lattice studies of dark matter

randy.lewis@yorku.ca

1. Lattice QCD studies of $\langle N|m_s\bar{s}s|N\rangle$.
   collected in Junnarkar and Walker-Loud, Phys Rev D87, 114510 (2013)

2. Putting the dark matter directly on the lattice:
   - SU(2) gauge theory. Lewis,Pica,Sannino, Phys Rev D85, 014504 (2012)
     Hietanen,Lewis,Pica,Sannino JHEP 07(2014)116
     Hietanen,Lewis,Pica,Sannino, arXiv:1308.4130
     Detmold,McCullough,Pochinsky, arXiv:1406.2276 and 1406.4116
   - SU(3) gauge theory. Appelquist et al (LSD collab), Phys Rev D88, 014502 (2013)
   - SU(4) gauge theory. Appelquist et al (LSD collab), Phys Rev D89, 094508 (2014)
   - SO(4) gauge theory. Hietanen,Pica,Sannino,Søndergaard, Phys Rev D87, 034508 (2013)

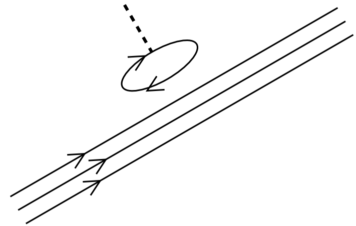
lattice QCD studies of $\langle N|m_s\bar{s}s|N\rangle$

Perhaps dark matter is a WIMP (weakly-interacting massive particle).
WIMP detection requires knowledge of WIMP-nucleon interactions.

The low-energy limit of a spin-independent interaction is scalar.
The scalar coupling to strangeness in a nucleon has been a challenge for theory.

Lattice QCD can determine the necessary matrix element,

$$f_s = \frac{\langle N|m_s\bar{s}s|N\rangle}{m_N}$$



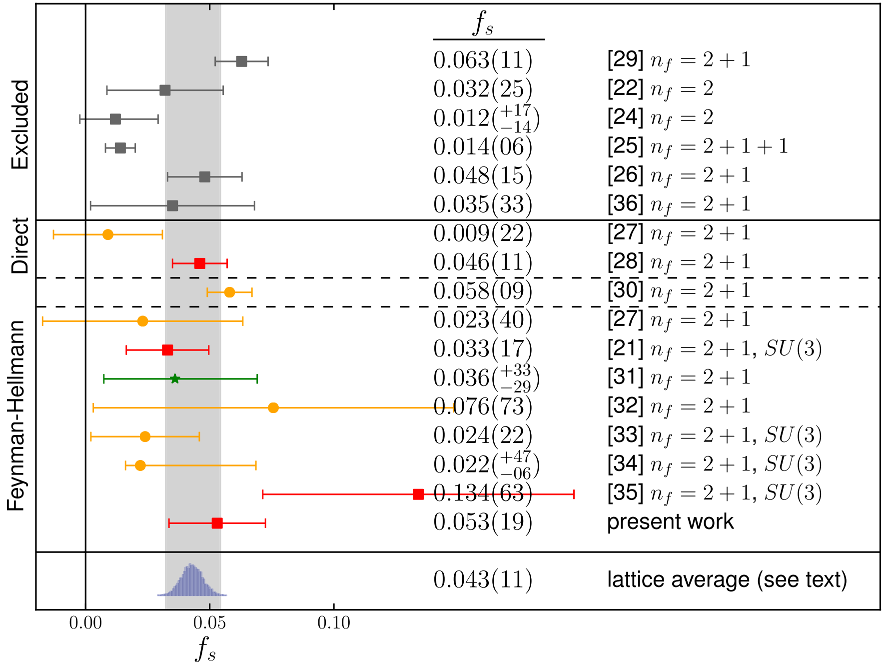
either directly or via the Feynman-Hellman theorem:

$$m_s\langle N|\bar{s}s|N\rangle = m_s\frac{\partial m_N}{\partial m_s}$$

Recent lattice results indicate that f_s is smaller than some previous estimates.

lattice QCD studies of $\langle N|m_s\bar{s}s|N\rangle$

Graph taken from Junnarkar and Walker-Loud, Phys Rev D87, 114510 (2013)



