Monopoles, scattering, generalized symmetries.

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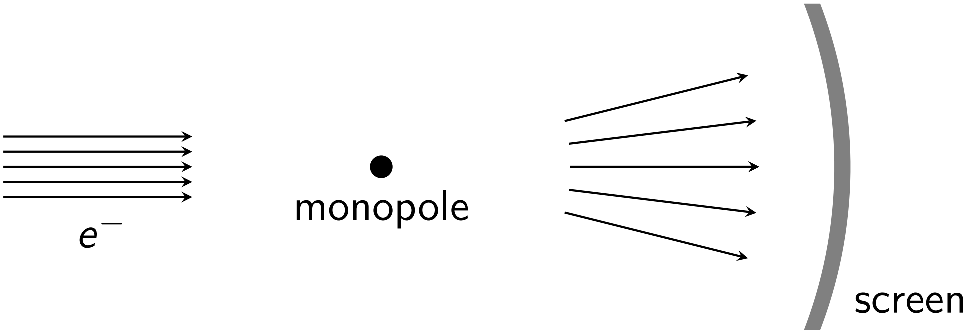
arXiv:2306.07318 with M. van Beest, P. Boyle Smith, Z. Komargodski, D. Tong.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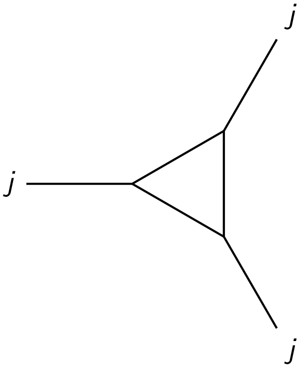
Background.

Background.

- Consider $4d$ fermions with a $U(1)$ flavor symmetry. Then,

$$\partial_\mu \langle j\, j\, j \rangle = p_\mu \Big(\quad \Big) \neq 0$$



- The conservation law for the current is anomalous. We call this an 't Hooft anomaly.

Background.

- More generally, we can have a symmetry G generated by j^μ. Then, $\partial_\mu j^\mu = 0$ if and only if
  - The symmetry G can be gauged.
  - The system admits G-preserving deformation to the empty theory.
  - One can latticize in a G-preserving way.
  - There exists at least one G-preserving boundary condition.
  - Etc.

Background.

- Even more information if we consider multiple symmetries at the same time.
- For example, if we have two symmetries $U(1)_1 \times U(1)_2$, we can consider mixed anomalies, where we put different currents j_1, j_2 at each vertex.
- If $\langle j_1 j_1 j_1 \rangle$ is non-anomalous, we can gauge this symmetry. The fate of $U(1)_2$ depends on mixed anomalies:
  - If $\langle j_1 j_2 j_2 \rangle$ is anomalous, the symmetry $U(1)_2$ becomes part of a 2-group. [Córdova-Dumitrescu-Intriligator]
  - If $\langle j_1 j_1 j_2 \rangle$ is anomalous, the symmetry $U(1)_2$ becomes non-invertible. [Choi-Lam-Shao, Córdova-Ohmori]

Background.

- A symmetry is in a 2-group if it mixes with a one-form symmetry. In $U(1)_1 \times U(1)_2$, it is the magnetic one-form symmetry.
  - "Sometimes conserved, sometimes not".
  - Constraints on UV completions: $U(1)_2$ is always emergent.
  - Constraints on IR dynamics: $T_{\mathrm{SF}} \geq T_{\mathrm{SC}}$.
- A symmetry is non-invertible if there is no U^{-1} such $UU^{-1} = 1$. In $U(1)_1 \times U(1)_2$, the symmetry acts invertibly on fermions but non-invertibly on monopoles.
  - We can cancel the anomaly by decorating $U(1)_2$ with a $3d$ TQFT (the Hall state, a.k.a. the boundary state of $\theta F \wedge F$).
  - The symmetry attaches a Wilson line to the monopole. Ultimately, just the Witten effect.

Monopole scattering.

Monopole scattering.

