Microlensing
with extended dark matter structures

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Dark matter substructure

Two things we may agree upon…

- (Unfortunately) all of our evidence for Dark Matter is gravitational
- Many dark matter models feature substructure

Primordial BHs	Boson stars	Subhalos	Miniclusters	Mirror stars

Dark matter substructure

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Primordial BHs	Boson stars	Subhalos	Miniclusters	Mirror stars

What else can we learn from gravitational interactions?

→ Microlensing surveys constrain primordial black holes

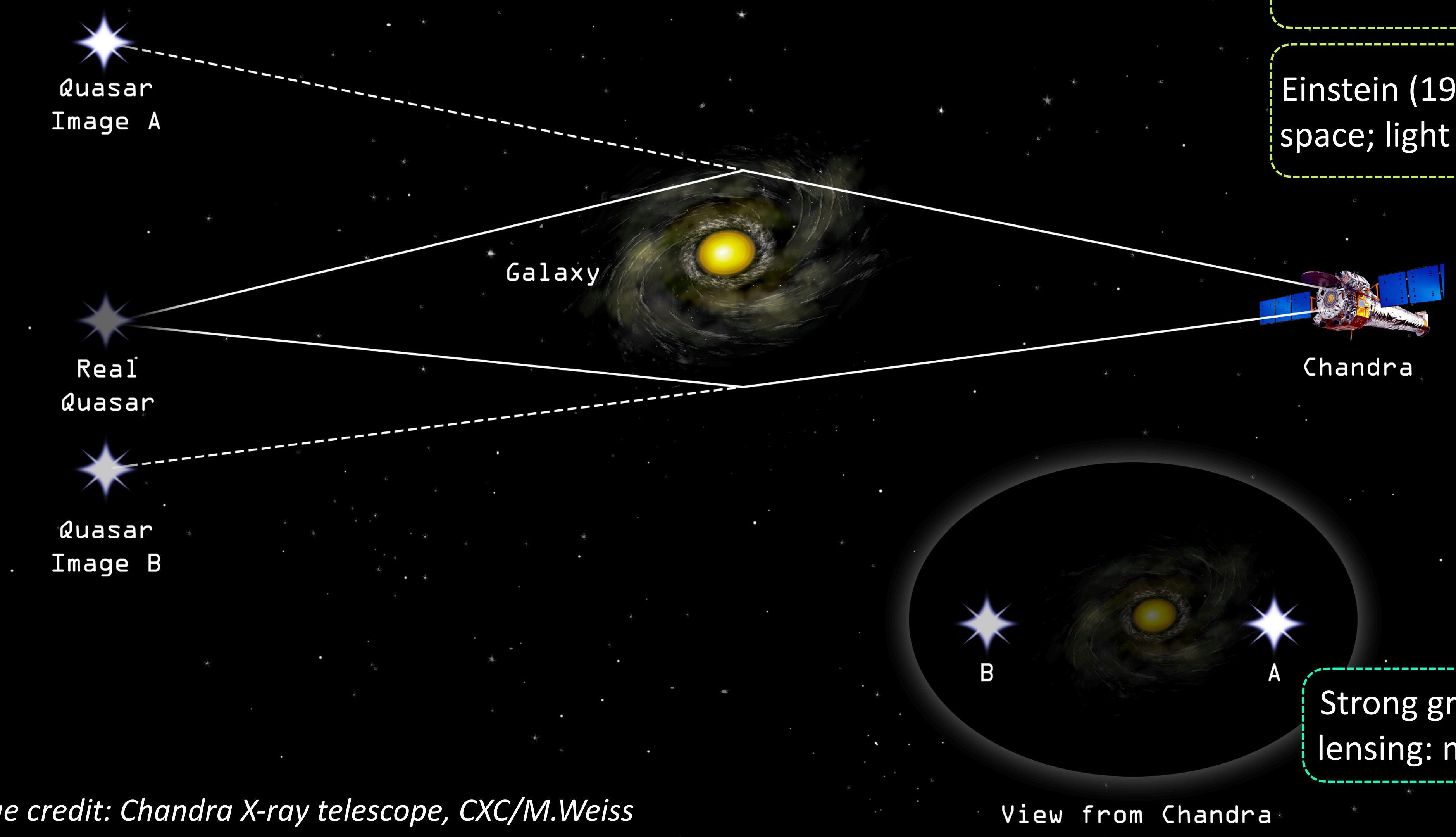
→ What about extended structures?

In this talk: Subaru-HSC, EROS-2 and OGLE-IV surveys

Strong gravitational lensing

Newton, Cavendish, Soldner (1704-1804): matter bends light (corpuscular theory of light)

Einstein (1911): matter bends space; light moves along geodesics



Strong gravitational lensing: multiple images

Image credit: Chandra X-ray telescope, CXC/M.Weiss

Gravitational <u>micro</u>lensing

*Micro*lensing: the lensed images are not individually resolved

Instead, there is a temporary magnification of the observed brightness

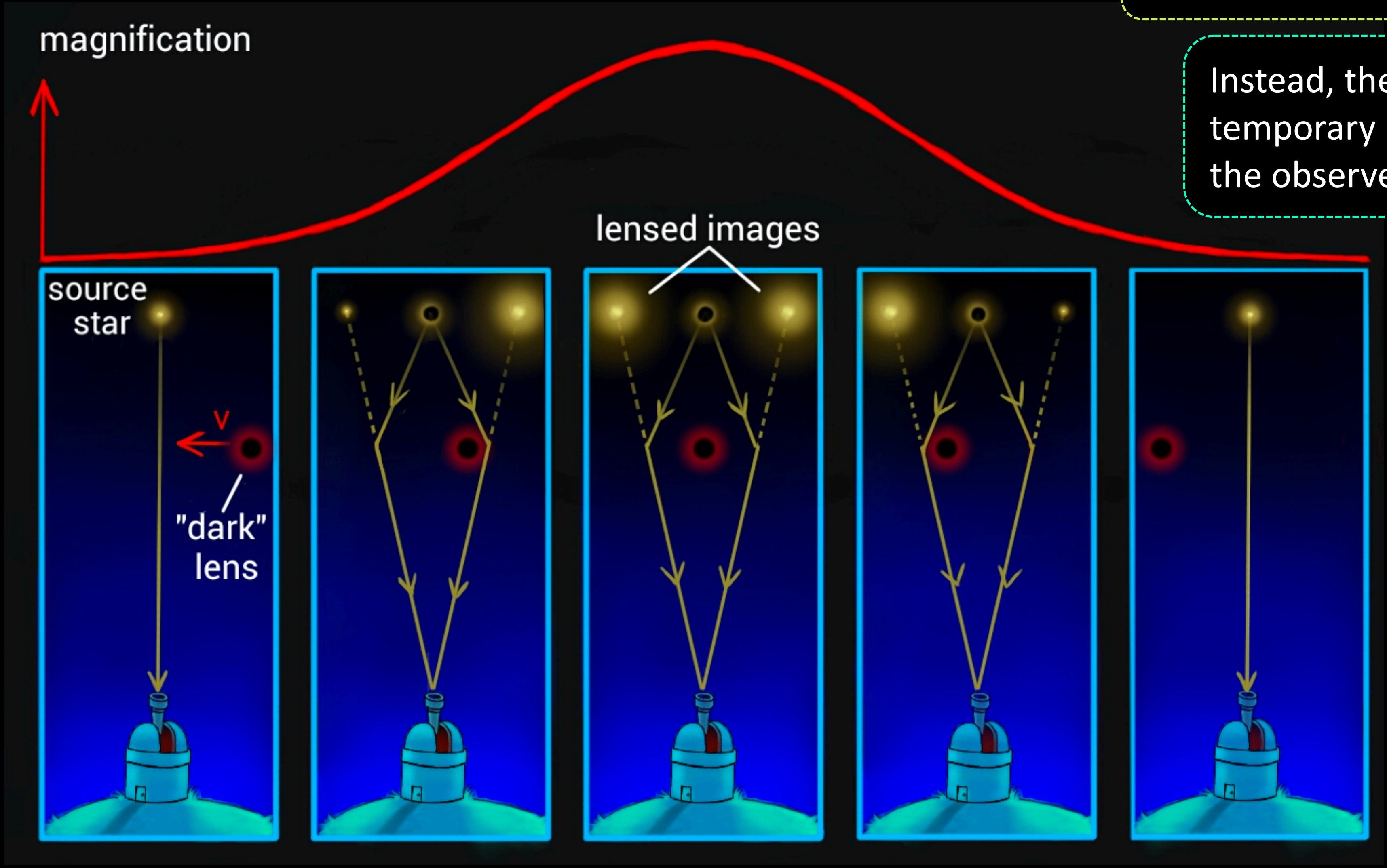


Image credit: Adam Rogers, theamateurrealist.wordpress.com

Gravitational microlensing

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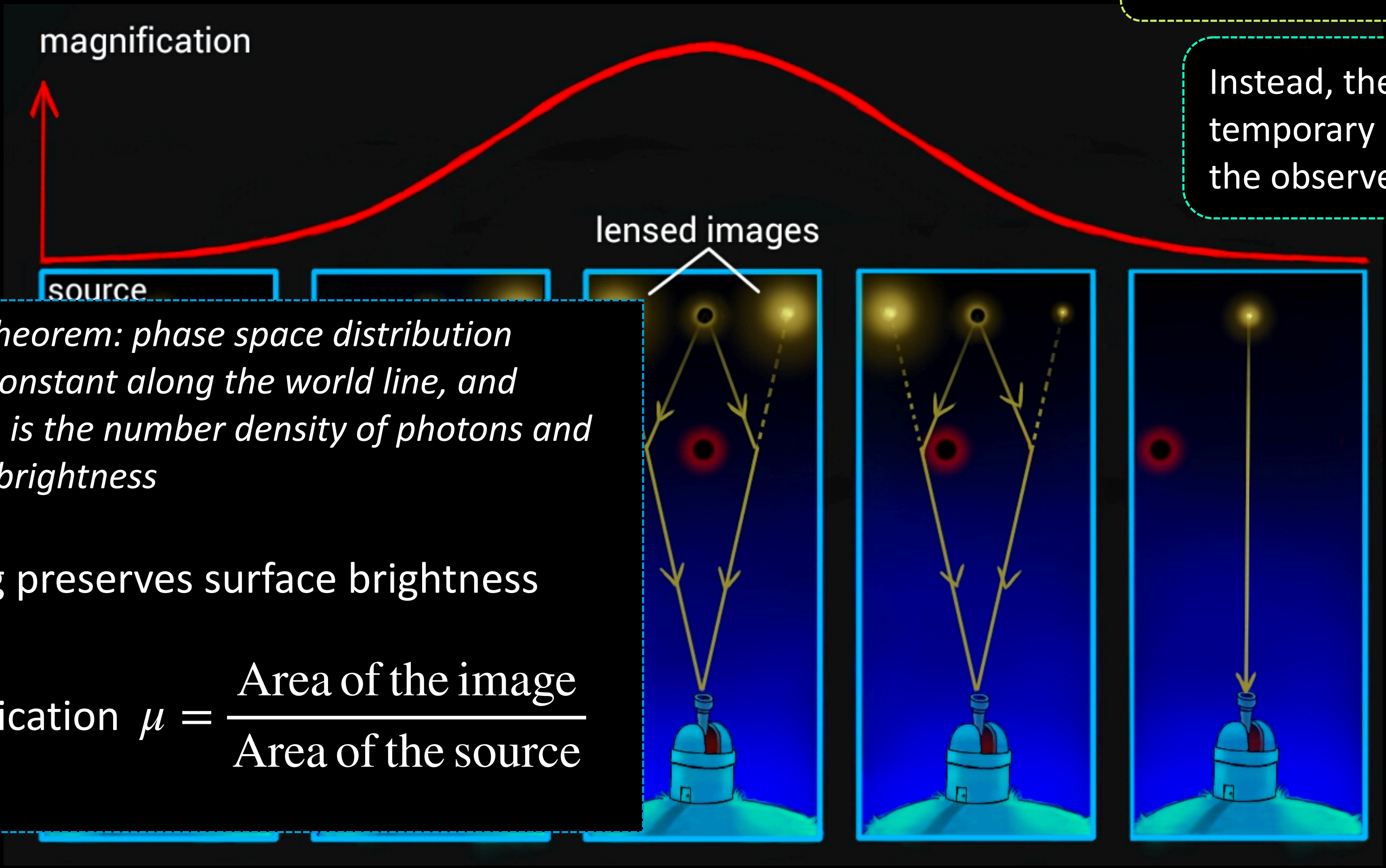


