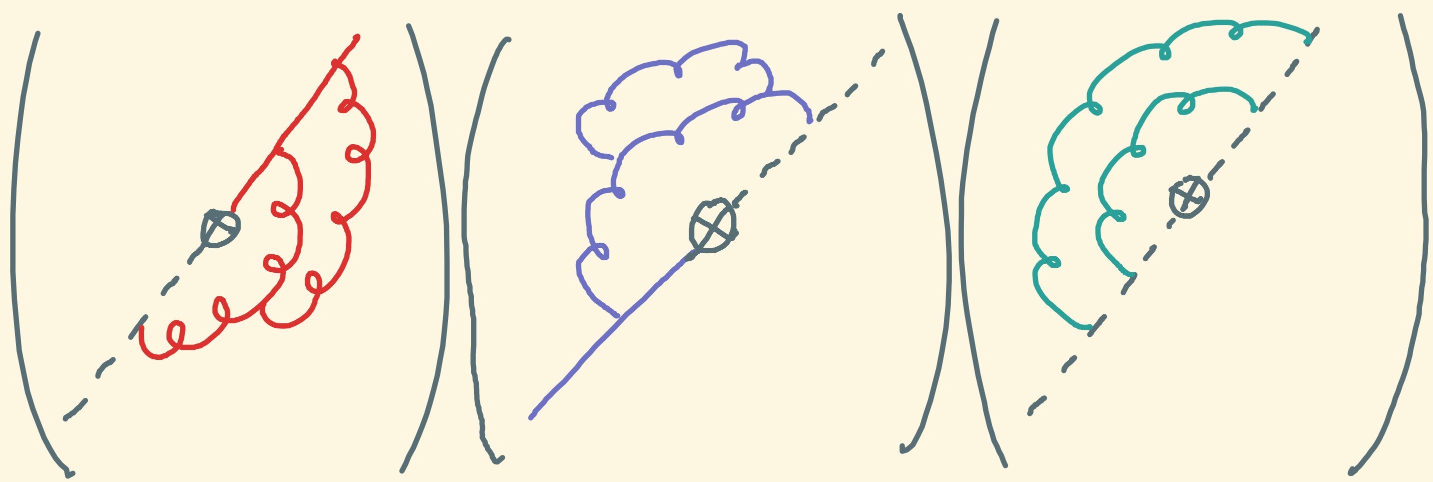
$k \cdot p_1 \sim \lambda^2 Q$

$k \cdot p_2 \sim \lambda^2 Q$

Soft-Collinear Effective Theory

$\sum_{\text{graphs}}$ 

Pick an assignment for each propagator as soft, n-collinear, or $\bar{n}$-collinear, sum over graphs:

$= C(\mu)$  $+\mathcal{O}(\lambda)$

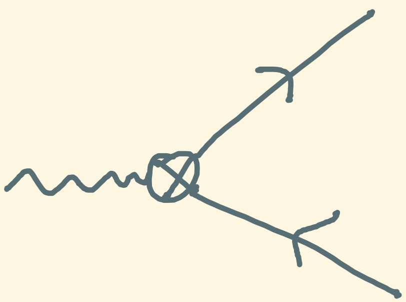
Soft-Collinear Effective Theory

Define SCET so that SCET matrix elements equal QCD matrix elements expanded in the soft and collinear limits

$$\mathcal{L}_{\text{SCET}} = \sum_{i \in \text{sectors}} \mathcal{L}^{i}_{\text{QCD}}$$

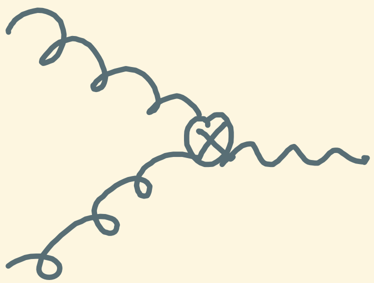
$$j^{\mu}_{\text{QCD}} \to C^{(0)} j^{(0)\mu}$$

Soft-Collinear Effective Theory



$$j^{\mu}_{\text{QCD}} \to C^{(0)} j^{(0)\mu}$$

$$\bar{\psi}\Gamma\psi \to \left[\bar{\psi}_n W_n\right] \Gamma \left[W^{\dagger}_{\bar{n}} W_n\right] \left[W^{\dagger}_{\bar{n}} \psi_{\bar{n}}\right]$$

