NEW RESULTS FOR STRONGLY COUPLED FIELD THEORIES

Carleton University
Department of Physics

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work in progress.

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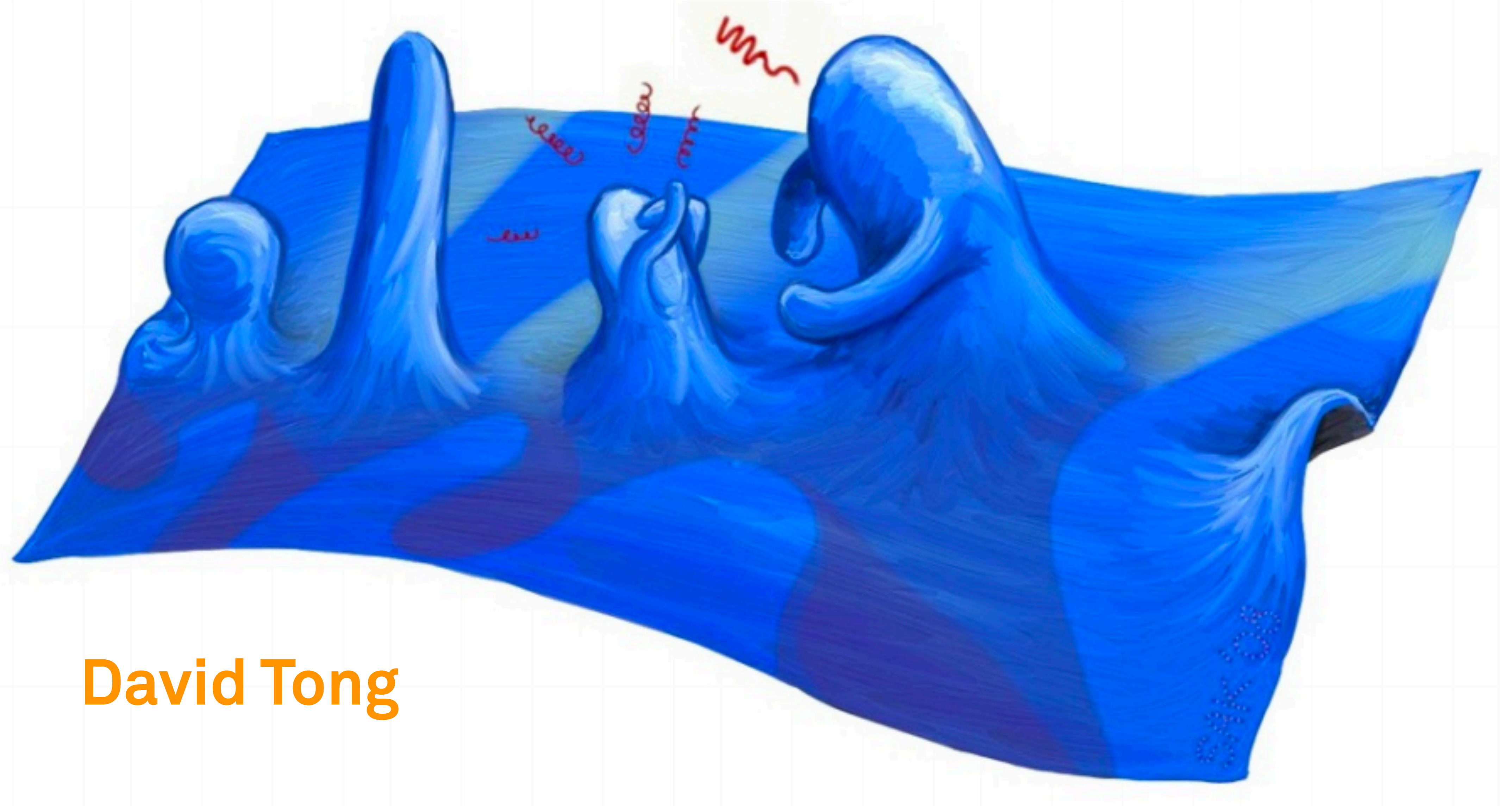
DANIEL STOLARSKI

University of Toronto Feb 7, 2025

QUANTUM FIELD THEORY

Quantum field theory works great!

Perturbation theory (Feynman diagrams) allow us to describe a huge range of phenomena.



QUANTUM FIELD THEORY

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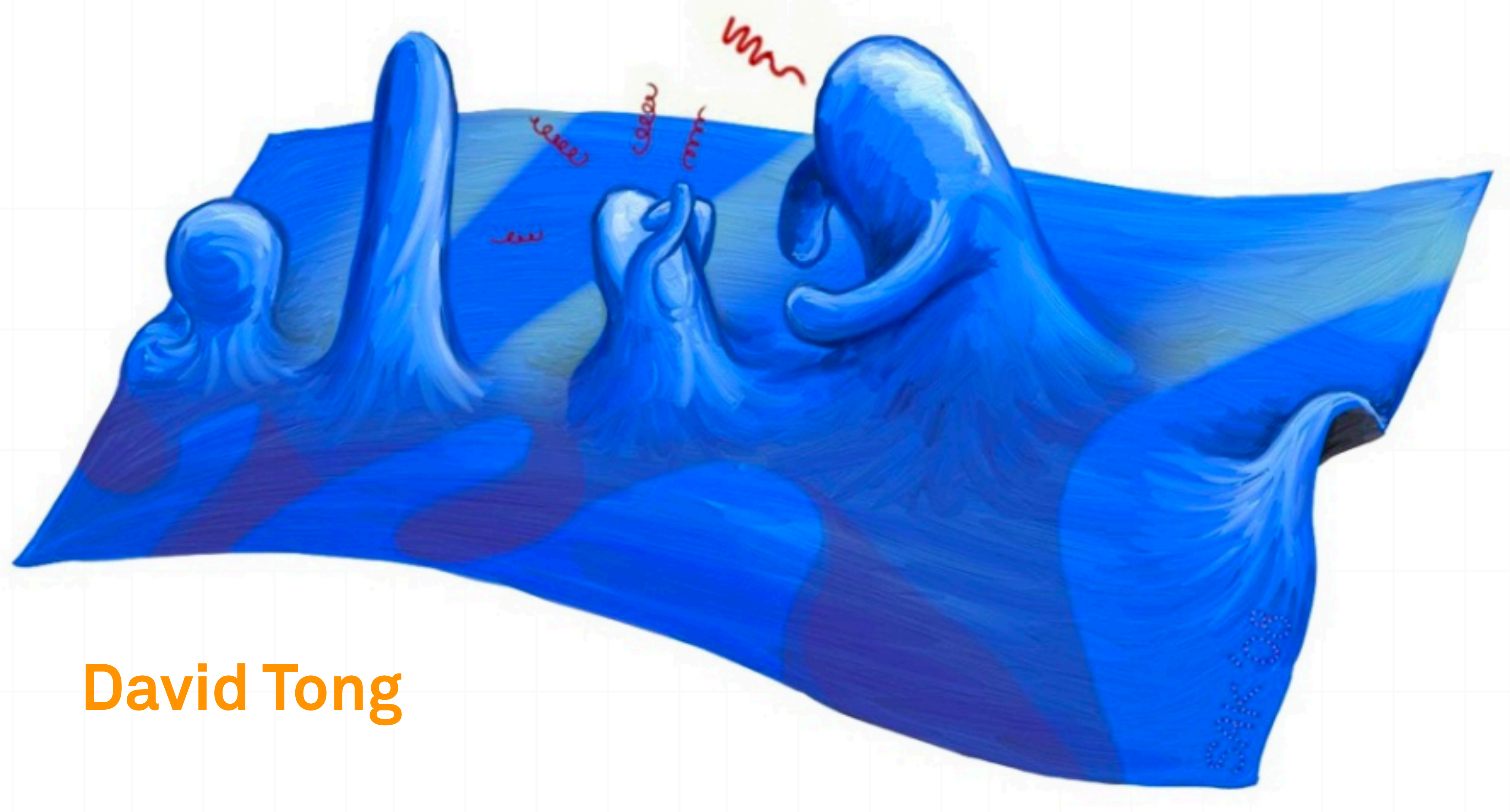
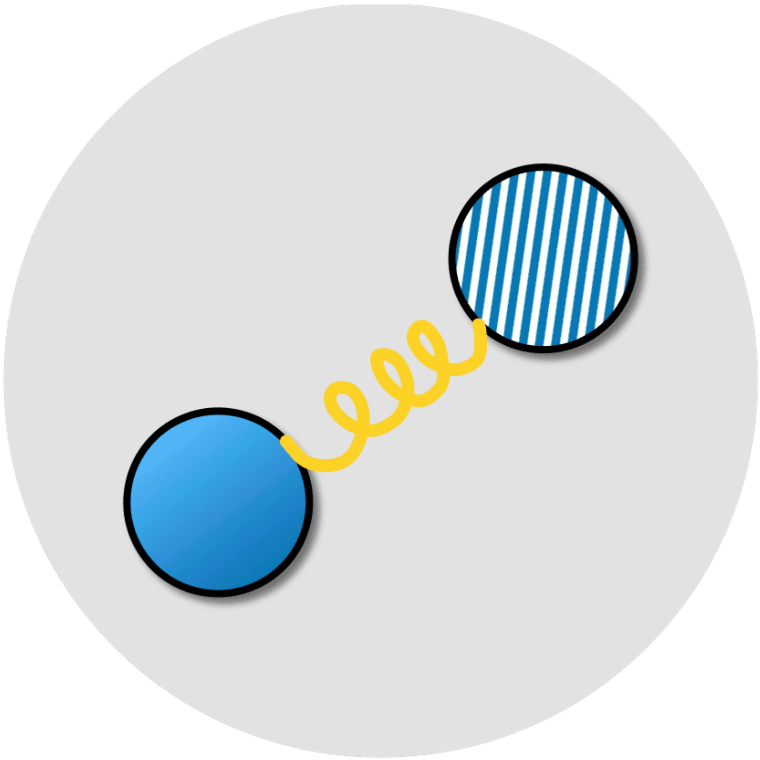
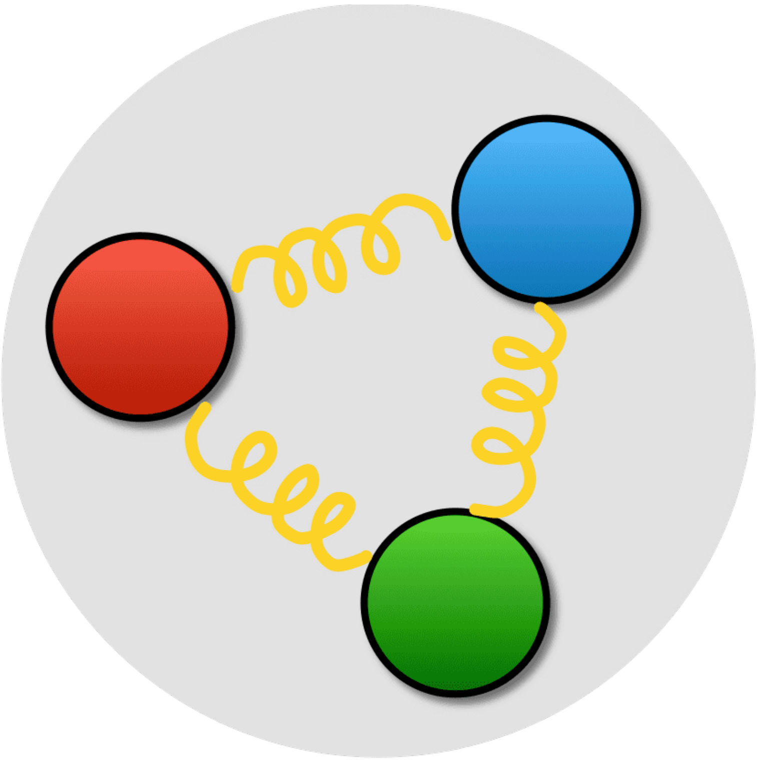
Perturbation theory (Feynman diagrams) allow us to describe a huge range of phenomena.

What about theories with $\alpha \sim 4\pi$? Cannot use perturbation theory.

QCD is such a theory.







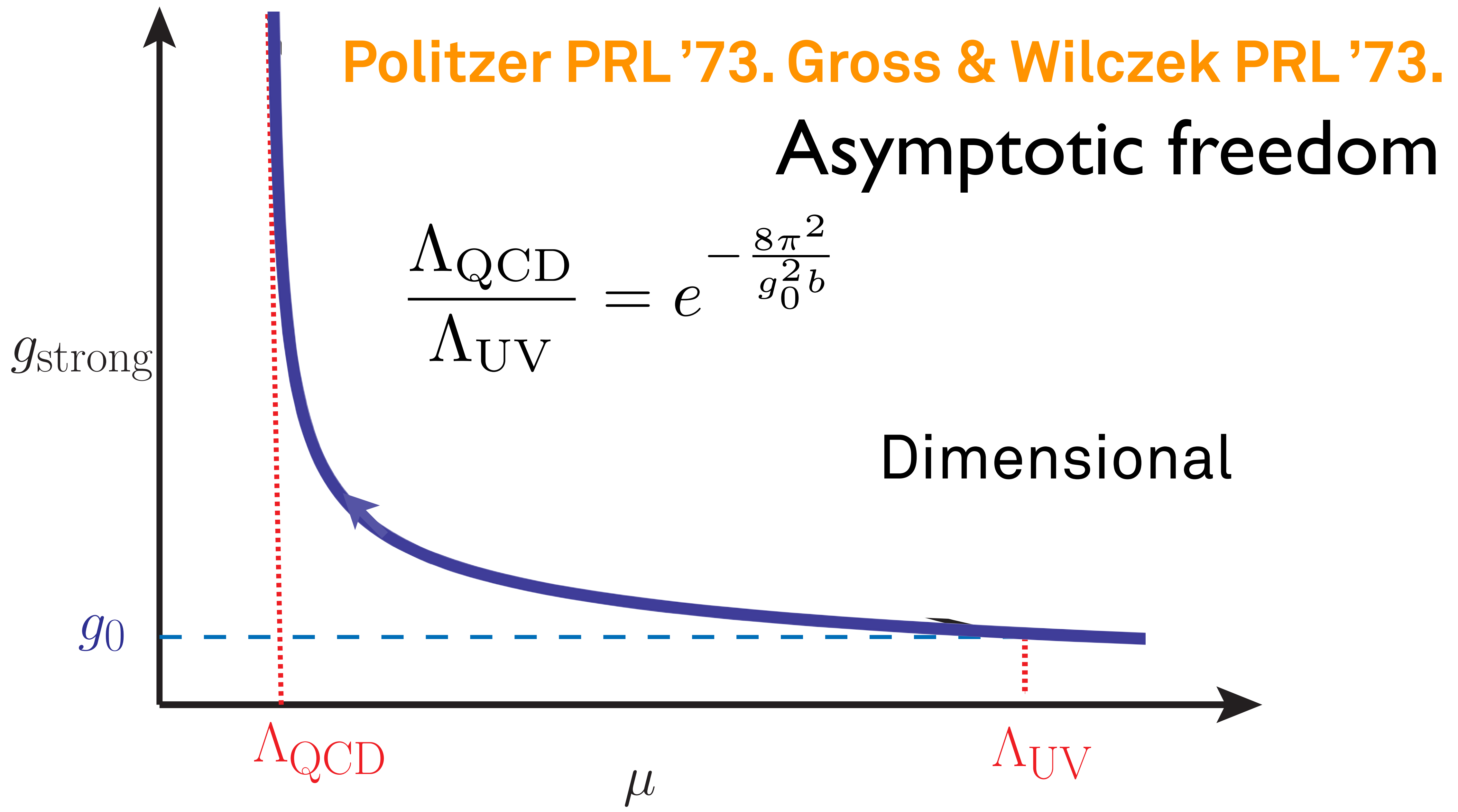
RUNNING COUPLINGS

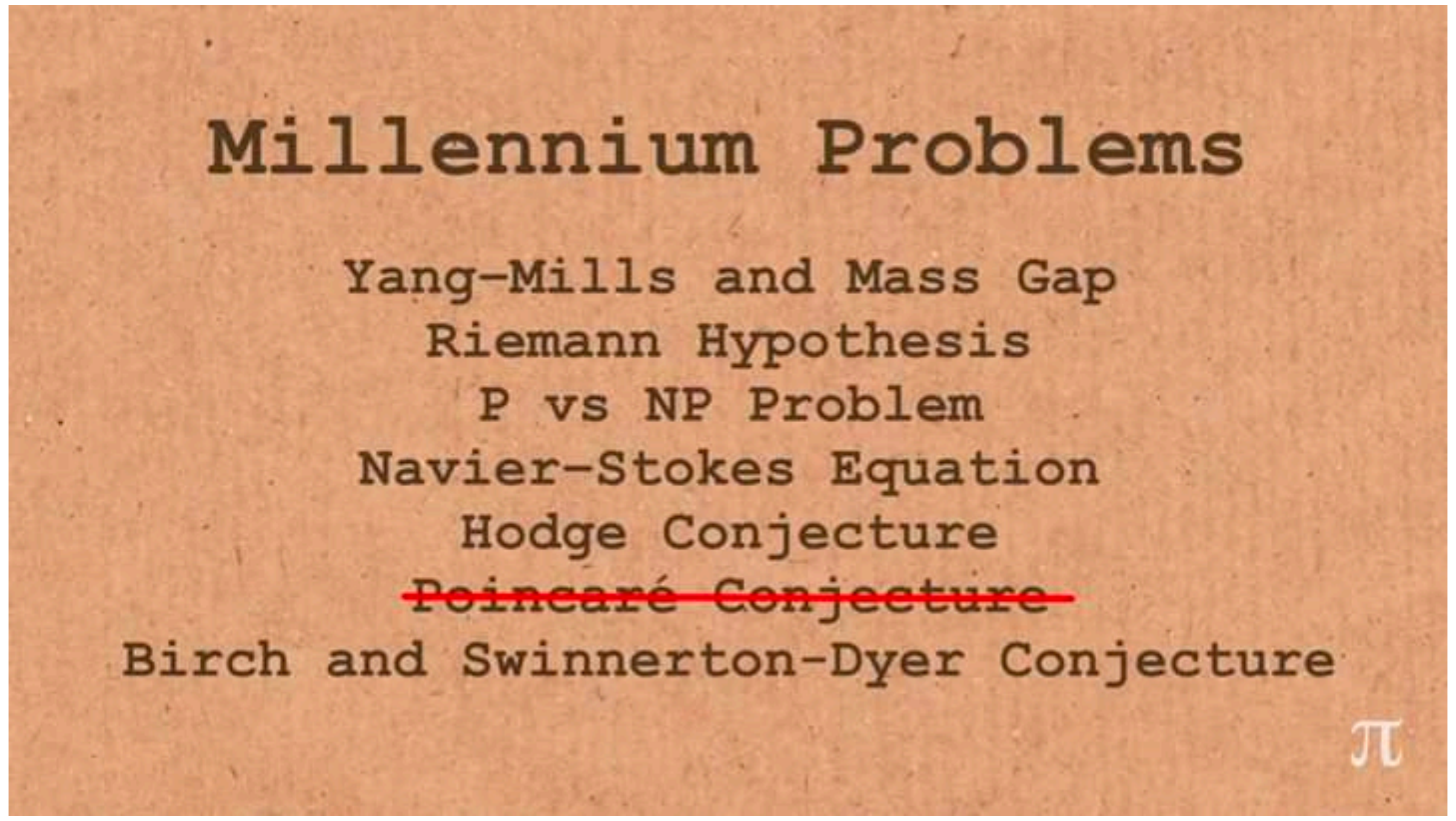
Strong coupling is quite generic in gauge theories.

Depends on sign of β_g .

For QCD we have data.

General case is an important open problem.





FIRST THEORY

What are the low energy dynamics of the following theory?

- SU(5) gauge theory
- 3 fermions in the $\bar{5}$ representation
- 3 fermions in the 10 representation

	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
A	10	3	1	1
$\overline{F}$	$\overline{5}$	1	3	-3

WHY SU(5)?

Fermion content of this theory I am working with has same fermion content as original SU(5) GUT.

Georgi, Glashow, PRL '74.

Theory has rich global symmetry.

Low energy theory of GUT if all scalars stabilized at the origin.

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Same global symmetry group as QCD!

CHIRAL THEORY

This is a chiral theory.

Very difficult to solve on the lattice.

No renormalizable operators consistent with gauge symmetry.

Only interaction is gauge interaction.

Gauge coupling is only parameter of the theory.

