Dark Energy from Causal Set Theory

Yasaman K. Yazdi (DIAS)

University of Toronto THEP Seminar

May 11, 2026

Taighde Éireann Research Ireland

DIAS
Institiúid Ard-Léinn Bhaile Átha Cliath | Dublin Institute for Advanced Studies

(Some) Open Questions in Cosmology

- **Early Universe:** Resolution of the big bang singularity and what happens shortly after (Exponential expansion? How and why?)

- **Origin of Dark Energy (cosmological constant puzzles):** Is it a cosmological constant? Is it dynamical? A fluid? stochastic?... Why is its value today so special and unnatural according to simple models of it as vacuum energy?

- **Fundamental and Microscopic Origin of (Cosmological) Horizon Entropy** $S = \frac{c^3}{\hbar}\frac{A}{4G}$: What are the correct degrees of freedom and how do we determine this.

(Some) Open Questions in Cosmology

- **Early Universe:** Resolution of the big bang singularity and what happens shortly after (Exponential expansion? How and why?)

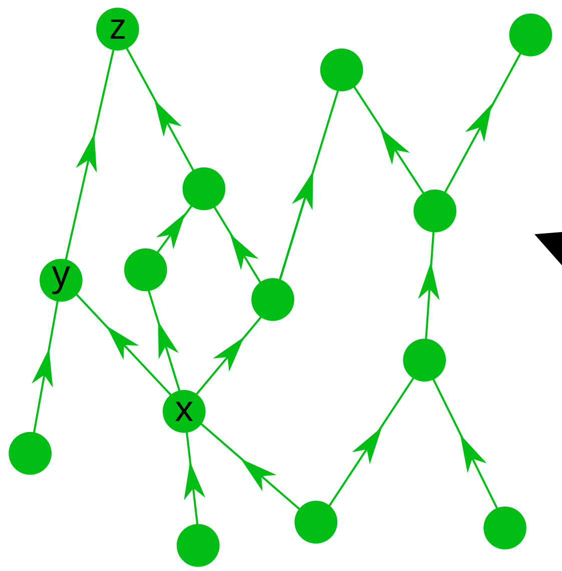
- **Origin of Dark Energy (cosmological constant puzzles):** Is it a cosmological constant? Is it dynamical? A fluid? stochastic?... Why is its value today so special and unnatural according to simple models of it as vacuum energy?

- **Fundamental and Microscopic Origin of (Cosmological) Horizon Entropy** $S = \frac{c^3}{\hbar}\frac{A}{4G}$: What are the correct degrees of freedom and how do we determine this.

Quantum Gravity

Causal Set Theory

Bombelli, Lee, Meyer, Sorkin, 1987, *Space-Time as a Causal Set*, PRL. 59, 521.

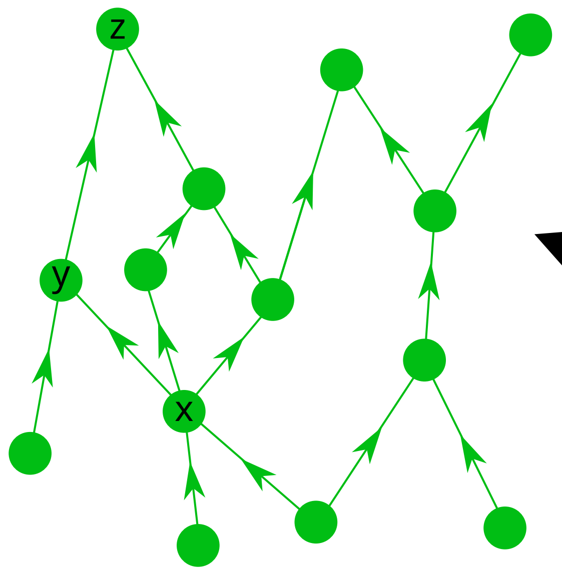


e.g.

A causal set is a fundamentally discrete spacetime with causal relations among some of its elements.

Causal Set Theory

Bombelli, Lee, Meyer, Sorkin, 1987, *Space-Time as a Causal Set*, PRL. 59, 521.



e.g.

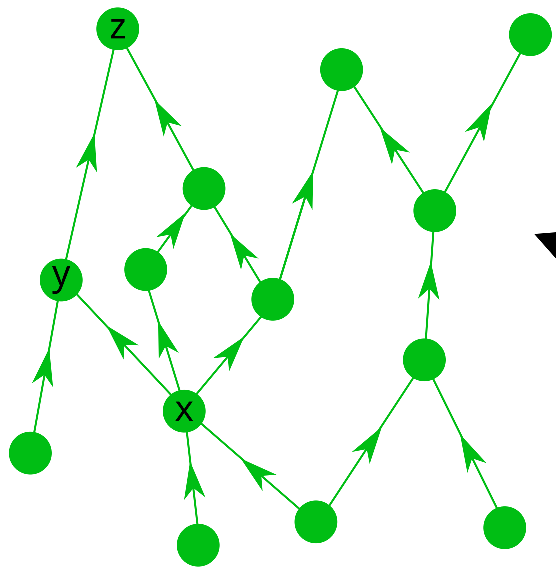
A causal set is a fundamentally discrete spacetime with causal relations among some of its elements.

A **locally finite partially ordered set**. The set $\mathscr{C}$ (of spacetime elements) and ordering relation $\preceq$ (causal precedence) satisfy:

- **Reflexivity**: for all $x \in \mathscr{C}, \quad x \preceq x$
- **Antisymmetry**: for all $x, y \in \mathscr{C}, x \preceq y \preceq x$ implies $x = y$
- **Transitivity**: for all $x, y, z \in \mathscr{C}, \quad x \preceq y \preceq z$ implies $x \preceq z$
- **Local Finiteness**: for all $x, y \in \mathscr{C}, |I(x, y)| < \infty$, where $|\cdot|$ denotes cardinality and $I(x, y)$ is the causal interval defined by $I(x, y) := \{z \in \mathscr{C} \,|\, x \preceq z \preceq y\}$

Causal Set Theory

Bombelli, Lee, Meyer, Sorkin, 1987, *Space-Time as a Causal Set*, PRL. 59, 521.



e.g.

A causal set is a fundamentally discrete spacetime with causal relations among some of its elements.

