

Quasi-dynamical symmetry; understanding persistent symmetry

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Abstract

1 Introduction

Let me precede my introductory remarks by showing you (in fig. 1) the results of a model calculation I did recently. The model concerns the pairing of interacting fermions at zero temperature and their condensation into a lowest single-particle level when the strength of the interaction between the fermions falls below a critical value. The details of the model are not important. What is interesting is that the results are characteristic of a number of systems that exhibit a persistent symmetry in the face of relatively strong symmetry breaking interactions. As the number of particles approaches infinity, the model exhibits a sharp second order phase transition at a critical strength of the interaction. Such a model lets one examine the approach to a phase transition in a finite system and try to understand the nature of persistent symmetries. I shall show several such examples in this talk and discuss the symmetries of the systems in the two phases. They will be shown to have some remarkable properties and lead to the concept of a quasi-dynamical symmetry. This is an example of how physics can lead to new mathematical concepts.

I suggest that without symmetry, in some form or other, the theoretical physicist would be completely stymied. This raises the questions: “is symmetry really prevalent in nature?” or “is its presence merely assumed by theoretical necessity?”. I believe it is prevalent in nature provided one extends the concept of symmetry to include quasi-dynamical symmetry.

Symmetry, in a dynamical sense, can be associated with the decoupling of degrees of freedom. It is the essential ingredient that separates the physical world into a hierarchical structure [1] and results in the objects of one level becoming the building blocks of the next. Thus, for example, condensed matter is described in terms of molecules, molecules in terms of atoms, atoms in terms

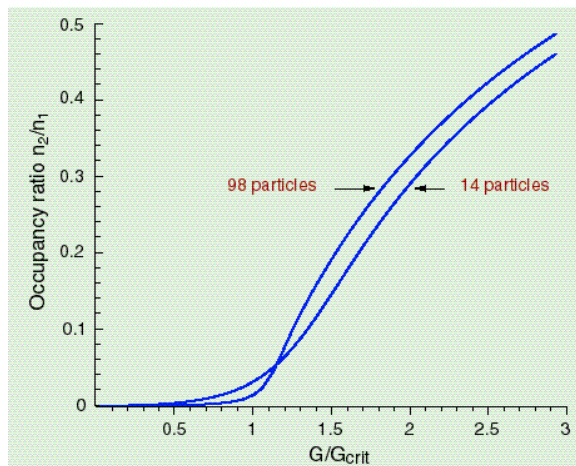


Figure 1: The fractional occupancy of the upper single particle level of a two-level model with interaction strength G in units of a critical value G_{crit} . One curve is for 14 particles and the other for 98. It is seen, when the particle number is large and $G < G_{\text{crit}}$, there is a clear reluctance of the system to give up the symmetry in which all particles occupy the lowest level.

of nuclei and electrons, and so on. Symmetry makes chemical and physical processes predictable. Without symmetry I cannot imagine that life, as we know it, could have any order or even exist. Certainly it is hard to envisage how we could relate to a physical world in which all degrees of freedom were intimately coupled and chaotic. So what is the mechanism by which the decoupling comes about? There are two important operational principles: adiabaticity and symmetry.

Adiabaticity has to do with differences of scale. For example, an atom comprises a nucleus and electrons. Nuclei are 4 to 5 orders of magnitude more massive than electrons and occupy a minute fraction of an atom's volume. Atomic excitation energies are ~ 1 eV whereas nuclear excitation energies are typically 100 keV. Thus, for nuclear physics, atomic electrons are like flies on the back of an elephant whereas, for atomic physics, the nucleus can be regarded as frozen and inert; there is an adiabatic separation of the nuclear and atomic degrees of freedom. Such a separation of dynamical degrees of freedom is the fundamental requirement for the validity of the renowned Born–Oppenheimer approximation [2].

Symmetry enters the picture because it is the essential mathematical language for describing adiabatically decoupled collective motions. However, we need to go beyond the elementary concepts of static symmetry. We need dynamical symmetry to construct and solve models. I shall argue that we also need quasi-dynamical symmetry to describe real physical systems.

In this talk, I shall first illustrate the concept of dynamical symmetry as a

mechanism for describing the adiabatic separation of rotational and vibrational motion for a diatomic molecule. I will then consider the soft, fluid-like, rotations of a nucleus for which quasi-dynamical symmetry is more appropriate. After that, I will show that the phenomenon of quasi-dynamical symmetry is robust and survives even when there are huge symmetry breaking interactions. I will conclude with a model having two fundamentally incompatible phases and show the emergence of superfluid rotational motions as the union of the two incompatible phases.

