Quantum critical dynamics: CFT, Monte Carlo & holography

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- E. Katz, S. Sachdev, E. Sørensen, WWK, PRB **90**, 2014 [Ed. Sugg.]
- WWK, E. Sørensen, S. Sachdev, Nat. Phys. **10**, 2014
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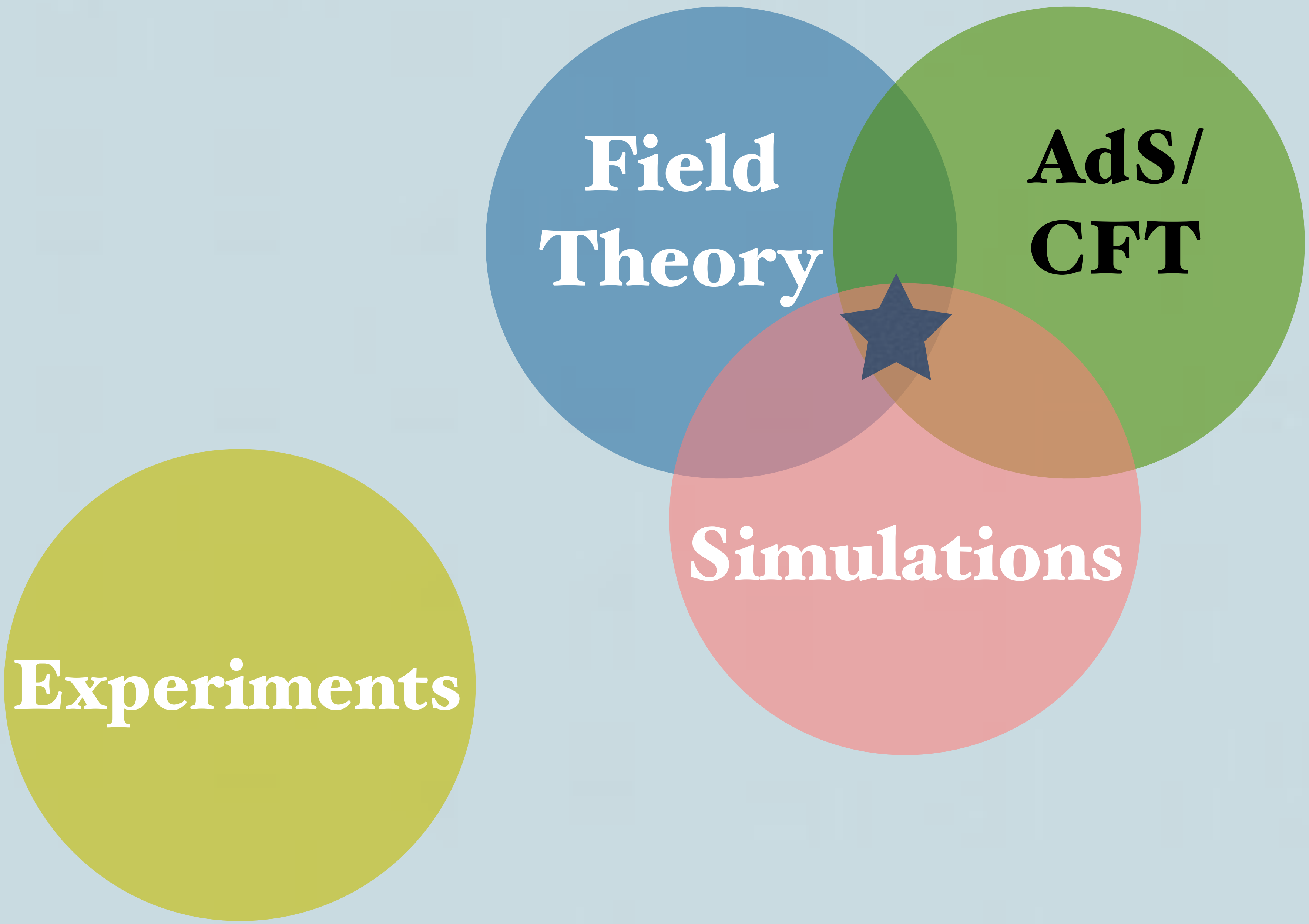
- WWK, S. Sachdev, PRB **86**, 2012

★ Real time finite *T* response of strongly correlated quantum fluids

- (quasi)particles: ∞ lifetime (poles)
- If have sharp (quasi)particles, get dynamics from Boltzmann Eq.
- Systems *without* quasiparticles:
  - Quantum critical phase transition
  - Strongly coupled gauge theories, etc.

