Asymptotic Safety: A (Non?)-Perturbative Route to Quantum Gravity

THEP Seminar

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UNIVERSITY OF TORONTO

Walter Benjamin-Programm

DFG Deutsche Forschungsgemeinschaft

QFT and the Standard Model

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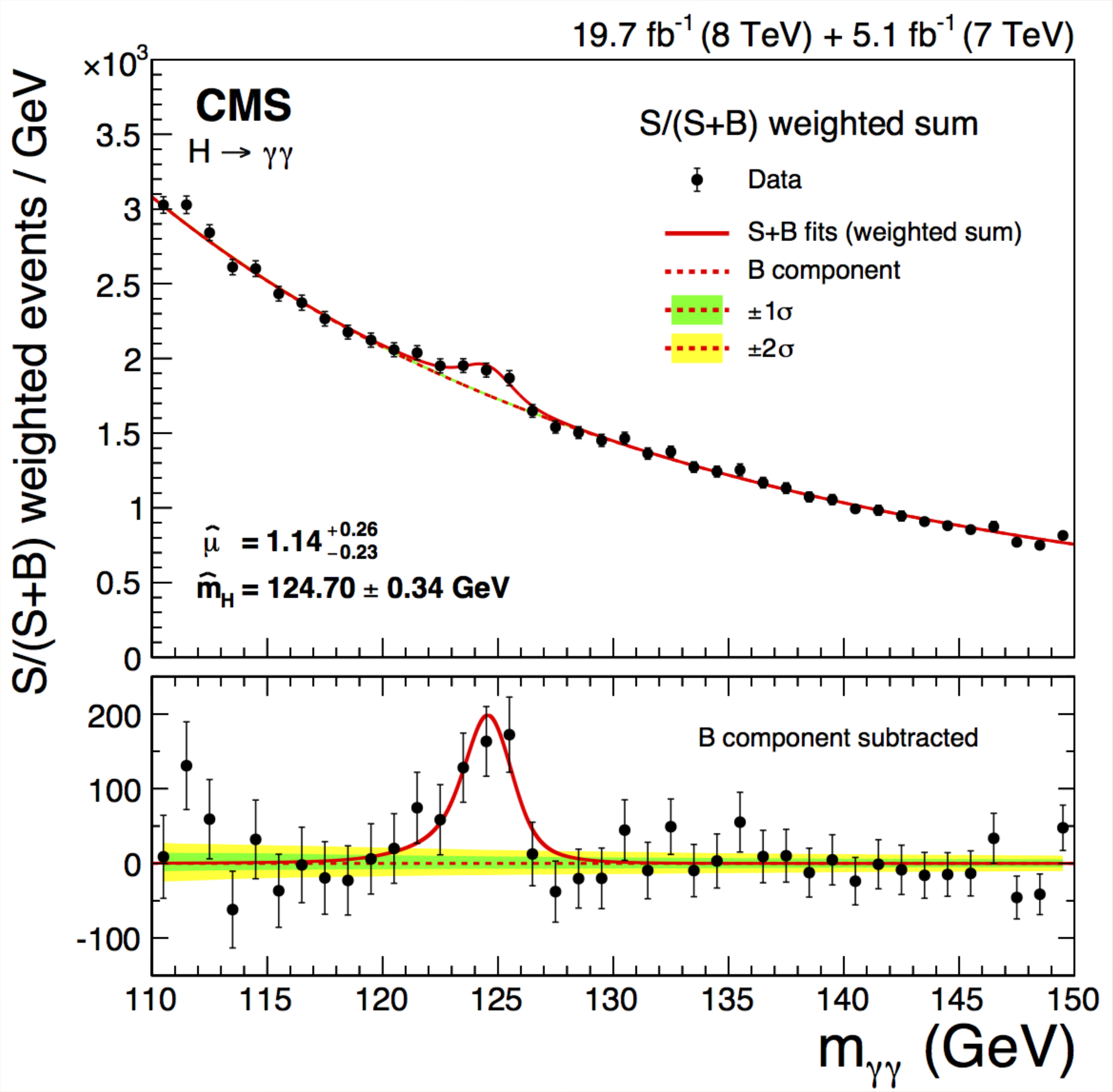

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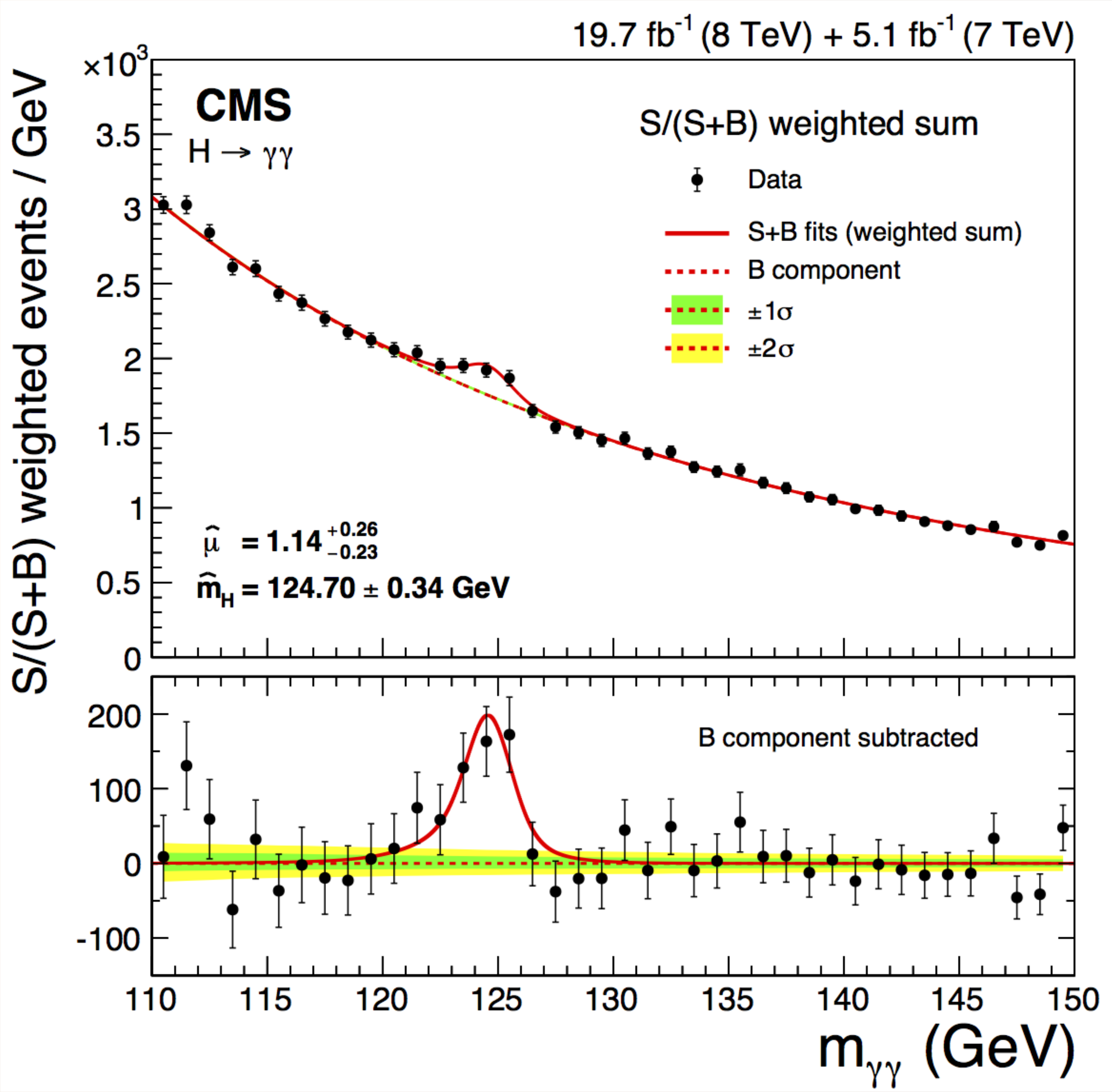
  - Effective Field Theories



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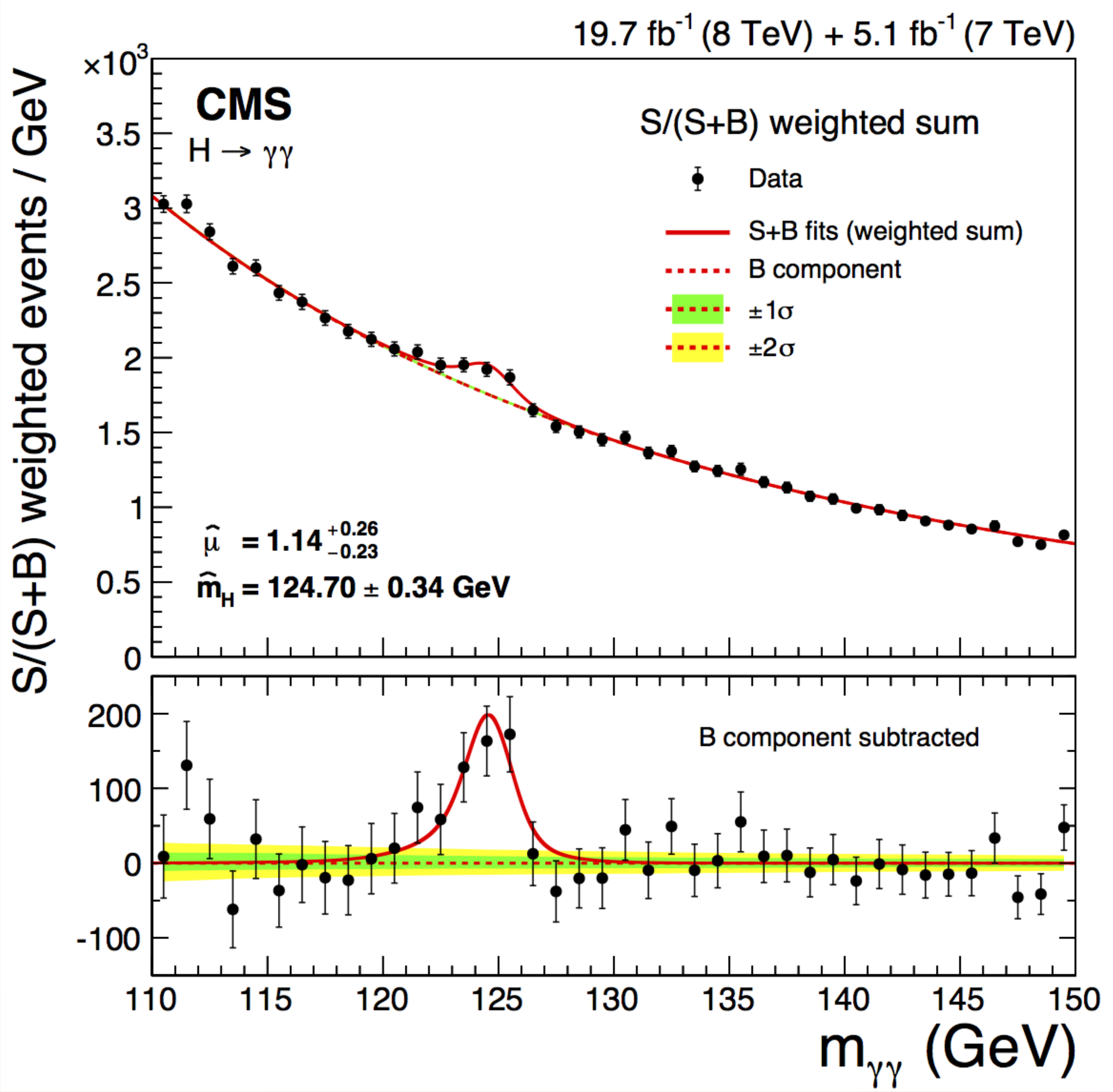

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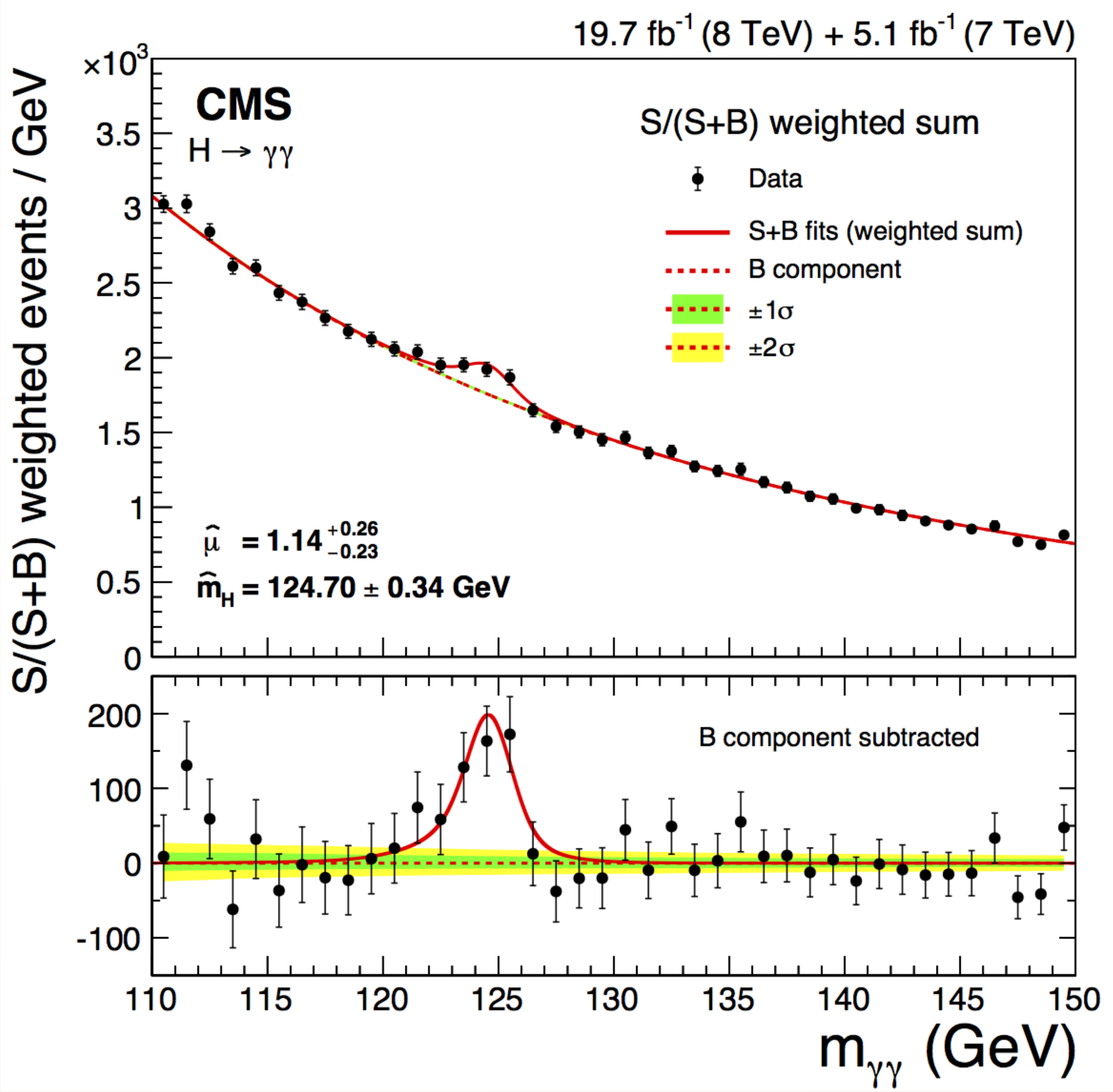

Is nature fundamentally a QFT?

