Magnetic Black Holes with Electroweak-Symmetric Coronas

Yang Bai

University of Wisconsin-Madison

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\begin{itemize}
\item Any interesting states in the electroweak sector?
\end{itemize}

Forcrand, et. al., arXiv:1503.08140
Dirac Monopole

- In E&M, we have learned that there is no monopole
- Dirac in 1931 proposed the possible existence of monopole

\[ \mathbf{B} = Q \frac{h \hat{r}}{4\pi r^2} \]

- \( Q = 1 \)
- \( h = \frac{2\pi}{e} \approx 68.5 \, e \)
t ‘Hooft-Polyakov Monopole

- Based on spontaneously broken gauge theory: SU(2)/U(1)

\[ \mathcal{L} = \frac{1}{2} (D_\mu \Phi)^2 - \frac{1}{4} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{\lambda}{4} \left( |\Phi|^2 - f^2 \right)^2 \]

\[ D_\mu \Phi^a = \partial_\mu \Phi^a + g \epsilon^{abc} A_\mu^b \Phi^c \]

\[ F_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c \]

- In the “hedgehog gauge” with \( A_0^a = 0 \) (spherically symmetric)

\[ \Phi^a = \hat{r}^a f \phi(r) \]

\[ A_i^a = \frac{1}{g} \epsilon^{aij} \hat{r}^j \left( \frac{1 - u(r)}{r} \right) \]

\[ Q = 2 \]
t ‘Hooft-Polyakov Monopole

- Classical equations of motion ($\bar{r} \equiv g f r = m_W r$)

$$\frac{d^2 \phi}{d\bar{r}^2} + \frac{2}{\bar{r}} \frac{d\phi}{d\bar{r}} = \frac{2u^2 \phi}{\bar{r}^2} + \frac{\lambda}{g^2} \phi(\phi^2 - 1)$$

$$\frac{d^2 u}{d\bar{r}^2} = \frac{u(u^2 - 1)}{\bar{r}^2} + u \phi^2$$

- Boundary conditions

$$\phi(0) = 0, \quad \phi(\infty) = 1, \quad u(0) = 1, \quad u(\infty) = 0$$

- Total energy or mass (finite)

$$M_M = \int 4 \pi r^2 \left( \frac{1}{2} B^a_i B^a_i + \frac{1}{2} (D_i \Phi^a)(D_i \Phi^a) + V(\Phi) \right)$$

$$= \frac{4\pi f g}{4} \int d\bar{r} \bar{r}^2 \left( \frac{\bar{r}^2 \phi'^2 + 2u^2\phi^2}{2 \bar{r}^2} + \frac{(1-u^2)^2 + 2\bar{r}^2u^2}{2 \bar{r}^4} + \frac{\lambda}{4g^2}(\phi^2 - 1)^2 \right)$$
t ‘Hooft-Polyakov Monopole

\[ M_\mathcal{M} \equiv \frac{4\pi f}{g} Y(\lambda/g^2) \quad Y(0) = 1 \quad Y(\infty) \approx 1.787 \]

- **Topological reason:** \( \pi_2[G/U(1)] = \pi_1[U(1)] = \mathbb{Z} \)
- **GUT monopole:** \( SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \)

\[ M_\mathcal{M}^{\text{GUT}} \sim 10^{17} \text{ GeV} \]
Monopole in the Standard Model

- In the SM: $SU(2)_W \times U(1)_Y \rightarrow U(1)_{\text{EM}}$ with a Higgs doublet

- Topological reason: $\pi_2[SU(2)_W \times U(1)_Y / U(1)_{\text{EW}}] = 0$, no finite-energy EW monopole

