Vertically Vibrated Ball-Chains in a Sinusoidal Potential

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Introduction

Ordinary ball-chains are seen to exhibit interesting coupled motions when vibrated on a vertically oscillating plate. Knots in such chains are seen to exhibit random walk behaviour [1], and at certain driving parameters they spontaneously coil into spirals [2]. This report discusses the behaviour of such chains in a sinusoidal potential, achieved by sinusoidally modulating the height of the plate.

The chain used was brass, with a minimum chain density of 3.03 ± 0.02 beads/cm and a maximum chain density of 4.06 +/- 0.02 beads/cm. Initially, the plate used was a square piece of plastic with sinusoidal ridges 4mm in wavelength and 1mm in depth, up to a 0.02 mm machining tolerance. Test runs were performed on this plate in order to determine appropriate groove sizes for a circular plate to be made.

While the 4mm wavelength proved large enough for the chain used, the acceleration amplitude required for kink formation was already quite low, and an aluminium plate would likely have an even lower critical amplitude due to less damping from the material. It was decided that 4mm would be the minimum wavelength used in the new plate. It was also decided that the plate should not exceed 14 inches in diameter, and that five grooves should be present for each groove measurement.

Four sets each with five sinusoidal grooves were machined into the plate, separated by 5mm flat regions. From the innermost to outermost sets, they had wavelengths of 5mm, 4mm, 6mm, and 10mm, and depths of 1.5mm, 1mm, 2mm, and 3mm respectively, each within the machining tolerance of 0.02mm. These measurements were selected to allow for sufficient availability of groove size (four types) and variability in their measurements (4mm to 10mm wavelength), while remaining within the constraint of a 14 inch diameter. Most of the runs were done on the outermost five grooves, which are 10.00 mm in wavelength and 5.00 mm in depth.

Care was taken to oscillate the plate at a frequency that avoided the formation of transverse standing waves in the chain. At some frequencies, transverse standing waves of large wavelength on the high end of the chain (7-10 beads) and short wavelength on the low end of the chain (3-4 beads) were formed, where the plate was not level. At 65Hz, where most data was collected, no such standing waves were formed.

Images of the chain were taken from directly above by a Nikon D70S camera at 2-3 second intervals. Python code was written which analyzes these images to locate the position of the chain and identify kinks. The code is given the coordiates of the center of the plate, as well as an upper bound for the plate radius in pixels. A mask is used which sets all pixels whose red and green values are above certain minimum values to 1, and all other pixels to 0. For each angle, a ray is traced from the center of the plate to the edge and the average position of the coordinates along that ray is found, weighted by the value of the nearest pixel. In this way, the center of the chain is found along that ray, within uncertainty of one pixel. One shortfall of the code is that it allows for only one radial position of the chain at a given angular position, though this is not always the case, as will be discussed later.

Features seen

The simplest feature seen is the kink, which is where the chain crosses from one groove to the next. Kinks must form in pairs, though the chain can be set up to initially have an odd number of kinks by securing the ends of the chain in different grooves. When two kinks of opposite sign come together, they can annihilate.

The next obvious feature is the double-kink, where the chain crosses over two grooves. This feature is created when two kinks of the same sign in the same grooves come together. What distinguishes a double-kink from two close kinks is that no section of chain runs laterally along the middle groove (Figure 1).

A kink that crosses n grooves outward as you increase angular position can be considered to have sign +n, and a kink that crosses n grooves inward can be considered to have sign -n. Within this framework, the overall sign of the chain is always conserved.

Figure 1: double-kink

One might imagine that kinks do not pass over each other in angle, in which case all features in the chain could be described as kinks of differing sign. However, this is not the case. When two kinks come together to form a double-kink, it is possible for them to pass by each other and form what will be referred to as a crossover (Figure 2).