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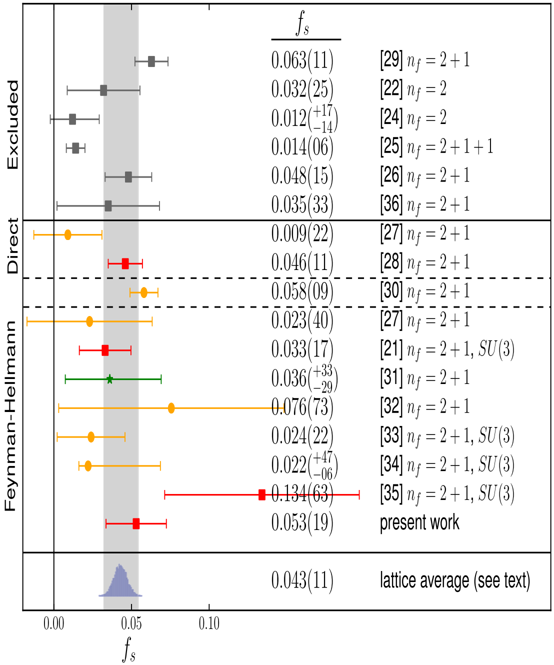
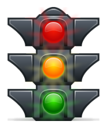


ASSESSING RELIABILITY:

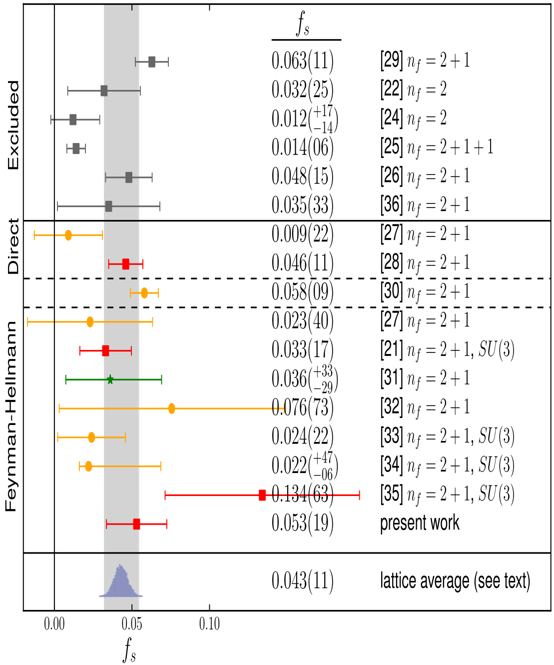
Were u,d quarks light enough?
Was the continuum limit taken?
Was the ∞ volume limit taken?



room for significant improvement
room for improvement
meets strictest constraints

lattice QCD studies of $\langle N|m_s\bar{s}s|N\rangle$

Graph taken from Junnarkar and Walker-Loud, Phys Rev D87, 114510 (2013)



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putting dark matter directly on the lattice

Dark matter is a BSM particle. Suppose it comes with a new strong interaction.

SU(2) gauge theory with 2 fundamental fermions is a minimal example.
— contains a dark matter candidate.
— produces electroweak symmetry breaking.
— accommodates a 125 GeV scalar.

Dynamical symmetry breaking, SU(4)→Sp(4), gives 5 Goldstone bosons:

$$\left.\begin{array}{c} \bar{U}\gamma_5 D \\ \bar{D}\gamma_5 U \\ \frac{1}{\sqrt{2}}(\bar{U}\gamma_5 U - \bar{D}\gamma_5 D) \end{array}\right\} \quad \text{eaten by } W^\pm \text{ and } Z$$

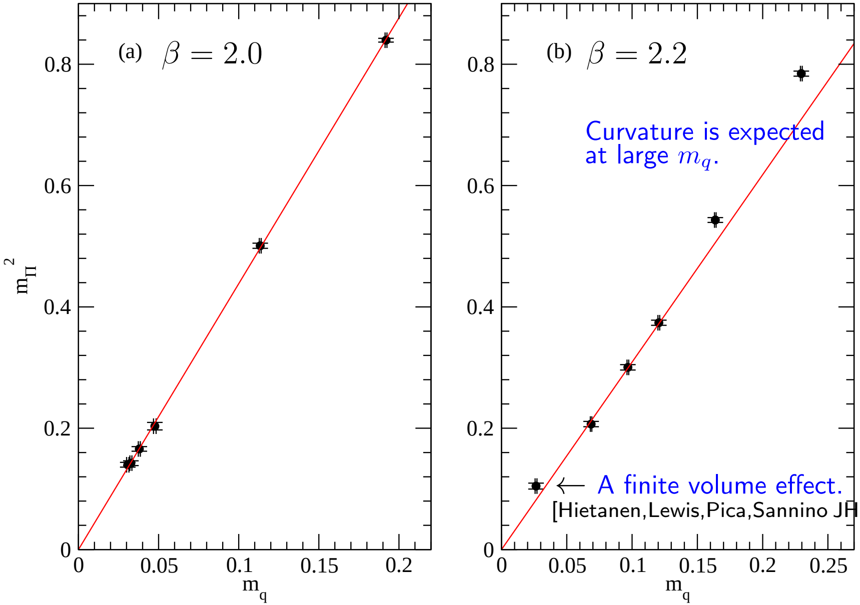
$$\left.\begin{array}{c} U^T(-i\sigma^2 C)\gamma_5 D \\ \bar{U}(-i\sigma^2 C)\gamma_5 \bar{D}^T \end{array}\right\}$$