- In more detail and again making a spherical configuration

$$H = \frac{v}{\sqrt{2}} \phi(r) \xi, \quad \xi = i \begin{pmatrix} \sin(\frac{\theta}{2}) e^{-i\phi} \\ -\cos(\frac{\theta}{2}) \end{pmatrix}$$

$$H^\dagger \sigma H = -\frac{v^2}{2} \phi(r)^2 \hat{r}$$

as the triplet case

$$A_i^a = \frac{1}{g} \epsilon^{aij} \hat{r}^j \left( \frac{1 - u(r)}{r} \right) \quad \longleftrightarrow \quad SU(2)_W$$

$$B_i = -\frac{1}{g_Y} (1 - \cos \theta) \partial_i \phi \quad \longleftrightarrow \quad U(1)_Y$$

Nambu, NPB130 (1977) 505
Cho, Maison, hep-th/9601028
Monopole in the Standard Model

\[ S = -4\pi \int dt \, dr \, r^2 \left( K + U \right) \]

\[ K = \frac{(u')^2}{g^2 \, r^2} + \frac{1}{2} v^2 (\phi')^2 \]

\[ U = \frac{u^2 - 1}{2 \, g^2 \, r^4} + \frac{v^2 \, u^2 \, \phi^2}{4 \, r^2} + \frac{\lambda_h \, v^4}{8} (\phi^2 - 1)^2 + \frac{1}{2 \, g Y \, r^4} \]

- The spherical EW monopole has an infinite mass
- Nambu’s monopole-anti-monopole dumbbell configuration
- Unstable! May be produced at a future collider
Introduce BSM physics to have a finite-energy monopole for instance, $U(1)_Y \subset SU(2)_R$

Or hide the divergence part behind the event horizon of a black hole

For the second avenue, no new BSM physics is needed. We just need to study the possible states based on

Standard Model + General Relativity
The stars’ orbits revealed that something invisible and heavy governed their paths at the heart of the Milky Way. Astronomers were able to map an entire orbit of less than 16 years for one of the stars, S2 (or S02). The closest it came to Sagittarius A* was about 17 light hours (more than 1000 million kilometres).

Stars closest to the centre of the Milky Way

Astronomers started mapping the path of S2 in 1992.

The S2 star’s radial velocity increases as it approaches Sagittarius A* and decreases as it moves away along its elliptical orbit. Radial velocity is the component of the star’s velocity that is in our line of sight.

Closest to Sagittarius A* (in 2002 and 2018), S2 reaches its maximum velocity of 7 000 km/s.

Figure 4. The stars’ orbits revealed that something invisible and heavy governed their paths at the heart of the Milky Way.
Black Holes

- **Schwarzschild black hole**

  \[
  ds^2 = - \left(1 - \frac{2 GM}{r}\right) dt^2 + \left(1 - \frac{2 GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
  \]

- **Charged or Reissner-Nordstrom black hole**

  \[
  ds^2 = - B_{\text{RN}}(r) dt^2 + B_{\text{RN}}(r)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
  \]

  \[
  B_{\text{RN}}(r) = 1 - \frac{2 GM}{r} + \frac{G \sqrt{Q_E^2 e^2 + Q_M^2 h^2}}{4\pi r^2}
  \]

- **The outer horizon radius is**

  \[
  r_+ = \left(\frac{M_{\text{cBH}} + \sqrt{M_{\text{cBH}}^2 - (Q_E^2 e^2 + Q_M^2 h^2)M_{\text{pl}}^2/4\pi}}{M_{\text{pl}}^2}ight)
  \]

  \[
  M_{\text{cBH}} = \frac{\sqrt{Q_E^2 e^2 + Q_M^2 h^2}}{\sqrt{4\pi}} M_{\text{pl}}
  \]
Hawking Radiation and PBH Lifetime

- According to the first law of the black hole thermal dynamics, the thermal radiation temperature has (for non-extremal BH)

\[ T = \frac{M_{\text{pl}}^2}{8\pi M_{\text{BH}}} \]

- Using the black body radiation formula, \( P \propto R^2 T^4 \), the lifetime of a Schwarzschild black hole is

\[ \tau \approx \frac{5120\pi}{g^*} \frac{M_{\text{BH}}^3}{M_{\text{pl}}^4} \]

