Origin of Mass of the Higgs Boson

Christopher T. Hill
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University of Toronto, April 28, 2015
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But, the Higgs Boson does NOT explain the origin of the electroweak mass-scale
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The Higgs Boson does NOT explain the origin of the electroweak mass-scale.

i.e., what is the origin of the Higgs Boson mass itself?
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The Higgs Boson does NOT explain the origin of the electroweak mass-scale

i.e., what is the origin of the Higgs Boson mass itself?

This is either very sobering, or it presents theoretical opportunities
Nature *may* be telling us something very profound:
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Point-like spin-0 bosons may be natural afterall!
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Isolated, point-like spin-0 bosons may be natural after all!

If true, it’s a serious challenge to our understanding of naturalness.
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What is the custodial symmetry?
The world of masslessness features a symmetry:

Scale Invariance
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**Scale Invariance**

Scale Invariance is (almost) always broken by quantum effects

Feynman Loops $\propto h$
Scale Symmetry in QCD is broken by quantum loops and this gives rise to:
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The Origin of the Nucleon Mass (aka, most of the visible mass in The Universe)
Gell-Mann and Low:

\[ \frac{dg}{d \ln \mu} = \beta(g) \]
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Gross, Politzer and Wilczek (1973):

$$\beta(g) = \beta_0 g^3 \quad \text{where} \quad \beta_0 = -\frac{\hbar}{16\pi^2} \left( \frac{11}{3} N_c - \frac{2}{3} n_f \right)$$
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“running coupling constant”

\[ \alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{|\beta_0| \ln(k^2/\Lambda^2)} \]
$\Lambda = 200 \text{ MeV}$

QCD running coupling constant

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S. Burby and C. Maxwell
A Puzzle: Murray Gell-Mann lecture ca 1975

Noether current of Scale symmetry

\[ S_\mu = x^\nu T_{\mu \nu} \]
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Current divergence

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Yang-Mills Stress Tensor

\[ T_{\mu\nu} = \text{Tr}(G_{\mu\rho}G^\rho_\nu) - \frac{1}{4} g_{\mu\nu} \text{Tr}(G_{\rho\sigma}G^{\rho\sigma}) \]
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Compute:

\[ \partial_\mu S^\mu = T^\mu_\mu = \text{Tr}(G_{\mu \nu} G^{\mu \nu}) - \frac{4}{4} \text{Tr}(G_{\mu \nu} G^{\mu \nu}) = 0 \]
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QCD is scale invariant!!!???
Resolution: The Scale Anomaly
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\[ \partial_\mu S^\mu = \frac{\beta(g)}{g} \text{Tr} \quad G_{\mu\nu} G^{\mu\nu} = \mathcal{O}(\hbar) \]

Origin of Mass in QCD

= Quantum Mechanics

See Murraypalooza talk:
Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill, hep-th/0510177
‘t Hooft Naturalness:

“Small ratios of physical parameters are controlled by symmetries. In the limit that a ratio goes to zero, there is enhanced symmetry ("custodial symmetry")."

\[
\frac{\Lambda}{M} = \exp \left( -\frac{8\pi^2}{|b_0|g^2(M)} \right) \quad b_0 \propto \hbar.
\]
Small ratios of physical parameters are controlled by symmetries. In the limit that a ratio goes to zero, there is enhanced symmetry ("custodial symmetry").

\[ \frac{\Lambda}{M} = \exp \left( -\frac{8\pi^2}{|b_0| g^2(M)} \right) \quad b_0 \propto \hbar. \]

Classical Scale Invariance is the "Custodial Symmetry" of \( \Lambda_{\text{QCD}} \).
Conjecture:

All mass is a quantum phenomenon.

Max Planck

Murraypalooza talk: Christopher T. Hill, hep-th/0510177
Many theories were proposed to imitate QCD for the electroweak scale.
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All of these featured “strong dynamics” and classical scale invariance as the custodial symmetry of $v_{\text{Weak}} \ll M_{\text{Gut, Planck}}$. 
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(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
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10. ...
Susy is still alive.
Susy is still alive?
If so, where is it?
How much fine tuning should we tolerate?
Weak Scale SUSY was seriously challenged before the LHC turned on (e.g. EDM’s)

MSSM now copes with severe direct limits; Some nMSSM models survive

If SUSY is the custodial symmetry we may see it in LHC RUN-II
Why EDM’s are so powerful:

\[ \frac{e m_e}{\Lambda^2} \bar{\Psi} \gamma_5 \sigma_{\mu\nu} F_{\mu\nu} \Psi \]

\[ d_e = \frac{e m_e}{\Lambda^2} = 0.2 \times 10^{-16} \text{ (e-cm)} \times \frac{(m_e/\text{MeV})}{(\Lambda / \text{GeV})^2} \]

Current limit: \[ d_e < 10^{-27} \text{ e-cm} \]

\[ \Lambda > 1.4 \times 10^5 \text{ GeV} \]
Are EDM's telling us something about SUSY?:

\[ \frac{e m_e}{\Lambda^2} \bar{\Psi} \gamma_5 \sigma_{\mu\nu} F_{\mu\nu} \Psi \]

\[ \frac{1}{(\Lambda)^2} = \left( \frac{\alpha \sin(\gamma)}{4\pi \sin^2 \theta} \right) \left( \frac{1}{M_{\text{selectron}}} \right)^2 \]

\[ M_{\text{selectron}} > 6.8 \times 10^3 \text{ GeV} \left( \sin(\gamma) \right)^{1/2} \]

Future limit: \( d_e < 10^{-29} \text{ e-cm} \quad -- \quad 10^{-32} \text{ e-cm} \)?
Can a perturbatively light Higgs Boson mass come from quantum mechanics?
Bardeen: Classical Scale Invariance could be the custodial symmetry of a fundamental, perturbatively light Higgs Boson in SU(3)xSU(2)xU(1).

The only manifestations of Classical Scale Invariance breaking by quantum loops are $d = 4$ scale anomalies.

Bardeen: Classical Scale Invariance could be the custodial symmetry of a fundamental, perturbatively light Higgs Boson in SU(3)xSU(2)xU(1).

The only manifestations of Classical Scale Invariance breaking by quantum loops are $d = 4$ scale anomalies. There is no meaning to the “quadratic divergence” as a source of scale breaking.

On naturalness in the standard model.
William A. Bardeen
The only manifestations of Classical Scale Invariance breaking by quantum loops are $d = 4$ scale anomalies.