either *light* asymmetric dark matter (technicolor limit)
or Higgs + *heavier* dark matter (little Higgs limit)
or an interpolation between these two limits

observing the Goldstone bosons in N_c=N_f=2

The expected behavior, $m_\Pi^2 \propto m_q$ for small m_q, is observed.
These plots apply to all five Goldstone bosons.

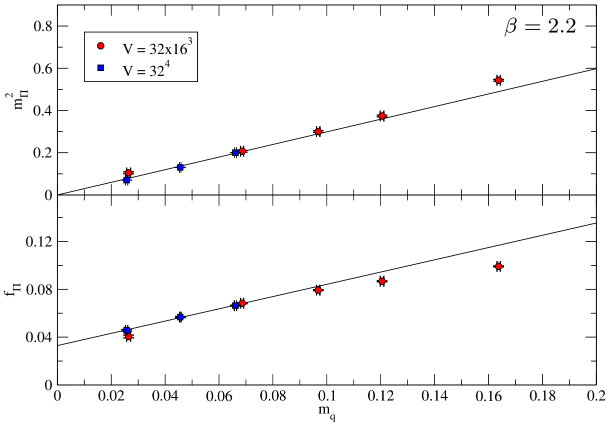
Lewis,Pica,Sannino, Phys Rev D85, 014504 (2012)

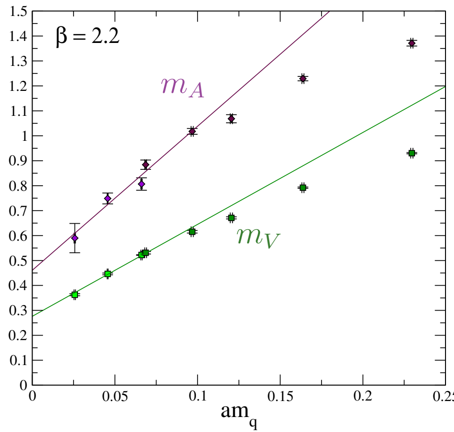


[Hietanen,Lewis,Pica,Sannino JHEP 07(2014)116]

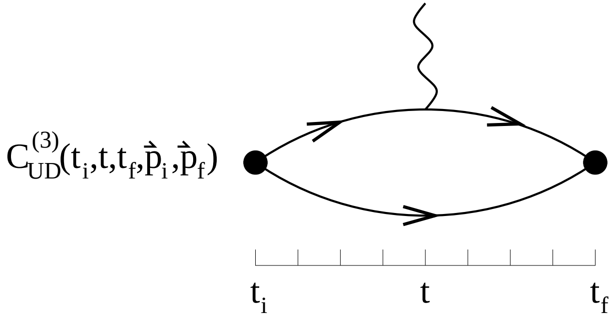
observing light hadrons in N_c=N_f=2

Hietanen,Lewis,Pica,Sannino JHEP 07(2014)116





relationships among Goldstone vector form factors in N_c=N_f=2



$$
\begin{aligned}
C^{(3)}_{UD}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= T^U - T^D \\
C^{(3)}_{\overline{UD}}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= -T^U + T^D \\
C^{(3)}_{U\overline{D}}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= T^U + T^D \\
C^{(3)}_{\overline{U}D}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= -T^U - T^D \\
C^{(3)}_{\overline{U}U+\overline{D}D}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= 0
\end{aligned}
$$

$$
T^X = \sum_{\vec{x}_i,\vec{x},\vec{x}_f} e^{-i(\vec{x}_f-\vec{x})\cdot\vec{p}_f} e^{-i(\vec{x}-\vec{x}_i)\cdot\vec{p}_i} \left\langle 0 \left| \mathcal{O}^{(\gamma_5)}_{UD}(x_f) V^X_\mu(x) \mathcal{O}^{(\gamma_5)\dagger}_{UD}(x_i) \right| 0 \right\rangle
$$

recalling resonance saturation in QCD

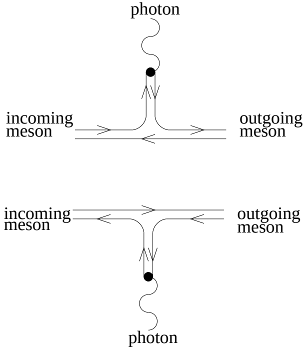
Lattice simulations with $m_U \neq m_D$ are expensive (photon hitting a vacuum loop doesn't cancel), but with $m_U = m_D$ the dark matter form factor vanishes.