Quantum critical dynamics

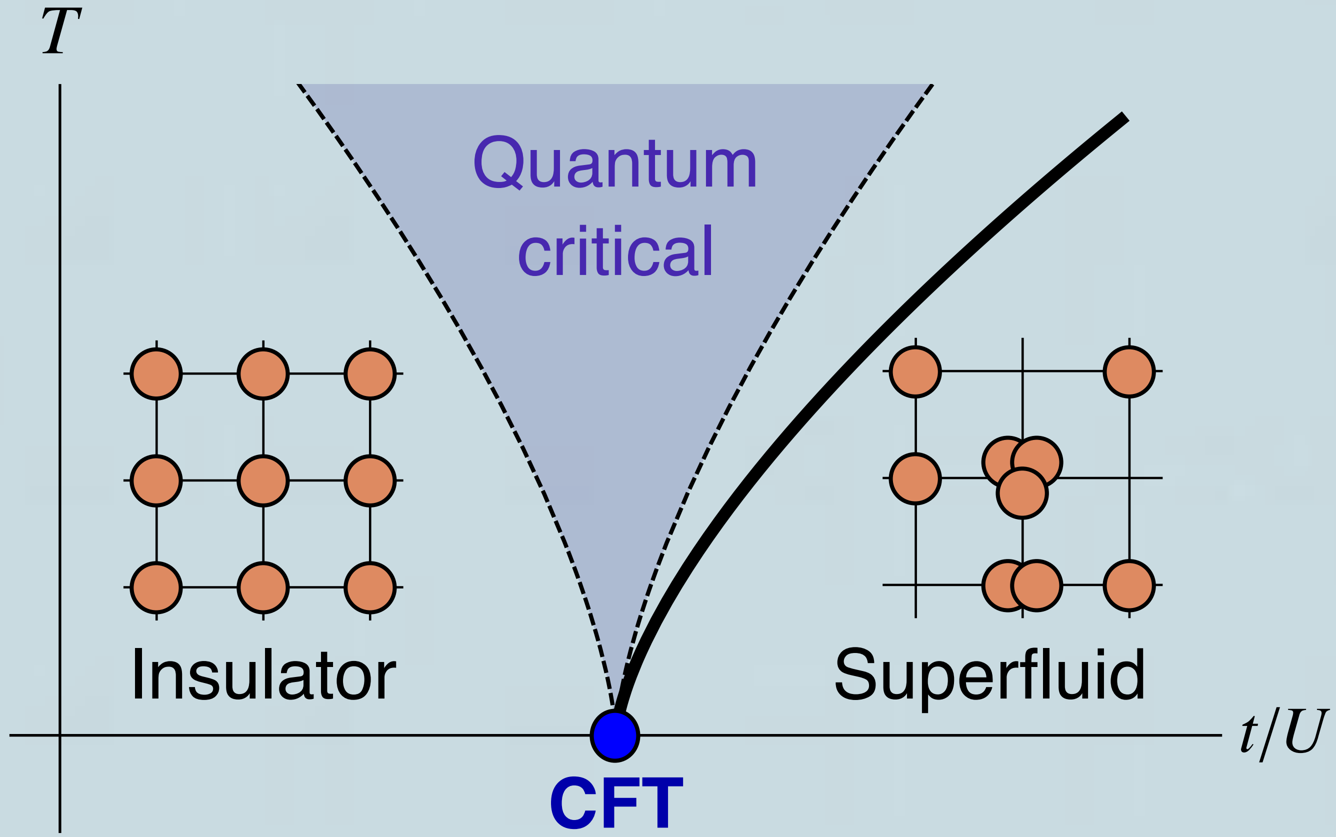


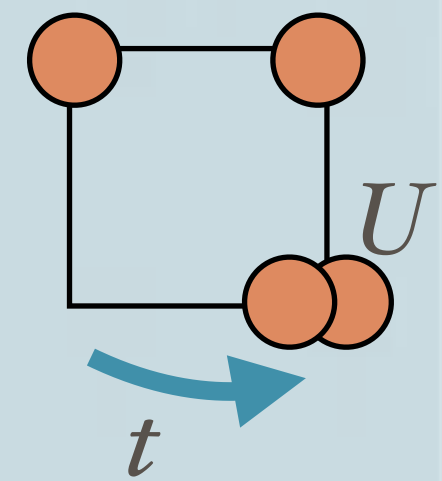
Conformal Field Theory

- Scale + Lorentz invariant QFT
- Best characterized quantum fluid w/out (quasi)particles
    - Many open questions: dynamics, etc
- Why care?
    - Experiments!
    - Realistic models: numerics
    - Gravity = CFT? AdS/CFT [Maldacena, etc]

Bosons in 2+1D

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$



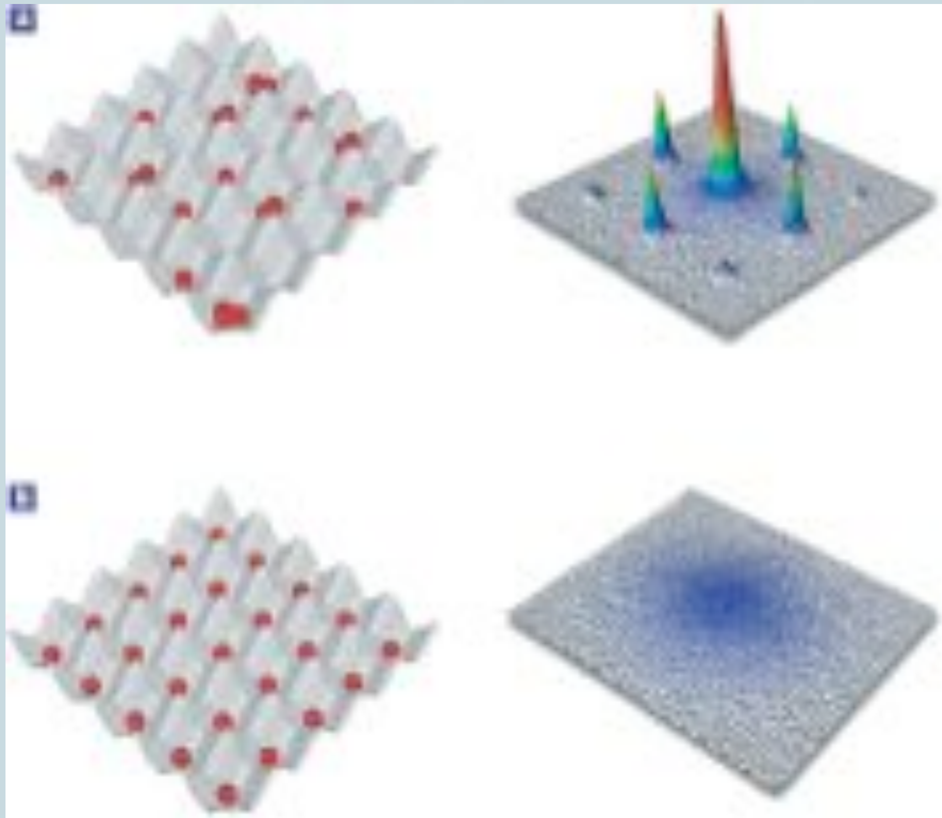


- O(2) universality class: Bose-Hubbard, quantum rotors, XY spins, etc

- SSB of O(2) order parameter

$$\mathcal{L} = (\partial_t \vec{\phi})^2 + (\nabla \vec{\phi})^2 + m^2 \phi^2 + u\phi^4$$