There are possible additive effects from higher mass scales:

$$\delta m_H^2 = \alpha^p M_{\text{GuT}}^2 + \alpha^q M_{\text{Planck}}^2.$$ 

But the existence of the low mass Higgs may be telling us that such effects are absent!

Something seems to be missing in our understanding of scale symmetry and fine-tuning.
The only manifestations of Classical Scale Invariance breaking by quantum loops are $d = 4$ scale anomalies.

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Something seems to be missing in our understanding of scale symmetry and fine-tuning.

This is profoundly important in sculpting our view of the physical world!
Treat this as a phenomenological question:

Is the Higgs Potential Generated by Infra-red (scale-breaking) Quantum Loop Effects?

i.e., is the Higgs potential a Coleman-Weinberg Potential?
Start with the Classically Scale Invariant Higgs Potential

\[ \frac{\lambda}{2} |H|^4 \]

\[ \langle H \rangle = v \]

Scale Invariance \( \rightarrow \) Quartic Potential \( \rightarrow \) No VEV
Quantum loops generate a logarithmic “running” of the quartic coupling

\[ \lambda(v) \propto \hbar \beta \log \left( \frac{v}{M} \right) \]

Nature chooses a particular trajectory determined by dimensionless cc’s.
Result: “Coleman-Weinberg Potential”

$$\tilde{\lambda}(v) \times v^4$$

Potential Minimum arises from running of $\lambda$

i.e. Quantum Mechanics
Result: “Coleman-Weinberg Potential

\[ \lambda(v) \times v^4 \]

Require: \( \beta > 0 \quad \lambda < 0 \) for a minimum, positive curvature

Boson Mass is determined by curvature at minimum
An Improved Coleman-Weinberg Potential

\[ S = \int d^4x \mathcal{L} = \int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]
An Improved Coleman-Weinberg Potential

\[ S = \int d^4 x \; \mathcal{L} = \int d^4 x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

Improved Stress tensor:
Callan, Coleman, Jackiw

\[ \tilde{T}_{\mu\nu} = T_{\mu\nu} + Q_{\mu\nu} \]

\[ = \frac{2}{3} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} \eta_{\mu\nu} \partial_\rho \phi \partial^\rho \phi - \frac{1}{3} \phi \partial_\mu \partial_\nu \phi \quad + \frac{1}{3} \eta_{\mu\nu} \phi \partial^2 \phi + \eta_{\mu\nu} V(\phi) \]
Trace of improved stress tensor:

\[ \tilde{T}_{\mu} = \phi \partial^2 \phi + 4V(\phi) = -\phi \frac{\delta}{\delta \phi} V(\phi) + 4V(\phi) \]

Traceless for a classical scale invariant theory:

\[ V(\phi) = \frac{\lambda}{4} \phi^4, \quad \tilde{T}_{\mu} = 0 \quad \text{Conserved scale current} \]
Trace of improved stress tensor:

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Traceless for a classical scale invariant theory:

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Running coupling constant:

\[ \frac{\delta}{\delta\phi} \lambda(\phi) = \beta(\lambda) \]

\[ \tilde{T}^\mu_\mu = -\frac{\beta(\lambda)}{\lambda} V(\phi) \]

Trace Anomaly associated with running coupling
Improved Coleman-Weinberg Potential is the solution to the equations:

\[ \phi \frac{\delta}{\delta \phi} V(\phi) - 4V(\phi) = \frac{\beta}{\lambda} V(\phi) \]

\[ \tilde{T}_\mu = -\frac{\beta(\lambda)}{\lambda} V(\phi) \]

\[ \frac{d\lambda(\mu)}{d \ln \mu} = \beta(\lambda) \]
Improved Coleman-Weinberg Potential is the solution to the equations:

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\[ \frac{d\lambda(\mu)}{d \ln \mu} = \beta(\lambda) \]

The solution is: \[ V(\phi) = \frac{1}{2} \lambda(\phi) \phi^4 \]

Phys Rev D.89. 073003.
Example: \( \phi^4 \) Field theory

\[
\frac{d\lambda}{d\ln(\phi)} = \beta(\lambda) = \frac{9\lambda^2}{32\pi^2}
\]

\[
V_{RG} = \frac{\lambda}{4} \phi^4 + \hbar \frac{9\lambda^2}{32\pi^2} \phi^4 \ln(\phi/M) = \hbar \frac{m_h^4}{32\pi^2} (\phi/\nu)^4 \ln(\phi/\tilde{M})
\]

agrees with CW original result \( \log \) (path Integral)
Example: $\phi^4$ Field theory

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agrees with CW original result \(\log\) (path Integral)

Example: Scalar Electrodynamics

\[
V(\phi) = \frac{\lambda_0}{2}|\phi|^4 + \frac{1}{16\pi^2}(5\lambda^2 - 6\lambda e^2 + 6e^4)|\phi|^4 \ln\left(\frac{|\phi|}{M}\right)
\]

\[
V(\phi_c') = \frac{\lambda_{\text{CW}}}{4!}\phi_c'^4 + \left(\frac{5\lambda_{\text{CW}}}{1152\pi^2} + \frac{3e^4}{64\pi^2}\right)\phi_c'^4 \ln\left(\frac{\phi_c^2}{M'^2}\right)
\]

\[\phi_c'^2 = 2|\phi|^2 \quad \text{and} \quad \frac{\lambda_{\text{CW}}}{4!}\phi_c'^4 = \frac{\lambda_0}{2}|\phi|^4\]

agrees with CW original result BUT with canonical normalization
The Renormalization Group generates the Coleman Weinberg potential

Expand about minimum:

\[ V_{CW}(h) = \frac{1}{2} \lambda (v + h/\sqrt{2})(v + h/\sqrt{2})^4 \]

\[ \frac{dV}{dh} \bigg|_{h=0} = \sqrt{2} v^3 \left( \lambda + \frac{1}{4} \beta \right) = 0 \]

\[ \beta_1(\lambda_i(v)) = -4 \lambda_1(v) \quad \text{at minimum} \]

\[ \frac{d^2V}{dh^2} = m_h^2 = \left( 3 \lambda + \frac{7}{4} \beta \right) v^2 \]

\[ m_h^2 = -4 \lambda v^2 = \beta v^2 > 0 \]
The Renormalization Group generates the Coleman Weinberg potential