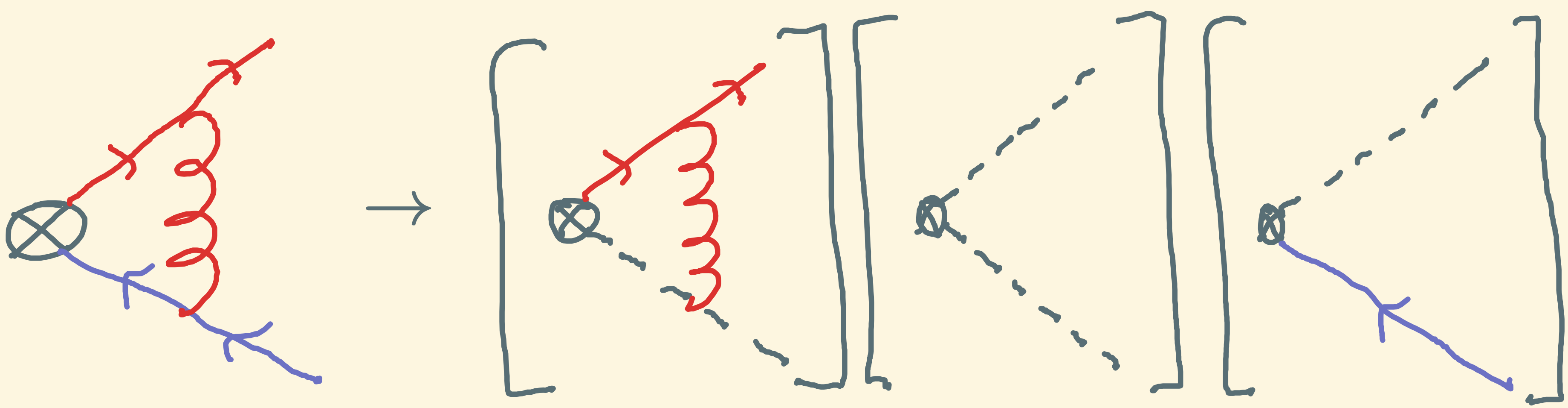


$$G^{a}_{\mu\nu}\Gamma G^{\mu\nu}_{a} \to \left[\bar{n} \cdot G^{a,\mu}_{n} W^{ab}_{n}\right] \Gamma \left[W^{bc\dagger}_{\bar{n}} W^{cd}_{n}\right] \left[W^{de}_{\bar{n}} \bar{n} \cdot G^{e,\mu}_{\bar{n}}\right]$$

Freedman, Luke (2012)

Soft-Collinear Effective Theory

$$\bar{\psi}\Gamma\psi \rightarrow \left[\bar{\psi}_n \boldsymbol{W}_n\right]\Gamma\left[\boldsymbol{Y}_n^\dagger \boldsymbol{Y}_{\bar{n}}\right]\left[\boldsymbol{W}_{\bar{n}}^\dagger \psi_{\bar{n}}\right]$$