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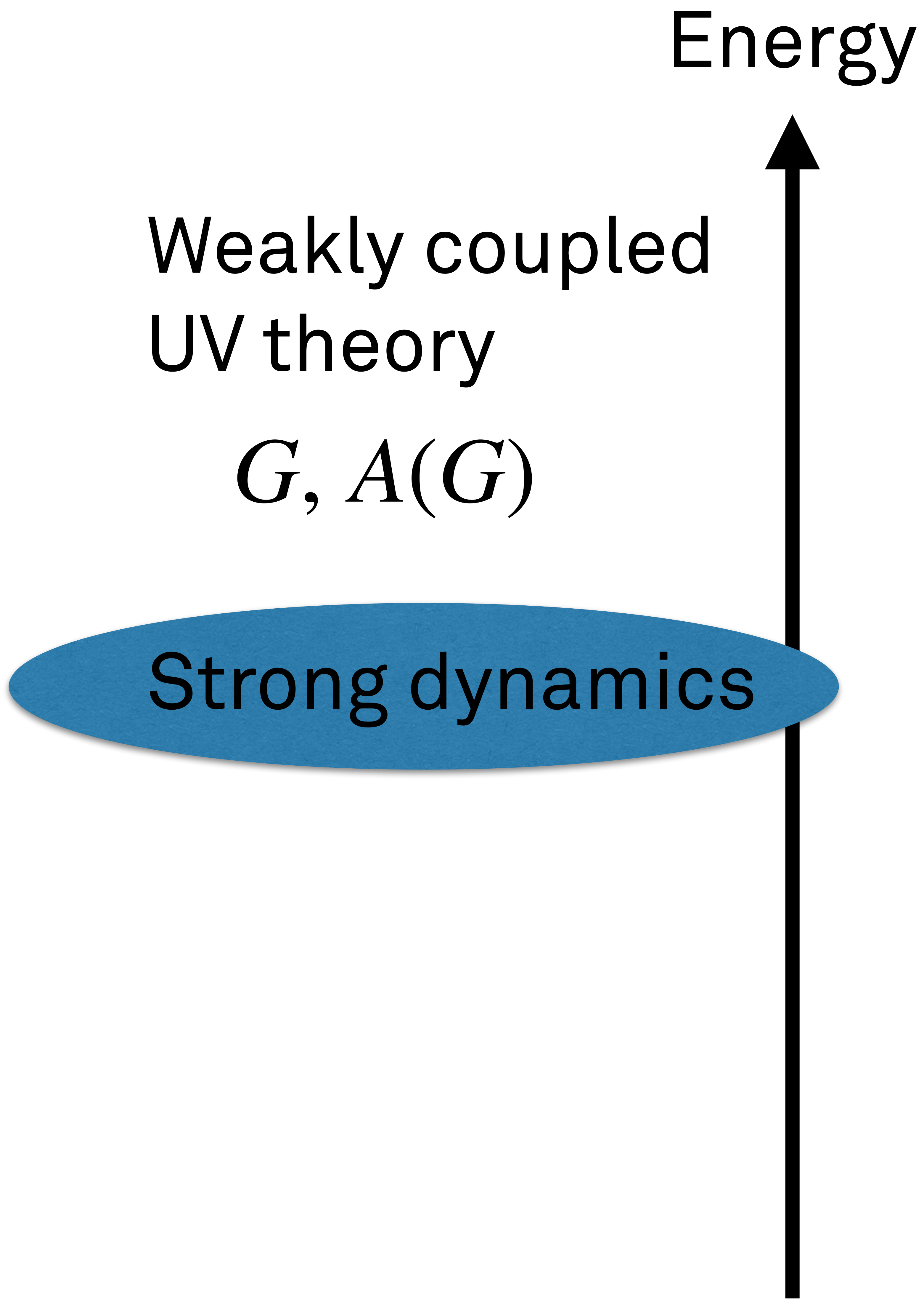
ANOMALIES

'T HOOFT ANOMALY MATCHING

Most robust tool to analyze strong interactions is 't Hooft anomaly matching. 't Hooft '80.



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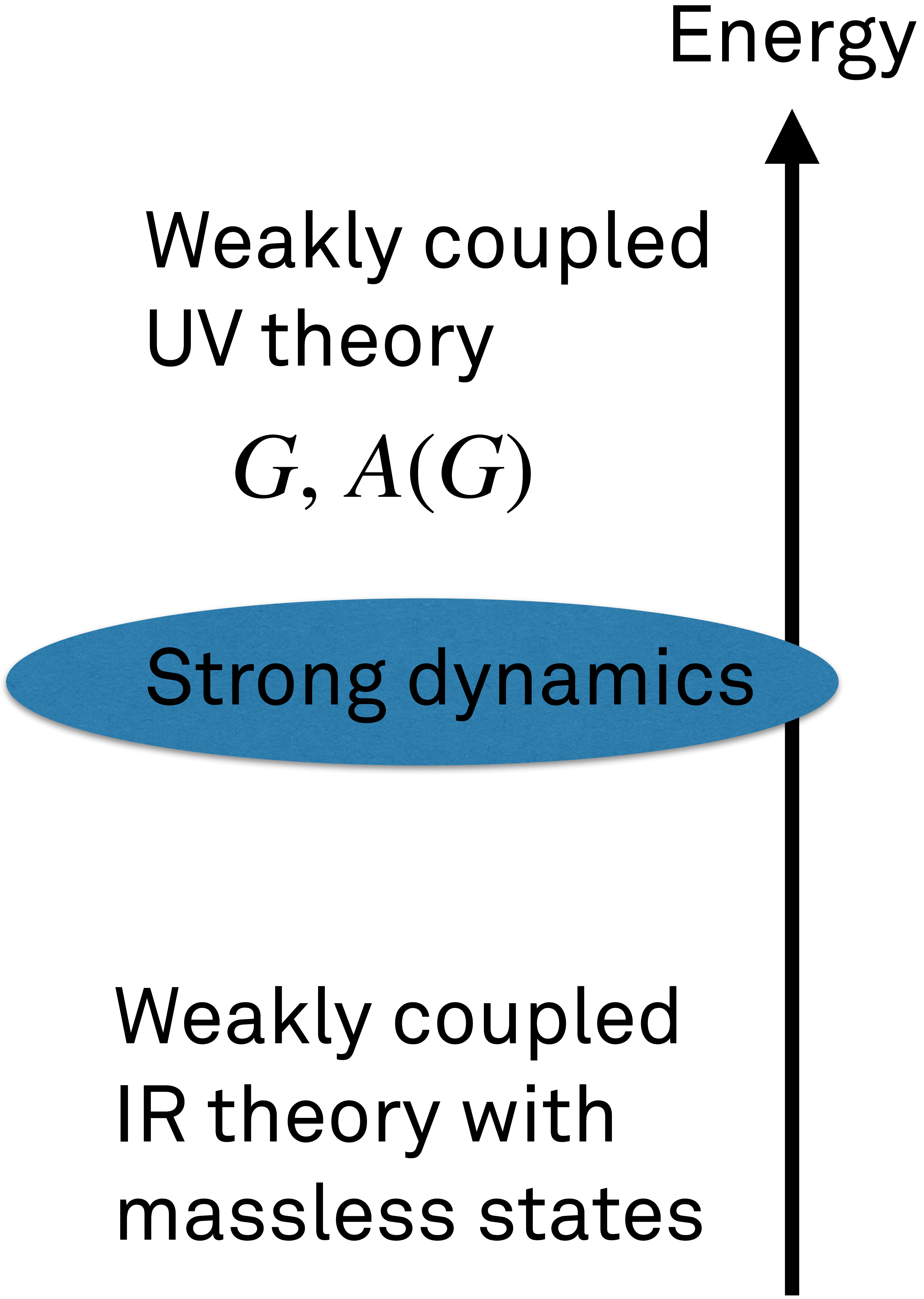

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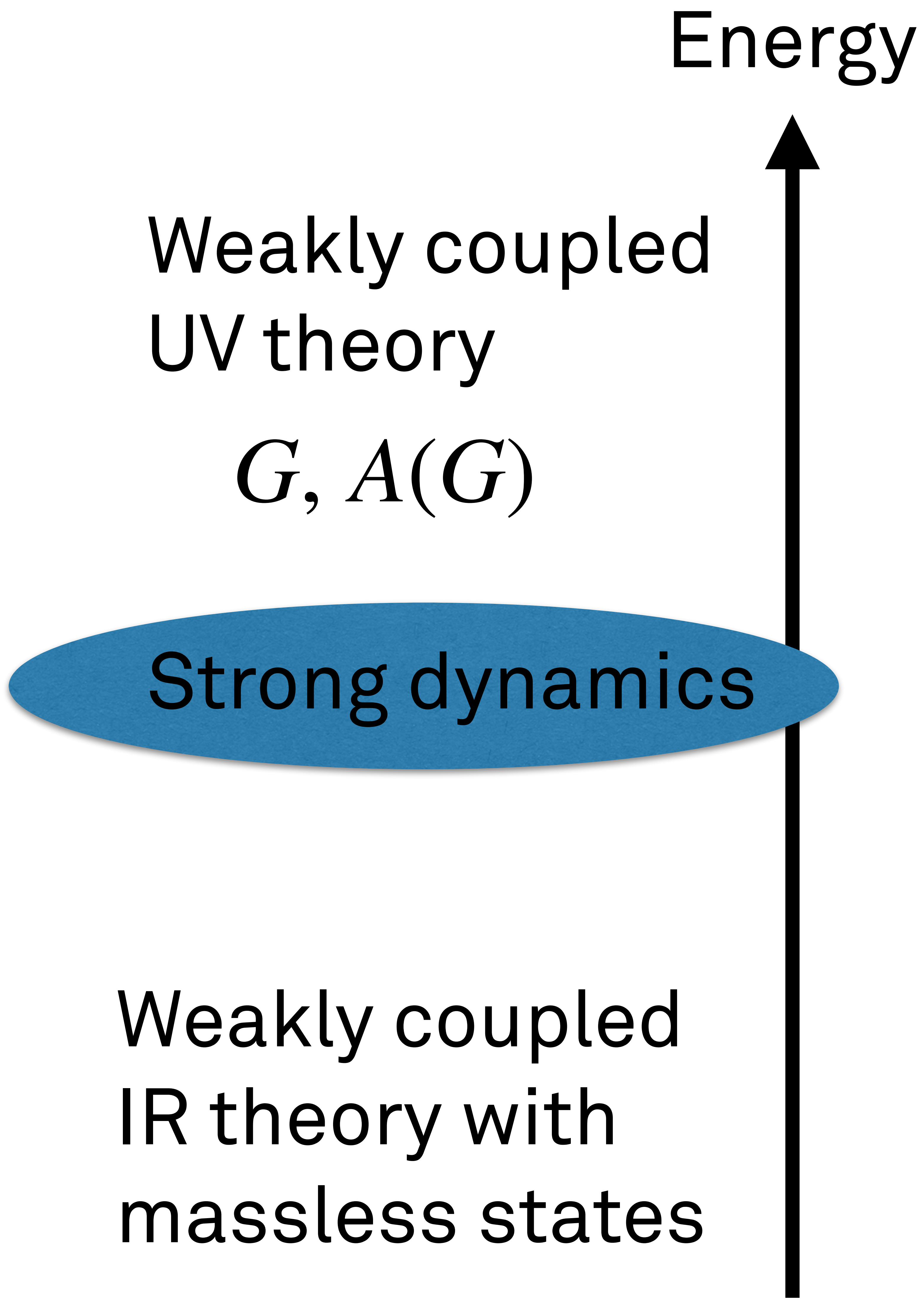
- Option 1: there are massless fermions to satisfy 't Hooft anomalies in the IR.
- Option 2: there are massless Goldstone bosons as a result of spontaneous symmetry breaking.



WARM UP: 1 GENERATION SU(5)

One generation theory:

	$[SU(5)]$	$U(1)_B$
A	10	1
$\overline{F}$	$\overline{5}$	-3

Dimopoulos, Raby, Susskind, NPB '80.
Seiberg, Stings 2019.
Csaki, Murayama, Telem, arXiv:2104.10171.

UV 't Hooft anomaly for $U(1) \times$ grav^2:

$$10 \cdot 1 + 5 \cdot (-3) = -5$$

UV 't Hooft anomaly for $U(1)^3$:

$$10 \cdot 1^3 + 5 \cdot (-3)^3 = -125$$

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$A\bar{F}\bar{F}$ bound state with B=-5.

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IR 't Hooft anomaly for $U(1) \times$ grav^2:

$$-5$$

IR 't Hooft anomaly for $U(1)^3$:

$$(-5)^3 = -125$$

LESSON FROM 1G THEORY

Find gauge invariant fermionic composite that satisfies ’t Hooft anomalies.

Example of “s-confinement”: theory confines but does not break any global symmetries. Seiberg, hep-th/9402044, hep-th/9411149. Csaki, Schmaltz, Skiba, hep-th/9610139.

Not a proof that this is the low energy theory:

- Could be other solutions to ’t Hooft anomaly matching.
- Could be spontaneous symmetry breaking.

ANOMALY MATCHING 3G THEORY

Six non-trivial 't Hooft anomalies:

	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
A	10	3	1	1
$\overline{F}$	$\overline{5}$	1	3	-3

$$A\left[SU(3)_A^3\right] = 10$$

$$A\left[SU(3)_{\bar{F}}^3\right] = 5$$

$$A\left[\text{grav}^2 \times U(1)_B\right] = -15$$

$$A\left[SU(3)_A^2 \times U(1)_B\right] = 10$$

$$A\left[U(1)_B^3\right] = -375$$

$$A\left[SU(3)_F^2 \times U(1)_B\right] = -15$$

IR ANOMALY MATCHING

Need to build gauge invariant fermionic bound states:

$$A\,\overline{F}\,\overline{F}$$

$$\overline{F}^5$$

$$A^4\overline{F}^3$$

$$A^5$$

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Need to build gauge invariant fermionic bound states:

$A\,\overline{F}\,\overline{F}$ $\qquad$ $A\,A\,\overline{F}^\dagger$

$\overline{F}^5$ $\qquad$ $A^3\overline{F}^{\dagger 4}$

$A^4\overline{F}^3$

A^5

NB: No SUSY (yet).

	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
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Need to solve 6 linear equations over integers.

There are infinitely many solutions.

IR ANOMALY MATCHING

Solutions are quite complicated.

A relatively simple example:

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	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
$(A\overline{F}\,\overline{F})^\dagger$	1	$\overline{3}$	3	5
$A\overline{F}\,\overline{F}$	1	3	6	-5
A^5	1	6	1	5
$\overline{F}^5$	1	1	$\overline{15}$	-15
$A^3\overline{F}^{\dagger 4}$	1	1	6	15
$A^3\overline{F}^{\dagger 4}$	1	1	$\overline{15}$	15
$2\times(A^3\overline{F}^{\dagger 4})^\dagger$	1	1	3	-15

IR ANOMALY MATCHING

Another example:

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	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
$(\overline{F}^5)^\dagger$	1	1	3	15
A^5 or $(A^4\overline{F}^3)^\dagger$	1	6	1	5
$\overline{F}(A^2)^\dagger$	1	3	3	-5
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Looks simpler, but $\left(\bar{F}^5\right)^\dagger$ state is problematic.

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Flavour = anti-symmetric

Spin = ?

Not enough states to satisfy Fermi statistics.

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Can work with orbital angular momentum, but weird.

CONCLUSIONS?

Is one of these the IR spectrum of the theory?

Probably not, but I cannot rule it out.

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Probably not, but I cannot rule it out.

Other alternative is to consider spontaneous symmetry breaking…
so many options.

SUPERSYMMETRY

SPHERICAL COW

Supersymmetric theories are the spherical cows of quantum field theory.

SUSY version of our model can be solved.

Deform away from SUSY.

Keenan Crane, Wikipedia.

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SUSY GG MODEL

Additional gaugino and additional global symmetry.

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W^α	24	1	1	0	1

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Holomorphy + extra symmetry means solution is unique.

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Dynamical superpotential is generated at low energy.

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$$W_{\rm dyn} = \lambda\, M^3 + \zeta\, B_2\, M\, B_1$$

Csaki, Schmaltz, Skiba, hep-th/9610139.

ANOMALY MATCHING

First question: why didn't we find this solution to the anomaly equations before?

$M = A^3\bar{F}$ so it is a boson, does not contribute to anomalies.

Mesino with at least one squark needed to satisfy anomalies.

	$SU(3)_A$	$SU(3)_F$	$U(1)_B$	$U(1)_R$
M	8	3	0	$\frac{2}{3}$
B_1	3	$\overline{3}$	-5	$\frac{4}{3}$
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Cannot get fermionic state with B=0 in non-SUSY theory.

WHAT IS THE VACUUM?

Assume vacuum doesn't break B:

$\langle B_1 \rangle = \langle B_2 \rangle = 0.$

Assume SUSY breaking is soft.

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$$W_{\rm dyn} = \lambda M^3 + \zeta B_2 M B_1$$

$$V \sim m^2 |M|^2 + \left(\kappa M^3 + \text{h.c}\right) + |\lambda|^2 |M|^4$$

$|m^2|, |\kappa|^2 \ll \Lambda^2_{\rm dyn}$ ensures "small" SUSY breaking.

SUM OF SQUARES

$$V = m^2|M|^2 + \left(\kappa M^3 + \text{h.c.}\right) + |\lambda|^2|M|^4$$

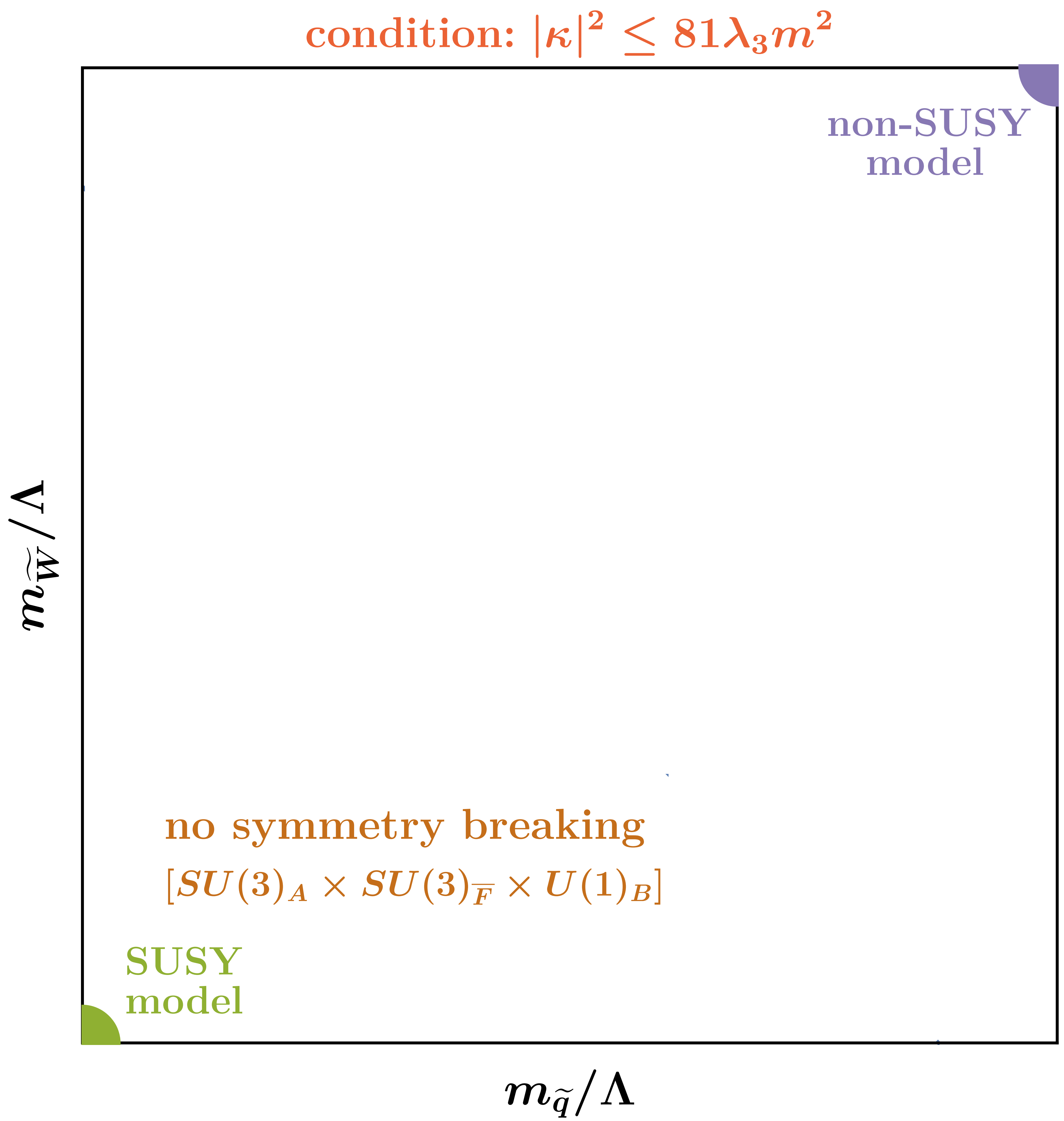
$$= |\lambda|^2 \,\text{tr}\left[\left(M^2 + \frac{\kappa^*}{|\lambda|^2}M^*\right)\left((M^2)^\dagger + \frac{\kappa}{|\lambda|^2}M^T\right)\right]$$

$$+ \left(m^2 - \frac{|\kappa|^2}{|\lambda|^2}\right)\text{tr}\left[MM^\dagger\right]$$

Rewrite the potential as the sum of squares.

$|\lambda|^2 m^2 \geq |\kappa|^2$ is a sufficient condition for $\langle M \rangle = 0$.

PHASE DIAGRAM



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If condition is satisfied, no symmetry breaking in SUSY model.

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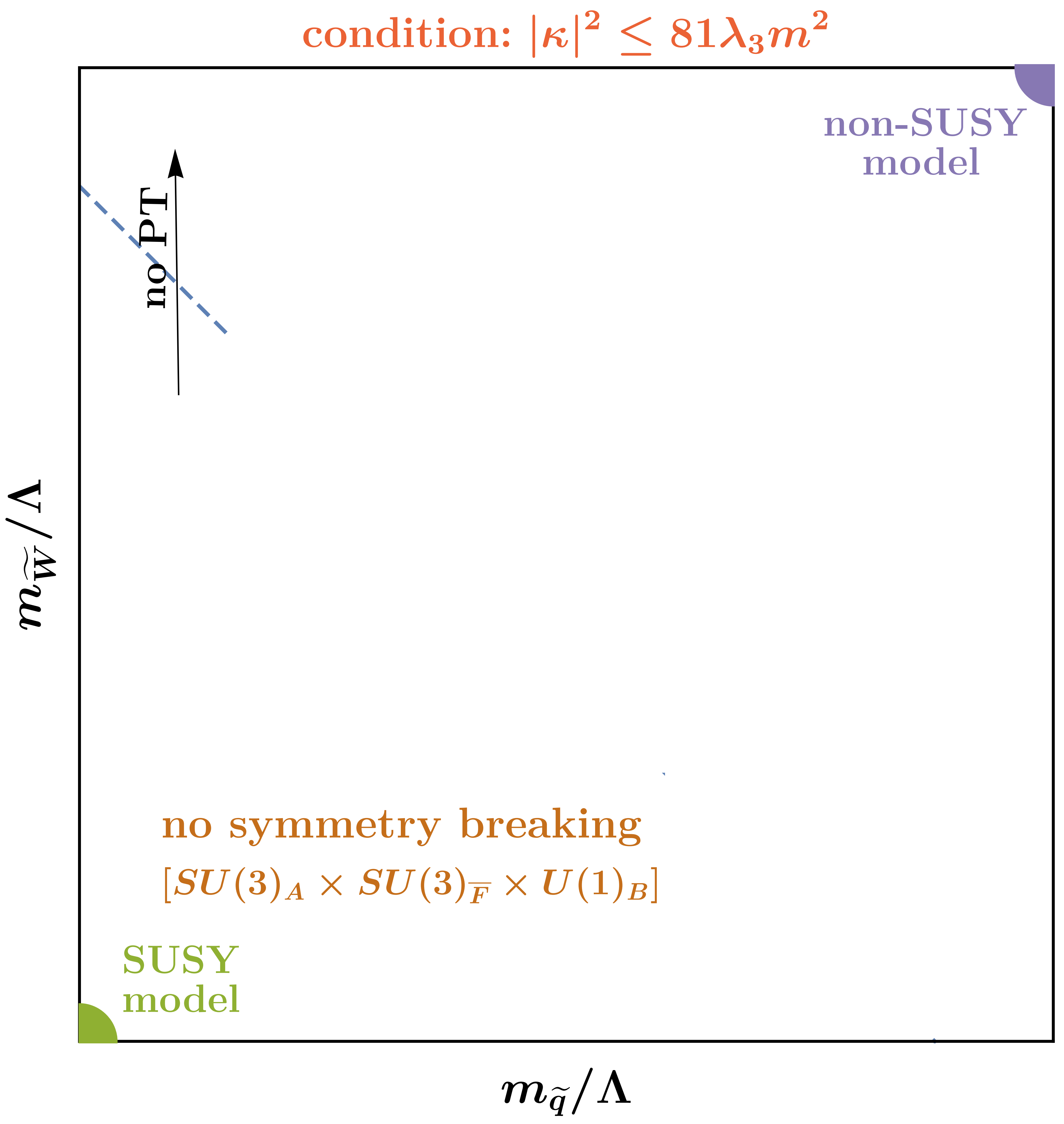

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If I take the gaugino mass to be large, anomalies can still match, no phase transition is expected.

