UF

6th April 2026

University of Toronto

Chiral Symmetry Breaking, Gravitational Waves, and the Origin of the Dark η' Mass in Confining Dark Sectors.

based on [2511:23467] & [26XX:XXXXX]

by Mia West, in collaboration with Rachel Houtz & Martha Ulloa.

Aims

1// Motivation / interesting things we can do with the η'.

2// Common Lore / Linear Sigma Model + Polyakov Loop Corrections.

2a// Improvements to the existing literature.

3// Anomaly Mediated SUSY Breaking & the η'.

4// Phenomenology / GW specrra & particle specrra.

Consider a Dark Confining Group with

- N colours
- F light quark flavours

1-loop beta function is negative (assuming no other active coloured scalars)

$$\beta(g) \sim -\left(\frac{11}{3}N - \frac{2}{3}F\right)\frac{g^2}{16\pi^2}$$

$$\Rightarrow \left(N - \frac{2}{11}F\right) > 0.$$

Confinement, Λ

Mesonic bound states form

$$SU(F)_L \times SU(F)_R \times U(1)_L \times U(1)_R$$

(classically)

$U(1)_A = U(1)_L - U(1)_R$ is explicitly broken

$U(1)_B = U(1)_L + U(1)_R$ relates to a conserved baryon number.

Chiral Symmetry Spontaneously Breaks, f_π

$$SU(F)_L \times SU(F)_R \longrightarrow SU(F)_V$$

Linear Sigma Model to model low energy meson dynamics

Mesons: • condensate of light quarks

• $\Phi = \frac{1}{\sqrt{2F}} \left((\varphi + i\eta') \mathbb{1}_F + (X^a + i\pi^a) T^a \right)$

• Under a Chiral transformation $U(F)_L \times U(F)_R$

$$\Phi \to U_L \Phi U_R^\dagger$$

=> has the same quantum numbers as fermion bilinear $\bar{q}_{Rj} q_{Li}$.

$$V_{LSM} = -m^2 \operatorname{tr}[\Phi^\dagger \Phi] + \frac{1}{2}(\lambda_0 - \lambda_a) \operatorname{tr}[\Phi^\dagger \Phi]^2 + \frac{F\lambda_a}{2} \operatorname{tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi]$$

$$-2(2F)^{FP/2-2}\, c\left((\det \Phi)^P + (\det \Phi^\dagger)^P\right)$$

Part III of talk

Mesonic Effective Masses

σ field-dependent masses

$$m_\sigma^2(\sigma) = -m^2 - \frac{cp}{F}(Fp-1)\sigma^{Fp-2} + \frac{3\lambda_\sigma}{2}\sigma^2$$

$$m_{\eta'}^2(\sigma) = -m^2 + \frac{cp}{F}(Fp-1)\sigma^{Fp-2} + \frac{\lambda_\sigma}{2}\sigma^2$$

$$m_x^2(\sigma) = -m^2 - \frac{cp}{F}\sigma^{Fp-2} + \frac{\lambda_\sigma}{2}\sigma^2$$

$$m_\pi^2(\sigma) = -m^2 + \frac{cp}{2}\sigma^{Fp-2} + \frac{1}{2}(\lambda_\sigma + 2\lambda_a)\sigma^2$$

Zero temperature masses

$$m_\sigma^2(f_\pi) = -\frac{cp}{F}(Fp-2)f_\pi^{F-2} + \lambda_\sigma f_\pi^2$$

$$m_{\eta'}^2(f_\pi) = cp^2 f_\pi^{Fp-2}$$

$$m_x^2(f_\pi) = \frac{2cp}{F}f_\pi^{Fp-2} + \lambda_a f_\pi^2$$

$$m_\pi^2(f_\pi) = 0$$

where $0 = -m^2 - \frac{cp}{F}f_\pi^{pF-2} + \frac{\lambda_\sigma}{2}f_\pi^2$

Part I: Phenomenology Example

Phenomenology example: Baryogenesis.

[M. Dichtl, J. Nava, S. Pascoli & F. Sala: arXiv: 2312.09282]

- Need a first order PT from a confining group.

- PT must be *supercooled*

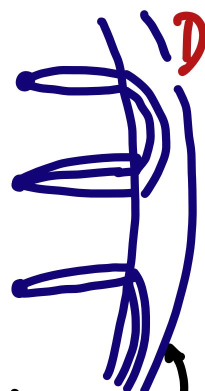
$$T_n < T_{eq} \qquad (\text{where } T_{eq} \text{ is when } \rho_{rad} = \Delta V)$$

- Make an abundance of *hadrons* through...

CONFINED

DECONFINED

fermion flux tubes



Bubble wall

If fermion energy > confinement scale in wall frame $\Rightarrow$

wall boost $\longrightarrow$ $\gamma T_{nuc} \gtrsim f_\pi$

flux tubes break

Quark ejected

Many hadrons & fermions produced

[I. Baldes, Y. Gouttenoire, F. Sala; arXiv: 2007.08440]

(also outside wall to conserve charge)

Many more hadrons produced from high energy fermion collisions.

$$\frac{\gamma_{coll} f_\pi^2}{M_{hadr}^2} \gtrsim$$ hadrons produced compared with non-confining PT.

Lorentz factor of confining fermions

Yield of a hadron Ψ

$$Y_\Psi \sim Y_F \times \frac{3\gamma_{coll} f_\pi^2}{M_{hadr}^2} Br(hadr \to \Psi) \times \left(\frac{T_{nuc}}{T_{eq}}\right)^3 \frac{T_{RH}}{T_{eq}}$$

$M_{hadr} \sim 4\pi f_\pi$

With a portal to the SM which destabilises Ψ & whose decay violates B, C & CP

$$\Rightarrow \quad Y_B \simeq \epsilon_\Psi \, \kappa_{sph} \, Y_\Psi$$

ϵ_Ψ: # of baryons on average per Ψ decay

κ_{sph}: 1 if before ew sphaleron is active, else < 1.

Part II : Standard Lore for GW (Gravitational Wave) Phenomenology

↳ from prev. equations, p = 1.

[R. Pasechnik, M. Reichert, F. Sannino & Z.-W. Wang ; arXiv: 2309.16755]

[D. Croon, R. Houtz, V. Sanz; arXiv: 1904.10967]

[A. Humboldt, J. Kubo, S. van der Woude ; arXiv: 1904.07891]

[L. Sagunski, P. Schicho, D. Schmitt ; arXiv: 2303.02450]

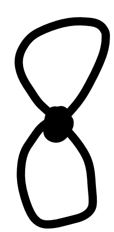
Steps:

1// σ acts as *order parameter* for the phase transition.

$$V_{\text{tree}} = -\frac{m^2}{2}\sigma^2 - \frac{c}{F^2}\sigma^F + \frac{\lambda_\sigma}{8}\sigma^4$$

(zero-temp vacuum: $0 = -m^2 - \frac{cp}{F} f_\pi^{F-2} + \frac{\lambda_\sigma}{2} f_\pi^2$).

2// Thermal corrections arise from CJT formalism



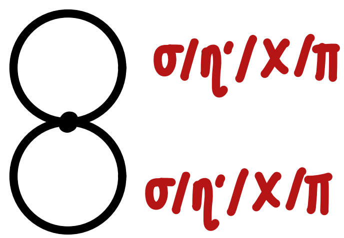
Consistent sum of 2-loop $V_{\text{eff},T}$.

3// Polyakov loop corrections due to gluonic contributions.

Two-loop thermal effective masses in CTJ:

Equivalent to daisy & superdaisy resummation

1// Hartree-Fock approximation:



Meson self-energies *independent* of momentum & energy.

2// Effective potential in CJT reads

tree-level propagator

$$V[\bar{\sigma},\bar{G}] = V[\bar{\sigma}] + \frac{1}{2}\int_k \ln \bar{G}^{-1}(k) + \frac{1}{2}\int_k \left[G^{-1}(k;\bar{\sigma})\,\bar{G}(k) - 1\right] + V_2[\bar{\sigma},\bar{G}]$$

dressed propagator

$$V_2[\bar{S},\bar{P},\bar{\sigma}] \xrightarrow{\text{Hartree}} V_2[\bar{S},\bar{P}]$$

Scalar loop

Pseudoscalar loop.

