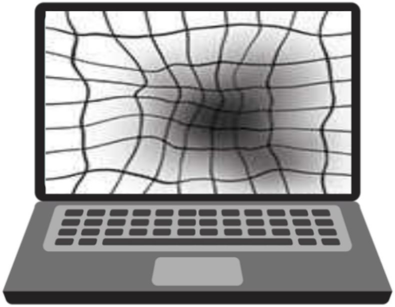
Non-Abelian lattice gauge theory on quantum computers

1. Motivation
2. Gauge fields on gated-based quantum computers
3. Gauge fields on a quantum annealer
4. Including quarks
5. Outlook



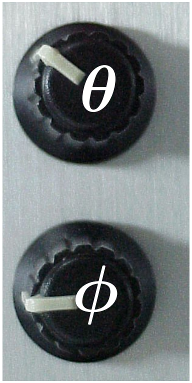
www.yorku.ca/lewisr

From bits to qubits

Classical computers use bits.
One bit is either $|0\rangle$ or $|1\rangle$.

$= |0\rangle$

Quantum computers use qubits.
One qubit is a <u>superposition</u> of $|0\rangle$ and $|1\rangle$.



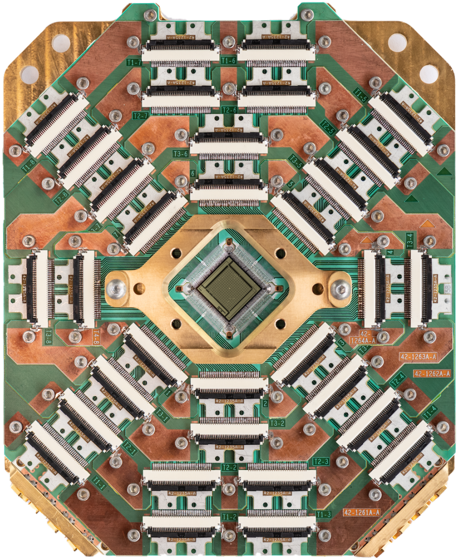
$$= \cos(\tfrac{\theta}{2})\,|0\rangle + e^{i\phi}\sin(\tfrac{\theta}{2})\,|1\rangle$$

Multiple bits act independently.

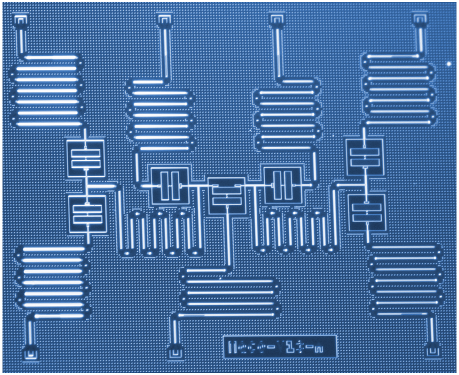
Multiple qubits can be <u>entangled</u>, so measuring one affects the others.

NOTE: Qubits can be in a *superposition of all* classically allowed states.

I will show results from qubits at IBM and D-Wave

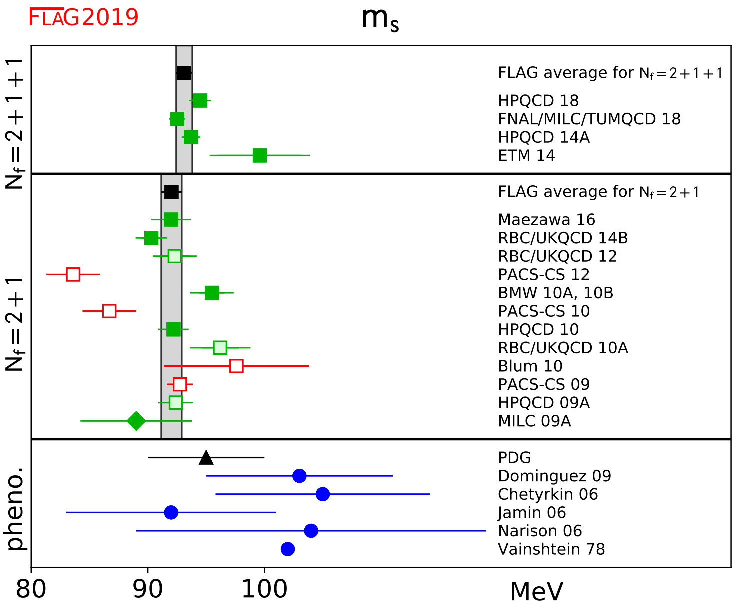


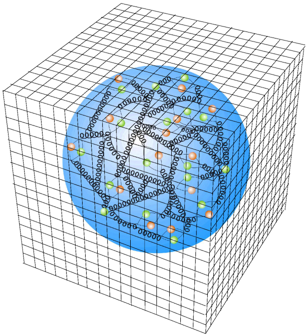
D-Wave, 5760 qubits, no gates



IBM, 7 qubits, universal gate set

Lattice gauge theory is very successful without qubits





http://flag.unibe.ch/2019/Quark%20masses

What qubits might do for lattice gauge theory

Quantum computers offer an efficient Hamiltonian-based approach that might...