Non-Renormalizability of Quantum Gravity

- Classical Gravity described by Einstein-Hilbert Lagrangian:

$$S_{EH} = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \sqrt{-g}\,(R - 2\Lambda)$$

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- UV divergencies induce counterterms in a quantum field theory of gravity:

$$S = \int \mathrm{d}^d x\sqrt{-g}\Big[\frac{1}{16\pi G_N}R - \frac{\Lambda}{8\pi G_N}$$ **Tree-Level**

$$+\lambda_{1,1}R^2 + \lambda_{1,2}R_{\mu\nu}R^{\mu\nu} + \sigma_E E$$ **One-Loop**

$$+\lambda_{3,1}R^3 + \ldots + \lambda_{C^3}C^{\rho\sigma}{}_{\mu\nu}C^{\mu\nu}{}_{\alpha\beta}C^{\alpha\beta}{}_{\rho\sigma} + \ldots\Big]$$ **Two-Loop**

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Two-Loop

[Goroff and Sagnotti, 1985]
[van de Ven, 1992]

First on-shell divergence beyond EH

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Do we have to give up QFT?

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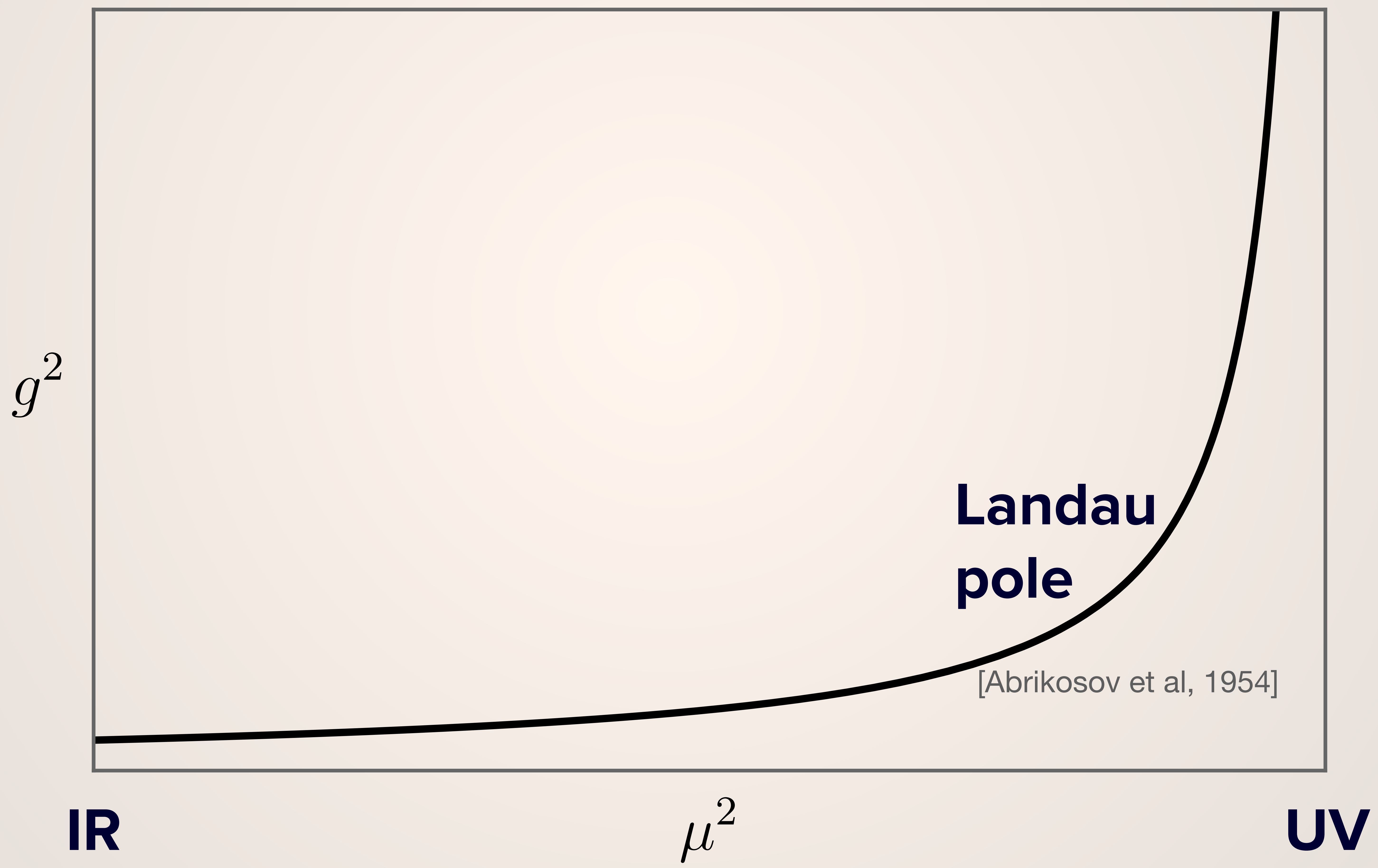
Are there UV complete QFTs?



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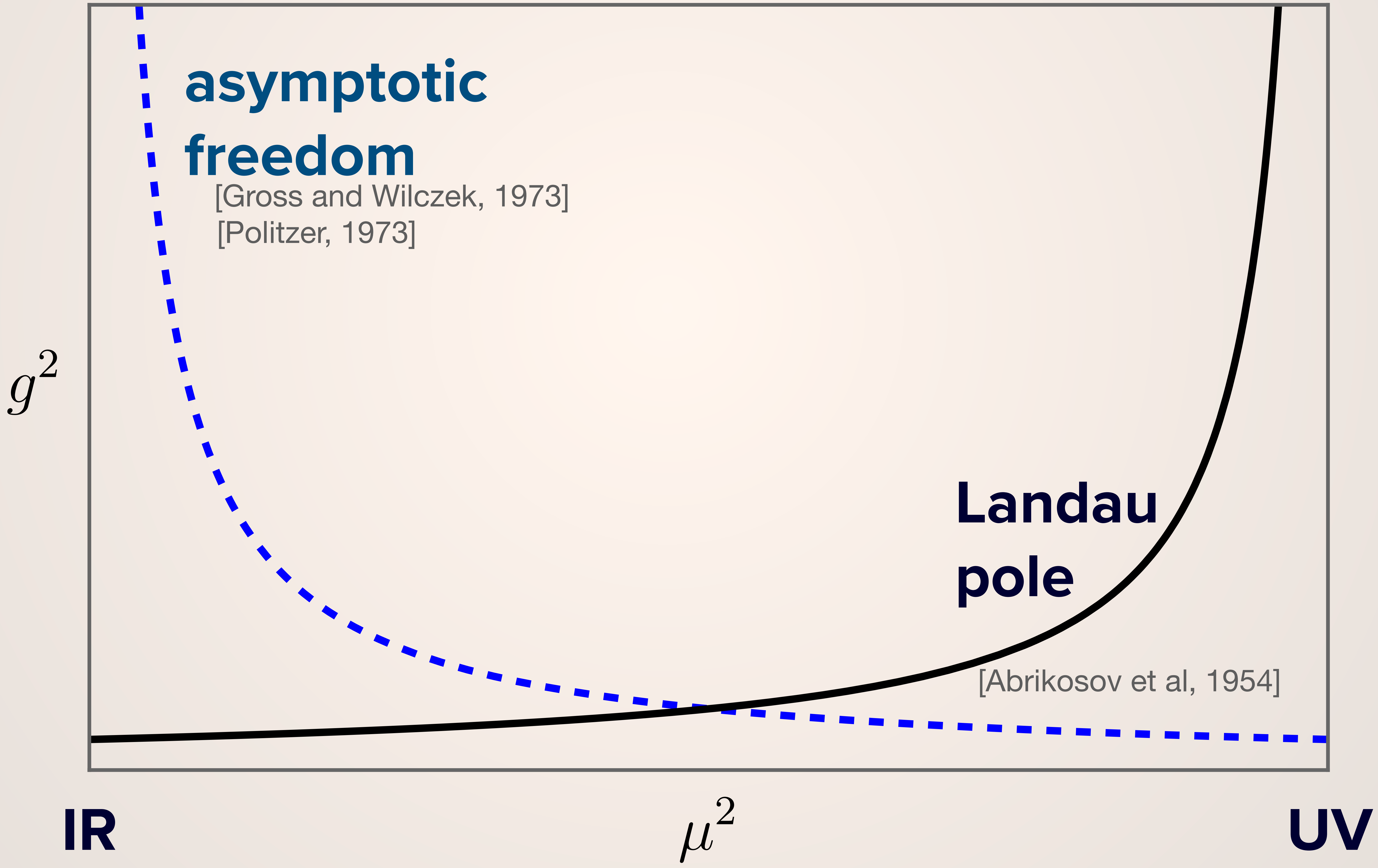

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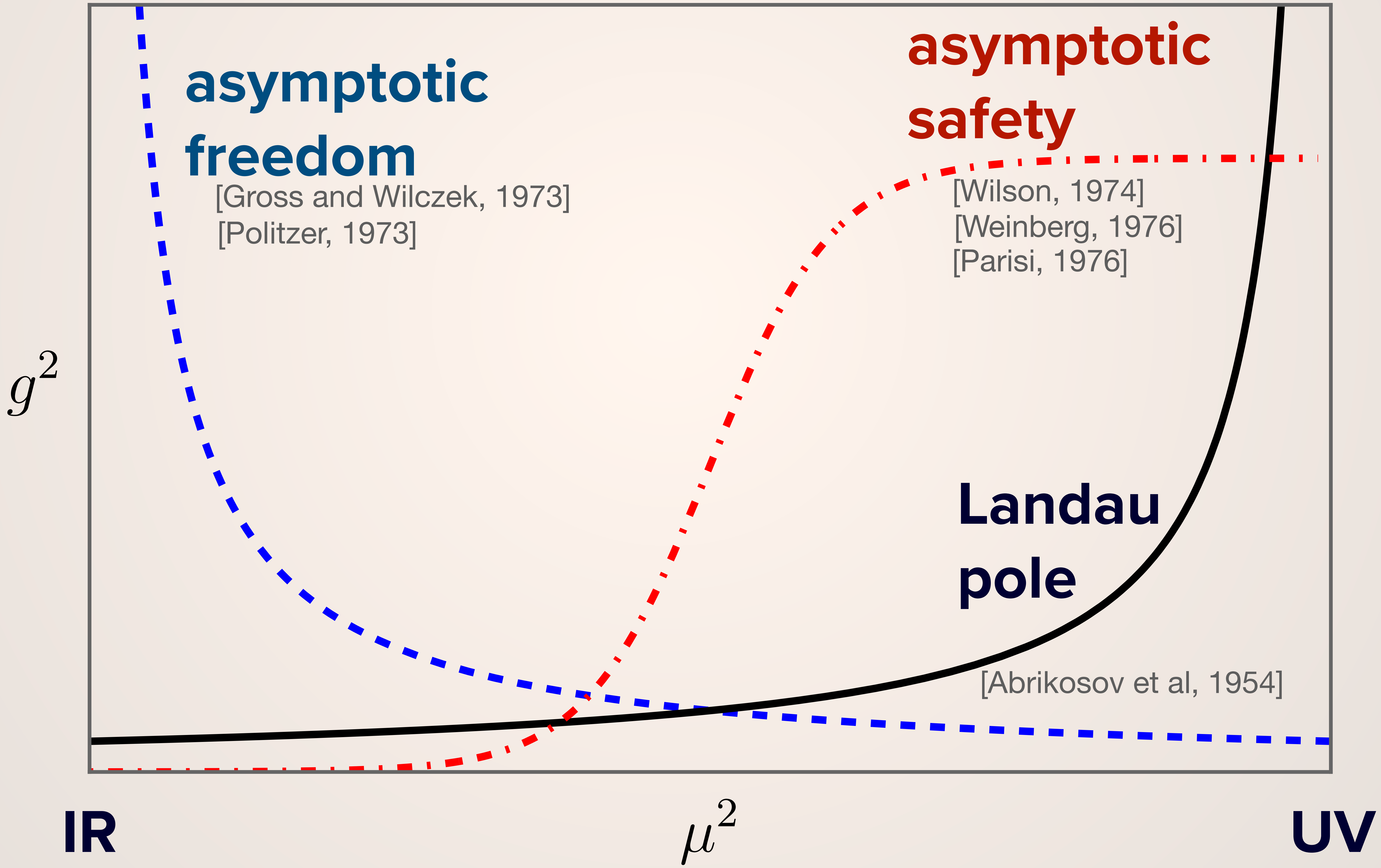
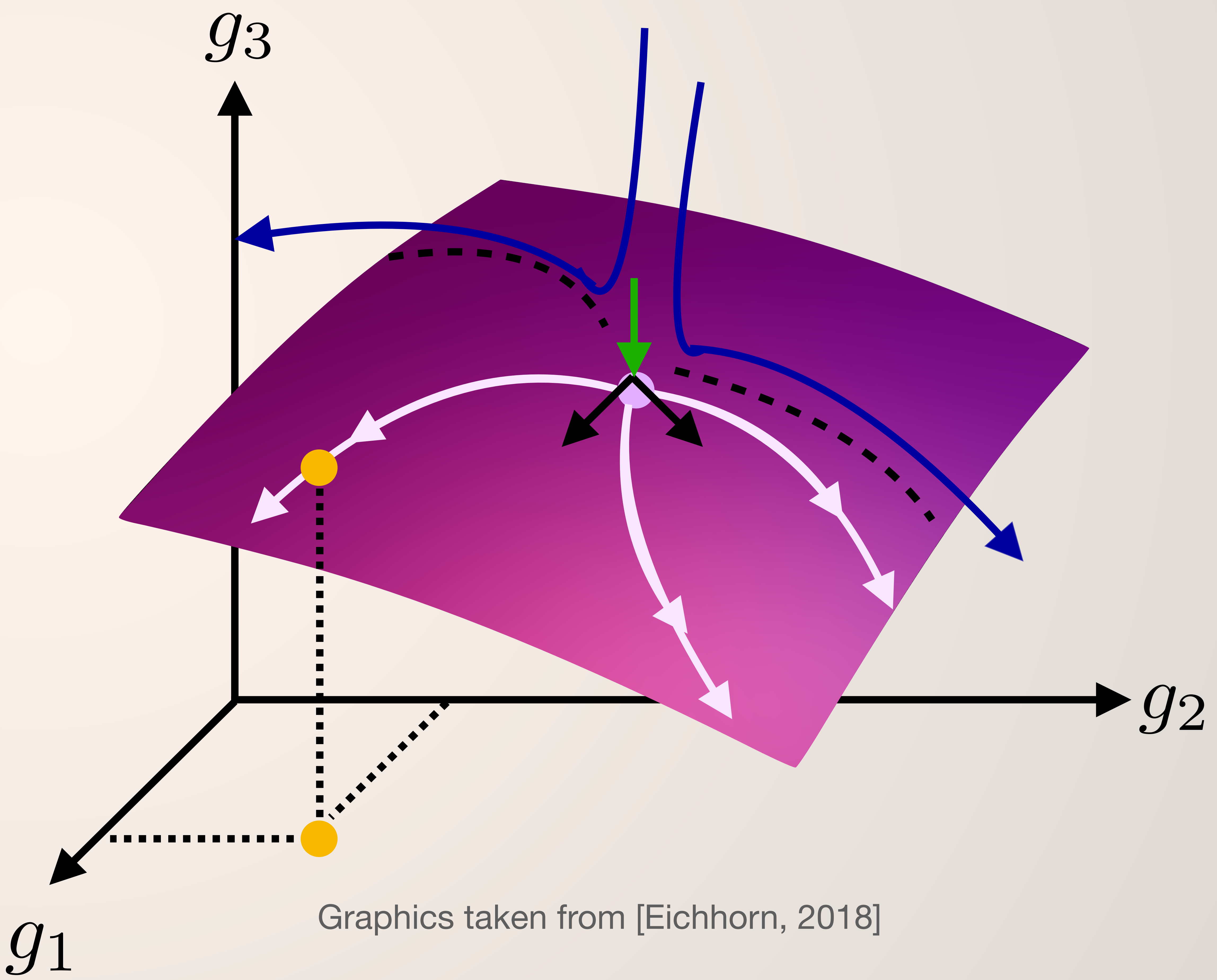