Figure 2: crossover

The existence of crossovers introduces a variety of complications. It is no longer true that at each angular position on the plate, the chain has one radial position, causing issues for the chain tracking software. It also becomes possible for the chain to collide with itself within a groove. Aside from directly affecting the dynamics of the chain's movement, this opens up the possibility of a chain crossing over itself, impeding movement of the lower piece of chain. It also allows the chain to lengthen greatly, by twice the length of separation of the two kinks involved. This has significant implications due to the lowering of chain density, which will be discussed further below.

In more extreme cases of off-levelling, the chain formed a feature similar to a crossover even in the absence of a double-kink (Figure 3). This began as a single kink, and is the result of the chain trying to minimize its gravitational potential energy by putting more beads on the low end of the plate.

Figure 3: extreme crossover

Effect of chain density

In this report, chain density refers to a measure of the number of beads per unit length. When a kink forms, the length of chain per unit angle increases. This results in an overall decrease in chain density. For any chain, there is a minimum and maximum chain density, which depends on the radius of the beads and the length of the rods connecting them. For an infinitely long, straight chain, when the chain density is towo low, it is impossible for kink pairs to form as to do so would require the chain density to decrease below its minimum value.

For a circular groove, there is the complication that inner grooves are shorter than outer grooves. Thus, at low chain densities, it may be impossible for outward kink pairs to form, but possible for inward kink pairs to form. In one test case, this caused the entire chain to shift inward by one groove[1].

If chain density is high, one would imagine that it is difficult for a kink to annihilate, as to do so requires the chain density to increase when the annihilation causes the chain length to

1 See run 2014_03_13-brass-10_3-65Hz-15.4V-2

decrease. It was also seen that high chain density increased the likelihood of crossover formation. It is unclear whether this effect was simply that below certain densities, there are insufficient beads to form crossovers, or whether at greater densities, the chain extends itself to gain "breathing room" and as a result forms more crossovers.

When a chain was used that achieved close to its maximum bead density, the acceleration amplitude for kink formation was significantly greater than the same type of chain at medium bead density. When a set of chains with bead densities varying from the minimum to maximum bead density were shaken on the plastic test plate, the ridge in which the chain was half-extended formed kink pairs most often (though it is uncertain whether this effect was statistically significant). One possible mechanism for the kink stifling effect at high chain densities is that the greater number of beads allows for more dissipation of the kinetic energy as heat. At low chain densities, it is less likely for enough beads to bunch up in one area in order to form a kink. It is also possible that beads in close proximity "push" each other to some extent into forming a kink, though this effect is overtaken by dissipation at high chain densities. Though measuring chain density from images was not undertaken within this investigation, doing so would be an interesting next step. It may be possible to quantify how chain density affects kink formation and annihilation probabilities.

At one point the chain was set up along the outermost groove, so that kinks could only form inwards, and the chain was vibrated near critical kink formation amplitude. When a kink pair did form, the decrease in chain density from the formation of the kink was dispersed throughout the length of the chain. As a result, the kink pair was resistant of annihilation, since to do so would require a large number of beads to shift position, in order to make room for the returning beads. The kink pair was also resistant of expansion, as the arc-length within the inner groove is less than that within the outer groove. Furthermore, the lack of expansion made it difficult for a double-kink to form, as the excess beads available in the inner groove to form a second kink pair was limited by the separation of the two initial kinks. The resulting stable structure is an example of the importance of chain density to kink dynamics.

Effect of off-levelling

Ideally for this investigation, the oscillating plate should be vertically levelled to avoid gravity pulling the chain horizontally. If the plate is not level, the system is no longer rotationally symmetric, but is acted on in the horizontal by gravitational forces dependent on the angle by which the plate is not level, φ, and the angular position along the plate. Let θ be the angular position where θ = 0 corresponds to the lowest point on the plate. The outward transverse component of this force is given by $g\sin(\phi)\cos(\theta)$, and the counter-clockwise longitudinal component of this force is given by $-g\sin(\phi)\sin(\theta)$.