...allow us to <u>avoid Euclidean time</u>, thus moving from statics to dynamics.

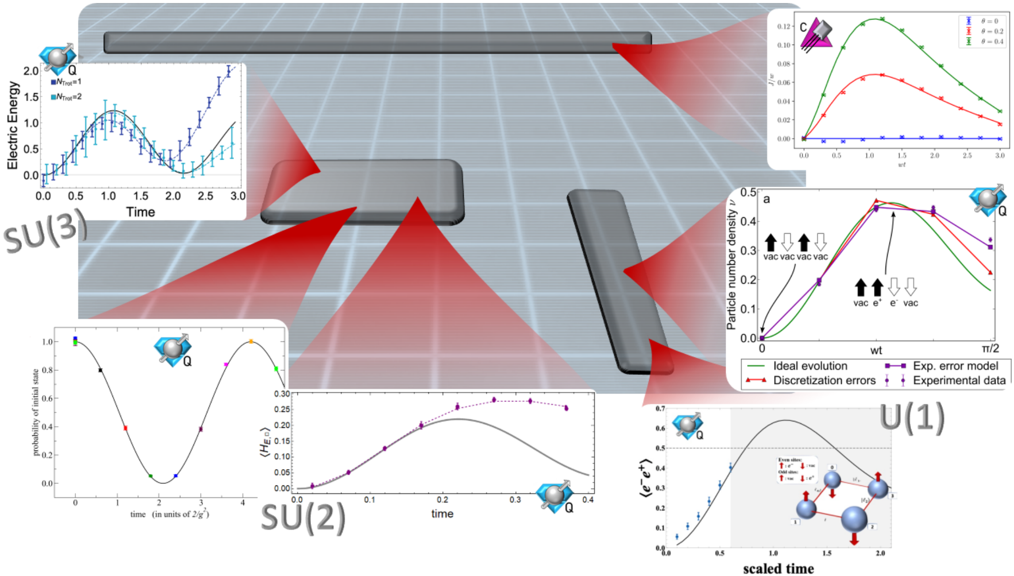
...allow us to <u>include a chemical potential</u>, thus reaching nuclear densities.

Lattice QCD at non-zero density would be valuable for heavy-ion collisions, the early Universe and neutron-star structure. In practice, simulations at finite μ suffer from a "sign problem" and are at a rudimentary stage.

— paraphrased from Particle Data Group, Review of Lattice QCD

Time evolution in gauge theories using qubits

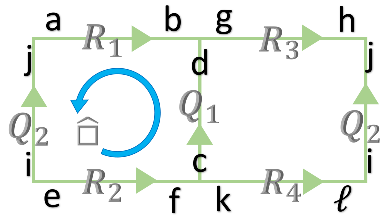
Figure 2 from Klco, Roggero and Savage, arXiv:2107.04769

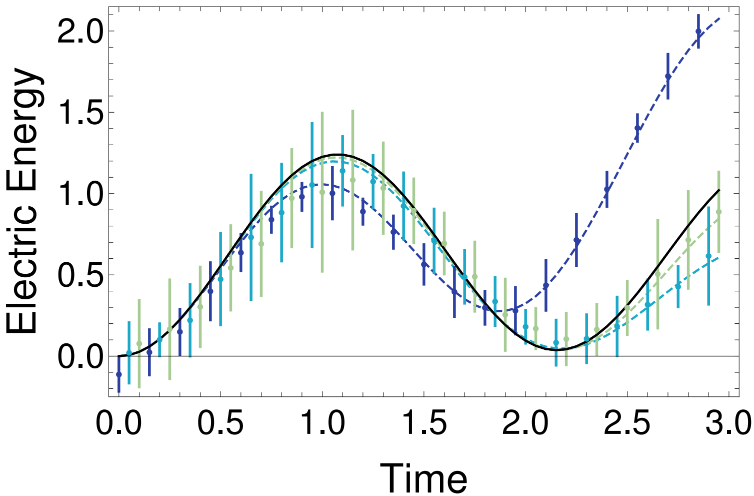


SU(3) pure gauge theory on gate-based hardware

Ciavarella, Klco, Savage, Phys.Rev.D103(2021)094501

Two plaquettes with a periodic boundary:





How the Hamiltonian was constructed

Ciavarella, Klco, Savage, Phys.Rev.D103(2021)094501

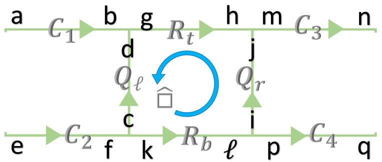
The Hamiltonian is

$$\hat{H} = \boxed{\frac{g^2}{2a^{d-2}} \sum_{b,\text{links}} \left|\hat{\mathbf{E}}^{(b)}\right|^2} + \boxed{\frac{1}{2a^{4-d}g^2} \sum_{\text{plaquettes}} \left[6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x})\right]}$$