2 The rotations of a diatomic molecule

The rotational motions of a diatomic molecule provides a simple example of adiabatically decoupled degree of freedom.

Consider the energy-level spectrum of the HCl molecule shown in fig. 2. It can be seen that the lowest states are rotational and the angular momentum has to exceed $50\hbar$ before the rotational energy becomes comparable to the energy of the first vibrational state. Thus, the lower rotational states are adiabatic.

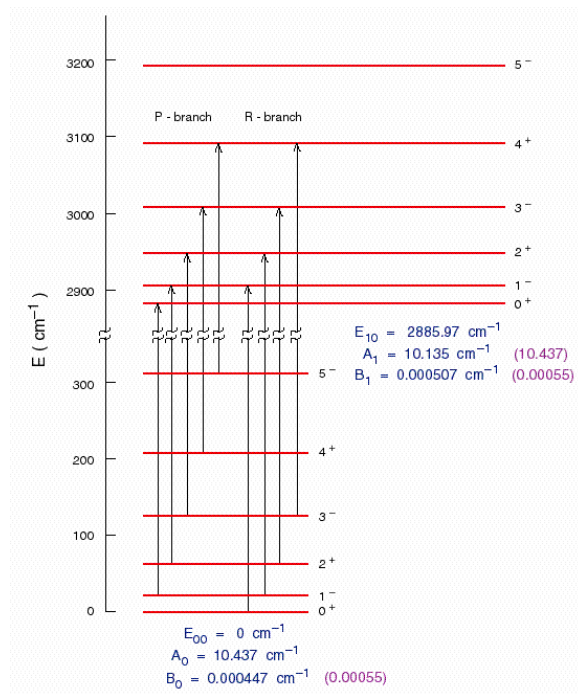


Figure 2: Energy levels of the ground-state rotational band and first excited vibrational band of the HCl molecule.

Solution of the Schrödinger equation for the molecule is possible because of a separation of variables for the radial and rotational degrees of freedom. This

separation of variables is associated with a direct product group $SU(1,1) \times O(3)$, where $SU(1,1)$ is a dynamical group for the Hamiltonian and $O(3)$ is a symmetry group in the following sense. A group G is said to be a *dynamical group* for a Hamiltonian on a Hilbert space \mathbb{H} if the Hamiltonian leaves all the G -invariant subspaces of \mathbb{H} invariant. G is a *symmetry group* of the Hamiltonian if it leaves the Hamiltonian invariant.

Analytical solutions are possible for a diatomic molecule when the interaction between the two atoms is approximated by the so-called Davidson potential [3]

$$V(r) = \frac{1}{2}m\omega^2 \left(r^2 + \frac{\varepsilon}{r^2} \right), \quad (1)$$

where m is the reduced mass for the two atoms; ω and ε are adjustable parameters. This potential is shown in fig. 3. It has a minimum, corresponding to the equilibrium separation distance of the two atoms, at $r_0 = \varepsilon^{1/4}$ and a strength proportional to ω^2 .

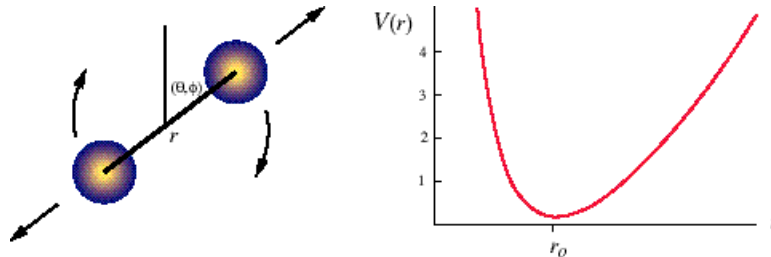


Figure 3: The Davidson potential for a diatomic molecule.

With addition of the relative kinetic energy for the two atoms, the Hamiltonian for the molecule becomes, in harmonic oscillator units of length,

$$H = \frac{1}{2}\hbar\omega \left(-\nabla^2 + r^2 + \frac{\varepsilon}{r^2} \right). \quad (2)$$

This Hamiltonian is invariant under the group $O(3)$ of inversions and rotations; thus it commutes with components of the angular momentum

$$\hat{\mathbf{L}} = -\mathbf{r} \times \nabla, \quad (3)$$

where the latter are the infinitesimal generators of $SO(3)$. Moreover, $H \propto Z_1 + Z_2$ is an element of the $SU(1,1)$ Lie algebra spanned by the operators

$$Z_1 = -\nabla^2 + \frac{\varepsilon}{r^2}, \quad Z_2 = r^2, \quad Z_3 = \frac{1}{2}(\mathbf{r} \cdot \nabla + \nabla \cdot \mathbf{r}). \quad (4)$$

Thus, its eigenvalues are easily determined [4] to be given by

$$E_{nl} = \left[2n + 1 + \sqrt{\left(l + \frac{1}{2} \right)^2 + \varepsilon} \right] \hbar\omega, \quad (5)$$

where n and l are radial and angular momentum quantum numbers, respectively. Eigenfunctions can also be derived without difficulty.