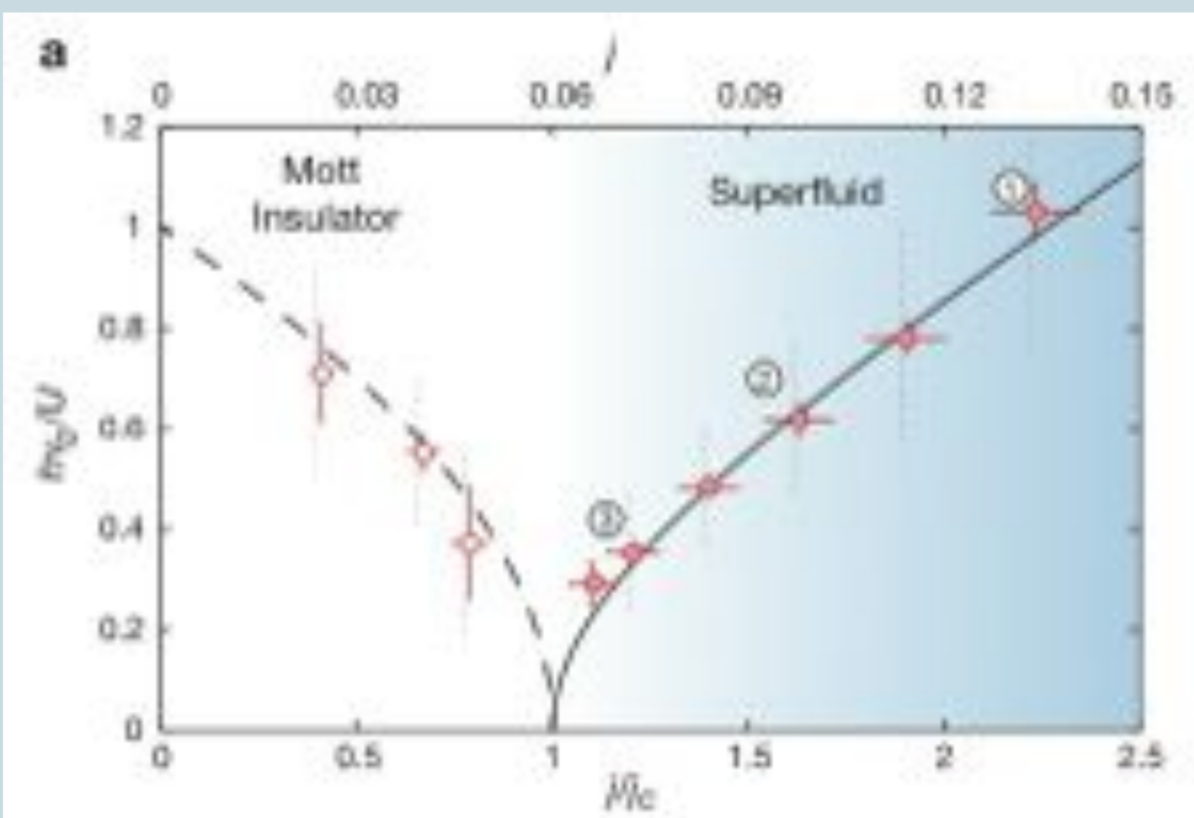
Strongly coupled fixed point
in 2d (Wilson-Fisher)



Superfluid

Insulator

[Bloch]



^{87}Rb

[Endres *et al*]

Break conformal symmetry

- Characteristic time scale: 1 / *T*

- **Short** times ω >> *T*: probing near vacuum

- **Long** times ω << *T*: excitations interact strongly with thermal background

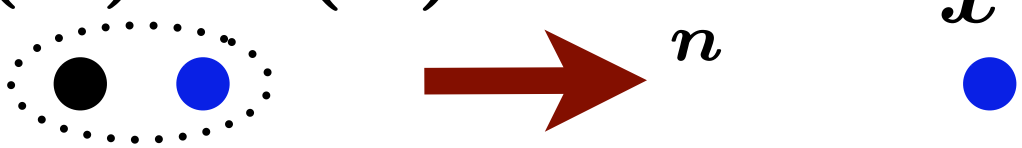
Operator Product Expansion

- A scalar primary op $O(x)$, w/ scaling dim Δ:

$$\langle O(x)\, O(0)\rangle = 1\, /\, x^{2\Delta}$$

- OPE:

$$\mathcal{O}(x)\,\mathcal{O}(0) \qquad \sum_n \frac{\mathcal{O}_n(0) + \text{desc.}}{x^{2\Delta - \Delta_n}}$$