The $\boxed{\text{chromoelectric term}}$ comes from

$$\sum_b \left|\hat{\mathbf{E}}^{(b)}\right|^2 |p,q\rangle = \frac{p^2+q^2+pq+3p+3q}{3} |p,q\rangle$$

The $\boxed{\text{chromomagnetic term}}$ comes from



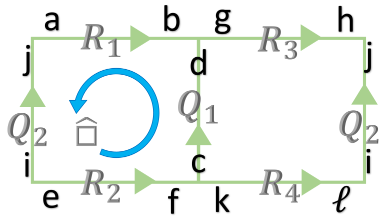
$$\left\langle \begin{pmatrix} \mathbf{C}_1, \mathbf{R}'_t, \mathbf{C}_3 \\ \mathbf{Q}'_\ell, \mathbf{Q}'_r \\ \mathbf{C}_2, \mathbf{R}'_b, \mathbf{C}_4 \end{pmatrix} \middle| \hat{\Box} \middle| \begin{pmatrix} \mathbf{C}_1, \mathbf{R}_t, \mathbf{C}_3 \\ \mathbf{Q}_\ell, \mathbf{Q}_r \\ \mathbf{C}_2, \mathbf{R}_b, \mathbf{C}_4 \end{pmatrix} \right\rangle =$$

$$\sqrt{\frac{\dim(\mathbf{R}_t)\dim(\mathbf{R}_b)}{\dim(\mathbf{R}'_t)\dim(\mathbf{R}'_b)\dim(\mathbf{Q}_\ell)\dim(\mathbf{Q}_r)\dim(\mathbf{Q}'_\ell)^3\dim(\mathbf{Q}'_r)^3}}$$

$$\begin{Bmatrix} \overline{\mathbf{R}}_t & \mathbf{C}_1 & \overline{\mathbf{Q}}_\ell \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \overline{\mathbf{R}}'_t & \mathbf{C}_1 & \overline{\mathbf{Q}}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}_r \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \mathbf{R}'_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}'_r \end{Bmatrix} \begin{Bmatrix} \overline{\mathbf{R}}_b & \mathbf{C}_2 & \mathbf{Q}_\ell \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \overline{\mathbf{R}}'_b & \mathbf{C}_2 & \mathbf{Q}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_b & \overline{\mathbf{C}}_4 & \mathbf{Q}_r \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{R}'_b & \overline{\mathbf{C}}_4 & \mathbf{Q}'_r \end{Bmatrix}$$

How the gauge links were truncated

Ciavarella, Klco, Savage, Phys.Rev.D103(2021)094501



Each of the 6 links is an irrep of SU(3): $1,\ 3,\ \bar{3},\ 6,\ \bar{6},\ 8,\ \ldots$

Truncating to only $\{1,3,\bar{3}\}$ gives $3^6 = \boxed{729}$ basis states for the lattice.

Enforcing Gauss's law at every vertex leaves only $\boxed{27}$ of those basis states.

$\boxed{9}$ of 27 are global singlet states. (Apply $\hat{\square}, \hat{\square}^\dagger$ to the strong-coupling vacuum.)

Spatial translation and parity block diagonalize the Hamiltonian: $9 \to \boxed{4}+2+2+1$.

$$\hat{H}^{(\mathbf{1 3 \bar{3}};++)} = \frac{g^2}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{16}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & 8 \end{pmatrix} + \frac{1}{2g^2}\begin{pmatrix} 6 & -2 & 0 & 0 \\ -2 & 5 & -\frac{\sqrt{2}}{9} & -\frac{\sqrt{2}}{3} \\ 0 & -\frac{\sqrt{2}}{9} & 6 & -\frac{2}{3} \\ 0 & -\frac{\sqrt{2}}{3} & -\frac{2}{3} & 6 \end{pmatrix}$$

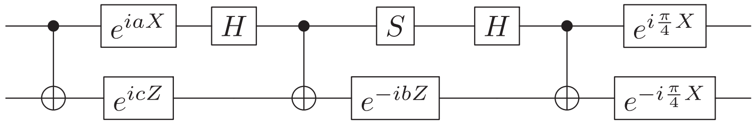
Note the various Clebsch-Gordan combinations.

How the circuit was implemented

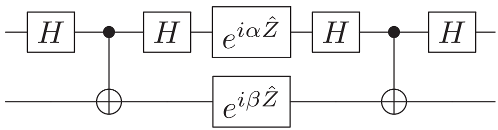
Ciavarella, Klco, Savage, Phys.Rev.D103(2021)094501

First-order Trotter is used. The circuit has <u>single qubit terms</u> and these:

$$e^{i(a\hat{X}\otimes\hat{X}+b\hat{Y}\otimes\hat{Y}+c\hat{Z}\otimes\hat{Z})} =$$



$$e^{i\left(\alpha\hat{Z}\otimes\hat{X}+\beta\hat{X}\otimes\hat{Z}\right)} =$$



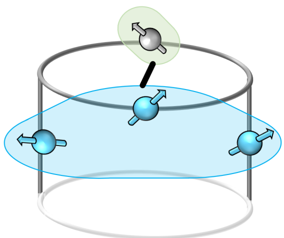
The IBM Athens chip has 5 qubits in total:

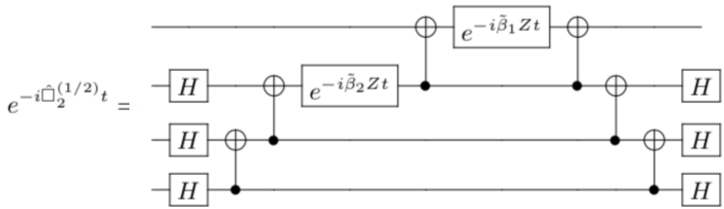
2 hold the state of the lattice.