Predictive Power of Asymptotic Safety

- Couplings must flow into UV fixed point
  - Not all trajectories reach UV fixed point
- **Only trajectories on UV critical surface (purple) are asymptotically safe**

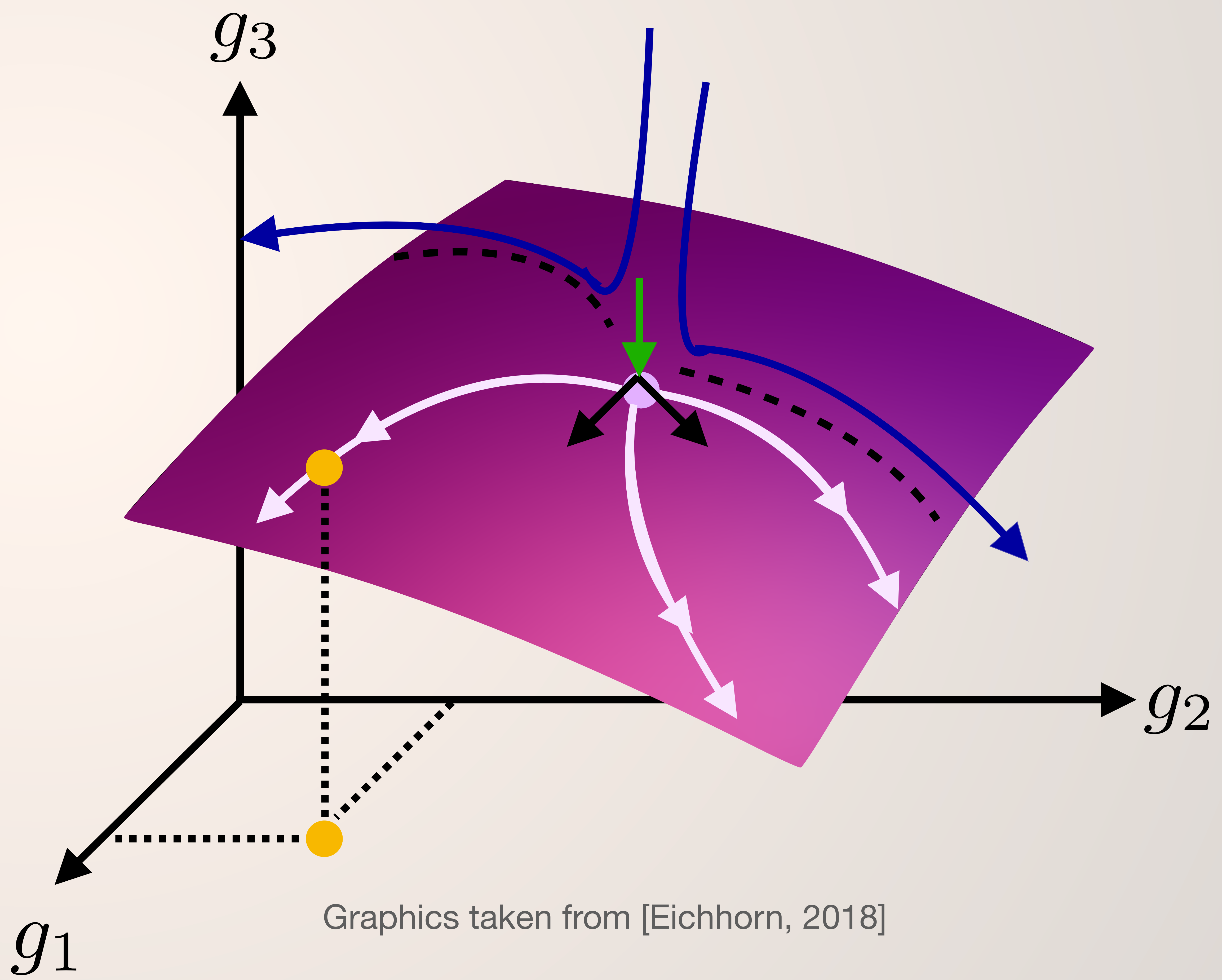


Graphics taken from [Eichhorn, 2018]

Predictive Power of Asymptotic Safety

- Count parameters by linearizing flow around fixed point

$$\beta_i = \frac{\partial \beta_i}{\partial \lambda_j}\bigg|_* (\lambda_j - \lambda_j^*) + \dots$$



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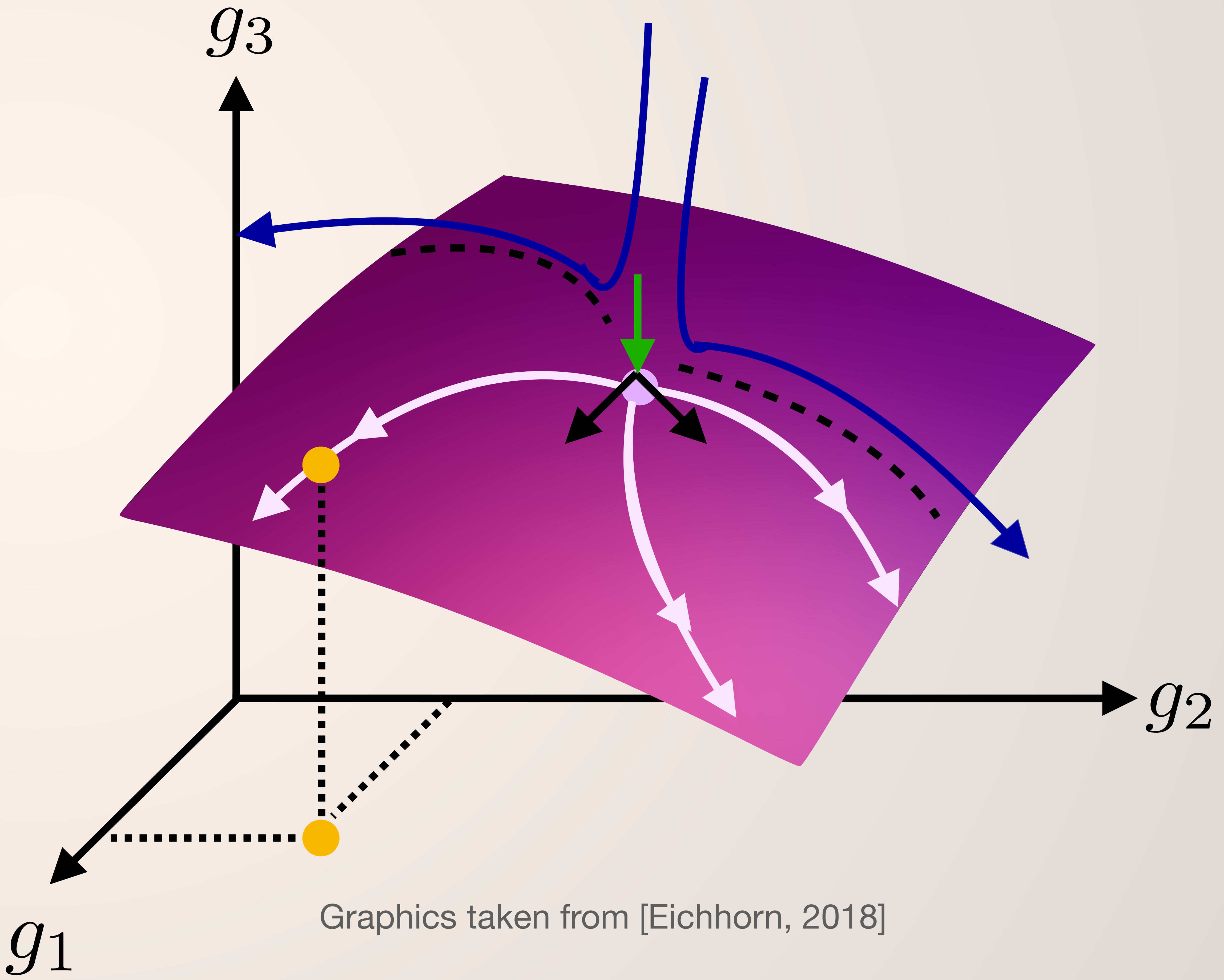
Predictive Power of Asymptotic Safety

- Count parameters by linearizing flow around fixed point

$$\left.\frac{\partial \beta_i}{\partial \lambda_j}\right|_* v_j^{(\ell)} = \theta_\ell v_i^{(\ell)}$$

- $\theta_\ell > 0$: irrelevant parameter
- $\theta_\ell < 0$: **relevant parameter**



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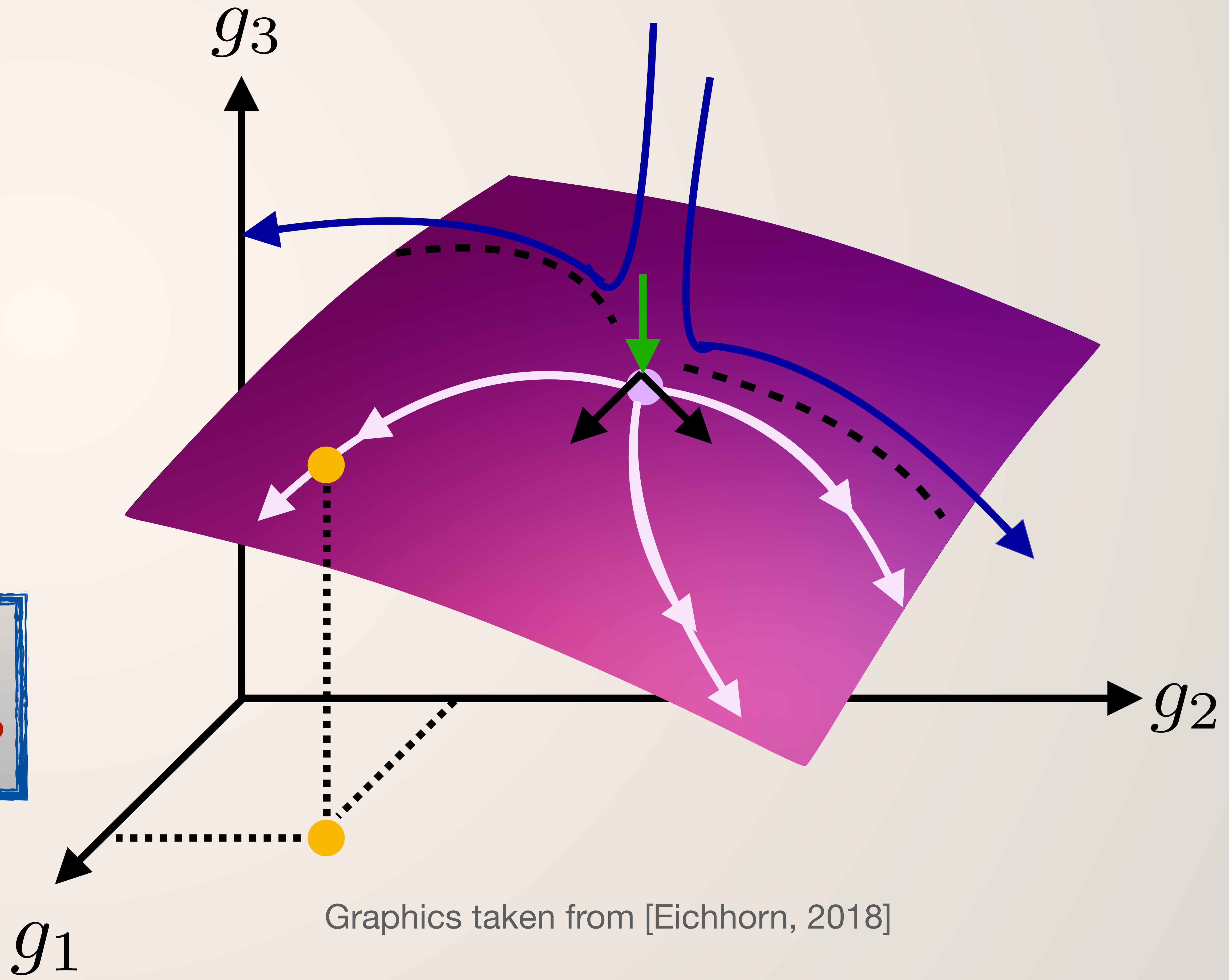
Does QG have a UV fixed point with finitely many relevant directions?