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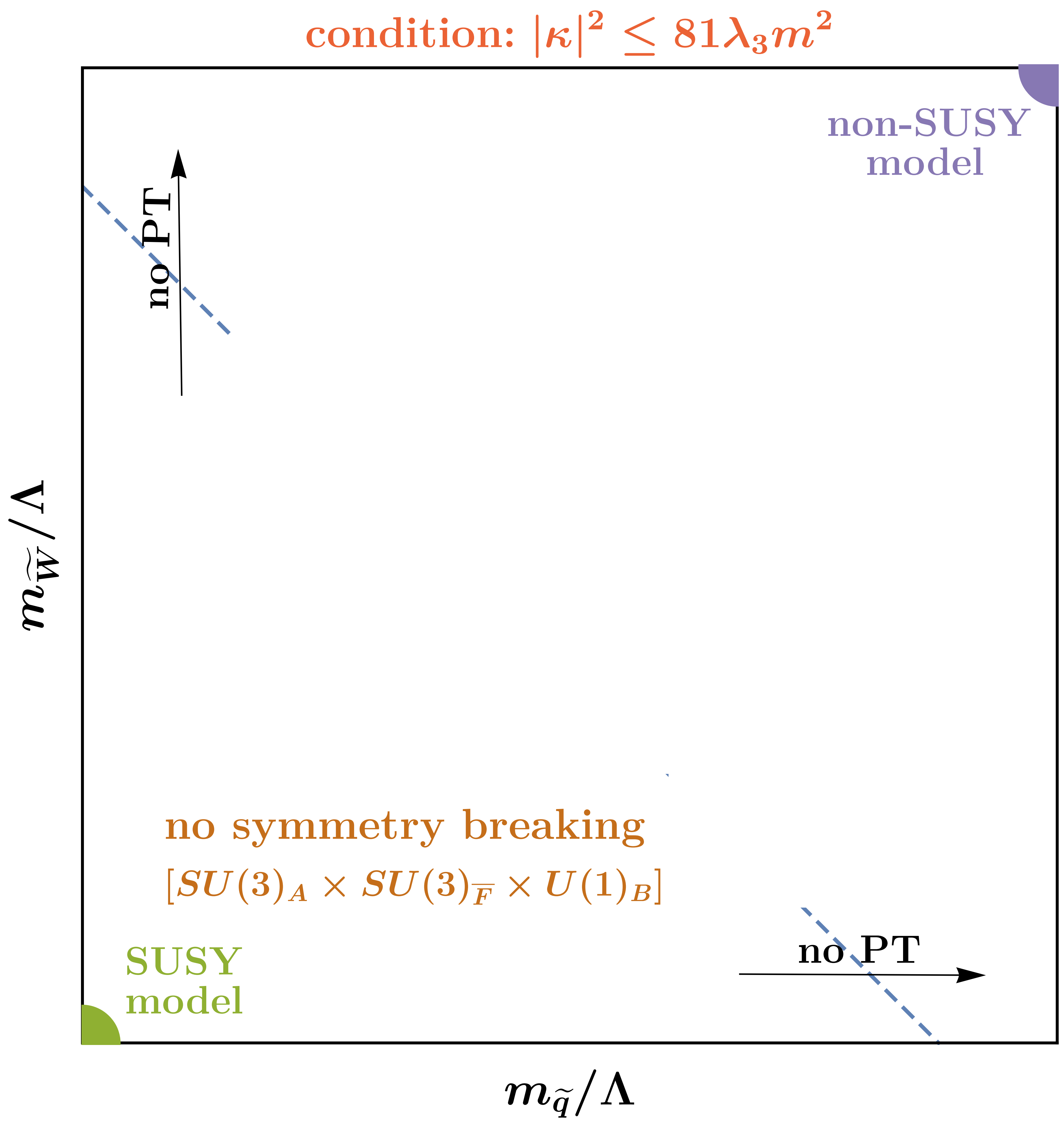

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If I keep the gaugino mass small but take squark masses to be large:

Replace mesino with $(A\widetilde{W})^3\bar{F}$, satisfy anomalies.

See also Dimopoulos & Preskill, NPB 1982.

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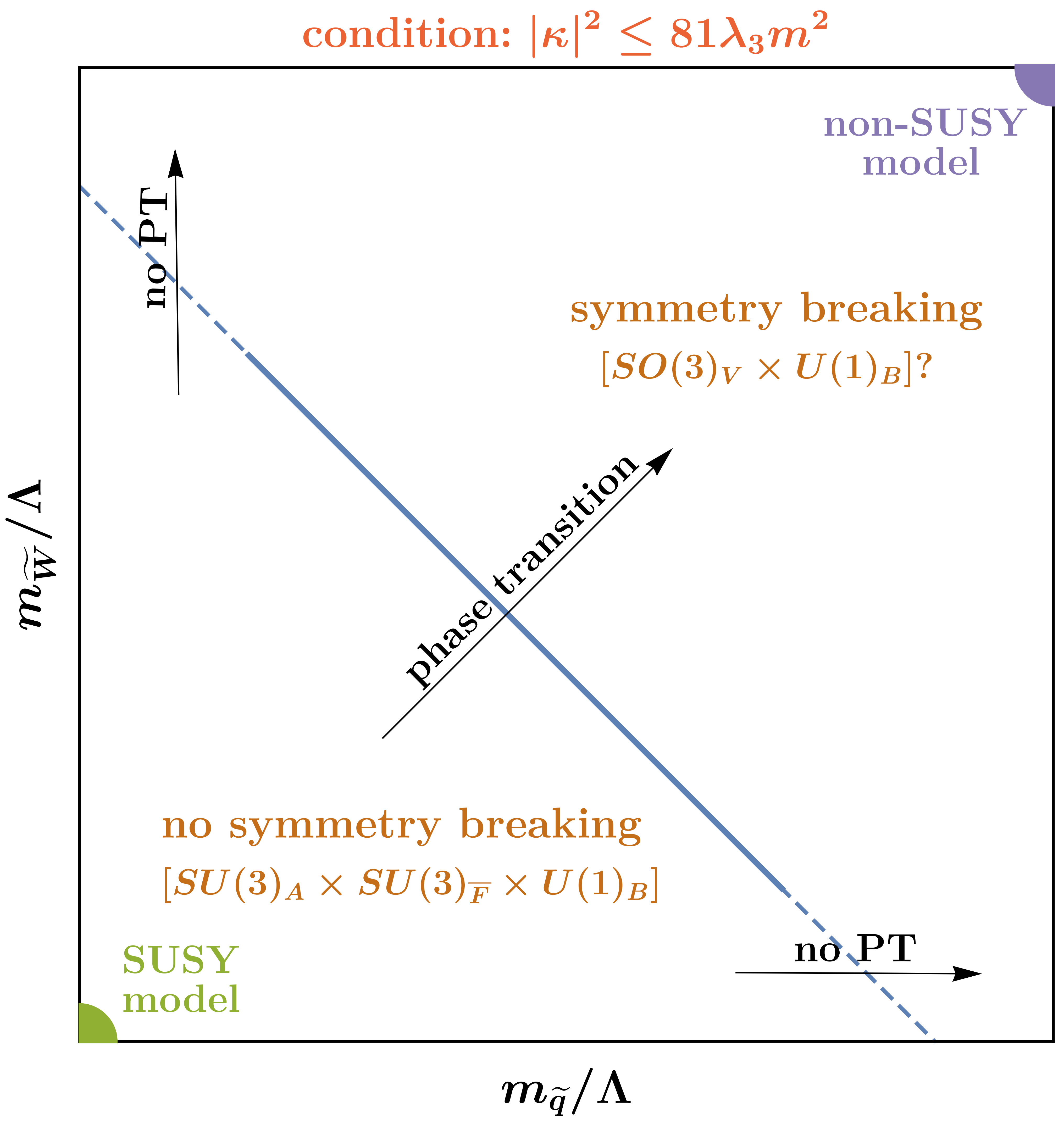

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If I take both SUSY breaking parameters to be large, cannot satisfy anomalies in same way.

Must have a phase transition.

SYMMETRY BREAKING

Let's now assume that condition is not satisfied, symmetry breaking will happen even for small SUSY breaking.

Which direction is symmetry broken?

Not enough symmetries to diagonalize M.

Let's guess.

$$M^{ai} = \left(\quad \right)$$

3

8

SUSY - I

$$\langle M^{ai} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f \\ 0 & 0 & 0 \end{pmatrix}$$

One option:

$$SU(3)_A \times SU(3)_{\bar{F}} \times U(1)_B \to SO(3)_V \times U(1)_B$$

Can compute fermion spectrum from superpotential:

Massless fermion: $(3)_{-5} \subset B_1 = A\,\bar{F}\,\bar{F}$

Massless bosons: $(5)_0 + (5)_0 + (3)_0 \subset M$

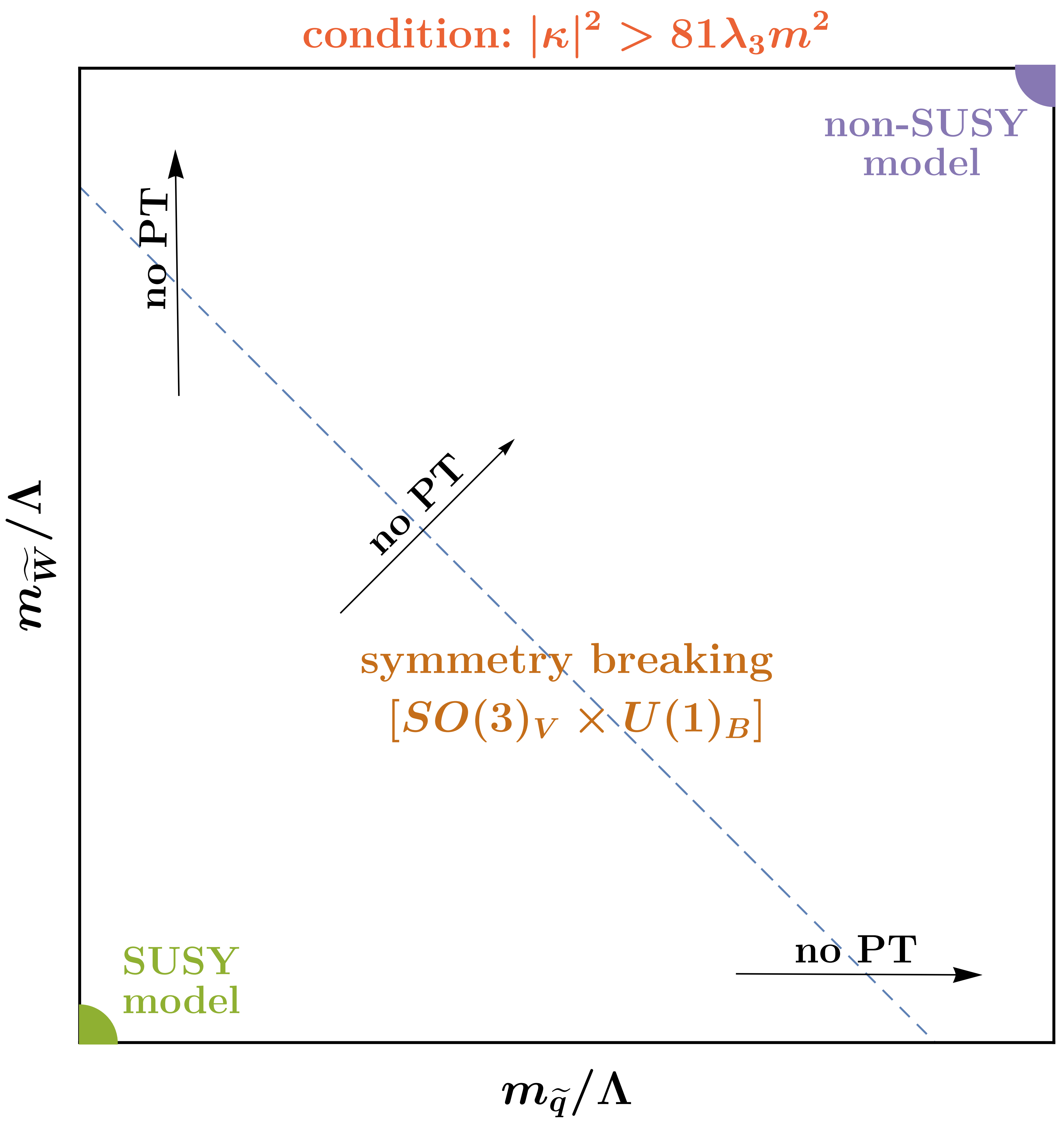
SUSY – I EVIDENCE

We think SUSY I the minimum for small SUSY breaking. The evidence:

- Spectrum satisfies ’t Hooft anomaly constraints.
- 3 generation analogue of 1 generation answer.
- Appears to be global minimum of potential as long as there is any symmetry breaking. Tried many other possibilities, did not find lower one.

More in the paper.

PHASE DIAGRAM



	$[SU(5)]$	$SU(3)_A$	$SU(3)_F$	$U(1)_B$	$U(1)_R$
A	10	3	1	1	0
$\overline{F}$	$\overline{5}$	1	3	-3	$\frac{2}{3}$
W^α	24	1	1	0	1
$M \equiv A^3\,\overline{F}$		8	3	0	$\frac{2}{3}$
$B_1 \equiv A\,\overline{F}\,\overline{F}$		3	$\overline{3}$	-5	$\frac{4}{3}$
$B_2 \equiv A^5$		6	1	5	0

With SUSY I, only B_1 states are massless.

Can be anywhere in the plane.

ANOMALY MEDIATION

ANOMALY MEDIATED SUSY BREAKING

Simplest method of breaking SUSY is anomaly mediation (AMSB).

Randall, Sundrum, hep-th/9810155.

Giudice, Luty, Murayama, Rattazzi, hep-ph/9810442.

All soft parameters dictated by one parameter: $m_{3/2}$.

$$V_{\text{tree}} = m_{3/2}\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right)$$

Soft parameters are RG invariant and UV insensitive.

Pomarol, Rattazzi, hep-ph/9903048.

Boyda, Murayama, Pierce, hep-ph/0107255.

AMSB + STRONG COUPLING

arXiv > hep-th > arXiv:2104.01179

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High Energy Physics – Theory

[Submitted on 2 Apr 2021 (v1), last revised 19 Jun 2021 (this version, v3)]

Some Exact Results in QCD-like Theories

Hitoshi Murayama

I propose a controlled approximation to QCD-like theories with massless quarks by employing supersymmetric QCD perturbed by anomaly-mediated supersymmetry breaking. They have identical massless particle contents. Thanks to the ultraviolet-insensitivity of anomaly mediation, dynamics can be worked out exactly when $m \ll \Lambda$, where m is the size of supersymmetry breaking and Λ the dynamical scale of the gauge theory. I demonstrate that chiral symmetry is dynamically broken for $N_f \leq \frac{3}{2}N_c$ while the theories lead to non-trivial infrared fixed points for larger number of flavors. While there may be a phase transition as m is increased beyond Λ, qualitative agreements with expectations in QCD are encouraging and suggest that two limits $m \ll \Lambda$ and $m \gg \Lambda$ may be in the same universality class.

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arXiv > hep-th > arXiv:2104.01179

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High Energy Physics – Theory

[Submitted on 20 Apr 2021 (v1), last revised 13 May 2021 (this version, v2)]

Some Exact Results in Chiral Gauge Theories

Csaba Csáki, Hitoshi Murayama, Ofri Telem

AMSB + STRONG COUPLING

arXiv > hep-th > arXiv:2104.01179

arXiv > hep-th > arXiv:2104.10171

arXiv > hep-th > arXiv:2105.03444

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High Energy Physics – Theory

[Submitted on 7 May 2021]

More Exact Results on Chiral Gauge Theories: the Case of the Symmetric Tensor

Csaba Csáki, Hitoshi Murayama, Ofri Telem

AMSB + STRONG COUPLING

arXiv > hep-th > arXiv:2104.01179

arXiv > hep-th > arXiv:2104.10171

arXiv > hep-th > arXiv:2105.03444

arXiv > hep-th > arXiv:2106.10288

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High Energy Physics – Theory

[Submitted on 18 Jun 2021 (v1), last revised 9 Sep 2021 (this version, v2)]

Demonstration of Confinement and Chiral Symmetry Breaking in $SO(N_c)$ Gauge Theories

Csaba Csáki, Andrew Gomes, Hitoshi Murayama, Ofri Telem

AMSB + STRONG COUPLING

arXiv > hep-th > arXiv:2104.01179

arXiv > hep-th > arXiv:2104.10171

arXiv > hep-th > arXiv:2105.03444

arXiv > hep-th > arXiv:2106.10288

arXiv > hep-th > arXiv:2107.02813

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High Energy Physics – Theory

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[Submitted on 6 Jul 2021 (v1), last revised 9 Sep 2021 (this version, v2)]

The Phases of Non-supersymmetric Gauge Theories: the $SO(N_c)$ Case Study

Csaba Csáki, Andrew Gomes, Hitoshi Murayama, Ofri Telem

AMSB + STRONG COUPLING

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Csaba Csáki, Andrew Gomes, Hitoshi Murayama, Ofri Telem

metry

arXiv > hep-th > arXiv:2111.09690

High Energy Physics – Theory

[Submitted on 18 Nov 2021]

Broken Conformal Window

Hitoshi Murayama, Bea Noether, Digvijay Roy Varier

AMSB + STRONG COUPLING

arXiv > hep-th > arXiv:2104.01179

arXiv > hep-th > arXiv:2111.09690

arXiv > hep-th > arXiv:2104.10171

arXiv > hep-th > arXiv:2202.01239

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arXiv > hep-th > arXiv:2105.03444

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arXiv > hep-th > arXiv:2106.10288

[Submitted on 2 Feb 2022]

On the Derivation of Chiral Symmetry Breaking in QCD-like Theories and S-confining Theories

Andrea Luzio, Ling-Xiao Xu

arXiv > hep-th > arXiv:2107.02813

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[Submitted on 6 Jul 2021 (v1), last revised 9 Sep 2021 (this version, v2)]

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arXiv > hep-th > arXiv:2209.09287

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High Energy Physics – Theory

[Submitted on 19 Sep 2022]

Dynamics of Simplest Chiral Gauge Theories

Dan Kondo, Hitoshi Murayama, Cameron Sylber

AMSB + STRONG COUPLING

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High Energy Physics – Theory

[Submitted on 6 Dec 2022]

A Guide to AMSB QCD

Csaba Csáki, Andrew Gomes, Hitoshi Murayama, Bea Noether, Digvijay Roy Varier, Ofri Telem

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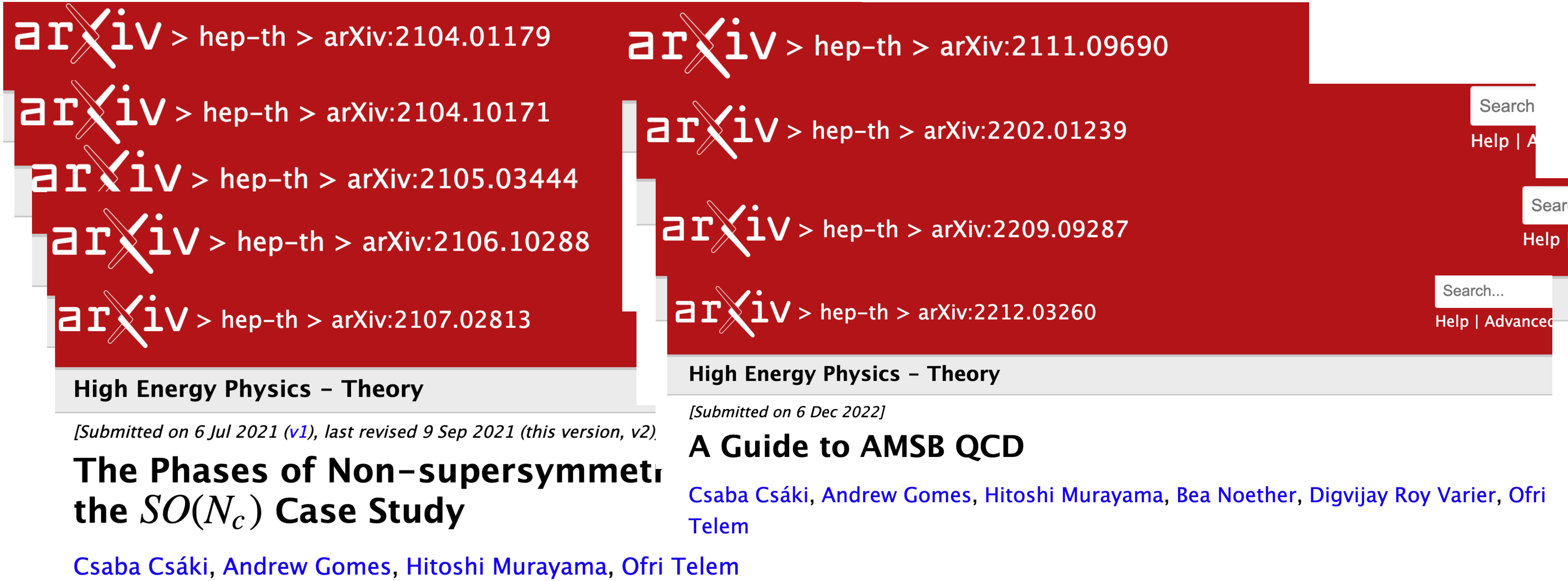
Csaba Csáki, Andrew Gomes, Hitoshi Murayama, Bea Noether, Digvijay Roy Varier, Ofri Telem

All consistent with smooth extrapolation to large $m_{3/2}$.