- Motivation is twofold: historical and modern.
- In the 80s people realized that monopoles induce proton decay (much larger rate than the BSM scale). [Callan, Rubakov]
- Interestingly, the scattering process has a very strange feature: the explicit computation indicates that a proton decays to half a positron plus half a pion!

$$e^{+}_{L} + M \to M + \tfrac{1}{2}e^{+}_{R} + \tfrac{1}{2}u_{1_R} + \tfrac{1}{2}u_{2_R} + \tfrac{1}{2}d_{3_L}$$ [Callan]

$$\mathrm{u}_{1\mathrm{R}} + \mathrm{M} \to \tfrac{1}{2}(\mathrm{u}_{1\mathrm{L}}\bar{\mathrm{u}}_{2\mathrm{R}}\bar{\mathrm{d}}_{3\mathrm{L}}\mathrm{e}^{+}_{\mathrm{L}}) + \mathrm{M},$$ [Peskin]

- What exactly did people mean by fractional particles?

Monopole scattering.

- The set of symmetries of gauge theories is much larger than people thought initially, newly discovered generalized symmetries are quite ubiquitous.
- Symmetries of QED:
  - 1940's: $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$.
  - 1960's [ABJ]: a mixed anomaly breaks $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ and $U(1)_A \to \varnothing$.
  - 2014 [GKSW]: there is a one-form symmetry $U(1)^{(1)}$ that acts on 't Hooft line operators.
  - 2018 [CDI]: $SU(N_f)_L \times SU(N_f)_R$ is not broken, instead it is part of a 2-group with $U(1)^{(1)}$.
  - 2022 [CLS-CO]: $U(1)_A$ is not broken either, but it is non-invertible and it acts on 't Hooft lines.
- All the original symmetries are resurrected, but they have a non-trivial interplay with magnetic charges.
- In regular scattering theory, with only electrically charged particles, the generalized symmetries behave like regular symmetries (usual Ward identities).
- In monopole scattering, these new features play an essential role, we need to understand them properly if we are to study the experiment we are after.

A simple toy model.

A simple toy model.

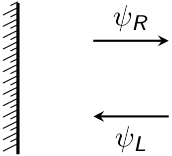
- Consider a bunch of free fermions in 1+1 dimensions. $2d$ fermions come in two chiralities, left-handed and right-handed; the Dirac equation for these reads

$$(\partial_t - \partial_x)\psi_L = 0 \qquad\qquad (\partial_t + \partial_x)\psi_R = 0$$

  Left-handed particles move to the left, and right-handed particles to the right.

- Take two left- and two right-movers, all complex. Then there is a $O(4)_L \times O(4)_R$ symmetry rotating them.

- Let us put the system on the half line, with some boundary condition at $x = 0$



A simple toy model.

- If we send ψ_L towards the boundary, it will bounce off and become some function of ψ_R

$$\psi_L \longrightarrow \mathcal{O}(\psi_R)$$

  whose details depend on the choice of boundary condition.

- Naive puzzle: such scattering processes seem incompatible with $O(4)_L \times O(4)_R$ conservation. The in-state is charged under $O(4)_L$ but not $O(4)_R$, and the other way around for the out-state. It is impossible to write an operator $\mathcal{O}(\psi_R)$ that has the same quantum numbers as ψ_L.

- The resolution is straightforward: the symmetry $O(4)_L \times O(4)_R$ has an 't Hooft anomaly, hence there are no symmetric boundary conditions. The boundary explicitly breaks this symmetry.

A simple toy model.

- A more subtle puzzle: consider the following subgroup of $O(4)_L \times O(4)_R$:

	ψ_1	ψ_2	$\tilde{\psi}_1$	$\tilde{\psi}_2$
$U(1)_1$	3	4	5	0
$U(1)_2$	4	-3	0	5

- This subgroup is anomaly-free:

$$\partial\langle j_1 j_1\rangle \propto 3^2 + 4^2 - 5^2 - 0^2 \equiv 0$$
$$\partial\langle j_1 j_2\rangle \propto 3 \times 4 + 4 \times (-3) - 5 \times 0 - 0 \times 5 \equiv 0$$
$$\partial\langle j_2 j_2\rangle \propto 4^2 + (-3)^2 - 0^2 - 5^2 \equiv 0$$

- As such, it does admit symmetric boundary conditions. It is possible to conserve this symmetry in scattering processes.

A simple toy model.

