1-form symmetry vs. large N QCD

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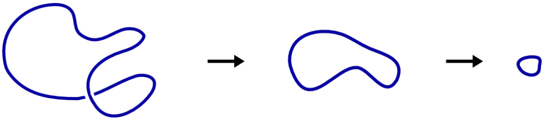


Based on `arXiv:2209.00027`

with Aleksey Cherman and Maria Neuzil

Two (opposing) facts about confinement

Quarks are confined in $SU(N)$ QCD at large N

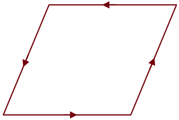
Confinement is associated with a (unbroken) 1-form symmetry

- the standard story in pure YM
  - indicators of confinement
  - connection to $\mathbb{Z}_N$ 1-form symmetry
- why large N QCD confines
- obstructions to 1-form symmetries
  - 2d example

Confinement in pure gauge theory

Pure $SU(N)$ YM theory **confines** fundamental-rep probe quarks

$$W_{\mathrm{F}}(C) = \frac{1}{N}\mathrm{tr}_{\mathrm{F}}\mathcal{P}\, e^{i\int_C a}$$

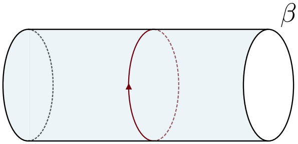


$$\lim_{|C|\to\infty}\langle W_{\mathrm{F}}(C)\rangle = 0$$

in *any* scheme

linear confining potential between far-separated $\bar{q}$ and q

$$P_{\mathrm{F}}(C) = \frac{1}{N}\mathrm{tr}_{\mathrm{F}}\mathcal{P}\, e^{i\int_0^\beta d\tau\, a_\tau}$$



$$\langle P_{\mathrm{F}}(C)\rangle = 0$$

free energy of an isolated quark is infinite

Symmetries of $SU(N)$ gauge theory

W_{F} and P_{F} are **order parameters** for a symmetry

- **center** symmetry: acts on Polyakov loops — Polyakov, 't Hooft late '70s; Weiss, Gross-Pisarski-Yaffe '81
- **1-form** symmetry: acts on all Wilson loops — Gaiotto, Kapustin, Seiberg, Willett '14

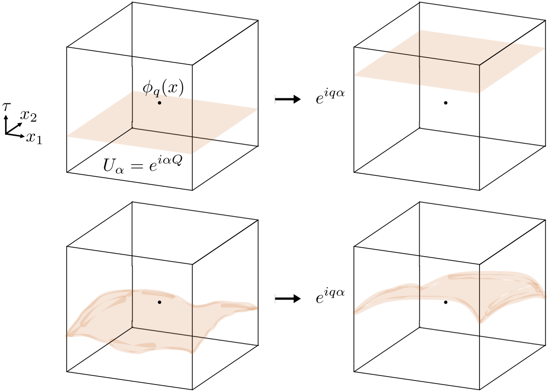
Why is this useful?

- sharp definition of confinement/deconfinement
- couple to backgrounds, 't Hooft anomalies
- part of nonperturbative definition of the theory
- easily derive selection rules

Modern perspective: 0-form symmetry

Ordinary, '0-form' symmetry:
e.g. $U(1)$ symmetry

operators which measure the **charges** of local operators



Modern perspective: 1-form symmetry

1-form symmetry in $SU(N)$ gauge theory

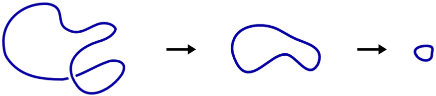
operators which measure the **charges** (representations) of Wilson lines **modulo screening** by gluons (adjoint rep)

- to measure the charge of a line, operator must be supported on a **codimension-2** closed manifold
- the operators can only detect the N**-ality** of a line ($\mathbb{Z}_N$-valued)

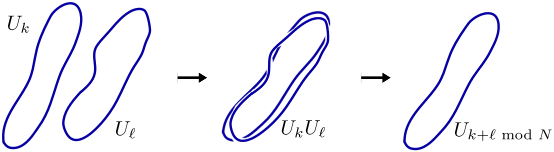
$\mathbb{Z}_N$ 1-form symmetry **generated by** $U_k(M_{d-2})$, with $k = 0, \ldots, N-1$