For a relatively large value of ε , the rotational motions for such a potential become adiabatic relative to the vibrational motions and the energies are given by the expansion

$$E_{nl} = E_0 + 2n\hbar\omega + Al(l+1) - Bl^2(l+1)^2 + \dots \quad (6)$$

This expression gives energy levels for the HCl molecule which the eye cannot distinguish from the measured values; the model values for the A and B parameters are shown on fig. 2. Details of the model are given in ref. [4].

3 Rotational states of a heavy nucleus

The emergence of rotational states in molecules is easy to understand because molecules are relatively rigid. In contrast, nuclei are more fluid-like. Nevertheless, numerous rotational bands are seen in a wide range of nuclei. The property that nuclear rotations share with molecular rotations is that, when they are seen, they are adiabatic relative to the competing degrees of freedom.

3.1 The nuclear symplectic model

Although it is not widely known, there is a partial separation of nuclear variables into collective coordinates and a complementary set of intrinsic coordinates [5, 6, 7]. It turns out that, in parallel with that for diatomic molecules, this separation is also associated with a direct product group. The group is $\text{Sp}(3, \mathbb{R}) \times \text{O}(A)$, where $\text{Sp}(3, \mathbb{R})$ is a dynamical group for a collective Hamiltonian [8] and $\text{O}(A)$, where A is the nucleon number, is a complementary intrinsic group [9, 10, 11]. Some of the remarkable “dual pair” properties of the group $\text{Sp}(3, \mathbb{R}) \times \text{O}(A)$ were identified in a nuclear physics context by Moshinsky and Quesne [12] and subsequently exploited widely in representation theory [13, 14].

The symplectic group $\text{Sp}(3, \mathbb{R})$ is basically the group of all linear canonical transformations of a set of position and momentum coordinates for a single particle in three dimensions. Thus, it acts on a many-nucleon phase space by transforming all nucleon coordinates in the same way. Thus, by definition, it is group of collective transformations.

The Lie algebra $\mathfrak{sp}(3, \mathbb{R})$ of $\text{Sp}(3, \mathbb{R})$ is spanned by the bilinear products of nucleon position and momentum coordinates summed over all nucleons; i.e., the combinations

$$\sum_{n=1}^A x_{ni}x_{nj}, \quad \sum_{n=1}^A p_{ni}p_{nj}, \quad \sum_{n=1}^A (x_{ni}p_{nj} + p_{nj}x_{ni}). \quad (7)$$

It contains, for example, the harmonic oscillator shell model Hamiltonian

$$H_0 = \sum_{n=1}^A \sum_{i=1}^3 \left[\frac{1}{2m} p_{ni}^2 + \frac{1}{2} m\omega^2 x_{ni}^2 \right] \quad (8)$$

and all components of the nuclear mass quadrupole tensor Q ;

$$Q_\nu = \sum_{n=1}^A r_n^2 Y_{2\nu}(\theta_n, \varphi_n), \quad \nu = 0, \pm 1, \pm 2. \quad (9)$$

Thus, it is possible to make collective Hamiltonians that are polynomials in the elements of the $\text{sp}(3, \mathbb{R})$ algebra and which, therefore, leave an $\text{Sp}(3, \mathbb{R})$ irrep invariant. Moreover, because the $\text{sp}(3, \mathbb{R})$ algebra contains the quadrupole operators, it is also possible to compute E2 transition rates between $\text{Sp}(3, \mathbb{R})$ model eigenstates.

A suitable Hamiltonian for the study of nuclear rotational bands with fully microscopic wave functions is the Hamiltonian

$$H = H_0 + V(Q), \quad (10)$$

where

$$V(Q) = \chi \left(Q \cdot Q + \frac{\varepsilon}{Q \cdot Q} \right) \quad (11)$$

is a Davidson potential with respect to quadrupole deformation degrees of freedom. The Davidson potential was first used in the nuclear collective model context by Elliott *et al.* [15]; it has a minimum when the nucleus has a spheroidal shape with a quadrupole moment equal to $\varepsilon^{1/4}$.

