COMPATIBLE AND INCOMPATIBLE SYMMETRIES IN THE THEORY OF NUCLEAR COLLECTIVE MOTION

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ABSTRACT

An elegant and economical way to express the dynamics of a system is by means of a dynamical symmetry group. To combine two sets of dynamical degrees of freedom, one would like to combine the corresponding dynamical symmetries into a larger structure. In doing this one generally finds that extra gauge degrees of freedom, of the Berry phase type, must be included. As an example, it is recalled that combining rotational and vibrational degrees of freedom gives an SO(3) gauge corresponding to vorticity. However, it often happens that the number of extra gauge degrees of freedom required is huge. This happens when the dimension of the smallest group which contains both of the given dynamical symmetry groups is very large. We then say that the symmetries are *incompatible.* The present analysis was motivated by the problem of combining Kerman's SU(2) quasispin model and Elliott's SU(3) model. SU(2) is a dynamical group for a simple model of superconductivity. SU(3) is a dynamical group for nuclear rotations. The incompatibility of these symmetries presents a major obstacle to the development of an algebraic theory of rotating, superconducting, nuclei. Some possibilities for overcoming this obstacle are discussed.

1. Introduction

In attempting to understand a complex physical system, it is necessary to focus on a few dynamical degrees of freedom at a time. However, having understood the relevant parts of a system separately, one would like to combine the pieces to form a model which allows couplings between the various degrees of freedom. Sometimes this is easy and there are no surprises.

Consider, for example, an object free to move along and rotate about an axis. The configuration of the object has the geometry of a cylinder. In this example, the translational and rotational subdynamics are simply additive. In fact, rotational and translational invariance ensures they are also decoupled. However, this example is not typical. In the generic situation, combining degrees of freedom involves new degrees of freedom with the result that unexpected and sometimes interesting structures emerge.

By their nature models invariably approximate the systems they represent. It is to be expected then that different models, representing different parts of a system, may not be compatible with one another. Models may be incompatible because they treat common aspect of a system in incompatible ways. However, my concern is with models that are completely consistent with one another but which, when

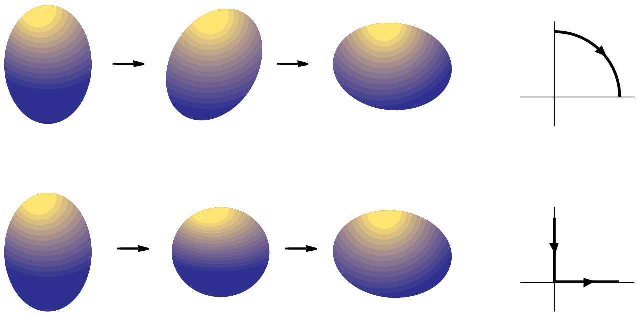

Fig. 1. A rotation of a nucleus effected by (a) a rigid rotations and (b) by a sequence of deformations.

combined, involve other, previously unmentioned, degrees of freedom of the system. I shall give an example in which adding extra degrees of freedom gives a closed dynamical system. In some cases the number of extra degrees of freedom required is very large; it may even be infinite. The situation is then analogous to the onset of chaos. It is conveniently analysed in terms of dynamical symmetries. We then talk of symmetries which are compatible, if they can be combined easily, and incompatible, if not.

The first example arises in the Bohr-Mottelson collective model.[1] The problem of giving the collective model a microscopic foundation resisted solution for many years until it was learned, through the analysis of Cusson, Weaver and Biedenharn[2] and subsequent developments,[3,4] that, to combine rotational and vibrational dynamics effectively, one needs extra gauge degrees of freedom corresponding to vorticity.

The second example, focuses on the longstanding problem of including pair correlations in the rotor model. It will be shown that the problem is a difficult one because the dynamical symmetries associated with rotations and with pairing are incompatible.

2. The rotations and vibrations of a two-dimensional nucleus

Consider first the volume conserving vibrations of an axially symmetric nucleus which is also free to rotate such that its symmetry axis remains in a plane. A configuration of such a nucleus is characterized by two polar coordinates (q, θ), where q is a quadrupole moment and θ an orientation angle. The rotational and vibrational dynamics are coupled by centrifugal and Coriolis forces. However, they also couple algebraically to a third intrinsic degree of freedom. One can see this geometrically.