- Requiring it to be longer than the age of our universe, one has a lower bound on PBH mass

\[ M_{\text{PBH}} \gtrsim 10^{15} \text{ g} \]
Extremal Black Hole

- The Hawking radiation is fourth power of $T$. One way to suppress $T$ is to make it extremal

$$T(M_{BH}, M_{eBH}) = \frac{M_{pl}^2}{2\pi} \frac{\sqrt{M_{BH}^2 - M_{eBH}^2}}{\left(M_{BH} + \sqrt{M_{BH}^2 - M_{eBH}^2}\right)^2}$$

- A PBH with a charge $Q$ will evolve towards a near extremal one, which has suppressed $T$

$$\frac{dM_{BH}}{dt} \approx - \frac{\pi^2}{120} g_* 4\pi r_+^2 \left[T(M_{BH}, M_{eBH})\right]^4$$
Evolution of the Black Hole Mass

\[ M_{\text{BH}}(t) = M_{\text{eBH}} + \frac{120\pi M_{\text{eBH}}^4}{g_* M_{\text{pl}}^4 t} \]

\[ T_{\text{eBH}} = \sqrt{\frac{60 M_{\text{eBH}}}{\pi g_* t}} \]

- The initial BH evaporation still generates lot of Hawking radiations
Electrically-Charged BH in SM

- The charged BH has a large electric field close to the event horizon
  \[ E = \frac{M_\text{pl}^3}{\sqrt{4\pi M_{\text{eBH}}}} \]
- The Schwinger effects can generate electrons and positrons from vacuum and discharge the eBH
The Schwinger discharge rate

\[
\frac{d\Gamma_{\text{Schwinger}}}{dV} = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{\pi n m_e^2}{e E} \right)
\]

This sets a lower bound on the eBH mass

\[
M_{\text{eBH}} > M_{\text{eBH}}^{\text{min}} \approx \frac{eM_{\text{pl}}^3}{2\pi^{3/2} m_e^2} \ln \left( \frac{e^3 M_{\text{pl}}^3 t_{\text{univ}}}{16 \pi^{7/2}} \right)
\]

Because the electron mass is small in SM, the minimum eBH mass is very large

\[
M_{\text{eBH}}^{\text{min}} \approx 10^8 M_\odot \quad \text{for} \quad m_e = 0.511 \text{ MeV}
\]

for dark electrically-charged BH, see YB, Orlofsky, arXiv: 1906.04858
Magnetically-Charged BH in SM

- Since there is no finite-energy magnetic monopole in the SM, no worry about Schwinger discharge

- If the GUT exists, it may worry its emission of GUT monopole

\[
B(R_{eBH}) = \frac{Q}{2e R_{eBH}^2} \approx \frac{e M_{pl}^2}{2 \pi Q}
\]
Electroweak Symmetry Restoration

- In a large B field background, the electroweak symmetry is restored

\[ \mathcal{E} = \frac{1}{2} |D_i W_j - D_j W_i|^2 + \frac{1}{4} g_f^2 + \frac{1}{4} Z_{ij}^2 + \frac{1}{2} g^2 \varphi^2 W_i W_i^\dagger + \left( \frac{g^2 \varphi^2}{4 \cos^2 \theta} \right) Z_i^2 \]

\[ + i g \left( f_{ij} \sin \theta + Z_{ij} \cos \theta \right) W_i^\dagger W_j + \frac{1}{2} g^2 \left[ (W_i W_i^\dagger)^2 - (W_i^\dagger)^2 (W_i)^2 \right] \]

\[ + (\partial_i \varphi)^2 + \lambda (\varphi^2 - \varphi_0^2)^2, \]

\[ (W_1^\dagger, W_2^\dagger) \begin{pmatrix} \frac{1}{2} g^2 \varphi_0^2 & i e f_{12} \\ -i e f_{12} & \frac{1}{2} g^2 \varphi_0^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \]