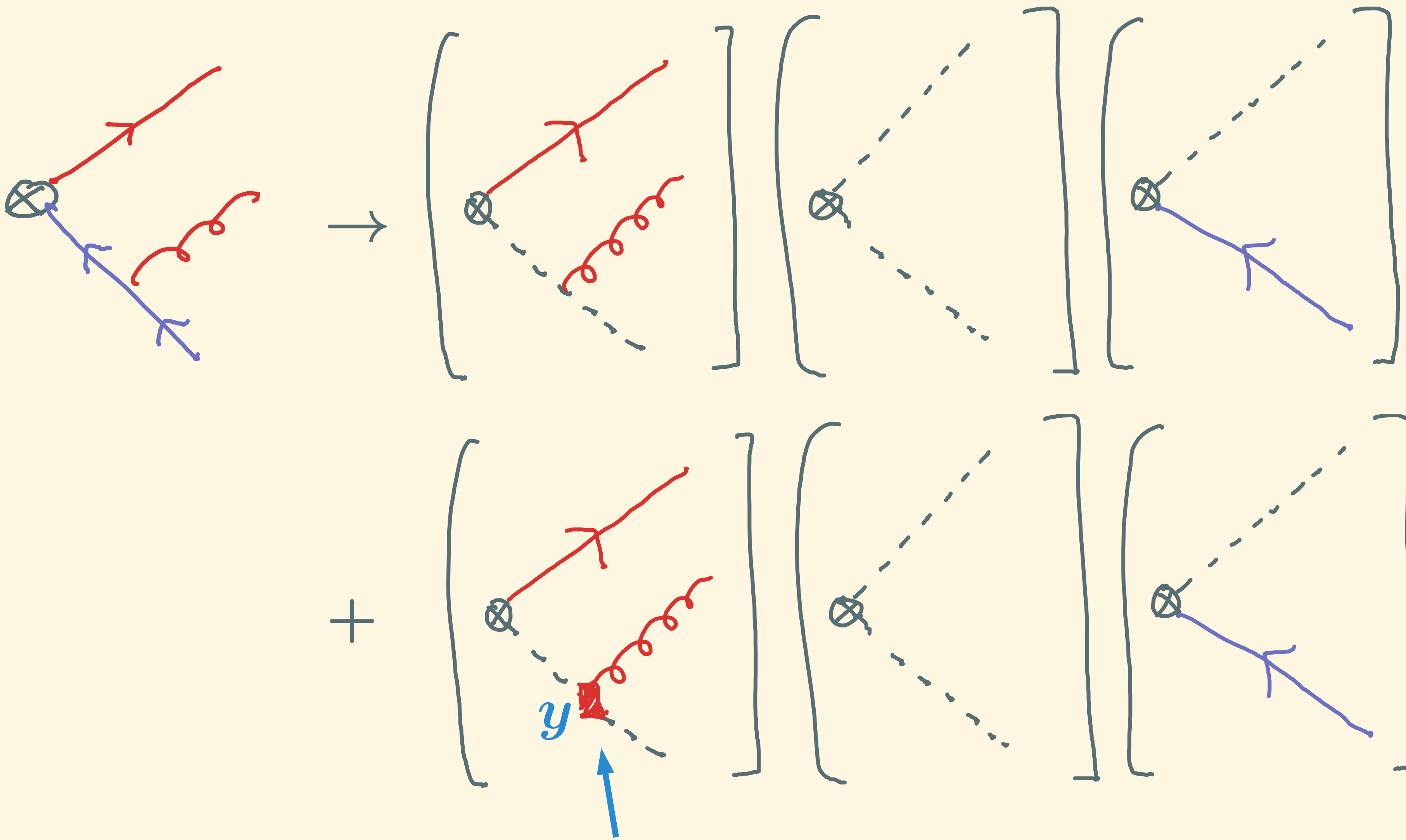


Subleading SCET

$$\langle j^{\mu}_{\text{QCD}} \rangle = C^{(0)} \langle j^{(0)\mu} \rangle + \mathcal{O}\left(\lambda\right)$$

$$\langle j^{\mu}_{\text{QCD}} \rangle = C^{(0)} \langle j^{(0)\mu} \rangle + \sum_i C^{(i)} \langle j^{(i)\mu} \rangle + \mathcal{O}\left(\lambda^2\right)$$

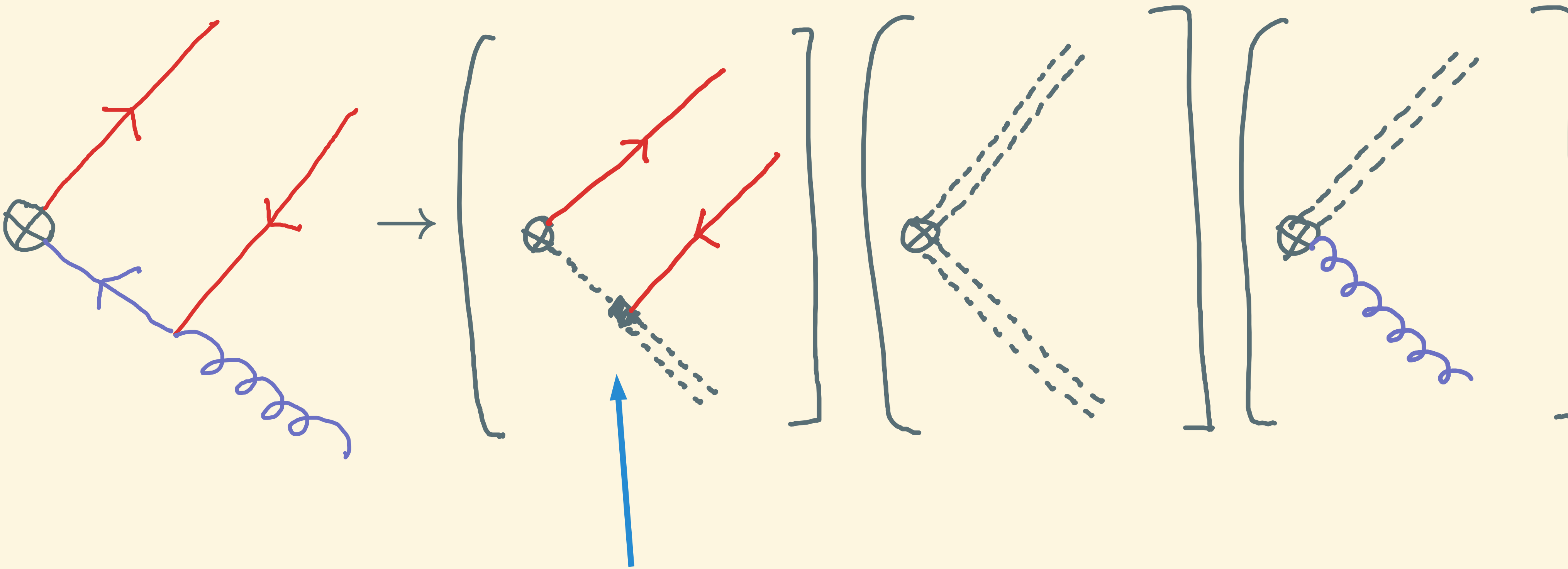
Subleading SCET



$$W(x, y)D(y)W(y, \infty)$$

Subleading SCET

New currents that did not appear at leading order:



$$\bar{\psi}(x)W(x,y)T^a\psi(y)W^{ab}(y,\infty)$$

SCET for Thrust

Returning to thrust, note:

$$q_1 \cdot q_2 \sim \lambda^2 Q \sim \tau Q$$

$$\rightarrow \lambda \sim \sqrt{\tau}$$

Generally in SCET:

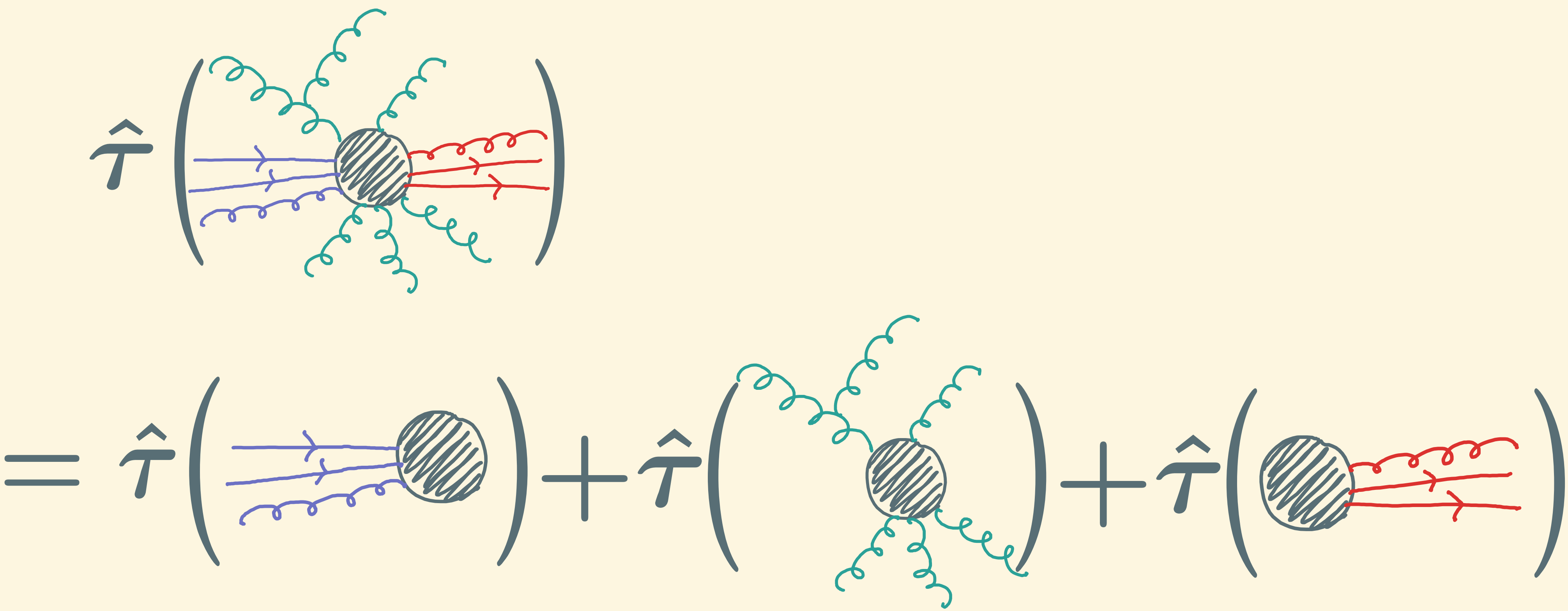
$$Q, \quad \lambda Q, \quad \lambda^2 Q$$

For thrust:

$$Q, \quad \sqrt{\tau} Q, \quad \tau Q$$

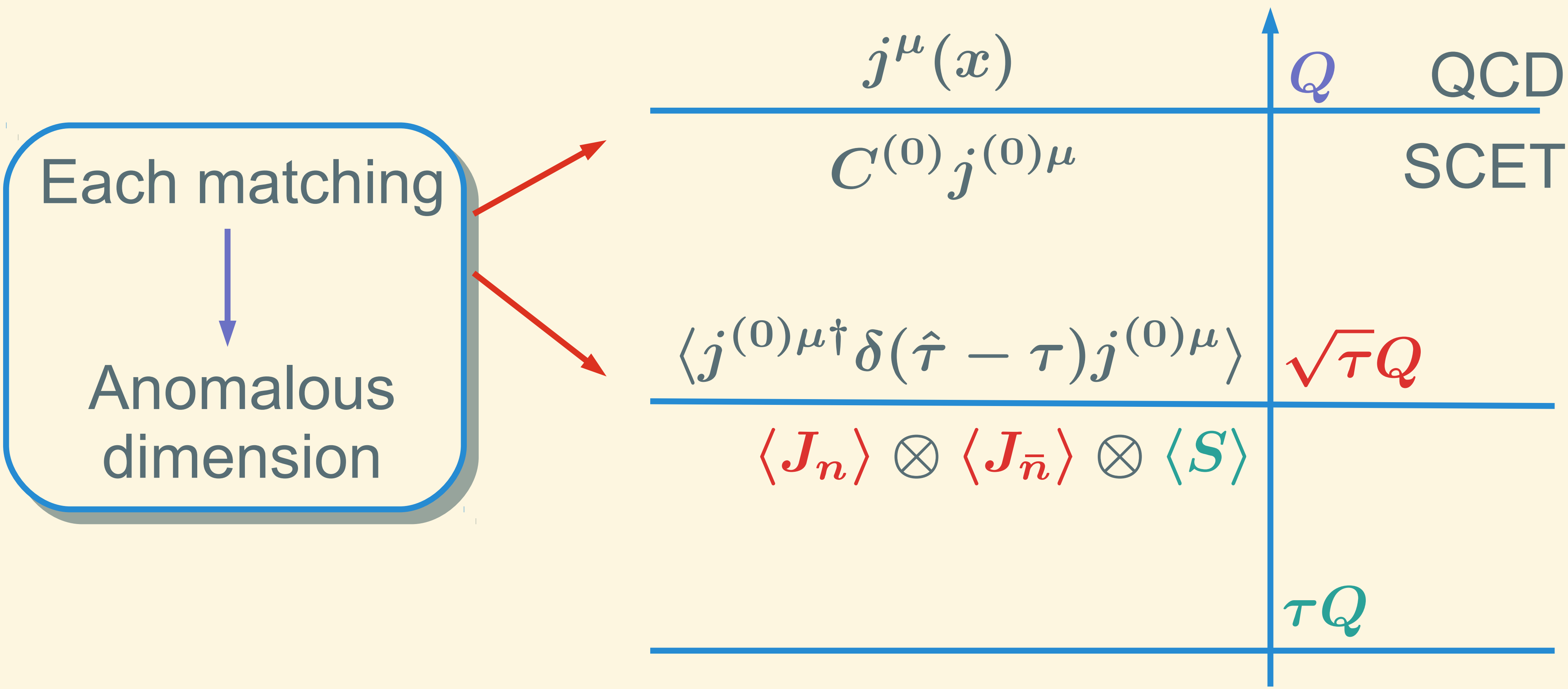
SCET for Thrust

Thrust factorizes in SCET. At leading-order:



SCET for Thrust

$$\frac{d\sigma}{d\tau} \propto \langle 0|\, j^{\mu\dagger}(x)\delta(\hat{\tau}-\tau)j^{\mu}(0)\,|0\rangle$$



SCET for Thrust

To include to subleading operators:

$$j^\mu(x)$$ — Q — QCD

$$C^{(i)} j^{(i)\mu}$$ — SCET

$$\langle j^{(i)\mu\dagger} \delta(\hat{\tau} - \tau) j^{(j)\mu} \rangle$$ — $\sqrt{\tau}Q$

$$\langle J_n^{(l)} \rangle \otimes \langle J_{\bar{n}}^{(m)} \rangle \otimes \langle S^{(n)} \rangle$$ — τQ

Each matching → Anomalous dimension

Freedman (2013)

SCET for Thrust

To sum subleading logs we need:

This work

1) Anomalous dimensions of the $\mathcal{O}(\lambda)$ currents

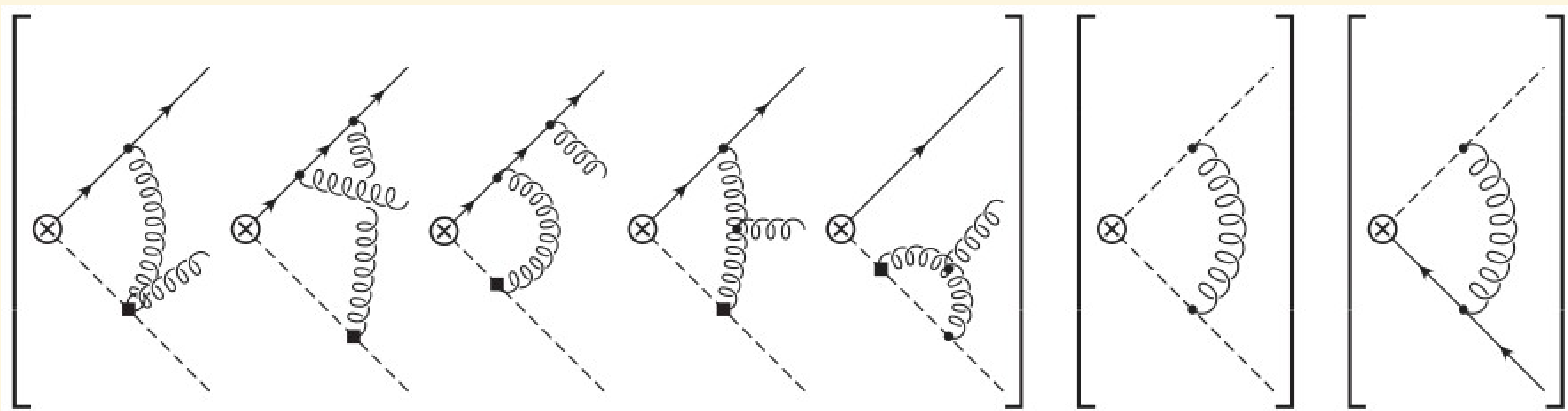
2) Anomalous dimensions of the $\mathcal{O}(\lambda^2)$ currents