The Hamiltonian of eqn. (10) is not analytically solvable. But, it can be diagonalized numerically in a sequence of subspaces of an infinite-dimensional $\text{Sp}(3, \mathbb{R})$ irrep, defined in terms of a partially-ordered basis of harmonic states (eigenstates of H_0), until convergence is obtained to within the desired level of accuracy.

The results of this model with adjusted values of χ and ε are shown in comparison with the observed energy level spectrum of ^{166}Er in fig. 4. Also shown for comparison are the results of a phenomenological rigid rotor model. The interesting result is that, although the parameters of the Davidson potential were adjusted to get the observed deformation of the nucleus, the Hamiltonian of the model contains the full microscopic kinetic energy. In contrast to the rigid-rotor model, it has no adjustable moments of inertia. Thus, the fact that the moments of inertia, that characterize the low-lying energy-level spacings, come out right is a remarkable result which supports the proposition that the dynamical content of the symplectic model is essentially correct. The figure also suggests that real nuclei are not as rigid as those of either the rigid rotor model or the symplectic model with a Davidson potential. Less rigid results can be obtained in the symplectic model with a more realistic potential.

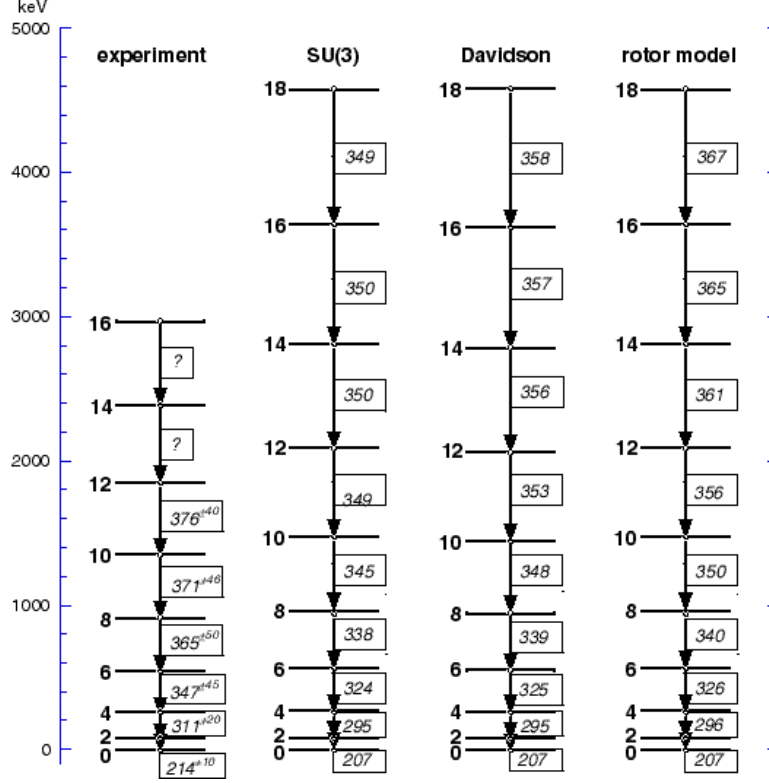


Figure 4: Energy levels of the ground-state rotational band of the ^{166}Er nucleus in the SU(3) model, the Davidson model, and the phenomenological rotor model. The numbers in boxes attached to arrows are E2 transition rates.

3.2 Symplectic model wave functions in an SU(3) basis

The symplectic model results shown in fig. 4 were computed in a shell model space with basis states that reduces the subgroup chain

$$\begin{array}{c} \text{Sp}(3, \mathbb{R}) \\ N_0(\lambda_0 \mu_0) \end{array} \supset \begin{array}{c} \text{U}(1) \times \text{SU}(3) \\ \rho \quad N \quad (\lambda \mu) \end{array} \supset \begin{array}{c} \text{SO}(3) \\ K \quad LM \end{array}, \quad (12)$$

where $\text{U}(3) \sim \text{U}(1) \times \text{SU}(3)$ is the symmetry group of the harmonic oscillator shell model Hamiltonian H_0 . Thus, basis states are labeled by the quantum numbers and multiplicity indices shown and the rotational states that emerge from the model are obtained as coefficients in the expansion

$$|\alpha LM\rangle = \sum_{\rho N(\lambda \mu) K} C_{\rho N(\lambda \mu) K L} |\rho N(\lambda \mu) K L M\rangle. \quad (13)$$

The coefficients are plotted for the angular momentum states $L = 0, \dots, 10$ in fig. 5