3/ Schwinger-Dyson equation for dressed propagator

[D. Röder, J. Ruppert, D. Rishke; arXiv: nucl-th/0301085]

Dressed propagators

$$\left.\begin{array}{l} S^{-1}_{ab}(k) = S^{-1}_{ab}(k;\bar{\sigma}) + \Sigma_{ab}(k) \\[2ex] P^{-1}_{ab}(k) = \underbrace{P^{-1}_{ab}(k;\bar{\sigma})}_{\text{tree-level propagators}} + \Pi_{ab}(k) \end{array}\right\} \xrightarrow{\text{Hartree approx.}} \begin{array}{l} S^{-1}_{ab}(k) = -k^2\delta_{ab} + M^2_{s\,ab}(k) \\[2ex] P^{-1}_{ab}(k) = -k^2\delta_{ab} + M^2_{p\,ab}(k) \end{array}$$

where...

$$\left.\begin{array}{l} \Sigma_{ab}(k) \equiv 2\,\dfrac{\delta V_2(\bar{\sigma},\bar{S},\bar{P})}{\delta \bar{S}_{ba}(k)}\Bigg|_{\bar{\sigma}=\sigma,\,\bar{S}=S,\,\bar{P}=P.} \\[4ex] \Pi_{ab}(k) \equiv 2\,\dfrac{\delta V_2(\bar{\sigma},\bar{S},\bar{P})}{\delta \bar{P}_{ba}(k)}\Bigg|_{\bar{\sigma}=\sigma,\,\bar{S}=S,\,\bar{P}=P.} \end{array}\right] \rightarrow \text{Self-energies.}$$

Omitting vacuum contributions to the integrals (ie we can avoid highly non-trivial renormalisation)

$$\int_q S(q) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\sqrt{\vec{q}^{\,2}+M^2}} \left(\exp\left\{ \frac{\sqrt{\vec{q}^{\,2}+M^2}}{T} \right\} - 1 \right)^{-1}$$

$$\int_q P(q) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\sqrt{\vec{q}^{\,2}+M^2}} \left(\exp\left\{ \frac{\sqrt{\vec{q}^{\,2}+M^2}}{T} \right\} - 1 \right)^{-1}$$

ie we have a system of coupled equations for the thermal dressed masses

$$M_\sigma^2(\sigma,T) = m_\sigma^2(\sigma) + \frac{T^2}{4\pi^2}\Big\{(3\lambda_\sigma - c_1)I_B(R_\sigma^2) + ((F^2-1)(\lambda_\sigma + 2\lambda_a) + c_2(F^2-1))I_B(R_X^2)$$
$$+ (\lambda_\sigma + c_1)I_B(R_{\eta'}^2) + ((F^2-1)\lambda_\sigma - c_2(F^2-1))I_B(R_\pi^2)\Big\}$$
$$M_{\eta'}^2(\sigma,T) = m_{\eta'}^2(\sigma) + \frac{T^2}{4\pi^2}\Big\{(3\lambda_\sigma - c_1)I_B(R_{\eta'}^2) + ((F^2-1)(\lambda_\sigma + 2\lambda_a) + c_2(F^2-1))I_B(R_\pi^2)$$
$$+ (\lambda_\sigma + c_1)I_B(R_\sigma^2) + ((F^2-1)\lambda_\sigma - c_2(F^2-1))I_B(R_X^2)\Big\}$$
$$M_X^2(\sigma,T) = m_X^2(\sigma) + \frac{T^2}{4\pi^2}\Big\{(\lambda_\sigma + 2\lambda_a + c_2)I_B(R_\sigma^2) + ((F^2+1)\lambda_\sigma + (F^2-4)\lambda_a - c_3)I_B(R_X^2)$$
$$+ (\lambda_\sigma - c_2)I_B(R_{\eta'}^2) + ((F^2-1)\lambda_\sigma + F^2\lambda_a + c_3)I_B(R_\pi^2)\Big\}$$
$$M_\pi^2(\sigma,T) = m_\pi^2(\sigma) + \frac{T^2}{4\pi^2}\Big\{(\lambda_\sigma + 2\lambda_a + c_2)I_B(R_{\eta'}^2) + ((F^2+1)\lambda_\sigma + (F^2-4)\lambda_a - c_3)I_B(R_\pi^2)$$
$$+ (\lambda_\sigma - c_2)I_B(R_\sigma^2) + ((F^2-1)\lambda_\sigma + F^2\lambda_a + c_3)I_B(R_X^2)\Big\}$$

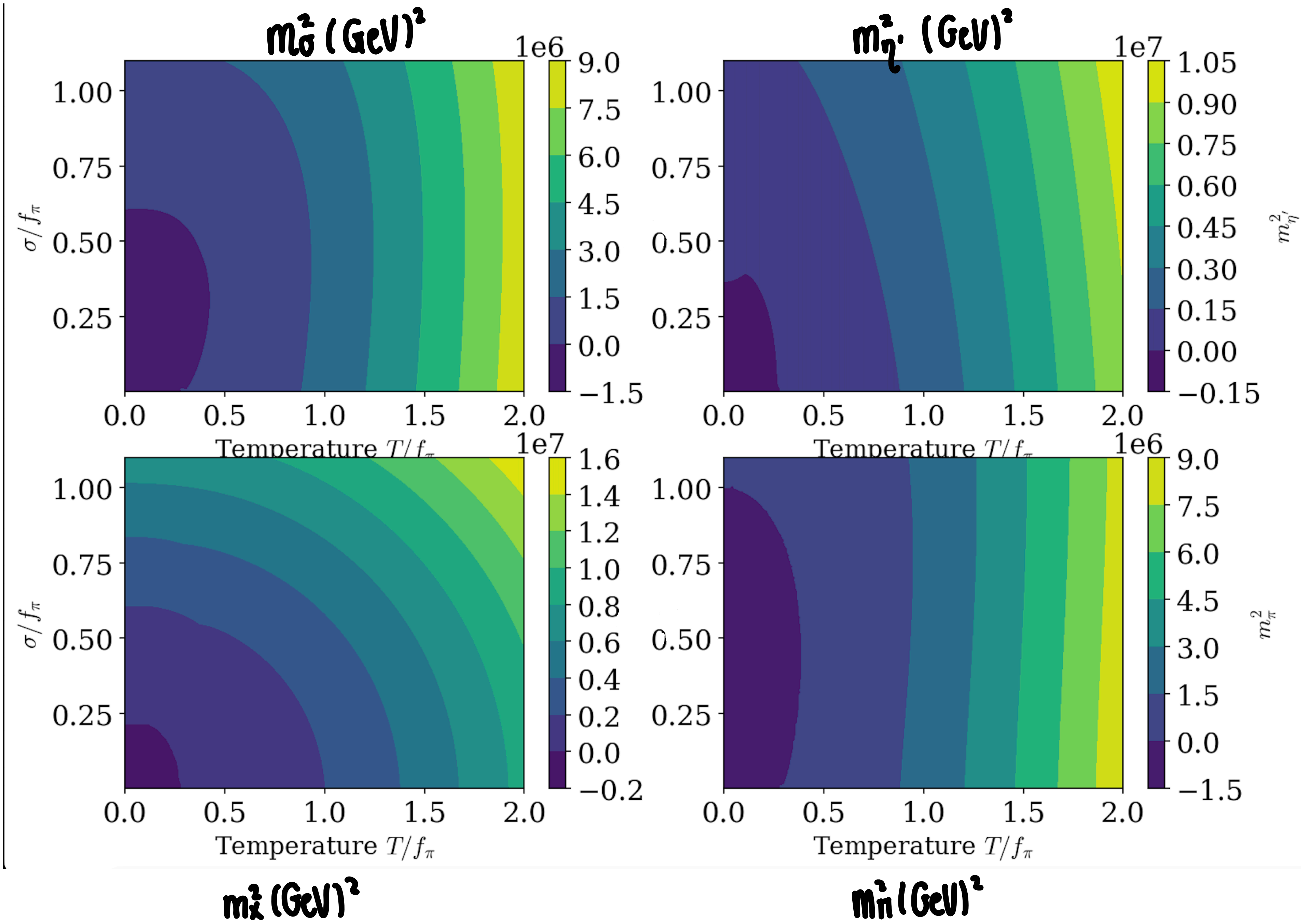
where

$$c_1 = \frac{c}{F^2}(pF)(pF-1)(pF-2)(pF-3)\delta(Fp,4)$$
$$c_2 = \frac{c}{F^2}(pF)(pF-2)(pF-3)\delta(Fp,4)$$
$$c_3 = \frac{c}{F^2}(pF)(pF^3-4F^3+pF+6)\delta(Fp,4)$$

with $R_i \equiv M_i(\sigma,T)/T$

$$I_B(R_i^2) = 2\frac{dJ_B(R^2)}{dR^2} = \int_0^\infty dx \, \frac{x^2}{\sqrt{x^2+R^2}} \, \frac{1}{e^{\sqrt{x^2+R^2}}-1}$$

Solved Numerically...