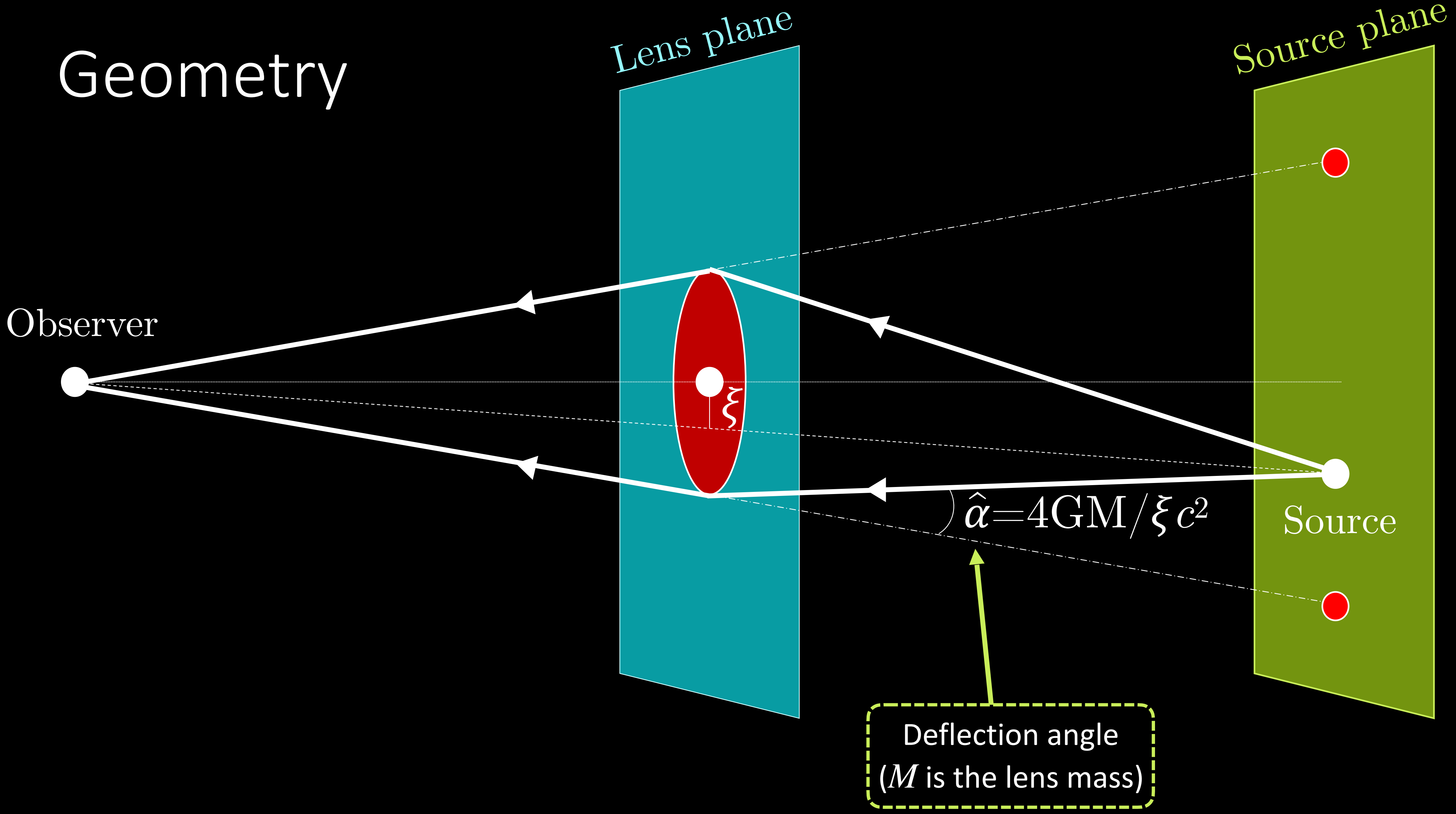
Liouiville's theorem: phase space distribution function is constant along the world line, and therefore so is the number density of photons and the surface brightness

→ Lensing preserves surface brightness

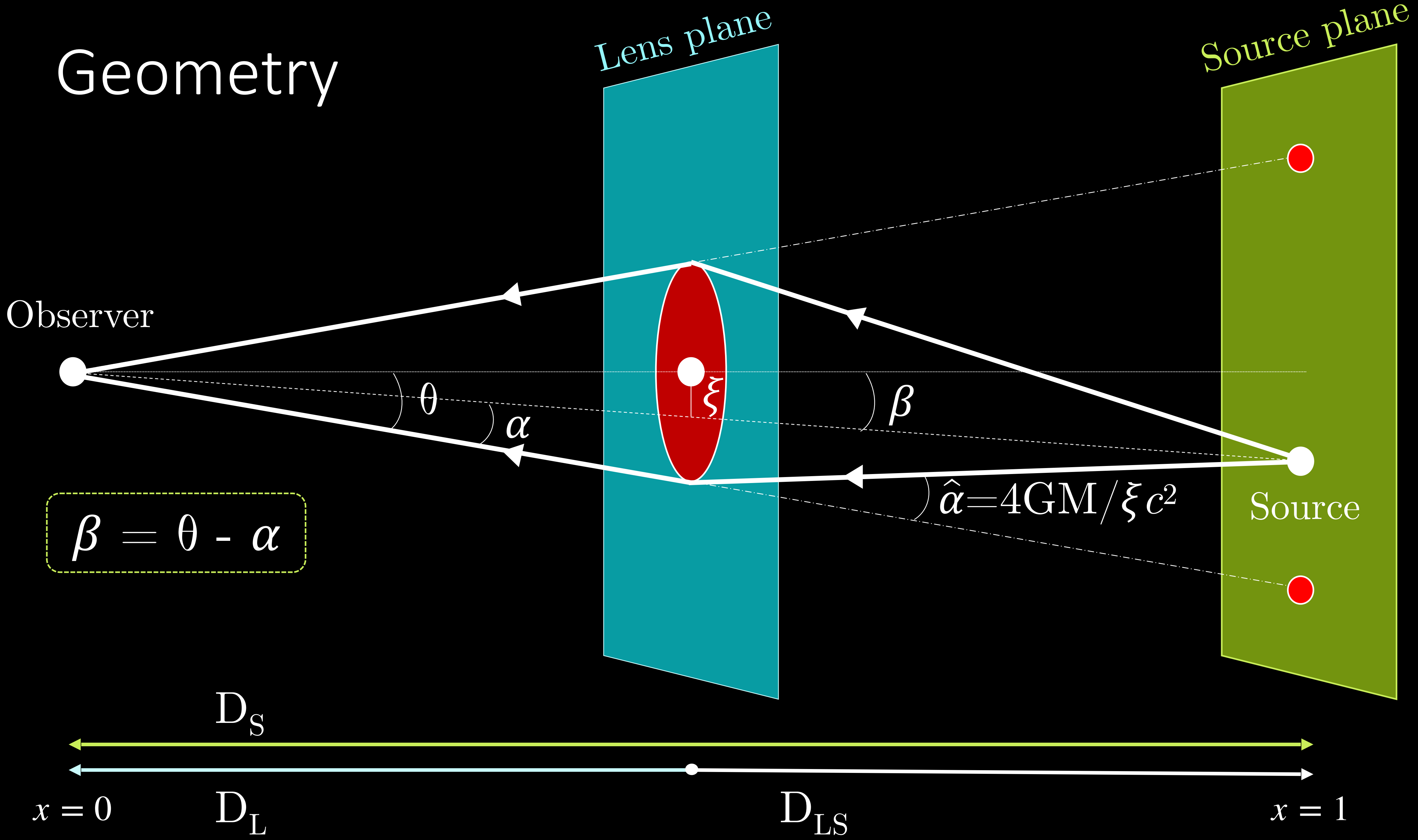
→ Magnification $\mu = \dfrac{\text{Area of the image}}{\text{Area of the source}}$

Image credit: Adam Rogers, theamateurrealist.wordpress.com

Geometry


Lens plane
Source plane
Observer
ξ
$\hat{\alpha} = 4GM/\xi c^2$
Source
Deflection angle
(M is the lens mass)

Geometry


Lens plane
Source plane
Observer
θ
α
ξ
β
$\hat{\alpha}=4GM/\xi c^2$
Source
$\beta = \theta - \alpha$
D_S
$x = 0$
D_L
D_{LS}
$x = 1$

The lensing equation (point-like lenses)

- The source position β and image position θ are related by

$$\beta = \theta - \alpha$$
$$= \theta - \frac{4GM(\theta)}{\theta c^2}\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}$$

$\beta = 0$

$M(\theta) = M$

Point-like lens right in front of the source

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}}$$

"Einstein angle" for a point-like lens

- The Einstein angle and corresponding radius define a characteristic scale for the source-lens system

A near perfect Einstein Ring with the Hubble Space telescope

The lensing equation (point-like lenses)

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$\beta = 0$
$M(\theta) = M$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$$