What to do?
Notice that vector meson dominance relates T^U to T^D in the large N_c limit (and is successful for $N_c = 3$ QCD):

$$F_{\pi^+} \approx \frac{2}{3}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right) + \frac{1}{3}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)$$

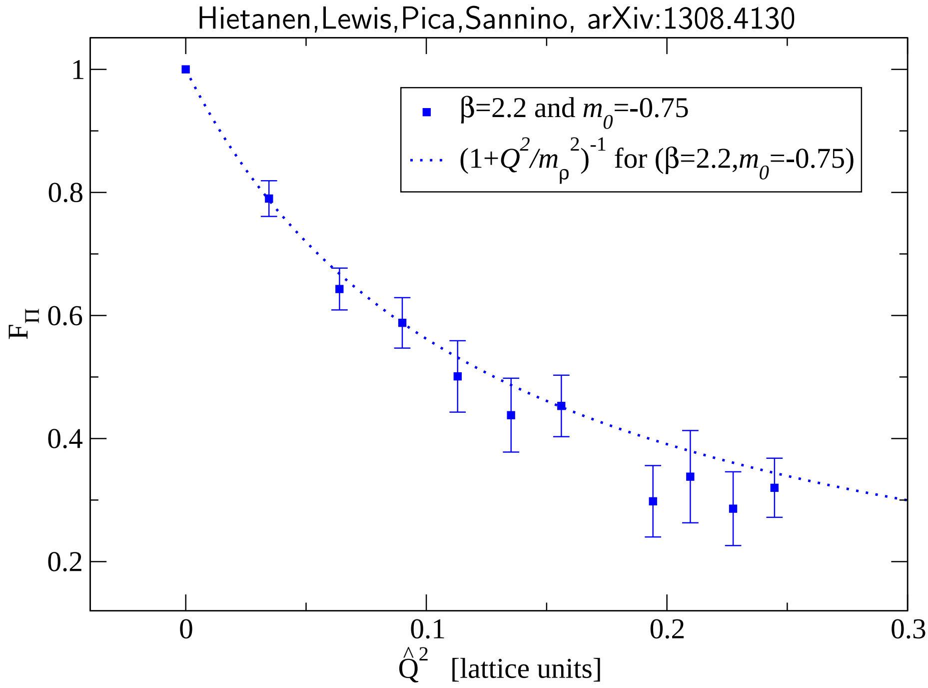
$$F_{K^+} \approx \frac{2}{3}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right) + \frac{1}{3}\left(\frac{m_\phi^2}{m_\phi^2+Q^2}\right)$$

$$F_{K^0} \approx -\frac{1}{3}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right) + \frac{1}{3}\left(\frac{m_\phi^2}{m_\phi^2+Q^2}\right)$$



We will test this VMD behaviour in SU(2) simulations with $m_U = m_D$.

observing resonance saturation in N_c=N_f=2



dark matter scattering by photon exchange in N_c=N_f=2

The coupling is due to the charge radius,

$$\mathcal{L}_B = ie\frac{d_B}{\Lambda^2}\phi^* \overleftrightarrow{\partial_\mu}\phi\,\partial_\nu F^{\mu\nu}$$

and we can calculate explicitly,

$$\frac{d_B}{\Lambda^2} = \lim_{Q^2\to 0}\frac{1}{Q^2}\left(\frac{1}{2}\frac{m_{\rho_U}^2}{m_{\rho_U}^2+Q^2} - \frac{1}{2}\frac{m_{\rho_D}^2}{m_{\rho_D}^2+Q^2}\right) = \frac{m_{\rho_U}^2 - m_{\rho_D}^2}{2m_{\rho_U}^2 m_{\rho_D}^2}$$

Therefore

$$\boxed{\Lambda = m_\rho} \quad \text{and} \quad \boxed{d_B = \frac{m_{\rho_U} - m_{\rho_D}}{m_\rho}}$$

The cross section for scattering from a proton is

$$\sigma_p^\gamma = \frac{\mu^2}{4\pi}\left(\frac{8\pi\alpha d_B}{\Lambda^2}\right)^2 \quad \text{where} \quad \mu = \frac{m_\phi m_N}{m_\phi + m_N}$$

Given $m_\phi > m_N$ and $|d_B| < 1$, we find $\boxed{\sigma_p^\gamma < 2.3 \times 10^{-44}\ \text{cm}^2}$.

adding the exchange of a composite Higgs

The dark matter candidate couples to a composite Higgs as follows:

$$\delta\mathcal{L} = \frac{d_1}{\Lambda} h \partial_\mu \phi^* \partial^\mu \phi + \frac{d_2}{\Lambda} m_\phi^2 h \phi^* \phi$$

We expect d_1 and d_2 to be of order unity.