A **locally finite partially ordered set**. The set $\mathscr{C}$ (of spacetime elements) and ordering relation $\preceq$ (causal precedence) satisfy:

- **Reflexivity**: for all $x \in \mathscr{C}, \quad x \preceq x$
- **Antisymmetry**: for all $x, y \in \mathscr{C}, x \preceq y \preceq x$ implies $x = y$
- **Transitivity**: for all $x, y, z \in \mathscr{C}, \ \ x \preceq y \preceq z$ implies $x \preceq z$
- **Local Finiteness**: for all $x, y \in \mathscr{C}, |I(x, y)| < \infty,$ where $|\cdot|$ denotes cardinality and $I(x, y)$ is the causal interval defined by $I(x, y) := \{z \in \mathscr{C} \,|\, x \preceq z \preceq y\}$

We know that we can recover all essential aspects of a continuous spacetime (**causal structure** + **volume (discreteness)**) from a causal set.

Zeeman, *Causality Implies the Lorentz Group*, J. Math. Phys. 5: 490-493 (1964). Hawking, King and McCarthy, *A New Topology for Curved Space-Time which Incorporates the Causal, Differential and Conformal Structures*, J. Math. Phys. 17: 174 (1976). Malament, *The Class of Continuous Timelike Curves Determines the Topology of Space-time*, J. Math. Phys 18: 1399 (1977)

Manifoldlike Causal Sets

What does it (roughly) mean for a causal set $\mathscr{C}$ to be well approximated by a spacetime $(\mathscr{M}, g)$?

- **Causal Relations** **$\mathscr{C}$ should reflect the causal structure of the spacetime**
- **Volumes** **The number of elements N within any arbitrary region with volume V should be statistically proportional to V (number-volume correspondence)**
- **Scale** **True at scales larger than the discreteness scale**

Manifoldlike Causal Sets

The best **number-volume correspondence** is achieved by the **Poisson distribution:** Aslanbeigi, Saravani, arXiv:1403.6429

$$P_N(V) = \frac{(\rho V)^N}{N!} e^{-\rho V} \tag{1}$$

$$\boxed{\begin{aligned} \langle N \rangle &= \rho V \\ \delta N &= \sqrt{\rho V} \end{aligned}}$$

Manifoldlike Causal Sets

The best **number-volume correspondence** is achieved by the **Poisson distribution:** Aslanbeigi, Saravani, arXiv:1403.6429

$$P_N(V) = \frac{(\rho V)^N}{N!} e^{-\rho V} \qquad (1)$$

$$\boxed{\begin{aligned} \langle N \rangle &= \rho V \\ \delta N &= \sqrt{\rho V} \end{aligned}}$$

How? Place points at random in $\mathscr{M}$ using a **Poisson process**: divide spacetime into small subregions with volume dV and place at most one point in each subregion, with probability ρdV.

Then, $P_N(V) = \binom{V/dV}{N} \left(\rho dV\right)^N (1 - \rho dV)^{V/dV - N}$

which becomes (1) when $dV \to 0$

(Poisson limit theorem)

Manifoldlike Causal Sets

The best **number-volume correspondence** is achieved by the **Poisson distribution:** Aslanbeigi, Saravani, arXiv:1403.6429

$$P_N(V) = \frac{(\rho V)^N}{N!} e^{-\rho V} \qquad (1)$$

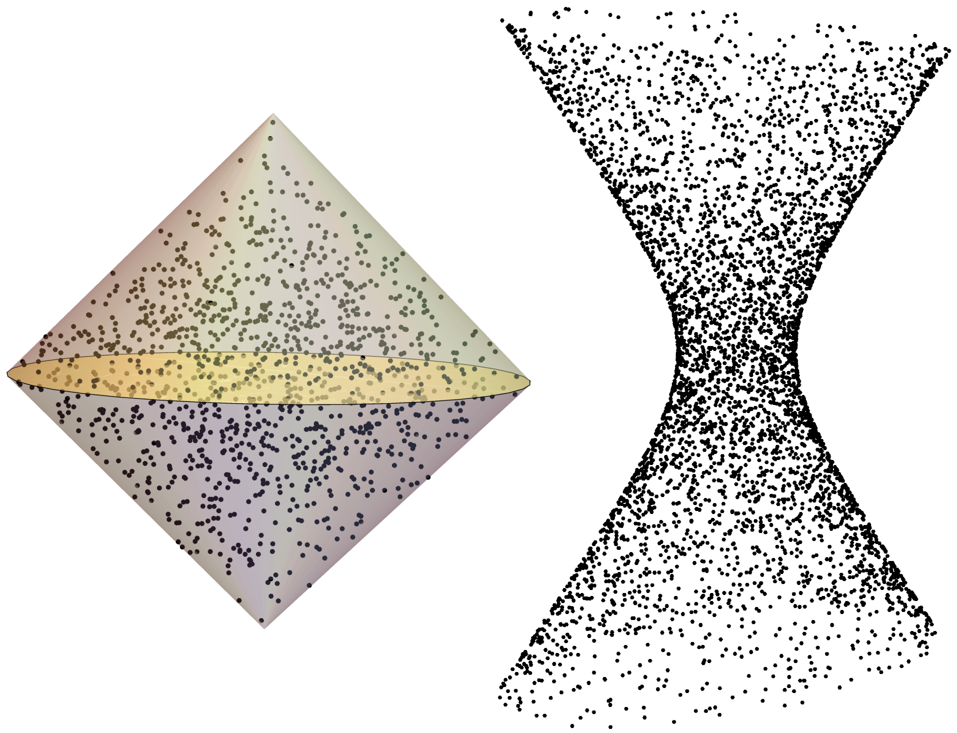
$$\boxed{\begin{aligned} \langle N \rangle &= \rho V \\ \delta N &= \sqrt{\rho V} \end{aligned}}$$

How? Place points at random in $\mathscr{M}$ using a **Poisson process**: divide spacetime into small subregions with volume dV and place at most one point in each subregion, with probability ρdV.