Thermal Corrections Look Like...

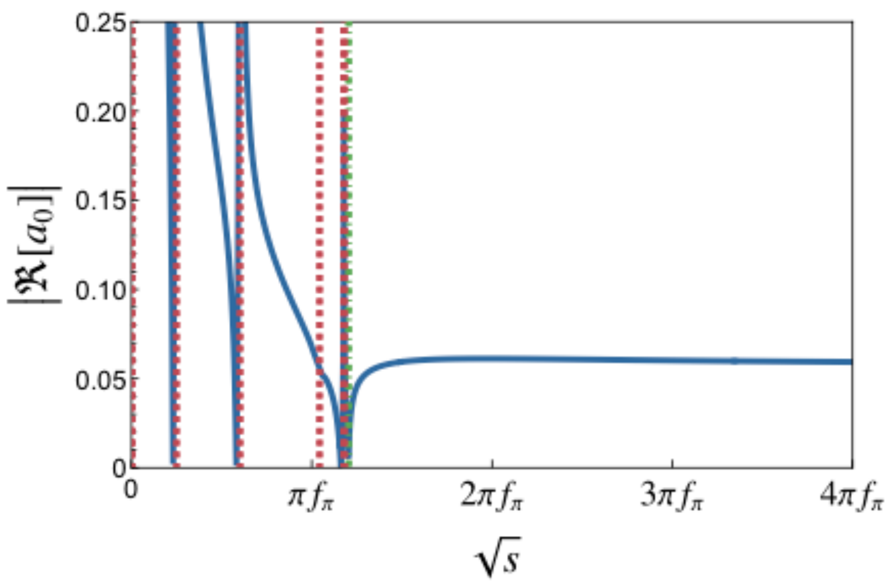
$$V_{CJT}(\bar{\sigma},T) = \frac{T^4}{2\pi^2}\sum_i \Big[\underbrace{J_B(R_i^2)}_{\text{“}\frac{1}{2}\int_k \ln \bar{G}^{-1}(k)\text{”}} \underbrace{- \frac{1}{4}(R_i^2 - r_i^2)\, I_B(R_i^2)}_{\text{“}V_2[\bar{G}]\text{”}} \Big]$$

$$-2V_2[S,P] = -\frac{1}{2}\int_k \Sigma(k;\bar{\sigma})\, S(k;\bar{\sigma}) - \frac{1}{2}\int_k \Pi(k,\bar{\sigma})\, P(k,\bar{\sigma})$$

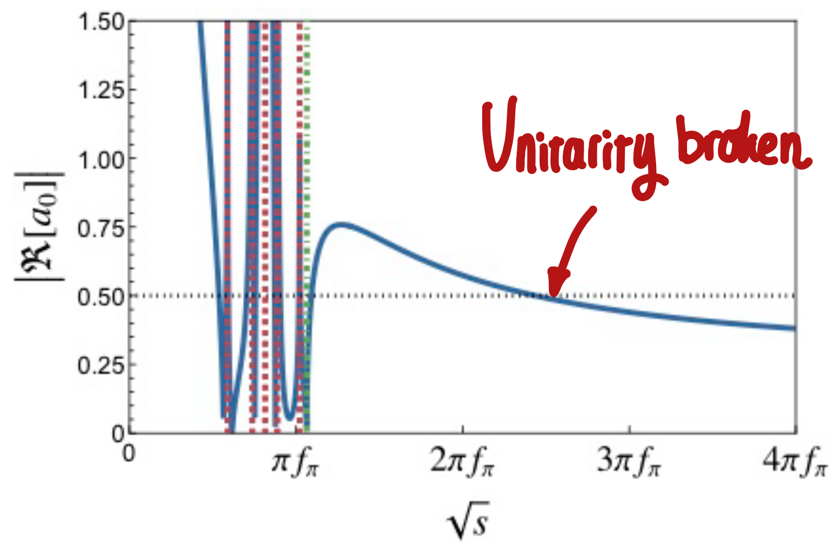
Limits of Applicability

1/ Unitarity bounds from 2-2 scattering:

a) $XX \to XX$



b) $\eta' \sigma \to \eta' \sigma$



for $F=4$: $m_\sigma^2 \sim 1\times10^6\,(\mathrm{GeV})^2$ $\quad m_X^2 \sim 5.8\times10^6\,(\mathrm{GeV})^2$

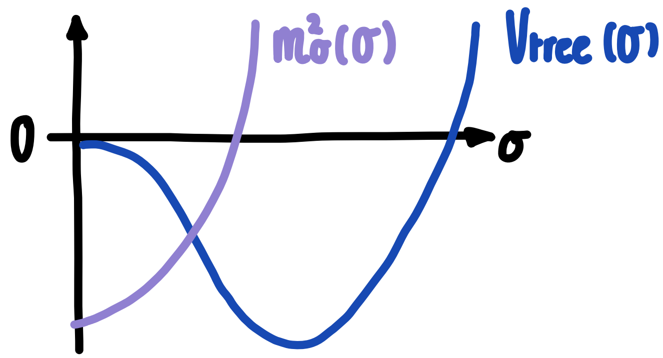
$m_{\eta'}^2 \sim 1.06\times10^7\,(\mathrm{GeV})^2$ $\quad f_\pi \sim 1270$ GeV

2// Matsubara Zero Modes:

at high-temperatures, infrared Bosonic modes become *highly occupied* and the effective loop expansion parameter grows like

$$g^2 \rightarrow g^2 n_B(E,T) = \frac{g^2}{e^{E/T}-1} \approx \frac{g^2 T}{E} \gg \frac{g^2 T}{M}$$