Modern perspective: 1-form symmetry

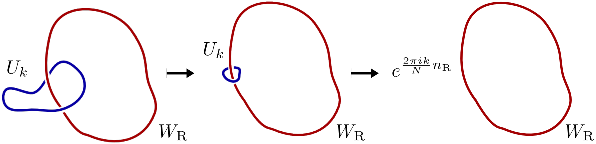
Topological:



$\mathbb{Z}_N$ **fusion:**

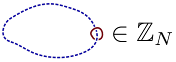


Action on Wilson lines:

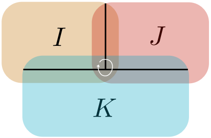


Topological symmetry operators

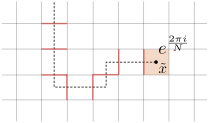
- Gukov-Witten operator: remove C from spacetime and impose 'twisted' BC's



- relax cocycle condition for gauge field transition functions
  - 't Hooft flux

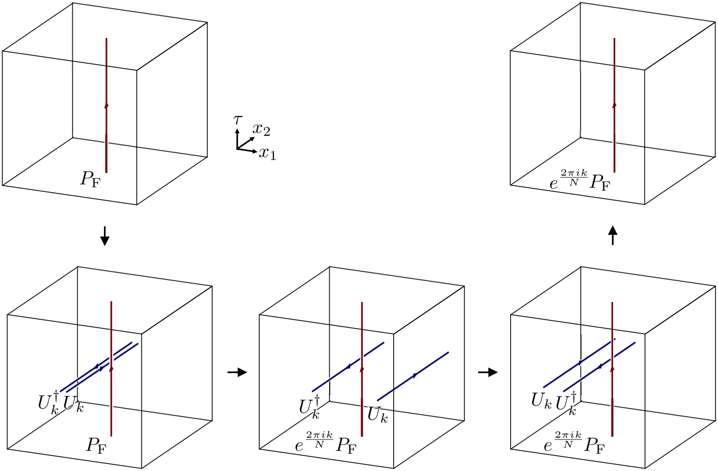


- on the lattice, 'thin center-vortex:' modify plaquettes pierced by closed codim-2 manifold on dual lattice



- couple to background 2-form $\mathbb{Z}_N$ gauge field B

Selection rules from topological operators



$$\langle P_{\mathrm{F}} \rangle = e^{\frac{2\pi i k}{N}} \langle P_{\mathrm{F}} \rangle \quad \Longrightarrow \quad \langle P_{\mathrm{F}} \rangle = 0$$

Motivating example: 2d YM

Tempting to say: *if* $\exists$ a selection rule, *then* $\exists$ a symmetry explanation

Selection rules in 2d YM

$$\left.\begin{array}{c}\langle W_{\mathrm{F}}(C)\rangle = e^{-g^2 c_{\mathsf{F}} A[C]} \\ \langle P_{\mathrm{F}}\rangle = 0\end{array}\right\} \mathbb{Z}_N \text{ 1-form symmetry}$$

$$\left.\begin{array}{c}\langle W_{\mathrm{adj}}(C)\rangle = e^{-g^2 c_{\mathsf{adj}} A[C]} \\ \langle P_{\mathrm{adj}}\rangle = 0\end{array}\right\}$$

non-invertible 1-form symmetry (Nguyen, Tanizaki, Ünsal '20)

QCD has no 1-form symmetry

e.g. 'real-world' QCD with light fundamental quarks

Effect on Wilson loops:

confining strings can break:
perimeter-law for large loops

Effect on symmetry operators:

some operators become ill-defined,
the rest are no longer *topological*

Expect *approximate* symmetry when quarks are heavy:

$$\langle W(C) \rangle = e^{-\sigma A} + c\, e^{-2m_q L}$$

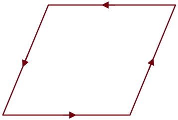
Area law dominates: $\Lambda^{-1} \ll L \ll \frac{2m_q}{\Lambda}\Lambda^{-1}$

Some codim-2 operator should be topological at long distances $\gg m_q^{-1}$

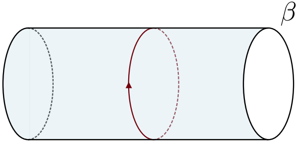
(Córdova, Ohmori, Rudelius '22)

Confinement in large N QCD

't Hooft large N limit ($N \to \infty$, $g^2 N = \lambda =$ fixed, $n_f = O(1)$)



$\langle W_{\rm F}(C)\rangle = e^{-\sigma A[C]}$



$\langle P_{\rm F}(C)\rangle = 0$

Screening by dynamical quarks is $1/N$-suppressed:

To leading order, gluonic observables don't see quarks

$\Longrightarrow$ Large N QCD has the same selection rules as pure YM!