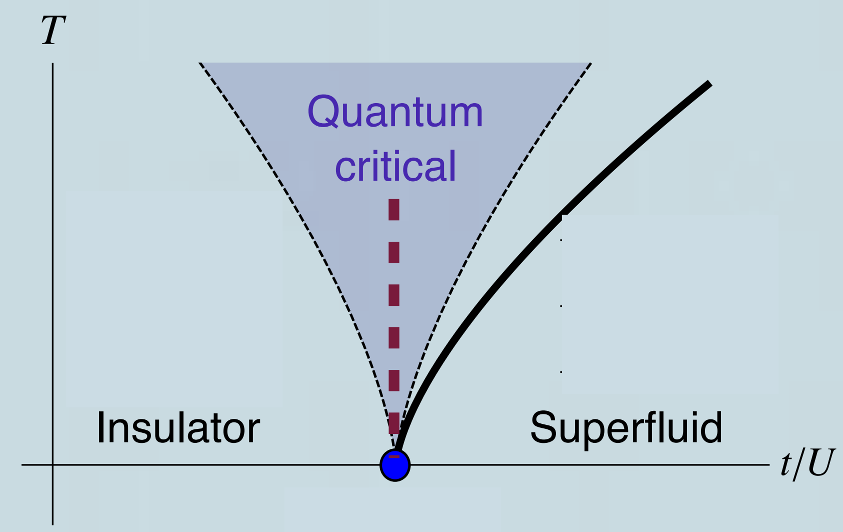


[Wilson; Polyakov; Ferrara *et al*; etc]

Current correlators

$$\partial_\mu J^\mu = 0$$

$$\sigma(\omega) = \frac{1}{i\omega} \langle J_x(\omega) J_x(-\omega) \rangle$$



Universal scaling function:

$$\sigma(\omega) = \Phi\left(\frac{\omega}{T}\right)$$

Current OPE

$$J_\mu(x)J_\nu(0) = \frac{I_{\mu\nu}\mathbb{1}}{x^{2\cdot 2}} + \mathcal{C}_{JJ\mathcal{O}}\frac{x_\mu x_\nu\,\mathcal{O}(0)}{x^{6-\Delta}} + \mathcal{C}_{JJT}\frac{T_{\mu\nu}(0)}{|x|} + \cdots$$

O(N) CFT:

Relevant

scalar $\mathcal{O} \sim \varphi^2$

Stress tensor

- Get OPE coefficients from $\langle JJ\mathcal{O}\rangle$

Conductivity via OPE

$$\lim_{|q|\gg|p|} J_x(q)J_x(-q+p) = -|q|\sigma_\infty\delta(p) - \mathcal{C}_{JJ\mathcal{O}}\frac{\mathcal{O}_g(p)}{|q|^{\Delta-1}}$$
$$+\frac{\mathcal{C}_{JJT}}{|q|^2}\left[T_{xx}-T_{yy}-12\gamma(T_{xx}+T_{yy})\right]\Big|_p+\cdots$$

Thermal average

$$\langle\mathcal{O}\rangle_T = BT^\Delta$$

$$\boxed{\sigma(i\omega_n) \overset{\omega_n\gg T}{=} \sigma_\infty + b_1\left(\frac{T}{\omega_n}\right)^\Delta + b_2\left(\frac{T}{\omega_n}\right)^3+\cdots}$$

Dominant op in OPE?

- For O(N) Wilson-Fisher, it's **SCALAR**:

$$O(x) = \varphi^2(x) \quad \text{since} \quad \Delta = 3 - 1/\nu$$

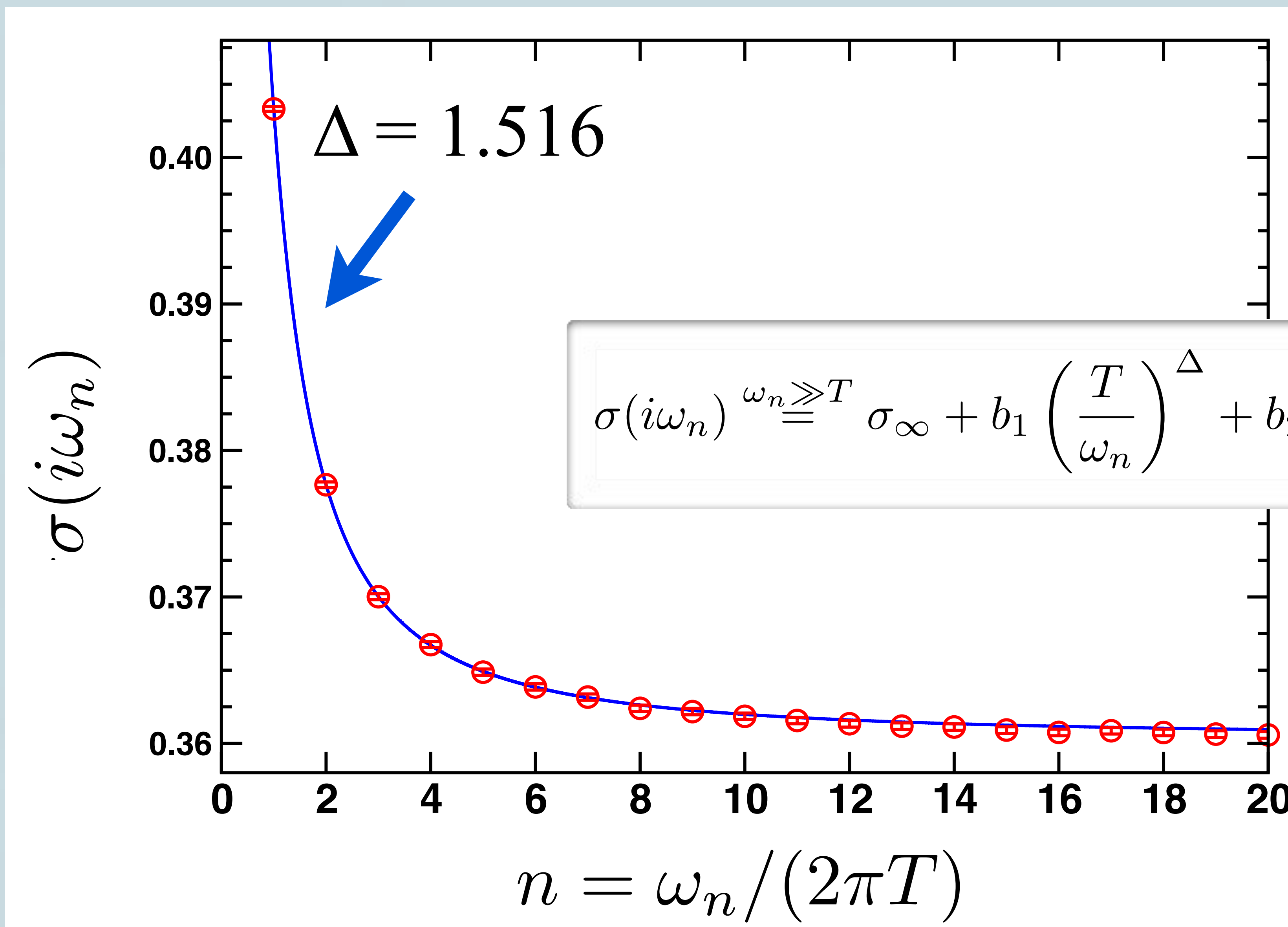
- $N = \infty$: $\Delta = 2$
  $N = 2$: $\Delta = 1.5106$