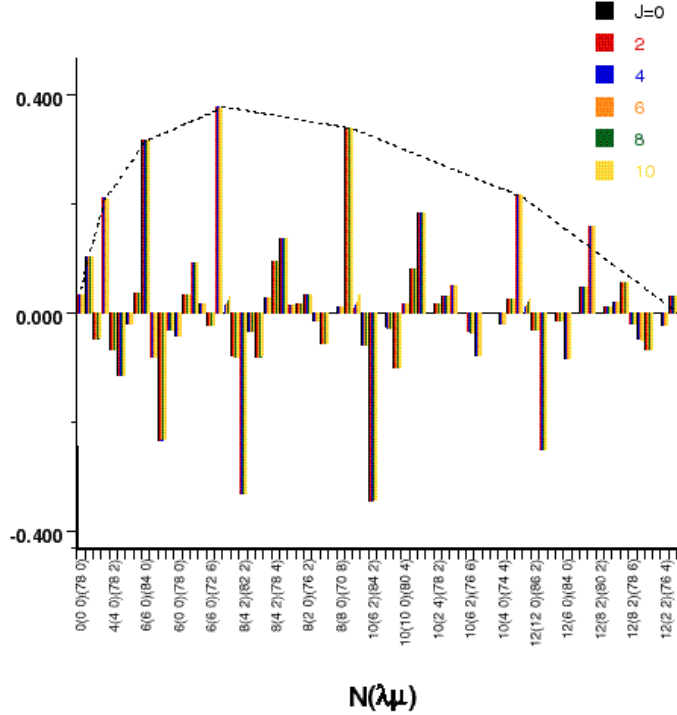


Figure 5: Histograms of the SU(3) amplitudes for the $L = 0, \dots, 10$ states.

The remarkable result is that the coefficients are essentially independent of L ; i.e., the rotational states are to a high degree of accuracy expressible in the L -independent form

$$|\alpha LM\rangle = \sum_{\rho N(\lambda\mu)K} C_{\rho N(\lambda\mu)K} |\rho N(\lambda\mu)KLM\rangle. \quad (14)$$

What is going on? Clearly U(3) is not a symmetry nor even a dynamical symmetry for the model; there is an enormous mixing of its irreps. However, the mixing occurs in a highly coherent way which indicates a completely new kind of symmetry; we call it quasi-dynamical symmetry. It turns out that this kind of symmetry is associated with a mathematically unusual kind of representation of a Lie algebra called an *embedded representation* [16].

4 Embedded representations; quasi-dynamical symmetry

An embedded representation of a Lie algebra is a true representation. But it is embedded in an unusual way in some other representation.

Definition: Let T be a (generally reducible) representation of a Lie algebra \mathfrak{g} on a vector space V . Let $E : U \rightarrow V$ be an embedding of a vector space U in V and let $\Pi : V \rightarrow U$ be the left inverse of E so that ΠE is the identity operator on U . Then if the set $\{S(X) = \Pi T(X)E; X \in \mathfrak{g}\}$ of transformations of U is a representation of \mathfrak{g} , i.e., if $S(X)S(Y) = S(X + Y)$, then S is said to be an embedded representation.

The embedding $E : U \rightarrow V$ identifies U with a subspace $E(U)$ of V . If the representation T leaves this subspace invariant, then the representation S is seen to be a subrepresentation of T . The interesting embedded representations are those that are not subrepresentations.

To make the concept appear less strange, recall that if a representation T of a Lie algebra contains a number of equivalent irreps, one can form subrepresentations by taking arbitrary linear combinations of the equivalent representations. Embedded representations are obtained by taking linear combinations of irreps that are merely similar to the extent that their matrix elements depend at most linearly on some representation labels. The Lie algebras which exhibit such irreps are the semidirect sums with Abelian ideals. However, many Lie algebras (perhaps most) contract in the limit of large-dimensional irreps to semidirect sums and, for them, embedded representations become possible as limiting situations.

Elementary examples of embedded representations are given for any Abelian Lie algebra. For example, if \mathfrak{g} is a real Abelian Lie algebra, then any irrep over the complex field is one-dimensional and defined by a linear function χ on \mathfrak{g} , i.e., a function such that $\chi(\alpha X + \beta Y) = \alpha\chi(X) + \beta\chi(Y)$, where X and Y are elements of \mathfrak{g} and α and β are real coefficients. Thus, if T is a reducible representation on a multidimensional Hilbert space V and $|\varphi\rangle$ is any state in V , then the function on \mathfrak{g} defined by

$$\chi(X) = \langle \varphi | T(X) | \varphi \rangle, \quad \forall X \in \mathfrak{g}, \quad (15)$$

is an irrep of \mathfrak{g} . However, unless $|\varphi\rangle$ is a common eigenstate of every $T(X)$, this irrep is not a subrepresentation of T .