"Einstein angle" for a point-like lens

- θ_E can be used to define a lensing tube with radius $r_E = \theta_E D_L$

Magnification: $\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta} = \sum_i \mu_i = \frac{u^2+2}{u\sqrt{u^2+4}}$

$\xrightarrow{u=1} 1.34 \ .$

Microlensing event is counted if $\mu > 1.34$

Images

$u \equiv \beta/\theta_{\rm E}$ Impact parameter

Part I: extended lenses

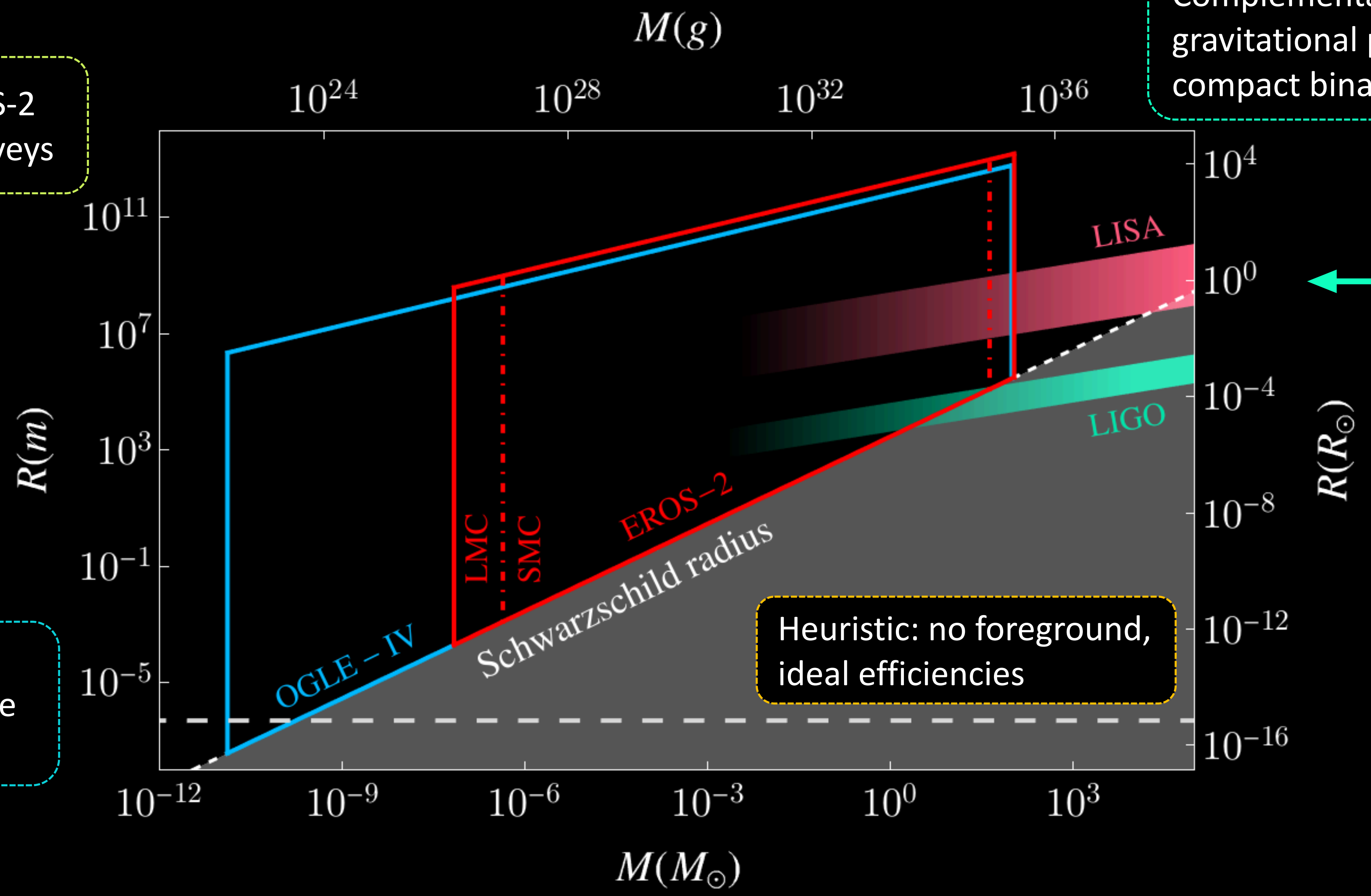
DC, D. McKeen, N. Raj, PRD, arXiv:2002.08962 [astro-ph.CO]

In this part: EROS-2 and OGLE-IV surveys

Sneak peak: substructure sensitivity

In this part: EROS-2 and OGLE-IV surveys

Complementary to other gravitational probes, like compact binary inspirals



Heuristic: no foreground, ideal efficiencies

Many decades in both substructure mass and size!

Lensing with finite sized objects

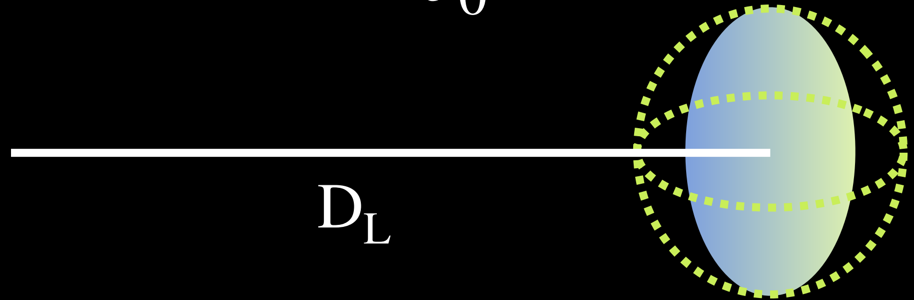
- Rewriting the lensing equation using the definition of θ_E,

$$\beta = \theta - \frac{\theta_E^2}{\theta}\frac{M(\theta)}{M}$$

- Here $M(\theta)$ gives the projection of the lens mass onto the lens plane,

$$M(\theta) = 2\pi D_{\mathrm{L}}^2 \int_0^\theta d\theta' \theta' \Sigma(\theta'), \qquad \Sigma(\theta) = \int_{-\infty}^{\infty} dz\, \rho\left(\sqrt{D_{\mathrm{L}}^2\theta^2 + z^2}\right)$$

Lens mass distribution



Lensing with finite sized objects

- Rewriting the lensing equation using the definition of θ_E,

$$\beta = \theta - \frac{\theta_E^2}{\theta}\frac{M(\theta)}{M}$$

- Using the new variables $u \equiv \beta/\theta_E$, $t \equiv \theta/\theta_E$, $m(t) \equiv M(\theta_E t)/M$ can rewrite this again to $u = t - \dfrac{m(t)}{t}$

Lensing with finite sized objects

- Can now also rewrite the magnification terms of the new variables

$$\mu = \left|1 - \frac{m(t)}{t^2}\right|^{-1} \left|1 + \frac{m(t)}{t^2} - \frac{1}{t}\frac{dm(t)}{dt}\right|^{-1}$$

- Where $m(t) \equiv M(\theta_E t)/M$ is given by

$$m(t) = \frac{\int_0^t d\sigma\sigma \int_0^\infty d\lambda\, \rho(r_E\sqrt{\sigma^2 + \lambda^2})}{\int_0^\infty d\gamma\gamma^2\rho(r_E\gamma)}$$

Will focus on different examples: more peaked and more diffuse objects

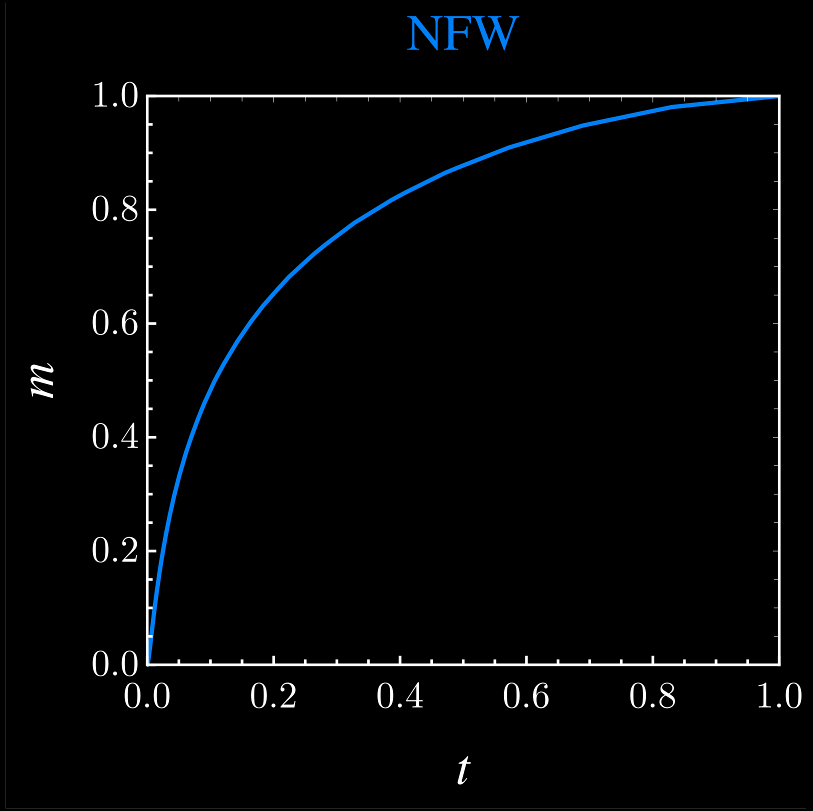
Will choose r_{90} — the radius enclosing 90% of the total mass — as the characteristic scale in both cases

Case study 1: NFW-halo mass profile

- Well known halo profile: $\rho(r) = \dfrac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$

- As the mass inclosed formally diverges, we cut it off at $R_{\rm cut} = 100\, R_{\rm sc}$
- Enclosed mass $\propto \log(\kappa + 1) - (\kappa/(\kappa + 1))$ where $\kappa = R_{\rm cut}/R_{\rm sc}$

- Computing $m(t)$ is then a trivial exercise:



Case study 2: Boson star mass profile

- The Schrodinger-Poisson equation,

$$\mu\Psi = -\frac{1}{2m_\phi}\left(\Psi'' + \frac{2}{r}\Psi'\right) + m_\phi \Phi \Psi$$