- For a large \( |f_{12}| \), a negative determinant leads to W-condensation and electroweak restoration. This happens when

\[ e B \gtrsim m_h^2 \]
Electroweak Symmetry Restoration

\[ B(R_{eBH}) = \frac{Q}{2 e R_{eBH}^2} \approx \frac{e M_{pl}^2}{2 \pi Q} \]
\[ e B(R_{eBH}) \gtrsim m_h^2 \]

Electroweak symmetry restoration happens for

\[ Q \lesssim Q_{\text{max}} \equiv \frac{e^2 M_{pl}^2}{2 \pi m_h^2} \approx 1.4 \times 10^{32} \]

Lee, Nair, Weinberg, PRD45(1992) 2751

For \( Q=2 \), one can obtain the spherically symmetric configuration

For \( Q > 2 \), a non-spherically symmetric configuration is anticipated, and requires complicated numerical calculations

Guth, Weinberg, PRD14(1976) 1660
Q=2: spherical solution

\[ ds^2 = -B(r)dt^2 + A(\frac{1}{r})dr^2 + r^2(\text{d}\theta^2 + \sin^2 \theta\text{d}\phi^2) \]

\[ S_{\text{matter}} = -4\pi \int dt\, dr\, r^2 \sqrt{A\, B} \left( \frac{K}{A} + U \right) \]

- Defining \( A(\frac{1}{r}) = \left( 1 - \frac{2\, G\, \mathcal{M}(r)}{r} \right) \), and solving the EOMs

\[ G\, v^2 = 10^{-2} \]
\[ R_e = 0.1 \]

\[ \mathcal{M}(r)/10 \]

\[ \phi(\bar{r}) \]
\[ u(\bar{r}) \]

\[ R_e \]

Lee, Nair, Weinberg, PRD45(1992) 2751

YB, Korwar, in progress
Non-extremal BHs are also relevant for phenomenology. They appear, than the gravitational one. The range of viable charges $M$ of the Universe for a term, so energetically it is preferable for an MeBH with a large charge to split into smaller subscripts to contain spiky features where vortex strings end on monopoles

$\mu = c \cos \theta$

For reference, the mass of the Earth is $M_{\oplus}$. Although the large-charged MeBH is metastable, its lifetime can be longer than the age of the Universe for MeBHs. The shape has not been worked out in detail, but may be expected to contain spiky features where vortex strings end on monopoles $\mu = c \cos \theta$.

We now give a more precise estimate for the mass. Including the contributions from both $\mu = c \cos \theta$ and $\mu = c \cos \theta$ states, so we will generally neglect it.

$M^{\text{tot}}_{\text{MeBH}}(Q) \approx c_W \frac{\sqrt{\pi} Q}{e} M_{\text{pl}} + \frac{4\pi}{3} R_{\text{EW}}^3 \frac{m_h^2 v^2}{8} = c_W \frac{\sqrt{\pi} Q}{e} M_{\text{pl}} + \frac{\pi}{12 \sqrt{2}} Q^{3/2} \frac{v^2}{m_h}$

$\equiv M_{\mu}(Q) + \frac{\pi}{12 \sqrt{2}} Q^{3/2} \frac{v^2}{m_h}$, $\quad M_{\mu}(Q) = c_W M_{e\text{BH}}^{\text{RN}}$

- For $Q < Q_{\text{max}} \approx 10^{32}$,
  \[ M_{\mu} \lesssim 9 \times 10^{51} \text{ GeV} \quad M_{\oplus} = 6.0 \times 10^{27} \text{ g} = 3.4 \times 10^{51} \text{ GeV} \]
2d Modes

- For non-extremal BH, the Hawking temperature is

\[
T(M_{\text{BH}}, M_*) = \frac{M_{\text{pl}}^2}{2\pi} \frac{\sqrt{M_{\text{BH}}^2 - M_*^2}}{(M_{\text{BH}} + \sqrt{M_{\text{BH}}^2 - M_*^2})^2}
\]