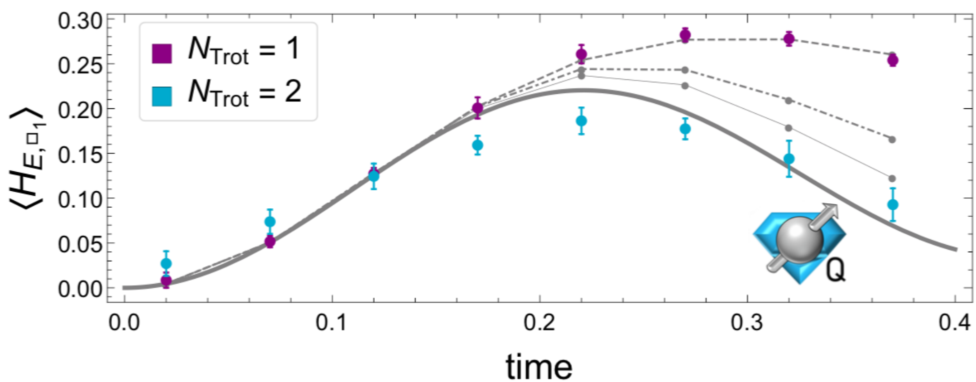
3 were used for post-selection error mitigation.

SU(2) pure gauge theory on gate-based hardware

Klco, Savage, Stryker, Phys.Rev.D101(2020)074512





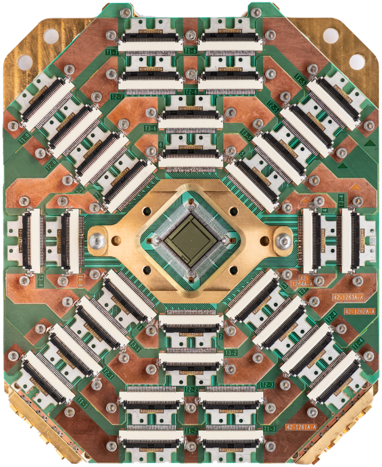


What a D-Wave quantum annealer calculates

The hardware moves quasi-adiabatically to the ground state of

$$H(q) = \sum_{i=1}^{N} h_i q_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij} q_i q_j$$

The user chooses any real h_i and J_{ij}.
Each q_i is either 0 or 1.



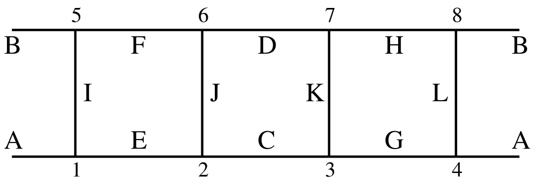
D-Wave Advantage, 5760 qubits
(usable as 180 all-to-all qubits)

A Rahman,Lewis,Mendicelli,Powell, Phys.Rev.D104(2021)034501

- Can this be used for a non-Abelian gauge theory? Yes, some aspects.
- Can it handle the various Clebsch-Gordan combinations? Yes!
- Will the number of qubits scale efficiently to large lattices?
  No, not with our method on today's hardware.

Constructing the SU(2) Hamiltonian

$$\hat{H} = \frac{g^2}{2}\left(\sum_{i=\text{links}} \hat{E}_i^2 - 2x \sum_{i=\text{plaquettes}} \hat{\Box}_i\right)$$



$$
\begin{aligned}
x &\equiv \frac{2}{g^4} \\
\langle\psi|\, \hat{\mathbf{E}}_i^2 \,|\psi\rangle &= j_i(j_i+1) \\
\langle\psi_{\text{final}}|\, \hat{\Box}_1 \,|\psi_{\text{initial}}\rangle &= (-1)^{j_A+J_E+j_I}\sqrt{(2j_I+1)(2J_E+1)}\begin{Bmatrix} j_A & j_E & j_I \\ \frac{1}{2} & J_I & J_E \end{Bmatrix} \\
&\quad (-1)^{j_C+J_E+j_J}\sqrt{(2j_E+1)(2J_J+1)}\begin{Bmatrix} j_C & j_E & j_J \\ \frac{1}{2} & J_J & J_E \end{Bmatrix} \\
&\quad (-1)^{j_D+J_F+j_J}\sqrt{(2j_J+1)(2J_F+1)}\begin{Bmatrix} j_D & j_F & j_J \\ \frac{1}{2} & J_J & J_F \end{Bmatrix} \\
&\quad (-1)^{j_B+J_F+j_I}\sqrt{(2j_F+1)(2J_I+1)}\begin{Bmatrix} j_B & j_F & j_I \\ \frac{1}{2} & J_I & J_F \end{Bmatrix}
\end{aligned}
$$

We also apply vertical reflection, horizontal reflection, and translation symmetries.