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We label all relevant coupling constants that enter in any order of the loop diagrams for the running of \( \lambda \) (e.g., \( g_{top}, g_2, g_{QCD} \), etc.) as \( \lambda_i \). We denote the scalar quartic (Higgs) coupling as \( \lambda \equiv \lambda_1 \) with \( \beta \)-function \( \beta_1(\lambda_i) \). Each \( \lambda_i \) has its own \( \beta_i \):

\[ \frac{d\lambda_i}{d\ln(\mu)} = \beta_i(\lambda_j) \]

\[ v\lambda'_1(v) = \beta_1 \]

\[ v^2 \lambda'_1(v) = \beta_j \frac{\partial \beta_1}{\partial \lambda_j} - \beta_1 \]

\[ v^3 \lambda''_1(v) = \beta_i \beta_j \frac{\partial^2 \beta_1}{\partial \lambda_i \partial \lambda_j} + \beta_j \frac{\partial \beta_i}{\partial \lambda_j} \frac{\partial \beta_1}{\partial \lambda_i} - 3\beta_j \frac{\partial \beta_i}{\partial \lambda_i} + 2\beta_1 \]

\[ v^4 \lambda'''_1(v) = \beta_i \beta_j \beta_k \frac{\partial^3 \beta_1}{\partial \lambda_i \partial \lambda_j \partial \lambda_k} + \beta_k \frac{\partial \beta_j}{\partial \lambda_j} \frac{\partial \beta_i}{\partial \lambda_k} \frac{\partial \beta_1}{\partial \lambda_i} + \beta_k \beta_j \beta_i \frac{\partial^2 \beta_1}{\partial \lambda_j \partial \lambda_k} \frac{\partial \beta_i}{\partial \lambda_i} \frac{\partial \beta_1}{\partial \lambda_j} - 6\beta_i \beta_j \frac{\partial^2 \beta_1}{\partial \lambda_i \partial \lambda_j} - 6\beta_j \frac{\partial \beta_i}{\partial \lambda_j} \frac{\partial \beta_1}{\partial \lambda_i} + 11\beta_i \frac{\partial \beta_1}{\partial \lambda_i} - 6\beta_1 \]

(21)
The Coleman-Weinberg Potential is completely determined by $\beta$-functions:

\[
V_{CW}(h) = -\frac{1}{8} \beta_1 v^4 + \frac{1}{2} v^2 h^2 \left( \beta_1 + \frac{1}{4} \beta_j \frac{\partial \beta_1}{\partial \lambda_j} \right) \\
+ \frac{5}{6} v h^3 \left( \beta_1 + \frac{9}{20} \beta_i \frac{\partial \beta_1}{\partial \lambda_i} + \frac{1}{20} \beta_j \beta_i \frac{\partial^2 \beta_1}{\partial \lambda_j \partial \lambda_i} \\
+ \frac{1}{20} \beta_j \frac{\partial \beta_i}{\partial \lambda_j} \frac{\partial \beta_1}{\partial \lambda_i} \right) \\
+ \frac{11}{48} h^4 \left( \beta_1 + \frac{35}{44} \beta_i \frac{\partial \beta_1}{\partial \lambda_i} + \frac{5}{22} \beta_j \beta_i \frac{d^2 \beta_1}{d \lambda_j d \lambda_i} \\
+ \frac{5}{22} \beta_j \frac{\partial \beta_i}{\partial \lambda_j} \frac{\partial \beta_1}{\partial \lambda_i} + \frac{1}{44} \beta_k \beta_j \beta_i \frac{d^3 \beta_1}{d \lambda_k d \lambda_j d \lambda_i} \\
+ \frac{1}{44} \beta_k \frac{\partial \beta_j}{\partial \lambda_k} \frac{\partial \beta_i}{\partial \lambda_j} \frac{\partial \beta_1}{\partial \lambda_i} + \frac{1}{44} \beta_j \beta_i \frac{d^2 \beta_1}{d \lambda_j d \lambda_i} \right) \\
+ \frac{3}{44} \beta_j \beta_k \frac{\partial \beta_i}{\partial \lambda_k} \frac{d^2 \beta_1}{d \lambda_j d \lambda_i} \right) \\
+ \frac{h^5}{40 \sqrt{2} v} \left( \beta + \frac{25}{12} \beta_i \frac{d \beta}{d \lambda_i} + \frac{35}{24} \beta_j \beta_i \frac{d^2 \beta}{d \lambda_j d \lambda_i} \\
+ \frac{35}{24} \beta_j \beta_i \frac{d \beta}{d \lambda_j d \lambda_i} + \frac{5}{12} \beta_k \beta_j \beta_i \frac{d^3 \beta}{d \lambda_k d \lambda_j d \lambda_i} \\
+ \frac{5}{12} \beta_k \frac{d \beta_j}{d \lambda_k} \frac{d \beta_i}{d \lambda_j} + \frac{5}{12} \beta_j \beta_i \frac{d^2 \beta}{d \lambda_j d \lambda_i} \\
+ \frac{5}{4} \beta_j \frac{d \beta_i}{d \lambda_j} \frac{d^2 \beta}{d \lambda_i} d \lambda_i + \frac{1}{24} \beta_i \beta_j \beta_k \frac{d^4 \beta}{d \lambda_i d \lambda_j d \lambda_k d \lambda_l} \\
+ \frac{1}{24} \beta_i \beta_j \beta_k \frac{d \beta}{d \lambda_i} \frac{d \beta}{d \lambda_j} \frac{d \beta}{d \lambda_k} \frac{d \beta}{d \lambda_l} + \frac{1}{4} \beta_i \beta_j \beta_k \frac{d \beta}{d \lambda_i} \frac{d \beta}{d \lambda_j} \frac{d \beta}{d \lambda_k} \frac{d \beta}{d \lambda_l} \\
+ \frac{1}{8} \beta_i \beta_k \frac{d \beta_j}{d \lambda_k} \frac{d^2 \beta}{d \lambda_j d \lambda_l} + \frac{1}{6} \beta_i \beta_j \frac{d \beta}{d \lambda_i} \frac{d \beta}{d \lambda_j} \frac{d \beta}{d \lambda_l} \\
+ \frac{1}{8} \beta_i \beta_k \frac{d \beta_j}{d \lambda_i} \frac{d \beta}{d \lambda_k} \frac{d \beta}{d \lambda_l} + \frac{1}{24} \beta_i \beta_k \beta_j \frac{d^2 \beta}{d \lambda_i d \lambda_j d \lambda_k} \frac{d \beta}{d \lambda_l} \\
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\]

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Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2} (\lambda^2 + \lambda g^2 - g^4) = \beta$$

$g$: top Yukawa $cc$

(I am ignoring EW contributions for simplicity of discussion)
Higgs Quartic coupling $\beta(\lambda)$

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$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2} (\lambda^2 + \lambda g^2 - g^4) = \beta$$

approximate physical values:

Higgs quartic cc: $\lambda = 1/4$
Top Yukawa cc: $g = 1$

$\beta = -5.2244 \times 10^{-2}$

$\beta$ is small and negative in standard model
No solution!
Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

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$$

approximate physical values:

Higgs quartic cc: $\lambda = 1/4$

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We require positive $\beta$ to have a Coleman-Weinberg potential
Higgs Quartic coupling $\beta(\lambda)$

Renormalization Group Equation:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2} (\lambda^2 + \lambda g^2 - g^4) = \beta$$

Approximate physical values:

- Higgs quartic cc: $\lambda = \frac{1}{4}$
- Top Yukawa cc: $g = 1$

$\beta$ is small and negative in standard model.