- In the existence of magnetic field, the massless 2d modes exist for a Dirac 4D massless fermion

\[
ds^2 = e^{2\sigma(t,x)} (-dt^2 + dx^2) + R^2(t, x) \left(d\theta^2 + \sin^2 \theta \, d\phi^2\right) \\
A_\phi = \frac{Q}{2} \cos \theta
\]

\[
dx = \frac{dr}{f(r)}, \quad e^{2\sigma(t,x)} = f(r) \equiv (1 - R_e/r)^2, \quad R(t, x) = r
\]

\[
\mathcal{D} \tilde{\chi} = m_\chi \tilde{\chi} \quad \tilde{\chi}_{\alpha\beta} = \frac{e^{-\frac{1}{2}\sigma}}{R} \psi_\alpha(t, x) \eta_\beta(\theta, \phi)
\]

\[
\left[ \sigma_y \frac{\partial_\phi - iA_\phi}{\sin \theta} + \sigma_x \left( \partial_\theta + \frac{\cot \theta}{2} \right) \right] \eta = 0, \\
(i\sigma_x \partial_t + \sigma_y \partial_x) \psi = m_\chi e^{\sigma} \psi.
\]

2d fermion
2d Modes

- **Solutions for** $Q > 0$,

  \[
  \eta_1 = 0, \\
  \eta_2 = \left( \sin \frac{\theta}{2} \right)^{j-m} \left( \cos \frac{\theta}{2} \right)^{j+m} e^{im\phi} = \frac{(1 - \cos \theta)^{\frac{q-m}{2}} (1 + \cos \theta)^{\frac{q+m}{2}}}{2^{q-\frac{1}{2}} (\sin \theta)^{\frac{1}{2}}} e^{im\phi}
  \]

  \[
  j = (|Q| - 1)/2 \equiv q - 1/2 \text{ and } -j \leq m \leq j
  \]

- **There are $|Q|$ massless modes for** $m_\chi = 0$

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(3) \times SU(2) \times U(1)$</th>
<th>Number of 2d modes (left - right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_L$</td>
<td>$(3, 2)_{\frac{1}{6}}$</td>
<td>$Q$</td>
</tr>
<tr>
<td>$u_R$</td>
<td>$(3, 1)_{\frac{2}{3}}$</td>
<td>$-2Q$</td>
</tr>
<tr>
<td>$d_R$</td>
<td>$(3, 1)_{-\frac{1}{3}}$</td>
<td>$Q$</td>
</tr>
<tr>
<td>$l_L$</td>
<td>$(1, 2)_{-\frac{1}{2}}$</td>
<td>$-Q$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>$(1, 1)_{-1}$</td>
<td>$Q$</td>
</tr>
</tbody>
</table>

2d Hawking radiation

- Fermions are massless (ignoring QCD vacuum) inside the EW-corona region

\[ P_2 = \frac{dE}{dt} = \frac{\pi g_*}{24} T^2 (M_{\text{BH}}, M_\odot) \]

- For high T, \( g_* = 18 |Q| \) for three-family fermions

- The 2d radiation is very fast; it reaches extremal very quickly

- 2d neutrino modes can not escape
- EM charged states can travel outside of coronas
2d Hawking radiation

- For $T < m_e$, the 2d radiation is suppressed. The 4D radiation dominants

  $$ P_4 = \frac{dE}{dt} \approx \frac{\pi^2 g_*}{120} (4\pi R_{EW}^2) T^4 (M_{BH}, M_{\odot}) $$

  with $g_* = 2$ for photon and $g_* = 21/4$ for neutrinos

- For $T > m_e$, the 2d radiation usually dominants over 4D
Primordial MBHs?