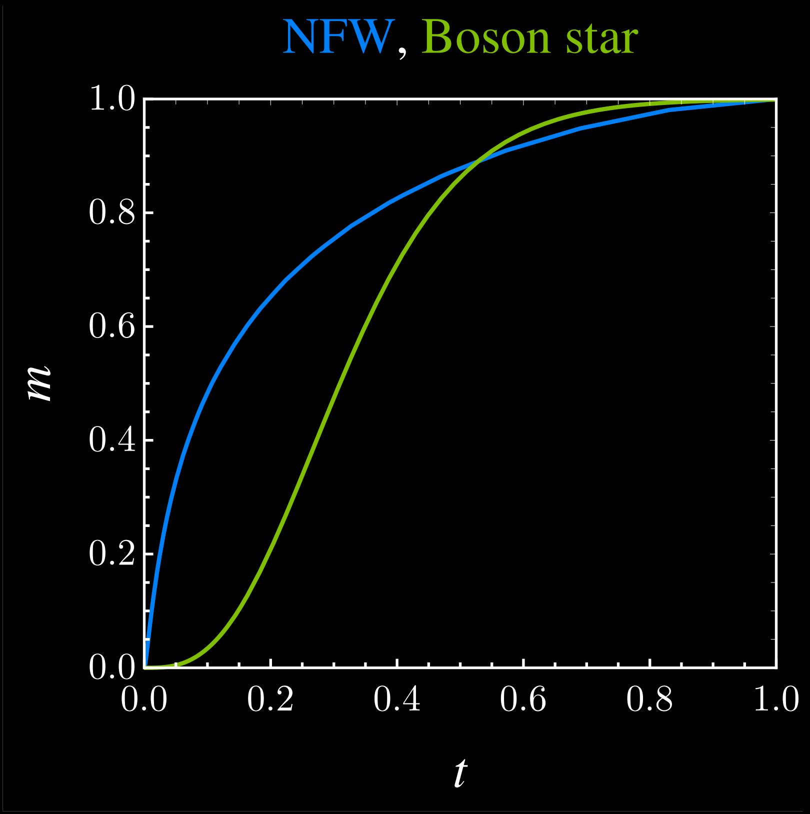
Describes the radial distribution

describes a *spherically symmetric ground state of a free scalar field in the non-relativistic limit*

- The mass enclosed is given by

$$M_{\mathrm{BS}}(r) = \frac{1}{m_\phi G}\int_0^{m_\phi r} dy\; y^2\; \Psi^2(y)$$

from which $m(t)$ may be computed



Comparing extended lenses

- For extended lenses, μ can not always be found analytically
- Define the threshold impact parameter $u_{1.34}$:

$$\mu_{\rm tot}(u \leq u_{1.34}) \geq 1.34$$

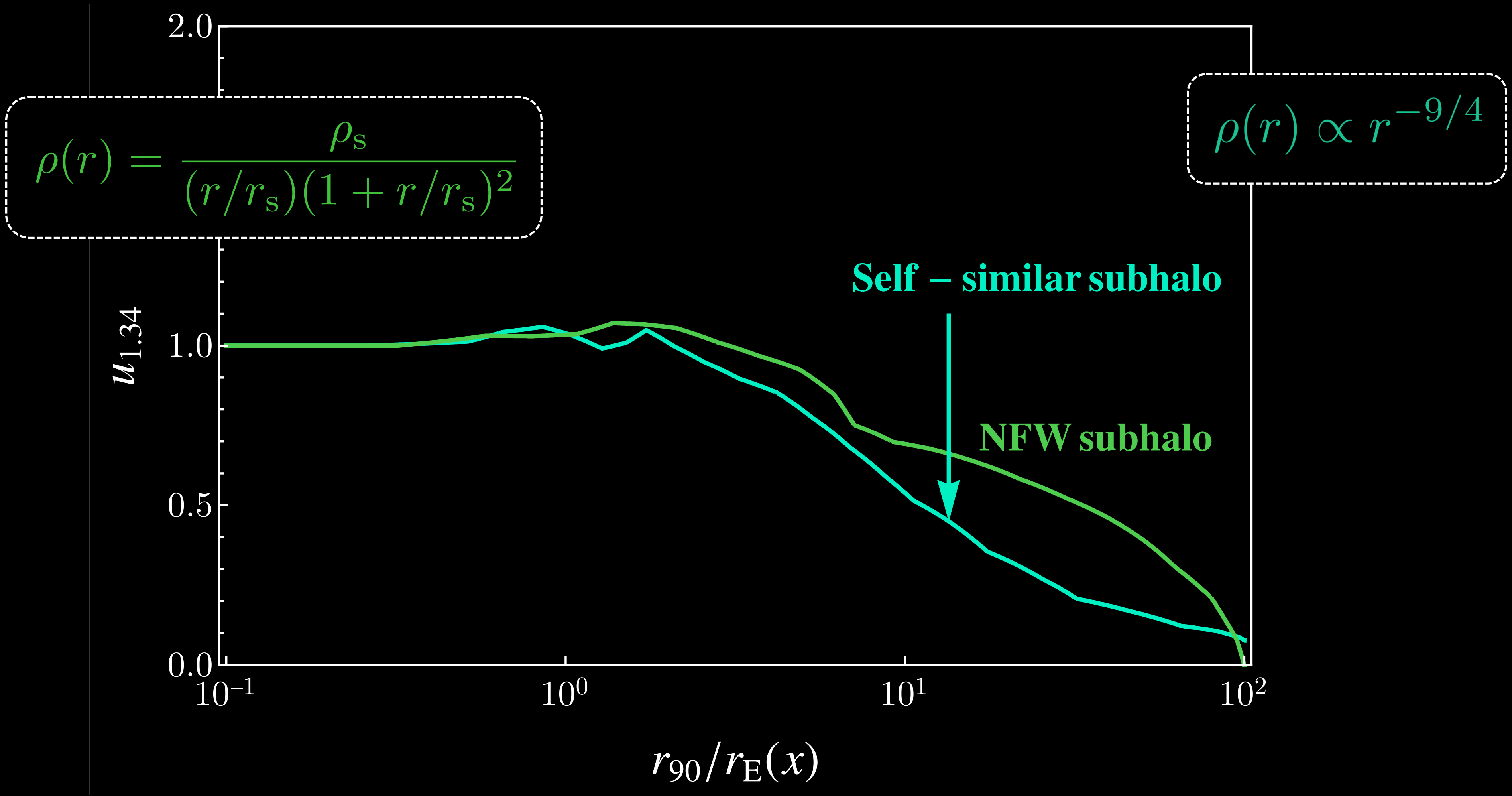
All smaller impact parameters produce a magnification above $\mu > 1.34$

- As we will see, the threshold impact parameter $u_{1.34}$ depends on different properties of the lens
  - Mass profile $M(\theta)/M$
  - Characteristic size r_{90}
  - Distances in the problem