The cross section for scattering from a proton is

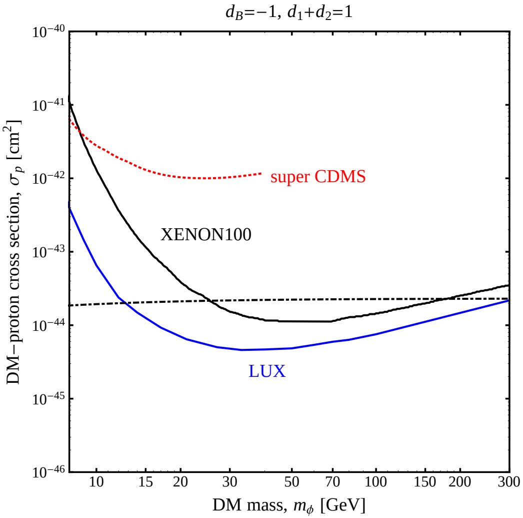
$$\sigma_p = \frac{\mu^2}{4\pi} \Bigg(\overbrace{\underbrace{\frac{(d_1 + d_2) f m_N m_\phi^2}{m_H^2 m_\phi v_{EW} \Lambda}}_{f_n} + \frac{8\pi\alpha d_B}{\Lambda^2}}^{f_p} \Bigg)^2 \quad \text{where} \quad \mu = \frac{m_\phi m_N}{m_\phi + m_N}$$

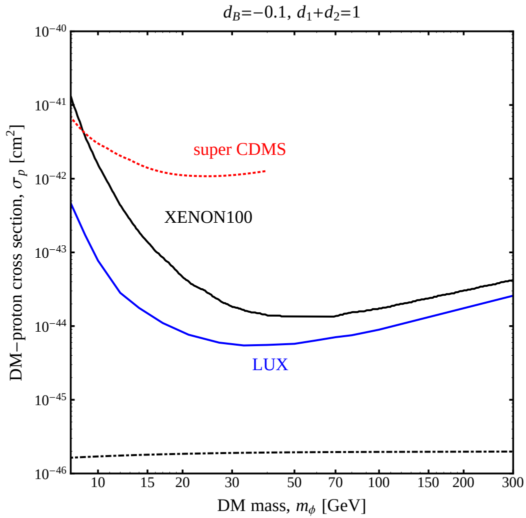
The Higgs to nucleon coupling is parametrized by $f \sim 0.3$.

This cross section is thus a function of m_ϕ and d_B. Compare to experiment...