3) Anomalous dimensions of $\langle S^{(n)} \rangle$

$\lambda \sim \sqrt{\tau}$

The Calculation

Graphs needed to renormalize the j^{1a} current:

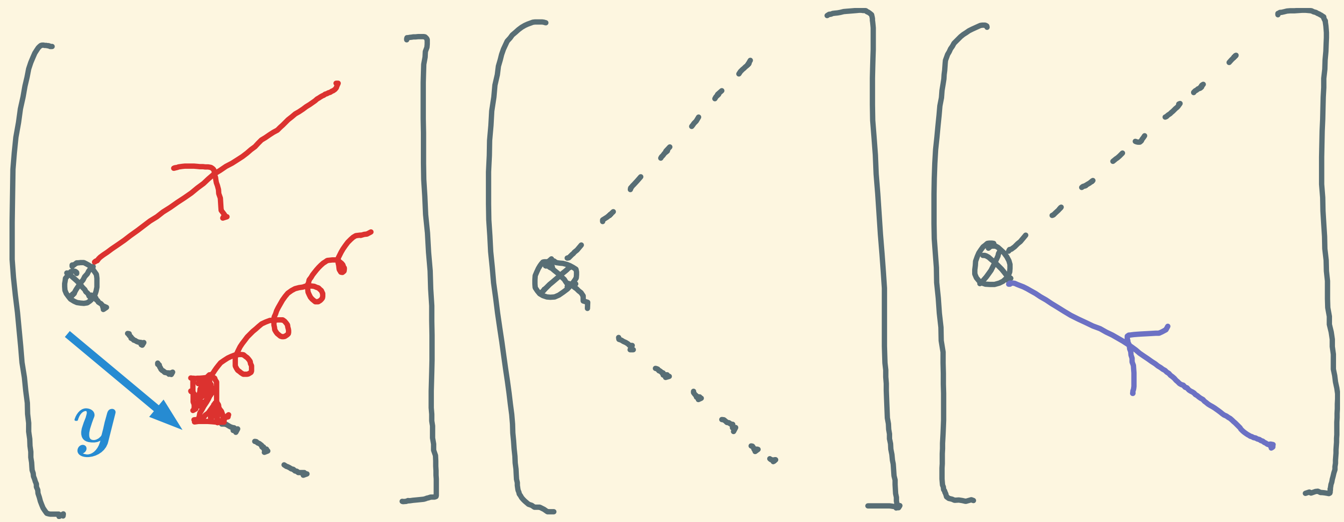


There are four independent anomalous dimensions, each requiring a similar number of graphs to compute.

The Calculation

Non-trivial mixing:

$$j^{1a}(y) =$$



$$j^{1a,\mathrm{ren}}(y) = \int dy' Z(y,y') j^{1a,\mathrm{bare}}(y')$$

Results

$$
\begin{aligned}
\gamma_{(1a_n)}(u,v) =& \frac{\alpha_s \delta(u-v)\theta(\bar{v})}{\pi}\left(C_F\left(\ln\frac{-Q^2}{\mu^2}-\frac{3}{2}+\ln\bar{v}\right)+\frac{C_A}{2}\right) \\
&+\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\bar{u}\left(\theta(1-u-v)\frac{uv}{\bar{u}\,\bar{v}}+\theta(\bar{u})\theta(\bar{v})\theta(u+v-1)\frac{uv+u+v-1}{uv}\right) \\
&-\frac{\alpha_s C_A}{2\pi}\bar{u}\left(\theta(\bar{u})\theta(u-v)\frac{\bar{v}-uv}{u\bar{v}}+\theta(\bar{v})\theta(v-u)\frac{\bar{u}-uv}{v\bar{u}}+\frac{1}{\bar{u}\,\bar{v}}\left[\frac{\bar{u}\theta(\bar{u})\theta(u-v)}{u-v}+\frac{\bar{v}\theta(\bar{v})\theta(v-u)}{v-u}\right]_+\right), \\
\gamma_{(1b_n)}(u,v) =& \gamma_{(1a_n)}(u,v), \\
\gamma_{(1B_n)} =& \frac{\alpha_s C_F}{\pi}\left(-\frac{3}{2}+\ln\frac{-Q^2}{\mu^2}\right)=\gamma_{(0)}, \\
\gamma_{(1c_n)}(u_2;u_1,v_1) =& \frac{\alpha_s \delta(u_1-v_1)\delta(u_2)}{\pi}\left(C_F\left(-\frac{3}{2}+\ln\frac{-Q^2}{\mu^2}\right)+\frac{C_A}{2}\ln v_1\right) \\
&-\frac{\alpha_s C_A \delta(u_2)}{\pi}\left(\left[\frac{\theta(v_1-u_1)\theta(u_1)}{v_1-u_1}+\frac{\theta(u_1-v_1)\theta(v_1)}{u_1-v_1}\right]_+-\frac{\theta(u_1-v_1)}{u_1}-\frac{\theta(v_1-u_1)}{v_1}\right), \\
\gamma_{(1d_n)}(u,v) =& \frac{\alpha_s \delta(u-v)}{\pi}\left(-\frac{C_F}{2}+C_A\left(\ln\frac{-Q^2}{\mu^2}+\ln(v)-\frac{1}{2}\right)\right)-\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{1}{v}\left[\frac{v\theta(u-v)\theta(v)}{u-v}+\frac{u\theta(v-u)\theta(u)}{v-u}\right]_+, \\
\gamma_{(1e_n)}(u,v) =& \frac{\alpha_s \delta(u-v)\theta(\bar{v})}{\pi}\left(\frac{C_F}{2}+C_A\left(\ln\frac{-Q^2}{\mu^2}+\ln(v)-1\right)\right)-\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{1}{v\bar{v}}\Big(\theta(\bar{v})\theta(v-u)u\bar{v} \\
&+\theta(\bar{u})\theta(u-v)v\bar{u}+\left[\frac{\bar{u}v\theta(\bar{u})\theta(u-v)}{u-v}+\frac{u\bar{v}\theta(\bar{v})\theta(v-u)}{v-u}\right]_+\Big),
\end{aligned} \tag{42}
$$

Conclusion

Punchline: We calculated the anomalous dimensions of subleading dijet operators in Soft-Collinear Effective Theory – a step towards improving theoretical precision in the thrust distribution, allowing for an improved determination of $\alpha_s(M_Z)$

Still need $\mathcal{O}\left(\lambda^2\right)$ and $\langle S(\mu)\rangle$ renormalization.

Renormalization group equations must be solved numerically

Bonus

Inclusive Drell-Yan: $p\bar{p} \to l^+ l^- X$

Collins, Soper, Sterman (1985)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2 dY} = \sum_{ij}\int_{x_a}^{1}\frac{d\xi_a}{\xi_a}\int_{x_a}^{1}\frac{d\xi_a}{\xi_a} f_i(\xi_a,\mu) f_j(\xi_b,\mu)$$

$$\times H_{ij}\left(\frac{x_a}{\xi_a},\frac{x_b}{\xi_b},q^2,\mu\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

$H_{ij}(\mu)$ → Sensitive only to Q

$f_i(\mu)$ → Sensitive to low energy, non-perturbative physics

Bonus

Introducing more scales requires further factorization:

Threshold Drell-Yan: $p\bar{p} \to l^+ l^- X_{\text{soft}}$

$$\tau = \frac{q^2}{E_{\text{cm}}} \approx 1 \xrightarrow{\text{new scale}} Q(1-\tau) \ll Q$$

$$\log\left(\frac{Q(1-\tau)}{Q}\right) = \boxed{\log(1-\tau) \gg 1}$$

Bonus

Threshold Drell-Yan: $p\bar{p} \to l^+ l^- X_{\text{soft}}$

Collins, Soper, Sterman (1987)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2} = \sum_{ij} \int_{x_a}^{1} \frac{d\xi_a}{\xi_a} \int_{x_a}^{1} \frac{d\xi_a}{\xi_a} f_i(\xi_a, \mu) f_j(\xi_b, \mu) H_{ij}(q^2, \mu)$$

$$\times\, S\left(1 - \frac{\tau}{\xi_a \xi_b}, \mu\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right) + \mathcal{O}\left(1 - \tau\right)$$