- Let us see if we can find the correct out-state,

$$\psi_1 \longrightarrow n_1\tilde{\psi}_1 + n_2\tilde{\psi}_2$$

- Conservation of $U(1)_1 \times U(2)_2$ implies the following constraints:

$$3 = 5n_1 + 0n_2$$
$$4 = 0n_1 + 5n_2$$

  with solution $n_1 = 3/5$, $n_2 = 4/5$.
- In other words, the scattering process apparently has a fractional out-state!

$$\psi_1 \longrightarrow \frac{3}{5}\tilde{\psi}_1 + \frac{4}{5}\tilde{\psi}_2$$

- There is no operator in the Fock space of ψ_R with charges 3 or 4,

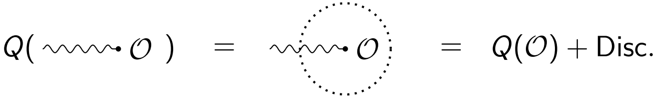
$$Q(\psi_R \cdots \psi_R) \propto 5$$

A simple toy model.

- Resolution: the spectrum of local excitations is much larger than the Fock space of ψ_L, ψ_R.
- In string theory, these additional states are known as the twisted sector, and their defining property is that they are multi-valued (their correlation functions have branch cuts).
- Impose twisted boundary conditions:

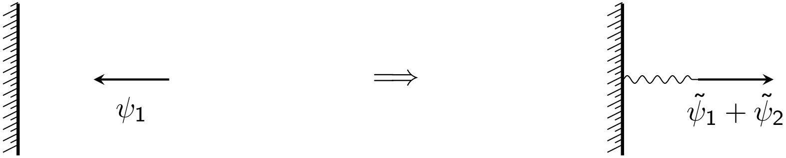
$$\tilde{\psi}_1(\sigma + 2\pi) = e^{2\pi i/10}\tilde{\psi}_1(\sigma)$$
$$\tilde{\psi}_2(\sigma + 2\pi) = e^{6\pi i/10}\tilde{\psi}_2(\sigma)$$

- The branch cut adds charge to the endpoint:



A simple toy model.

- One can show that there is a unique twist operator with the same charge as ψ_1, namely $\rightsquigarrow \tilde{\psi}_1 \tilde{\psi}_2$.
- The scattering process then looks like this:



- The S-matrix is somewhat non-standard: it turns regular (local) operators into twist fields.
- This is fine: twist fields behave, for the most part, like regular fields, the only difference are extra phases as we move them around each other.

A simple toy model.

Conclusions:

- Anomalous symmetries are explicitly broken by boundaries. They are not conserved in scattering experiments.
- Anomaly-free symmetries can be preserved by boundaries. If we choose symmetric boundary conditions, then the scattering process conserves charge.
- If the chiral fields carry different quantum numbers under the symmetry, the scattering process is subtle: usually, no operator in the out-going Fock space carries the same charges as the in-going states.
- Naive charge conservation seems to lead to fractional out-states. The correct interpretation is the appearance of branch cuts, which add charge to the endpoints.
- The twist fields are locally indistinguishable from regular fields, but they live in a different sector of the theory.

Monopole scattering.

Monopole scattering.

- Consider N_f Dirac fermions in $3+1d$. The symmetries are

$$(SU(N_f)_L \times SU(N_f)_R \times U(1)_A) \ltimes U(1)_m^{(1)}$$

- A Dirac fermion has two chiral components, e_L, e_R, whose charges under the gauge group and symmetry group are

	e_L	e_R
$U(1)_{\text{EM}}$	1	-1
$SU(N_f)_L$	$\square$	$\bullet$
$SU(N_f)_R$	$\bullet$	$\square$
$U(1)_A$	1	1

Monopole scattering.

- Let us take a heavy monopole and place it at the origin. We send a lepton, either e_L or e_R, and measure the outcome. The scattering process is

$$\psi + M \longrightarrow M + \mathcal{O}$$

where $\mathcal{O}$ is some operator with the same charges as ψ. Our task is to identify this operator.

- The Dirac equation reads

$$(i\partial\!\!\!/ + q_i A\!\!\!/)\psi_i = 0, \qquad A_\phi = \frac{m}{r}(1-\cos\theta)$$

where $m \in \mathbb{Z}$ is the magnetic charge of the monpole and q_i are the electric charges of the fermions.

- No gauge fluctuations around monopole. We could add those, no big changes.