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- **Gauge-Yukawa Theories in 4d** [Litim and Sannino, 2014]

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left[\bar{\psi} i \not{D} \psi\right] - y\left[\bar{\psi}\phi\psi\right] + \left[\partial^\mu \phi^\dagger \partial_\mu \phi\right] - m^2\left[\phi^\dagger\phi\right] - u\left[\phi^\dagger\phi\phi^\dagger\phi\right] - v\left[\phi^\dagger\phi\right]\left[\phi^\dagger\phi\right]$$

  - Conventionally Renormalizable
  - Rigorous UV fixed point under perturbative control $\alpha_g \propto \epsilon$ with $\epsilon = \frac{n_F}{N_c} - \frac{11}{2}$

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- **Gross-Neveu model in 3d** [Gross and Neveu, 1974]

$$\mathscr{L}_{\mathrm{GN}} = \bar{\psi}_0\left(i\not{\partial} - M_0\right)\psi_0 + \frac{G_0}{2}(\bar{\psi}_0\psi_0)^2$$

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    - Large N, conformal bootstrap, FRG, … [Rosenstein and Warr and Park, 1989] [de Calan et al, 1991]

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    - **Blueprint for Quantum Gravity**

Asymptotic Safety from the FRG

Functional Renormalization Group

- Introduce scale dependent Legendre effective action Γ_k (effective average action)
- RG-scale dependence introduced by IR regulator in path integral

$$\mathcal{I}[J,\bar{g}] = \int \mathcal{D}h_{\mu\nu}\, e^{-\int \mathrm{d}^d x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k^{\mu\nu\rho\sigma}[\bar{g}] h_{\rho\sigma} - \mathcal{S}[\bar{g}+h] + \int \mathrm{d}^d x \sqrt{\bar{g}}\, J^{\mu\nu} h_{\mu\nu}}$$

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- Functional Renormalization Group

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left\{ \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \right\}$$ [Wetterich, 1993]

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[Wetterich, 1993]

- Reproduces Perturbation Theory
- Successful in non-perturbative territory (QCD, Wilson-Fisher, ...)

Non-Perturbative Approximations

- FRG is non-perturbative in couplings
    - Various approximation schemes can be applied that are not based on expansions in coupling

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- FRG is non-perturbative in couplings
  - Various approximation schemes can be applied that are not based on expansions in coupling
- Approximate by truncating form of effective action:

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g} \left[\frac{\Lambda}{8\pi G_N} - \frac{R}{16\pi G_N} + \lambda_{R^2} R^2 + \lambda_{R^{\mu\nu}R_{\mu\nu}} R^{\mu\nu} R_{\mu\nu} + \lambda_E E \right.$$

$$+ \lambda_{R^3} R^3 + \lambda_{C^3} C^3 + \dots$$

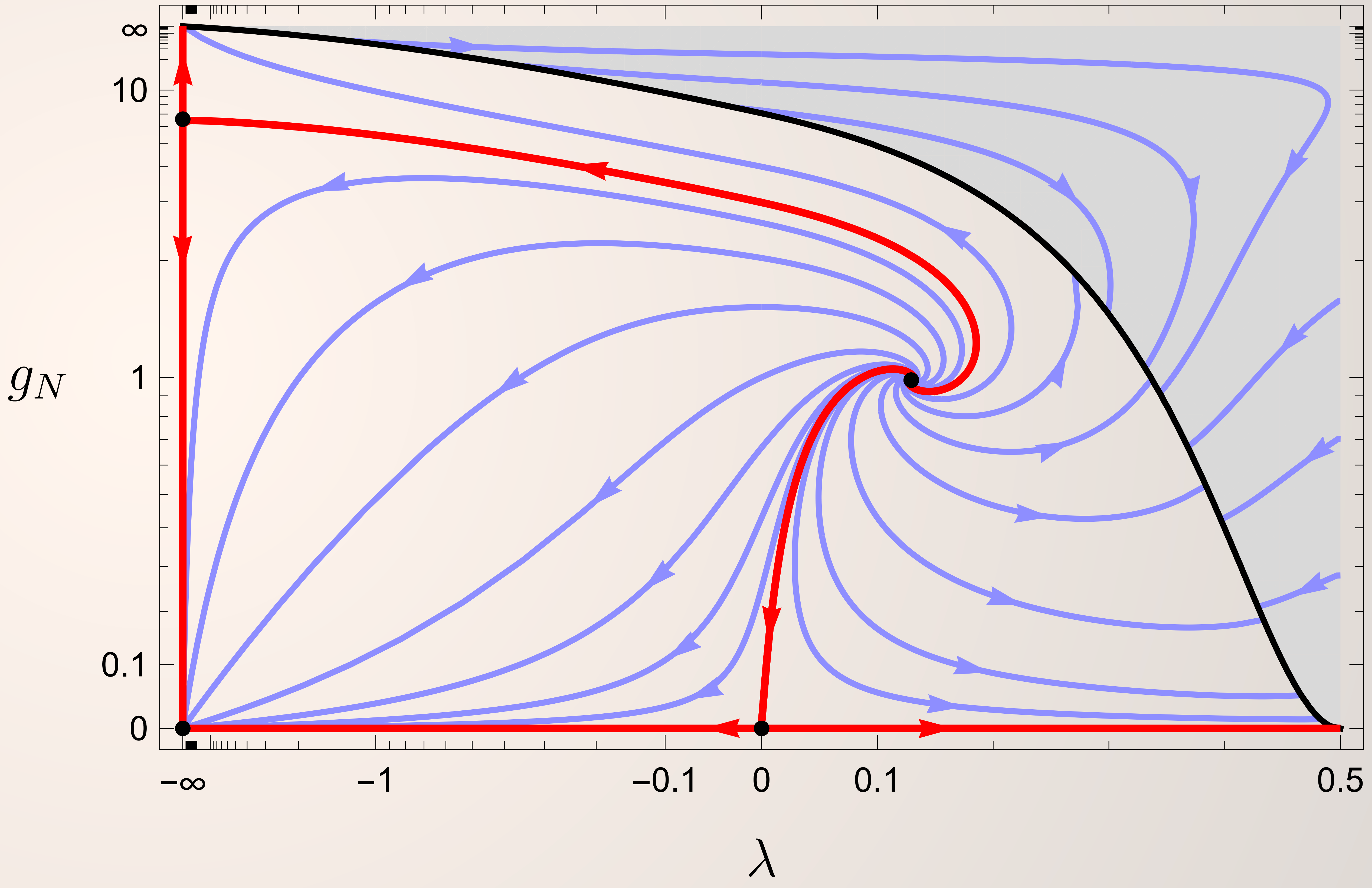
$$\left. + \lambda_{R^4} R^4 + \dots \right]$$

Einstein-Hilbert Gravity

- Fixed Point can already be seen in Einstein-Hilbert [Reuter, 1996]

$$\lambda^* = 0.11867 \quad g_N^* = 0.94305$$

$$\theta = -2.4113 \pm 1.9471 I$$



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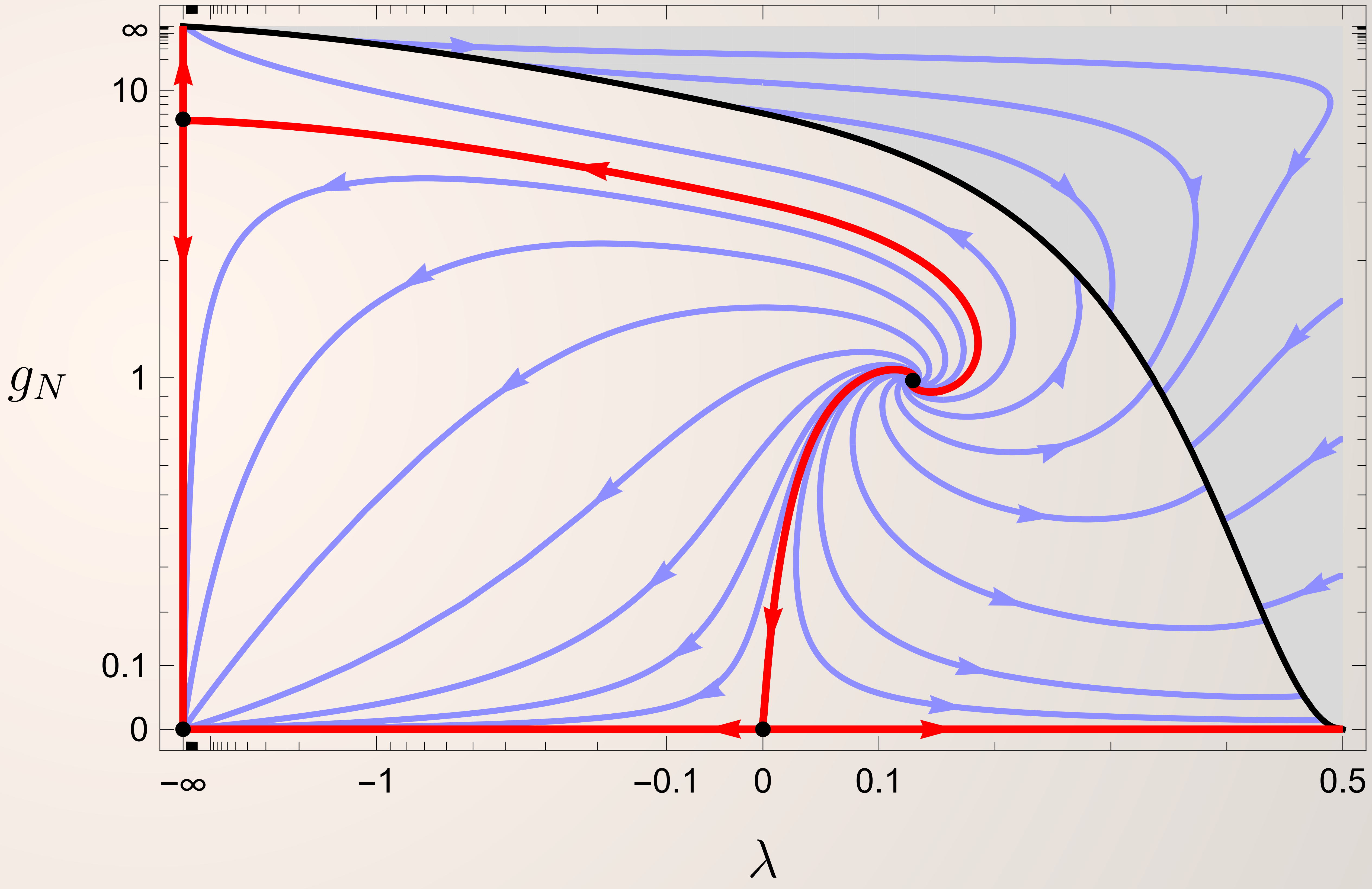


- Is this a predictive theory of quantum gravity?

Curvature Invariants in QG

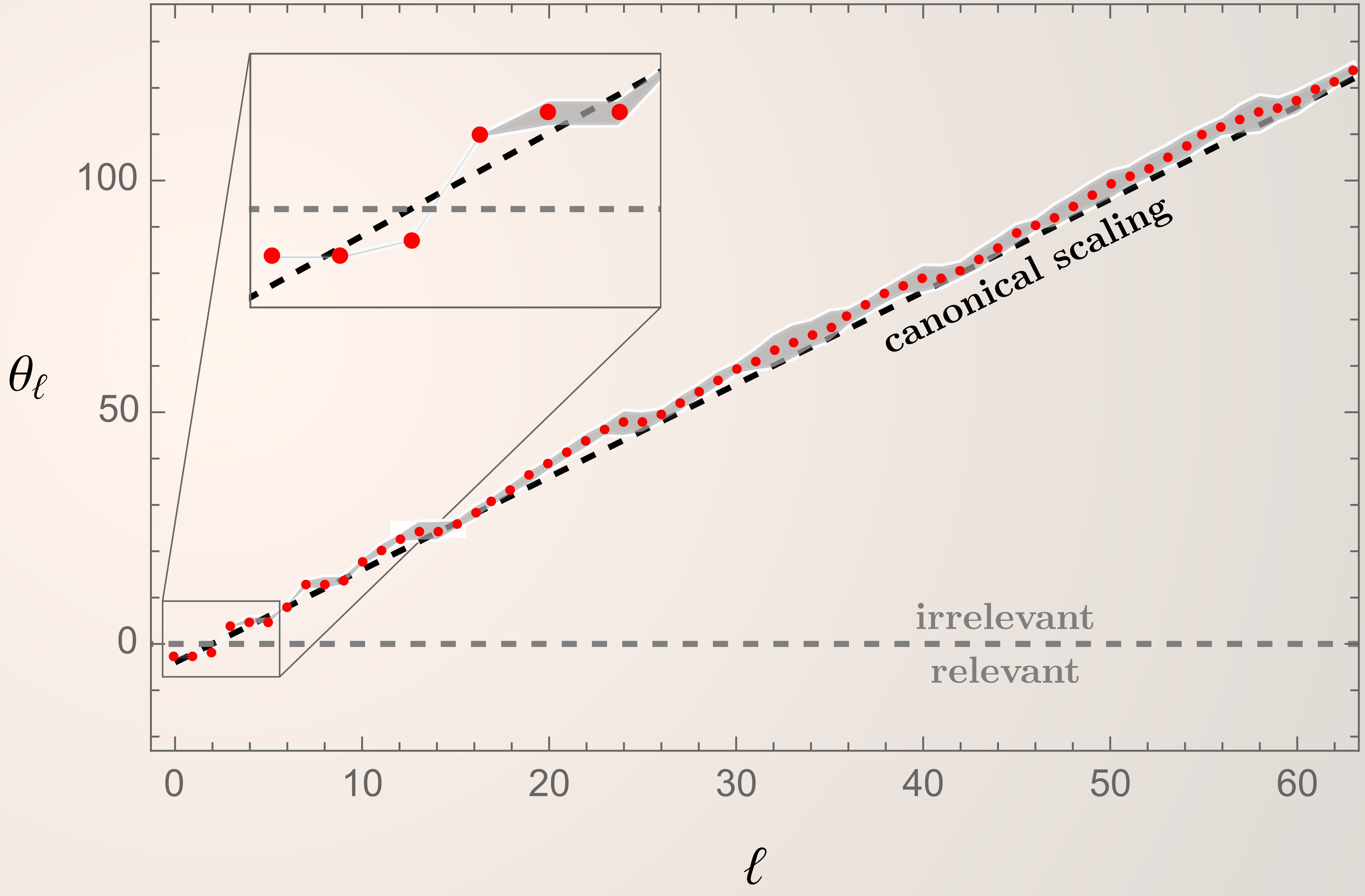
	Curvature Invariants
dim-0	$\mathbb{1}$
dim-2	R
dim-4	$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, \Box R$
dim-6	$R^3\,, RR_{\mu\nu}R^{\mu\nu},\, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu},\, R_{\mu\nu}R^{\nu\rho}R^{\mu}_{\rho},\, R_{\mu\rho}R_{\nu\sigma}R^{\mu\nu\rho\sigma},\, R^{\mu}_{\nu}R_{\nu\alpha\beta\gamma}R^{\mu\alpha\beta\gamma},$ $R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu},\, R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}R^{\nu\alpha\sigma\beta}, \nabla_{\mu}R\nabla^{\mu}R,\, \nabla_{\rho}R_{\mu\nu}\nabla^{\rho}R^{\mu\nu}$
$\vdots$	