The quantum annealer eigensolver (QAE)

Recall the variational method: $E_0 \leq \dfrac{\langle\psi|\, H\, |\psi\rangle}{\langle\psi|\psi\rangle}$.

Recall that D-Wave finds the minimum of $H(q) = \sum_{i=1}^{N} h_i q_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij} q_i q_j$.

If the $|\psi\rangle$ vector has only 0 and 1 as entries, then those are basically the same.

$$\begin{array}{rcl} q & \rightarrow & |\psi\rangle \\ h_i & \rightarrow & \text{on-diagonals of } H \\ J_{ij} & \rightarrow & \text{off-diagonals of } H \end{array}$$

All Hamiltonian entries can be entered directly and easily.

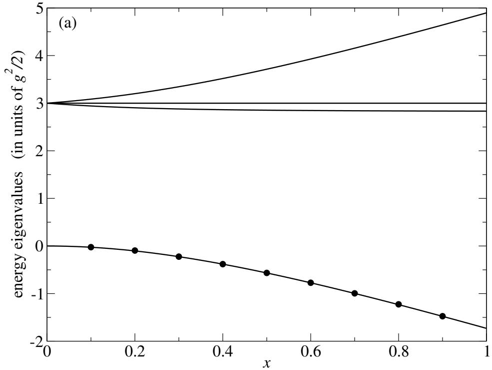
QAE handles a general vector (fixed-point representation) and the normalization.
It uses one penalty term (called λ) to avoid the null vector.

`Teplukhin,Kendrick,Babikov,J.Chem.Theory&Comp15,4555(2019)`

We built an adaptive QAE to use fewer qubits and solve larger Hamiltonians.
Its only parameter is the λ from original QAE.

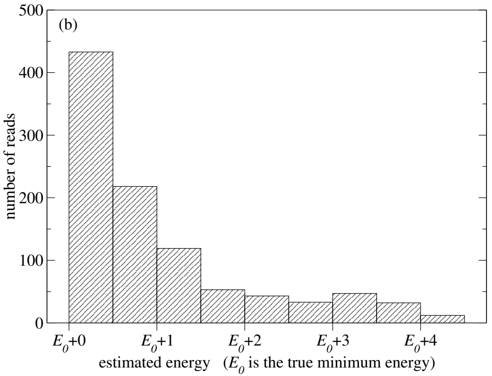
Ground state eigenvalue for two plaquettes and $j_{\max} = \frac{1}{2}$

A Rahman, Lewis, Mendicelli, Powell, Phys.Rev.D104(2021)034501



Data points are from QAE.

Curves are exact eigenvalues.



Raw data for $x = 0.5$ in the graph above.

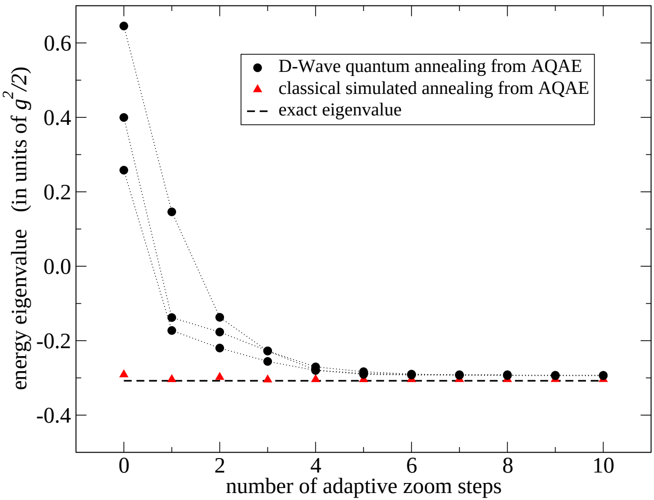
1000 anneals were used.

Each anneal took 20 microseconds.