We require positive $\beta$ to have a Coleman-Weinberg potential.

Requires New Bosonic physics beyond the standard model.
Higgs Quadratic coupling $\beta(\lambda)$

Introduce a new field: $S$

Higgs-Portal Interaction $\lambda' |H|^2 |S|^2$
Higgs Quartic coupling $\beta(\lambda)$

Introduce a new field: $S$

Higgs-Portal Interaction $\lambda' |H|^2 |S|^2$

Two possibilities:

1. Modifies RG equation to make $\beta > 0$:

$$\frac{d\lambda(v)}{d\ln(v)} = \frac{12}{16\pi^2} \left( \lambda^2 + \lambda g^2 - g^4 + c \lambda'^2 \right)$$

2. $S$ develops its own CW potential, and VEV $\langle S \rangle = V'$ and Higgs gets mass, $\lambda' V'$
Simplest hypotheses:

S may be:

(1) A singlet field with or without VEV
   e.g., Ultra-weak sector, Higgs boson mass, and the dilaton

(2) A new doublet NOT coupled to
   SU(2) x U(1) (inert) w or wo VEV

S sector is Dark Matter
Simplest hypotheses:

S may be:

1. A singlet field with or without VEV
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3. New doublet COUPLED to
   SU(2)xU(1) with no VEV (dormant)
   Is the Higgs Boson Associated with
   Coleman-Weinberg Dynamical Symmetry Breaking?
Simplest hypotheses

$S$ may be:

A new doublet **NOT** coupled to $SU(2) \times U(1)$ (inert) w or wo VEV

e.g., Hambye and Strumia Phys.Rev. D88 (2013) 055022;
S. Iso, and Y. Orikasa, PTEP (2013) 023B08; …
“Ultra-weak sector, Higgs boson mass, and the dilaton,”
Light Dark Matter, Naturalness, and the Radiative Origin of
the Electroweak Scale, W. Altmannshofer, W. Bardeen, M Bauer,

Many, many papers on this approach!

A New doublet **COUPLED** to $SU(2) \times U(1)$ with no VEV (dormant)

e.g., Is the Higgs Boson Associated with
Coleman-Weinberg Dynamical Symmetry Breaking?
Massless two doublet potential

\[ V(H_1, H_2) = \frac{\lambda_1}{2} |H_1|^4 + \frac{\lambda_2}{2} |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2} \left[ (H_1^\dagger H_2)^2 e^{i\theta} + h.c. \right] \]

Two doublet RG equations

\[ 16\pi^2 \frac{d\lambda_1(\mu)}{d\ln(\mu)} = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_1(3g_2^2 + g_1^2) + \frac{3}{2}g_4^2 + \frac{3}{4}(g_1^2 + g_2^2)^2 + 12\lambda_1 g_t^2 - 12g_t^4 \]

\[ 16\pi^2 \frac{d\lambda_2(\mu)}{d\ln(\mu)} = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_2(3g_2^2 + g_1^2) + \frac{3}{2}g_4^2 + \frac{3}{4}(g_1^2 + g_2^2)^2 + 12\lambda_2 g_b^2 - 12g_b^4 \]

\[ 16\pi^2 \frac{d\lambda_3(\mu)}{d\ln(\mu)} = (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 - 3\lambda_3(3g_2^2 + g_1^2) + \frac{9}{4}g_4^2 + \frac{3}{4}g_1^4 - \frac{3}{2}g_1^2 g_2^2 + 6\lambda_3(g_t^2 + g_b^2) - 12g_t^2 g_b^2 \]

\[ 16\pi^2 \frac{d\lambda_4(\mu)}{d\ln(\mu)} = 2(\lambda_1 + \lambda_2)\lambda_4 + 4(2\lambda_3 + \lambda_4)\lambda_4 + 8\lambda_5^2 - 3\lambda_4(3g_2^2 + g_1^2) + 3g_1^2 g_2^2 - 12g_t^2 g_b^2 \]

\[ 16\pi^2 \frac{d\lambda_5(\mu)}{d\ln(\mu)} = \lambda_5 [2(\lambda_1 + \lambda_2) + 8\lambda_3 + 12\lambda_4 - 3(3g_2^2 + g_1^2) + 2(g_t^2 + g_b^2)] \]
The observed Higgs boson mass implies:

\[ m_h^2 = -4\lambda v^2 = \beta v^2 \geq 0 \]

\[ \beta = \left( \frac{126}{175} \right)^2 = 0.5184 \]

Note that \( \lambda \) is negative:

\[ \lambda = -(0.25)(0.5184) = -0.1296 \]

Can now solve for \( \lambda_3 \):

\[ \beta = \frac{1}{16\pi^2} \left( 12\lambda^2 + 12\lambda g^2 - 12g^4 + 4\lambda_3^2 \right) \]

\[ g = g_{\text{top}} \approx 1 \]

Solution is: \( \lambda_3 = 4.8789 \)
Mass of New Doublet: \[ \sqrt{4.8789} \times (175) = 386.54 \text{ GeV} \]

\[ M^2 \text{ is determined} \rightarrow \text{ heavy “dormant” Higgs doublet} \]

No VEV but coupled to SU(2) x U(1):
“Dormant” Higgs Doublet (vs. “Inert”)

Production, mass, and decay details are model dependent

If Dormant Higgs couples to SU(2) x U(1) but not fermions

Parity \( H_2 \rightarrow -H_2 \) implies stability: \( H_2^+ \rightarrow H_2^0 + (e^+\nu) \) if \( M^+ > M^0 \)

Then \( H_2^0 \) is stable dark matter WIMP
CalcHEP estimates

The Dormant Doublet is pair produced above threshold near $2M_H \approx 800 \text{ GeV}$

$p + p \rightarrow H^+_2, H^-_2$ at 14 TeV cms; $\sigma_{\text{total}} = 2.8 \text{ fb}$

$pp \rightarrow X + (\gamma^*, Z^*, W^*, h^*) \rightarrow X + H H^*$

$pp \rightarrow H^0 H^0 \quad \sigma = 1.4 \text{ fb} \quad \Gamma_{H^0 \rightarrow bb} = 45 \text{ GeV} \quad \text{Assume } g_{b'}^H = 1$

$pp \rightarrow H^+ H^- \quad \sigma = 2.8 \text{ fb} \quad \Gamma_{H^+ \rightarrow tb} = 14 \text{ GeV}$

$pp \rightarrow H^+ H^0 \quad \sigma = 0.9 \text{ fb}$

Maybe Run II?
TABLE I: Predicted decay widths and production cross-sections for the dormant Higgs bosons. We used CalcHep, and production runs CTEQ61 proton structure functions, $1.64 \times 10^5$ calls. All cross-sections are evaluated at 14 TeV cm/s energy with the mass of $H_2$ doublet set to 380 GeV/c^2. Model dependent processes have rates or cross-sections that are indicated as $\propto (g'_b)^2$.