Predictivity of AS

- Fixed point persists in models with larger number of curvatures, e.g. $f(R)$ gravity

$$f(R) = \sum_{\ell=0}^{\infty} \lambda_\ell R^\ell$$

- 3 relevant directions
    - **Eigenvalues follow canonical scaling**



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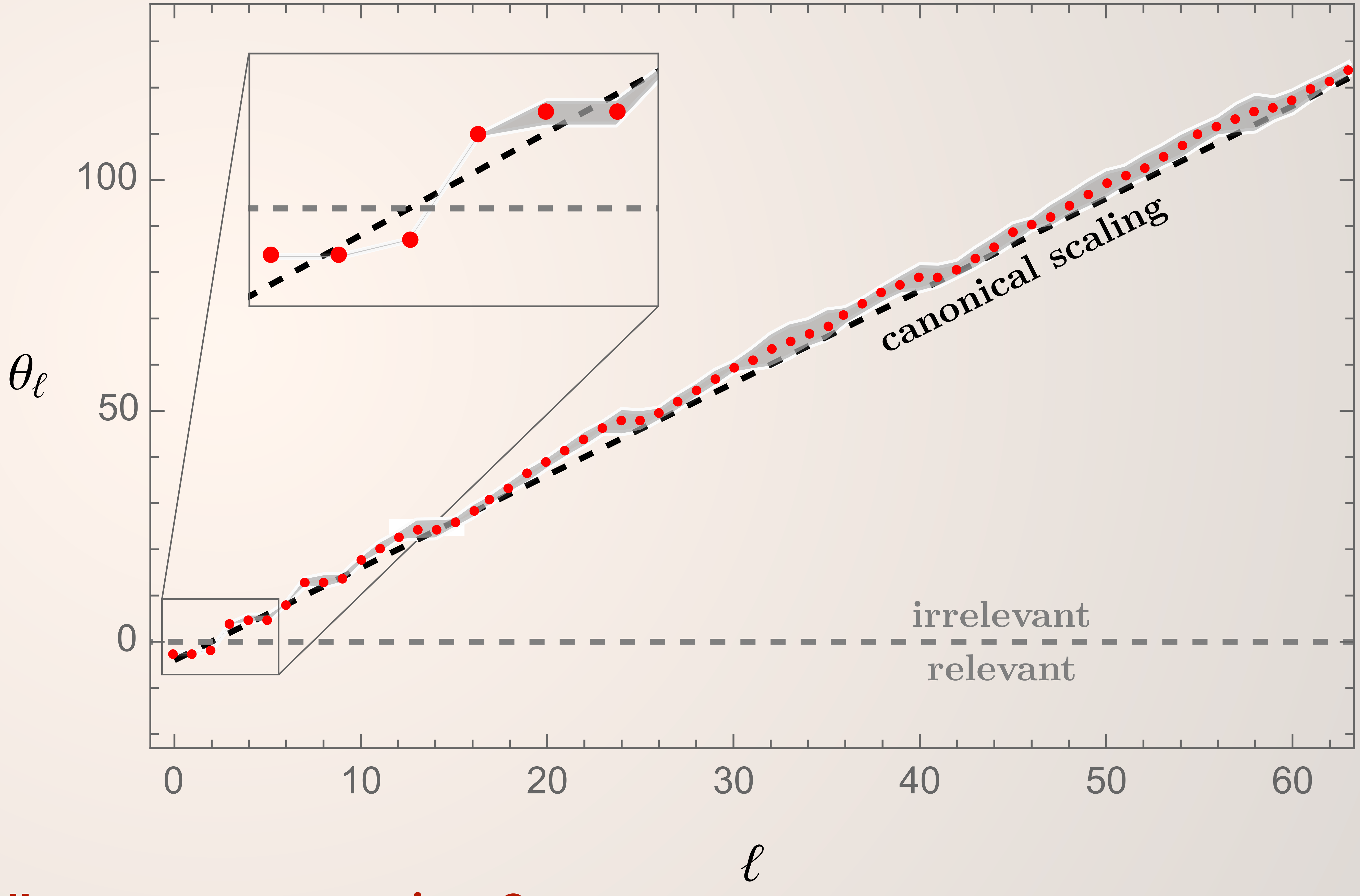
small quantum corrections?



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[Baldazzi and Falls and **YK** and Knorr, 2023]

AS in cubic Gravity

- Using field redefinitions, a complete derivative expansion to sixth order was performed
  - Action only needs to include on-shell terms

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left[\frac{\rho}{8\pi} - \frac{1}{16\pi G_N}R + \sigma_E E + G_{C^3}C^{\rho\sigma}{}_{\mu\nu}C^{\mu\nu}{}_{\alpha\beta}C^{\alpha\beta}{}_{\rho\sigma}\right]$$

  - Off-shell terms absorbed by field redefinitions [Ben Ali-Zinati and Baldazzi and Falls, 2021]

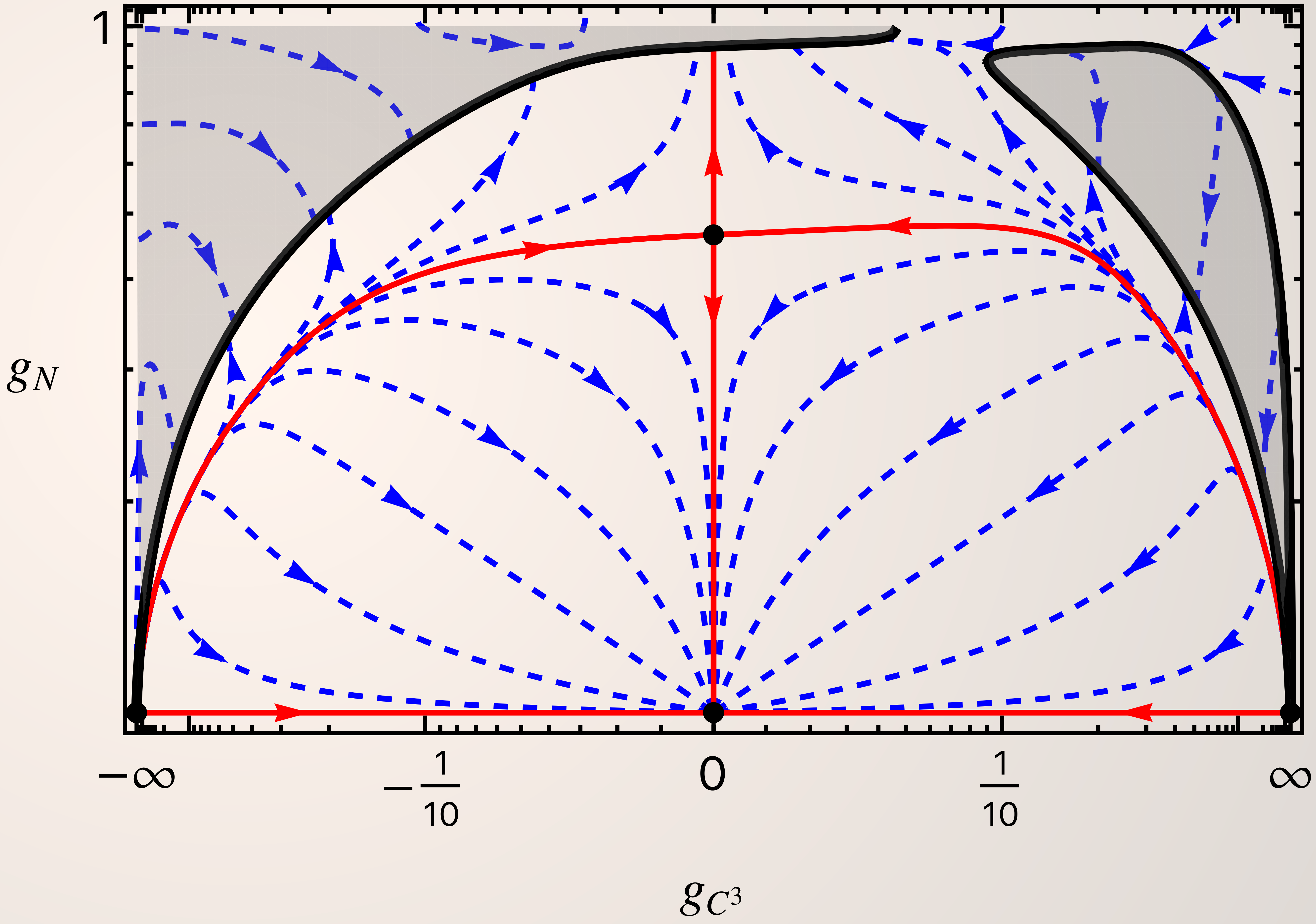
$$\langle \partial_t g_{\mu\nu} \rangle = \gamma_g\, g_{\mu\nu} + \gamma_R\, R\, g_{\mu\nu} + \gamma_S\, S_{\mu\nu} + \gamma_{R^2}\, R^2\, g_{\mu\nu} + \ldots$$

AS in cubic Gravity

- Unique non-trivial fixed point

$$g_N^* = 0.364$$
$$g_{C^3}^* = 4.490 \cdot \mathbf{10^{-7}}$$

- Only one fixed point!



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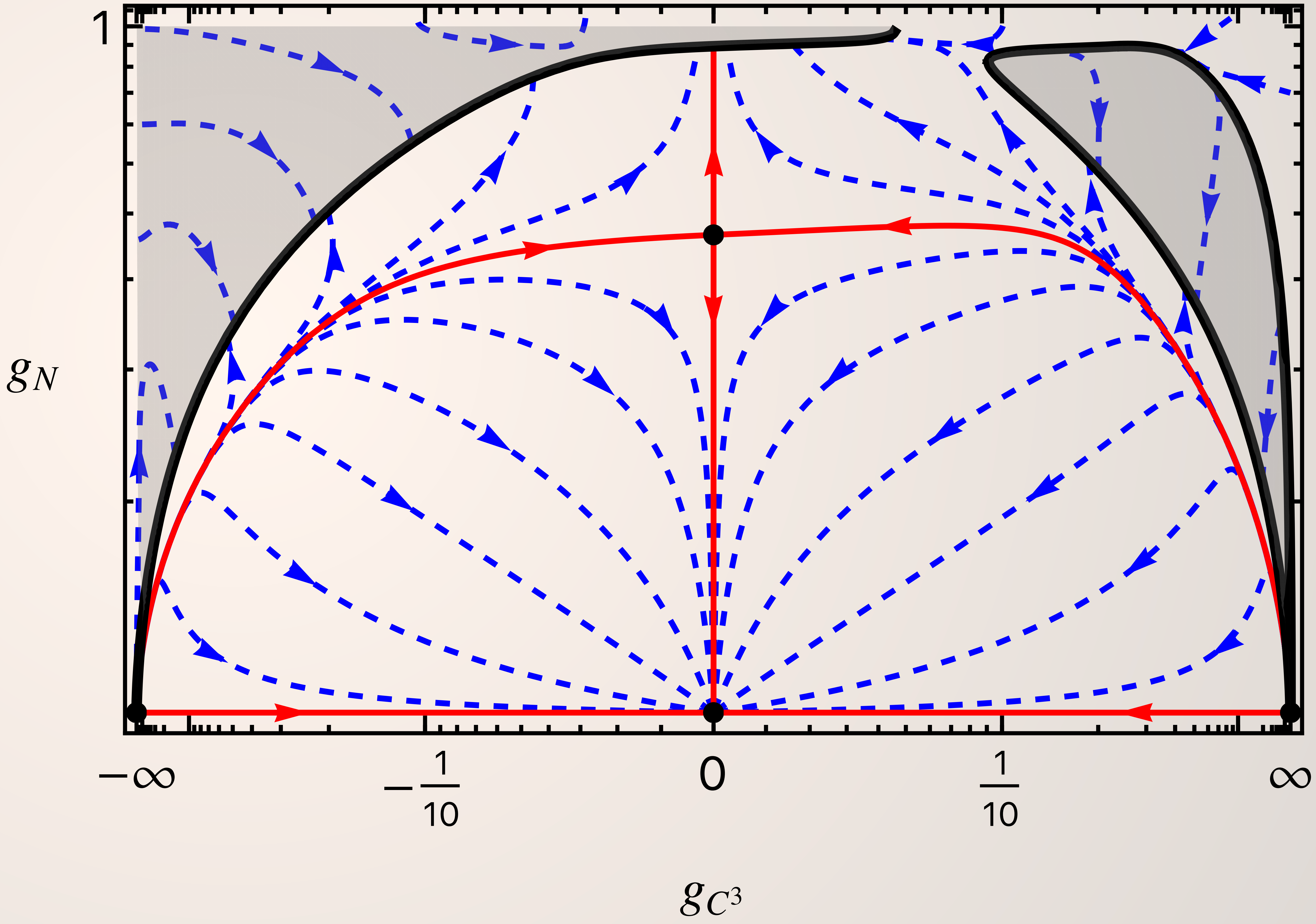


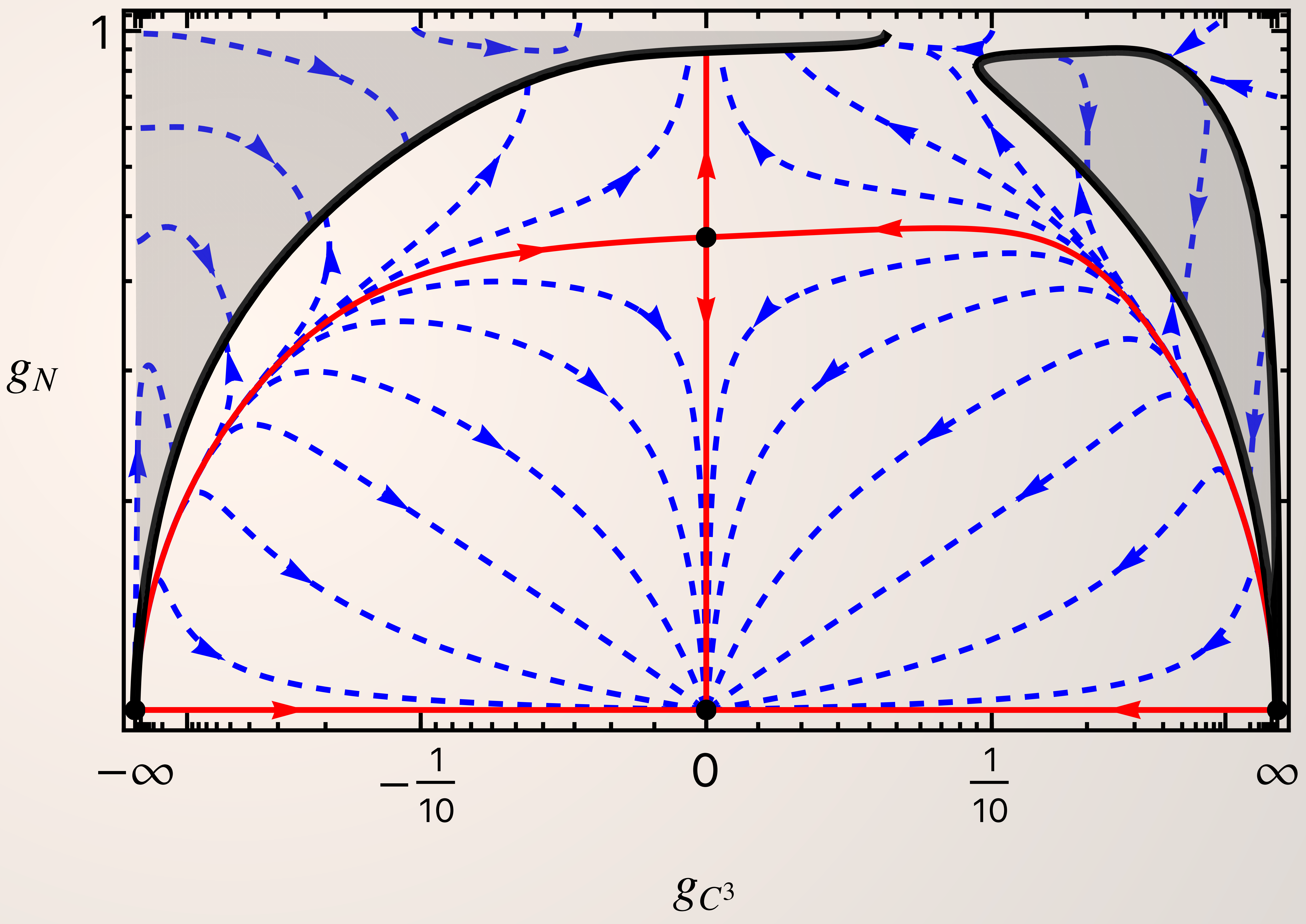
Near-Perturbative?