Then, $P_N(V) = \binom{V/dV}{N} \left(\rho dV\right)^N (1 - \rho dV)^{V/dV - N}$

which becomes (1) when $dV \to 0$

(Poisson limit theorem)



Poisson Sprinkling

Frame independent (arXiv:gr-qc/0605006, arXiv:1909.06070)

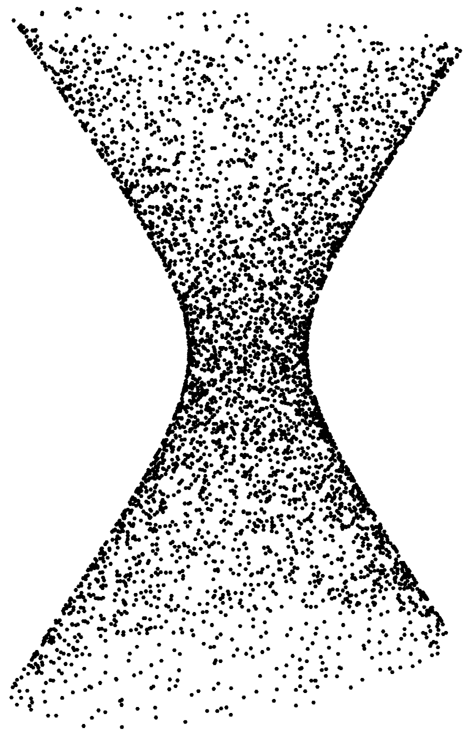
Manifoldlike Causal Sets

The best **number-volume correspondence** is achieved by the **Poisson distribution:** Aslanbeigi, Saravani, arXiv:1403.6429

$$P_N(V) = \frac{(\rho V)^N}{N!} e^{-\rho V} \qquad (1)$$

$$\boxed{\begin{aligned} \langle N \rangle &= \rho V \\ \delta N &= \sqrt{\rho V} \end{aligned}}$$

- **Fundamental and Microscopic Origin of (Cosmological) Horizon Entropy** $S = \frac{c^3}{\hbar}\frac{A}{4G}$: what are the correct degrees of freedom and how do we determine this.



In manifoldlike causal sets, UV cutoff is $\left(\frac{V}{\langle N \rangle}\right)^{1/d} = \left(\frac{1}{\rho}\right)^{1/d}$ **in any geometry**

Covariant regularization of entanglement entropy

Jones, YKY, Spectral Spacetime Entropy for Quasifree Theories, arXiv:2602.16782.
Sorkin, YKY, Entanglement Entropy in Causal Set Theory, CQG 35 074004 (2018), arXiv:1611.10281.

Dynamical Models

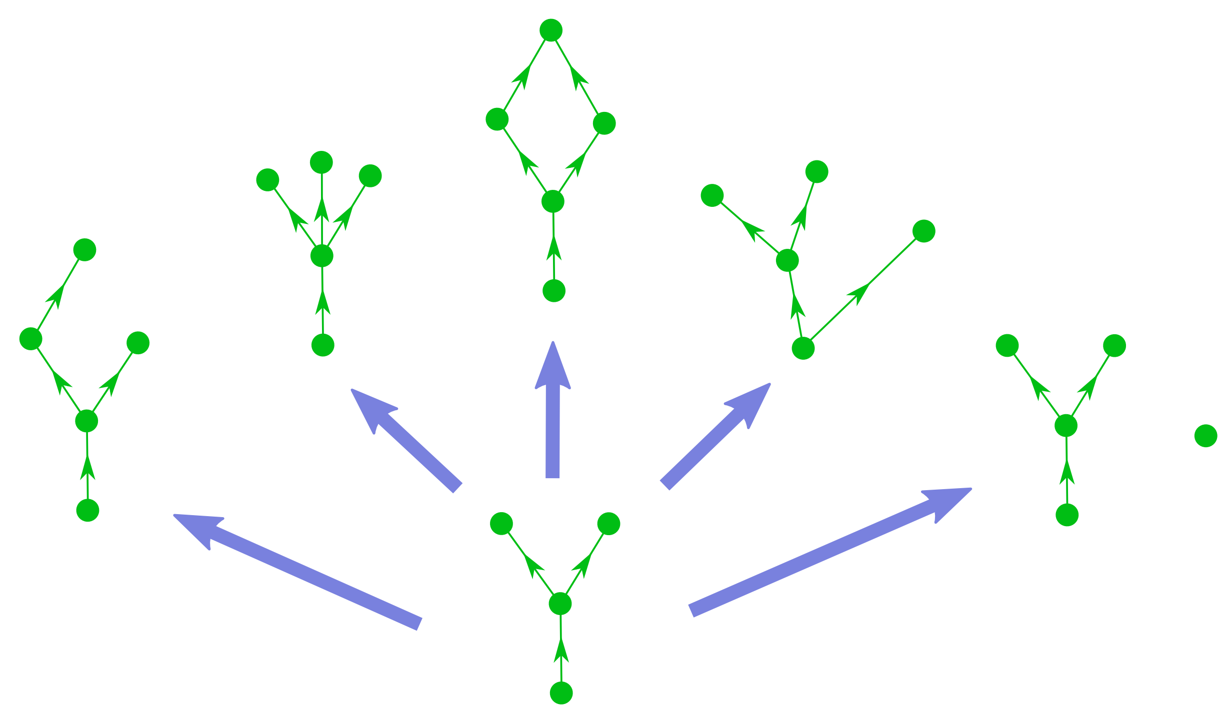
In a quantum sum over histories dynamics:

$$\boxed{\mathcal{Z}(N) = \sum_{|\mathscr{C}|=N} e^{iS_{cst}[\mathscr{C}]}}$$

number of elements plays a similar role to time.

In classical dynamical models, a causal set **grows element by element** sequentially. At stage 1, the first element is born, and subsequently at each stage n we have an n-element causal set.

Rideout, Sorkin, arXiv:gr-qc/9904062.