<table>
<thead>
<tr>
<th>Process</th>
<th>value</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(H^+ \rightarrow t + \bar{b}) = \Gamma(H^- \rightarrow b + \bar{t})$</td>
<td>$14.5 (g'_b)^2 \pm 5 \times 10^{-5}%$ GeV</td>
<td></td>
</tr>
<tr>
<td>$\Gamma(H^0 \rightarrow b + \bar{b}) = \Gamma(A^0 \rightarrow b + \bar{b})$</td>
<td>$5.67 (g'_b)^2 \pm 5 \times 10^{-5}%$ GeV</td>
<td></td>
</tr>
<tr>
<td>$\Gamma(H^0 \rightarrow 2h, 3h) = \Gamma(A^0 \rightarrow 2h, 3h)$</td>
<td></td>
<td>absent in model</td>
</tr>
<tr>
<td>$pp \rightarrow (\gamma, Z^0) \rightarrow H^+H^-$</td>
<td>$\sigma_t = 1.4$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow (\gamma, Z) \rightarrow H^0H^0$</td>
<td></td>
<td>absent in model</td>
</tr>
<tr>
<td>$pp \rightarrow (\gamma, Z) \rightarrow A^0H^0$</td>
<td>$\sigma_t = 1.3$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow (\gamma, Z) \rightarrow A^0A^0$</td>
<td></td>
<td>absent in model</td>
</tr>
<tr>
<td>$pp(gg) \rightarrow h \rightarrow H^0H^0$ or $A^0A^0$</td>
<td>$\sigma_t = 1.7 \times 10^{-5}$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow W^+ \rightarrow H^0H^+$</td>
<td>$\sigma_t = 1.8$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow W^+ \rightarrow A^0H^+$</td>
<td>$\sigma_t = 1.8$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow W^- \rightarrow H^0H^-$</td>
<td>$\sigma_t = 0.74$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow W^- \rightarrow A^0H^-$</td>
<td>$\sigma_t = 0.74$ fb</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow b + \bar{b} + H^0$ or $A^0$</td>
<td>$\sigma_t = 1.8 (g'_b)^2$ pb $\pm 2.4%$ No $p_T$ cuts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 67 (g'_b)^2$ fb $\pm 5%$ $p_T(b)$ and $p_T(\bar{b}) &gt; 50$ GeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 9.6 (g'_b)^2$ fb $\pm 3.5%$ $p_T(b)$ and $p_T(\bar{b}) &gt; 100$ GeV</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow t + \bar{b} + (H^-)$</td>
<td>$\sigma_t = 220 (g'_b)^2$ fb No cuts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 44 (g'_b)^2$ fb $p_T(t), p_T(\bar{b}) &gt; 50$ GeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 14 (g'_b)^2$ fb $p_T(t), p_T(\bar{b}) &gt; 100$ GeV</td>
<td></td>
</tr>
<tr>
<td>$pp \rightarrow \bar{t} + b + (H^+)$</td>
<td>$\sigma_t = 270 (g'_b)^2$ fb No cuts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 46 (g'_b)^2$ fb $p_T(\bar{t})$ $p_T(b) &gt; 50$ GeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_t = 14 (g'_b)^2$ fb $p_T(\bar{t})$ $p_T(b) &gt; 100$ GeV</td>
<td></td>
</tr>
</tbody>
</table>
The trilinear, quartic and quintic Higgs couplings will be significantly different than in SM case.

\[ V_{CW}(H) = \frac{1}{2}m_h^2h^2 + \frac{5}{6\sqrt{2}v}h^3\left(\beta_1 + \frac{9}{20}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \frac{11}{48v^2}h^4\left(\beta_1 + \frac{35}{44}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \frac{1}{40\sqrt{2}v}h^5\left(\beta_1 + \frac{25}{12}\beta_3\frac{\partial\beta_1}{\partial\lambda_3}\right) + \ldots \]

- **trilinear**: \[ \frac{5}{3}\left(1 + \frac{v^2}{5m_h^2}\frac{\lambda_3^3}{8\pi^4}\right) \approx 1.75 \]
- **quadrilinear**: \[ \frac{11}{3}\left(1 + \frac{35v^2}{44m_h^2}\frac{\lambda_3^3}{8\pi^4}\right) \approx 4.43 \]
- **quintic**: \[ \frac{3}{5}\left(\frac{\beta_1}{\hat{\beta}} + \frac{25}{12\hat{\beta}}\frac{\lambda_3^3}{6\pi^4}\right) \approx -8.87 \]

This may be doable at LHC!
Problem:
UV Landau Pole
implying strong scale

\[ \lambda_3(175 \text{ GeV}) = 4.79 \quad \text{(black)} \]
\[ \lambda_1(175 \text{ GeV}) = -0.1 \quad \text{(red)} \]
\[ \lambda_2(175 \text{ GeV}) = 0.1 \quad \text{(green)} \]
\[ g_{\text{top}} = 1 \quad \text{(blue)} \]
\[ \lambda_4 = \lambda_5 = 0 \]

Landau Pole = 10 - 100 TeV

Landau Pole ->
Composite \( H_2 \)
New Strong Dynamics

e.g. Higgs mass from compositeness at a multi-TeV scale,
Hsin-Chia Cheng Bogdan Dobrescu, Jiayin Gu
e-Print: arXiv:1311.5928
$S$ develops a Coleman-Weinberg potential and VEV $v_2$