The importance of our adaptive algorithm

A Rahman,Lewis,Mendicelli,Powell, Phys.Rev.D104(2021)034501

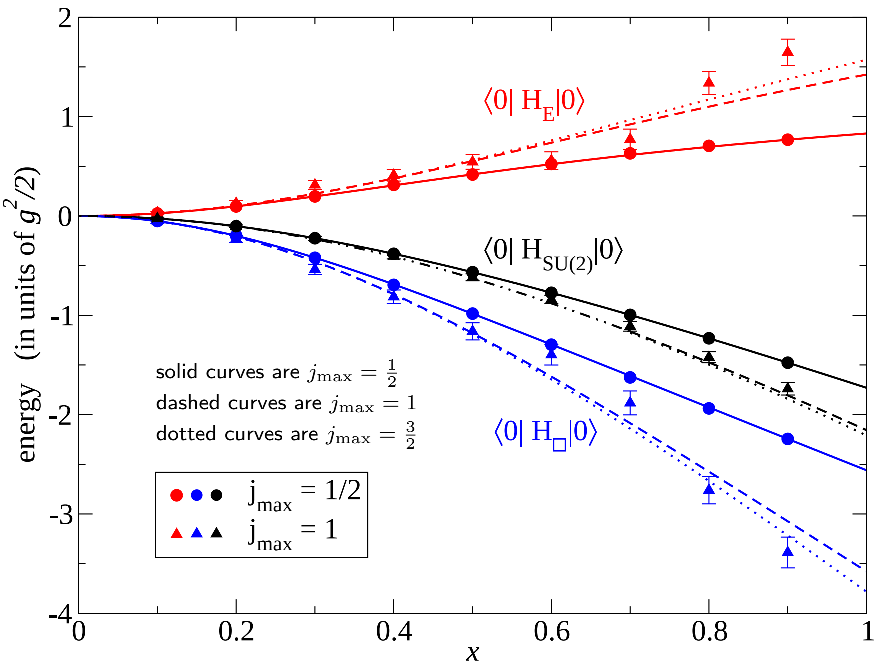
$\boxed{\text{six-plaquette lattice}}$ at $x = 0.2$



The original QAE has no adaptive step, so zoom=0.
Our AQAE is helpful on a classical simulator.
Our AQAE is necessary for larger Hamiltonians on noisy quantum hardware.

Assessing the gauge truncation

A Rahman,Lewis,Mendicelli,Powell, Phys.Rev.D104(2021)034501



Time evolution as a minimization problem

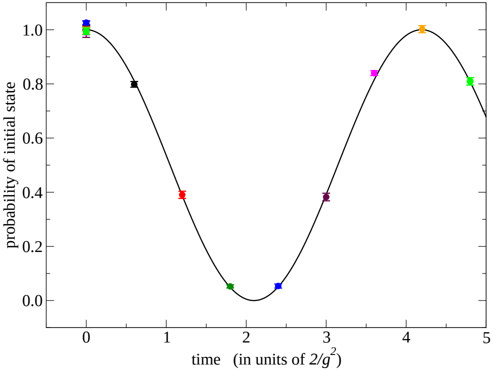
The TEDVP algorithm minimizes this functional:

$$\mathcal{L} = \sum_{t,t'} \langle t'| \langle \Psi_{t'}| \mathcal{C} |\Psi_t\rangle |t\rangle - \lambda\Big(\sum_{t,t'} \langle t'| \langle \Psi_{t'}|\Psi_t\rangle |t\rangle - 1\Big)$$

$$\mathcal{C} = C_0 + \frac{1}{2}\sum_t \Big(I \otimes |t\rangle\langle t| \; - \; e^{-i\epsilon H_t} \otimes |t+\epsilon\rangle\langle t| \; - \; e^{i\epsilon H_t} \otimes |t\rangle\langle t+\epsilon| \; + \; I \otimes |t+\epsilon\rangle\langle t+\epsilon|\Big)$$

McClean,Parkill,Aspuru-Guzik,Proc.Natl.Acad.Sci.110,E3901(2013)

For D-Wave hardware, we
- express H_t as imaginary so coefficients are real.
- use a combined QAE+TEDVP algorithm.



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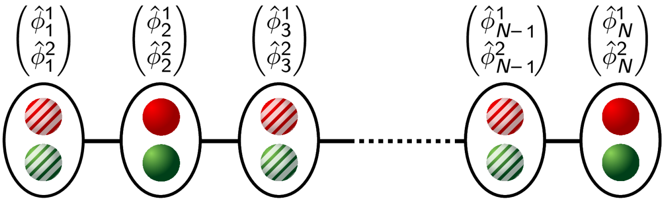
Including quarks

Atas, Zhang, Lewis, Jahanpour, Haase, Muschik, accepted for publication.
SU(2) hadrons on a quantum computer via a variational approach

Consider a one-dimensional lattice. It will have no colour-magnetic fields.

Put quarks and antiquarks on alternating sites ("staggered fermions").

We need two qubits per lattice site.