i.e. => for the sigma

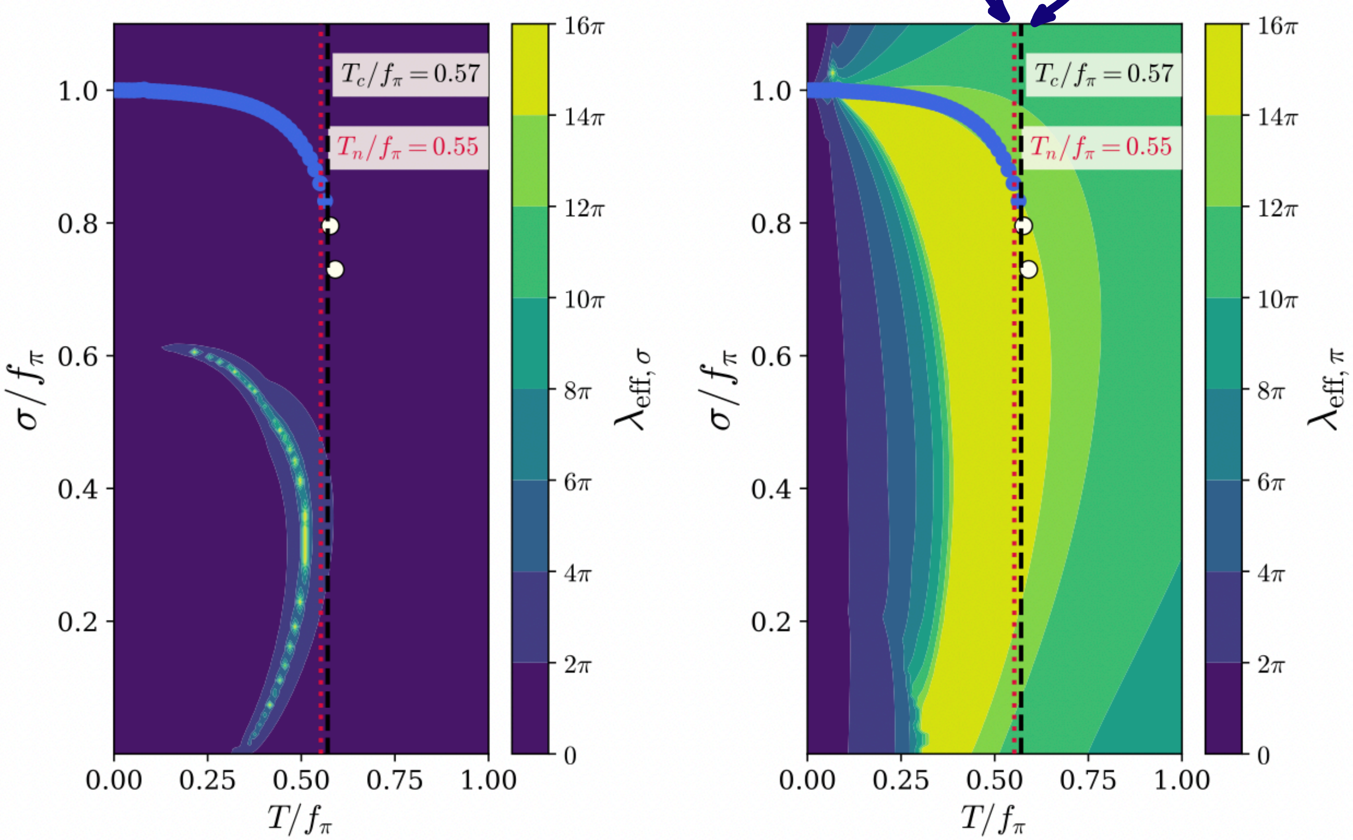


CJT increases $M(\sigma,T)$ s.t. in the phenomenologically interesting region (usualy)

CRITERIA: at most 10% of $\lambda_{\text{eff},\sigma}$ & $\lambda_{\text{eff},\pi}$ exceeds 16π.

Nucleation temp

Critical temp



$$\lambda_{\text{eff},i} \sim \left[\text{4pt coupling}\right] \times \frac{T}{\sqrt{|M_i(\sigma,T)^2|}}$$

Benchmark:

F=3 N=3

$f_\pi = 1102$ GeV.

Polyakov Loop Improvement

Build an EFT to model a confining PT near to the Chiral Symmetry Breaking PT.

Expect a $\mathbb{Z}_N$ centre symmetry to distinguish confined / deconfined phases in pure gluon sector.

$\ell(x)$ acts as an *order parameter* for this phase transition,

$$\ell(\vec{x}) = \frac{1}{N} \mathrm{Tr}_c\left[\vec{L}(\vec{x})\right]$$

$\ell \simeq 0$ is *confined phase*

$\ell \simeq 1$ is *deconfined phase*

Thermal Wilson Line $\vec{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} d\tau \, A_4(\vec{x}, \tau) \right]$

Path Ordering

temporal component

Thermal Effective Potential of gluon sector preserving $\mathbb{Z}_N$ symmetry is

$$V_{PLM} = T^4\left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \ldots - b_3(\ell^N + \ell^{*N})\right)$$

$$b_2(T) = a_0 + a_1\left(\frac{T_\Lambda}{T}\right) + a_2\left(\frac{T_\Lambda}{T}\right)^2 + a_3\left(\frac{T_\Lambda}{T}\right)^3 + a_4\left(\frac{T_\Lambda}{T}\right)^4$$

fit parameters from Lattice

Table 1. The parameters for the best-fit points.

N	3	3 log	4	5	6	8
a_0	3.78	4.26	9.58	11.4	11.2	20.1
a_1	-5.48	-6.53	-8.81	-12.2	-29.1	-52.3
a_2	8.47	22.8	10.1	4.41	67.1	121
a_3	-9.47	-4.10	-12.2	-0.148	-95.6	-172
a_4	0.222		0.475	-8.29	32.9	59.2
b_3	2.36	-1.77		-7.03		
b_4	4.49		-2.37	-14.7	-29.3	-52.8
b_6			3.18		38.2	68.8
b_8					-12.6	-22.7

[R. Pasechnik, M. Reichert, F. Sannino & Z-W. Wang: 2309.16755]

V_{medium} connects the pure gluonic & chiral components

$$V_{med} = -2FT \int \frac{d^3p}{(2\pi)^3} \, \mathrm{Tr}\left\{ \ln\left[1 + L_R e^{-E_p/T}\right] + \ln\left[1 + L_R^{\dagger} e^{-E_p/T}\right] \right\}$$

assuming zero chemical potential

where $E_p = \sqrt{|\vec{p}|^2 + m_q^2}$ is the *energy of the quarks.*

Quark mass term $m_q \sim \underbrace{g_q}_{O(1)\ \text{constant}} \sigma \sim \sigma$.

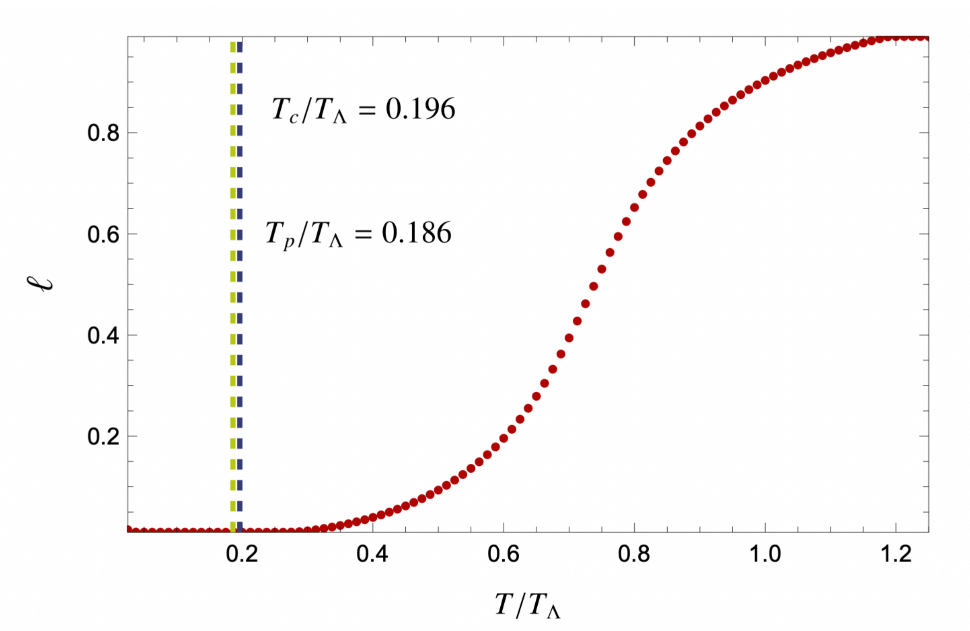
[K. Fukushima, V. Skokov: arXiv: 1705.00718]

NB// $t = t^*$ assuming zero chemical potential

Technically $\ell(x)$ acts as an *order parameter* for this phase transition,

commonly just use Mean Field Approx. for ℓ. ie

$$V_{PLM}(\sigma,T) + V_{med}(\sigma,T) = \underset{0\leq\ell\leq 1}{\mathrm{Min}} \left[V_{PLM}(\sigma,T,\ell) + V_{med}(\sigma,T,\ell) \right]$$



Benchmark $F=4$ $N=3$

Confining PT at $T_\Lambda \sim 2f_\pi$

$m_\sigma^2 \sim 1\times10^6\,(\mathrm{GeV})^2$

$m_{\eta'}^2 \sim 1.54\times10^7\,(\mathrm{GeV})^2$

$m_x^2 \sim 5.8\times10^6\,(\mathrm{GeV})^2$

$f_\pi \sim 2121.32$ GeV.