Reasonable hypothesis:
dilute lenses give $u_{1.34} < 1$

Threshold impact parameter

For some lenses, as expected, the larger the lens, the smaller $u_{1.34}$

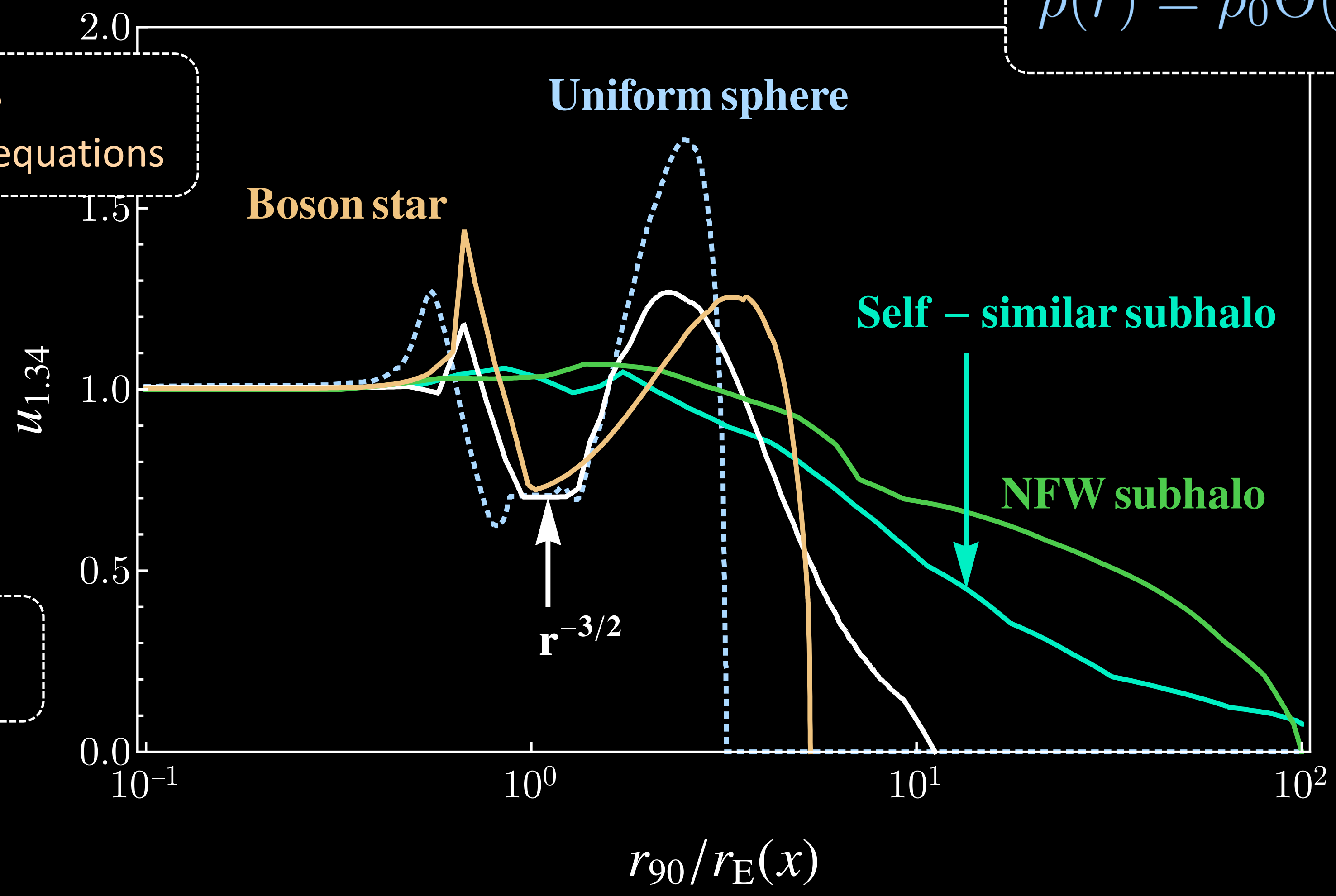


Threshold impact parameter

But for others, something else happens…

Numerically solve the Schrodinger-Poisson equations

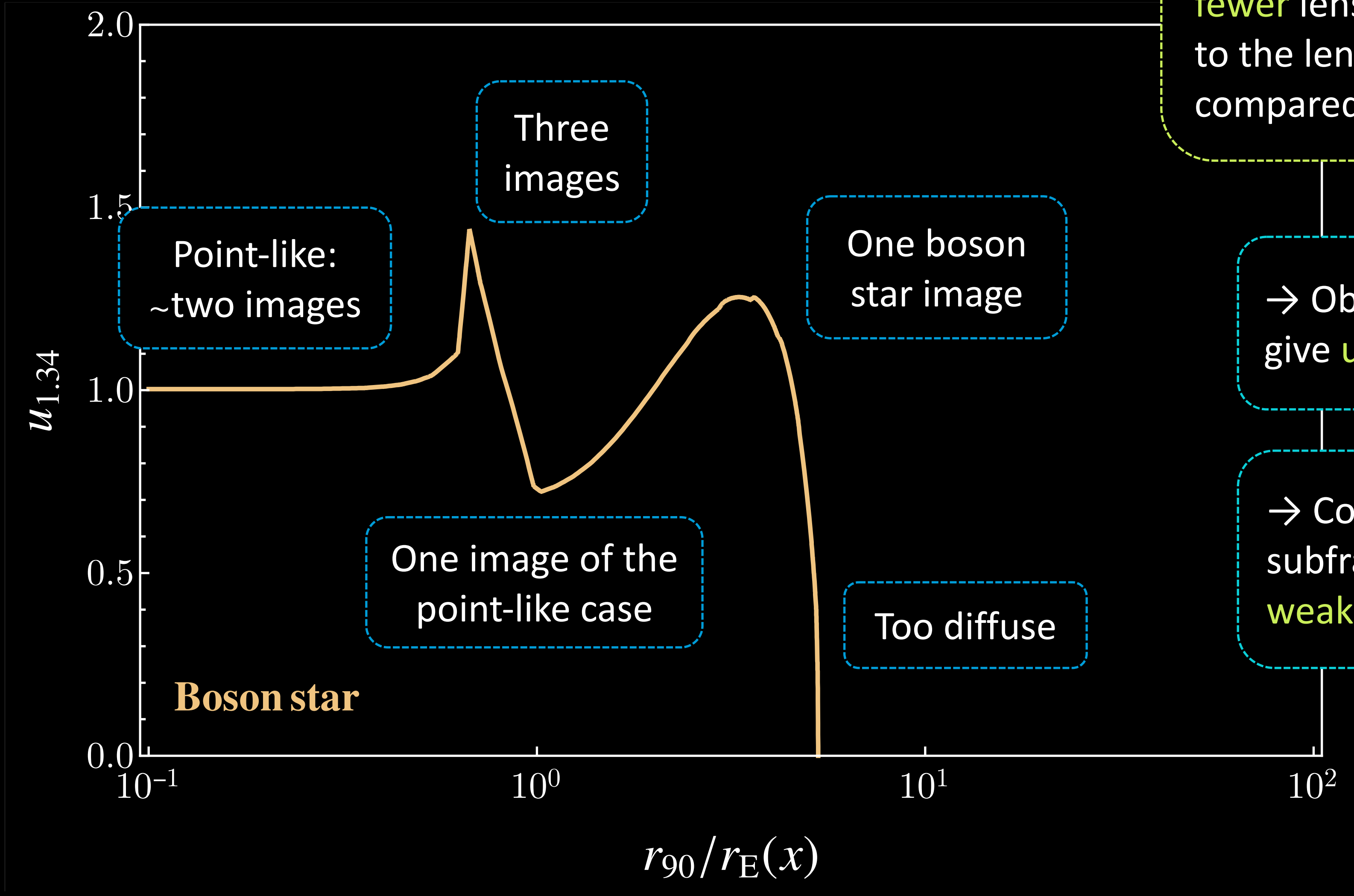
$$\rho(r) = \rho_0 \Theta(r_{\rm m} - r)$$



$$\rho(r) \propto r^{-3/2}$$

Caustics

What's going on in this plot?



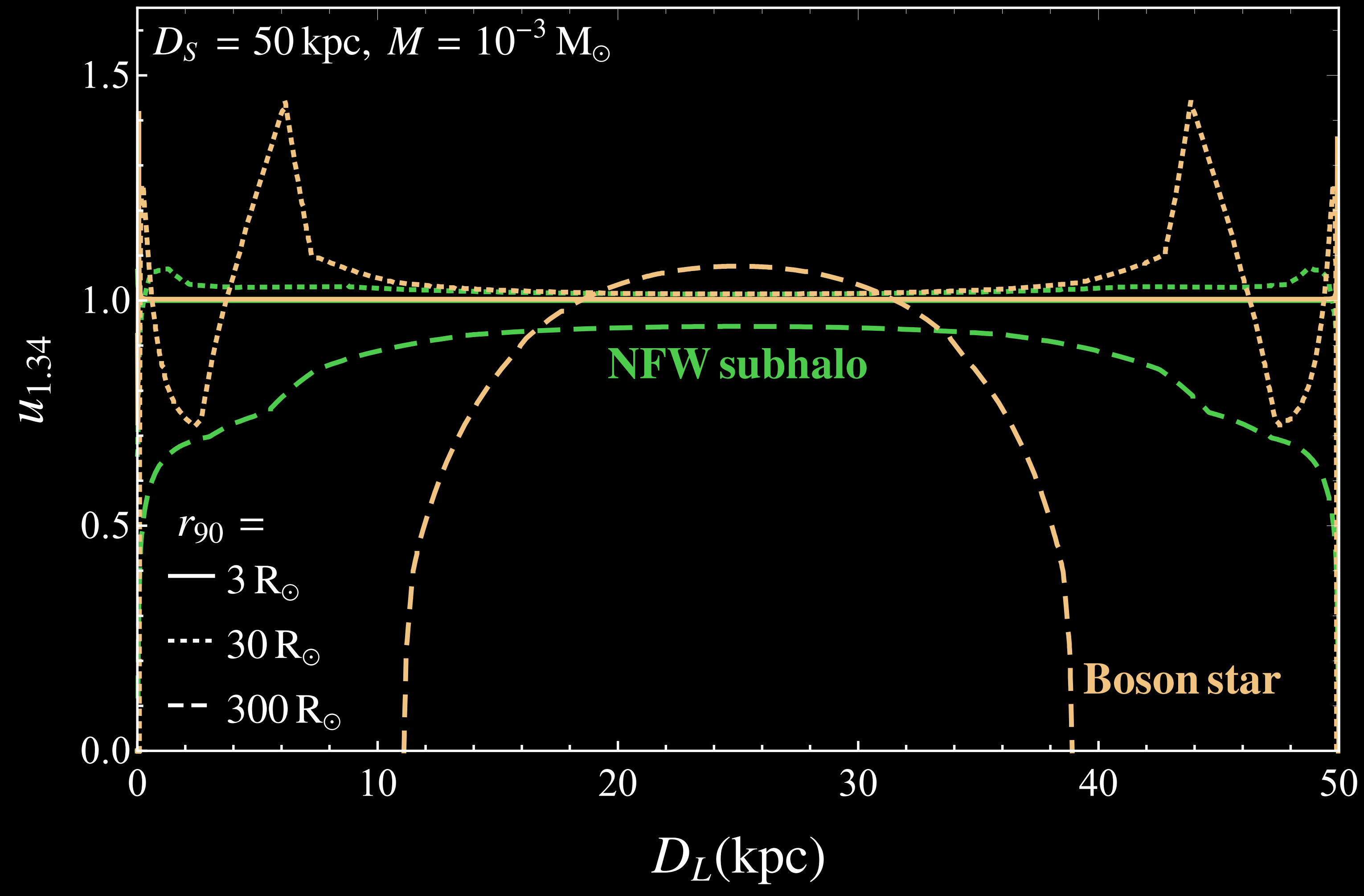
Sufficiently flat density profiles can give more or fewer lens images (solutions to the lens equation) compared to a point-like lens

→ Objects such as boson stars may give unique microlensing signals

→ Constraints on the dark matter subfraction may be stronger or weaker than for point-like lenses

Caustics

Consequence: the Einstein tube is not a tube; not ellipsoidal



→ Depending on the source, experiments may be more or less sensitive to extended objects compared to point sources in different locations

Constraining extended objects

The differential event rate contains all the essential physics

$$x = \frac{D_{\rm L}}{D_{\rm S}}$$

Fraction of $\Omega_{\rm DM}$

$$v_{\rm E}(x) \equiv 2u_{1.34}(x) r_{\rm E}(x)/t_{\rm E}$$

$$\frac{d^2\Gamma}{dx\,dt_{\rm E}} = \varepsilon(t_{\rm E}) \frac{2D_{\rm S}}{v_0^2 M} f_{\rm DM}\,\rho_{\rm DM}(x)\, v_{\rm E}^4(x)\, e^{-v_{\rm E}^2(x)/v_0^2}$$

$220\ {\rm km/s}$

Efficiency of the experiment

Halo profile: isothermal

Constraining extended objects

The total number of expected events depends on the experiment

$$N_{\text{events}} = N_\star T_{\text{obs}} \int_0^1 dx \int_{t_{\mathrm{E,min}}}^{t_{\mathrm{E,max}}} dt_{\mathrm{E}} \frac{d^2\Gamma}{dx\, dt_{\mathrm{E}}}$$

Maximum and minimum transit time

Number of observed stars

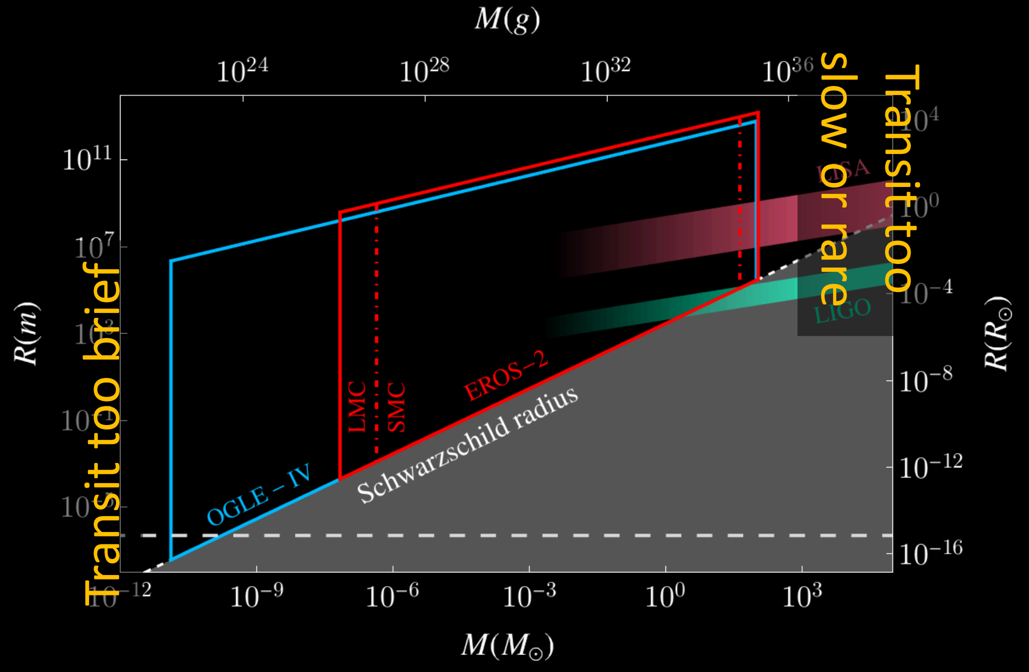
EROS-2 LMC: 5.49×10^6

OGLE-IV: 4.88×10^7

Observation time

EROS-2 LMC: 2500 days

OGLE-IV: 1826 days



Obtaining constraints

To obtain limits, we have to account for the observed events

- EROS-2: 3.9 events at 90% CL
- OGLE-IV: $\mathcal{O}(1000)$ astrophysical events, $\kappa = 4.61$ at 90% CL