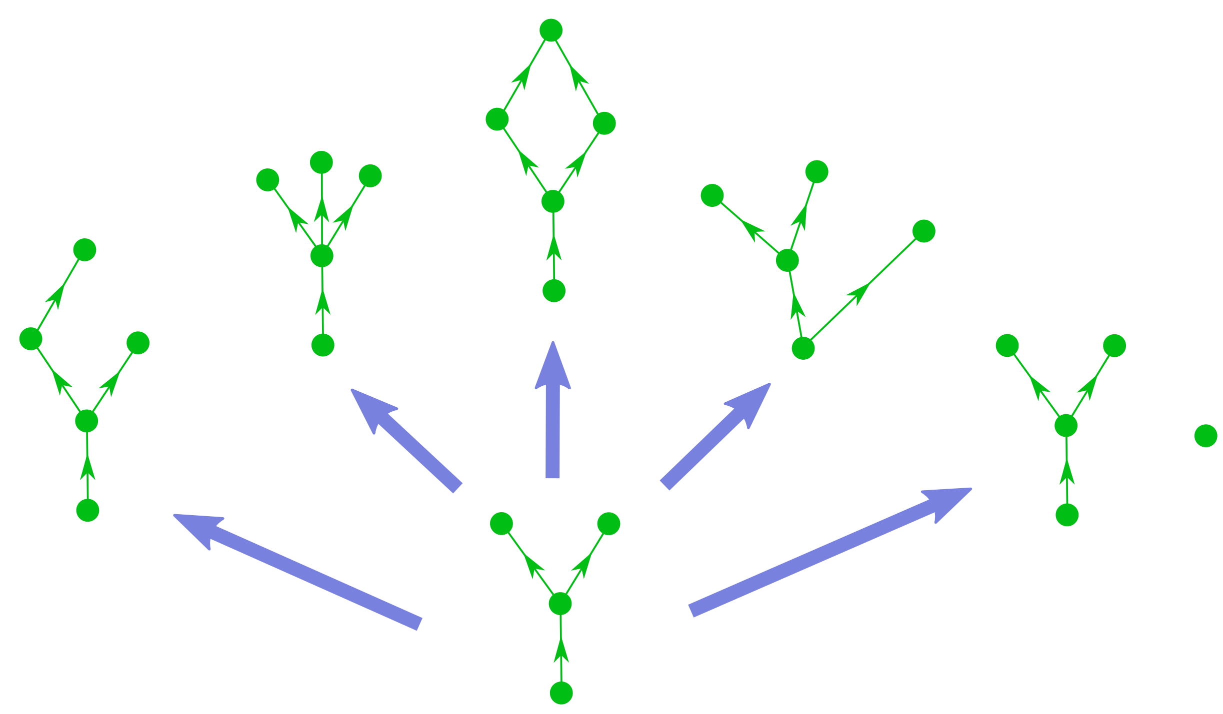
Early Rapid Growth in Dynamical Models

YKY, arXiv:2311.14881.

- **Early Universe:** Resolution of the big bang singularity and what happens shortly after (Exponential expansion? How and why?)

In classical dynamical models, a causal set **grows element by element** sequentially. At stage 1, the first element is born, and subsequently at each stage n we have an n-element causal set.

Rideout, Sorkin, arXiv:gr-qc/9904062.



Everpresent Λ

$$\mathscr{Z}(V) \sim \int_{Vol(\mathscr{M})=V} \mathscr{D}g_{\mu\nu}\; e^{iS_G[g]}$$

Everpresent Λ

$$\mathcal{Z}(V) \sim \int_{Vol(\mathcal{M})=V} \mathcal{D}g_{\mu\nu}\ e^{iS_G[g]} \xrightarrow{\text{Fourier Transform}} \mathcal{Z}(\Lambda) = \int dV\ e^{-i\Lambda V} \mathcal{Z}(V)$$

$$\mathcal{Z}(\Lambda) \sim \int \mathcal{D}g_{\mu\nu} \exp\left(iS_G[g] - i\Lambda \int d^4x \sqrt{-g} \right)$$

Therefore, spacetime volume V and Λ are quantum mechanically conjugate

$$\frac{\delta\Lambda}{8\pi G} \cdot \delta V \geq \frac{\hbar}{2}$$

Everpresent Λ

$$\mathscr{Z}(V) \sim \int_{Vol(\mathscr{M})=V} \mathscr{D}g_{\mu\nu}\ e^{iS_G[g]} \xrightarrow{\text{Fourier Transform}} \mathscr{Z}(\Lambda) = \int dV\ e^{-i\Lambda V} \mathscr{Z}(V)$$

$$\mathscr{Z}(\Lambda) \sim \int \mathscr{D}g_{\mu\nu} \exp\left(iS_G[g] - i\Lambda \int d^4x\sqrt{-g} \right)$$

Therefore, spacetime volume V and Λ are quantum mechanically conjugate

$$\frac{\delta\Lambda}{8\pi G} \cdot \delta V \geq \frac{\hbar}{2}$$

A causal set with fixed N can be approximated by continuum spacetimes with mean $\langle V \rangle = N$ and standard deviation $\delta V = \sqrt{N} \sim \sqrt{V}$. Also: $N, V \gg 1$

$$\delta\Lambda\ \delta V \sim 1$$

Everpresent Λ

$$\mathcal{Z}(V) \sim \int_{Vol(\mathcal{M})=V} \mathcal{D}g_{\mu\nu}\ e^{iS_G[g]} \xrightarrow{\text{Fourier Transform}} \mathcal{Z}(\Lambda) = \int dV\ e^{-i\Lambda V}\mathcal{Z}(V)$$

$$\mathcal{Z}(\Lambda) \sim \int \mathcal{D}g_{\mu\nu} \exp\left(iS_G[g] - i\Lambda \int d^4x\sqrt{-g} \right)$$

Therefore, spacetime volume V and Λ are quantum mechanically conjugate

$$\frac{\delta\Lambda}{8\pi G} \cdot \delta V \geq \frac{\hbar}{2}$$

A causal set with fixed N can be approximated by continuum spacetimes with mean $\langle V \rangle = N$ and standard deviation $\delta V = \sqrt{N} \sim \sqrt{V}$. Also: $N, V \gg 1$

$$\delta\Lambda\ \delta V \sim 1 \xrightarrow{\text{Assume } \langle\Lambda\rangle = 0} \Lambda \sim \delta\Lambda \sim 1/\delta V = 1/\sqrt{V} \sim H^2 \sim 10^{-122}!$$

Everpresent Λ

$$\Lambda \sim \frac{1}{\delta V} = \frac{1}{\sqrt{V}} \sim H^2 \sim 10^{-122}$$

1987 **Sorkin, "A Modified Sum-Over-Histories for Gravity"** reported in the article by Brill and Smolin: "Workshop on quantum gravity and new directions", in Proceedings of the International Conference on Gravitation and Cosmology, Goa, India, 14–19 December **1987**, pp. 184–186, 1988.