A more interesting example is provided by the SGA of a two-dimensional rotor; i.e., an algebra with basis $\{x, y, L\}$. This algebra has infinite-dimensional unitary irreps, $\{T^R\}$, labeled by a continuously variable real number R ; they are carried by the square integrable functions on the circle, $\mathcal{L}^2(S_1)$. Thus, a basis for an irrep is given by the functions $\{\psi_m; m = 0, \pm 1, \pm 2, \dots\}$, where

$$\psi_m(\theta) = e^{im\theta} \quad (16)$$

and

$$\begin{aligned} [T^R(x)\psi_m](\theta) &= R \cos \theta \psi_m(\theta), \\ [T^R(y)\psi_m](\theta) &= R \sin \theta \psi_m(\theta), \\ [T^R(L)\psi_m](\theta) &= -i \frac{\partial}{\partial \theta} \psi_m(\theta) = m \psi_m(\theta). \end{aligned} \quad (17)$$

The states of such an irrep can be interpreted as those of a particle moving on a circle of radius R .

Now suppose that T is the representation of this same algebra on $V = \mathcal{L}^2(\mathbb{R}^2)$, the square integrable functions on the Euclidean plane, with action

$$\begin{aligned} [T(x)\Psi](r, \theta) &= r \cos \theta \Psi(r, \theta), \\ [T(y)\Psi](r, \theta) &= r \sin \theta \Psi(r, \theta), \\ [T(L)\Psi](r, \theta) &= -i \frac{\partial}{\partial \theta} \Psi(r, \theta). \end{aligned} \tag{18}$$

Suppose that E embeds the Hilbert space $\mathcal{L}^2(S_1)$ in V as the subspace of functions spanned by the set $\{\Psi_m = E(\psi_m); m = 0, \pm 1, \pm 2, \dots\}$ where

$$\Psi_m(r, \theta) = f(r) \psi_m(\theta), \tag{19}$$

with f a fixed radial function. Then, $S = \Pi T E$ acts on $\mathcal{L}^2(S_1)$ by

$$\begin{aligned} [S(x)\psi_m](\theta) &= [\Pi T(x)\Psi_m](\theta) = \bar{R} \cos \theta \psi_m(\theta), \\ [S(y)\psi_m](\theta) &= [\Pi T(y)\Psi_m](\theta) = \bar{R} \sin \theta \psi_m(\theta), \\ [S(L)\psi_m](\theta) &= [\Pi T(L)\Psi_m](\theta) = m \psi_m(\theta), \end{aligned} \tag{20}$$

where

$$\bar{R} = \int |f(r)|^2 r^2 dr. \tag{21}$$

S is seen to be an irrep isomorphic to $T^{\bar{R}}$. However, it is not a subrepresentation of T ; it is an embedded representation.

These concepts have a clear physical interpretation. For example, a description of the rotational states of a diatomic molecule by a basis for an irrep of a rotor model algebra would amount to making the assumption that the radial wave function is a delta function, thereby implying a precise value for the distance separating the two atoms. This may be a reasonable assumption for some, relatively rigid, molecules but it makes little sense for a softer rotor. Nevertheless, if the rotations of even a relatively soft rotor are slow enough the Coriolis and centrifugal forces may effect only small couplings of the internal and rotational degrees of freedom. It is seen that describing a rotor by the states of an irreducible embedded representation corresponds to turning off the Coriolis and centrifugal forces; i.e., treating the rotating frame as an inertial frame. The effects of these neglected forces can subsequently be restored by mixing different embedded representations. One then expects to see that the low angular momentum states of a rotational band, for which the rotational motion is adiabatic, are rather well described by the states of a single embedded irrep but that the higher angular momentum states progressively exhibit more effects of Coriolis and centrifugal coupling perturbations.

It is also clear that the expansion coefficients for the states of an embedded irrep in terms of rigid rotor states have a physical interpretation in terms of vibrational wave functions.

Systems whose states are described (maybe in some approximation) by embedded representations are said to have a quasi-dynamical symmetry.

5 A superconducting to rotational phase transition

I now give another example of a system that exhibits a quasi-dynamical symmetry. The example illustrates what can happen in a system with two fundamentally incompatible collective degrees of freedom.