The nucleus can go from one configuration to another by many different paths. However, different paths are not equivalent. Consider, for example, the two paths shown in Fig. 1. The path shown in (a), in which q is held constant and θ goes from 0 to $\pi/2$, corresponds to a *rigid-body* rotation. The path shown in (b), corresponding to a sequence of deformations, appears as a “rotation” of the nuclear shape but involves no circulation of the nuclear matter. In fact, a deformable object can

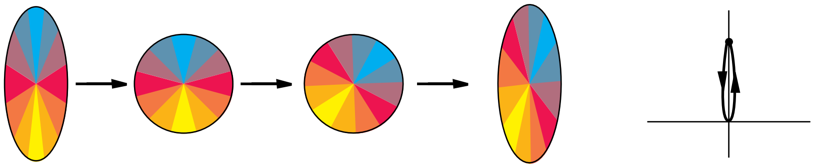

Fig. 2. A vortex rotation.

change its orientation in space in this way without any angular momentum. To differentiate between the two situations, an extra gauge degree of freedom of the Berry phase type[5] is required.

One can also have paths which correspond to an internal circulation without a change in shape orientation as illustrated in Fig. 2. Such flows are described as *vortex* flows.

Consider now a slightly more general two-dimensional model of a nucleus whose shape and orientation is characterized by a quadrupole matrix

$$Q = \begin{pmatrix} \sum x^2 & \sum xy \\ \sum yx & \sum y^2 \end{pmatrix} , \tag{1}$$

in which the sums are over all nucleon $x - y$ coordinates. The dynamics of the model involve four basic structures:

(i) *A configuration space*

Since Q is real and symmetric, it can be brought to diagonal form by a rotation and expressed in the form

$$Q = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} . \tag{2}$$

Thus, a configuration of the model is specified by three cooordinates (q_1, q_2, θ).

(ii) *A motion group*

If we forget volume conservation, then for vibrations without rotations (in 2 dimensions), the motion group is

$$G_1 = \left\{ \begin{pmatrix} a & \\ & b \end{pmatrix} ; a, b > 0 \right\} . \tag{3}$$

It is a group with two degrees of freedom. For rotations without vibrations, the group is SO(2); it has one degree of freedom. One finds that the smallest group that contains both G_1 and SO(2) is GL(2,R). Thus, we consider GL(2,R) to be the motion group for the model. It has four degrees of freedom.

(iii) *A gauge group*

An element g of GL(2,R) transforms the configuration space according to the equation

$$Q \to gQ\tilde{g} , \tag{4}$$

where $\tilde{g}$ is the transpose of the matrix g. Since GL(2,R) has dimension four, whereas the configuration space is three-dimensional, it follows that there is a one-dimensional subgroup H of elements of GL(2,R) that leaves a configuration Q invariant;

$$H = \{h \in \mathrm{GL}(2,R); hQ\tilde{h} = Q\}\,. \tag{5}$$

This subgroup is called the *stability subgroup* at Q. In the present context it is also the gauge group. Consider, for example, the stability subgroup at Q_0, where

$$Q_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

is the quadrupole matrix for a spherical nucleus. The gauge group

$$H = \{h \in \mathrm{GL}(2,R); hQ_0\tilde{h} = Q\} \tag{7}$$

is now SO(2), the subgroup of rotations.

(iv) *A dynamical group*
The motion group is not a dynamical group for the model. It is defined as a group of transformations of the configuration space for the model. To define a Hamiltonian dynamics, the configuration space must be extended to a phase space. If we regard GL(2,R) as a group of general linear canonical point transformations, we determine that a suitable dynamical group is the group Sp(2,R) of general linear canonical transformations of the corresponding phase space. More details can be found in refs. 6-7.

The configuration space, the motion group and the gauge group are related as follows. First observe that the configuration space can be generated by starting with some particular configuration, e.g. Q_0, and transforming it with all elements of the motion group GL(2,R). Thus, the configuration space is characterized by the set of quadrupole matrices

$$\{gQ_0\tilde{g}; g \in \mathrm{GL}(2,R)\}\,. \tag{8}$$

Next observe that any element $g \in$ GL(2,R) can be expressed in the factored form

$$g = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}. \tag{9}$$

It then follows that

$$gQ_o\tilde{g} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{10}$$

which is identical to the general form given by Eq. (2) with $a^2 = q_1$ and $b^2 = q_2$. As anticipated, the quadrupole matrices generated in this way do not depend on the SO(2) gauge angle φ. In group theoretical language, the configuration space is isomorphic to the factor space GL(2,R)/SO(2).