1987 **Sorkin**, **"Role of Time in the Sum-Over-Histories Framework for Gravity",** International journal of theoretical physics 33 (1994), no. 3 523–534. The text of a talk at the conference, "The History of Modern Gauge Theories," in Logan, Utah, in July **1987**.

1990 **Sorkin, "First Steps with Causal Sets"**, in Proceedings of the Ninth Italian Conference of the same name, in Capri, Italy, September, **1990**, pp. 68–90 (World Scientific, Singapore, 1991).

1993 **Sorkin, "Forks in the Road, on the Way to Quantum Gravity"**, talk given at the conference entitled "Directions in General Relativity", held at College Park, Maryland, May, **1993**; Int. J. Th. Phys. 36 : 2759–2781 (1997)

2004 **Ahmed**, **Dodelson**, **Greene**, and **Sorkin**, **"Everpresent Λ"**, PRD 69, no. 10 103523, **2004**.

2013 **Ahmed** and **Sorkin**, **"Everpresent Λ II: Structural Stability"**, PRD 87, **2013**.

2018 **Zwane**, **Afshordi**, and **Sorkin**, "**Cosmological tests of Everpresent Λ"**, CQG 35194002, **2018**.

2023 **Das**, **Nasiri**, and **YKY**, **"Aspects of Everpresent Λ (I): A Fluctuating Cosmological Constant from Spacetime Discreteness"**, *JCAP* 10 **2023** 047.

2024 **Das**, **Nasiri**, and **YKY**, **"Aspects of Everpresent Λ (II): Cosmological Tests of Current Models"**, *JCAP* 10 **2024** 076.

2025 **Moradi**, **YKY**, and **Zilhão** **"Fluctuations and Correlations in Causal Set Theory"**, CQG *42 045017,* **2025**.

A Phenomenological Model of Everpresent Λ

Ahmed, Dodelson, Greene, and Sorkin, "Everpresent Λ", PRD 69, no. 10 103523, 2004.

$$|\Lambda| \sim 1/\sqrt{V}$$

Each causal set element contributes a random variable with standard deviation α to $S_\Lambda = \Lambda V$:

$$S_\Lambda|_{1\ element} = \begin{cases} \alpha, & p = \frac{1}{2} \\ -\alpha, & p = \frac{1}{2} \end{cases}$$

A Phenomenological Model of Everpresent Λ

Ahmed, Dodelson, Greene, and Sorkin, "Everpresent Λ", PRD 69, no. 10 103523, 2004.

$$|\Lambda| \sim 1/\sqrt{V}$$

Each causal set element contributes a random variable with standard deviation α to $S_\Lambda = \Lambda V$:

$$S_\Lambda|_{1\,element} = \begin{cases} \alpha, & p = \frac{1}{2} \\ -\alpha, & p = \frac{1}{2} \end{cases}$$

The effective S_Λ at time t is then the sum of all these single-element S_Λ's from elements in the **past lightcone**

of an observer at t. From the

central limit theorem $|S_\Lambda| \sim \alpha\sqrt{V}$

and $|\Lambda| = |S_\Lambda|/V \sim \alpha/\sqrt{V}$

A Phenomenological Model of Everpresent Λ

Ahmed, Dodelson, Greene, and Sorkin, "Everpresent Λ", PRD 69, no. 10 103523, 2004.

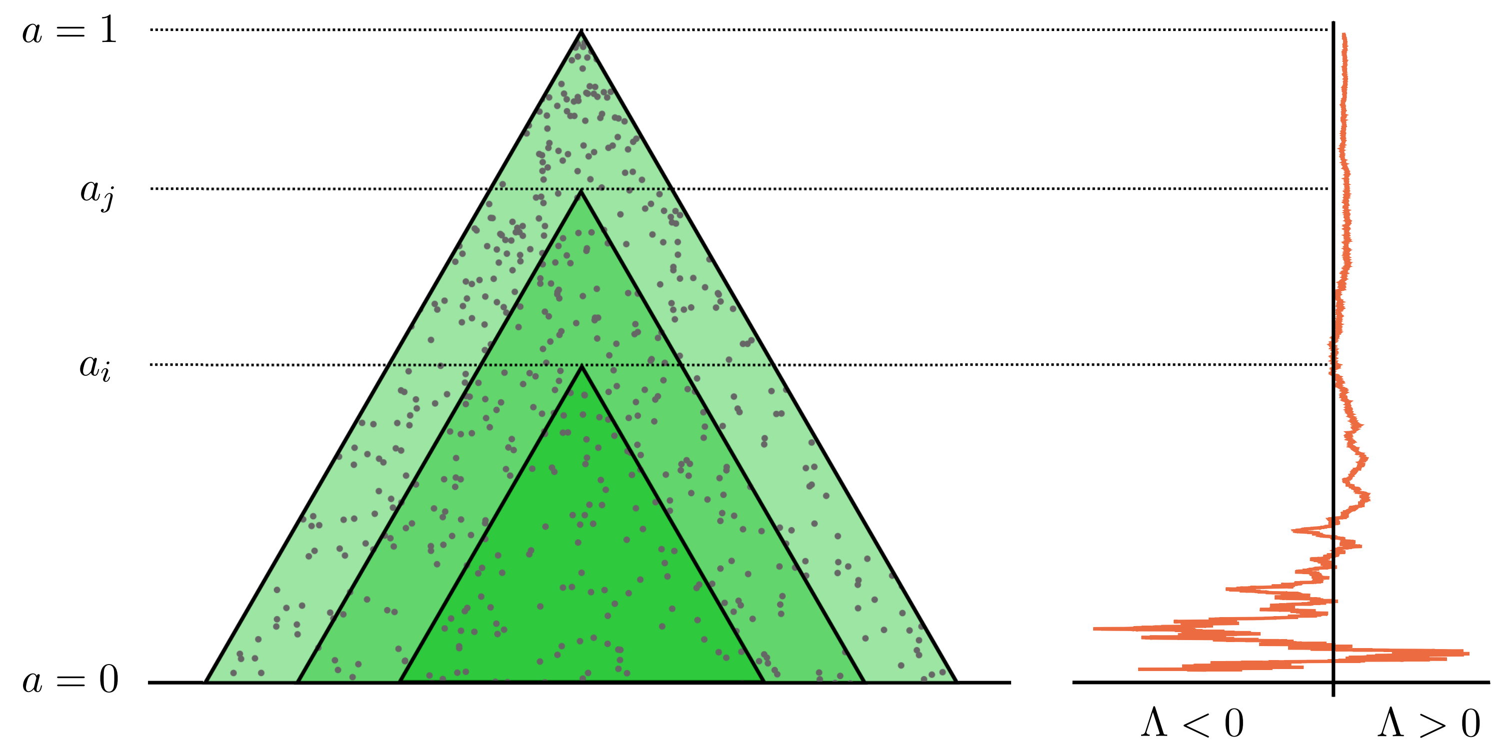
$$|\Lambda| \sim 1/\sqrt{V}$$

Each causal set element contributes a random variable with standard deviation α to $S_\Lambda = \Lambda V$:

$$S_\Lambda|_{1\,element} = \begin{cases} \alpha, & p = \frac{1}{2} \\ -\alpha, & p = \frac{1}{2} \end{cases}$$