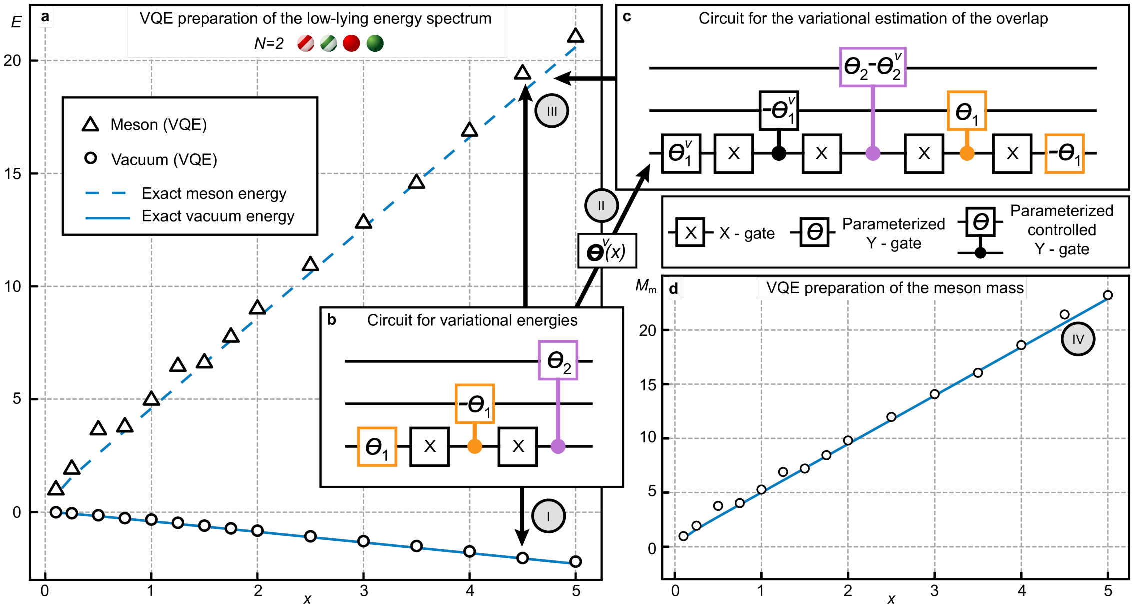
Absorbing the gauge fields

There are two physics parameters: the gauge coupling and the quark mass.

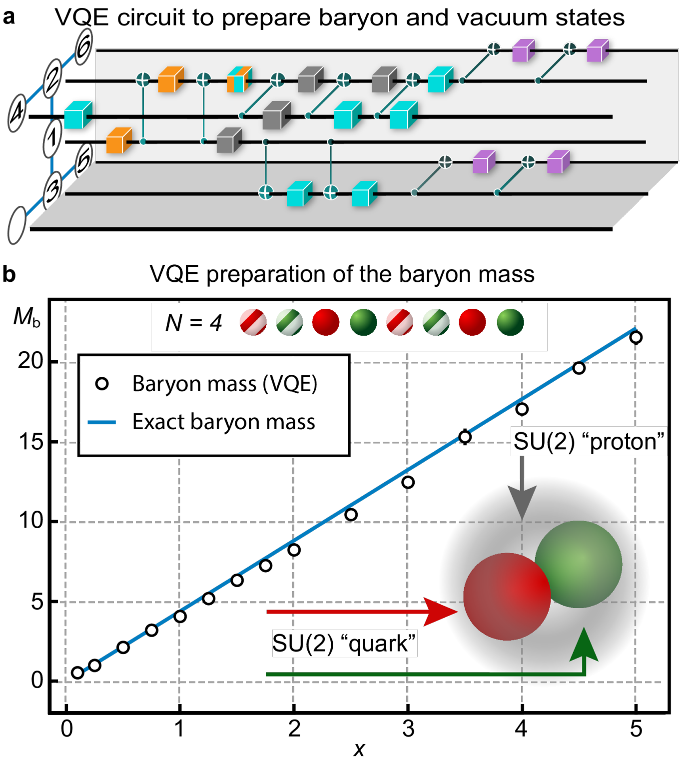
With open lattice boundaries, gauge field effects are long-range quark interactions.

$$
\begin{aligned}
\hat{H} &= x\tilde{m}\hat{H}_{\mathsf{m}} + \hat{H}_{\mathsf{el}} + x\hat{H}_{\mathsf{kin}} \\
\hat{H}_{\mathsf{m}} &= 2\sum_{n=1}^{N}\left(\frac{(-1)^n}{2}\left(\hat{\sigma}^z_{2n-1} + \hat{\sigma}^z_{2n}\right) + 1\right) \\
\hat{H}_{\mathsf{kin}} &= -\sum_{n=1}^{N-1}\left(\hat{\sigma}^+_{2n-1}\hat{\sigma}^z_{2n}\hat{\sigma}^-_{2n+1} + \hat{\sigma}^+_{2n}\hat{\sigma}^z_{2n+1}\hat{\sigma}^-_{2n+2} + \mathsf{h.c.}\right) \\
\hat{H}_{\mathsf{el}} &= \frac{3}{8}\sum_{n=1}^{N-1}(N-n)(1-\hat{\sigma}^z_{2n-1}\hat{\sigma}^z_{2n}) \\
&\quad + \frac{1}{8}\sum_{n=1}^{N-2}\sum_{m>n}^{N-1}(N-m)\left(\hat{\sigma}^z_{2n-1} - \hat{\sigma}^z_{2n}\right)\left(\hat{\sigma}^z_{2m-1} - \hat{\sigma}^z_{2m}\right) \\
&\quad + \sum_{n=1}^{N-2}\sum_{m>n}^{N-1}(N-m)\left(\hat{\sigma}^+_{2n-1}\hat{\sigma}^-_{2n}\hat{\sigma}^+_{2m}\hat{\sigma}^-_{2m-1} + \text{h.c.}\right)
\end{aligned}
$$