comparison of N_c=N_f=2 to experiments

Hietanen,Lewis,Pica,Sannino, arXiv:1308.4130

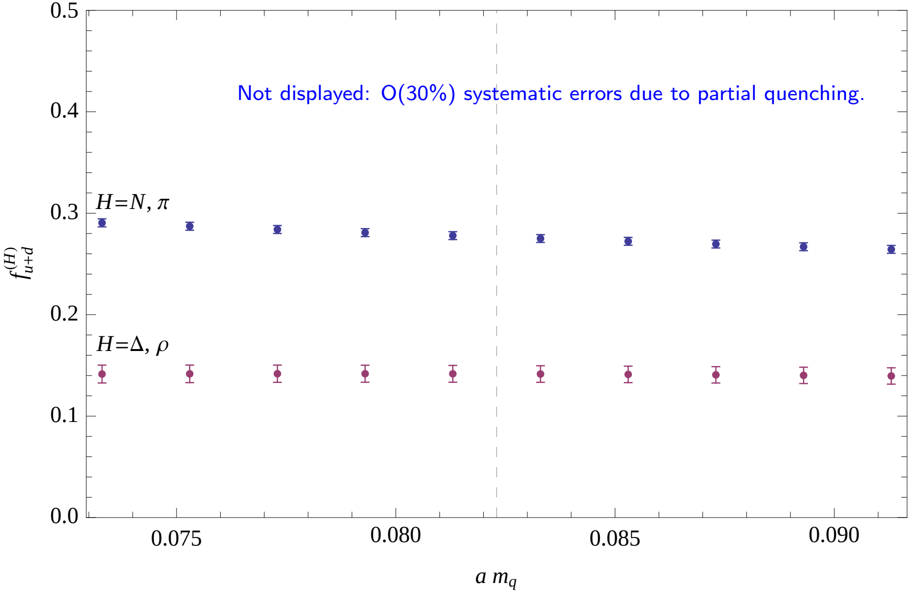




scalar couplings in N_c=N_f=2

Detmold,McCullough,Pochinsky, arXiv:1406.4116

$$f_q^{(H)} = \frac{m_q}{M_H}\frac{\partial M_H}{\partial m_q} = \frac{\langle H|m_q\bar{q}q|H\rangle}{M_H}$$



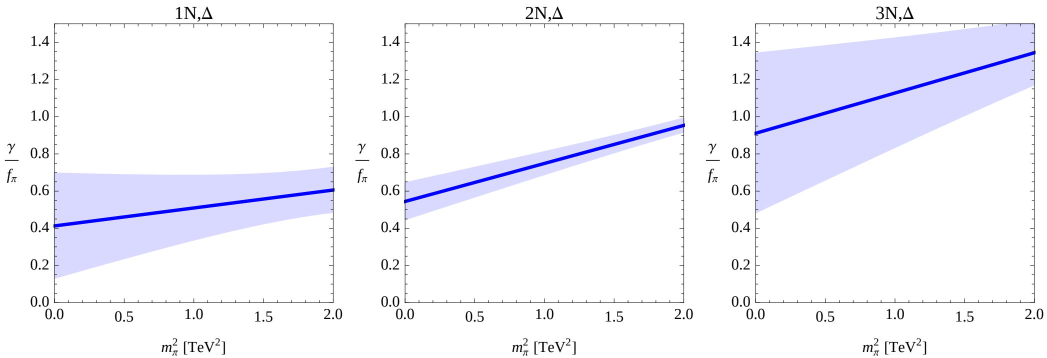
dark nuclei in N_c=N_f=2

Detmold,McCullough,Pochinsky, arXiv:1406.4116

For scattering states, $\Delta E(L) \propto 1/L^3 + \ldots$.

For bound states, $$\Delta E(L) = -\frac{\gamma^2}{2\mu}\left[1 + \frac{12\hat{C}}{\gamma L} e^{-\gamma L}\right]$$

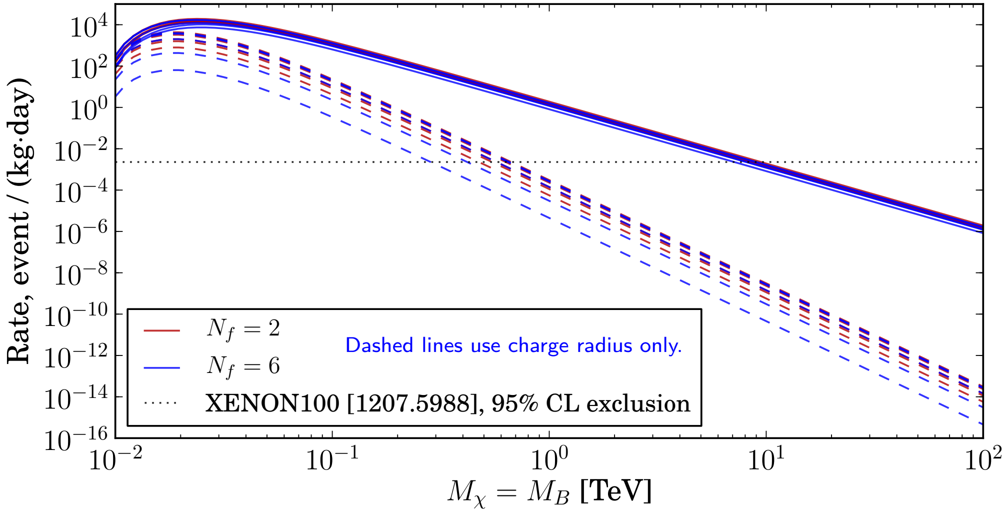
Bound states are observed for $J^P = 1^+$ in $N\Delta$ and $NN\Delta$ and perhaps $NNN\Delta$:



(This study uses $m_\rho/2 < m_\pi < m_\rho$ and $f_\pi = 246$ GeV.)