The longitudinal gravitational force decreases the bead density where the plate is high and increases it on the opposite low side. This impacts probabilities for kink formation, as previously discussed. In experiments where the plate is significantly off-levelled, the chain is pulled inward by gravity on the high end, and outward on the low end. This lowers the critical acceleration amplitude for inward kink formation on the high end of the plate. The decrease in bead density on the high end of the plate also decreases this critical amplitude. As a result, when the chain is shaken at certain acceleration amplitudes, kink pairs will only form inward at the high end of the plate.

One would also expect that the critical amplitude for kink formation on the low end of the plate should be decreased, as the transverse gravitational force and the high bead density caused by the longitudinal force both promote outward kink formation. However, the kink stifling effect of high bead density seems to outweigh the outward kink promoting effect of high bead density.

Once a kink pair has formed at the high end of the plate under these conditions, the two kinks immediately separate and move towards the low end of the plate. This cannot be solely explained by the transverse gravitational force pulling the chain inwards in the upper region of the plate, as the kinks can be pulled significantly further than the two sides of the plate. It is necessary to consider that the gravitational force pulls the kink itself downward along the plate. Another way to consider this is that in the region of the kink, there is a greater amount of chain at the same angular position. In order to minimize gravitational potential energy, this region needs to move downward along the plate. In the lower region of the plate this is countered by the fact that moving the kink downwards means moving the chain itself upwards, and so an equilibrium point is found (Figure 4).

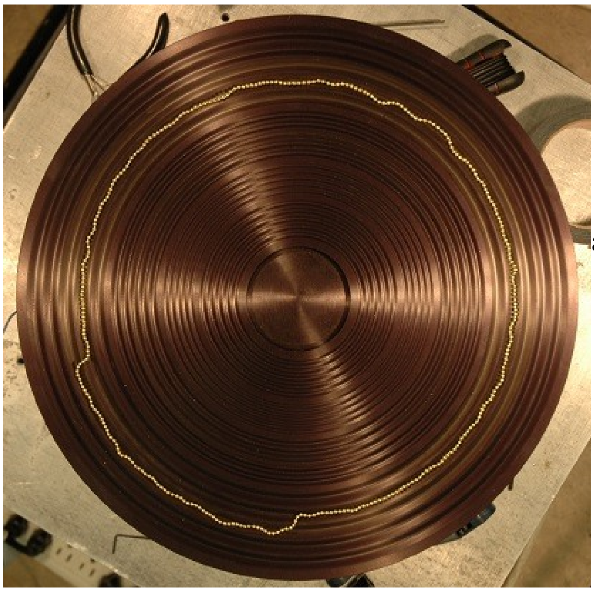
Figure 4: unlevel plate

Ideal forcing parameters

One point of inquiry for this system is to determine whether kinks affect each others' motions, and in what ways. In order to achieve this goal, it is apparent that the oscillating plate must be vertically levelled, to avoid the significant effects that off-levelling has on kink formation, kink movement, and chain density in lower and higher regions.

Aside from this, it is apparent that the formation of crossovers should be avoided. Having the plate level is a positive step in achieving this. Crossovers could be avoided entirely by having a level plate and using only two grooves, so that double-kinks are not possible, though this requires modification of the apparatus. If more than two grooves are used, the chain density can be lowered to decrease the probability of crossover formation.

It is possible to halt any runs where crossovers are formed, though this must be accounted for when analysis takes place. For example, do crossovers form when local chain density is above a certain threshold? If this is the case, sampled data will be biased against having high chain densities in regions near double-kinks.

Conclusion

Kink dynamics are more complicated than was initially thought. Because kinks can pass by one another and form crossovers, much more complicated behaviour is possible than can be described by moving charges of positive or negative sign. Chain density is of much more importance than initially thought, and can constrain whether formation or annihilation of kinks is likely or even possible.

There is much room for further exploration. Using a vertically levelled plate would allow for the analysis of interactions between kinks. Images can be analyzed not only to locate the positions of the chain, but also to calculate the chain density along the chain. It is likely possible to set up the system in such a way that crossovers do not form, which would prevent their interference with an analysis of simple kinks.

References

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