AMSB + STRONG COUPLING



All consistent with smooth extrapolation to large $m_{3/2}$.

See also Dine, Yu, arXiv:2205.00115 and Karasik, Onder, Tong, arXiv:2208.07842.

AMSB SCALAR POTENTIAL

Can compute full potential analytically.

Can do same complete the square technique.

$$V_{\rm susy} + V_{\rm \not{susy}} = \left|\frac{dW_\lambda}{dM^{ai}} + \frac{dW_\zeta}{dM^{ai}} + \frac{1}{3}A_1^* M^{ai*}\right|^2 + \left|\frac{dW_\zeta}{dB_2^{\beta\delta}} + (A_2^* - \frac{1}{3}A_1^*)B_2^{\beta\delta*}\right|^2 + \left|\frac{dW_\zeta}{dB_1^{\gamma i}}\right|^2$$

$$+ m_1^2 \sum_{\gamma i} |B^\gamma_{1\,i}|^2 + \left(m_2^2 - \left|A_2 - \frac{1}{3}A_1\right|^2\right) \sum_{\beta\delta} |B_2^{\beta\delta}|^2 + \left(m_3^2 - \left|\frac{1}{3}A_1\right|^2\right) \sum_{ai} |M^{ai}|^2$$

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Sufficient condition for no symmetry breaking:

$$m_1^2 \geq 0$$

$$m_2^2 - \left|A_2 - \frac{1}{3}A_1\right|^2 \geq 0$$

$$m_3^2 - \left|\frac{1}{3}A_1\right|^2 \geq 0$$

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$$V_{\text{susy}} + V_{\text{su}\not{\text{s}}\text{y}} = \left| \frac{dW_\lambda}{dM^{ai}} + \frac{dW_\zeta}{dM^{ai}} + \frac{1}{3} A_1^* M^{ai*} \right|^2 + \left| \frac{dW_\zeta}{dB_2^{\beta\delta}} + (A_2^* - \frac{1}{3} A_1^*) B_2^{\beta\delta *} \right|^2 + \left| \frac{dW_\zeta}{dB_1^{\gamma i}} \right|^2$$

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Sufficient condition for no symmetry breaking:

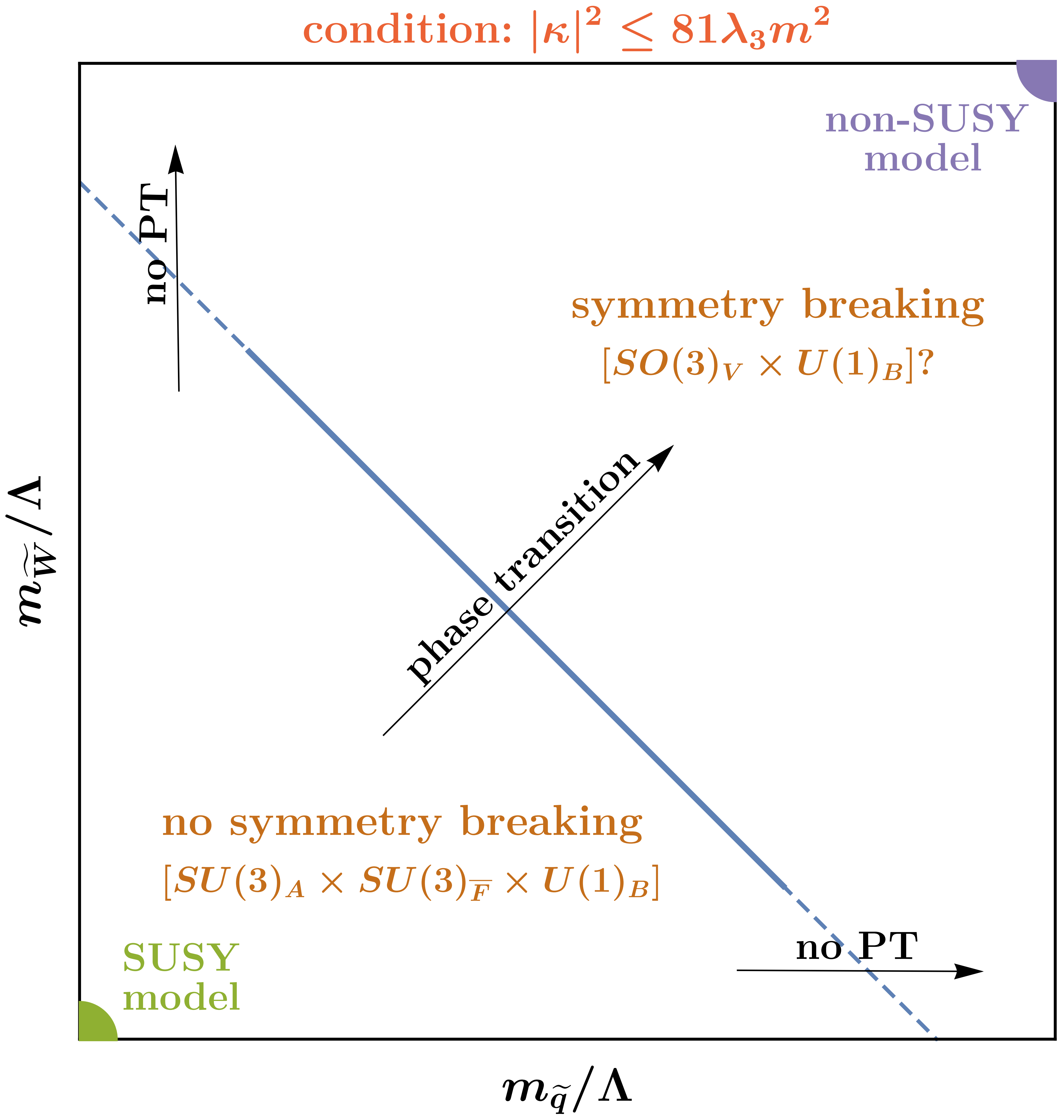
$$\begin{aligned} m_1^2 &\geq 0 \\ m_2^2 - \left| A_2 - \frac{1}{3} A_1 \right|^2 &\geq 0 \\ m_3^2 - \left| \frac{1}{3} A_1 \right|^2 &\geq 0 \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} 69|\zeta|^4 + 4|\zeta|^2|\lambda|^2 &\geq 0 \\ 19|\zeta|^4 + 4\,|\zeta|^2|\lambda|^2 &\geq 0 \\ 1161|\zeta|^4 + 216|\zeta|^2|\lambda|^2 + 80|\lambda|^4 &\geq 0 \end{aligned}$$

AMSB CONCLUSIONS

There is no symmetry breaking in AMSB with $m_{3/2} \ll \Lambda$.

This conclusion is exact, in the sense of the other AMSB papers.

Must be a phase transition as $m_{3/2}$ is increased.



TWO FLAVOURS?

Now understand SU(5) theory for $N_F = 1, 3$. What about others?

2 flavour theory has different anomaly matching.

Witten, PLB 1982.

In non-SUSY theory can match anomalies with two states.

	$[SU(5)]$	$SU(2)_A$	$SU(2)_{\bar{F}}$	$U(1)_B$
A	10	2	1	1
$\bar{F}$	$\bar{5}$	1	2	-3

In SUSY case, anomaly matching is impossible.

Proves χSB!

MSc thesis by Jonathan Ponnudurai.

SUSY QCD

SQCD WITH $N_c = 2$ AND $N_f = 3$

This theory is qualitatively different than with $N_c \geq 3$.

	$[SU(2)]$	$SU(6)$	$U(1)_R$
q	2	6	$1/3$
W^α	3	1	1

SQCD WITH $N_c = 2$ AND $N_f = 3$

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$B = \bar{B} = M \sim q^2$.

Enhanced symmetry relative to usual QCD.

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W^α	3	1	1
$M = q^2$		15	2/3

$$W_{\text{dyn}} = \epsilon^{abcdef} M_{ab} M_{cd} M_{ef} \equiv \text{Pf}\, M$$

Seiberg, hep-th/9402044.

AMSB + SQCD WITH 2 COLOURS

$$W_{\text{dyn}} = \kappa \,\text{Pf}\, M$$

Hassan Easa PhD thesis, Csaki et. al., arXiv:2212.03260. de Lima, DS, arXiv:2307.13154.

Add anomaly meditated SUSY breaking to theory. Need one loop correction.

$$V = |\kappa|^2 \left| \frac{\partial \text{Pf}\, M}{\partial M_{ij}} \right|^2 + \frac{27|\kappa|^4}{1024\pi^4} |m_{3/2}|^2 |M|^2 - \frac{9}{32\pi^2} \kappa\, m_{3/2}\, \text{Pf}\, M$$

Can use complete the square trick and get sum of positive terms:

$$\langle M \rangle = 0 \,.$$

NON-SUSY QCD WITH 2 COLOURS

With only quarks, gauge invariant bound states have even number of fermions.

Gauge invariant bound states all bosonic, no way to satisfy $SU(6)$ 't Hooft anomaly.

Must be symmetry breaking.

	$[SU(2)]$	$SU(6)$	$U(1)_R$
q	2	6	$1/3$
W^α	3	1	1
$M = q^2$		15	$2/3$

This theory also has a phase transition as you increase $m_{3/2}$.

HIGHER ORDER CORRECTIONS

Result is leading order in $m_{3/2}/\Lambda$.

Next term in perturbation expansion appears in Kahler potential:

$$K \sim \frac{(M^\dagger M)^2}{\Lambda^2}$$

Can we see phase transition at NLO?

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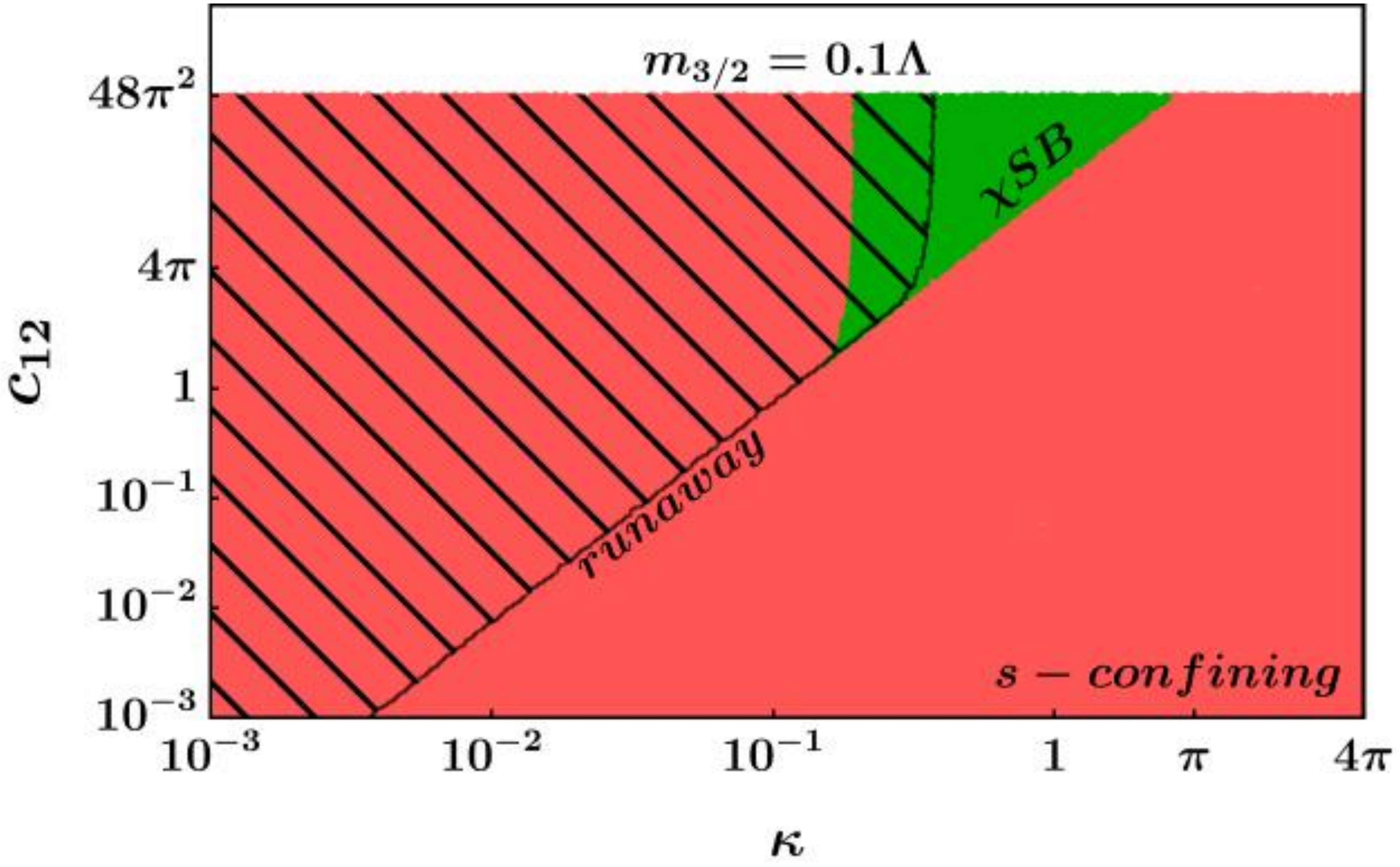


Kind of.

de Lima, DS, arXiv:2307.13154.

QCD WITH MORE FLAVOURS

QCD with $N_f = N_c + 1$ is also s-confining and described by dynamical superpotential in terms of mesons and baryons. Seiberg, hep-th/9402044.

$$W_{\text{dyn}} = \kappa\, BM\bar{B} - \frac{\lambda}{\Lambda^{N_f - 3}} \det M$$

Perturbative analysis points to QCD-like vacuum:

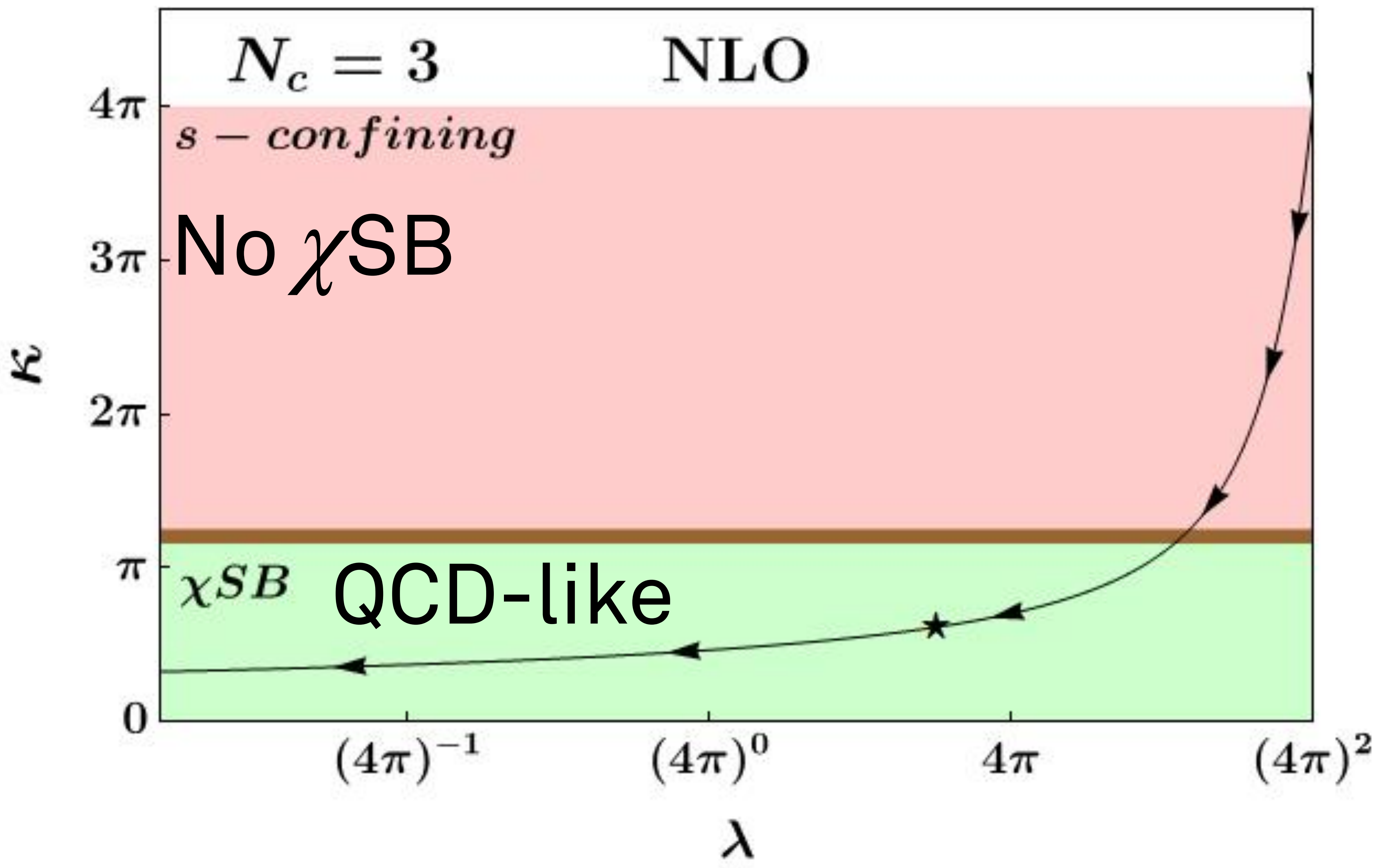
$\langle M \rangle \sim \mathbf{1}$ and $\langle B \rangle = \langle \bar{B} \rangle = 0.$

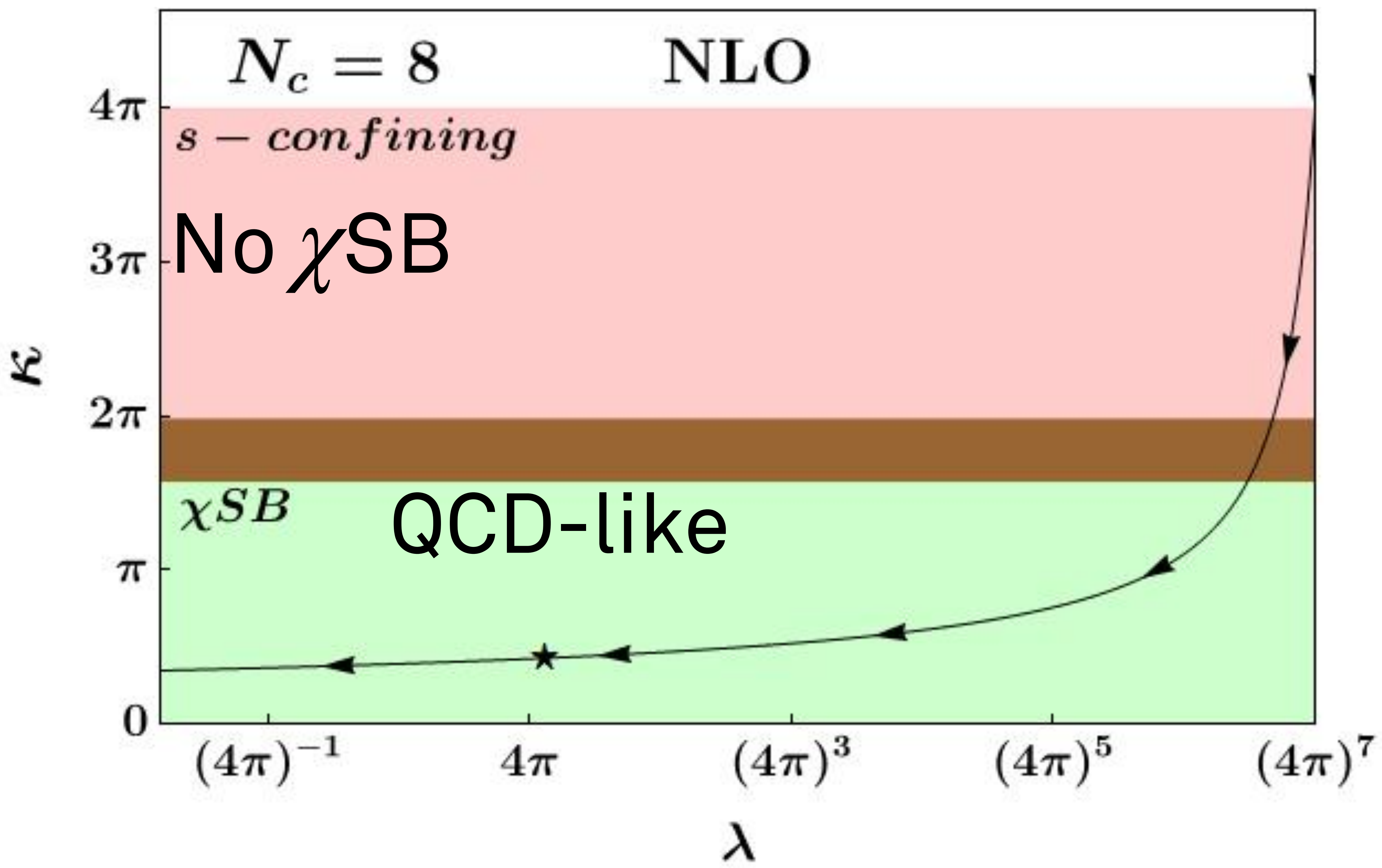
Murayama, 2104.01179. Csaki et. al., arXiv:2212.03260.

NUMERICAL EXPLORATION

Explore potential numerically:

$\star$ - $m_{3/2}/\Lambda = 0.1$





Appears to be no dependence on λ. No direct dependence on $m_{3/2}$!