AS in cubic Gravity

- Critical exponents are stable between different orders

order	θ_1	θ_2
$\mathcal{O}(\partial^2)$	2.239	—
$\mathcal{O}(\partial^4)$	2.228	—
$\mathcal{O}(\partial^6)$	2.225	−3.850

- **Goroff-Sagnotti term irrelevant** [Gies et al, 2016]



AS from the FRG

- Evidence for existence of fixed point in quantum gravity
    - Finite number of relevant directions
    - Operators with large mass dimensions are irrelevant

- Eigenvalues follow canonical scaling
- Perturbatively small fixed point for Goroff-Sagnotti coupling

- Challenges:
    - Diffeomorphism symmetry?
    - Lorentzian signature?
    - Negative norm states?

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Could AS be perturbative?

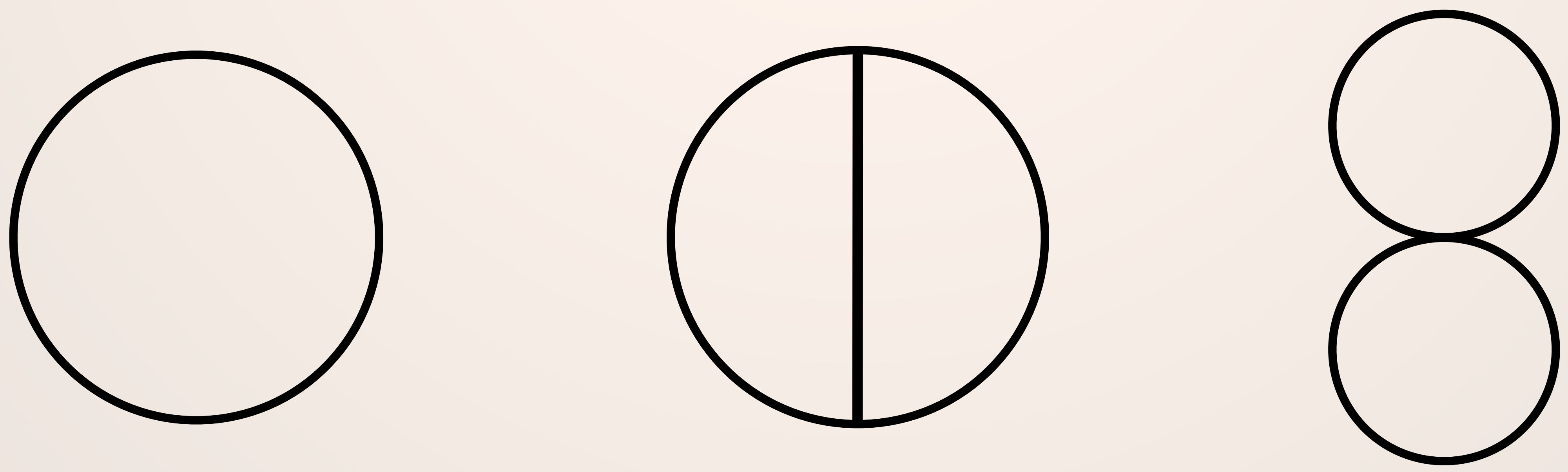
Based on: arXiv:2409.09252
arXiv:2602.xxxxx

A Perturbative Approach to Quantum Gravity

- Start with action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{16\pi G_0} - \frac{\Lambda_0}{8\pi G_0} + \lambda_E^{(0)} E + \lambda_{C^3}^{(0)} C^3 + \ldots \right]$$

- Calculate observables as perturbative series in Newton coupling
- Compute Feynman diagrams to calculate observables



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Could be as small as 10^{-7} at the fixed point!

[Baldazzi, Falls, **YK**, Knorr, 2023]

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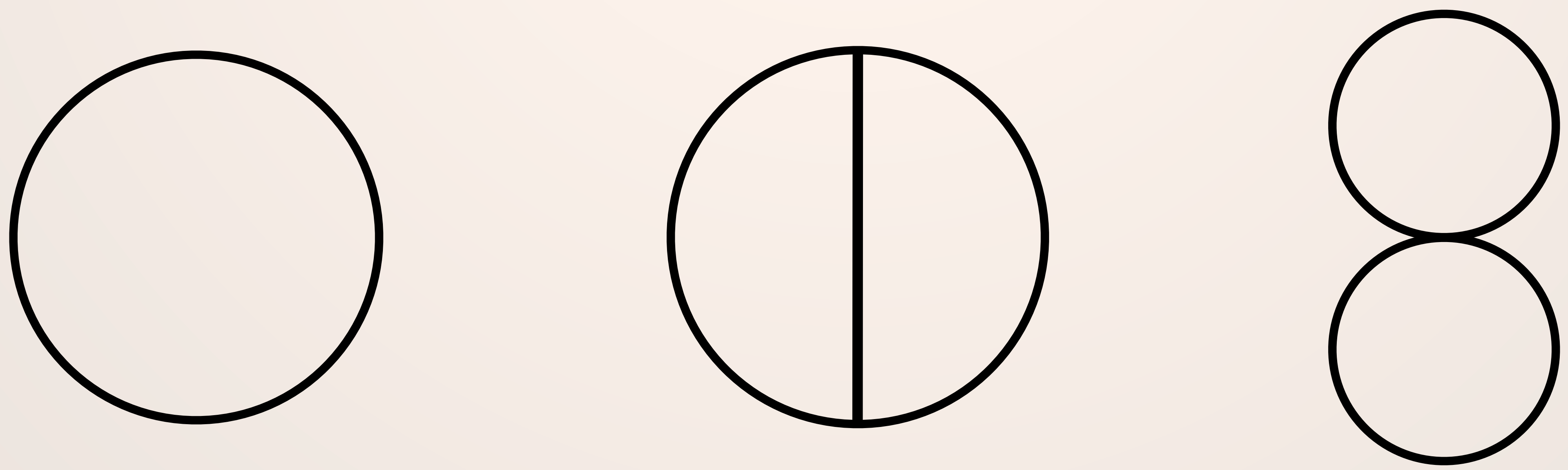

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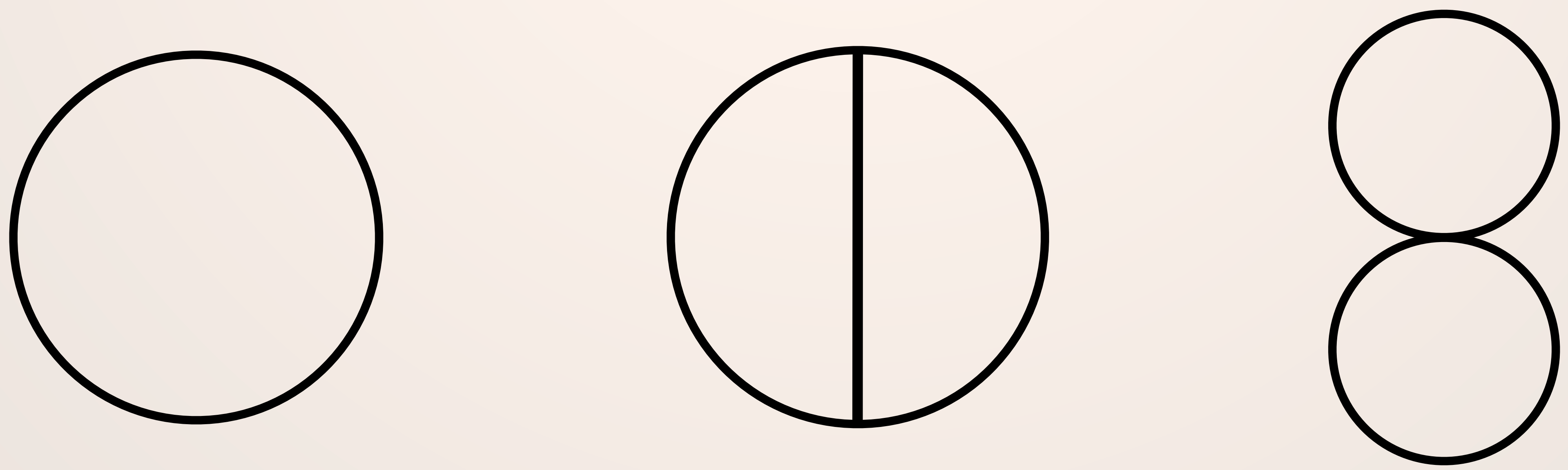

- **But can we see a fixed point for the Einstein-Hilbert terms?**

Quantum Gravity in $d = 2 + \varepsilon$

- First indications for asymptotic safety from perturbation theory in $d = 2 + \varepsilon$

[Kawai and Ninomiya, 1990]

[Jack and Jones, 1991]

[Kawai and Kitazawa and Ninomiya, 1993]

⋮

$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}\left(g^3\right)$$

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$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}\left(g^3\right)$$

- Extrapolate to $\varepsilon \to 2$
    - UV fixed point!

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$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = \varepsilon g - B_1 g^2 + \mathcal{O}\left(g^3\right)$$

- Extrapolate to $\varepsilon \to 2$ **Are we allowed to do this?**
  - UV fixed point!

Running Couplings in Quantum Gravity

- β-functions in **4d** one-loop quantum gravity in **dimensional regularization with MS**

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  - **No one-loop fixed point**

Running Couplings in Quantum Gravity

- β-functions in 4d one-loop quantum gravity

quart div

$$\mu \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -2\lambda - g\left(A_1 + A_2\lambda + A_3\lambda^2\right) + \mathcal{O}\left(g^2\right)$$

quad div

log div

$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = 2g - g^2\left(B_1 + B_2\lambda\right) + \mathcal{O}\left(g^3\right)$$

Running Couplings in Quantum Gravity

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  - **No power-law divergences**

Is dimensional regularization blind to power-law divergences?

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Power-Law Sensitive Dimensional Regularization

- How to identify power-law divergences in dimensional regularization?

$$\int \frac{\mathrm{d}^d p}{\pi^{d/2}} \frac{1}{p^2 + m^2} = \int \mathrm{d}p\, p^{d-1} \int \mathrm{d}\Omega_d \frac{1}{p^2 + m^2} = m^{d-2} \Gamma\left(1 - \frac{d}{2}\right)$$

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- The right-hand side is divergent in $d = 2,4,6,\ldots$
  - **Power-law divergences lead to divergences in $d < 4$** [Veltman, 1981]

Power Divergence Subtraction:

Renormalize bare couplings to remove divergences in $d \leq 4$

[Weinberg, 1979]
[Jack and Jones, 1990]
[Al-Sarhi and Jack and Jones, 1990]
[Al-Sarhi and Jack and Jones, 1992]
[Kaplan et al, 1998]
[**YK**, 2024]

PDS Renormalization in the Gross-Neveu model

- Let's test PDS with an easier theory

$$\mathscr{L}_{\mathrm{GN}} = \bar{\psi}_0 \left(i\partial\!\!\!/ - M_0\right) \psi_0 + \frac{G_0}{2} (\bar{\psi}_0 \psi_0)^2$$

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- Compute renormalization parameters including al divergences in $d \leq 3$

$$\psi_0 = \sqrt{Z_\Psi}\, \psi \qquad M_0 = \mu\, Z_M\, m \qquad G_0 = \mu^{2-d}\, Z_G\, g$$

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- Standard particle physics computation (QGRAF, FORM, FIRE, MATAD)

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$$Z_m = 1 + \frac{g}{2\pi} \frac{N-1}{d-2} + \frac{g^2}{4\pi^2} \left[\frac{7(N-1)}{24} \frac{m^2}{d-3} - \frac{N-1}{4(d-2)} + \frac{(N-1)(2N-3)}{2(d-2)^2} \right]$$

$$Z_\psi = 1 + \frac{g^2}{4\pi^2} \left[\frac{1}{4} \frac{N-1}{d-2} - \frac{5(N-1)}{48} \frac{m^2}{d-3} \right]$$

$$Z_g = 1 + \frac{g}{2\pi} \frac{N-2}{d-2} + \frac{g^2}{4\pi^2} \left[\frac{13N-31}{12} \frac{m^2}{d-3} - \frac{N-2}{2(d-2)} + \frac{(N-2)^2}{(d-2)^2} \right]$$

Gross-Neveu Fixed Points

$$\frac{\beta_g}{g} = 1 - (N-2)\frac{g}{2\pi} + \left(N - 2 - \frac{13N-31}{6}\,m^2\right)\frac{g^2}{4\pi^2}$$

$$\frac{\beta_m}{m} = -1 - (N-1)\frac{g}{2\pi} - \frac{N-1}{12}(7m^2-6)\frac{g^2}{4\pi^2}$$