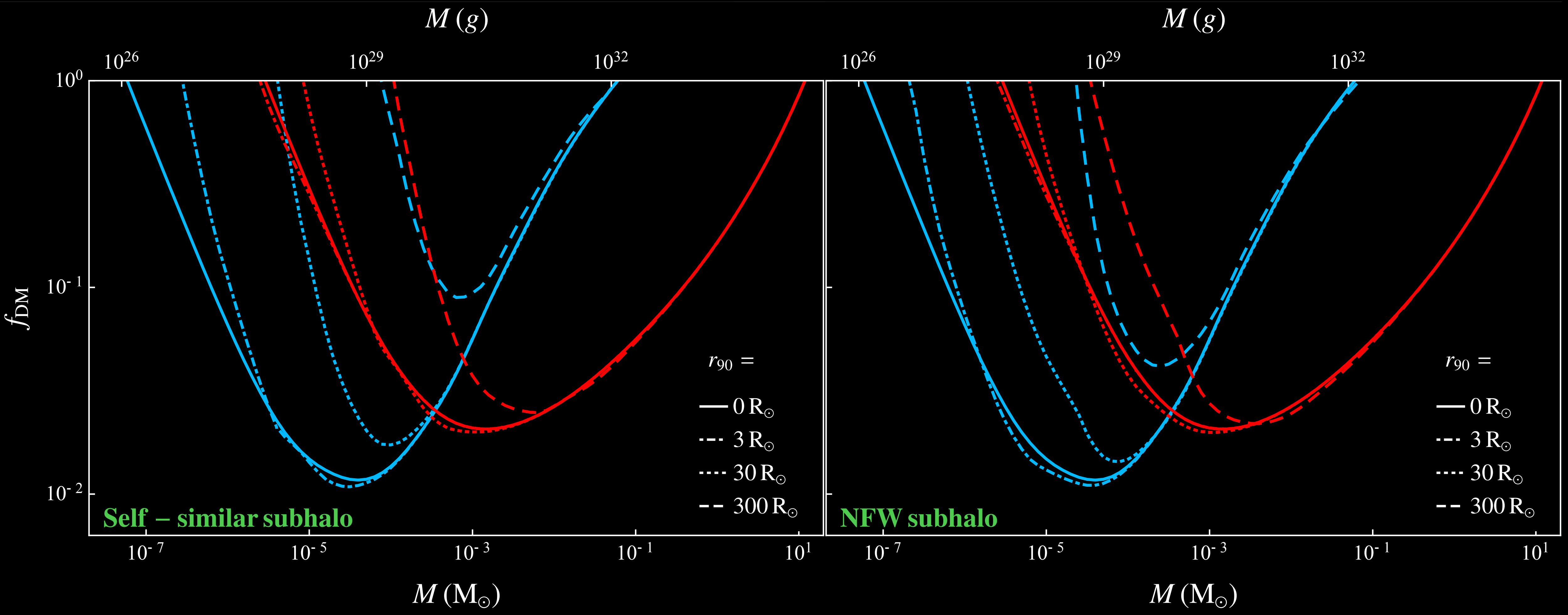
Bin events in $\mathrm{t_E}$

$$N_i^{\mathrm{SIG}} \equiv N_i^{\mathrm{FG}} + N_i^{\mathrm{DM}}$$

$$\kappa = 2 \sum_{i=1}^{N_{\mathrm{bins}}} \left[N_i^{\mathrm{FG}} - N_i^{\mathrm{SIG}} + N_i^{\mathrm{SIG}} \ln \frac{N_i^{\mathrm{SIG}}}{N_i^{\mathrm{FG}}} \right]$$

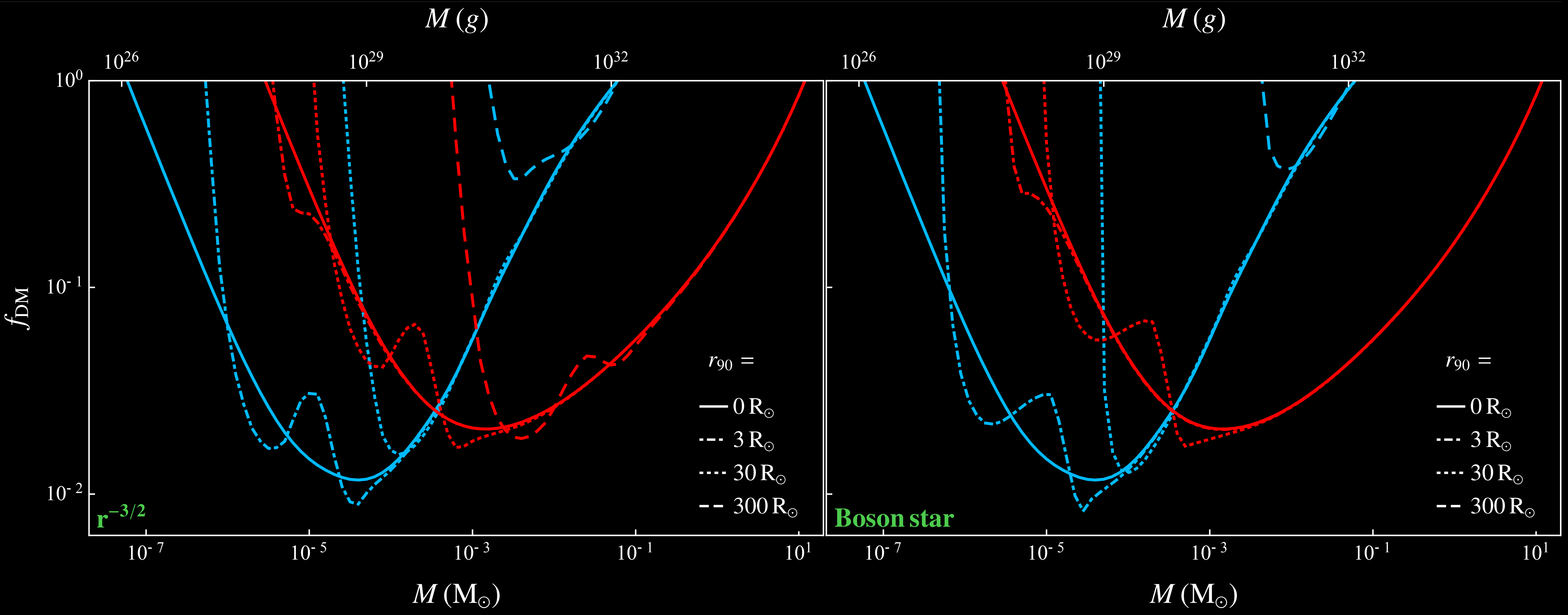
Constraints on DM fraction

Generally, constraints on extended objects are weaker…



Constraints on DM fraction

But for sufficiently flat density profiles, caustics change the constraints



In this part: the
Subaru-HSC survey

Part II: extended lenses and sources

DC, D. McKeen, N. Raj, Z. Wang, PRD, arXiv:2007.12697 [astro-ph.CO]

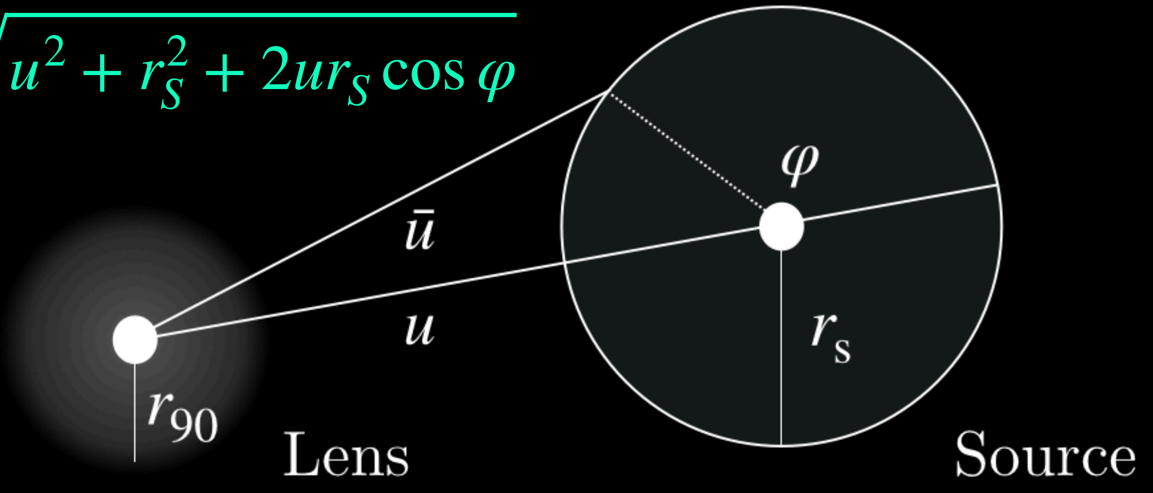
Lensing geometry

- Up to this point, we have assumed that the sources are point-like
- This approximation breaks down when $r_E = \theta_E D_L \sim r_S$

- Geometry in the lens plane:

$$r_s \equiv xR_\star / r_E$$

$$\bar{u}(\varphi) = \sqrt{u^2 + r_S^2 + 2ur_S \cos\varphi}$$



For point-like lenses, see for example, Witt and Mao, Astrophys. J (1994); Montero-Camacho, Fang, Vasquez, Silva, Hirata, [JCAP, arXiv:1906.05950]; Smyth, Profumo, English, Jeltema, McKinnon, Guhathakurta [PRD, arXiv:1910.01285];