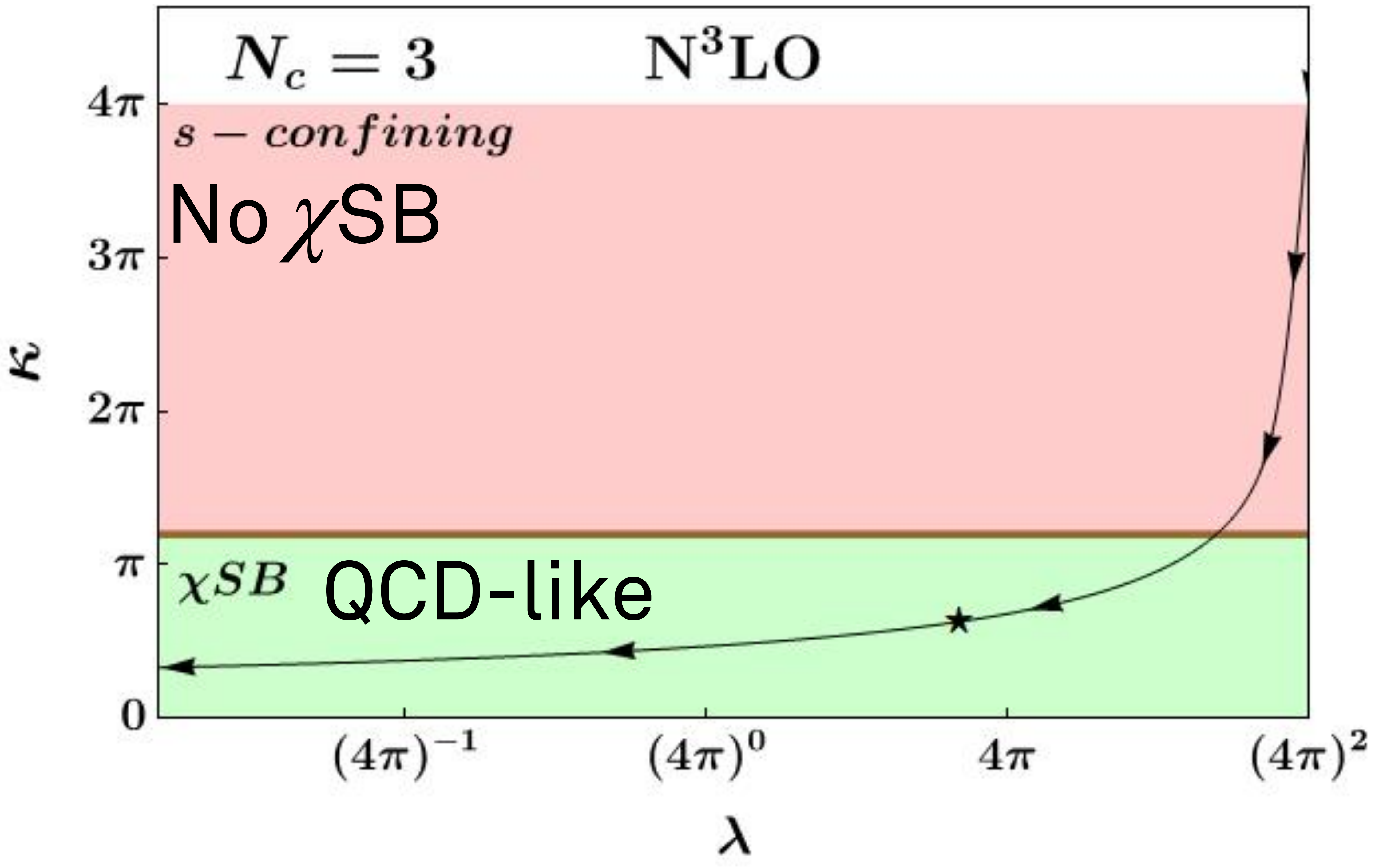
A phase boundary in κ.

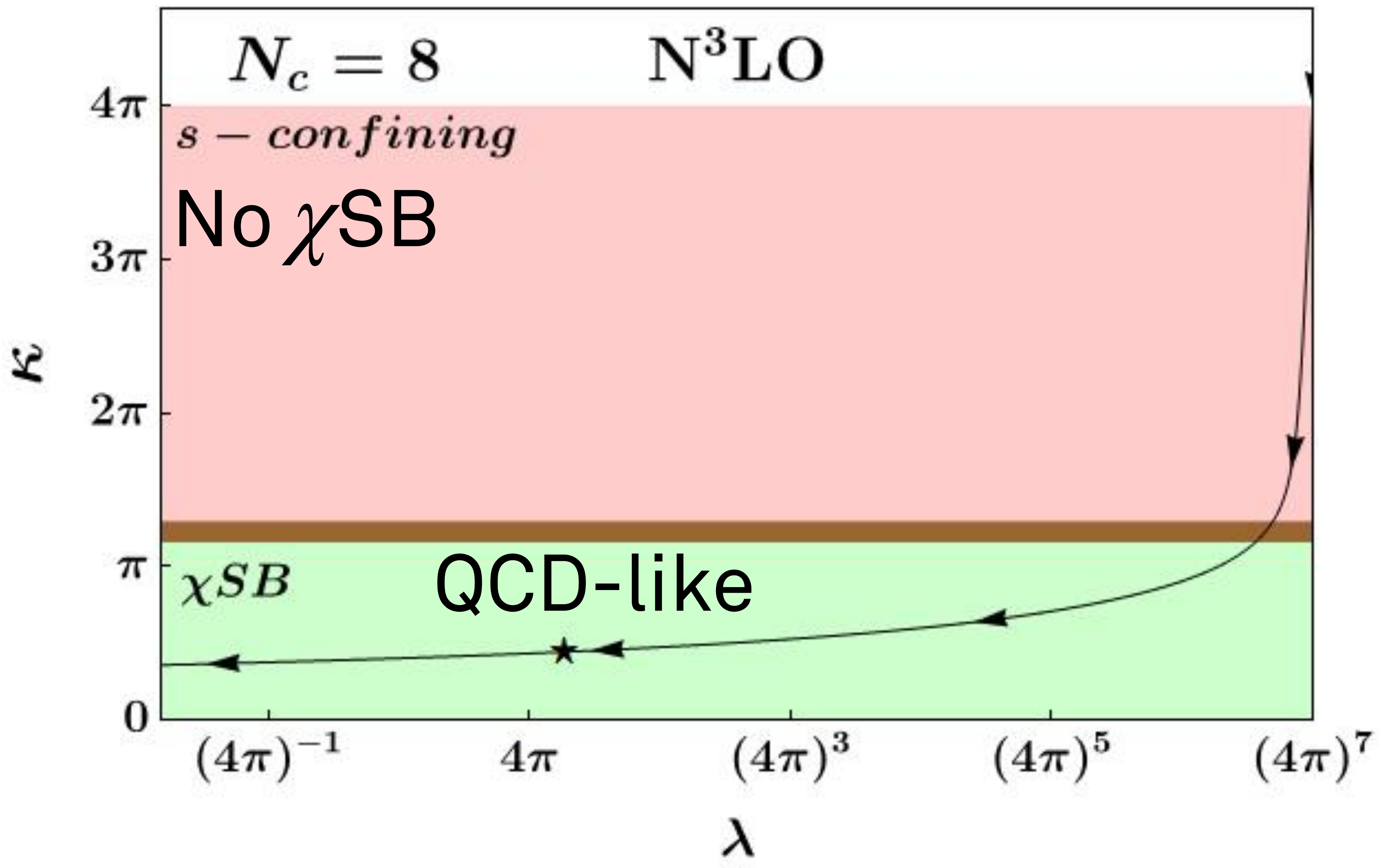
de Lima, DS, arXiv:2307.13154.

NUMERICAL EXPLORATION

Can extend analysis to higher loops:

$\star$ - $m_{3/2}/\Lambda = 0.1$

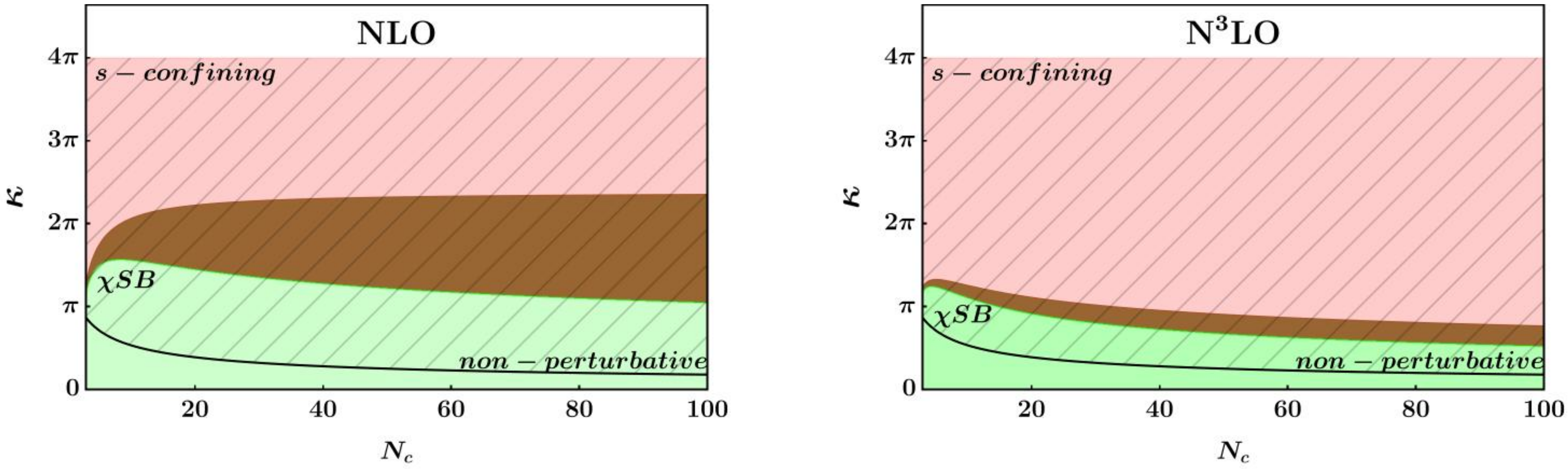




Does not change conclusions.

PERTURBATIVITY?

Can compare perturbative series order by order:



Phase transition does not appear to be under control.

NLO KAHLER?

What about higher order terms in $m_{3/2}/\Lambda$?

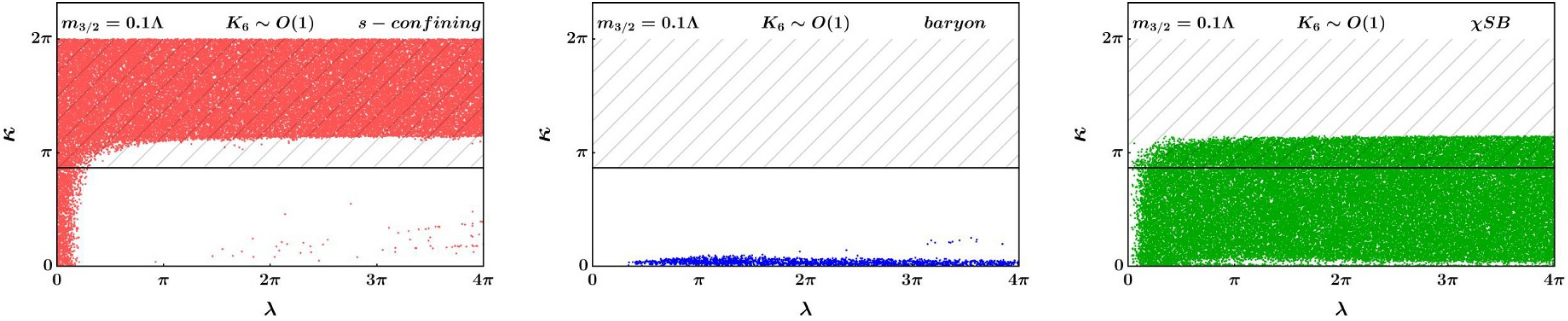
$$\Lambda^2 K_6 = \frac{c_{M_1}}{N_f^2}\,\mathrm{Tr}\left(M^\dagger M\right)^2 + \frac{c_{M_2}}{N_f}\,\mathrm{Tr}\left(M^\dagger M M^\dagger M\right) + \frac{c_B}{N_f}\left((B^\dagger B)^2 + (\tilde{B}^\dagger \tilde{B})^2\right)$$
$$+ \frac{c_{B\tilde{B}}}{N_f}(B^\dagger B)(\tilde{B}^\dagger \tilde{B}) + \frac{c_{MB}}{N_f^2}\,\mathrm{Tr}\left(M^\dagger M\right)\left(B^\dagger B + \tilde{B}^\dagger \tilde{B}\right)$$
$$+ \frac{c_{BMMB}}{N_f}\left(BMM^\dagger B^\dagger + \tilde{B}MM^\dagger \tilde{B}^\dagger\right),$$

Six new parameters 😢.

Let’s do a numerical scan varying parameters between 0 and 1.

NLO KAHLER?

Can see a modified vacuum structure. $N_c = 3 \qquad \frac{m_{3/2}}{\Lambda} = 0.1$



No χSB

Spontaneous baryon number violation

QCD-like

Hard to draw concrete conclusions without knowing κ, λ.

HOLOGRAPHY?

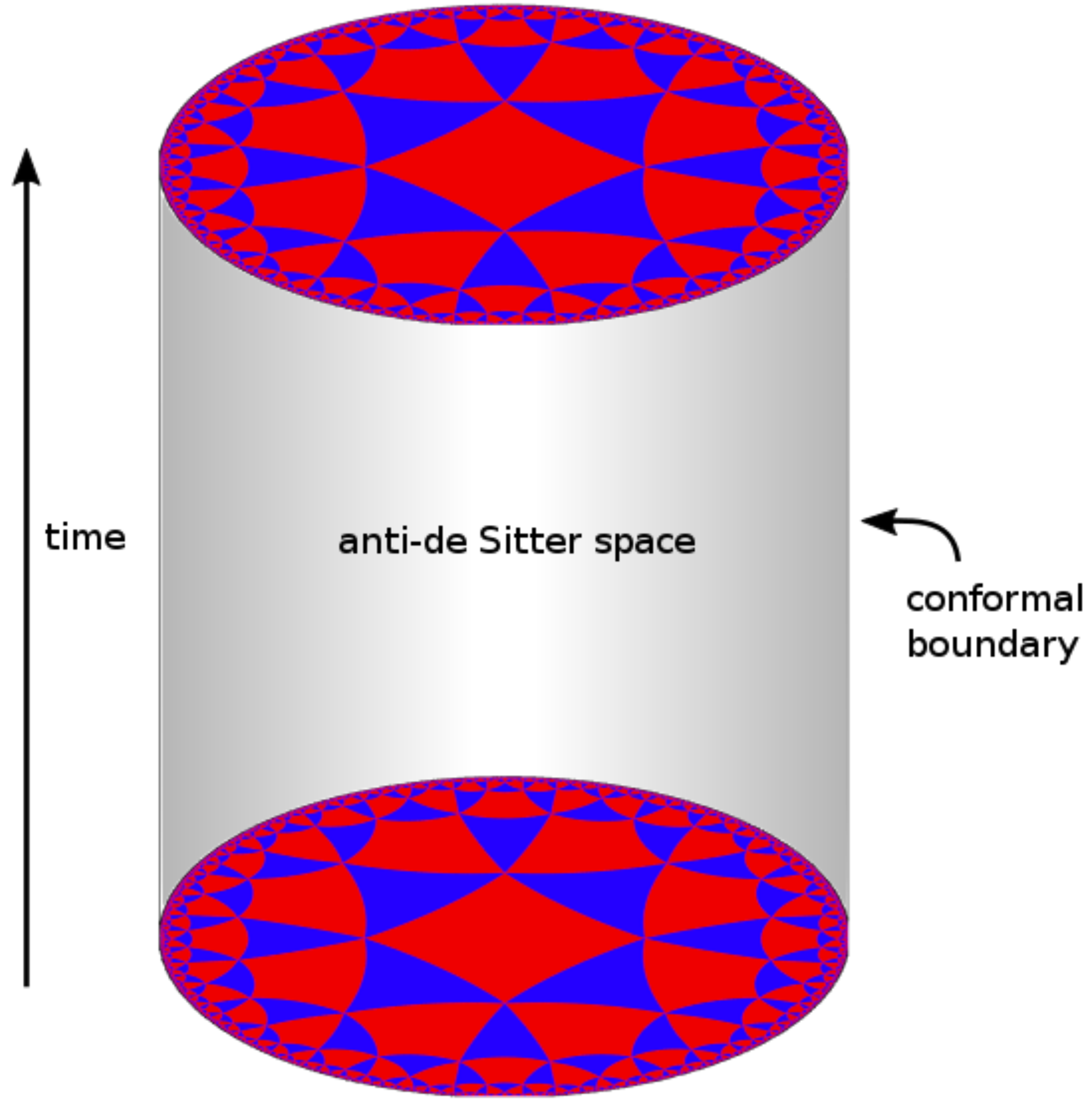
AdS/CFT correspondence says conformal field theories are dual to gravitational theories in 5 dimensions.

Maldacena, hep-th/9711200. Witten, hep-th/9802150.

Very supersymmetric theories work great in AdS/CFT.

What is gravitational dual of Anomaly mediation?

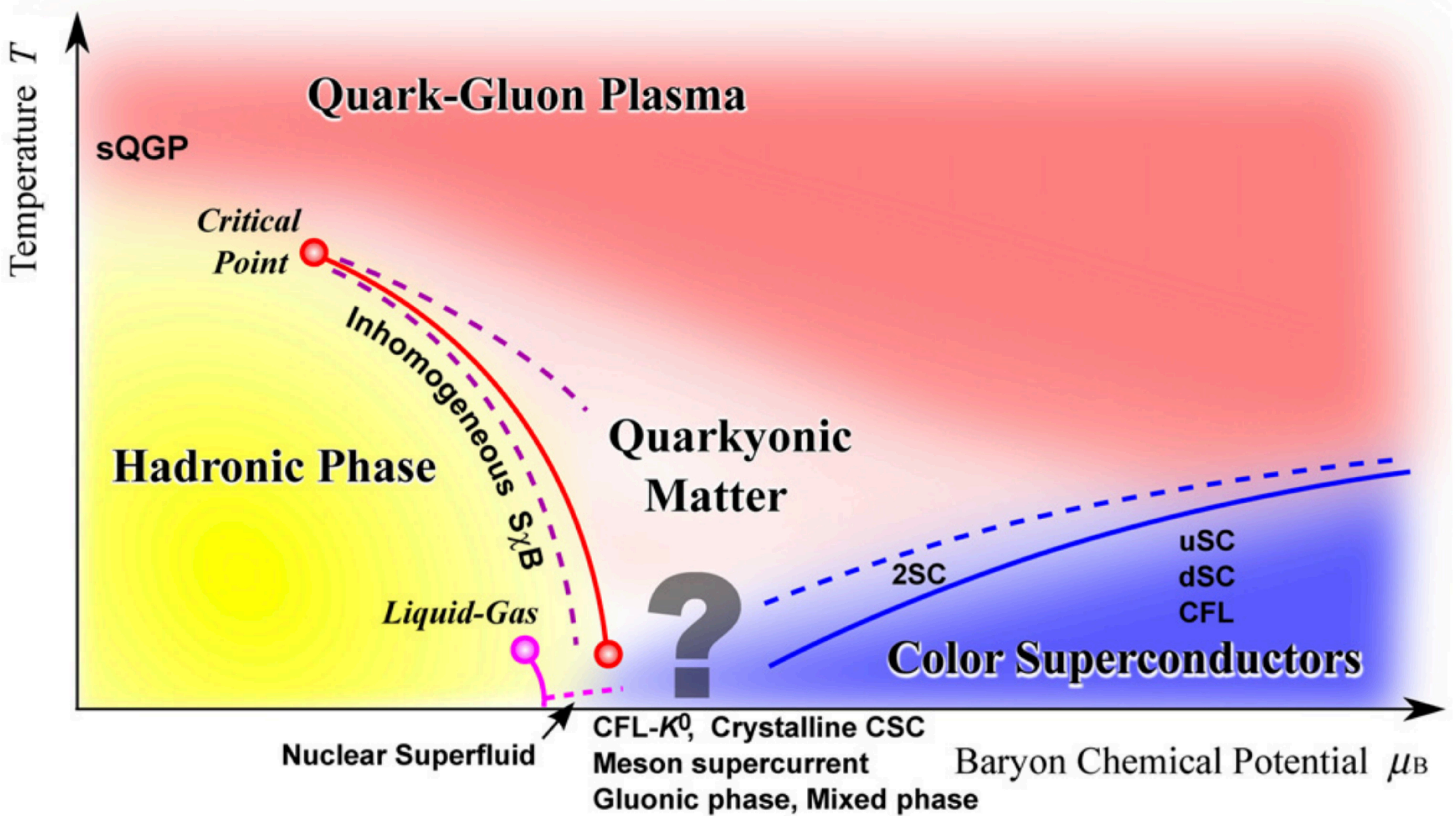
Work in progress with Cyrus Robertson Orkish.



FINITE BARYON DENSITY

PHASE DIAGRAM

What is the phase diagram of QCD?



Longstanding difficult problem with only partial results.

Fukushima and Hatsuda, '11.

Conjectured QCD phase diagram with boundaries that define various states of QCD matter based on SχB patterns.