3. The symplectic model

For the collective vibrational and rotational motions of a three-dimensional nucleus, the corresponding motion group is GL(3,R), the gauge group is SO(3), the configuration space is GL(3,R)/SO(3), and the dynamical group is Sp(3,R). The gauge group, corresponding to internal vortex flow, is now non-Abelian.

One may well ask "what is achieved by making the Bohr-Mottelson collective model more complicated by the addition of vorticity?" After all, the model was already successful. The simple answer is that it gives the collective model a microscopic foundation. By adding vorticity degrees of freedom and expressing the model in terms of a dynamical group, the collective model becomes a submodel of the nuclear shell model. To see this, one has only to observe that the Lie algebra sp$(3, R)$ of the symplectic group Sp$(3, R)$ consists of simple one-body operators. The collective model expressed in this way is known as the symplectic model.[6,7]

Giving the collective model a microscopic foundation is of vital importance for the development of nuclear structure theory. For if a model can be embedded in a microscopic theory, it becomes a part of a much more complete description and it gains the possibility of being related and combined with other such models in building up a complete understanding of the nucleus. Moreover, if a phenomenological model can be given microscopic wave functions, it gains predictive power. This is because the full arsenal of microscopic observables can be invoked to investigate its dynamical content. For example, the symplectic model can make predictions of rotational current flows and moments of inertia, whereas in the Bohr-Mottelson model, the moments of inertia are adjustable parameters.

To illustrate, we recall the phenomenogical sympletic model calculations of Park, Carvalho, et al.[8] They considered the Hamiltonian

$$H = \frac{1}{2m}\sum_{ni} p_{ni}^2 + \frac{1}{2}m\omega^2 \sum_{ni} x_{ni}^2 + \chi V(\beta,\gamma) \tag{11}$$

with the potential

$$V(\beta,\gamma) = \left[(Q\times Q)_0\right]^2 + a(Q\times Q\times Q)_0\,, \tag{12}$$

shown in Fig. 3. This Hamiltonian contains three parameters: ω, χ and a. The harmonic oscillator frequency ω can be chosen to give the correct nuclear mean square radius. The parameter a can be fixed such that the potential has a minimum at the experimentally observed deformation. Finally, the depth of the potential, proportional to χ, can be adjusted to ensure that the deformation of the nuclear density is also consistent with experiment and to get the observed moment of inertia. Note that the full many-nucleon kinetic energy is an element of the sp$(3, R)$ Lie algebra so that no a priori assumptions need be made regarding the rotational kinetic energy.

The spectrum of the above Hamiltonian can be calculated because it is a polynomial in the Lie algebra sp$(3, R)$ of the dynamical group Sp(3,R). Results for the lowest four energy levels of four deformed nuclei are shown in Fig. 4.

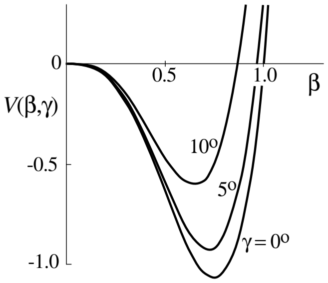

Fig. 3. A model potential.