- There are various ways to form primordial black holes
  - Large primordial fluctuations
  - Phase transitions, boson stars, ……
- Produce large number of monopoles and anti-monopoles (maybe Nambu’s dumbbell configurations)
- The formation of black holes eat totally $N$ objects
- Anticipate the net BH magnetic charge: $\sim \sqrt{N}$

YB, Orlofsky, arXiv: 1906.04858

- To be studied more. Let’s discuss how to search for them
Parker Limits

- Requiring the domains of coherent magnetic field are not drained by magnetic monopoles

\[ M_\bullet/Q = c_W \sqrt{\pi} M_{\text{pl}}/e \approx 5.1 M_{\text{pl}} \]

- The mass per charge is much larger than GUT monopoles

- PMBH flux: \[ F_\bullet \approx (9.5 \times 10^{-21} \text{ cm}^{-2} \text{sr}^{-1} \text{s}^{-1}) \times f_\bullet \left( \frac{10^{26} \text{ GeV}}{M_\bullet} \right) \left( \frac{\rho_{\text{DM}}}{0.4 \text{ GeV cm}^{-3}} \right) \left( \frac{v}{10^{-3}} \right) \]

- Mean energy gained by PMBHs for the regeneration time is smaller than the energy stored in B

\[ \Delta E \times F_\bullet \times (4\pi \ell_c^2) \times (\pi \text{ sr}) \times t_{\text{reg}} \lesssim \frac{B^2}{8\pi} \frac{4\pi \ell_c^3}{3} \]

\[ \Delta E \approx M_\bullet \Delta v^2/2 \quad \Delta v \approx B h_Q \ell_c/(M_\bullet v) \]

\[ f_\bullet \lesssim 3.8 \times \frac{v_{-3}}{\rho_{0.4} \ell_{21} t_{15}} \]

\[ \rho_{0.4} = \rho_{\text{DM}}/(0.4 \text{ GeV cm}^{-3}) \]

\[ v_{-3} = v/(10^{-3}) \]

\[ t_{15} = t_{\text{reg}}/(10^{15} \text{ s}) \]

\[ \ell_{21} = \ell_c/(10^{21} \text{ cm}) \]

Turner, Parker, Bogdan, PRD26(1982) 1296
Parker Limit from M31

\[ \ell_c \sim 10 \, \text{kpc} \Rightarrow \ell_{21} \sim 30 \text{ and } t_{\text{reg}} \sim 10 \, \text{Gyr} \Rightarrow t_{15} \sim 300 \]

\[ f_{\star} \lesssim 4 \times 10^{-4} \]

which is independent of PMBH mass
PMBHs Captured by Astrophysical Objects

- Taking the finite size into account, the PMBH power loss rate is

\[
\frac{dW}{dt} = -\frac{\mu_0 V^2}{4\pi R^2} \left[ \log \left( \frac{R}{l} \right) + \frac{1}{4} \right]
\]

\[V = 10^{-3} \quad l = \pi^{-1/4} (v_{\text{th}}/V)^{1/2} \omega_p^{-1} \approx 3 \times 10^{-6} \text{ cm} \quad R_{\text{EW}} = \sqrt{\frac{Q}{2 m_h}} \approx (10^{-8} \text{ cm}) \sqrt{\frac{Q}{10^{16}}} \]

**attenuation length**

<table>
<thead>
<tr>
<th></th>
<th>(n_e)</th>
<th>electron (v_{\text{th}}) or (v_F)</th>
<th>(Q_{\text{stop,min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>(10^{24} \text{ cm}^{-3})</td>
<td>(v_{\text{th}} = 0.058) (from (T = 10^7 \text{ K}))</td>
<td>30 [53, 55]</td>
</tr>
<tr>
<td>Earth</td>
<td>((5.5 \text{ cm}^{-3}) \frac{Z}{A} N_A \sim 1.7 \times 10^{24} \text{ cm}^{-3})</td>
<td>(v_F \sim \sqrt{\frac{2(1 \text{ eV})}{(0.511 \text{ MeV})}} \sim 2 \times 10^{-3})</td>
<td>1900</td>
</tr>
<tr>
<td>Neutron star</td>
<td>(6 \times 10^{37} \text{ cm}^{-3})</td>
<td>(v_F \sim 1)</td>
<td>1</td>
</tr>
<tr>
<td>White dwarf</td>
<td>(6 \times 10^{29} \text{ cm}^{-3})</td>
<td>(v_F \sim 0.7)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Physical quantities relevant for stopping.