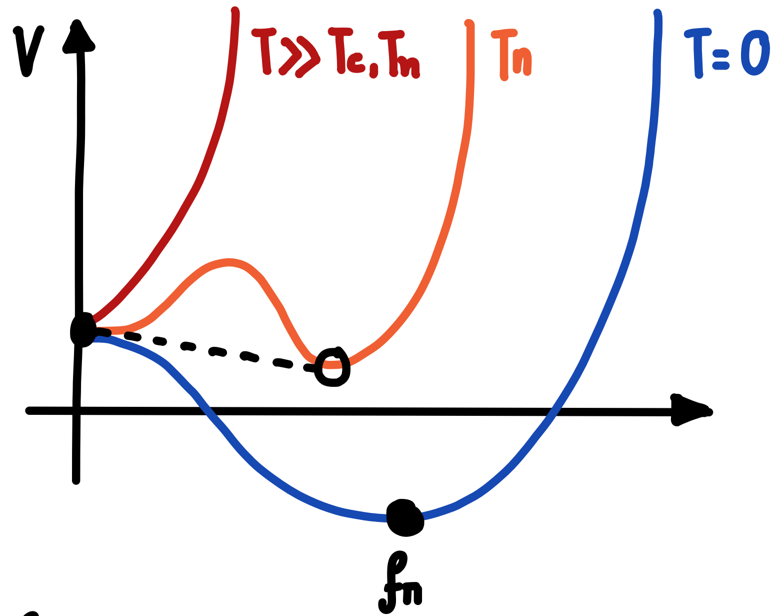
full potential

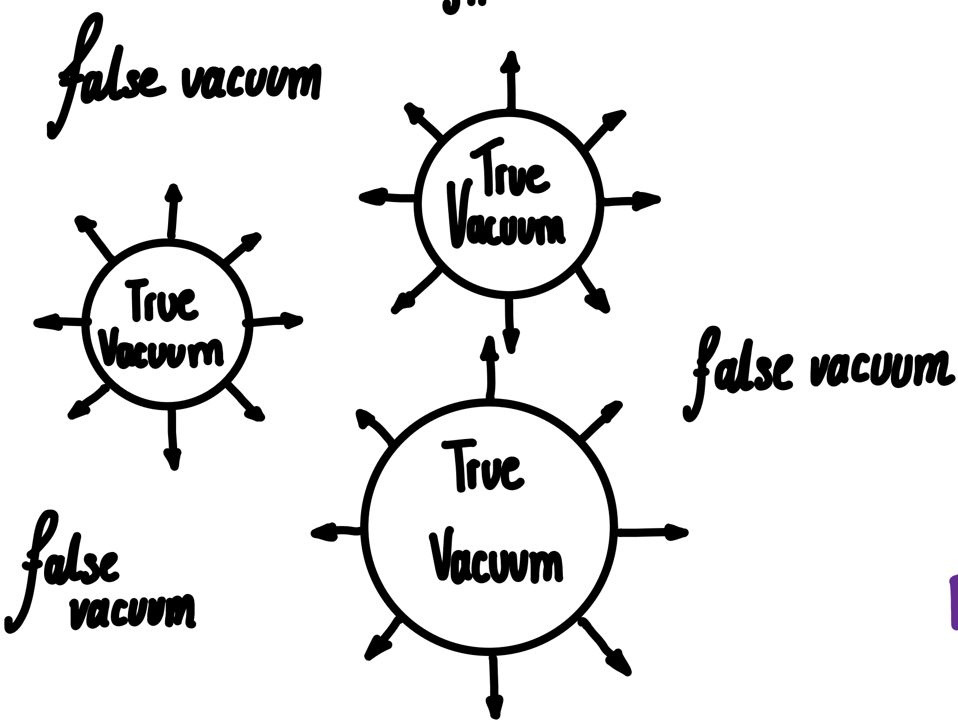
NB// Ignoring vacuum contributions

$$V_{eff}(\sigma,T) = V_{LSM}(\sigma) + \min_{0\leq \ell \leq 1}\left[V_{PLM}(\ell,T) + V_{med}(\sigma,\ell,T\right]$$

$$+ V_{CJT}(\sigma,T).$$

First Order PT





Where the *false-vacuum probability* is

$$P(T) = e^{-I(T)}$$

$$I(T) = \frac{4\pi}{3}\int_T^{T_c} dT' \, \frac{\Gamma(T')}{H(T')T'^4}\left(\int_T^{T'} dT'' \, \frac{v_w(T'')}{H(T'')}\right)^3$$

Weight function

Nucleation/Percolation occurs when:

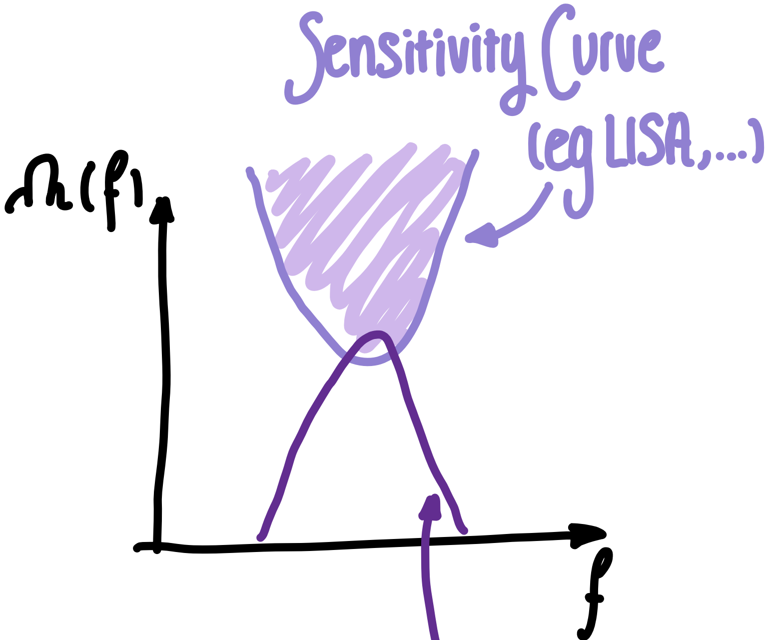
$$I(T_p \approx T_n) = 0.34$$

or

$$P(T_p) = 0.7.$$

[J. Ellis, M. Lewicki, J.M. No; arXiv: 1809.08242]

GW Spectra [Caprini et. al.; arXiv:1512.06239]



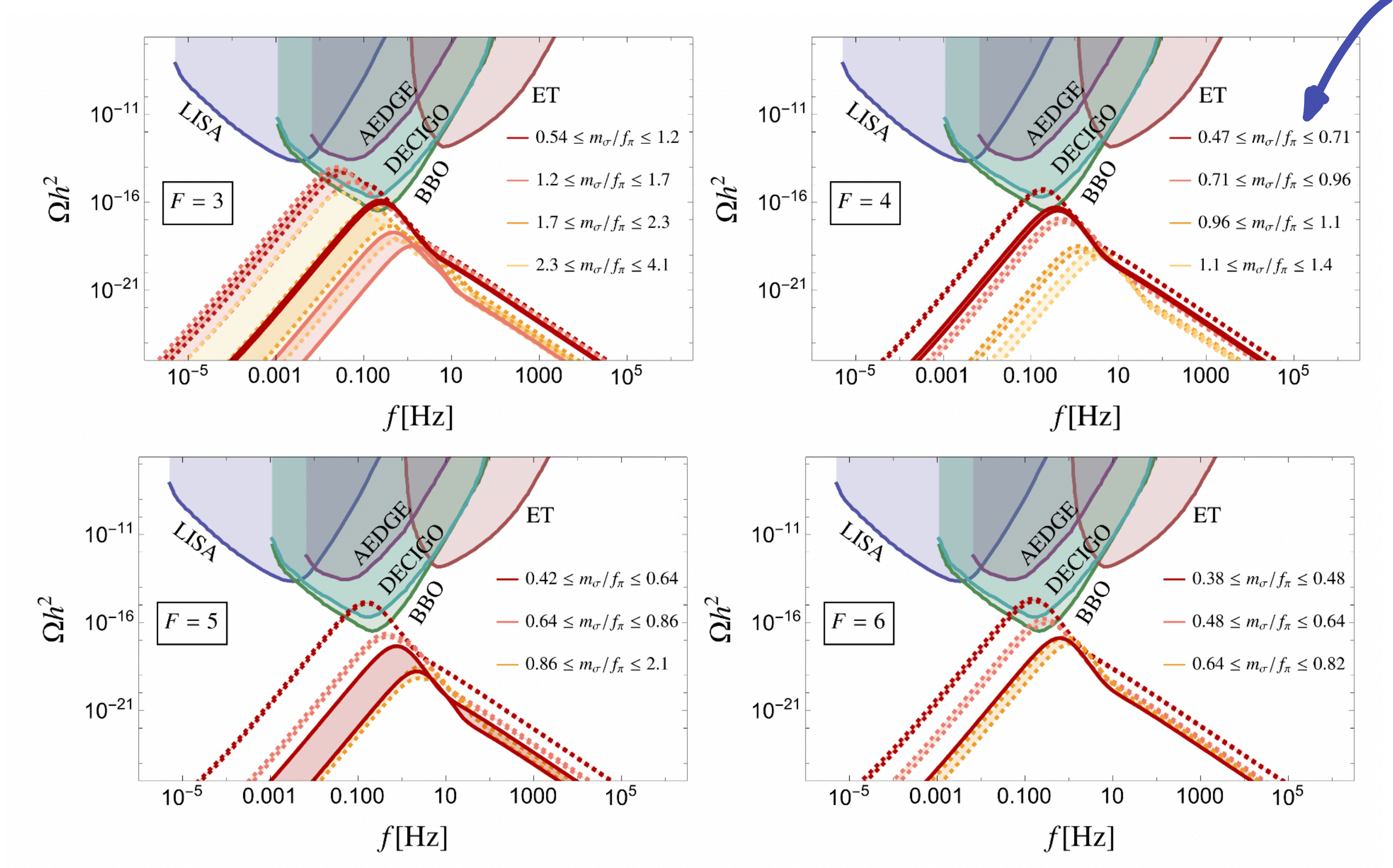
- Sum of many waves of different frequencies
- Looks like a stochastic background in a GW interferometer (eg LISA)