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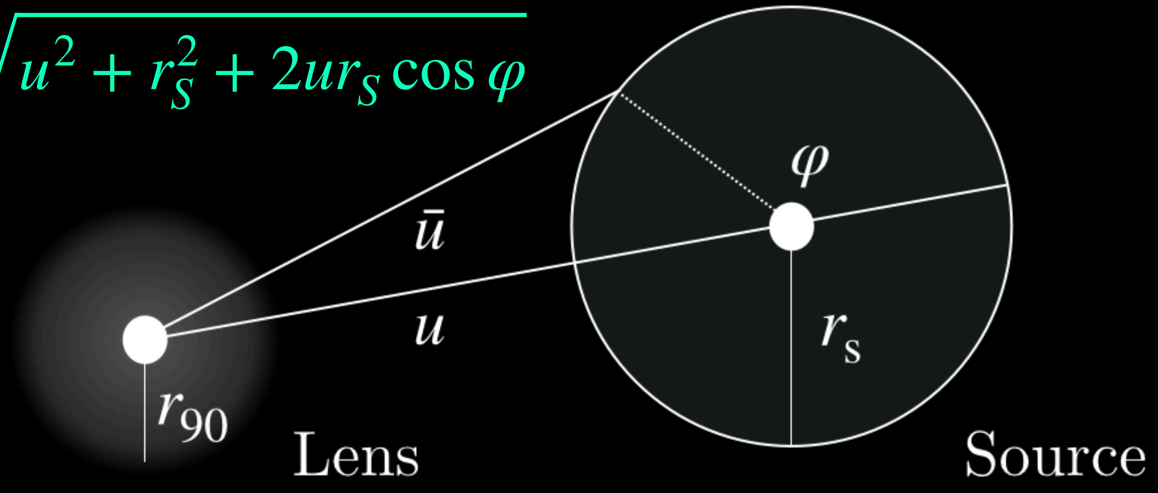


Lensing equation:

$$\bar{u}(\varphi) = t(\varphi) - \frac{m(t(\varphi))}{t(\varphi)}$$

Image parity

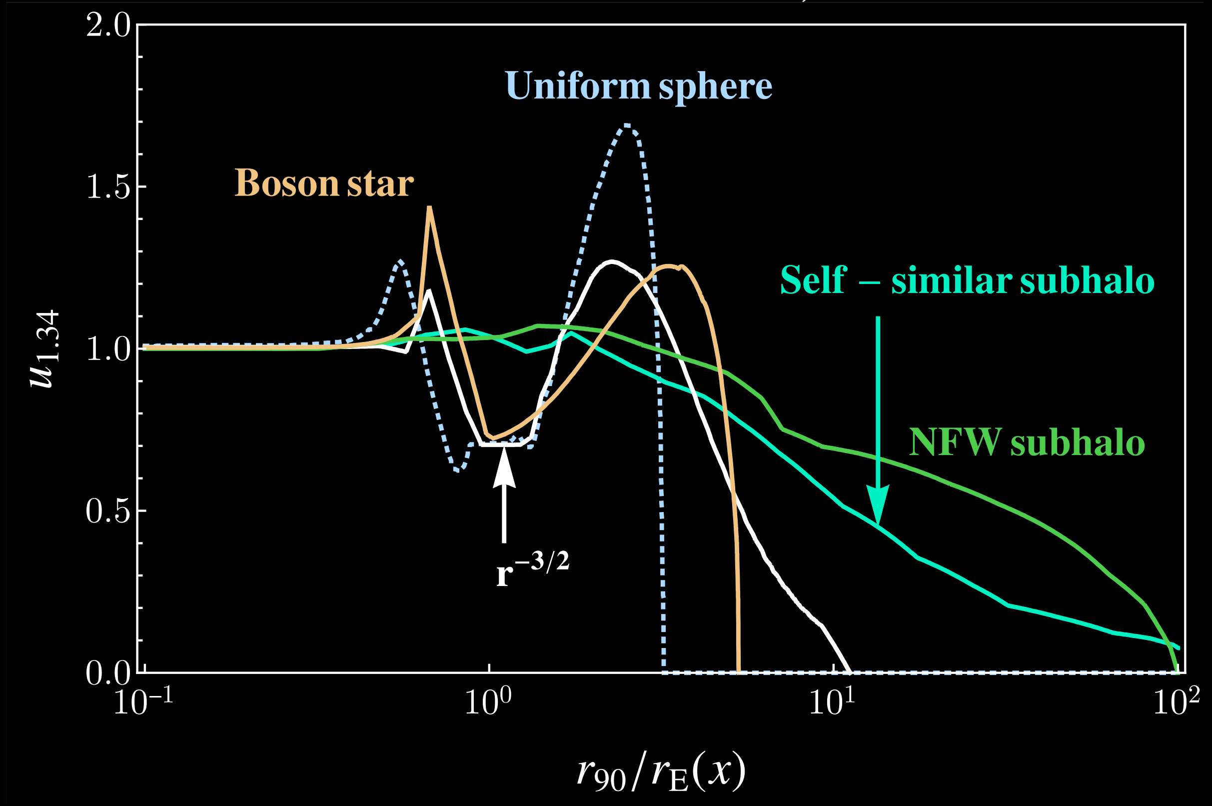
Image position

$$\mu_i = \eta \frac{1}{\pi r_S^2} \int_0^{2\pi} d\varphi \, \frac{1}{2} t_i^2(\varphi)$$

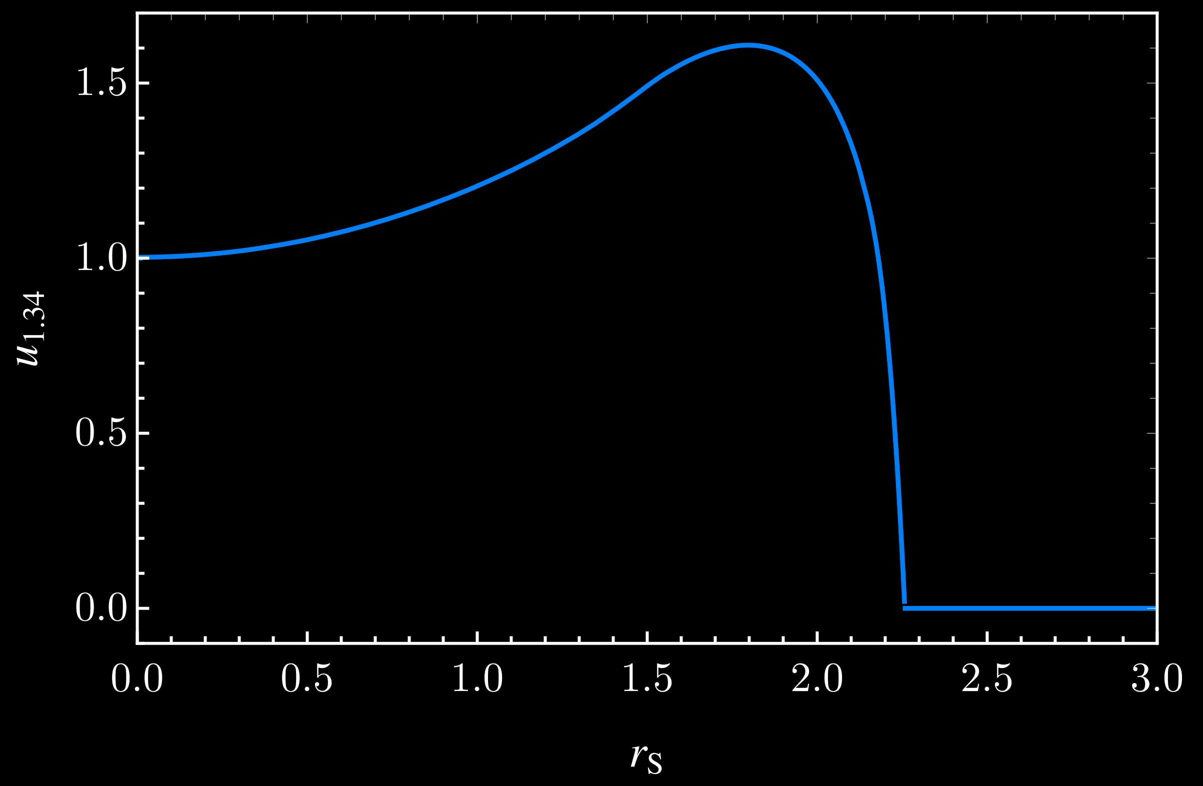
Threshold impact parameter

Same procedure as before, but now $u_{1.34}$ is a function of both r_{90} and r_S