PMBHs inside the Sun

- The capture rate is
  
  \[ C_{\text{cap}} \approx \epsilon \pi R_\odot^2 \left[ 1 + \left( \frac{v_{\text{esc}}}{v} \right)^2 \right] \ 4\pi F_\star \approx \left( 9.2 \times 10^3 \text{s}^{-1} \right) \epsilon f_\star M_{26}^{-1} \]

- Then, it drifts to the center region with a time scale
  
  \[ t_{\text{drift}} \sim \frac{R_\odot}{v_{\text{drift}}} \sim \frac{R_\odot^3}{M_\odot c_w^2 m_e v_{\text{th}}} M_\star \sim \left( 8 \times 10^4 \text{s} \right) M_{26} \]

- Force-balance equation:
  
  \[ 0 = F = B \frac{2\pi Q}{e} - \frac{4\pi}{3} G \rho_c M_\star z - \frac{G N_\star M_\star^2}{(2 z)^2} \]
PMBHs inside the Sun

\[
0 = F = B \frac{2\pi Q}{e} - \frac{4\pi}{3} G A \rho c M \star z - \frac{G N \star M^2 \star}{(2z)^2}
\]

- For \( N < N^{\text{crit}} \) \[\approx \frac{18 M^3_{\text{pl}} B^3}{\sqrt{\pi} c^3_{W} M \star \rho_c^2} = (3.8 \times 10^{10}) B^{3}_{100} M_{26}^{-1} \] the first two terms are important

\[
z_B \approx \frac{3 B M_{\text{pl}}}{2 \sqrt{\pi} c_W \rho_c} = (2.0 \times 10^{3} \text{ cm}) B_{100}
\]

- For \( N > N^{\text{crit}} \), an equilibrium is quickly reached between capture and annihilation rates with

\[
\Gamma_A = \frac{1}{2} C_A N \star^2 \approx \frac{1}{2} C_{\text{cap}} = (4.6 \times 10^{3} \text{ s}^{-1}) f \star M_{26}^{-1}
\]
Annihilation Products

- For two eBHs with $Q_1$ and $-Q_2$ charges, the merge product has
  \[ Q = Q_1 - Q_2 \]
  \[ M_{BH} \approx c_W \sqrt{\pi (Q_1 + Q_2) M_{pl}/e} \]

- It is a non-extremal MBH with
  \[ T_{BH} \approx \frac{M_{pl}^2}{2\pi} \frac{1}{8 M_\ast(Q_1)} = (2.8 \times 10^{10} \text{ GeV}) M_{26}^{-1} \]

- For $T_{BH} > m_e$, it has quick 2d Hawking radiation to reach the extremal state

- The radiated charged particles can decay into photons, neutrinos and protons; only (not too high-energy) neutrinos can easily propagate out of the Sun
Solar $\nu$ from PMBH Annihilation

- To satisfy the neutrino energy cut,

$$M_\odot \lesssim M_{\text{max}, E} = (2.8 \times 10^{35} \text{ GeV}) \left( \frac{10 \text{ GeV}}{E_\nu^{\text{cut}}} \right)$$

- To have the time interval of two events shorter than the experimental operation time

$$M_\odot \lesssim M_{\text{max}, t} = (2.1 \times 10^{37} \text{ GeV}) f_\odot \left( \frac{t_{\text{exp}}}{532 \text{ day}} \right)$$

- The generated neutrino flux is

$$E_\nu \simeq \langle E_f \rangle/\eta_\nu \approx (1.19/\eta_\nu) T_{\text{BH}}$$

$$I_\nu \approx \frac{N_\nu \Gamma_A}{4\pi d_\odot^2} \approx (5.5 \times 10^{-9} \text{ cm}^{-2} \text{ s}^{-1}) M_{26} \eta_\nu f_\odot$$

$$f_\odot \lesssim \begin{cases} 1.4 \times 10^{-7}, & 2 \times 10^{21} \text{ GeV} \lesssim M_\odot \lesssim 2.9 \times 10^{30} \text{ GeV}, \\ M_\odot/(2.1 \times 10^{37} \text{ GeV}), & 2.9 \times 10^{30} \text{ GeV} \lesssim M_\odot \lesssim 2.8 \times 10^{35} \text{ GeV}, \end{cases}$$