frequency spectrum $\left\{ \begin{array}{l} \alpha: \text{latent heat} \\ \beta/H: \text{inverse duration of PT} \\ T_n: \text{nucleation temperature} \\ v_w: \text{bubble wall velocity} \end{array} \right.$

[W.-Y. Ai, B. Laurent, J. Van de Vis; arXiv:2303.10171]

Numerical Results

Smaller mass increases signal strength



N=4 colours, envelope is minmax for $T_\Lambda \sim \xi f_\pi$ $\xi = [0,1,2,5]$.

- $m_\chi^2 \in [1, 25] \times 10^6\ (\mathrm{GeV})^2$
- $f_\pi \in [0.5, 1.5]\sqrt{\frac{F}{2}} \times 10^3\ \mathrm{GeV}$

Part III

Anomaly Mediated SUSY Breaking (AMSB)

Based on arguments from

[C. Csaki, R. Tito D'Agnolo, R. Gupta, E. Kuflik, T. Roy; arXiv:2307.04809]

[D. Kondo, H. Murayama & B. Noether; arXiv:2505.18138]

[H. Murayama; arXiv:2104.01179]

[C. Csaki, H. Murayama, O. Telem; arXiv:2104.10171]

Comparison with the Large N limit & the Origin of the η' mass

- Classically when $m_q \sim 0$;

$$U(F)_L \times U(F)_R = SU(F)_L \times SU(F)_R \times U(1)_L \times U(1)_R$$

- Quantum mechanically

$$U(1)_A = U(1)_L - U(1)_R$$

is explicitly broken.

- Anomaly:

$$\partial_\mu j^\mu_A = F \frac{g^2}{32\pi^2} \mathrm{Tr}[G\tilde{G}]$$

(assumed only due to F fundamental fermions & SU(N) gauge group)

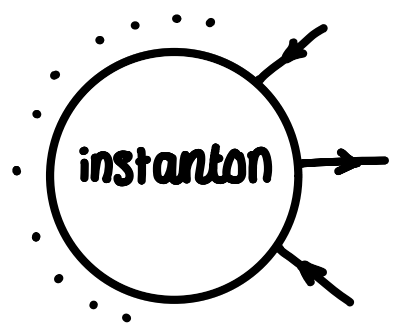
gives mass to the Goldstone boson η' (associated to $U(1)_A$ current).

η' mass in QCD is much higher than the other pseudo-Goldstone pions.

mass generated from t'Hooft instanton term?

[G. 't Hooft; Phys Rev Lett, 37 8 (1967)
Phys Rev D, 14, 3432 (1976)].

2F massless fermions



(breaks axial charge conservation)

$$\text{Coupling} \sim C g^{-8} \underbrace{e^{-8\pi^2/g^2}}_{\text{Tell-tale instanton coupling!}} \underbrace{\det\left[\bar{\psi}(1+\gamma_5)\psi\right]}_{\sim \det \Phi}$$

• In the large N limit, the anomaly vanishes:

$$\partial_\mu j^\mu_A \sim F \frac{g^2}{16\pi^2} \mathrm{Tr}[G\tilde{G}] \sim \frac{\lambda}{16\pi^2} \frac{F}{N} \mathrm{Tr}[G\tilde{G}] \to 0$$

where $\lambda = g^2 N$ is the 't Hooft coupling which is held fixed.

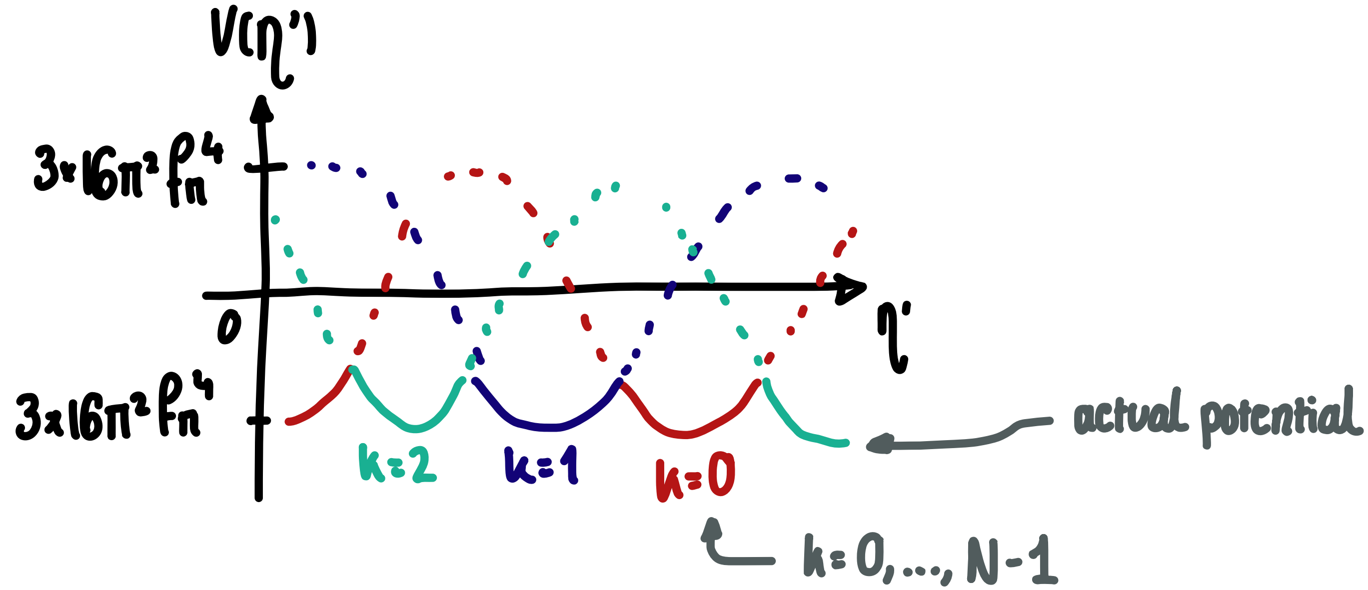
• However for the Chiral Lagrangian to recover $m^2_{\eta'} \sim \frac{1}{N}$ scaling,

$$V_{\cancel{U(1)}_A} \sim \left((\det \Phi)^{1/N} + h.c. \right)$$

correctly matches the expectation from the large N limit.

[E. Witten; Nucl Phys B 149 (1979) 285-320]
[G. Veneziano; Nucl Phys B 159 (1979) 213-224]

Such a potential would have a *branched structure*



which cannot be generated by instantons.