Consider a many-fermion model with Hamiltonian

$$H(\alpha) = H_0 + (1 - \alpha)V_{\text{SU}(2)} + \alpha V_{\text{SU}(3)}, \quad (22)$$

defined such that, when the parameter $\alpha=0$, the system has an $\text{SU}(2)$ dynamical symmetry and a spectrum characteristic of a finite spherical superconductor and, when $\alpha=1$, it has an $\text{SU}(3)$ dynamical symmetry and an adiabatic rotational spectrum. For intermediate values of α , it may not be possible to compute the spectrum of H . The reason for this is that the two dynamical symmetry groups $\text{SU}(2)$ and $\text{SU}(3)$ are realized in very different ways with the result that the smallest group that contains both $\text{SU}(2)$ and $\text{SU}(3)$ as subgroups may be too large, in general, to be useful; it could even be infinite. The two dynamical symmetry groups $\text{SU}(2)$ and $\text{SU}(3)$ are then said to be *incompatible* [17, 18].

To investigate what happens when α lies between 0 and 1, we considered the special case in which the smallest Lie group that contains both $\text{SU}(2)$ and $\text{SU}(3)$ is the unitary symplectic group $\text{USp}(3)$. Even then, for a large fermion number ($N = 48$ in the case considered), the dimensions of the representation are large and the diagonalization of H is far from trivial.

The energy levels that result are shown in Figure 6. The left hand side of the figure shows the non-adiabatic spectrum characteristic of the $\alpha = 0$ limit. Note that only the lowest energy state of each angular momentum in the range $L = 0, \dots, 8$ is shown. The spectrum resembles that of a vibrator; in fact the ground state corresponds to all the fermions bound together in Cooper pairs; the excited states correspond to one or more broken pairs. The right hand side of the figure shows an adiabatic rotational spectrum characteristic of the $\text{SU}(3)$ model [19]. Now, if the Hamiltonian were simply

$$H_1(\alpha) = (0.75 - \alpha)[H_0 + V_{\text{SU}(2)}], \quad (23)$$

the energy levels would decrease linearly with α and follow the straight lines tangential to the $\alpha = 0$ energies shown in the figure. (Note that the factor 0.75 instead of 1.0 in this expression is an adjustment, needed to get the observed slope; it allows for the fact that $V_{\text{SU}(3)}$ makes a non-zero contribution to the $\text{SU}(2)$ energy levels.) On the other hand, if the Hamiltonian were

$$H_2(\alpha) = \alpha[H_0 + V_{\text{SU}(3)}], \quad (24)$$

the energy levels would increase linearly from the origin to the $\alpha = 1$ energies shown. The results of the computation show that, for the full Hamiltonian H , the energies flip from one behavior to the other within a relatively narrow transition region. In fact, if the number N is increased, the transition region

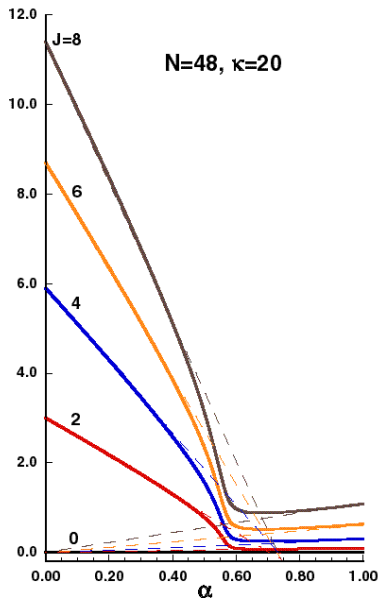


Figure 6: Energies of the lowest $J = 0, \dots, 8$ states as a function of α for $H(\alpha)$.

becomes progressively more narrow until, in the limit, there is a sharp phase transition.

At first sight, it would appear that the model is well described by the Hamiltonian $H_1(\alpha)$ for α well below its critical value and by $H_2(\alpha)$ for α well above. However, the wave functions reveal a more interesting situation. They are shown as histograms of the coefficients of the energy eigenstates in an expansion on the SU(3) basis in Figure 7. If the states were simply eigenstates of $H_2(\alpha)$ for α above the transition, the coefficients would all be equal to those of the $\alpha = 1$ limit; i.e., only a single SU(3) irrep would have a non-zero coefficient. This is far from the situation. For α below the critical value, the wave functions are very complicated (in the SU(3) bases); they would be simpler in the SU(2) bases. However, even though there is much mixing of SU(3) irreps for values of α just above the critical value, the mixing is remarkably coherent. All the states of angular momentum in the range $L = 0, \dots, 8$, for which the coefficients are plotted, are seen to be essentially identical. This is an indication that SU(3) is a remarkably good quasi-dynamical symmetry. Thus, in spite of the huge mixing of SU(3) irreps, the energy levels and E2 transition rates could be fitted very well by an SU(3) model with a single irrep. In the limit as the particle number approaches infinity, it can be shown that the irreps of SU(3) contract to those of a rigid rotor algebra, by an Inönü–Wigner contraction process [20], and that the states of the rotational band that emerges belong to an embedded representation.