- **All** CFTs have $1/\omega^3$ term because *there's always a stress tensor*
  - Sometimes it dominates (Dirac CFT, QED3, $\mathcal{N}$=4 super-Y-M, etc)

Quantum Monte Carlo

[WWK, **Sorensen**, Sachdev, Katz]

- Simulate lattice model for O(2) QCP ➛ **imaginary** frequencies



$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^{\Delta} + b_2 \left(\frac{T}{\omega_n}\right)^{3} + \cdots$$

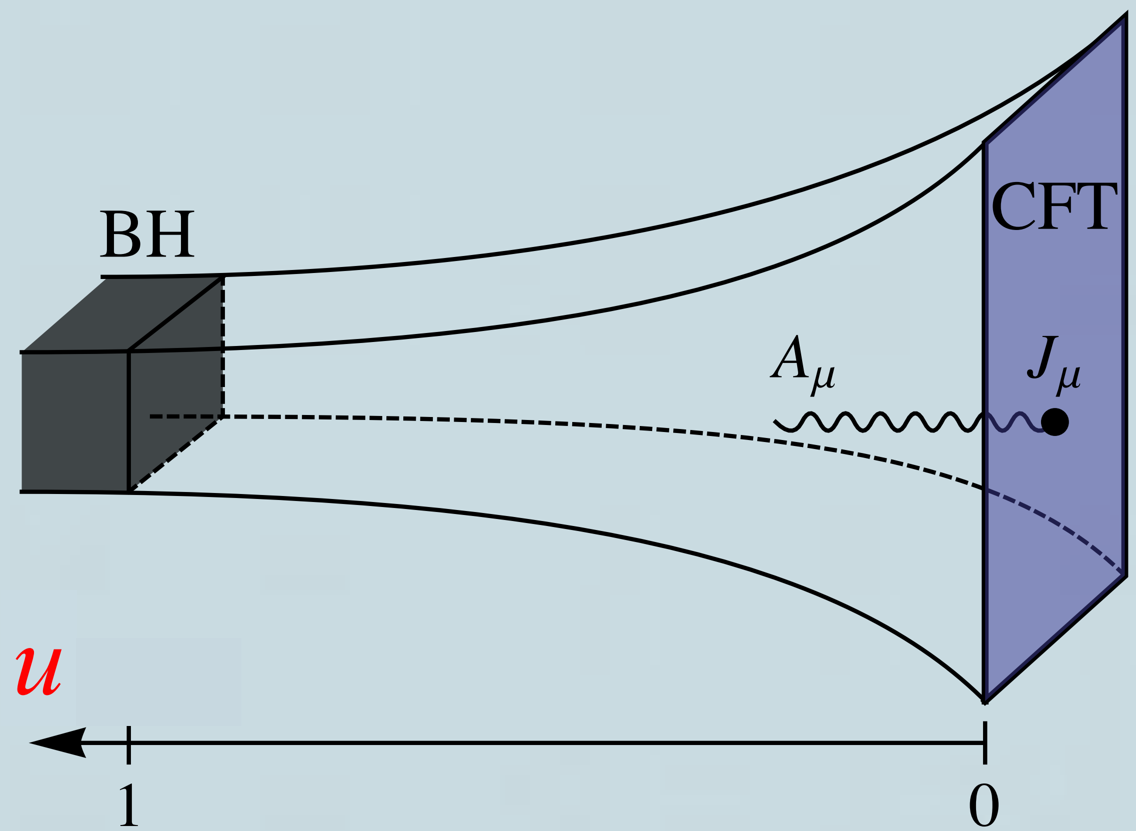
Long times &
analytic continuation

★Use **AdS/CFT** to generate a family of physically motivated (constrained) variational functions $\sigma(\omega/T)$