Is there any (emergent) 1-form symmetry in large N QCD?

Obstructions to 1-form symmetry in large N QCD

Expectation:

There exist operators $U_k(M_{d-2})$ in QCD
which are topological *up to* $1/N$ corrections

Obstructions:

Existence of open Wilson lines

Failure of quark loop suppression

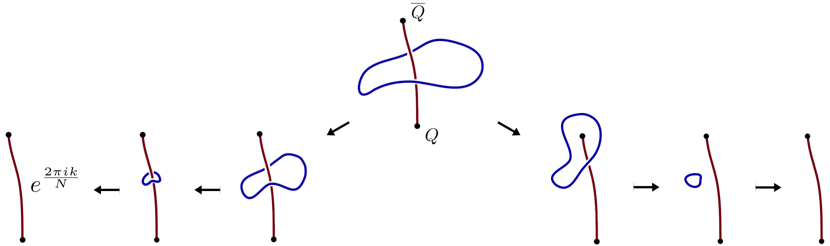
Endability

Higher-form symmetries are broken when charged operators are 'endable'

(Rudelius, Shao '20)

Wilson lines are **endable** in QCD: $W_{xy} = \frac{1}{N}\,\overline{Q}(x)\,e^{i\int_x^y a}\,Q(y)$

Suppose a codim-2 operator is topological in the planar limit:



Consistent if $\langle W_{xy}\rangle = O(\frac{1}{N})$. In reality, $\langle W_{xy}\rangle = O(1)$.

Quark loop non-suppression

Standard large N counting: $\langle \mathcal{O}_{\text{glue}} \rangle_{\text{QCD}} = \langle \mathcal{O}_{\text{glue}} \rangle_{\text{YM}} + O(\frac{1}{N})$

What are the interesting operators $\mathcal{O}_{\text{glue}}$ at large N?

Wilson loops $W_{\mathrm{R}}(C)$

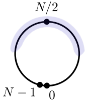
$\mathrm{R} \in \mathrm{F}^m \otimes \bar{\mathrm{F}}^\ell$, $m, \ell = O(1)$

Ensures that $n_{\mathrm{R}} = O(1)$

Symmetry generators $U_k(M_{d-2})$

$k = O(N)$

Ensures that operator acts non-trivially



$$U_k(M_{d-2}) W_{\mathrm{R}}(C) = \exp\left(\frac{2\pi i\, k\, n_{\mathrm{R}}}{N}\right) W_{\mathrm{R}}(C)$$

To have a non-trivial $\mathbb{Z}_N$ 1-form symmetry, require that U_k with $k \sim N$ are topological

Quark loop non-suppression

With an exact $\mathbb{Z}_N$ symmetry:

$$\langle U_k \rangle = \langle U_1^k \rangle = \langle U_1 \rangle^k$$

Generically, $\langle U_k(M_{d-2}) \rangle = e^{\frac{2\pi i k}{N}\ell} \in \mathbb{Z}_N$
If M_{d-2} is contractible, $\langle U_k(M_{d-2}) \rangle = 1$.

In large N QCD, expect:

$$\langle U_1 \rangle = 1 + O(\tfrac{1}{N}), \quad \langle U_1^2 \rangle = \langle U_1 \rangle \langle U_1 \rangle + O(\tfrac{1}{N})$$

What about U_k, $k = O(N)$?

$$\langle U_k \rangle \sim \langle \underbrace{U_1 U_1 \cdots U_1}_{k} \rangle = \langle U_1 \rangle^k + O(\tfrac{k^2}{N})$$

Quark loops are **not** suppressed, and
expectation values deviate from $\mathbb{Z}_N$-constraints:

$$\langle U_k \rangle - \langle U_1 \rangle^k = \begin{cases} O(1) & \text{for } k \sim \sqrt{N}, \\ O(N) & \text{for } k \sim N \end{cases}$$

Recap

- in pure YM theory, confinement is associated to a $\mathbb{Z}_N$ 1-form symmetry
- large N QCD also confines, but

There is no emergent $\mathbb{Z}_N$ 1-form symmetry which acts on Wilson loops

(There are no codimension-2 topological operators in large N QCD)