From before : Point source, extended lens



Point-like lens, extended source



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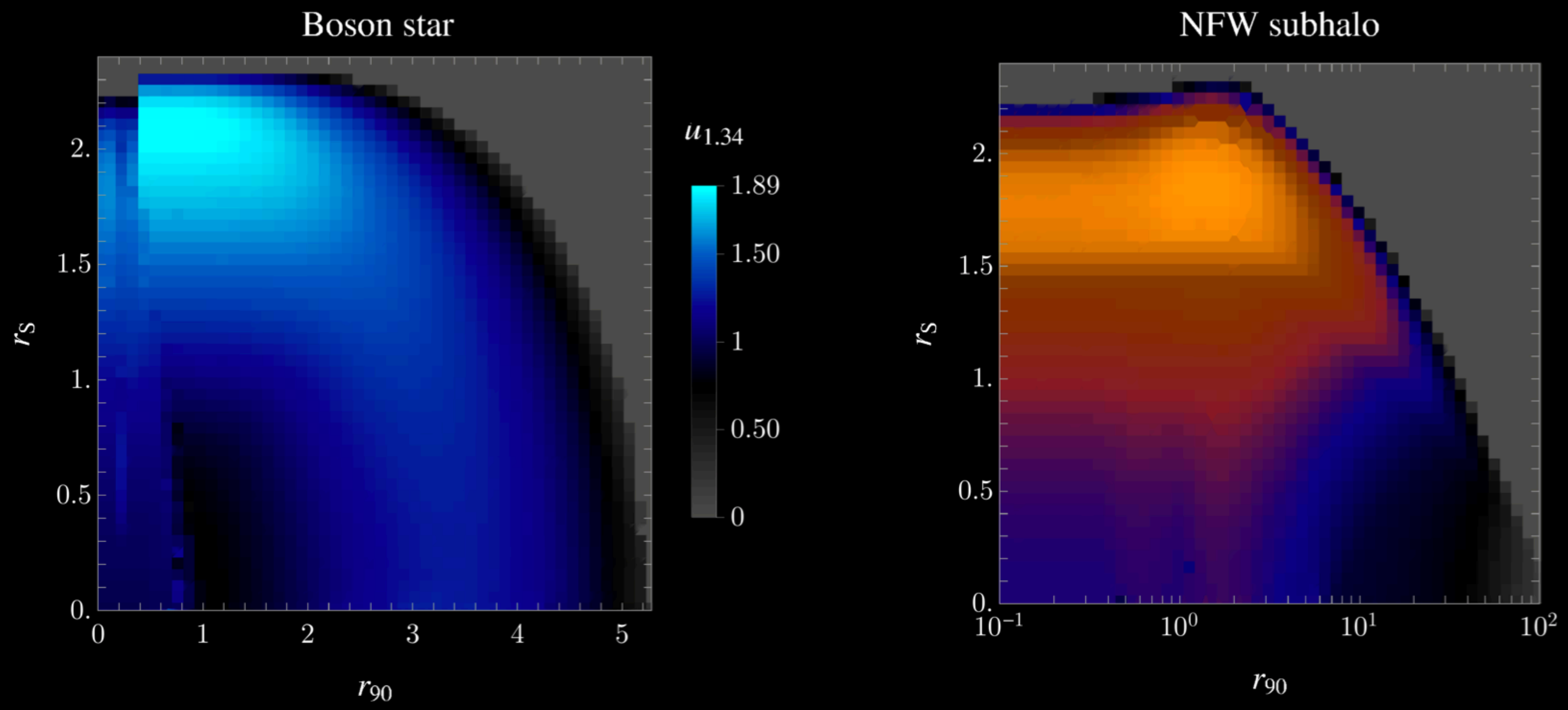
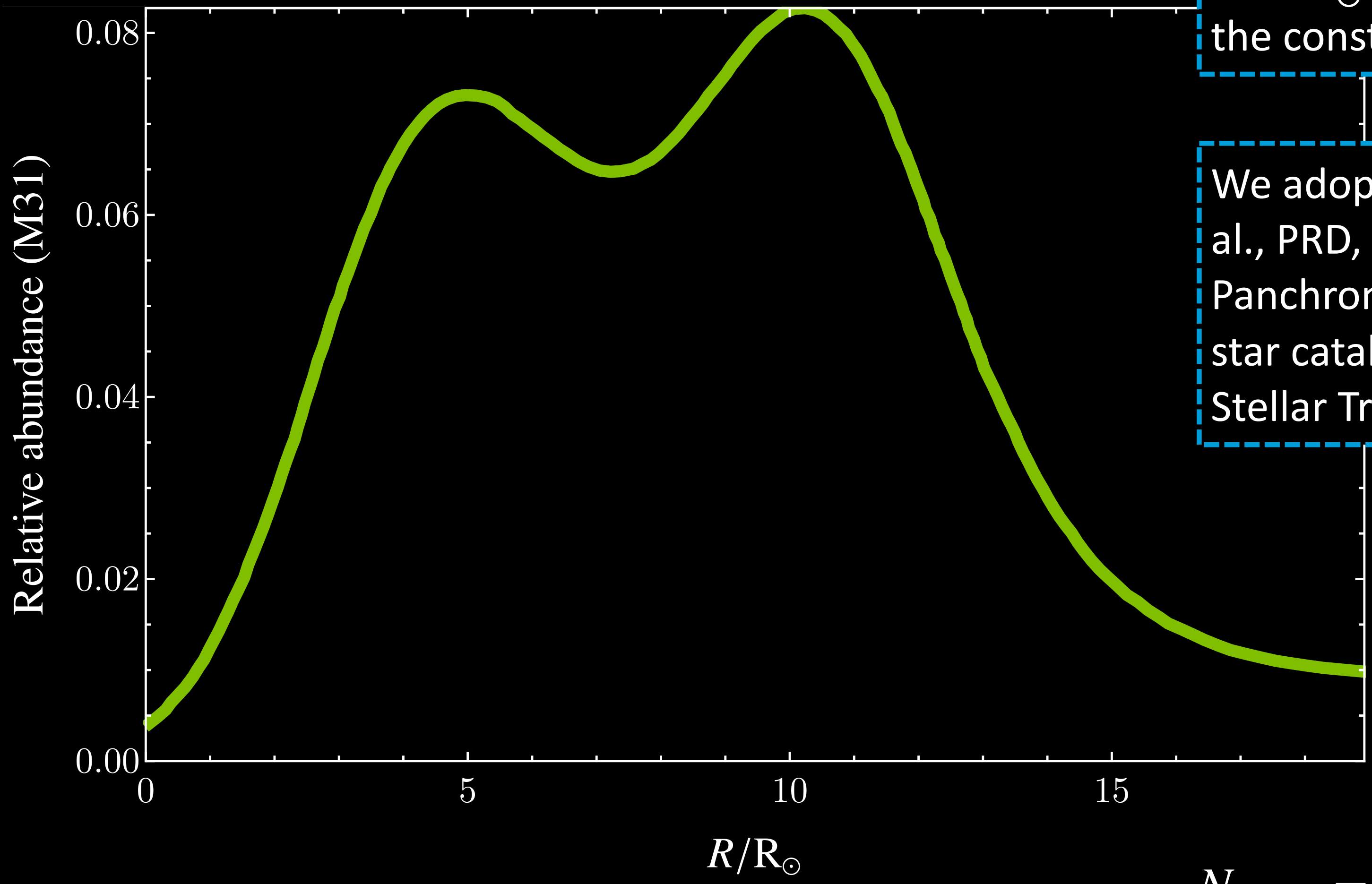


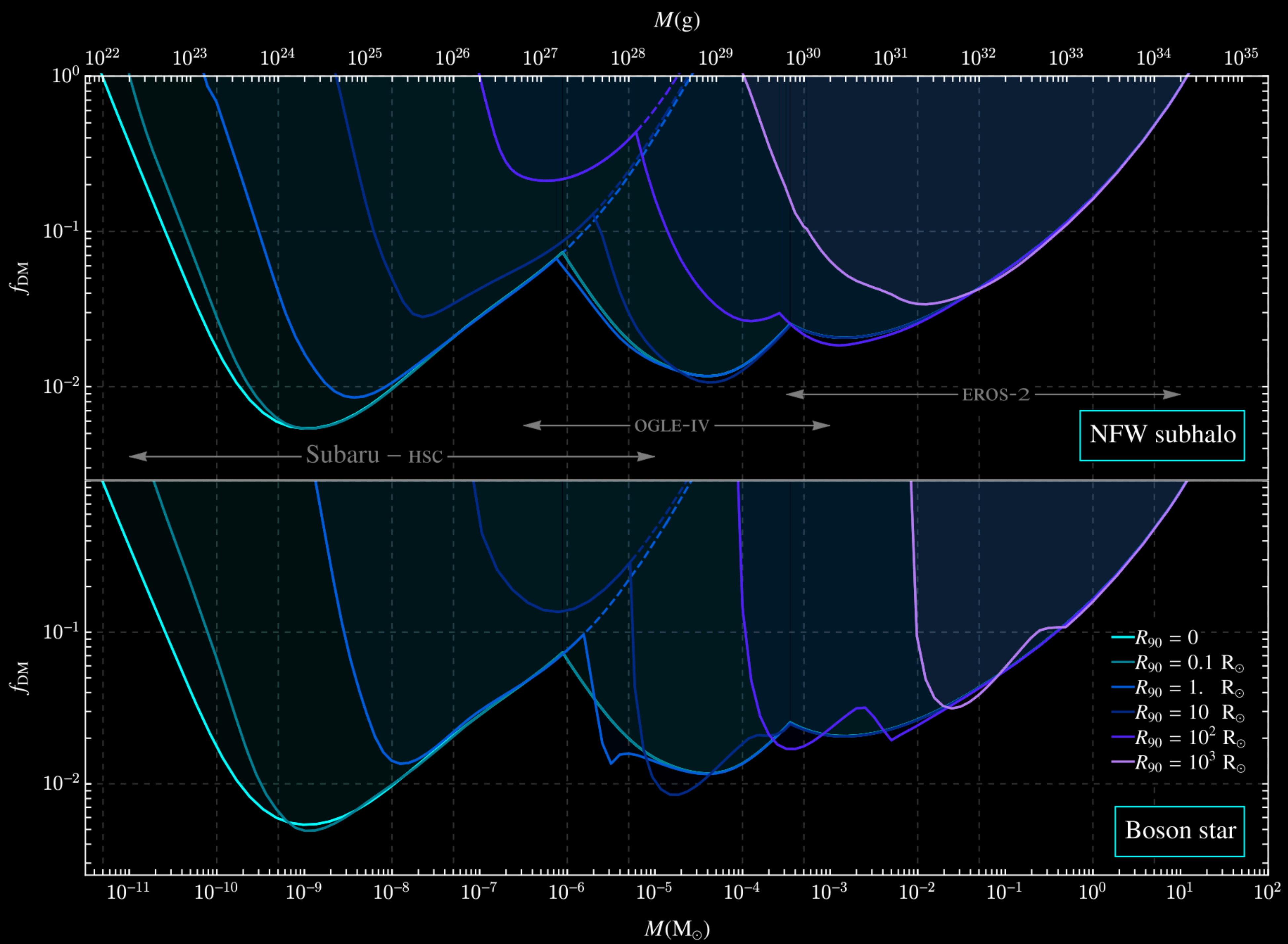
Star sizes in M31



Initially, the Subaru-HSC collaboration used $R = \mathrm{R}_\odot$ for all stars, but this overestimates the constraints on the dark matter fraction

We adopt the distribution derived in [Smyth et al., PRD, arXiv:1910.01285] using the Panchromatic Hubble Andromeda Treasury star catalogue and the $\mathtt{MESA}$ Isochrones and Stellar Tracks stellar evolution package

$$N_{\mathrm{events}} = N_\star T_{\mathrm{obs}} \int dt_{\mathrm{E}} \int dR_\star \int_0^1 dx \frac{d^2\Gamma}{dx\,dt_{\mathrm{E}}} \frac{dn}{dR_\star}$$



M(g)
10^{22}
10^{23}
10^{24}
10^{25}
10^{26}
10^{27}
10^{28}
10^{29}
10^{30}
10^{31}
10^{32}
10^{33}
10^{34}
10^{35}
$f_{\rm DM}$
10^{0}
10^{-1}
10^{-2}
EROS-2
OGLE-IV
Subaru – HSC
NFW subhalo
$R_{90} = 0$
$R_{90} = 0.1\ \mathrm{R}_\odot$
$R_{90} = 1.\ \mathrm{R}_\odot$
$R_{90} = 10\ \mathrm{R}_\odot$
$R_{90} = 10^2\ \mathrm{R}_\odot$
$R_{90} = 10^3\ \mathrm{R}_\odot$
Boson star
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
$M(\mathrm{M}_\odot)$

To conclude,

- All of our current evidence for Dark Matter is gravitational
- Many dark matter models feature substructure such as miniclusters, microhaloes, and Bose Einstein condensates

- Microlensing provides a way to look for dark matter substructure of a large range of sizes and masses
  - →Extended objects may give unique microlensing signatures

- Gravitational probes of dark matter are filling in the composite dark matter parameter space

Thank you!

…ask me anything you like!

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