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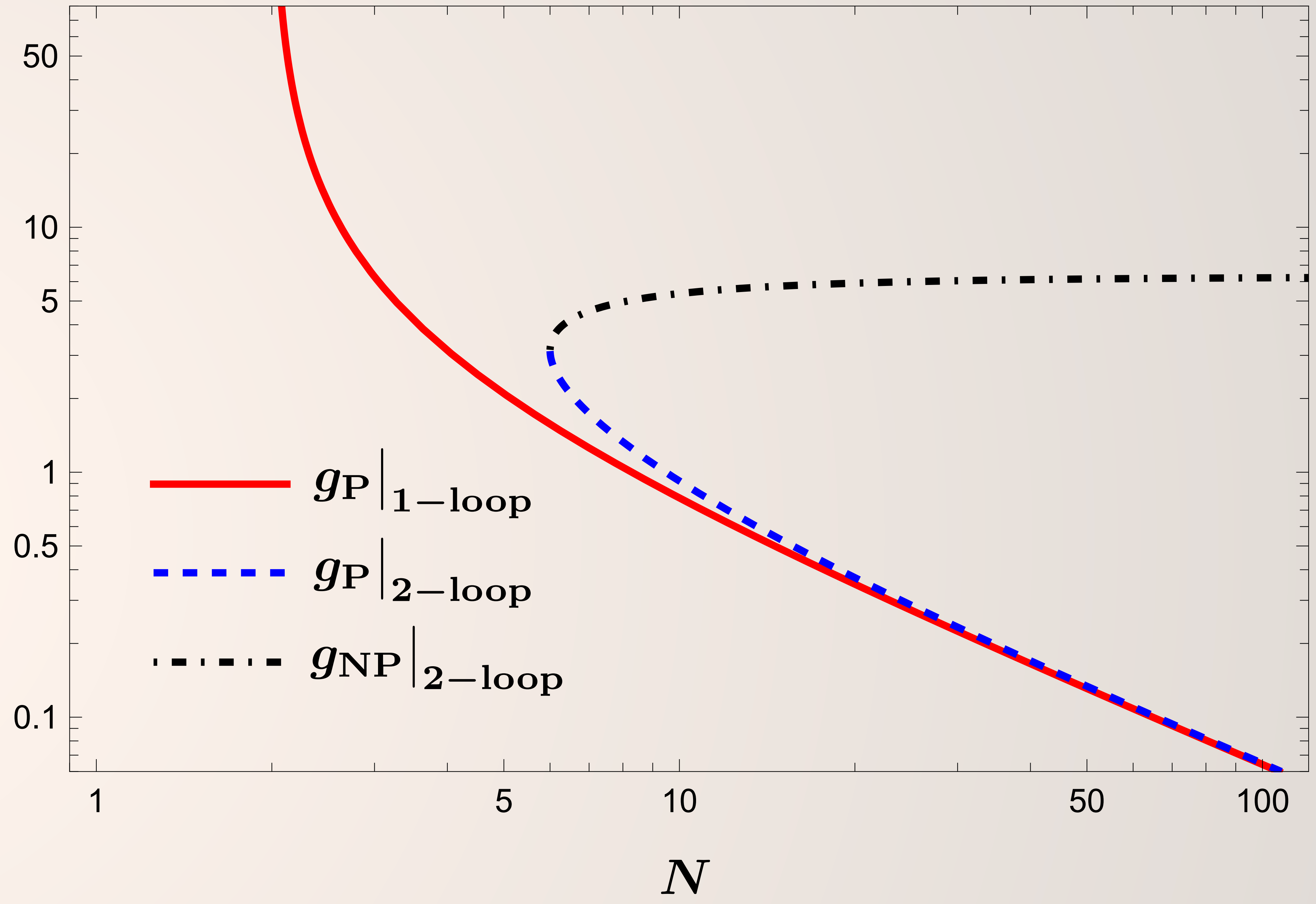

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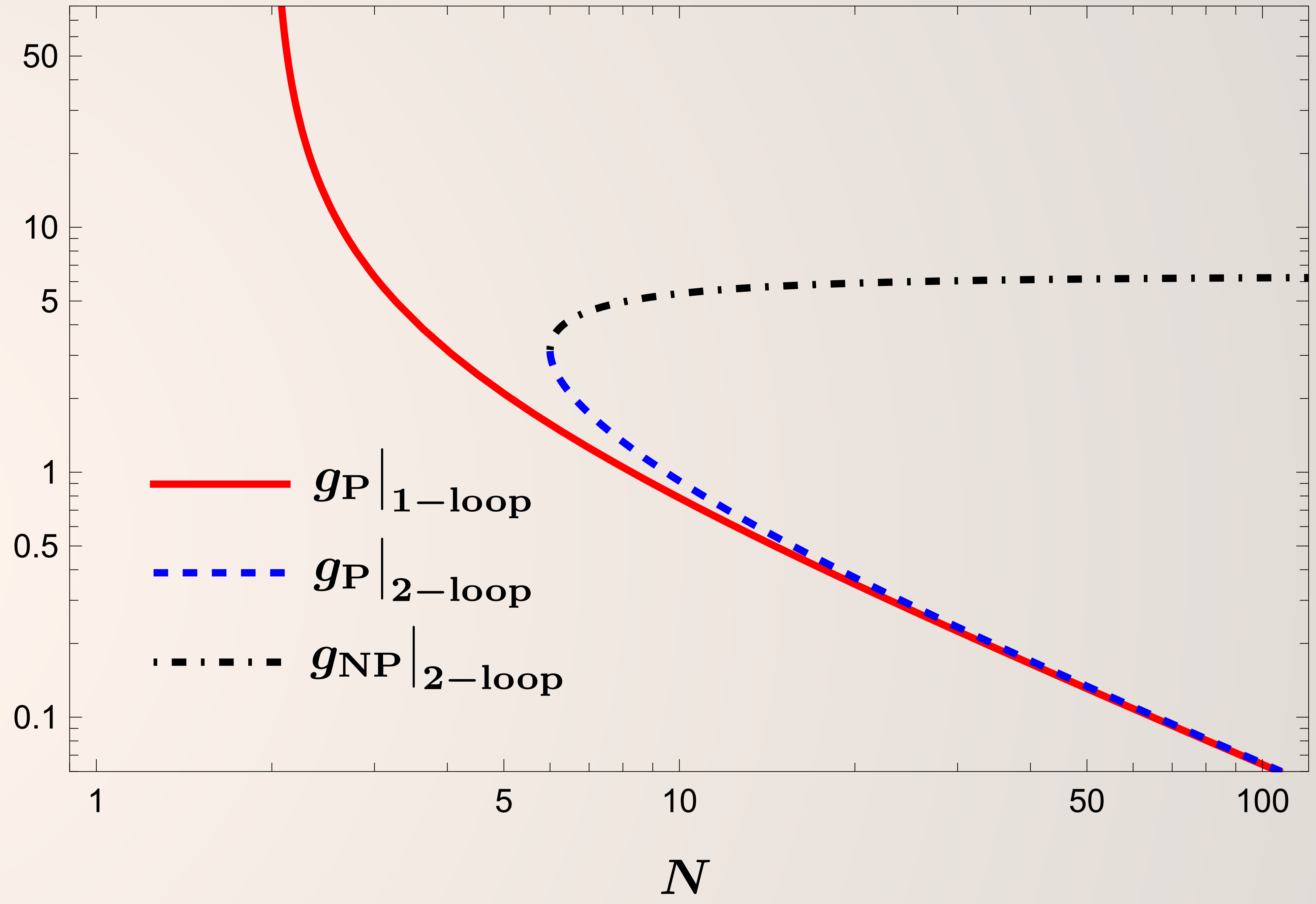

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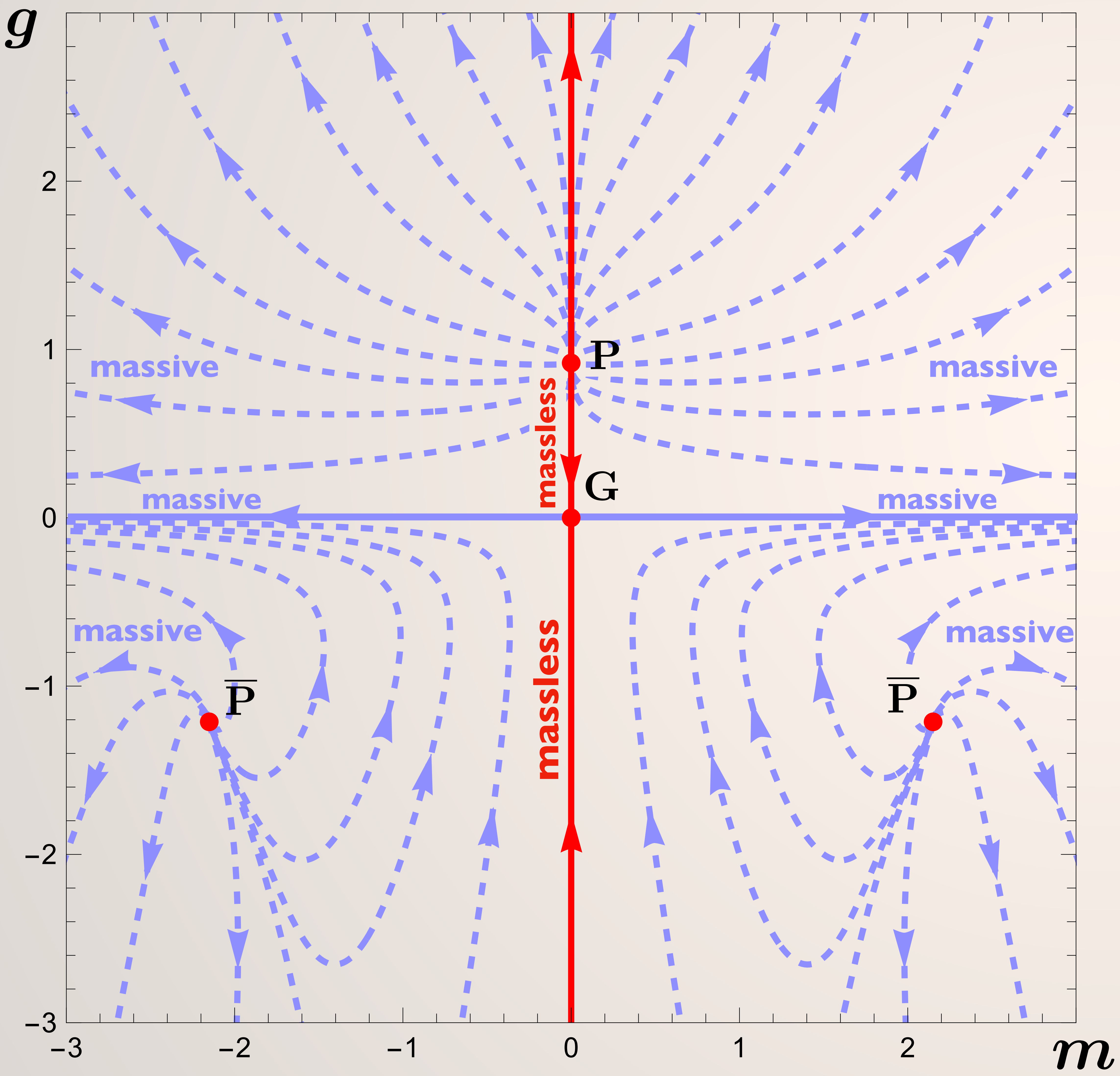
[Cresswell-Hogg and **YK** and Litim, 2026]

Phase Diagram



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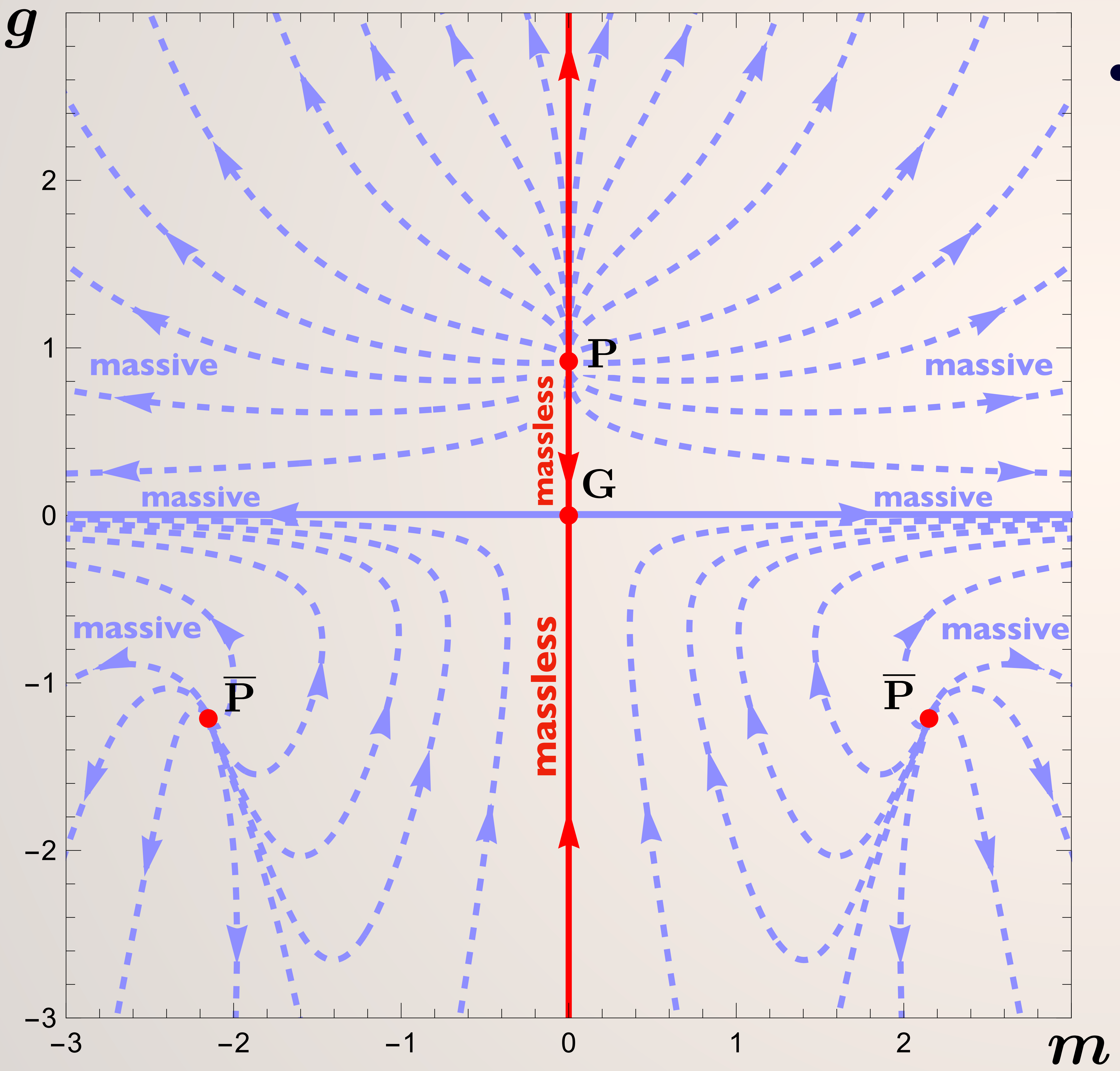

- Additional fixed points with non-vanishing m

$$g_{\overline{\mathrm{P}}/\overline{\mathrm{NP}}} = \frac{\pi}{6N-17}\left(19N - 48 \mp \sqrt{\Delta}\right)$$

$$m^2 = \frac{6}{7}\left(1 - \frac{4\pi}{g} - \frac{2}{N-1}\frac{4\pi^2}{g^2}\right) \quad \theta\big|_{\overline{\mathrm{P}}} = -\frac{99}{38} \pm i\frac{\sqrt{231}}{38} + O(N^{-1})$$