[C. Csaki, et. al. ; arXiv:2307.04809]

Story from AMSB

[C. Csaki, et. al.; arXiv:2307.04809]

Tree-level potential for the squarks: if Kähler potential is *not* quadratic & superpotential is *not* cubic.

$$V_{tree} = \partial_i W g^{ij^*} \partial_j^* W^*$$

$$+ m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K\right)$$

$$+ m \left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + h.c.$$

Vector superfield $W_\alpha = \lambda_\alpha + \ldots$ gaugino

$\Rightarrow g_{ij^*} = \partial_i \partial_j^* K$ is Kähler metric

Idea: To connect with QCD, must take limit:

$$\Lambda \ll m \to \infty$$

QCD confinement scale SUSY breaking scale

[D. Kondo, H. Murayama & B. Noether; arXiv:2505.18138] [H. Murayama; arXiv:2104.01179]
[C Csaki, T. Roy, M. Ruhdorfer, T. Youn; arXiv: 2505.07593]
[C. Csaki, H. Murayama, O. Telem; arXiv: 2104.10171]

CASE 1 $F<N$: ADS Superpotential [C. Csaki, et. al.; arXiv:2307.04809]

$$W = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/N-F} + \mathrm{Tr}[m_Q M]$$

also see eg [M. Dine, P. Draper, L. Stephenson-Haskins & D. Xu; arXiv:1612.05770]

where $M_{ff'} = \bar{Q}_f Q_{f'}$ are $\sim$ the mesons.

Scalar Potential:

$$V \sim (\text{constants})\,(\det U)^{-1/(N-F)} + h.c.$$

where $U = e^{i\eta'} e^{i\pi^a T^a}$

Recall Higgs $H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ (h+v) + i\phi_4 \end{pmatrix} \xrightarrow{\text{field redefinition}} \begin{pmatrix} 0 \\ (\tilde{h}+v)/\sqrt{2} \end{pmatrix} U(\varphi)$

CASE II : $F = N, N+1$:

also see eg [N. Seiberg; arXiv: hep-th/9509066] No change

[C. Csaki, A. Gomes, H. Murayama, B. Noether, D.R. Varier & O. Telem; arXiv: 2212.03260]

Protected from Runaways to Incalculable Minima

CASE III : $N+1 < F \lesssim 3N/2$ Gaugino condensation in the dual gauge group

$$W_d = \frac{1}{\mu} q_i M_{ij} \bar{q}_j + \mathrm{Tr}(m_Q M)$$

Relates scales of original & dual theories

$$\Lambda^{3N-F} \tilde{\Lambda}^{3\tilde{N}-F} = (-1)^{F-N} \mu^F$$

fundamentals of $SU(F-N)$

and results in the same scalar potential

$$V \sim (\text{constants}) (\det U)^{-1/(F-N)} + h.c.$$

[C. Csaki, et. al.; arXiv: 2307.04809]

Important!!

- Impossible to restore Chiral symmetry. (Ie write V as a function of Φ)

Note: We want to do this to investigate symmetry breaking.

- The term to try would be

$$\sim -c\,(\det \Phi)^p + c.c.$$

where $p = 1/(F-N)$ (for $F > N+1$) but...

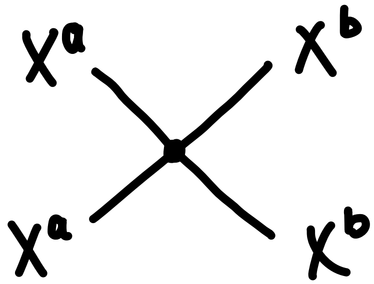
TERM IS NOT ANALYTIC AT $\Phi = 0$!

$\Rightarrow$ Symmetry cannot be linearly realised

[S. Coleman, J. Wess & B. Zumino: Structure of phenomenological Lagrangians 1. Phys. Rev 177 (1969)]

Only have $U(1)_V \times SU(F)_V$ symmetry linearly realised!

Can also see this in scattering amplitudes



$$= \frac{cp}{2F}\,\bar{\sigma}^{Fp-4}\,(F^2-1)\left[-4F^2+6+pF^3+pF\right]$$

Position of the vacuum

$$\xrightarrow{p=1} \frac{c}{2F}\,\sigma^{F-4}\,(F^2-1)(F-3)(F-2)(F-1).$$

Cannot take $\bar{\sigma}\to 0$ for p not integer!

NB Such dynamics familiar in HEFT [A. Falkowski, R. Rattazzi; arXiv: 1902.05936], [R. Alonso, J.C. Criado, R. Houtz, MW; arXiv: 2312.00881], etc

How can we investigate such a term with GWs?

Chiral symmetry already *mildly* broken explicitly by quark masses

- Add small* degenerate quark mass term

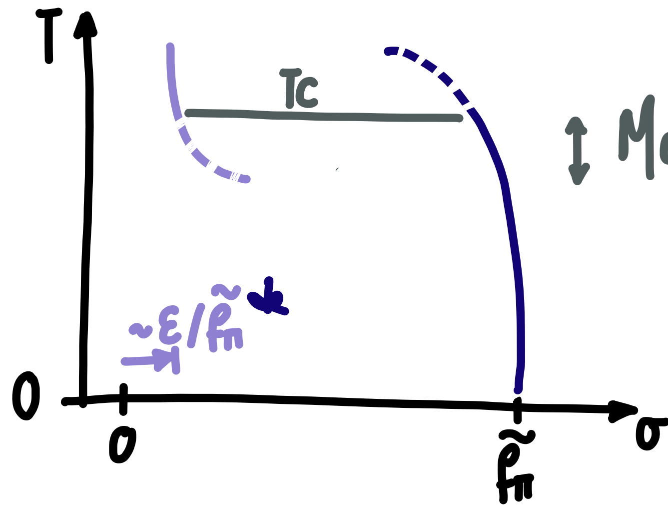
$$V_{LSM} = -\frac{\varepsilon}{\sqrt{2F}} \left(\mathrm{Tr}[\Phi] + \mathrm{Tr}[\Phi^\dagger] \right)$$

- New tree-level potential for σ

$$V_{\text{tree}}(\sigma) = -\varepsilon\sigma - \frac{m^2}{2}\sigma^2 - \frac{c}{F^2}\sigma^{Fp} + \frac{\lambda_\sigma}{8}\sigma^4$$

(Zero temp, tree-level vacua $0 = -\varepsilon - m^2 \tilde{f}_\pi - \frac{cp}{F}\tilde{f}_\pi^{\,Fp-1} + \frac{\lambda_\sigma}{2}\tilde{f}_\pi^{\,3}$).

- Vacuum evolution now looks like:



↕ May have nucleation

- Keeps us away from the non-analytic, would-be 'symmetric point'.

↳ Must be very careful about this!

eg shift everything around $\sigma = \tilde{f}_\pi$

$$V^0_{LSM} = -\varepsilon(\delta\sigma + \tilde{f}_\pi) - \frac{m^2}{2}(\delta\sigma + \tilde{f}_\pi)^2 - \frac{c}{F^2}(\delta\sigma + \tilde{f}_\pi)^{F_P} + \frac{\lambda_0}{8}(\delta\sigma + \tilde{f}_\pi)^4$$

Aside

Zero temperature masses:

$$m_\sigma^2(0) = \frac{\varepsilon}{\tilde{f}_\pi} - \frac{cp}{F}(Fp-2)\tilde{f}_\pi^{Fp-2} + \lambda_\sigma \tilde{f}_\pi^2$$

$$m_{\eta'}^2(0) = \frac{\varepsilon}{\tilde{f}_\pi} + cp^2 \tilde{f}_\pi^{Fp-2}$$

$$m_X^2(0) = \frac{\varepsilon}{\tilde{f}_\pi} + \frac{2cp}{F} f_\pi^{Fp-2} + \lambda_a f_\pi^2$$