Simplest example: 2d scalar QCD

Regulate on the lattice: links u_ℓ, plaquettes u_p, sites ϕ_x

$$Z = \prod_\ell \int du_\ell \prod_x \int d\phi_x d\phi_x^\dagger \prod_p e^{-s_{\mathsf{YM}}(u_p)} \prod_\ell e^{-s_{\mathsf{H}}(\phi_x^\dagger u_\ell \phi_{x'})} \prod_x e^{-m^2 \phi_x^\dagger \phi_x}$$

Large mass phase:

Expand in ‘hopping’ parameter $s_{\mathsf{H}} = -\kappa\, \phi_x^\dagger u_\ell \phi_{x'} + \text{h.c.}$

Far from continuum limit, but allows controlled expansion:

$$\langle \mathcal{O}_{\mathsf{glue}} \rangle = \langle \mathcal{O}_{\mathsf{glue}} \rangle_{\mathsf{YM}} + \sum_{\substack{\text{hopping loops} \\ \Gamma}} \left(\frac{\kappa}{m^2} \right)^{|\Gamma|} \left\langle \mathcal{O}_{\mathsf{glue}}\ \mathrm{tr} \prod_{\ell \in \Gamma} u_\ell \right\rangle_{\mathsf{YM}}$$

Look at $\mathcal{O}_{\mathsf{glue}} \longrightarrow W_{\mathrm{F}}(C), U_k(\tilde{x})$

Small $\kappa \Longleftrightarrow$ large m^2 in lattice units

Simplest example: 2d scalar QCD

$$Z = \prod_\ell \int du_\ell \prod_x \int d\phi_x d\phi_x^\dagger \prod_p e^{-s_{\mathsf{YM}}(u_p)} \prod_\ell e^{-s_{\mathsf{H}}(\phi_x^\dagger u_\ell \phi_{x'})} \prod_x e^{-m^2 \phi_x^\dagger \phi_x}$$

$$\langle \mathcal{O}_{\mathsf{glue}} \rangle = \langle \mathcal{O}_{\mathsf{glue}} \rangle_{\mathsf{YM}} + \sum_{\substack{\text{hopping loops} \\ \Gamma}} \left(\frac{\kappa}{m^2} \right)^{|\Gamma|} \left\langle \mathcal{O}_{\mathsf{glue}} \ \mathrm{tr} \prod_{\ell \in \Gamma} u_\ell \right\rangle_{\mathsf{YM}}$$

To compute pure YM correlation functions, use heat-kernel action:

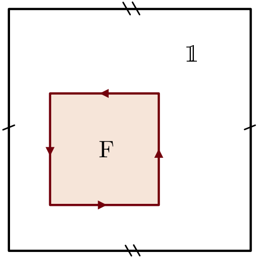
$$e^{-s_{\mathsf{YM}}(u_p)} = \sum_{\text{irreps } \alpha} d_\alpha \, \chi_\alpha(u_p) \, e^{-g^2 c_\alpha}$$

Migdal '75, Menotti-Onofri '81, Witten '91

Subdivision-invariant $\Longrightarrow$ continuum answers on coarse lattices

Wilson loop in pure YM

$$\langle W_{\mathrm{F}}(C)\rangle_{\mathrm{YM}} =$$

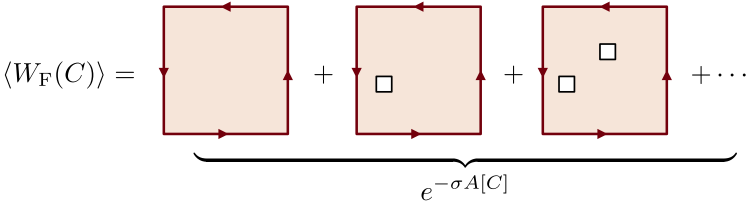


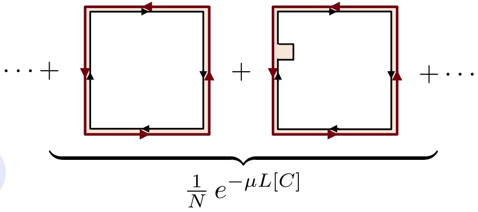
$$= \frac{1}{N}\left(d_{\mathrm{F}}\, e^{-g^2 c_{\mathrm{F}} A[C]}\right)\left(d_{\mathbb{1}}^{-1}\, e^{-g^2 c_{\mathbb{1}} (A - A[C])}\right)$$

$$= e^{-g^2 c_{\mathrm{F}} A[C]}$$

Despite absence of propagating gluons,
pure YM in 2d confines:
string tension $\sigma = \lambda/2$ at large N