The effective S_Λ at time t is then the sum of all these single-element S_Λ's from elements in the **past lightcone** of an observer at t. From the central limit theorem $|S_\Lambda| \sim \alpha\sqrt{V}$ and $|\Lambda| = |S_\Lambda|/V \sim \alpha/\sqrt{V}$

$$\Lambda(t_n) = \frac{\Lambda(t_{n-1})V(t_{n-1}) + \alpha\sqrt{\Delta V_n}\,\xi_n}{V(t_n)}$$



A Phenomenological Model of Everpresent Λ

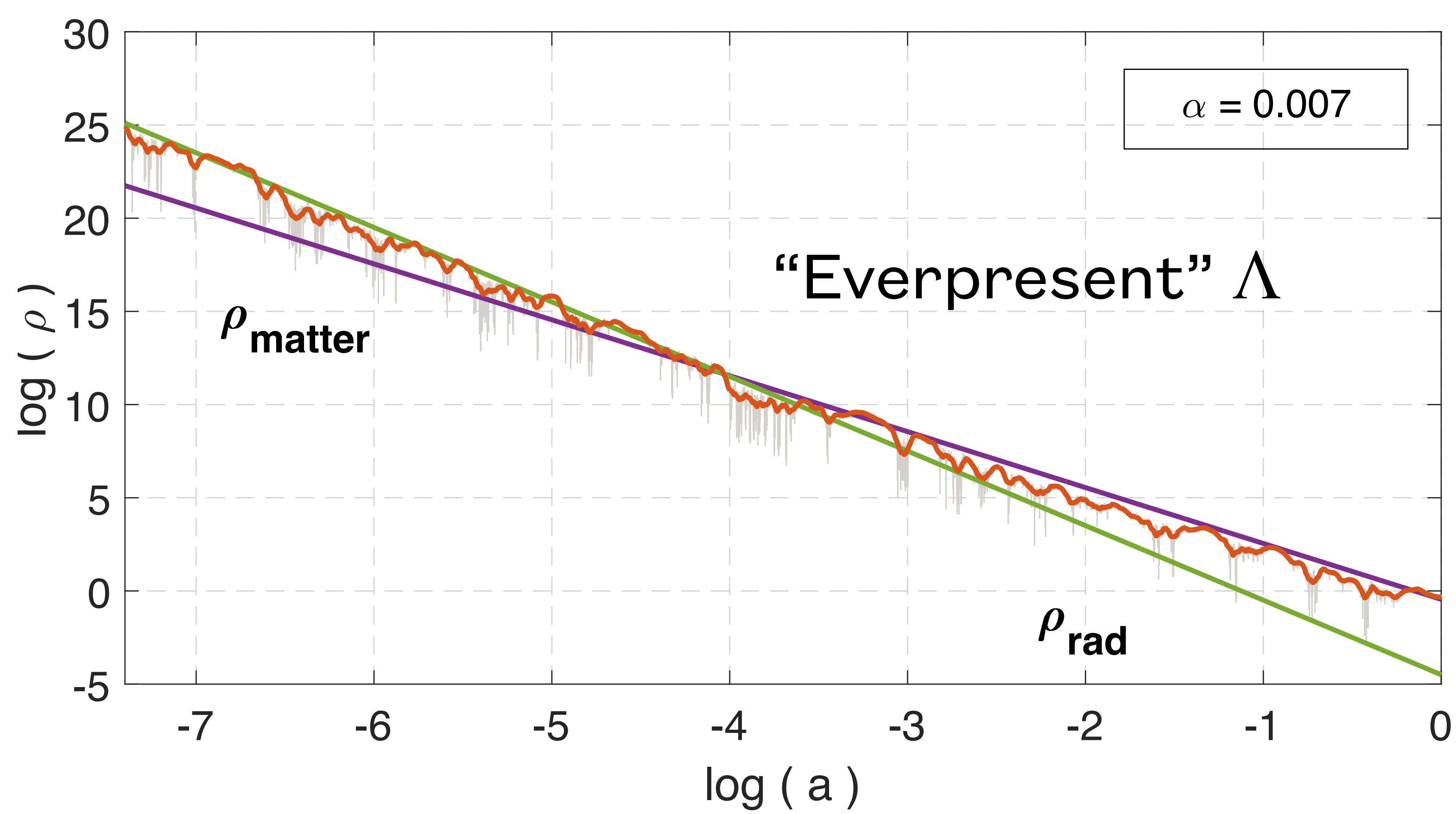
Assume a spatially flat, isotropic and homogeneous FLRW universe:

$$ds^2 = a(\tau)^2(-d\tau^2 + dr^2 + r^2 d\Omega^2)$$

Evolve V's using Friedmann eq:

$$\boxed{\Lambda(t_n) = \frac{\alpha \sum_{i=1}^{n} \sqrt{\Delta V_i}\, \xi_i}{V(t_n)}}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r\right) + \frac{\Lambda}{3}$$



Das, Nasiri, YKY, arXiv:2304.03819.