$\lambda_3$ is negative and gives the Higgs boson mass $-m^2 = \lambda_3 |S|^2$

The model does not require large quartic cc's, has sensible UV behavior

$H_2$ and associated gauge fields become viable dark matter

But, hard to detect!
Ultra-Weak Sector, dilaton, axion, K. Allison, CTH, G. G. Ross, PL B738 191 (2015), NP B891, 613 (205)

\[ V(H, \sigma) = \frac{\lambda}{2}(H^\dagger H)^2 + \frac{\zeta_1}{2} \sigma^2 H^\dagger H + \frac{\zeta_2}{4} \sigma^4 \]

\[ V(\sigma, \phi_i, \lambda_i, \zeta_i) = V_1(\phi_i, \lambda_i) + V_2(\sigma, \phi_i, \zeta_i) \]

\( \sigma \) is a complex singlet

Here the full potential decomposes into components \( V_1 \) and \( V_2 \) where \( \frac{\delta}{\delta \sigma_i} V_1 = \frac{\delta}{\delta \zeta_i} V_1 = 0 \), and \( \frac{\delta}{\delta \lambda_i} V_2 = 0 \).

\[ \beta_\lambda = \frac{d \lambda(\mu)}{d \ln(\mu)} = \frac{1}{16\pi^2} \left( 12\lambda^2 - 3\lambda(3g_2^2 + g_1^2) + \frac{3}{4}(g_1^2 + g_2^2)^2 + \frac{3}{2}g_2^4 + 12\lambda g_t^2 - 12g_t^4 + \zeta_1^2 \right), \]

\[ \begin{align*}
\beta_1 &= \frac{d \zeta_1(\mu)}{d \ln(\mu)} = \frac{1}{16\pi^2} \left( 6\zeta_1 \zeta_2 + 6\zeta_1 \lambda + 4\zeta_1^2 \\
&\quad - \frac{3}{2}\zeta_1(3g_2^2 + g_1^2) + 6\zeta_1 g_t^2 \right), \\
\beta_2 &= \frac{d \zeta_2(\mu)}{d \ln(\mu)} = \frac{1}{16\pi^2} \left( 18\zeta_2^2 + 2\zeta_1^2 \right).
\end{align*} \]

The \( \zeta_i \) are technically naturally small: shift symmetry

\[ \langle \sigma \rangle = f \] can be very large, e.g. GUT scale
Ultra-weak sector, dilaton, axion,

\[ f \gtrsim 10^{10} \text{ GeV}. \]

Incorporates the axion, GUT scale breaking, \( f \), yields the Higgs boson mass

\[ m_{\text{axion}} \sim \Lambda_{\text{QCD}}^2 / f \]

\[ m_{\text{Dilaton}} \sim m_{\text{Higgs}}^2 / f \]

\[ m_\sigma \approx 0.179 \left( \frac{10^{10} \text{ GeV}}{f} \right) \text{ keV}. \]

The model therefore predicts a low mass \( 0^+ \) particle for \( f \gtrsim 10^{10} \text{ GeV} \).

The \( \zeta_i \) are technically naturally small. Extend to include right-handed neutrinos;

\( (\sigma, \nu_R) \) can form an \( N = 1 \) SUSY multiplet; SUSY broken by \( \zeta_i \) allows tiny neutrino Dirac masses.
A Conjecture:

All mass is a quantum phenomenon.

\[ h \rightarrow 0 \] Classical scale symmetry

Conjecture on the physical implications of the scale anomaly: M. Gell-Mann 75th birthday talk: C. T. Hill hep-th/0510177
Musings:
What if it’s true?

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The $\hbar \to 0$ limit of nature is exactly scale invariant.

(a heretic)
We live in $D=4$!  

$$ T^\mu_\mu = \text{Tr} \, G_{\mu\nu}G^{\mu\nu} - \frac{D}{4} \text{Tr} \, G_{\mu\nu}G^{\mu\nu} $$

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions!
“Predictions” of the Conjecture:

We live in $D=4!$

$$T_{\mu}^{\mu} = \text{Tr} \, G_{\mu\nu} G^{\mu\nu} - \frac{D}{4} \text{Tr} \, G_{\mu\nu} G^{\mu\nu}$$

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Testable in the Weak Interactions!

Does the Planck Mass Come From Quantum Mechanics?

Can String Theory be an effective theory?

... or Weyl Gravity? (A-gravity?)

Weyl Gravity is Renormalizable!

Weyl Gravity is QCD-like:

$$\frac{1}{\hbar^2} \sqrt{-g} \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)$$
Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill (Fermilab).

We live in $D=4!$

\[ T_\mu^\mu = \text{Tr} \, G_{\mu\nu} G^{\mu\nu} - \frac{D}{4} \text{Tr} \, G_{\mu\nu} G^{\mu\nu} \]

Cosmological constant is zero in classical limit

$D$ scale is generated in this way; Hierarchy naturally generated

Testable in the Weak Interactions!

$\rightarrow$ **String Theory RULED OUT** (classical string scale)

$\rightarrow$ **Weyl Gravity?**

Weyl Gravity is Renormalizable in $D=4!$
Weyl Gravity in $D=4$ is QCD-like:

\[ R_{\mu\nu} - \frac{1}{3} R \, \delta_{\mu\nu} \]

$\rightarrow$ **The Planck Mass Comes From Quantum Mechanics!**

See: Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill (Fermilab). hep-th/0510177 (and refs.therein)
“Predictions” of the Conjecture:

We live in $D=4$!

\[ T^{\mu}_{\mu} = \text{Tr } G_{\mu \nu} G^{\mu \nu} - \frac{D}{4} \text{Tr } G_{\mu \nu} G^{\mu \nu} \]

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy naturally generated

Testable in the Weak Interactions!

String Theory RULED OUT (classical string scale)

Weyl Gravity:

Weyl Gravity is Renormalizeable! Predicts $D=4!$

Weyl Gravity in $D=4$ is QCD-like:

\[ \frac{1}{\hbar^2} \sqrt{-g}(R_{\mu \nu} R^{\mu \nu} - \frac{1}{3} R^2) \]

The Planck Mass Comes From Quantum Mechanics!

We Live in a Scaloplex !!!
The “Scaloplex” (scale continuum) is infinite, uniform, and classically isotropic.
Physics is determined by local values of dimensionless coupling constants.
Physics is determined by local values of dimensionless coupling constants.

\[ g_0 = g(10^{1000\mu}) \]

an equivalent universe \( 10^{1000} \times \)

Hubble Scale’  Planck Scale’

-\( \infty \)  Log(\( \mu \))  \( \infty \)
Physics is determined by local values of dimensionless coupling constants.