$H_{ij}(\mu)$ ⟶ Sensitive only to Q

$S(\mu)$ ⟶ Sensitive only to $Q(1-\tau)$

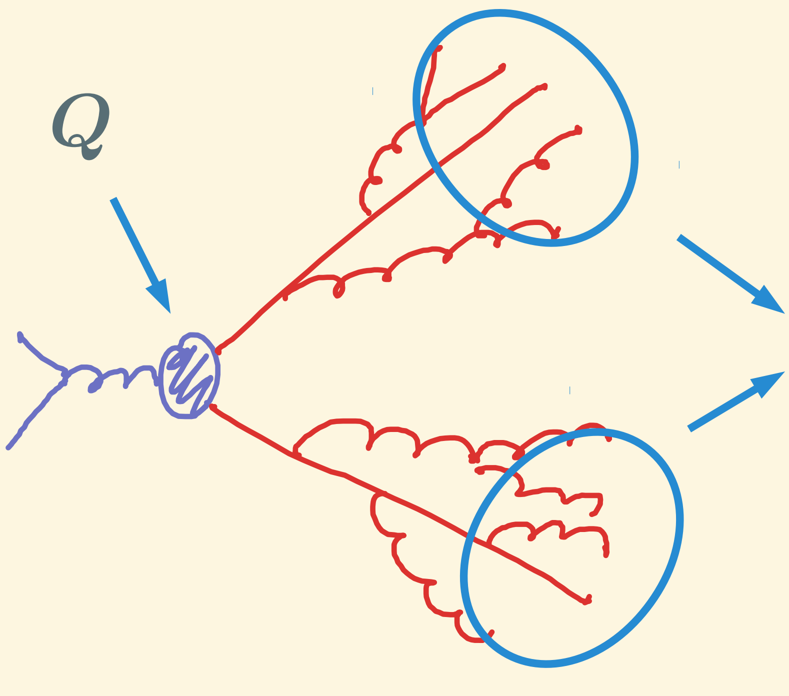
Bonus

$$\boldsymbol{W_{n_i}(x,\infty) = P\,\mathbf{exp}\left(-ig\int_0^\infty ds\bar{n}\cdot A\left(x+\bar{n}_i s\right)\right)}$$

Factorization

Factorization Theorem:

$$d\sigma = \sum_{ij} d\sigma_{ij}^{\text{part}} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$ (inclusive)



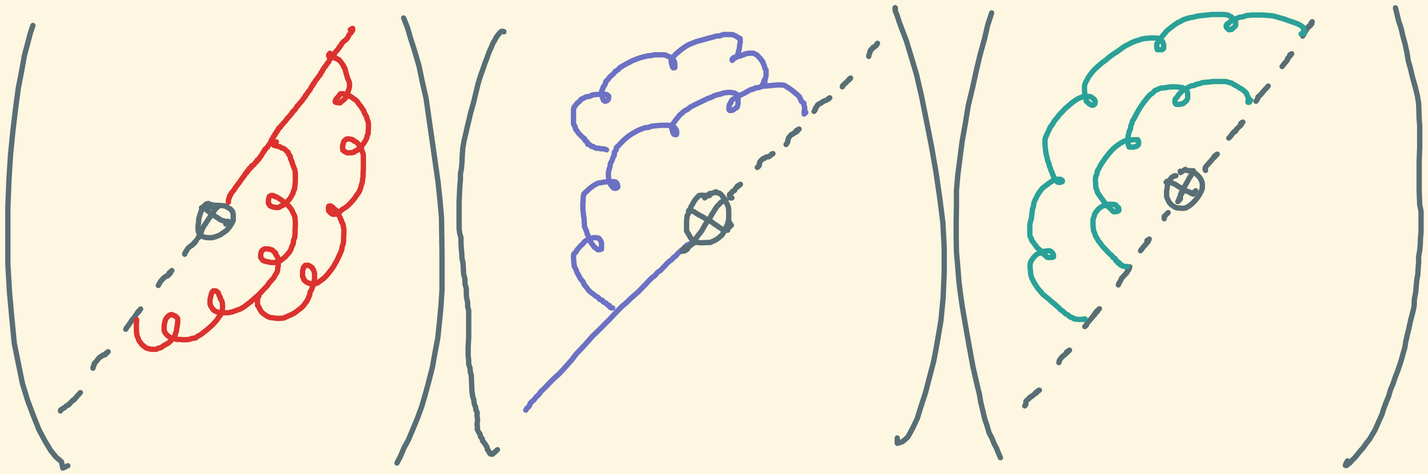
$$m \ll Q \rightarrow \log\left(\frac{m}{Q}\right) \gg 1$$

EFT's and Factorization

$$\langle j_{\text{Full}}(\mu)\rangle\Big|_{\mu=\Lambda_1} = C(\mu)\langle j_{\text{Eff}}(\mu)\rangle\Big|_{\mu=\Lambda_1}$$
$$+\frac{1}{\Lambda_1}C^{(1)}(\mu)\langle j^{(1)}_{\text{Eff}}(\mu)\rangle\Big|_{\mu=\Lambda_1} + \mathcal{O}\left(\frac{\Lambda_2^2}{\Lambda_1^2}\right)$$

Renormalizing sums subleading logs.

Soft-Collinear Effective Theory

$= C(\mu)$  $+\mathcal{O}(\lambda)$

$$1 \quad - \quad g\frac{\bar{n}\cdot A}{\bar{n}\cdot q_1} \quad + \quad g^2\frac{\bar{n}\cdot A\,\bar{n}\cdot A}{\bar{n}\cdot(q_1+q_2)\bar{n}\cdot q_2} + \cdots$$