A Phenomenological Model of Everpresent Λ

Assume a spatially flat, isotropic and homogeneous FLRW universe:

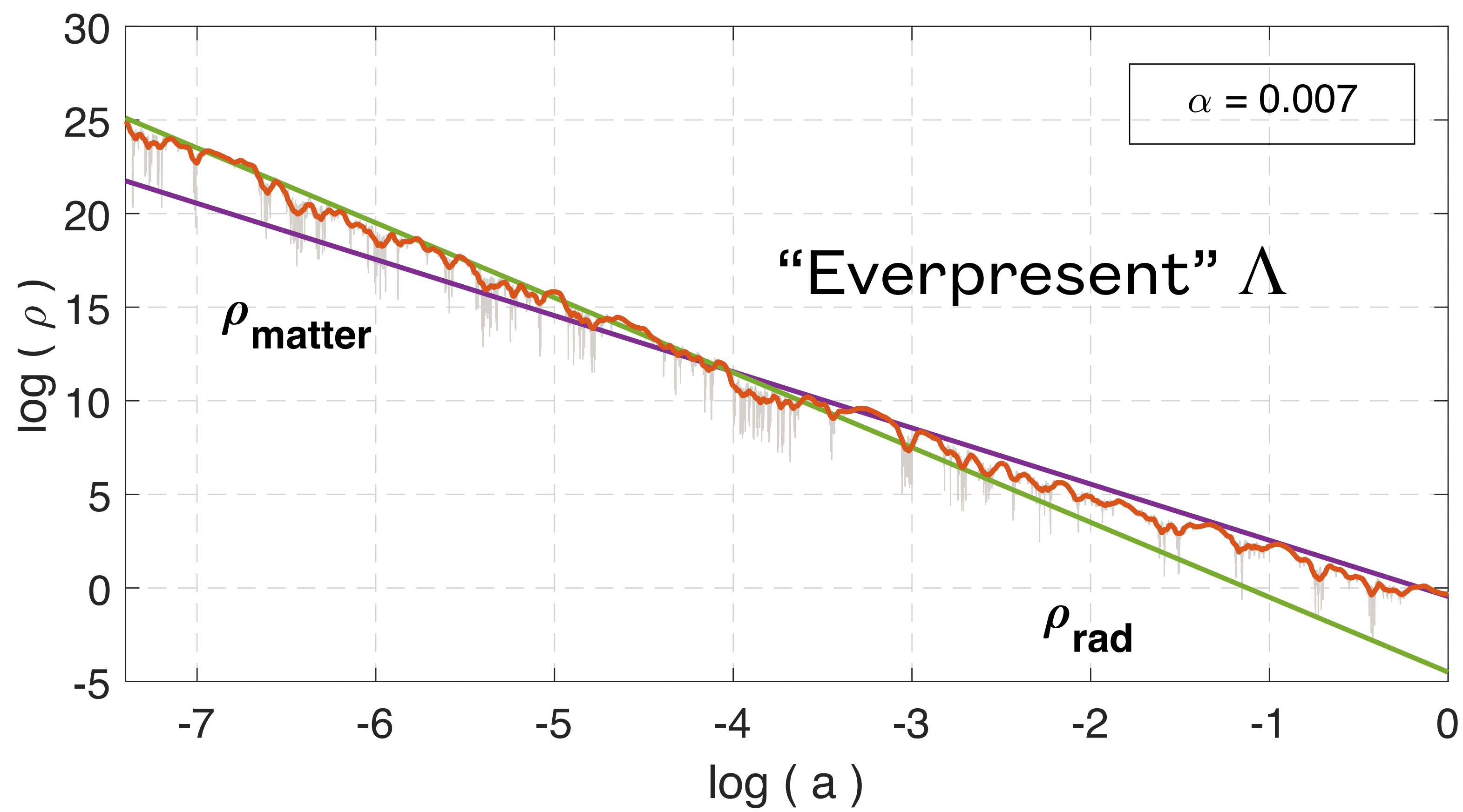
$$ds^2 = a(\tau)^2(-d\tau^2 + dr^2 + r^2 d\Omega^2)$$

Evolve V's using Friedmann eq:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r\right) + \frac{\Lambda}{3}$$

$$\boxed{\Lambda(t_n) = \frac{\alpha \sum_{i=1}^{n} \sqrt{\Delta V_i}\, \xi_i}{V(t_n)}}$$

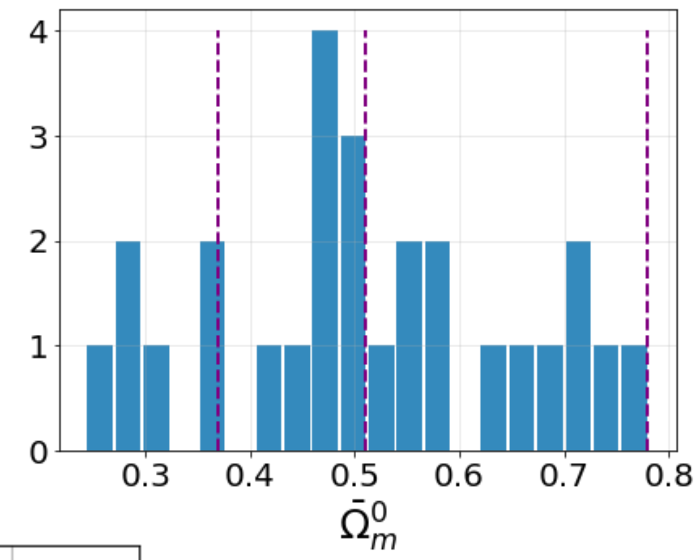
Sometimes, we encounter $H^2 < 0$ before the present. At such points, our evolution eq. is no longer valid. We discard such realizations, but finding the replacement evolution law for these cases is an **important open question.**