Event rate for XENON100 from a N_c=3 dark matter model

Appelquist et al (LSD collab), Phys Rev D88, 014502 (2013)



All dark quarks are weak singlets. $Q_U = \frac{2}{3}$, $Q_D = -\frac{1}{3}$. Disconnected lines omitted.

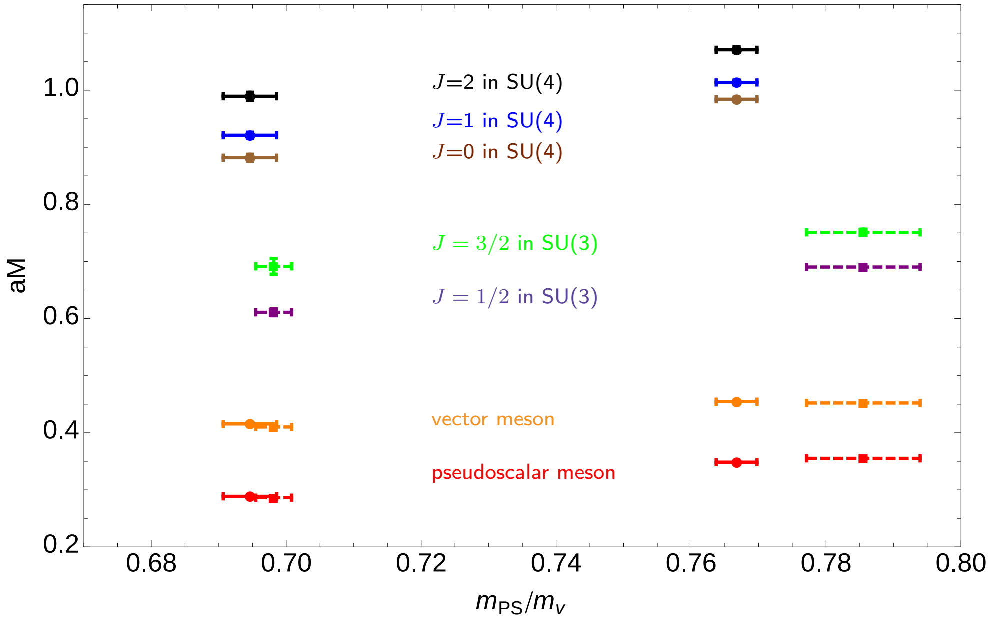
The $N_f^2 - 1$ Goldstones are assumed unstable so "neutron" is the DM candidate.

Caution: $\langle r_E^2 \rangle_{\text{neutron}} \approx$ experiment/10. Decreasing m_q might clarify this.

hadron mass spectrum in $N_c = 4$ dark matter model

Appelquist et al (LSD collab), Phys Rev D89, 094508 (2014)

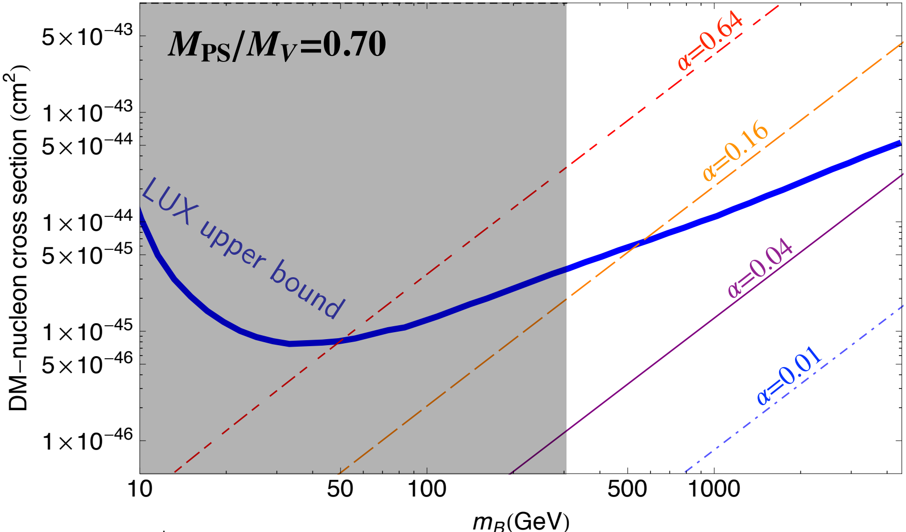
This is a quenched exploration. It has $m_f \sim \Lambda_4$.



bounds on fermion-Higgs coupling in $N_c = 4$ dark matter model

Appelquist et al (LSD collab), Phys Rev D89, 094508 (2014)

This is a quenched exploration. It requires $m_{\rm PS} > 100$ GeV due to LEP.



$$\alpha = \frac{v}{m_f} \frac{\partial m_f(h)}{\partial h}\bigg|_{h=v}$$ is the coupling of a dark baryon to the Higgs.