There is a natural physical interpretation of what is going on. For α below

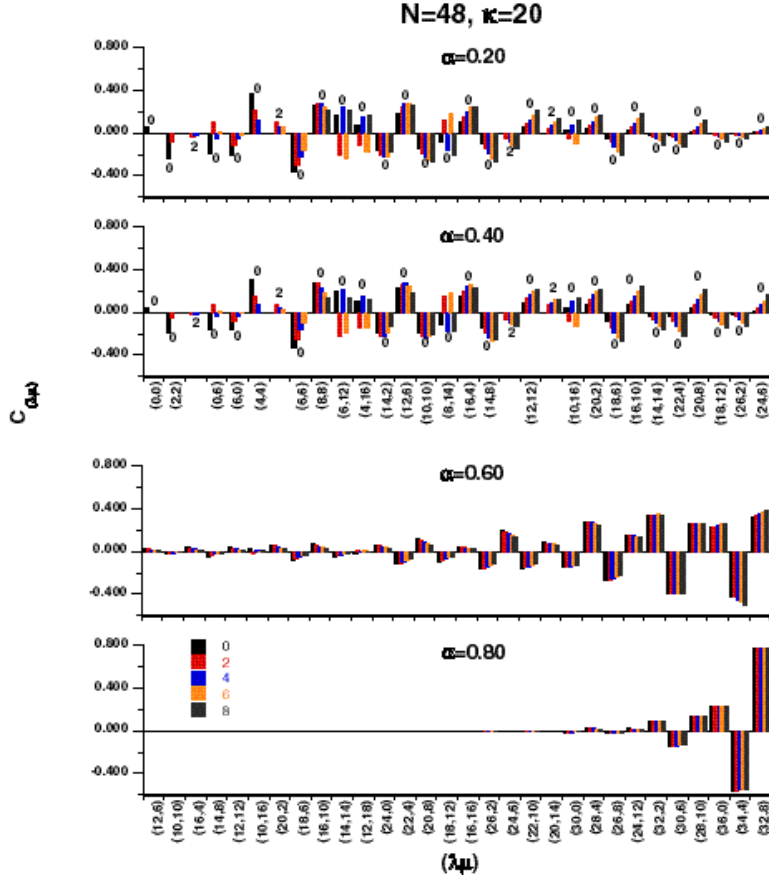


Figure 7: Wave functions in an $SU(3)$ basis for the angular-momentum $L = 0, 2, \dots, 8$ states of the Hamiltonian $H(\alpha)$ for four values of α .

the critical value, the model represents a spherical superconductor and its low-energy collective modes are only center-of-mass translations (not shown). The energy levels shown are the relatively high energy excitations characteristic of the energies required to break Cooper pairs. However, as the strength of the (short-range) pair-coupling interaction is increased and replaced by a long-range interaction of the $SU(3)$ type, there comes a point at which the spherical symmetry is broken, the model assumes a deformed equilibrium shape, and a low-energy (adiabatic) rotational collective mode emerges. However, the pair correlations persist and are large above the critical value of α , implying that it is appropriate to think of the ground state band as that of a rotor with superfluid flows. This interpretation is supported by the full spectrum, which exhibits excited rotational bands associated with broken pairs.

Details of the model are given in ref. [18].

6 Conclusions

The identification of quasi-dynamical symmetry as a mechanism for organizing the emergence of collective phenomena provides an intuitive understanding of the phenomena in microscopic terms. In particular, it helps explain why simple models work as well as they do even in the face of large residual interactions that might be expected to destroy the validity of a model. It explains why collective effects are as robust and as insensitive to the details of the interactions as they appear to be. It also emphasizes that one should not take the dynamical symmetry of successful collective models too literally. The model may be working in an average, effective, manner of a quasi-dynamical symmetry.

The success of quasi-dynamical symmetry to explain the emergence of rotational bands in nuclear physics, is only an example of the many phenomena and systems one could wish to investigate using the concept. It would appear that much is to be gained, for example, by examining phase transitions and the emergence of collective phenomena in large but finite systems, in which the number of particles can be varied. For such systems, it becomes feasible to make realistic or semi-realistic model investigations which can also be tested against experiment so that one can understand in considerable detail the way correlations and coherence properties develop. For such purposes nuclear physics is invaluable; indeed, it is essentially unique in exhibiting many-body phenomena that can only be observed elsewhere in infinite condensed matter systems.

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