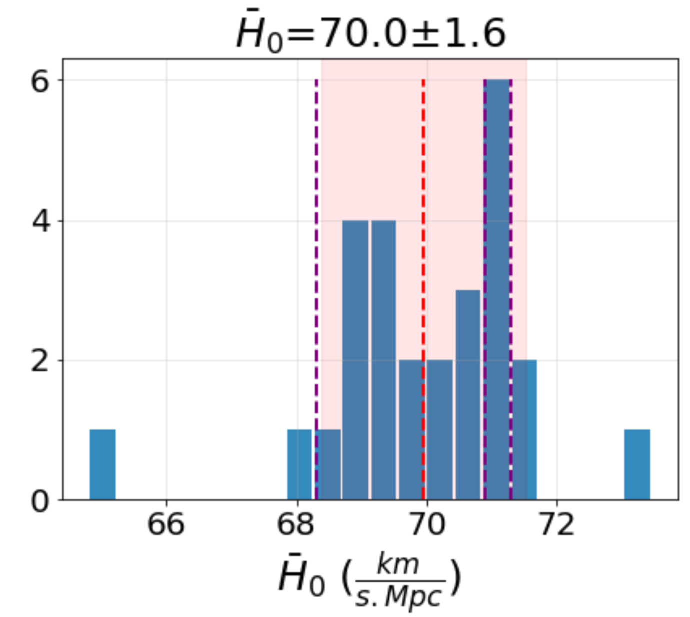


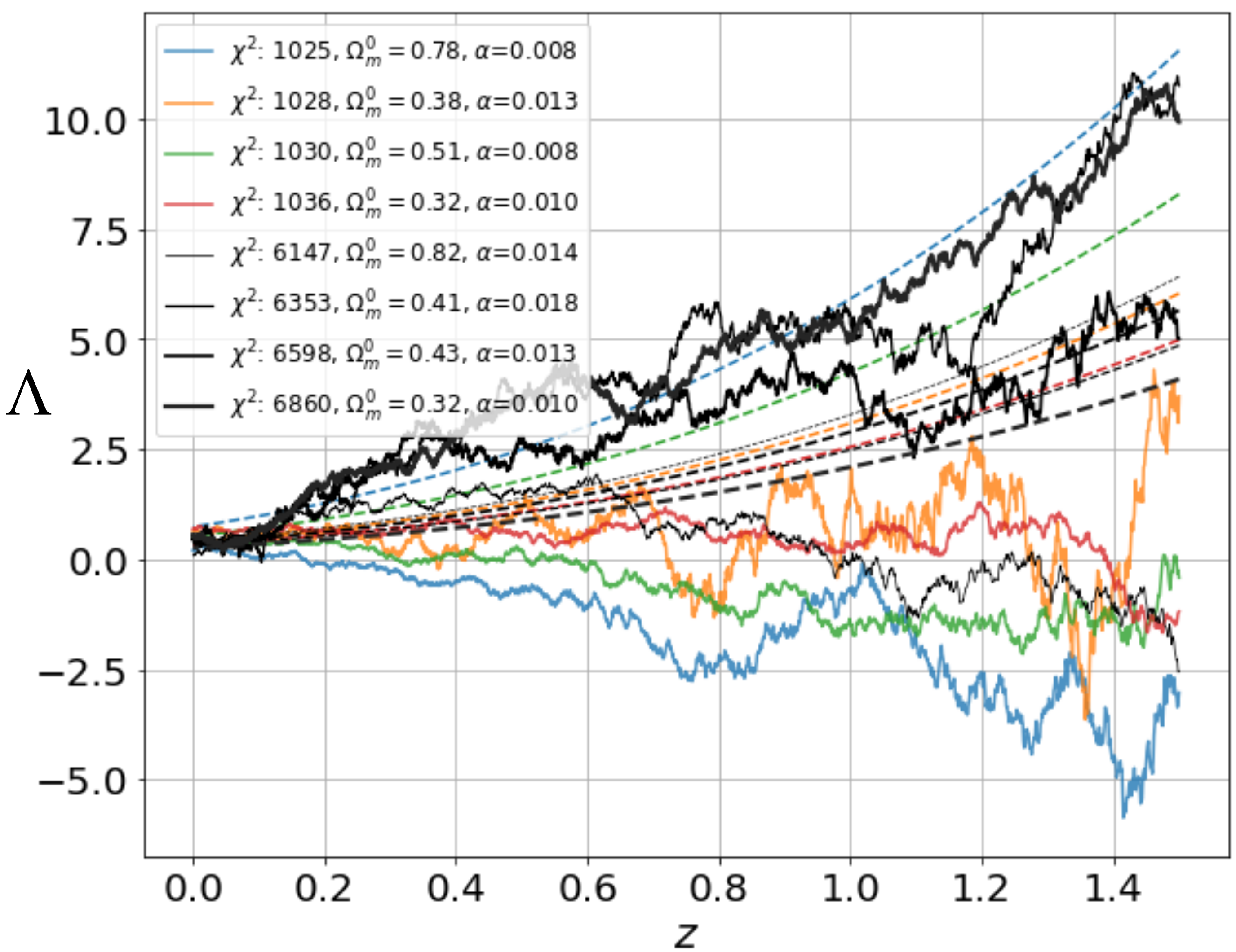
Das, Nasiri, YKY, arXiv:2304.03819.
Das, Nasiri, YKY, arXiv:2307.13743.

Type Ia Supernovae

Out of a sample of 20,000 seeds, 3 Everpresent Λ histories (dashed lines) yielded a better χ^2 (1025, 1028, 1030) than ΛCDM (1033).







Dark energy densities that are smaller than matter density at small redshifts (colored) are favored over those with comparable or larger dark energy densities (black).

Das, Nasiri, YKY, arXiv:2307.13743.

General Features of Everpresent Λ

$$\boxed{|\Lambda| \sim 1/\sqrt{V}}$$

- Λ fluctuates in magnitude and in sign
- UV-IR mixing
- Dynamical, as favored by recent DESI data
- Hubble Tension
- No extra fields needed

Phenomenological Models of Everpresent Λ

- Fluctuations of the causal set action

Benincasa, Dowker, arXiv:1001.2725.
Moradi, YKY, Zilhão, arXiv:2407.03395.

- Truncate negative random numbers beyond one standard deviation
- Shifting the mean: If $\Lambda_0 > 0$ initially (at very high redshift), V undergoes exponential growth

Das, Nasiri, YKY, arXiv:2304.03819.

  - Exit to radiation dominated era when Everpresent Λ overtakes Λ_0
  - Value of Λ_0 can potentially be related to non-manifoldlike causal sets

- Modifications to the Friedmann equation
- Suggestions welcome

Phenomenological Models of Everpresent Λ

- Fluctuations of the causal set action

Benincasa, Dowker, arXiv:1001.2725.
Moradi, YKY, Zilhão, arXiv:2407.03395.

- Truncate negative random numbers beyond one standard deviation

- Shifting the mean: If $\Lambda_0 > 0$ initially (at very high redshift), V undergoes exponential growth

Das, Nasiri, YKY, arXiv:2304.03819.

    - Exit to radiation dominated era when Everpresent Λ overtakes Λ_0

    - Value of Λ_0 can potentially be related to non-manifoldlike causal sets

- Modifications to the Friedmann equation

- Suggestions welcome

Thank You