σ via AdS/CFT

[Maldacena *etc*]

spacetime:
B-Hole in AdS$_4$



$$A_\mu(t,x,y;u) \leftrightarrow J_\mu^{\mathrm{CFT}}(t,x,y)$$

- Solve **classical** EoM for A $\longrightarrow$ get J-correlator in **CFT**

$$\sigma(\omega/T) = \frac{T}{\omega} \left. \frac{\partial_u A_y(\omega,\vec{0};u)}{A_y(\omega,\vec{0};u)} \right|_{u=0}$$

1st try

$$S_{\rm bulk}[A_\mu] = \int d^4x\sqrt{-g}\,\frac{1}{g_4^2}\left[F^2 + \gamma C^{abcd}F_{ab}F_{cd}\right]$$

C = traceless part of Riemann

- Derivative expansion

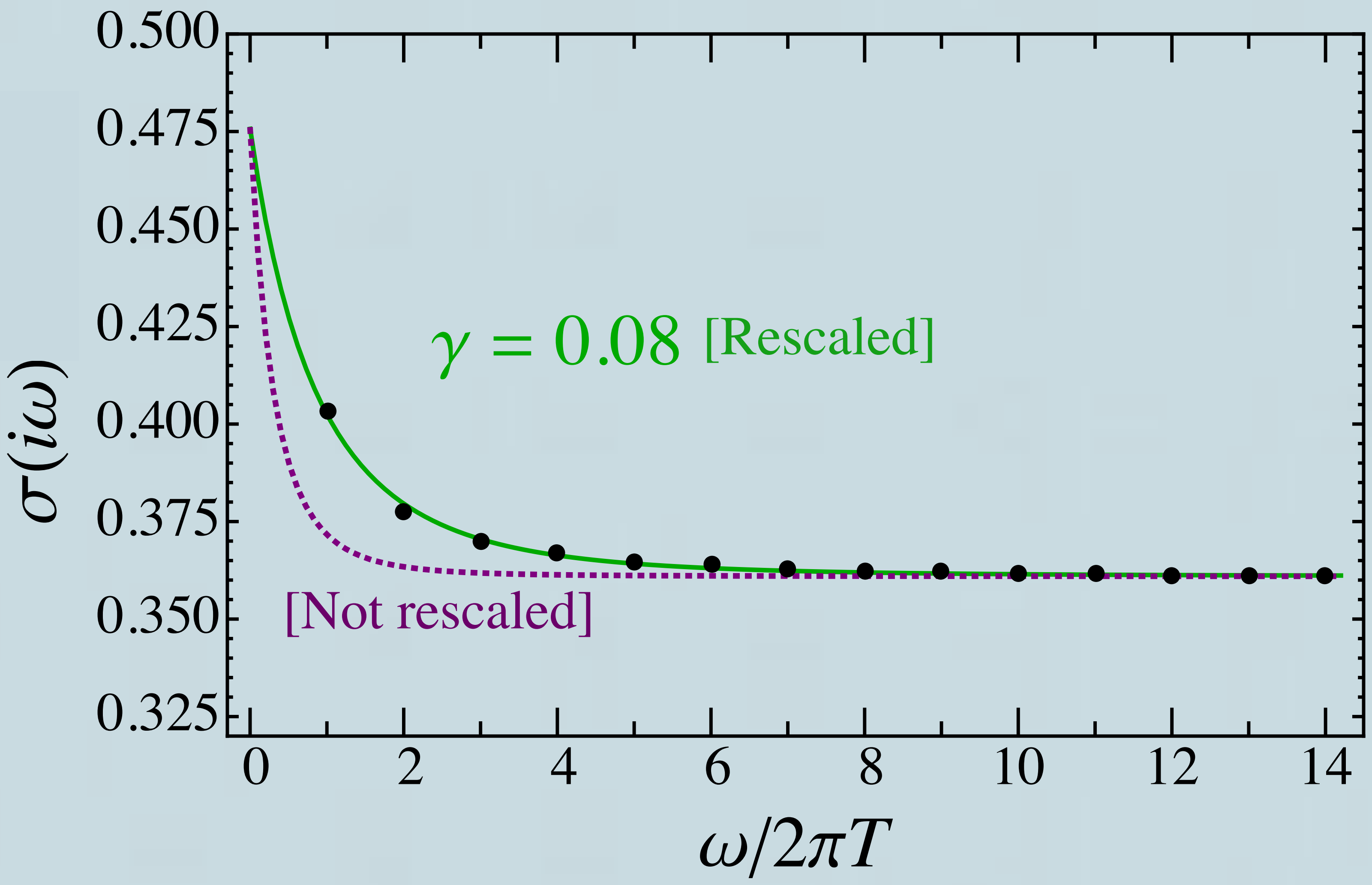
- Asymptotics [WWK]: $\sigma \sim \sigma_\infty + (T/\omega_n)^3 + \ldots$

CFT	AdS
$T_{\mu\nu}$	$g_{\mu\nu}$

[Herzog, Kovtun, Sachdev, Son; Ritz, Ward; Myers, Sachdev, Singh]

Holographic fit: take 1

[WWK, Sorensen, Sachdev]



$$\sigma\left(\kappa\,\frac{\omega}{T};\gamma\right)$$

2nd try

$$S_{\text{bulk}}[A_\mu] = \int d^4x \, \frac{1}{g_4^2} \left[1 + \alpha \, \varphi(x)\right] F_{ab}F^{ab}$$

- Add **scalar field** (dilaton)