Monopole scattering.

- If we were doing regular scattering theory we would look at plane waves of the form

$$\int e^{ipx} a_p \psi(x)$$

- The monopole breaks translation invariance. We only have rotations and energy conservation,

$$\int e^{iEt} Y_{j\mu}(\Omega) a_{Ej\mu} \psi(t, r, \Omega)$$

  where Y are the spherical harmonics. Angular momentum is bounded by $j \geq \frac{|qm|-1}{2}$.

- Key aspect of monopole scattering: the mode with the lowest angular momentum, $j = j_0$, satisfies

$$(\partial_t + \text{sign}(qm)\partial_r) \int_{S^2} \psi \equiv 0$$

  so this mode describes incoming radiation if $qm > 0$ and outgoing radiation if $qm < 0$.

Monopole scattering.

- The j_0 wave carries the following quantum numbers:

		$U(1)_{\mathrm{EM}}$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_A$
Incoming:	$\int_{S^2} e_L$	1	$\square$	$\bullet$	1
Outgoing:	$\int_{S^2} e_R$	-1	$\bullet$	$\square$	1

- Formally identical to our toy model: we have perturbations that move in a single direction, but they carry different quantum numbers. The monopole plays the role of the boundary.
- Here we face out first puzzle. The incoming wave is charged under $SU(N_f)_L$, but the outgoing one is not, so the out-state will never conserve $SU(N_f)_L \times SU(N_f)_R$!

Monopole scattering.

- Resolution: the symmetry $SU(N_f)_L \times SU(N_f)_R$ is in a 2-group with $U(1)^{(1)}$.
- This implies that the symmetry is not conserved in scattering processes involving magnetically charged matter. The monopole explicitly breaks this symmetry.
- Only the anomaly-free subgroup $SU(N_F)_V$ is conserved. So we should look at

	$U(1)_{\text{EM}}$	$SU(N_f)_V$	$U(1)_A$
Incoming: $\int_{S^2} e_L$	1	$\square$	1
Outgoing: $\int_{S^2} e_R$	-1	$\square$	1

- Hence, the first puzzle disappears: we do have candidate out-states that conserve $SU(N_f)_V$, for example

$$M + e_L \longrightarrow M + e_R$$

Monopole scattering.

- But this does not work either, because even though $SU(N_f)_V$ is conserved, electric charge is not: the in-state has charge $+1$ while the out-state has charge -1.

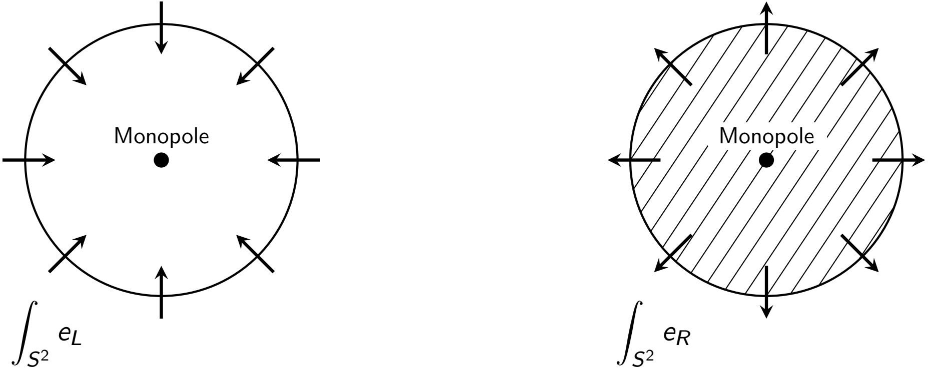
- We could fix this by writing instead

$$M + e_L \longrightarrow M + e_R^\dagger$$

but this does not preserve $SU(N_f)_V$ nor $U(1)_A$. It seems impossible to conserve all symmetries at the same time!

Monopole scattering.

- Exact same puzzle as in the toy model. People in the 80s proposed fractional out-states.
- Our claim: the out-state is a twist field. We propose an out-state of the form



Monopole scattering.