An equivalent universe $10^{-1000} \times$

Hubble Scale" Planck Scale"

$g_0 = g(10^{-1000}\mu)$

$\text{-infinity} \quad \text{Log}(\mu) \quad \text{infinity}$
Lack of additive scales: Is the principle of scale recovery actually a “Principle of Locality” in Scale?

Physical Mass Scales, generated by e.g. Coleman-Weinberg or QCD-like mechanisms, are Local in scale, and do not add to scales far away in the scaloplex.

E.g, “shining” with Yukawa suppression in extra dimensional models.

Does Coleman-Weinberg mechanism provide immunity from additive scales?
Conjecture on a solution to the Unitarity Problem of Weyl Gravity

$M_{\text{Planck}}$ arises via QCD-like mechanism. Theory becomes Euclidean for $\mu > M_{\text{Planck}}$ (infinite temperature or instanton dominated). Time is emergent for $\mu \ll M_{\text{Planck}}$

Passage through the Planck Scale

$D=4$ quantum theory $T = 0$ $\rightarrow$ $D=5$ quantum theory $T = 0$ $\rightarrow$ $D=5$ $T \rightarrow \mu$ theory $\rightarrow$ $D=5$ $T \rightarrow$ infinity theory $\rightarrow$ $D=4$ Euclidean Theory

Log($\mu$): $T < M_{\text{Planck}}$ Euclidean $D=4$

Hawking-Hartle Boundary Condition?
I think this is a profoundly important scientific question:

Is the Higgs potential Coleman-Weinberg?

- Examined a “maximally visible” scheme
- Dormant Higgs Boson from std 2-doublet scheme
  \[ M \approx 400 \text{ GeV} \]
- May be observable, LHC run II, III?
- Higgs trilinear and quartic couplings non-standard
  - UV problem \( \Rightarrow \) new strong scale \(< 100 \text{ TeV} \)
  - or New bosons may be dark matter

Perhaps we live in a world where all
Mass comes from quantum effects
No classical mass input parameters.
Conclusions:
An important answerable scientific question:
Is the Higgs potential Coleman-Weinberg?

• We examined a “maximally visible” scheme
• Dormant Higgs Boson from std 2-doublet scheme
  \( M \approx 386 \text{ GeV} \)
  • May be observable, LHC run II, III?
• Higgs trilinear ... couplings non-standard
  or New bosons may be dark matter

Perhaps we live in a world where all mass comes from quantum effects
No classical mass input parameters.

Everyone is still missing the solution to the scale recovery problem!
End
Why do couplings run with field VEV?

\[ S = \int d^4x \, \mathcal{L} = \int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

Equation of motion

\[ \partial^\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = \partial^2 \phi + V'(\phi) = 0 \quad V'(\phi) = \frac{\delta}{\delta \phi} V(\phi) \]

\[ x'^\mu = x^\mu - \zeta^\mu, \quad \delta d^4x = -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu) d^4x^\lambda \]

\[ \delta \partial_\mu = (\partial_\nu \zeta_\mu) \partial_\nu \quad \delta d^4x = -(\partial_\mu \zeta^\mu) d^4x \]

\[ \delta S = \int d^4x \left[ \frac{1}{2} (\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial_\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi \right. \]

\[ \left. + (\partial_\mu \zeta^\mu) V(\phi) \right] \equiv -\frac{1}{2} \int d^4x \left[ (\partial_\mu \zeta_\nu) T^{\mu\nu} \right] \]
Why do couplings run with field VEV?

\[ S = \int d^4 x \, \mathcal{L} = \int d^4 x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

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Infinitesimal translation (diffeomorphism)

\[ x'^\mu = x^\mu - \zeta^\mu, \]
\[ \phi'(x') = \phi(x) \]

\[
\begin{align*}
\delta d^4 x & = -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu) \, d^4 x^\lambda \\
\delta \partial_\mu & = (\partial_\nu \zeta_\mu) \partial_\nu \\
\delta d^4 x & = -(\partial_\mu \zeta^\mu) d^4 x
\end{align*}
\]
Why do couplings run with field VEV?

\[ S = \int d^4x \mathcal{L} = \int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

Equation of motion

\[ \partial_\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = \partial^2 \phi + V'(\phi) = 0 \quad V'(\phi) = \frac{\delta}{\delta \phi} V(\phi) \]

Infinitesimal translation (diffeomorphism)

\[ x'^\mu = x^\mu - \zeta^\mu \]
\[ \phi'(x') = \phi(x) \]

\[ \begin{align*}
\delta d^4x &= -(\partial_\mu \zeta^\mu) d^4x \\
\delta \partial_\mu &= (\partial^\nu \zeta_\mu) \partial_\nu \\
\delta d^4x &= -(\partial_\mu \zeta^\mu) d^4x \\
\end{align*} \]

\[ \delta S = \int d^4x \left[ -\frac{1}{2} (\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi + (\partial_\mu \zeta^\mu) V(\phi) \right] \]

\[ \equiv -\frac{1}{2} \int d^4x \left[ (\partial_\mu \zeta_\nu) T^{\mu\nu} \right] \]
Stress tensor: 
\[ T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \eta_{\mu\nu} \left( \frac{1}{2} \partial_\rho \phi \partial^\rho \phi - V(\phi) \right) \]

\[ \partial^\mu T_{\mu\nu} = \partial^2 \phi \partial_\nu \phi + \partial_\mu \phi \partial^\mu \partial_\nu \phi - \partial_\nu \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

Equation of motion:
\[ = \partial_\nu \phi \left( \partial^2 \phi + V'(\phi) \right) \]

can choose \( \zeta^\mu = -\epsilon x^\mu \)

\[ \delta S = \frac{1}{2} \int d^4x \left[ (\partial_\mu \epsilon x_\nu) T^{\mu\nu} \right] \]

Scale Current:
\[ \frac{\delta S}{\partial_\mu \epsilon} \equiv S^{\mu} = x_\nu T^{\mu\nu} \]
\[ \partial_\mu S^{\mu} = T^{\mu}_{\mu} \]
Scale Current not conserved with canonical stress tensor:

$$\partial_\mu S^\mu = T^\mu_\mu$$

$$T^\mu_\mu = -\partial_\rho \phi \partial^\rho \phi + 4V(\phi)$$

The “Improved Stress Tensor”

$$S \rightarrow S + S_2$$

$$S_2 = \xi \int d^4 x \ \partial^2 \phi^2$$

$$\delta S_2 = \xi \int d^4 x [-(\partial_\mu \zeta^\mu) \partial^2 \phi^2 + \partial^\mu ((\partial^\nu \zeta_\mu) \partial_\nu \phi^2] \equiv \int d^4 x (\partial_\mu \zeta_\nu)[Q^{\mu\nu}]$$

$$Q^{\mu\nu} = \xi (\partial_\mu \partial_\nu \phi^2 - \eta^{\mu\nu} \partial^2 \phi^2)$$