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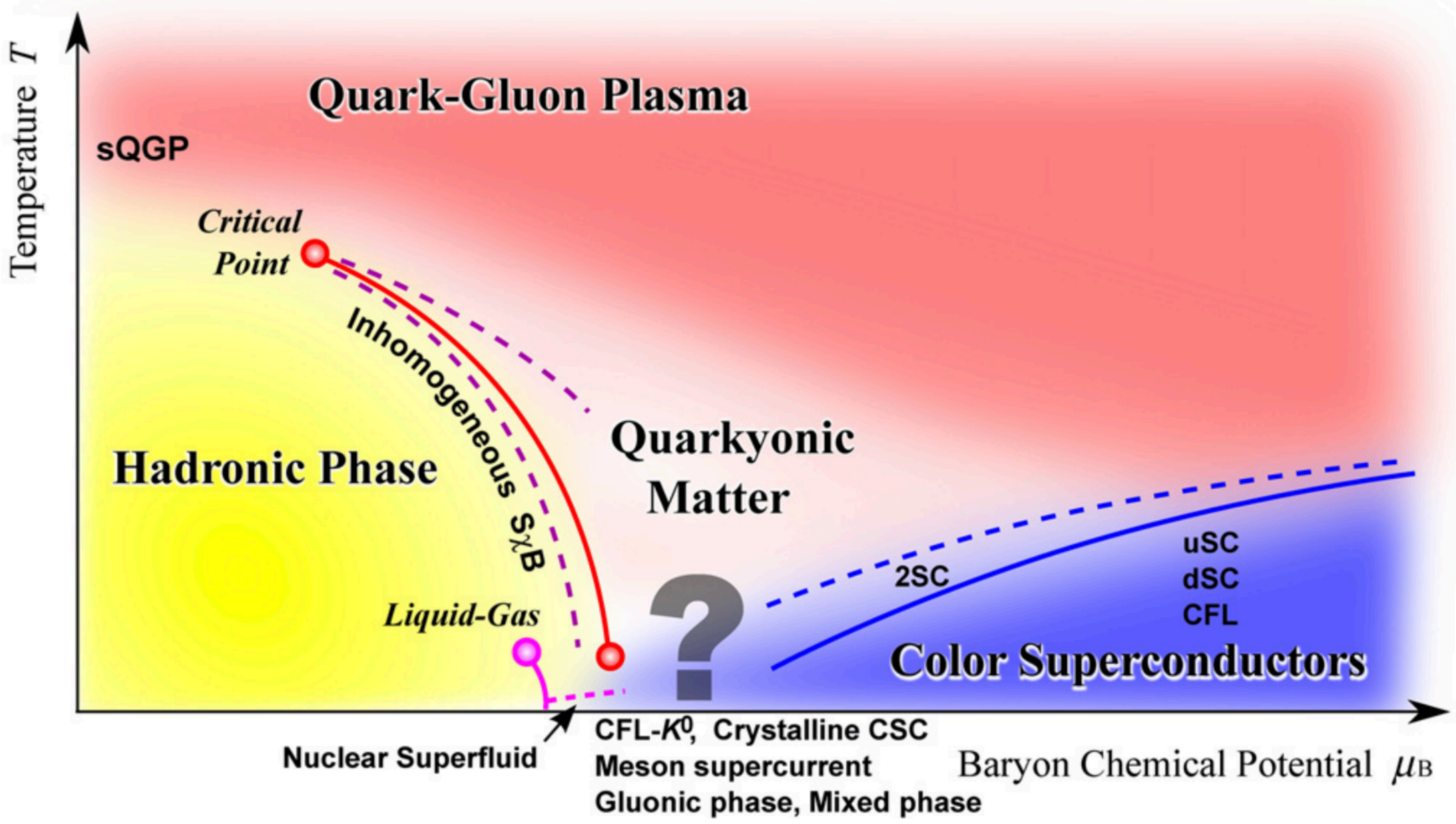

Conjectured QCD phase diagram with boundaries that define various states of QCD matter based on SχB patterns.

Longstanding difficult problem with only partial results.

Focus on $T = 0$.

Fukushima and Hatsuda, '11.

BARYON CHEMICAL POTENTIAL

Look at s-confining SQCD + AMSB + finite baryon density.

Adding baryon chemical potential gives negative mass squared to scalars containing baryon number.
Harnik, Larsen, Murayama, '03.

Start in low energy (hadron) theory:

$$m_B^2 = \frac{9}{64\pi^4}|\kappa|^4|m_{3/2}|^2 - 9\mu_B^2 .$$

Fix $N_c = 3$ and $N_f = 4$

QCD-like vacuum destabilized for sufficiently large μ_B.

BARYON VEV

To find baryon vev, need quartic, comes from F-term:

$$V = m_B^2 \left(|B|^2 + |\widetilde{B}|^2 \right) + \left| \kappa B \widetilde{B} \right|^2 \quad \text{with} \quad m_B^2 < 0$$

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Problem: potential is unstable in the $\widetilde{B}$ direction!

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$$V = m_B^2\left(|B|^2 + |\widetilde{B}|^2\right) + \left|\kappa B \widetilde{B}\right|^2 \quad \text{with} \quad m_B^2 < 0$$

Problem: potential is unstable in the $\widetilde{B}$ direction!

What now?

QUARK PHASE

Theory runs to high energy quark phase. AMSB still predicts the potential:

$$V = m_q^2 \left(|q|^2 + |\bar{q}|^2 \right) + \frac{g^2}{6} \left(q^\dagger T^a q - \bar{q}^\dagger T^a \bar{q} \right)^2$$

$$m_q^2 = \frac{5g^4}{24\pi^4} |m_{3/2}|^2 - \mu_B^2 \lesssim 0$$

Same problem in the direction where $\langle q \rangle = \langle \bar{q} \rangle$.

Theory appears to run to infinity!

KAHLER TERMS

Maybe we were too quick going over the transition.

As we approach $B \simeq \Lambda$, higher order Kahler terms become important.

$$\Lambda^2 K_6 = \frac{c_{M_1}}{N_f^2}\,\mathrm{Tr}\Big(M^\dagger M\Big)^2 + \frac{c_{M_2}}{N_f}\,\mathrm{Tr}\Big(M^\dagger M M^\dagger M\Big) + \frac{c_B}{N_f}\Big((B^\dagger B)^2 + (\tilde{B}^\dagger \tilde{B})^2\Big)$$
$$+ \frac{c_{B\tilde{B}}}{N_f}(B^\dagger B)(\tilde{B}^\dagger \tilde{B}) + \frac{c_{MB}}{N_f^2}\,\mathrm{Tr}\Big(M^\dagger M\Big)\Big(B^\dagger B + \tilde{B}^\dagger \tilde{B}\Big)$$
$$+ \frac{c_{BMMB}}{N_f}\Big(BMM^\dagger B^\dagger + \tilde{B}MM^\dagger \tilde{B}^\dagger\Big),$$

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$$+ \frac{c_{BMMB}}{N_f}\Big(B M M^\dagger B^\dagger + \tilde{B} M M^\dagger \tilde{B}^\dagger\Big)\,,$$

Only one term affects runaway direction.

KAHLER TERMS

Have 5 parameters:

- κ
- λ
- c_B
- $m_{3/2}$
- μ_B

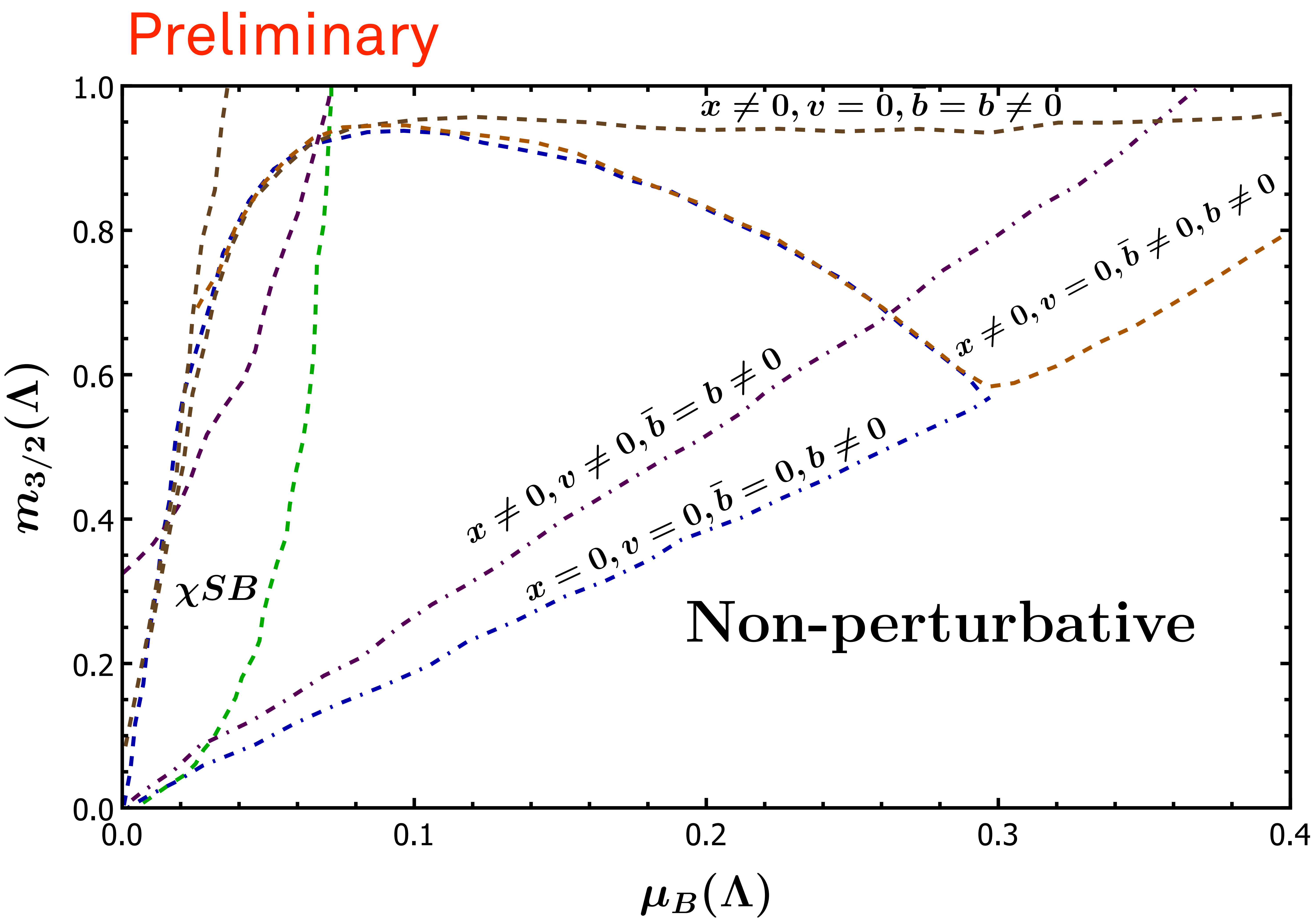
Have (at least) 4 field directions.

$$M = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & v & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 0 & v \end{pmatrix} \quad B = \begin{pmatrix} b \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \widetilde{B} = \begin{pmatrix} \tilde{b} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Intractable analytically,
annoying numerically.

PHASE DIAGRAM

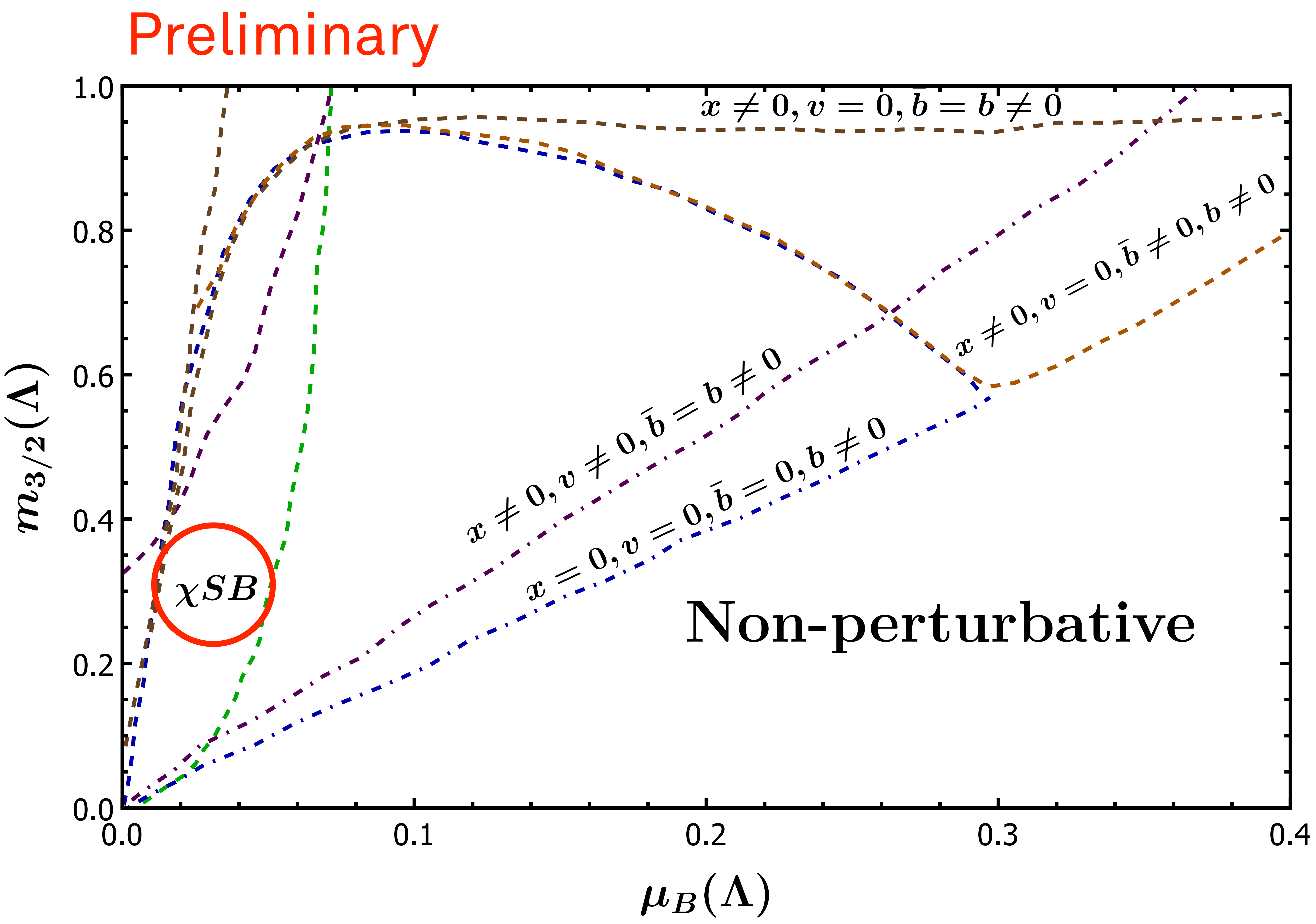
Vaccua we found:



PHASE DIAGRAM

Vaccua we found:

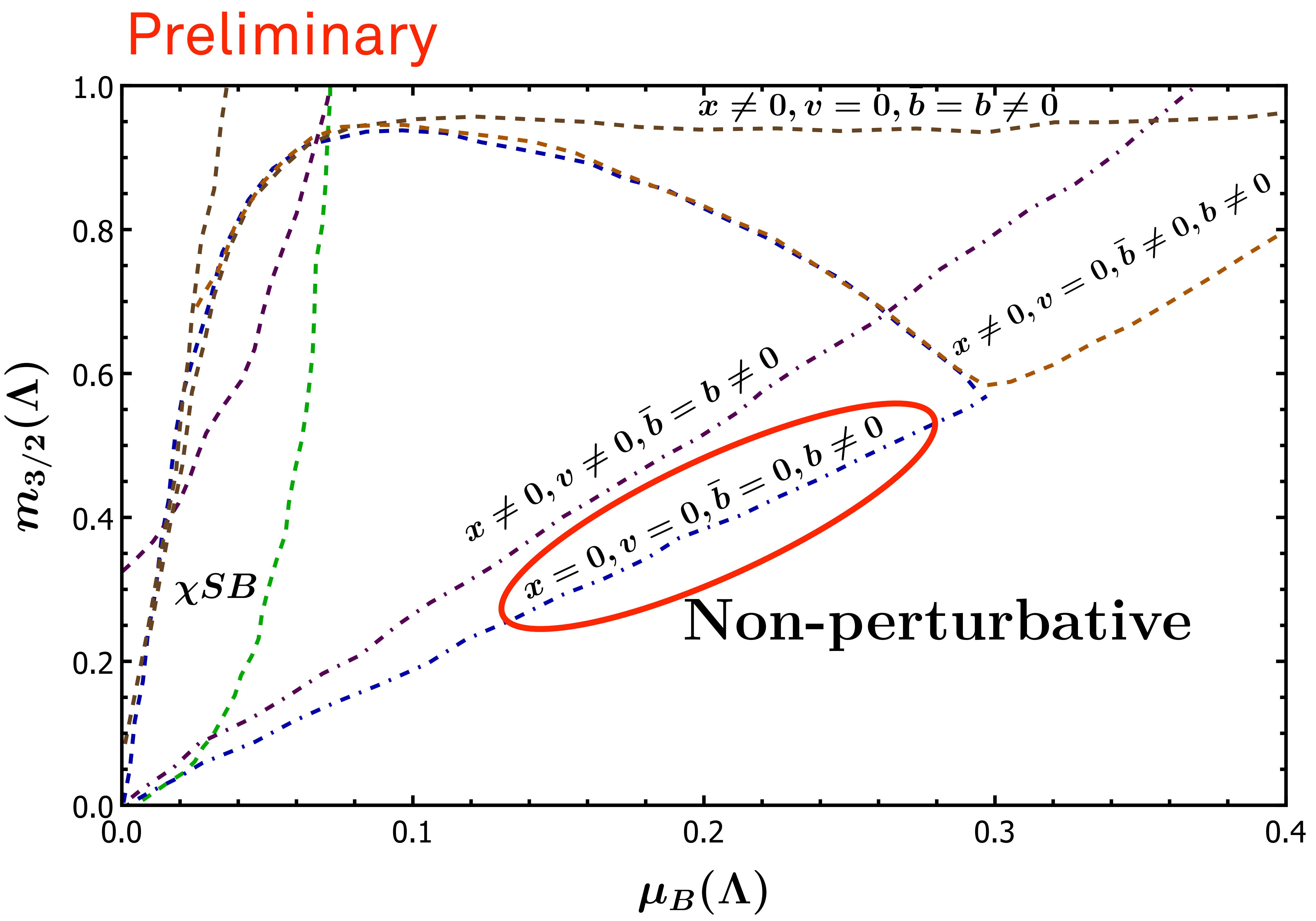
- QCD-like



PHASE DIAGRAM

Vaccua we found:

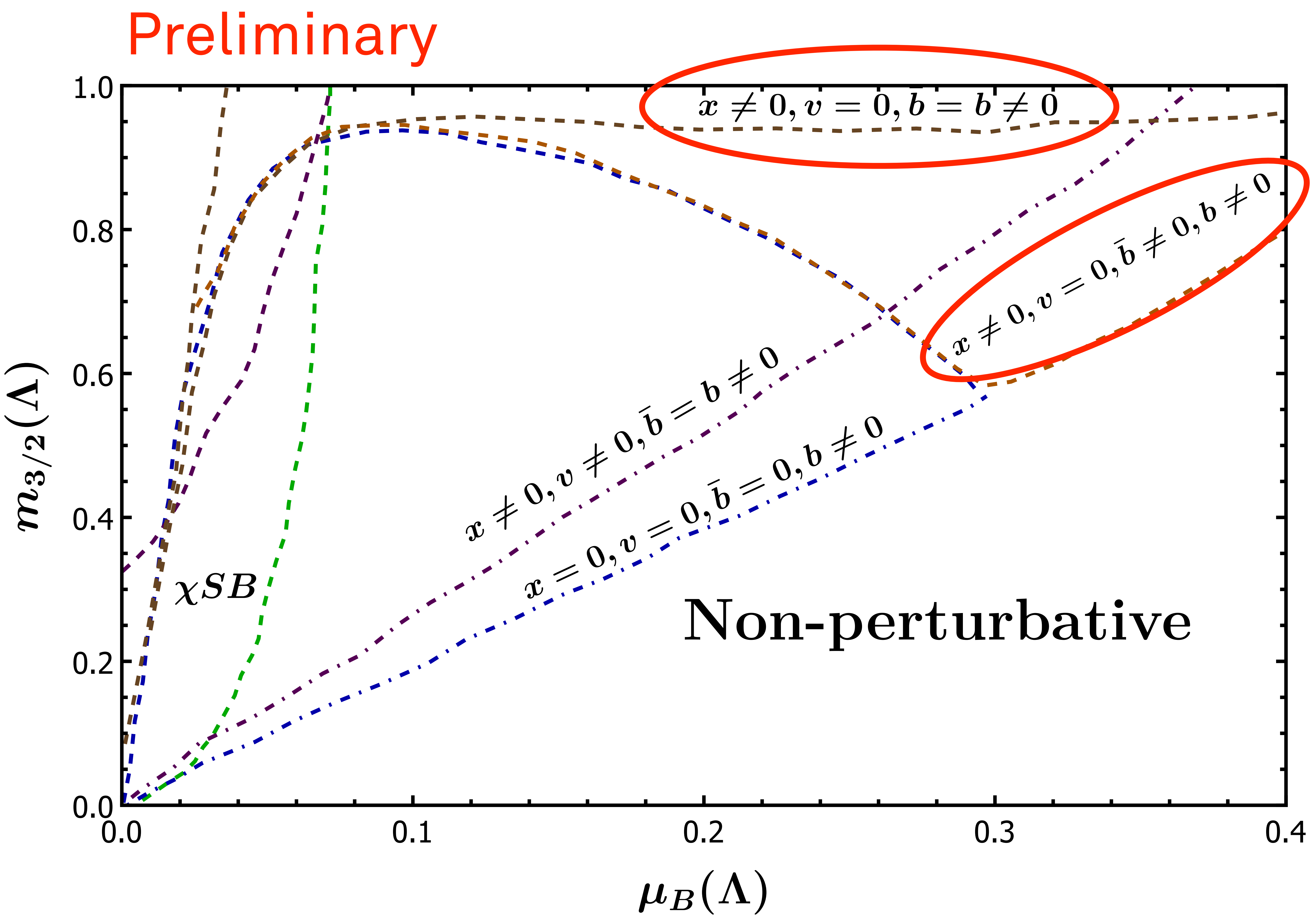
- QCD-like
- Baryon-number breaking



PHASE DIAGRAM

Vaccua we found:

- QCD-like
- Baryon-number breaking
- $SU(4)^2 \to SU(3)^2$



PHASE DIAGRAM

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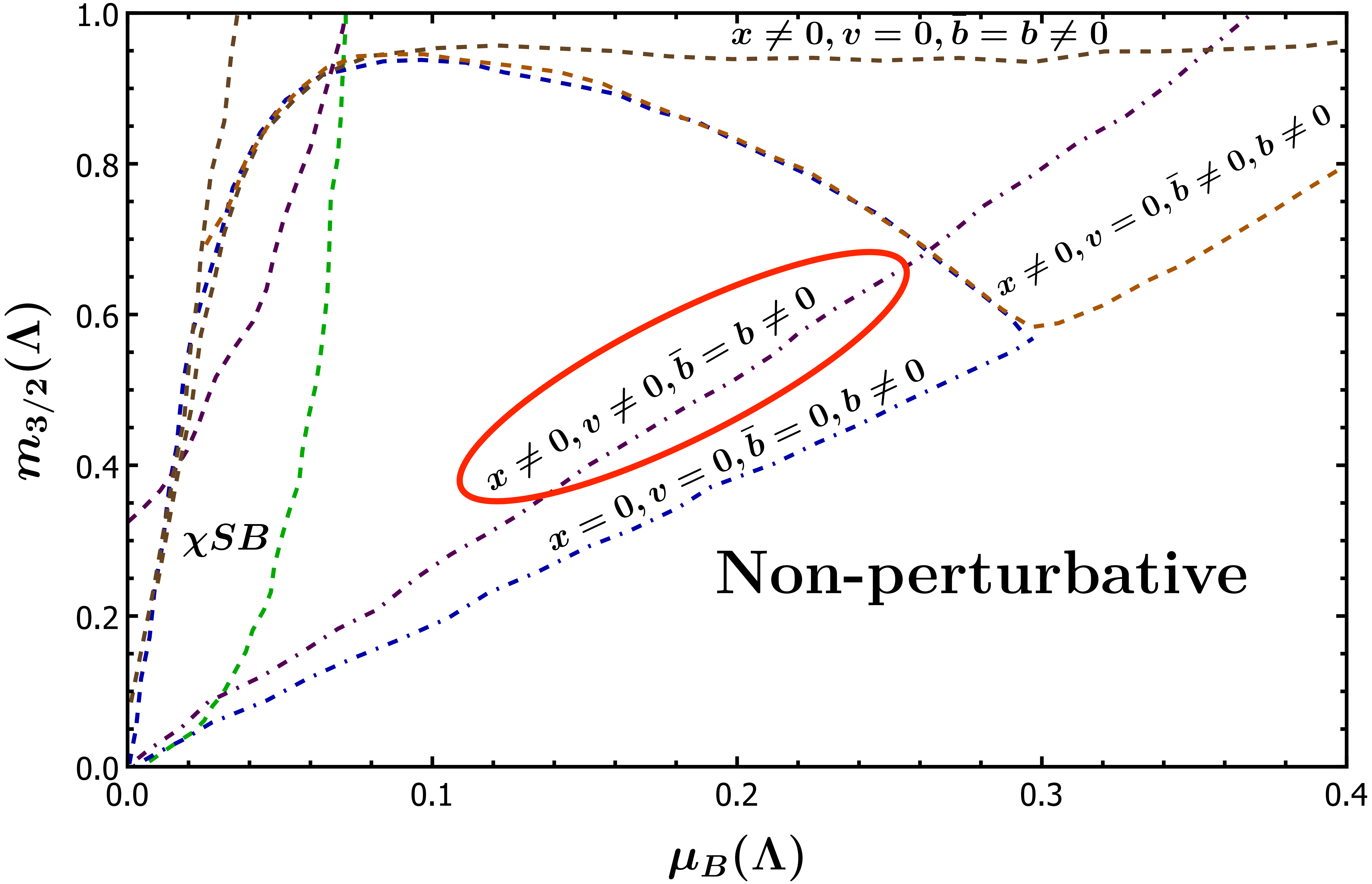
- $SU(4)^2 \to SU(3)^2$
- $SU(4)^2 \to SU(3)$

Preliminary



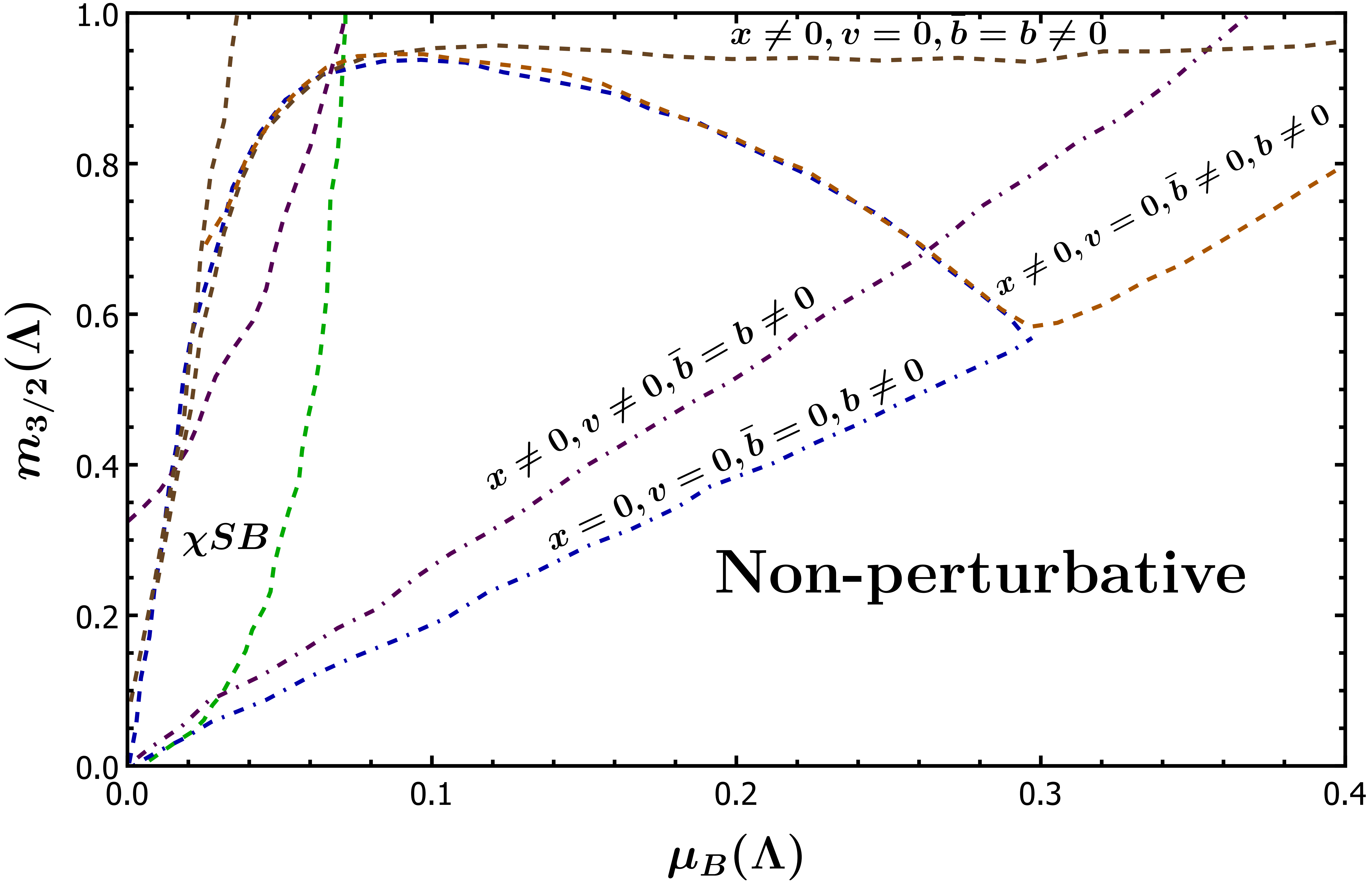
PHASE DIAGRAM

Vaccua we found:

- QCD-like
- Baryon-number breaking
- $SU(4)^2 \rightarrow SU(3)^2$
- $SU(4)^2 \rightarrow SU(3)$

Last two are not in the QCD literature (I think).

Preliminary



CONCLUSIONS

Explored interesting chiral SU(5) theory: likely vacuum has SO(3)xU(1) symmetry with 3 fermions and 13 bosons.

If SUSY theory is coupled to anomaly mediation, there must be a phase transition as $m_{3/2}$ increases.

Explored dynamics of s-confining SQCD-like theories including higher order Kahler terms and finite baryon density. Find some interesting phases.

Finite temperature likely also interesting; to do!

THANK YOU

WHY LOW ENERGY?

“Low energy” is where we do experiments.

Can ignore non-renormalizable “irrelevant” operators.

Ultimately this is our goal as field theorists.

Dynamics almost always become simpler at low energy.

•QED -> free Maxwell theory.

•QCD -> theory of pions -> trivial theory.

VECTOR VS CHIRAL

QCD is a vector-like theory.

- Mass terms are allowed

- Can make progress on the lattice.

	$[SU(3)]$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$
$Q_L = (u_L \;\; d_L)$	3	2	1	1
$Q_R = (u_R \;\; d_R)$	$\overline{3}$	1	2	-1

VECTOR VS CHIRAL

QCD is a vector-like theory.

Electroweak theory is chiral.

- Mass terms forbidden.
- Interesting dynamics if it confines.

Kuzmin, Shaposhnikov, Tkachev, PRD ’92.
Quigg, Shrock, 0901.3958.
Konstandin, Servant, 1104.4791.
Bai, Long, 1804.10249.
And many others.

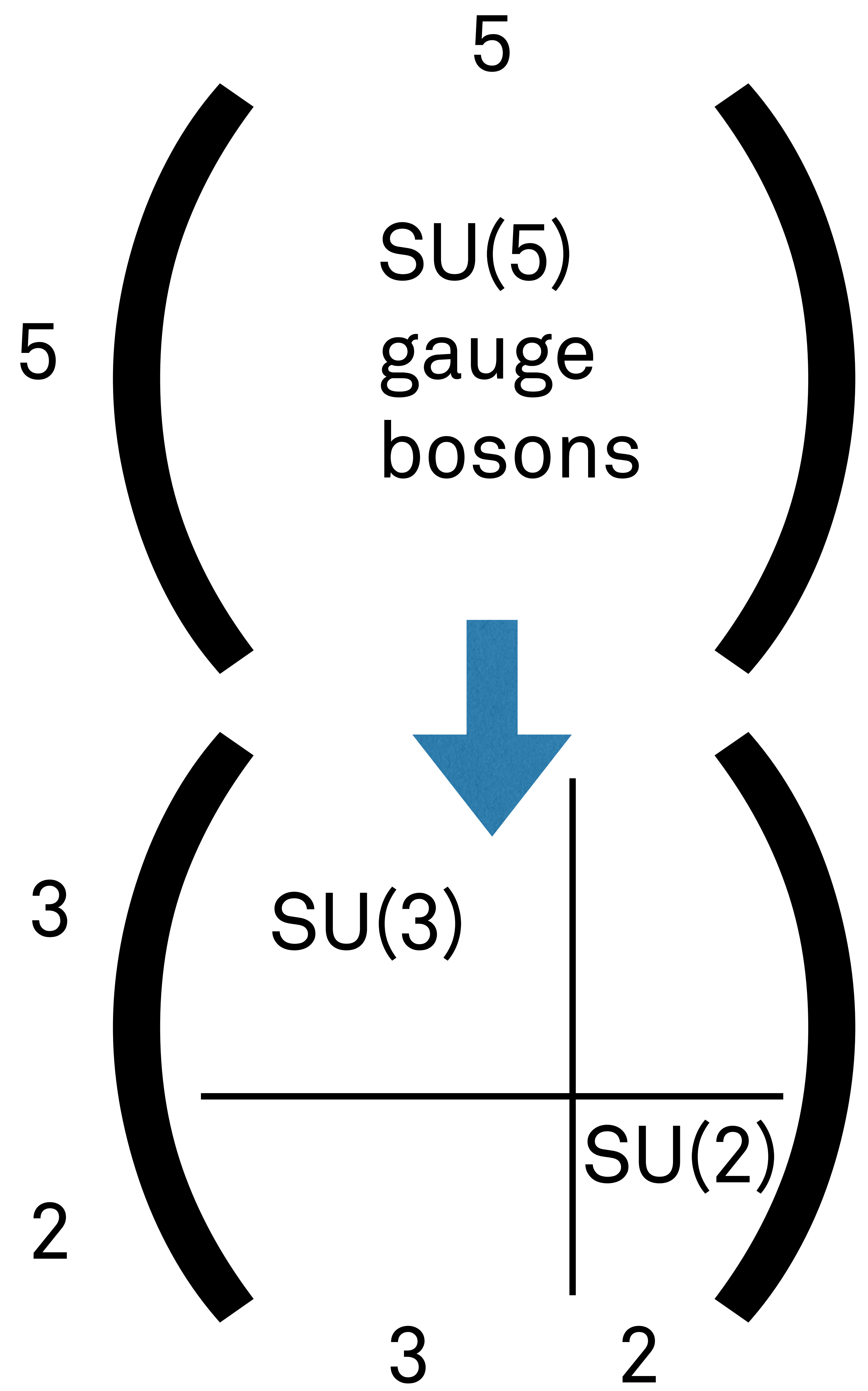
	$[SU(3)]$	$[SU(2)_L]$	$[U(1)_Y]$	$U(1)_{B-L}$
Q_L	3	2	$1/6$	$1/3$
u_R	$\overline{3}$	1	$-2/3$	$-1/3$
d_R	$\overline{3}$	1	$1/3$	$-1/3$
L_L	1	2	$-1/2$	-1
e_R	1	1	1	$+1$

GEORGI GLASHOW GUT

Simplest "Grand Unified Theory" (GUT) is Georgi Glashow SU(5).

Georgi, Glashow, PRL '74.

SU(5) breaks down to SU(3)xSU(2)xU(1).



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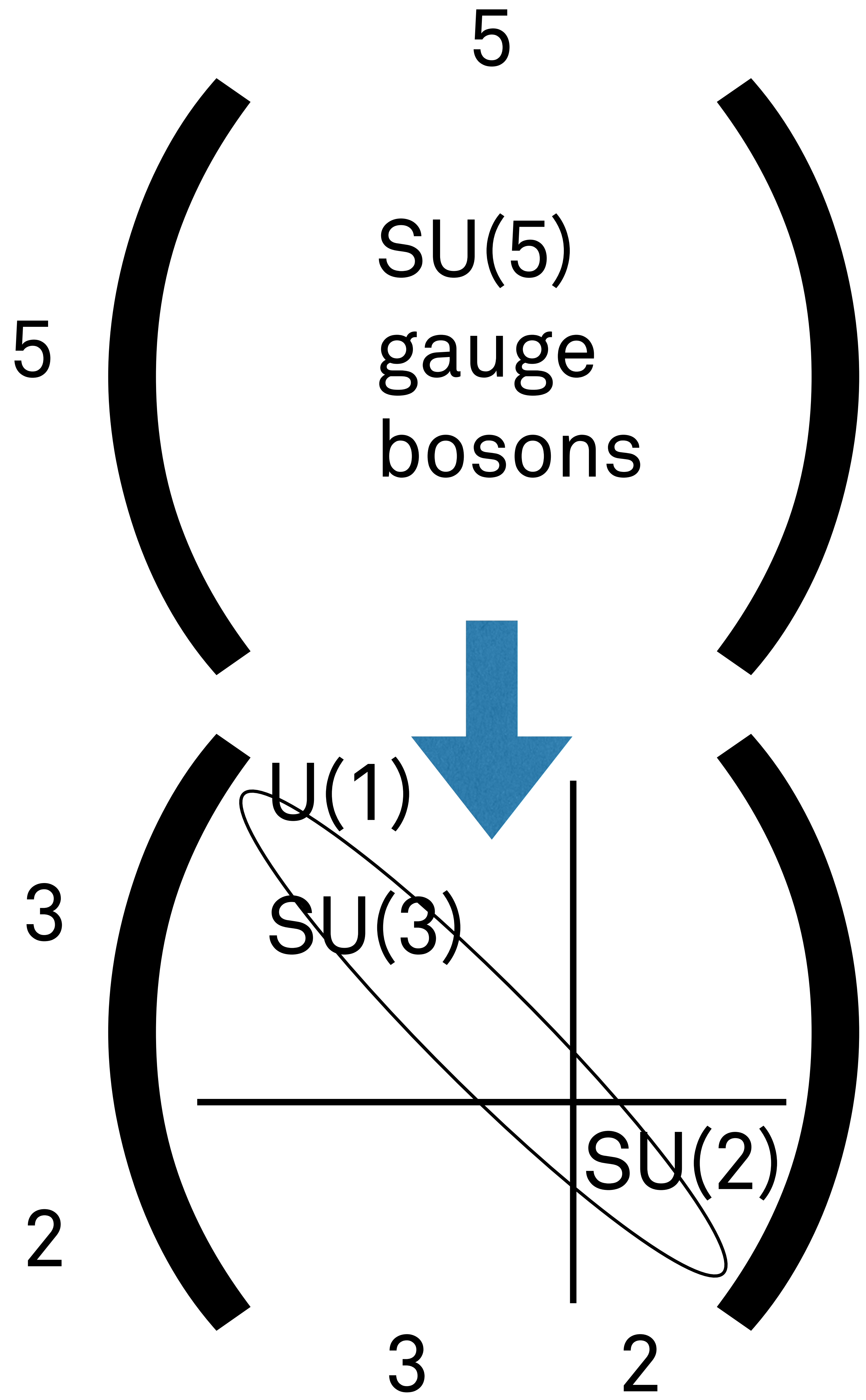


SU(5) REPRESENTATIONS

All SM fermions fit into either $\bar{5}$ or 10 representation.

Georgi, Glashow, PRL '74.

$$\bar{5} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ L_1 \\ L_2 \end{pmatrix}$$

Get exactly the SM fermions and no others.

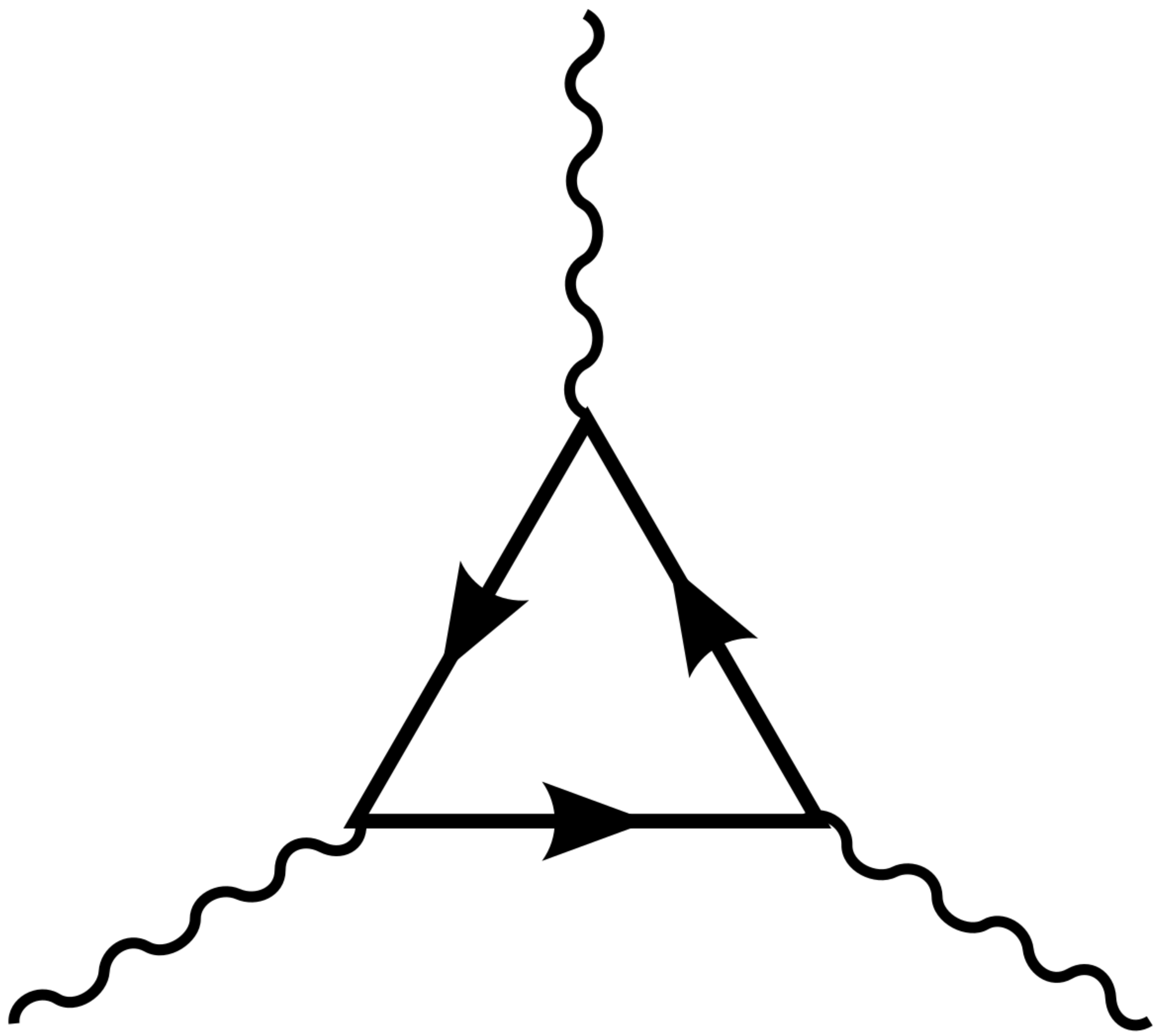
$$10 = \begin{pmatrix} 0 & u_1 & u_2 & Q_{11} & Q_{12} \\ -u_1 & 0 & u_3 & Q_{21} & Q_{22} \\ -u_2 & -u_3 & 0 & Q_{31} & Q_{32} \\ -Q_{11} & -Q_{21} & -Q_{31} & 0 & e \\ -Q_{12} & -Q_{22} & -Q_{32} & -e & 0 \end{pmatrix}$$

Scalars are a bit more complicated.

TRIANGLE ANOMALIES

Anomalies are a quantum breaking of a classical symmetry.

Can be represented in terms of triangle diagrams.