CFT	AdS
O	φ

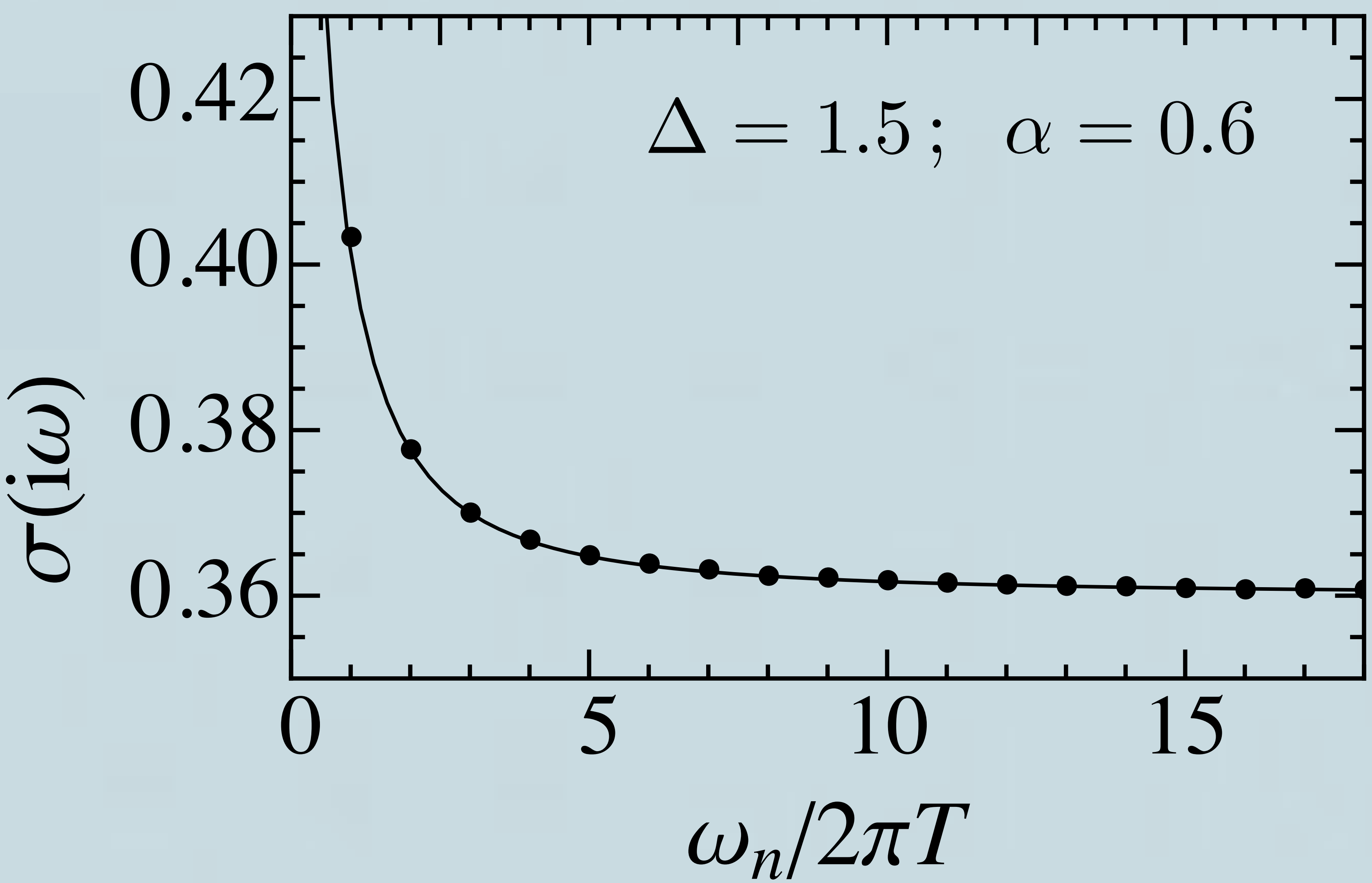
- Simplest **Ansatz**:
  fix profile using OPE of O(2) Wilson-Fisher CFT