- In the interior of S^2 we place a topological defect that implements a rotation

$$e_L \mapsto e^{2\pi i/mN_f} e_L, \qquad e_R \mapsto e^{2\pi i/mN_f} e_R$$

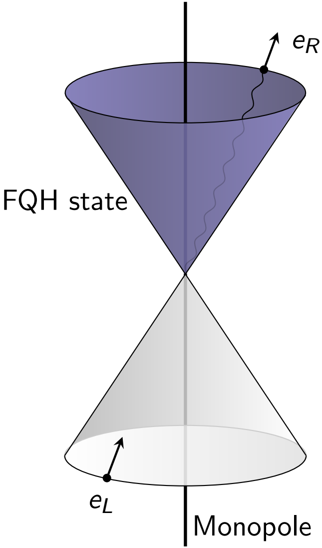
- The defect is just an axial rotation by an angle $1/mN_f$. This defect is generically non-invertible.

- In the paper we give two arguments for this: 1) we compute the charge carried by Wilson lines in the 3d Hall state TQFT and 2) we reduce on S^2 to yield a 2d problem essentially identical to the toy model from before.

Monopole scattering.

- The take-home-message is: monopole scattering requires the full machinery of generalized symmetries.
- Without these new symmetries there is an apparent paradox in which there is no possible out-state consistent with the conservation laws.
- If we take into account the full set of symmetries, a consistent answer does exist, albeit a rather non-trivial one: the S-matrix maps the regular Fock space into a twisted Fock space.
- In other words, incoming radiation formed by regular leptons becomes outgoing radiation formed by a field in a twisted sector, and there is a topological defect trailing it.
- This defect is non-invertible and hosts a $3d$ TQFT inside (the Hall state).

Monopole scattering.



Thanks!

Extra slides.

Non-invertible symmetry.

- The $U(1)_A$ symmetry had a mixed anomaly in $\langle j_{EM}\, j_{EM}\, j_A \rangle$, which gives rise to an anomalous conservation law
$$\mathrm{d}j_A = \frac{N_f}{2\pi}\mathrm{d}A \wedge \mathrm{d}A$$
- As j_A is not conserved, the associated charge is not topological
$$U_\theta = \exp\left(2\pi\theta \int j_A\right)$$
- Topological superconductors provide a fix. If $\theta \in \mathbb{Q}$, there exists a $3d$ TQFT (the Hall state),
$$L[a, A] := \frac{1}{4\pi} a^t K \mathrm{d}a - \frac{1}{2\pi} v a \wedge \mathrm{d}A$$
where a is a $3d$ $U(1)$ gauge field, K is a certain integral matrix and v an integral vector, such that $\theta = v^t K^{-1} v$.

Non-invertible symmetry.

- We can consider a defect

$$D_\theta := U_\theta \int [\mathrm{d}a] \exp\left(2\pi i \, 2N_f \int L[a, A]\right)$$

- The equations of motion for a are $Ka - vA = 0$, i.e., $a = K^{-1}vA$. Plugging this back into $L[a, A]$ yields

$$D_\theta \sim \exp\left(2\pi\theta \int \left(j_A - \frac{N_f}{2\pi} A \wedge \mathrm{d}A\right)\right)$$

- This is now topological since the current $j_A - \frac{N_f}{2\pi} A \wedge \mathrm{d}A$ is conserved.

Non-invertible symmetry.

- The TQFT commutes with local operators so D_θ acts on them the same way U_θ would, i.e., a regular axial rotation. But the TQFT does not commute with 't Hooft lines, so it acts on them in a non-trivial way.
- Consider a monopole singularity in A, $A_\phi \sim m/r$. This gives $\int_{S^2} \mathrm{d}A = 2\pi m$, and therefore

$$\int_{\mathbb{R}\times S^2} L[a, A] = \int_{\mathbb{R}\times S^2} \frac{1}{4\pi} a^t K \mathrm{d}a - mv \int_{\mathbb{R}} a$$

- The monopole in A gives rise to a Wilson line in a, with coefficient $2mN_f v$. Given that $a = K^{-1} vA$, this Wilson line couples to A with coefficient $2mN_f v K^{-1} v \equiv 2mN_f\theta$.
- This shows that the defect D_θ acts on an 't Hooft line as $H \mapsto HW$, where W is a Wilson line with charge $2mN_f\theta$.
- Given that the electric charge of e_L differs from the electric charge of e_R by two units, this Wilson line must carry the missing charge, and therefore $\theta = 1/mN_f$, as claimed.