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu} + Q^{\mu\nu} \quad \xi = 1/6$$

$$= \frac{2}{3} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} \eta^{\mu\nu} \partial_\rho \phi \partial^\rho \phi - \frac{1}{3} \phi \partial_\mu \partial_\nu \phi + \frac{1}{3} \eta^{\mu\nu} \phi \partial^2 \phi + \eta^{\mu\nu} V(\phi)$$
Trace of improved stress tensor

\[ \widetilde{T}^\mu_\mu = \phi \partial^2 \phi + 4V(\phi) = -\phi \frac{\delta}{\delta \phi} V(\phi) + 4V(\phi) \]

Traceless for a scale invariant theory

\[ \phi \frac{\delta}{\delta \phi} V(\phi) = DV(\phi) \]

D = 4 \rightarrow \quad V(\phi) = \frac{\lambda}{4} \phi^4, 

In general, \( D = 4 + \gamma \)

\[ \frac{\delta}{\delta \phi} \lambda(\phi) = \beta(\lambda) \]

\[ \widetilde{T}^\mu_\mu = -\frac{\beta(\lambda)}{\lambda} V(\phi) \]

Trace anomaly associate with running coupling
A Canonical Picture of Scale Breaking

Stress tensor:

\[ T_{\mu\nu} = \partial_{\mu} \phi \partial_{\nu} \phi - \eta_{\mu\nu} \left( \frac{1}{2} \partial_{\rho} \phi \partial^{\rho} \phi - V(\phi) \right) \]

Scale Current:

\[ \frac{\delta S}{\partial_{\mu} \epsilon} \equiv S^\mu = x_\nu T^{\mu\nu} \quad \partial_{\mu} S^\mu = T^\mu_\mu \]

Scale Current not conserved with canonical stress tensor:

\[ \partial_{\mu} S^\mu = T^\mu_\mu \quad \quad T^\mu_\mu = -\partial_{\rho} \phi \partial^{\rho} \phi + 4V(\phi) \]
Derivation of stress tensor: (diffeomorphism, constant metric)

\[ S = \int d^4x \mathcal{L} = \int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

\[ x'^\mu = x^\mu - \zeta^\mu, \]

\[ \phi'(x') = \phi(x) \]

\[ \begin{align*}
\delta d^4x &= -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu) d^4x^\lambda \\
\delta \partial_\mu &= (\partial^\nu \zeta_\mu) \partial_\nu \\
\delta d^4x &= -\partial_\mu \zeta^\mu d^4x
\end{align*} \]

\[ \delta S = \int d^4x \left[ -\frac{1}{2} (\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi \\
+ (\partial_\mu \zeta^\mu) V(\phi) \right] \]

\[ \equiv -\frac{1}{2} \int d^4x \left[ (\partial_\mu \zeta_\nu) T^{\mu\nu} \right] \]
Derivation of stress tensor: (diffeomorphism, constant metric)

\[ S = \int d^4x \mathcal{L} = \int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

\[ x'^\mu = x^\mu - \zeta^\mu, \]

\[ \phi'(x') = \phi(x) \]

\[ \begin{align*}
\delta d^4x &= -d\zeta^\mu(x) = -(\partial_\lambda \zeta^\mu) d^4x^\lambda \\
\delta \partial_\mu &= (\partial^\nu \zeta_\mu) \partial_\nu \\
\delta d^4x &= -(\partial_\mu \zeta^\mu) d^4x 
\end{align*} \]

\[ \begin{align*}
\delta S &= \int d^4x \left[ -\frac{1}{2} (\partial_\rho \zeta^\rho) \partial_\mu \phi \partial^\mu \phi + (\partial^\rho \zeta_\mu) \partial_\rho \phi \partial^\mu \phi + (\partial_\mu \zeta^\mu) V(\phi) \right] \\
&= -\frac{1}{2} \int d^4x \left[ (\partial_\mu \zeta_\nu) T^{\mu\nu} \right]
\end{align*} \]
Derivation of the “Improved Stress Tensor”

\[ S \rightarrow S + S_2 \quad S_2 = \xi \int d^4x \, \partial^2 \phi^2 \]

\[ \delta S_2 = \xi \int d^4x \left[ -\left( \partial_\mu \zeta^\mu \right) \partial^2 \phi^2 + \partial^\mu \left( \partial^\nu \zeta_\mu \partial_\nu \phi^2 \right) \right] \]

\[ \equiv \int d^4x \left( \partial_\mu \zeta_\nu \right) [Q^{\mu\nu}] \]

\[ Q_{\mu\nu} = \xi \left( \partial_\mu \partial_\nu \phi^2 - \eta_{\mu\nu} \partial^2 \phi^2 \right) \quad \tilde{T}_{\mu\nu} = T_{\mu\nu} + Q_{\mu\nu} \quad \xi = 1/6 \]

\[ = \frac{2}{3} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} \eta_{\mu\nu} \partial_\rho \phi \partial^\rho \phi - \frac{1}{3} \phi \partial_\mu \partial_\nu \phi + \frac{1}{3} \eta_{\mu\nu} \phi \partial^2 \phi + \eta_{\mu\nu} V(\phi) \]
Classical Standard Model

Higgs Potential

\[ m_{Higgs}^2 \approx \frac{1}{2} m_{top}^2 \quad m_{top} \approx v_{weak} \]

\[ v_{weak} \approx 175 \text{ GeV} \]

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} + g_t \bar{\psi}_L t_R H + \text{h.c.} - \frac{\lambda}{2} (H^\dagger H - v_{weak}^2)^2 \]

\[ g_t \approx 1, \quad \lambda \approx \frac{1}{4} \]
It is possible that we need only the strongest coupled SUSY partners to the Higgs Boson to be nearby in mass.

e.g., “Natural SUSY” : A Light Stop

The More minimal supersymmetric standard model
Quantum loops generate a logarithmic “running” of the quartic coupling

\[ \lambda(v) \propto \hbar \beta \log \left( \frac{v}{M} \right) \]

running couplings have many possible trajectories, each parameterized by some \( M \)
Quantum loops can generate a logarithmic “running” of the quartic coupling

\[ \lambda(v) \propto \hbar \beta \log (v/M) \]

this is the relevant behavior
\[ \tilde{\lambda} \] passing from \(< 0\) to \(> 0\) requires \(\beta > 0\)