$$\varphi(u) = u^{\Delta}$$

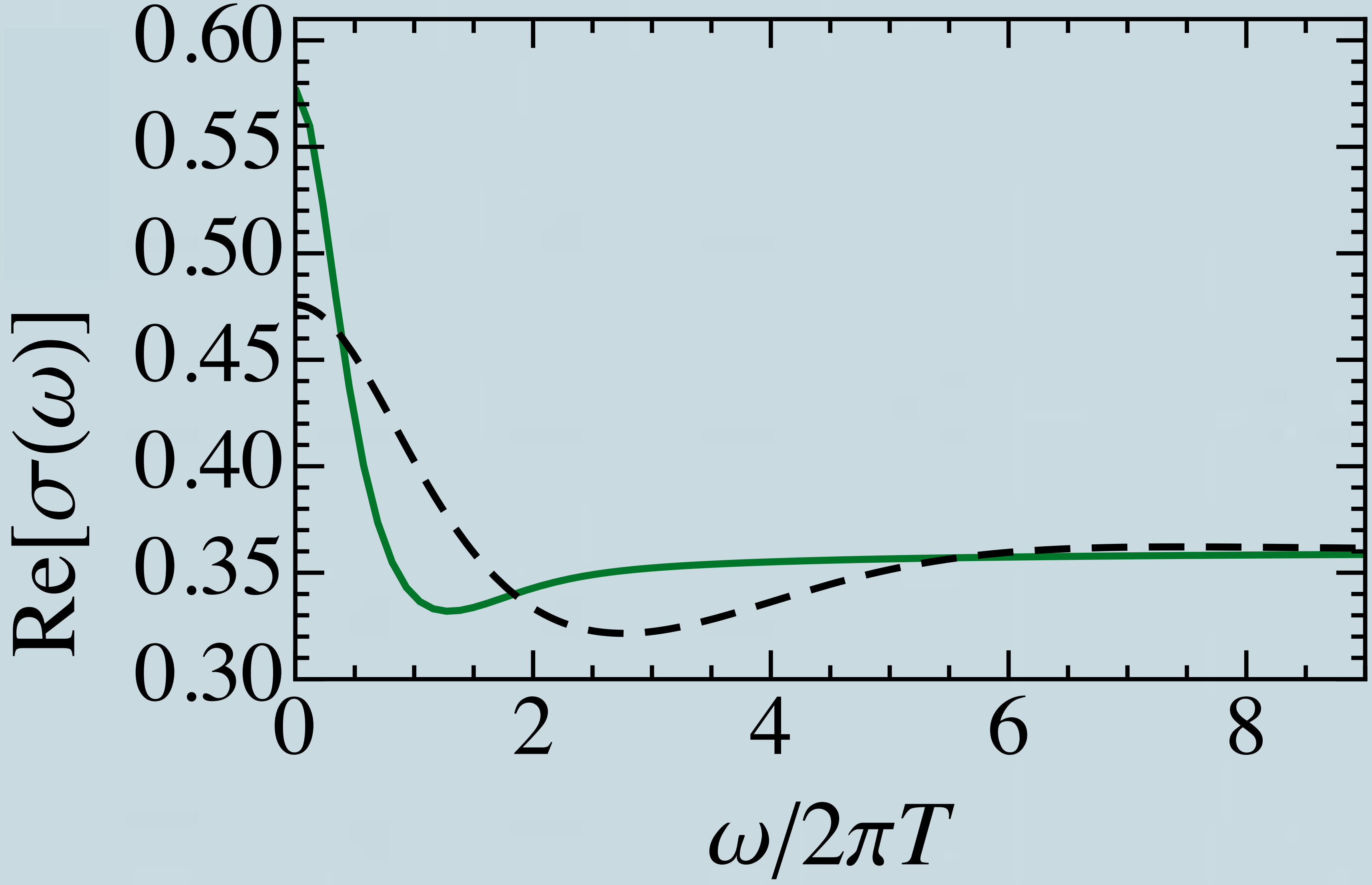
[Katz, Sachdev, Sorensen, WWK]

Holographic fit: take 2

[Katz, Sachdev, Sorensen, WWK]



Real frequencies



Why dabble with black holes?

- Physical properties:
  - ***Tailored*** holographic description to match asymptotics
  - **Sum rules**

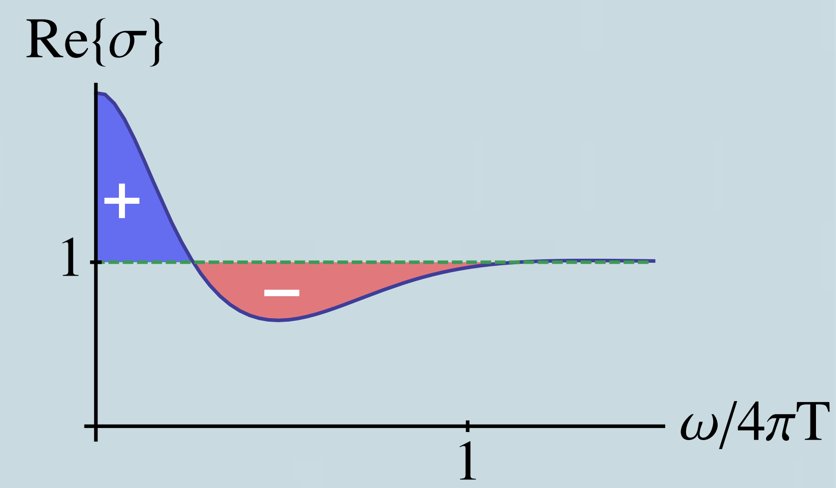
$$\int_0^\infty d\omega \, \mathrm{Re}[\sigma(\omega/T) - \sigma(\infty)] = 0$$

  - **Analytic properties** in common with field theory results
  - **Practical**: real-time, small number of params, fast numerics

Sum rules

[WWK, Sachdev; Gulotta, Herzog, Kaminski]

$$\int_0^\infty d\omega \, [\mathrm{Re}\, \sigma(\omega/T) - \sigma(\infty)] = 0$$



❖ S-dual version:

$$\int_0^\infty d\omega \left[\mathrm{Re} \left\{ \frac{1}{\sigma(\omega/T)} \right\} - \frac{1}{\sigma(\infty)} \right] = 0$$

✔ $\mathcal{N}$=8 supersymmetric ABJM

✔ O(N) CFT @ N=∞

✔ Dirac CFT

Sum rule proof via OPE

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^\Delta + b_2 \left(\frac{T}{\omega_n}\right)^3 + \cdots$$

- σ analytic in upper half-plane
- $\Delta > 1 \;\Rightarrow\;$ Integrable $\;\Rightarrow\;$ Close contour in UHP ✔

Conclusions

- Quantum critical dynamics (CFTs) in 2+1D
- OPE to constrain *short time* physics
  - Large ω conductivity $$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^{\Delta} + b_2 \left(\frac{T}{\omega_n}\right)^{3} + \cdots$$
- Input OPE data of CFT into simple holographic ansatz
- Can match Monte Carlo data of O(2) CFT w/out unphysical tweaks

Outlook

- Apply this program (OPE, sum rules) to other correlators & CFTs [WWK, arXiv:1501.xxxxx]
- Go beyond simplest holographic Ansatz
  [in progress with T. Sierens & R. Myers]
- Unitarity in proof of sum rules?

Thanks!

- E. Katz, S. Sachdev, E. Sørensen, W.Witczak-Krempa, PRB **90**, 2014 [Ed. Sugg.]
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