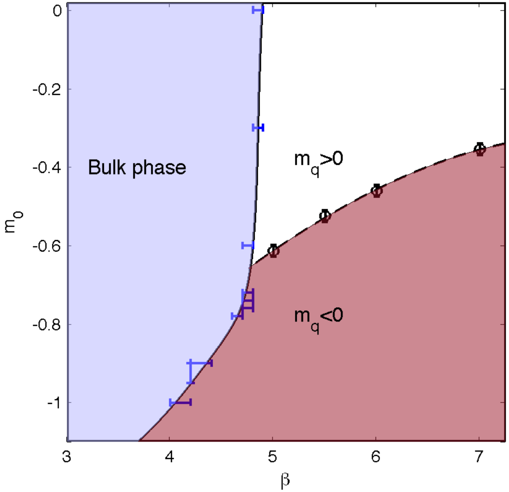
Lattice spacing, volume, and some range of $m_{\rm PS}/m_V$ were studied.

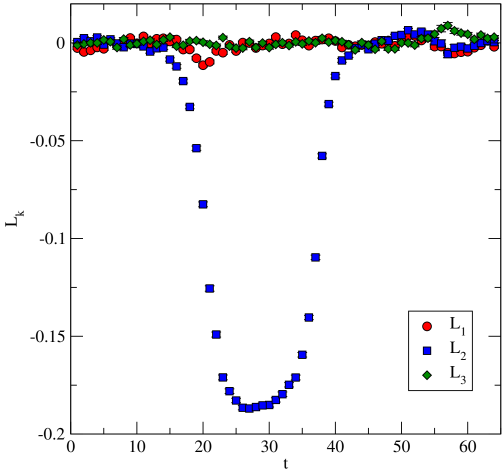
phase structure of SO(4) with 2 fermions

Hietanen,Pica,Sannino,Søndergaard, Phys Rev D87, 034508 (2013)

Lattice dark matter beyond SU(N): step one is to explore the phases.

This study uses 2 (Wilson) Dirac fermions in the vector representation.





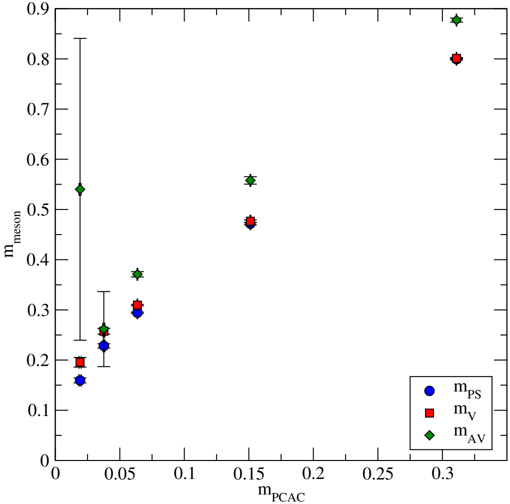
Surprising Polyakov multi-phase phenomenon not observed for larger volumes.

hadron masses in SO(4) with 2 fermions

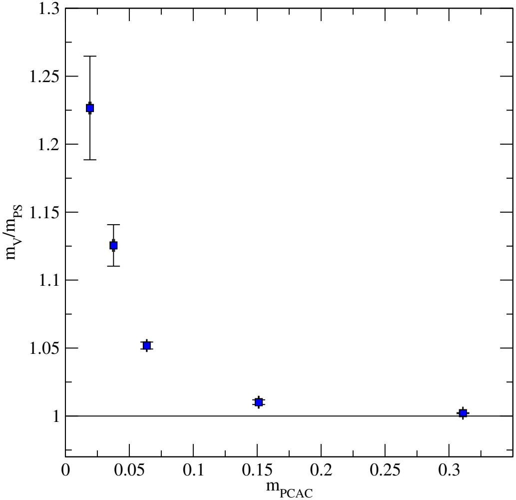
Hietanen,Pica,Sannino,Søndergaard, Phys Rev D87, 034508 (2013)

Expected global symmetry breaking is SU(4)→SO(4). Therefore 9 Goldstones.
The isospin=0 Goldstone boson is the dark matter candidate.

meson masses



chiral symmetry breaking



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   - SO(4) gauge theory. Hietanen,Pica,Sannino,Søndergaard, Phys Rev D87, 034508 (2013)

All of this is just the beginning...