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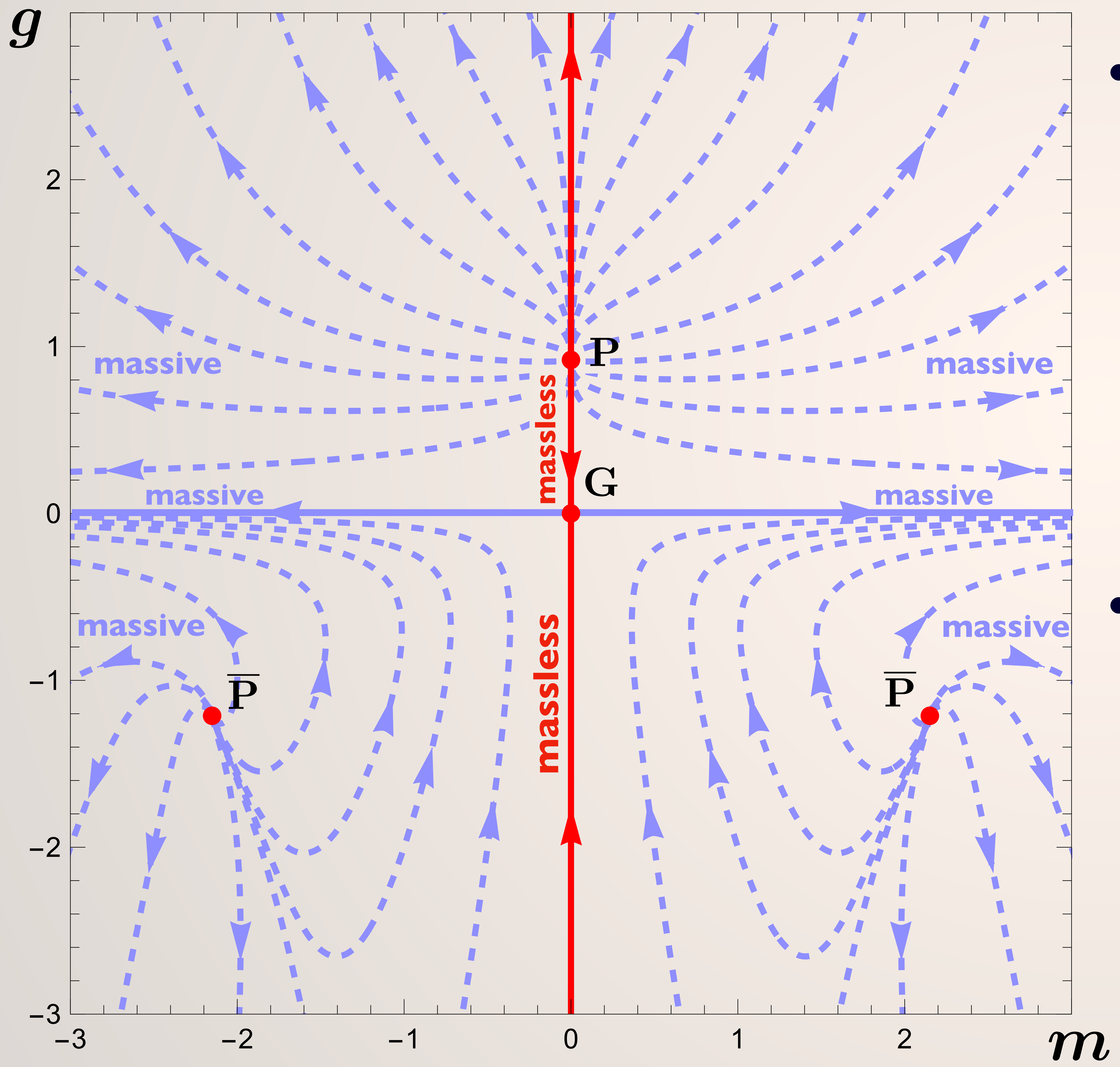

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- g_P under perturbative control
  - UV safe + trajectories to Gaussian FP

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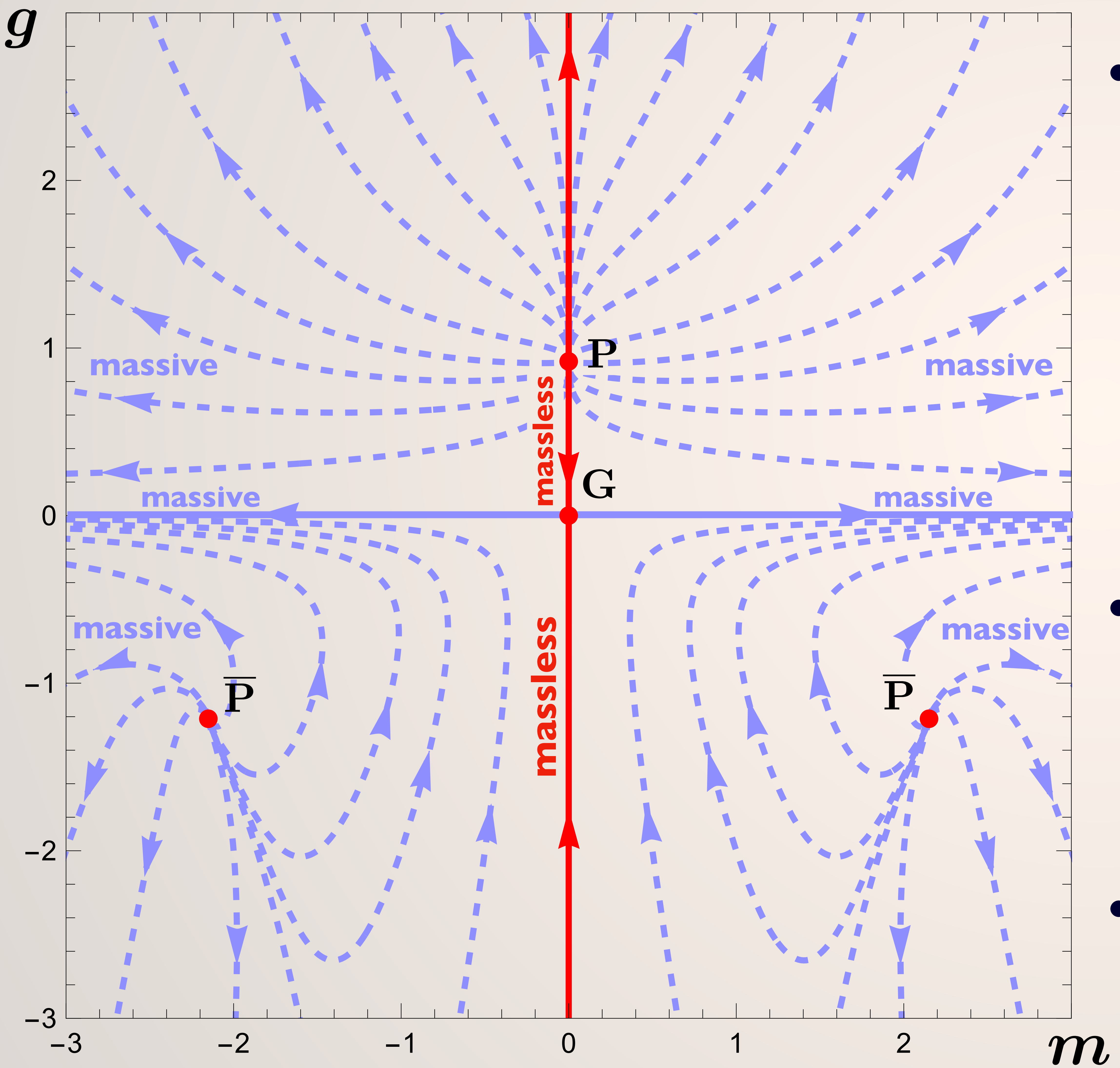

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- g_P under perturbative control
  - UV safe + trajectories to Gaussian FP

- Asymptotic Safety from perturbation theory!

One-Loop Quantum Gravity

- Starting from

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{16\pi G_0} - \frac{\Lambda_0}{8\pi G_0} \right]$$

we compute one-loop divergences

$$\Gamma^{(1)} = \frac{i}{2} \mathrm{Tr} \log\{S^{(2)}\} - i \, \mathrm{Tr} \log\{S^{(2)}_{\text{gh}}\}$$

in $d = 4,2,0$

One-Loop Divergences

- Logarithmic divergences, i.e. $d = 4$

$$\Gamma^{(1)}\Big|_{d=4-2\epsilon} = \frac{1}{(4\pi)^2\epsilon}\left[-10\Lambda_0^2 + \frac{13}{3}\Lambda_0 R - \frac{53}{90}E - \frac{7}{20}R_{\mu\nu}R^{\mu\nu} - \frac{1}{120}R^2\right] + \mathcal{O}(1)\,.$$

  - These are universal

One-Loop Divergences

- Quadratic divergences, i.e. $d = 2$

$$\Gamma^{(1)}\Big|_{d=2-2\epsilon} = \frac{1}{(4\pi)\epsilon}\left[-2\Lambda_0 + \frac{13}{6}R\right] + \mathcal{O}(1)\,.$$

- Quadratic Divergences
  - Missed in MS!

One-Loop Divergences

- Quartic divergences, i.e. $d = 0$

$$\Gamma^{(1)}\Big|_{d=-2\epsilon} = \mathcal{O}(1)\,.$$

  - No quartic divergence present
    - Trace of heat kernel cancels pole in $d = 0$
    - Consequence of using regulator preserving diffeomorphism invariance? [Akhmedov, 2002] [Ossola and Sirlin, 2003]

Renormalization

- One-loop renormalization leads to β-functions

$$\beta_\lambda = \mu \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -2\lambda - \frac{28}{3} g\lambda + \frac{4}{3\pi} g\lambda^2$$

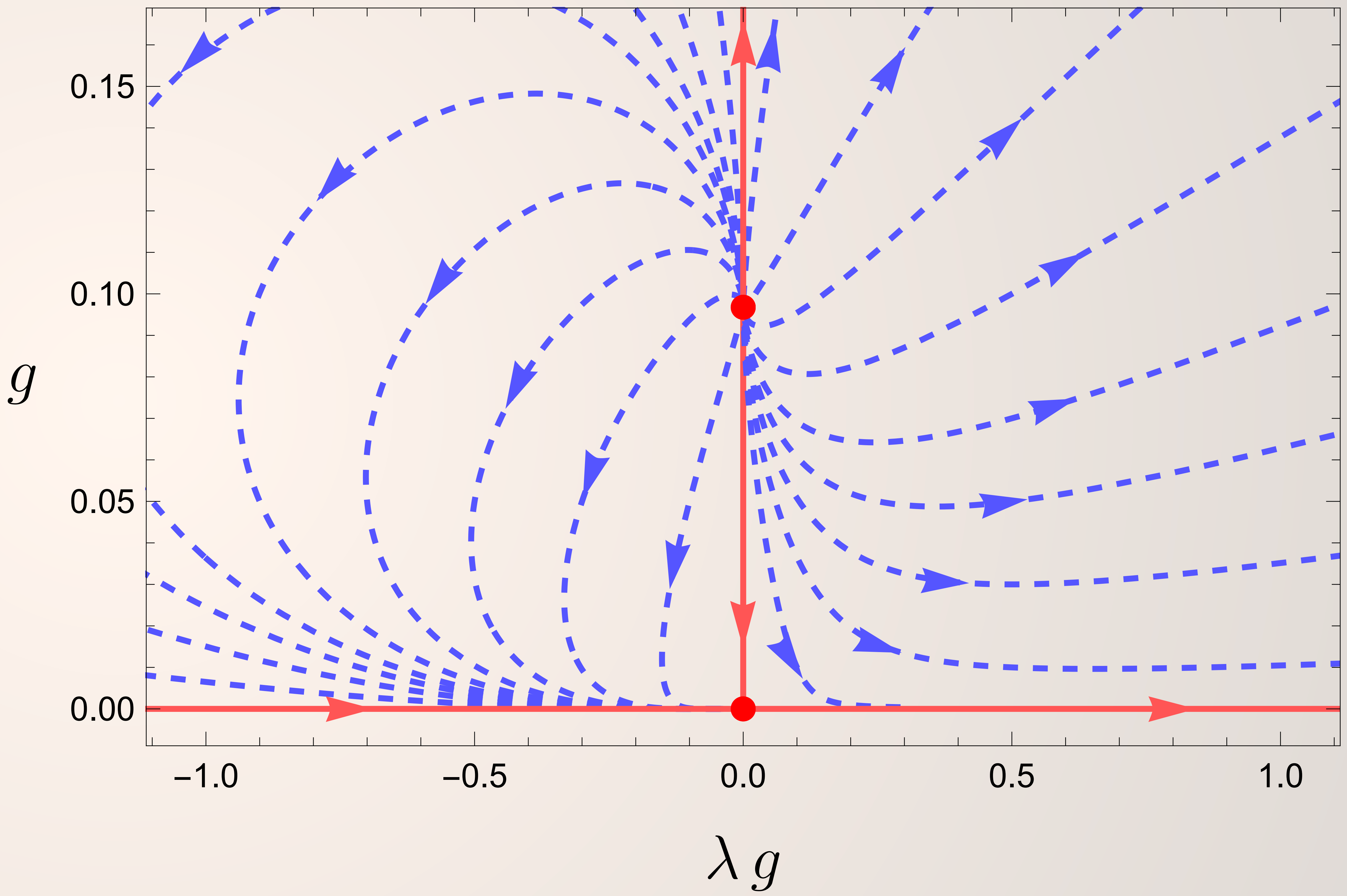
$$\beta_g = \mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = 2g - \frac{2 \cdot 26}{3} g^2 - \frac{26}{3\pi} g^2 \lambda$$

- β-function in $d = 2$ with c scalar fields:

$$\beta_g = -(26 - c)\frac{2}{3} g^2$$ [Falls, 2015]

Phase Diagram of One-Loop Gravity

	λ	g	θ_0	θ_1
$\mathbf{FP}_{\text{Gauss}}$	0	0	2	-2
$\mathbf{FP}_{\text{UV}}$	0	$\frac{3}{26}$	3.077	2
$\mathbf{FP}_{\text{unphys}}$	$-\frac{40\pi}{11}$	$-\frac{11}{78}$	$0.658 + 2.662i$	$0.658 - 2.662i$



- One-loop fixed point exists
  - Critical exponents similar to FRG
  - $\lambda = 0$ is a fixed point of the RG flow

Some Remarks

- Fixed point is generated by quadratic divergences
  - Divergences in $d = 2$ are determining its properties
  - Is the UV of gravity two-dimensional?
- Quartic divergences are absent in this scheme
  - $\lambda = 0$ is a fixed point

Conclusions…

- Non-Perturbative techniques lead to indications for UV fixed point in quantum gravity
  - Near-perturbative properties!
- Study of Asymptotic Safety with power-law sensitive renormalization schemes
  - Dimensional regularization expresses power-law divergences as divergences in lower dimensions
- This provides perturbative UV fixed points, such as in 4d quantum gravity

...Outlook

- PDS allows for applications to higher orders
    - Fixed points in two-loop quantum gravity?
- What happens if matter is included?
- PDS in other non-renormalizable field theories?

Any Questions…?