Wilson loop and quark suppression





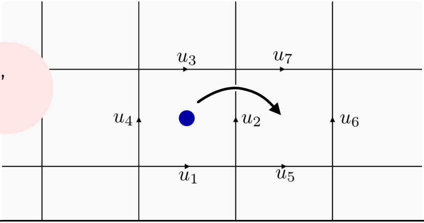
String tension $\sigma = \sigma_{\text{YM}} + O(1/N)$
$\Longrightarrow$ confining theory!

$$\langle W_{\mathrm{F}}(C) \rangle = e^{-\left(\frac{\lambda}{2} + \frac{1}{N}\left(\frac{\kappa}{m^2}\right)^4 2 \sinh(\lambda/2)\right) A[C]} + \frac{1}{N}\, e^{-\log(m^2/\kappa) L[C]}$$

Center-vortices

On the lattice, 1-form symmetry generators of YM
are realized as 'center-vortices' $U_k(\tilde{x})$:

On the plaquette p dual to $\tilde{x}$,
$s_{\mathsf{YM}}(u_p) \to s_{\mathsf{YM}}(e^{-\frac{2\pi i k}{N}} u_p)$



$$\cdots e^{-s_{\mathsf{YM}}(e^{-\frac{2\pi i k}{N}} u_1 u_2 u_3^\dagger u_4^\dagger)} \, e^{-s_{\mathsf{YM}}(u_5 u_6 u_7^\dagger u_2^\dagger)} \cdots$$

$$\langle U_k(\tilde{x}) \rangle_{\mathsf{YM}} = 1 \text{ on } \mathbb{R}^2$$

Center-vortices and quark non-suppression

$$\langle U_k(\tilde{x})\rangle = \frac{\langle\langle U_k(\tilde{x})\rangle\rangle}{Z} = \frac{\bullet + (A-1)\,\bullet\square + \boxdot + \cdots}{1 + A\,\square + \cdots}$$

$$\bullet = 1$$

$$\bullet\,\square = 2N\left(\frac{\kappa}{m^2}\right)^4 e^{-g^2 c_{\mathrm{F}}}$$

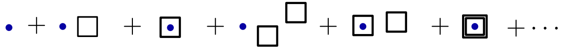
$$\boxdot = 2N\left(\frac{\kappa}{m^2}\right)^4 e^{-g^2 c_{\mathrm{F}}} \cos\left(\frac{2\pi k}{N}\right)$$

$$= 1 - \left(\frac{\kappa}{m^2}\right)^4 2N\left(1 - \cos\left(\frac{2\pi k}{N}\right)\right) + O(\kappa^6)$$

Already see that quark loops are not suppressed for $k \sim \sqrt{N}$

Center-vortices and quark non-suppression

Resum single plaquette contributions:



$\Longrightarrow$ Exponentiation: $\langle U_k(\tilde{x})\rangle = \exp\left[-\left(\frac{\kappa}{m^2}\right)^4 2N\left(1-\cos\left(\frac{2\pi k}{N}\right)\right) + O(\kappa^6)\right]$

$$\text{Large } N\text{: } \langle U_k(\tilde{x})\rangle = \begin{cases} e^{-\#\frac{1}{N}} \to 1 & \text{for } k \sim 1, \\ e^{-\#} \in [0,1] & \text{for } k \sim \sqrt{N}, \\ e^{-\#N} \to 0 & \text{for } k \sim N. \end{cases}$$

Smooth large N limit, but trivial ($k \sim 1$)

Smooth large N limit, but inconsistent with $\mathbb{Z}_N$ fusion ($k \sim \sqrt{N}$)

Not a smooth large N limit ($k \sim N$)

Quark loops are not suppressed for center-vortex operators

$\Longrightarrow$ no emergent $\mathbb{Z}_N$ 1-form symmetry

Outlook

Is there a symmetry principle that explains the selection rules in large N QCD?

If **yes**, it would be some further generalization. Color-flavor-center symmetry? Something genuinely new?

Our considerations also apply to

- emergent non-invertible symmetry in large N YM
- emergent $\mathbb{Z}_N$ 1-form symmetry from orientifold equivalence

Is this specific to large N?

- Yaffe '82: large N as classical dynamics—symmetry operators are not part of description!