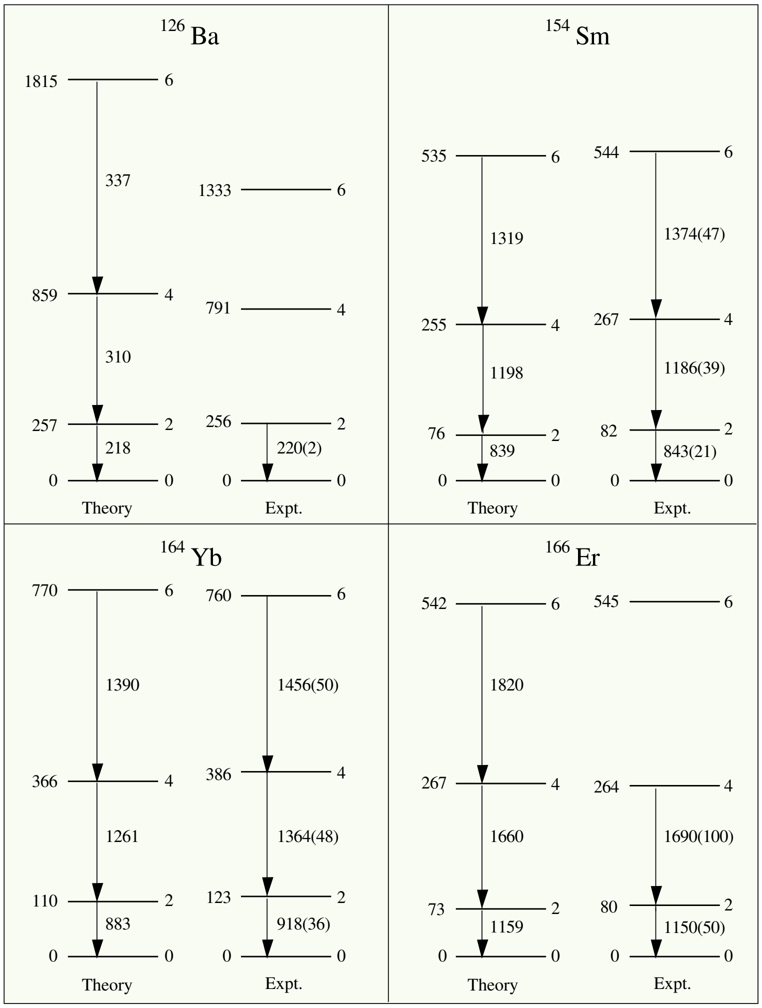

Fig. 4. Low energy states of deformed nuclei for a symplectic model Hamiltonian (calculations by Park et al.[8])

What is remarkable about the results is that they give microscopic shell model states with properties which are extremely close (closer than experiment) to those of pure rotational bands. Moreover, they give correct B(E2) values (without the use of effective charges) and moments of inertia. Thus, the results show that the microscopic symplectic model contains the nuclear rotor model with experimentally observed deformations and moments of inertia. Getting the observed moments of inertia suggests that, with the inclusion of vorticity, the rotational dynamics is given correctly in the symplectic model.

These results are important not just because they explain rotational bands in terms of interacting nucleons, but because they confirm that the symplectic model is a suitable microscopic framework for asking questions about rotational dynamics. In this context, it is emphasized that the symplectic shell model[9] is an exact formal framework in the same sense as the shell model with major-shell mixing. Thus, in principle, one can do shell model calculations in bases of states classified by the representation labels of $\mathrm{Sp}(3,R)$ and its subgroups. Since the above phenomenological symplectic model calculations confirm that rotor bands are easily obtainable within the space of a single irrep, one could, in principle, repeat the calculations with a realistic microscopic Hamiltonian to see if observed rotational bands emerge from such a calculation with no adjustable parameters. One could then investigate the dynamical implications of the results. One could, in principle, include other effects (e.g., pairing correlations) by allowing mixing of irreps, to discover the contributions of different irreps in the low-energy states of rotational nuclei. What one needs for such investigations is the facility to carry out shell-model calculations in an $\mathrm{Sp}(3,R)$ basis. The technology for doing such calculations, within a single irrep in a generator coordinate scheme, has been developed, for example, by Vassanji.[10] Sophisticated shell model calculations in an SU(3) basis, as a prelude to mixed shell $\mathrm{Sp}(3,R)$ calculations, have been developed by Bahri and Draayer.[11]

One of the questions we would like to answer is “what is the relative importance of vorticity and pairing correlations in fixing the dynamics of rotational flows in deformed nuclei?”

4. The vorticity content of a symplectic rotor band

We first recall that a rotor band, in the symplectic model, can be regarded as an SU(3) band that is renormalized by coupling to giant monopole and quadrupole resonance degrees of freedom. Thus a symplectic band is characterized by the U(3) $\sim$ U(1) $\times$ SU(3) quantum numbers $(N\lambda\mu)$ of the associated $\mathrm{SU}(3) \subset \mathrm{U}(3)$ band. The vorticity content of a symplectic band is then expressed in terms of a theorem, given (without proof) several years ago.[12]

Theorem: The vorticity angular momentum $\mathcal{L}$ takes the same values in an sp(3,R) representation $(N\lambda\mu)$ as the ordinary angular momentum takes in an su(3) representation $(\lambda\mu)$; e.g., for λ even and $\mu = 0$, $\mathcal{L}$ takes the values $0, 2, 4, \ldots, \lambda$.