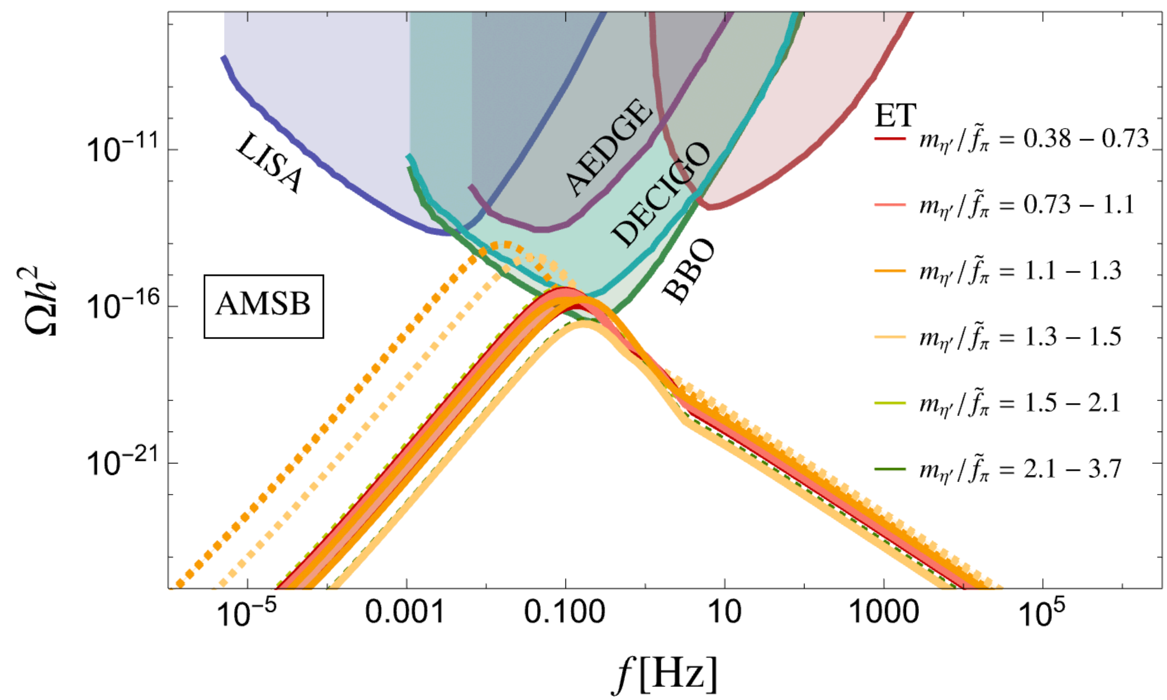
$$m_\pi^2(0) = \frac{\varepsilon}{\tilde{f}_\pi}$$

shifted equations for dressed masses:

$$c_1 = \frac{c}{F^2}\tilde{f}_\pi^{\;pF-4}(pF)(pF-1)(pF-2)(pF-3)$$
$$c_2 = \frac{c}{F^2}\tilde{f}_\pi^{\;pF-4}(pF)(pF-2)(pF-3)$$
$$c_3 = \frac{c}{F^2}\tilde{f}_\pi^{\;pF-4}(pF)(pF^3-4F^3+pF+6)$$

Preliminary Results

F=6 N=4



$m_0^2 \sim (0.5, 2.5) \times 10^6$

$m_0^2 \sim (0.5, 1.3) \times 10^6$ ← Unitary & perturbative

$m_\pi^2 \sim (0.001, 0.1) \times 10^6$ (GeV)^2

$m_\pi^2 \sim (0.001, 0.05) \times 10^6$ (GeV)^2

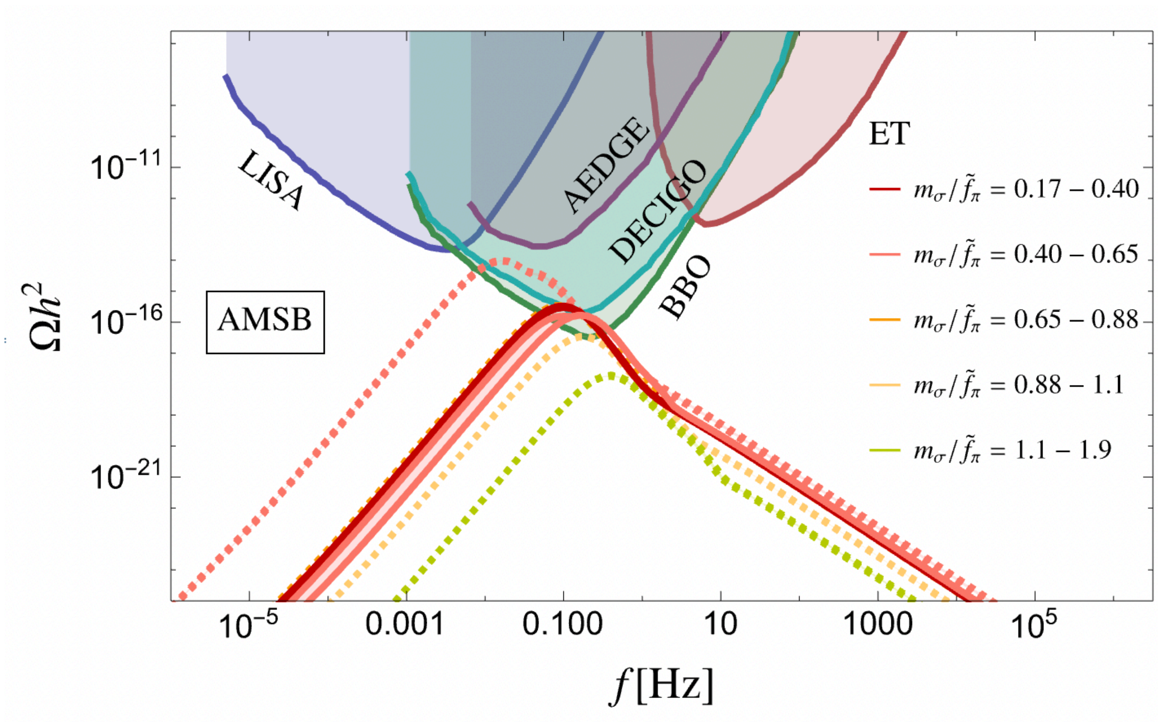
$f_\pi \sim (0.6, 1.8) \times 10^3$ (GeV)

$f_\pi \sim (0.9, 1.8) \times 10^3$ (GeV)

⟹ Tuned Masses?

Preliminary Results

F:6 N:4



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$m_{\eta'}^2 \sim (0.5, 5)\times10^6$ (GeV)^2

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Conclusions

1// Interesting to test out these AMSB → QCD arguments.

↳ Questions about applicability eg reversing confinement & SUSY breaking scales.

2// Gravitational wave phenomenology may be one angle, perhaps in combination with a meson mass spectrum?

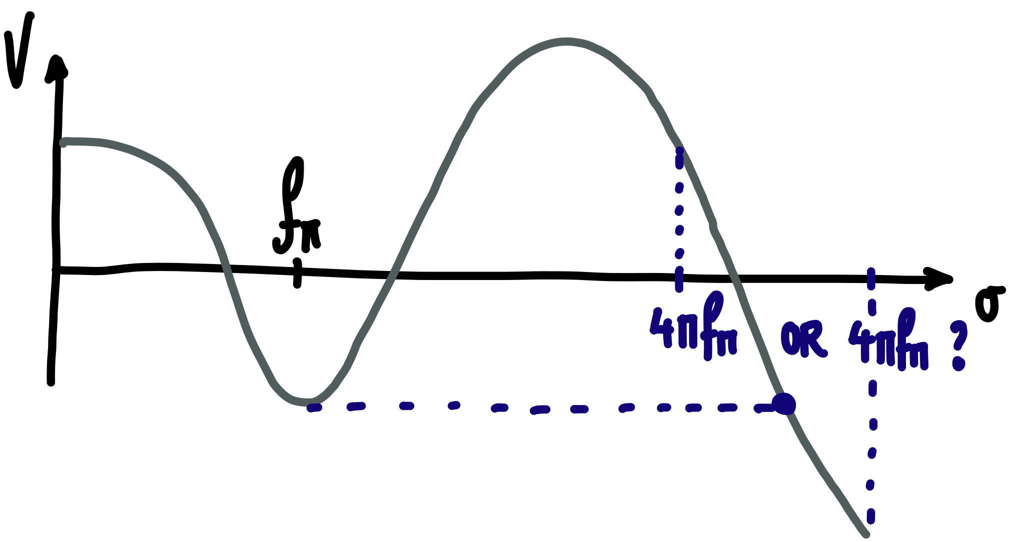
3// Nevertheless, there are *many* unknowns & uncertainties

↳ always check your EFT is unitary!

Backup: Boundedness from Below

What happens when $F > 4$?

$$V_{tree} = -\frac{m^2}{2}\sigma^2 - \frac{c}{F^2}\sigma^F + \frac{\lambda_\sigma}{8}\sigma^4$$



The EFT cutoff is at $4\pi f_\pi$.

- If $V(\sigma = 4\pi f_\pi) > V(\sigma = f_\pi)$
  potential is *bounded from below*

- If $V(\sigma = 4\pi f_\pi) < V(\sigma = f_\pi)$
  potential *NOT bounded from below*

within EFT validity.