(IceCube)

- Super-K probes even heavier masses because a smaller energy cut
PMBH inside Earth

- Similar story as the Sun, the capture rate is

\[ C_{\text{cap}} \approx \epsilon \pi R_\odot^2 4\pi F_\odot \approx (0.15 \text{s}^{-1}) \epsilon f_\odot M_{26}^{-1} \]

- Other than the neutrino signals, the total power generated from BH annihilation is

\[ P_A \approx (2.4 \times 10^{15} \text{W}) f_\odot \]

- The internal heat of the Earth is \( P_\oplus \approx 4.7 \times 10^{13} \text{W} \), so

\[ f_\odot \lesssim 0.02 \quad \text{(Earth heat)} \]

for \( 1.2 \times 10^{23} \text{ GeV} \lesssim M_\odot \lesssim 1 \times 10^{37} \text{ GeV} \)

\( t_{\text{drift}} < t_\oplus \)
PMBH inside Neutron Stars

- The capture rate is

\[ C_{\text{cap}} \approx \epsilon \pi R^2 \left[ \frac{1 + (v_{\text{esc}}/v)^2}{1 - v^2_{\text{esc}}} \right] \approx 4 \pi F \approx (0.11 \text{ s}^{-1}) f M^{-1} \]

\[ N_{\cdot}^{\text{NS}} = C_{\text{cap}} \tau_{\text{NS}} \sim (3.3 \times 10^{16}) f R_{10}^2 M^{-1} \tau_{10} \]

\[ \tau_{\text{NS}} = \tau_{10} \times 10^{10} \text{ yr} \]

- The inner core of a neutron star is anticipated to be a proton superconductor

  Gezerlis, et. al, arXiv:1406.6109

- The magnetic field of PMBH is confined to flux tubes with

\[ \lambda = \left( \frac{m_p}{e^2 n_p} \right)^{1/2} \sim 10^{-12} \text{ cm} \]

\[ B_{\Phi} \sim \frac{\Phi}{\pi \lambda^2} \sim 10^{16} \text{ gauss} \]

\[ F_T \sim B_{\Phi}^2 \pi \lambda^2 \ln (\lambda/\xi) \sim 10^4 \text{ N} \]
Fraction of PMBH over dark matter

Figure 3: Bounds on PMBH abundance as a fraction of the dark matter abundance. In green is the Parker bound using M31/Andromeda. Red and blue show constraints from the Sun and the Earth, respectively, due to neutrino observations at IceCube (IC), Super-Kamiokande (SK), and Earth heating. Orange dashed lines show constraints from neutron stars (NS) assuming a total baryon number violation energy on emitted photon luminosity of either $r = 10^3$ or $r = 10^4$. See details and caveats in the text. Purple regions are excluded by direct searches from MACRO and ancient mica. Brown displays constraints from microlensing at Subaru/HSC (HSC), Kepler (K), and MACHO/EROS/OGLE (M/E/O). The dotted black vertical lines show where $Q = 2$, $Q_{\text{min}} = 10^6$ (assuming the existence of a GUT monopole), and $Q_{\text{max}}' = 10^32$ (above which there is no EWS corona).

If PMBHs are indeed primordial, then they can form binaries in the early Universe that merge today, giving high energy neutrinos and gamma rays throughout the sky. An estimate of this signal is given in [21], but more detailed numerical work is needed, particularly on binary...
Conclusions

- Magnetic black holes have electroweak-symmetric coronas
- It has a fast 2d Hawking radiation rate and can reach the extremal state quickly
- Because of their heavy masses, they require astrophysical objects to infer their existence
- It is unlikely to account 100% of dark matter abundance, because of the Parker limit and neutrino and photon signals from the Sun, Earth and NS captures
- It does not require unknown new physics. More studies are deserved to search for them
Thanks!