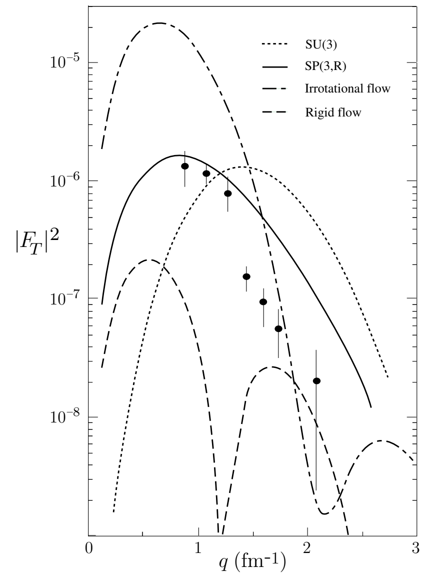

Fig. 5. Transverse form factors for inelastic electron scattering to the first 2^+ rotational state of ^{24}Mg. The experimental results are those of Hotta et al.[13] They are compared with form factors, calculated by Carvalho et al.,[15] for rigid flow, irrotational flow, and in the SU(3) and symplectic models.

For example, the ground state of ^{16}O in the shell model belongs to a symplectic band with

$$(\lambda\mu) = (0,0), \quad \mathcal{L} = 0\,. \tag{13}$$

The flow is irrotational. There are only giant resonance states in the band – no $L \neq 0$ su(3) states and no rotational band. However, the lowest excited state in ^{16}O is the band head of a highly deformed rotational band with

$$(\lambda\mu) = (8,4), \quad \mathcal{L} = 0, 2^2, 3, 4^3, 5^2, 6^3, 7^2, 8^3, 9^2, 10^2, 11, 12\,. \tag{14}$$

Thus, we find that vorticity is essential for the appearence of rotational bands.

It is then of interest to ask if vorticity can be measured? The simplest way to probe nuclear current flows is by means of inelastic electron scattering. However, longitudinal form factors only give information on nuclear densities. To obtain information on currents, one needs transverse form factors and not many have been measured in rotational nuclei. Fig. 5 shows the results of Hotta et al.[13] for ^{24}Mg in comparison to currents calculated for rigid and irrotational flows, and in the SU(3) and symplectic models. The microsocpic wave functions for these calculations were determined by Vassanji et al.[14] One sees that the data lie between the rigid and irrotational flow limits and that they are fit reasonably well in the symplectic model. The latter fit is far from perfect, but this is to be expected because, in a more realistic calculation, one should include contributions from different symplectic irreps, which was not done here.

5. Adiabatic mixing of irreps – quasi-dynamical symmetry

A feature of the above results that is disturbing at first sight is the fact that correct moments of inertia and current flows are given unexpectedly well in the symplectic model without any account being taken of pairing correlations. In view of the long-held view that superfluid flow in a many-fermion system is a consequence of pairing, this needs explanation.

First observe that the whole shell model Hilbert space is a direct sum of symplectic model irreps. This means that a complete set of shell model states can be classified by the quantum numbers of the symplectic irreps to which they belong. We next claim from experience that, if a reasonable Hamiltonian is diagonalized within the space of one symplectic irrep, the states that emerge generally have the structure of a rotational band with giant resonance excitations built upon them.

The effect of a short range (e.g., pairing) force will be to mix different rotor bands. However, if we consider a nucleus which has a well-defined ground state rotational band, we should expect the mixing to be *adiabatic* in the following sense.

Because of centrifugal and Coriolis forces, a rotating frame is not an inertial frame. Consequently, the intrinsic state of a rotational band varies with the rotational angular momentum to a degree which depends on the ratios of rotational and intrinsic excitation energies. However, if the rotational angular velocity is small enough, and if the intrinsic excitation energies of a rotor are large enough compared to rotational energies, a rotating frame may be a good approximation to an inertial frame. The low angular momentum states of the rotor band are then characterized by a common intrinsic state and the rotations are said to be *adiabatic*.

The adiabaticity of nuclear rotational motions underlies the Born-Oppenheimer mechanism for explaining the separation of rotational degrees of freedom. Moreover, it is believed to be essential for the existence of well-defined rotational bands. It is appropriate then that adiabaticity should also have a natural expression in the language of dynamical symmetry. Adiabaticity of a set of rotational states means that all the states (of a subset of rotational states) have a common intrinsic state, which does not change with increasing angular momentum. Thus, even though there may be mixing of rigid-rotor bands by various interactions, the mixing coefficients will be the same for each angular momentum state within the subset of rotational states for which the rotations are deemed to be adiabatic. In other words, the whole set of adiabatic rotational states can be generated (by angular momentum projection) from a single intrinsic state just as if the intrinsic state belonged to a single irrep of the rotor model algebra. The important characteristic of such adiabatic mixing of irreps is that many of the essential simplifying properties of the dynamical symmetry survive the mixing; we say that the model with mixing of irreps has a *quasi-dynamical* symmetry. The concept of a quasi-dynamical symmetry is given a firm mathematical foundation in terms of an *embedded representation*.[16]