Computing the meson mass



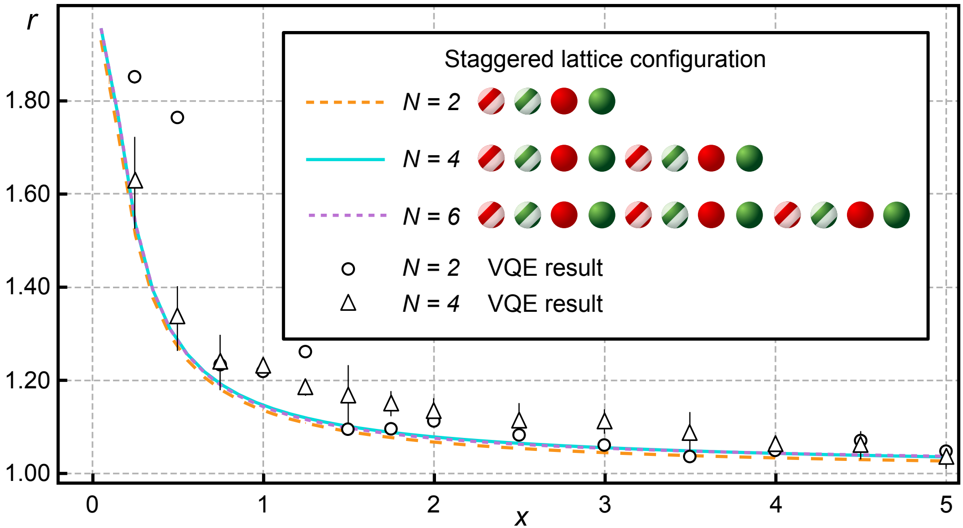
Computing the baryon mass



Computing the meson-to-baryon mass ratio

For continuum SU(2), the meson and baryon are exactly degenerate.

Our staggered lattice calculation is consistent with this continuum limit.



Comparing several formulations

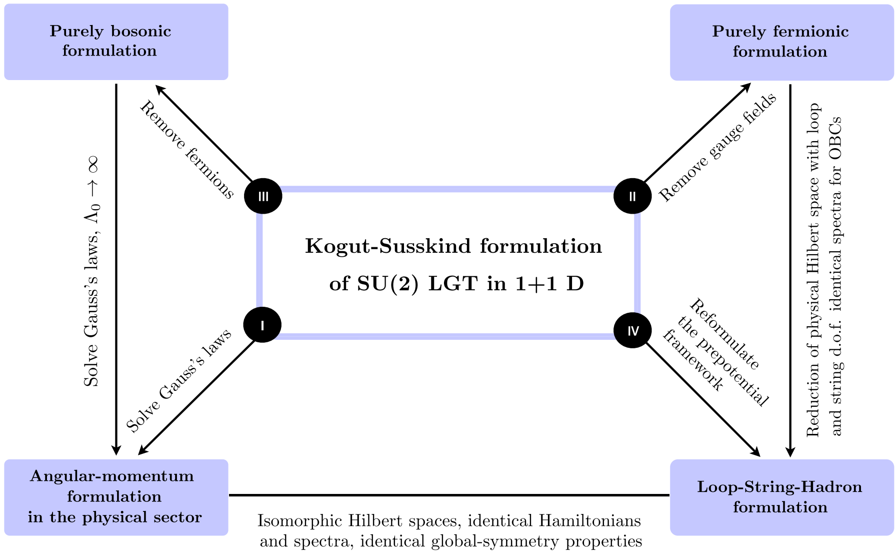
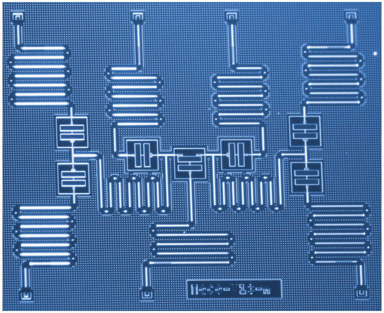
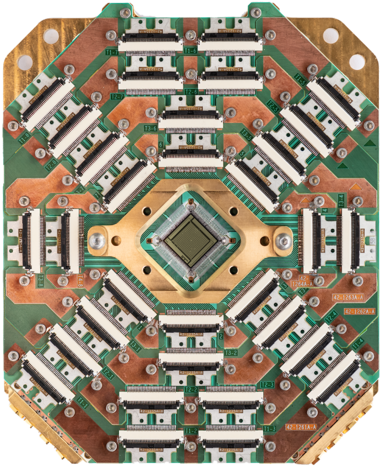


FIG. 1. Various formulations of the KS SU(2) LGT in 1 + 1D studied in this work and the connection among them.

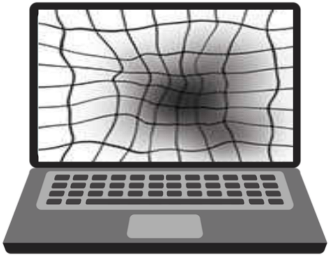
Many ideas remain to be explored



IBM, 7 qubits, universal gate set



D-Wave, 5760 qubits, no gates



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