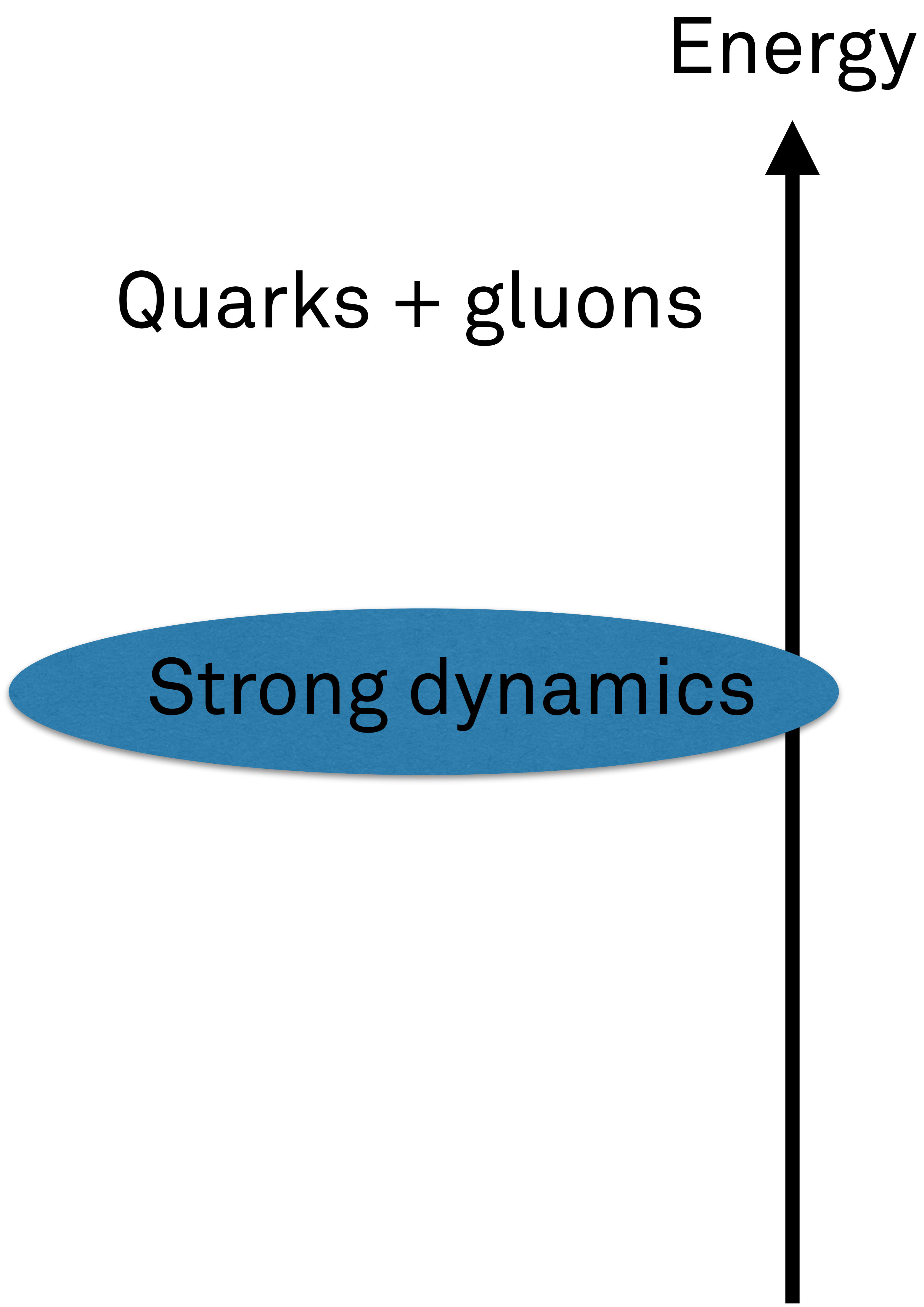
Computed by counting massless fermions charged under a symmetry.

EXAMPLE: 2-FLAVOUR QCD (MASSLESS)

	$[SU(3)]$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$
$Q_L = (u_L \;\; d_L)$	3	2	1	1
$Q_R = (u_R \;\; d_R)$	$\overline{3}$	1	2	-1

In massless limit, $SU(2)_L \times SU(2)_R$ is exact.

Both SU(2)'s have odd number of fermions so both have 't Hooft anomalies.



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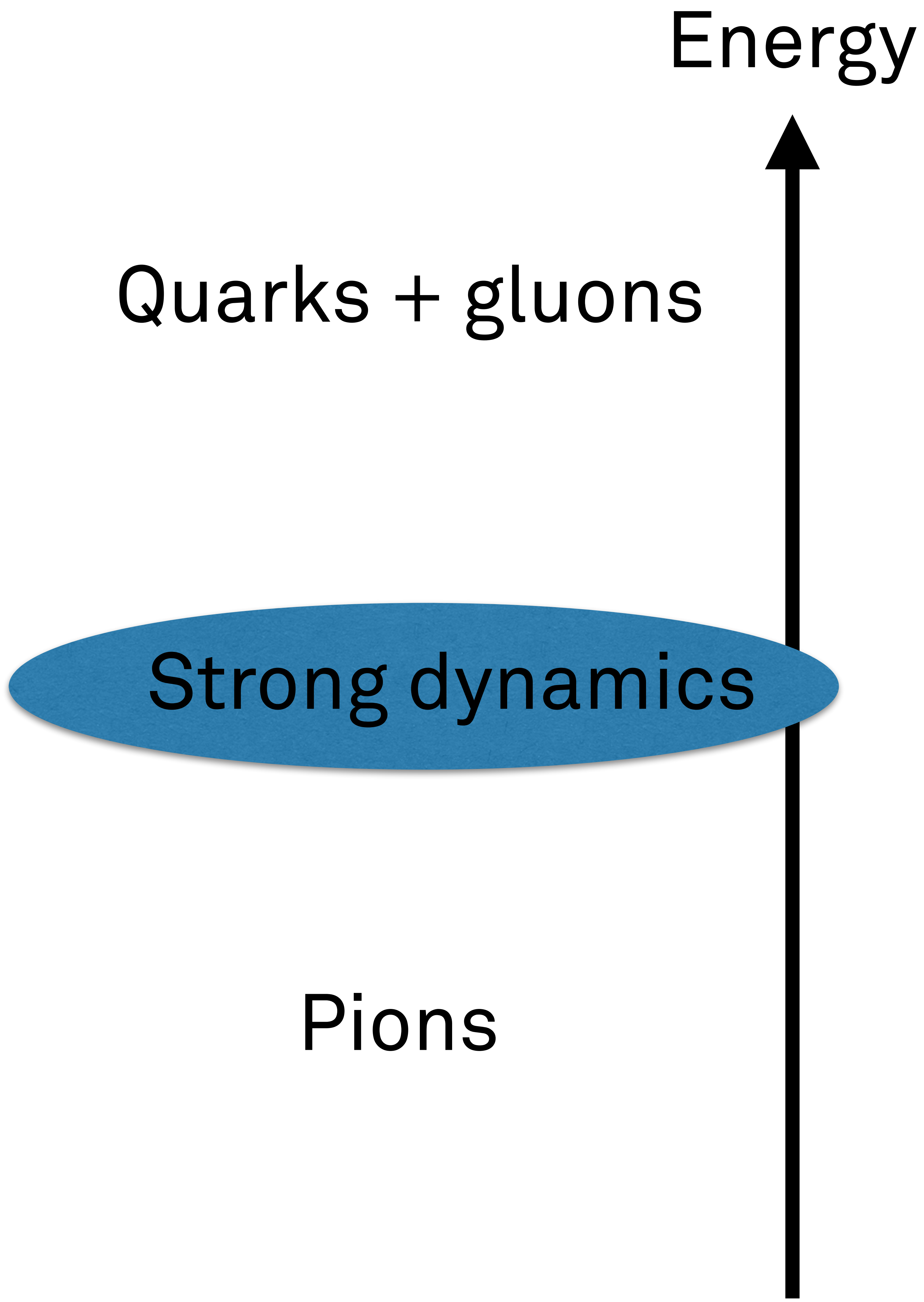
In massless limit, $SU(2)_L \times SU(2)_R$ is exact.

Both SU(2)'s have odd number of fermions so both have 't Hooft anomalies.

Break $SU(2)_L \times SU(2)_R \to SU(2)_V$, pions are massless Goldstone bosons.



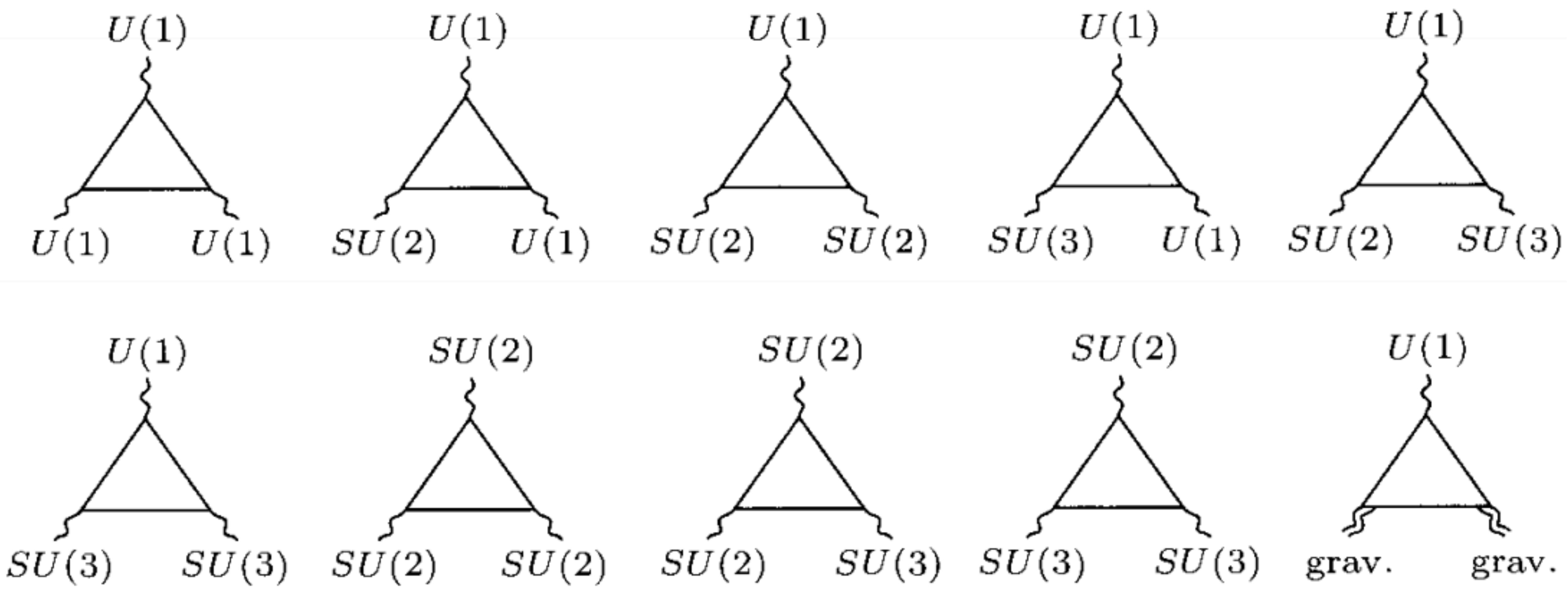
ASIDE: GAUGE ANOMALIES

't Hooft anomalies are not gauge anomalies.

Gauge anomalies = bad.

SM gauge anomaly cancellation appears miraculous.

	$[SU(3)]$	$[SU(2)_L]$	$[U(1)_Y]$	$U(1)_{B-L}$
Q_L	3	2	$1/6$	$1/3$
u_R	$\overline{3}$	1	$-2/3$	$-1/3$
d_R	$\overline{3}$	1	$1/3$	$-1/3$
L_L	1	2	$-1/2$	-1
e_R	1	1	1	$+1$



ASIDE: SU(N) GAUGE ANOMALIES

For general $SU(N)$:

$$A(\bar{\square}) = -1$$

$$A\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right) = N - 4$$

$SU(5)$ with $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \bar{\square} = 10 + \bar{5}$ is (simplest?) anomaly free chiral gauge theory.

$SU(5)$ GUT can explain miraculous anomaly cancellation of SM.

ASIDE: AMSB IN THE MSSM

AMSB is extremely predictive.

$$m_i^2 \propto -\frac{d\,\gamma(\phi_i)}{dt} \propto -\,g\beta_g$$

$\beta_g > 0$ for SU(2) and U(1).

Sleptons are tachnyonic, AMSB is excluded.

UV insensitivity means it is very difficult to fix this problem.

AMSB FOR SU(5)

Start with small AMSB.

$$V_{\text{tree}} = m_{3/2}\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right)$$

$$W_{\text{dyn}} = \lambda M^3 + \zeta B_2 M B_1$$

$$V_{\text{tree}} = 0$$

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$$V_{\text{tree}} = m_{3/2}\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right)$$

$$W_{\text{dyn}} = \lambda M^3 + \zeta B_2 M B_1$$

$$V_{\text{tree}} = 0$$

A = Scale anomaly, effect is due to change of couplings with scale.

Yukawa couplings are marginal, do not change with scale at tree level.

Do change at loop level.

HOW TO BREAK SUSY?

In UV, can have squark mass and gaugino mass.

In IR, scalar (hadrino) mass and A terms.

Symmetry breaking condition depends on IR terms.

Relation between IR and UV is non-perturbative.

	$[SU(5)]$	$SU(3)_A$	$SU(3)_F$	$U(1)_B$	$U(1)_R$
A	10	3	1	1	0
$\overline{F}$	$\overline{5}$	1	3	-3	$\frac{2}{3}$
W^α	24	1	1	0	1
$M \equiv A^3\,\overline{F}$		8	3	0	$\frac{2}{3}$
$B_1 \equiv A\,\overline{F}\,\overline{F}$		3	$\overline{3}$	-5	$\frac{4}{3}$
$B_2 \equiv A^5$		6	1	5	0

$$W_{\rm dyn} = \lambda\, M^3 + \zeta\, B_2\, M\, B_1$$

AMSB AT 1 LOOP

$$V_{\text{soft}} = m_1^2 |B_1|^2 + m_2^2 |B_2|^2 + m_3 |M|^2 + A_1 \lambda M^3 + A_2 \zeta B_1 M B_2$$

$$A_1 = -3\gamma(M)\, m_{3/2}$$

$$A_2 = -\left\{\gamma(M) + \gamma(B_1) + \gamma(B_2)\right\}\, m_{3/2}$$

$$m_i^2 = -\frac{d\,\gamma(\phi_i)}{dt} |m_{3/2}|^2$$

All loop effects can be written in terms of anomalous dimensions.

Straightforward to calculate.

SUSY - II

$$\langle M^{ai}\rangle = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Another option:

$$SU(3)_A \times SU(3)_{\bar{F}} \times U(1)_B \to SU(2)_V \times U(1)_A \times U(1)_B$$

Massless fermions:

$$(4)_{1,-5} \subset B_1$$
$$(1)_{-4,5} \subset B_2$$
$$(3)_{0,0} \subset M$$

Massless bosons: $(5)_{0,0} + (3)_{0,0} + (2)_{3,0}$

SUSY - III

$$\langle M^{ai} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f\sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f\cos\theta \end{pmatrix}$$

Another option:

$$SU(3)_A \times SU(3)_{\bar{F}} \times U(1)_B$$
$$\rightarrow U(1)_{A3} \times U(1)_{A8} \times SU(2)_F \times U(1)_B$$

Massless fermions: $6 \subset B_1$

$3 \subset B_2$

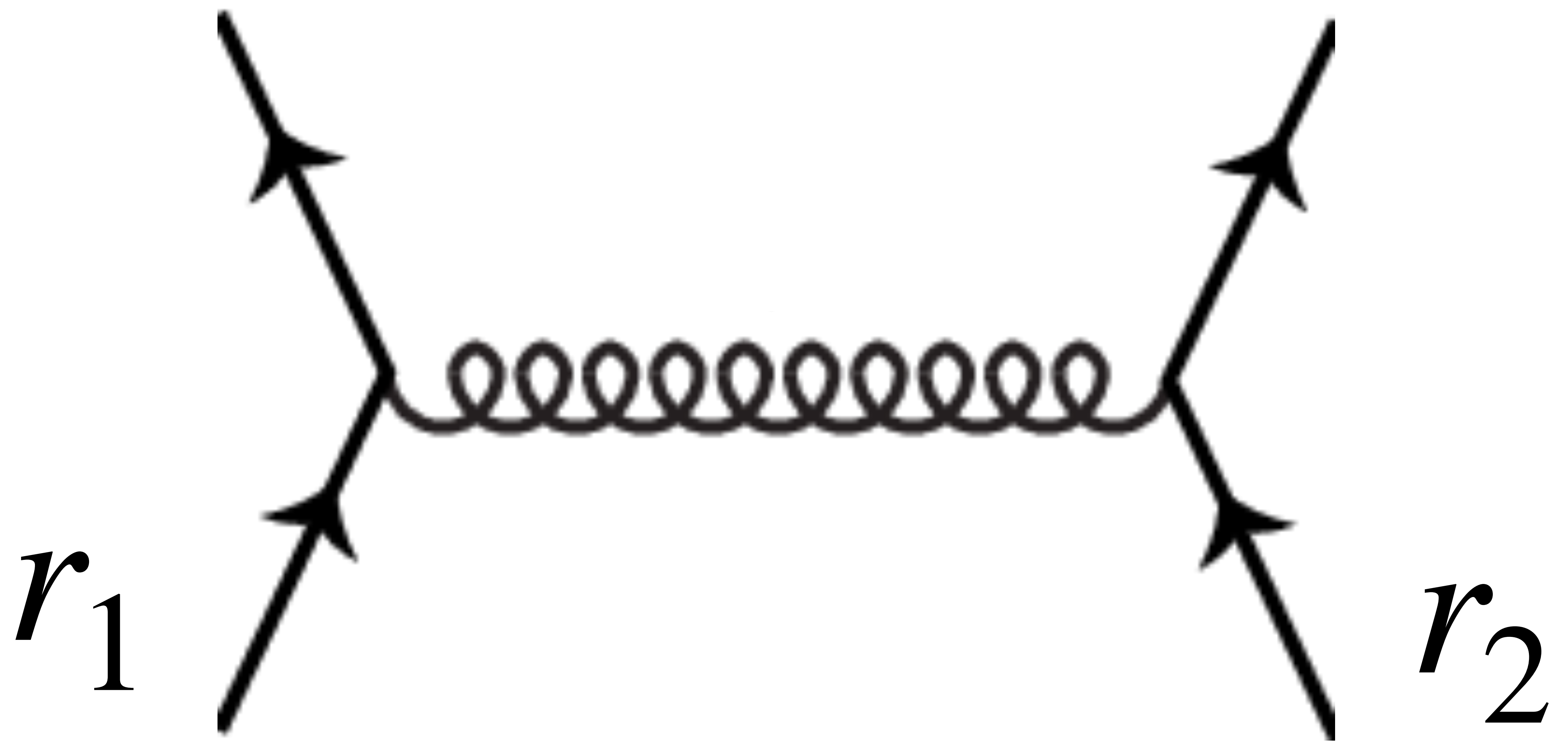
$12 \subset M$

Massless bosons: 12 (one too many?)

MAXIMALLY ATTRACTIVE CHANNEL

Can estimate fermion condensate by seeing which of these diagrams is most negative (MAC).

Dimopoulos, Raby, Susskind, NPB '80.



Can compute in terms of group theory factors.

$$\max C_2(r_1) + C_2(r_2) - C_2(r_c)$$

MAC = $\langle AA \rangle \rightarrow \bar{5}$.

See also Bolognesi, Konishi, arXiv:1906.01485.

MAC + COMPLEMENTARITY

Can parameterize symmetry breaking by scalar with same quantum numbers.

Fradkin, Shenker, PRD ’79.

$H \sim (\bar{5},6,1)_2$ order parameter.

Not enough symmetries to diagonalize.

	$[SU(5)]$	$SU(3)_A$	$SU(3)_{\overline{F}}$	$U(1)_B$
A	10	3	1	1
$\overline{F}$	$\overline{5}$	1	3	-3

MAC I

Guess:

$$\langle H^a_{ij} \rangle = \delta^{a5}\,\delta_{ij}\,f$$

Breaking:

$$[SU(5)] \times SU(3)^2 \times U(1) \to [SU(4)] \times SO(3)_A \times SU(3)_F \times U(1)_{B'}$$

$$A = (6,3,1)_0 + (4,3,1)_{5/2}$$

$$F = (\bar{4},1,3)_{-5/2} + (1,1,3)_{-5}$$

$[SU(4)]$ is vectorlike and confines.

Breaks global symmetry down to $SO(3)_V \times U(1)_B$.

MAC I = SUSY I.

MAC II

Guess:

$$\langle H^a_{ij} \rangle = \delta^{a5}\, \delta_{i3}\, \delta_{j3}\, f$$

Breaking:

$$[SU(5)] \times SU(3)^2 \times U(1) \to [SU(4)] \times SU(2)_A \times SU(3)_F \times U(1)_{B'}$$

$$A = (6,2,1)_0 + (6,1,1)_0 + (4,2,1)_{5/2} + (4,1,1)_{5/2}$$

$$F = (\bar{4},1,3)_{-5/2} + (1,1,3)_{-5}$$

$[SU(4)]$ confinement further breaks to $U(1)_V \times U(1)_{B'}$.

MAC II SPECTRUM

Guess:

$$\langle H^a_{ij} \rangle = \delta^{a5}\, \delta_{i3}\, \delta_{j3}\, f$$

Breaking:

$$[SU(5)] \times SU(3)^2 \times U(1) \to U(1)_V \times U(1)_{B'}$$

Fermions: $(\pm 1, -5)\ +\ (0, -5)$

Bosons: 15 GBs.

MAC III

Guess:

$$\langle H_{ij}^{a} \rangle = \delta_i^a \, \delta_{ij} \, f$$

Breaking:

$$[SU(5)] \times SU(3)^2 \times U(1) \to [SU(2)] \times SU(3)_F \times U(1)_A^2 \times U(1)_{B'}$$

$[SU(2)]$ confines and could break $SU(3)_F \to SU(2)_F$.

Fermions: 15

Bosons: 11 GBs.

SUSY QCD

Supersymmetric $SU(N_c)$ gauge theory with N_f flavours of quarks and anti-quarks understood by Seiberg long ago. Seiberg, hep-th/9402044.

SQCD + AMSB analyzed for $N_c \geq 3$ in first paper. Murayama, 2104.01179.

If $N_f = N_c + 1$, theory is s-confining and described by dynamical superpotential in terms of mesons and baryons.

$$W_{\text{dyn}} = BMB - \det M$$

MESON AND BARYON

For $N_f = N_c + 1$:

$$(M)^i_j = \bar{q}^{\alpha i} q_{\alpha j}$$

$$B^i = \det q$$

$$\bar{B}_i = \det \bar{q}$$

$$W_{\text{dyn}} = BMB - \det M$$

In terms of the low energy fields, first term is tri-linear and classically scale invariant.

2nd term: $\det M \sim M^{N_f}$. Only scale invariant if $N_f = 3$ (and $N_c = 2$).

SU(3) GELL MANN MATRICES

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$