If a given rotor band of states is adiabatic then, within the framework of the symplectic model, the corresponding rotational wave functions are of the form

$$\Psi_{\alpha JM} = \sum_{\nu} C_{\alpha\nu}\psi_{\nu JM}\,, \tag{15}$$

where $\psi_{\nu JM}$ is the rotor wave function coming from the symplectic irrep labelled by the index ν and the mixing coefficients $\{C_{\alpha\nu}\}$ are independent of the angular momentum J. We may then enquire into the effect of such adiabatic mixing of irreps on moments of inertia. Let us assume for the moment that rotational energies are basically kinetic. Thus, if the nuclear Hamiltonian is a sum of kinetic and potential energies, we assume that the states of a rotor band are characterized by a common intrinsic state and a common potential energy. The energy differences of states belonging to the same rotor band then come only from the kinetic components of their total energies.

We now observe that the many-nucleon kinetic energy operator

$$T = \frac{1}{2m}\sum_{n} p_n^2 \tag{16}$$

is an element of the sp(3,R) Lie algebra. This means that T has only nonvanishing matrix elements between states of the same irrep. Thus, for adiabatic states of the type given by Eq. (15), the kinetic energies are given by

$$\langle\Psi_{\alpha JM}|T|\Psi_{\alpha JM}\rangle = \sum_{\nu} |C_{\alpha\nu}|^2 \langle\psi_{\nu JM}|T|\psi_{\nu JM}\rangle\,. \tag{17}$$

The implication is that, for adiabatic rotational bands, the moments of inertia are the averages of the values given by the contributing symplectic irreps. Thus, to the extent that the assumption of adiabaticity is valid and rotor wave functions are of the type given in Eq. (15), one does not expect the mixing of representations to qualitatively change moments of inertia. This, of course, presupposes that moments of inertia vary smoothly and slowly with the representation labels, which we know from experience to be the case.

The above argument is based on several assumptions and has weak points. In the first place, nuclear rotational energies may not be purely, or even mostly, kinetic. For example, in the SU(3) model, where the states of an SU(3) band all belong to the same harmonic oscillator shell, one obtains an $L(L+1)$ energy-level spectrum in which all the energy differences come from the potential energy. It would not be unreasonable then to suppose that, in a more realistic symplectic model calculation involving states from higher shells, the rotational energies would have contributions from both the kinetic and potential energies. If such is the case, the concept of a moment of inertia in connection with nuclear rotational bands is misleading and we should revise our intuitive understanding of rotational dynamics. In particular, the successes of the Inglis-Belyaev cranking model[17] and the improved agreements that result from including pair correlations, would have to be regarded as fortuitous. This may well be the case. From the perspective of the symplectic model, if rotational energies come in part from the potential energy, there could be coherent cross terms coming from admixtures of different irreps; moments of inertia might then not be averages of those of the contributing irreps.

6. The pair-coupling model in spherical nuclei

We consider first a simple Hamiltonian of the type

$$H = \sum \varepsilon_\nu a^\dagger_\nu a_\nu - G S_+ S_- \,, \tag{18}$$

where $S_+ = \sum_{jm>0} a^\dagger_{jm} a^\dagger_{j\bar{m}}$ and $S_- = \sum_{jm>0} a_{j\bar{m}} a_{jm}$. By defining the elements of a set of su(2) algebras (one for each j value)

$$S^j_+ = \sum_{m>0} a^\dagger_{jm} a^\dagger_{j\bar{m}} \,, \quad S^j_- = \sum_{m>0} a_{j\bar{m}} a_{jm} \,, \quad S^j_0 = \tfrac{1}{2} \sum_{m>0} (a^\dagger_{jm} a_{jm} - a_{j\bar{m}} a^\dagger_{j\bar{m}}) \,, \tag{19}$$

the above Hamiltonian can be expressed (to within an unimportant constant) in terms of these su(2) algebras

$$H = 2 \sum_j \varepsilon_j S^j_0 - G \sum_{ij} S^i_+ S^j_- \,. \tag{20}$$

Thus, we have a dynamical algebra for H, which is a direct sum of su(2) subalgebras, and we can find the eigenstates and energy levels for H exactly. However, it is useful to consider the pair-coupling approximation in order to exhibit more clearly the structure of the solutions.

First observe that, if all the active single particle states are degenerate in energy, so that H reduces to the simple form

$$H = -G S_+ S_- \,, \tag{21}$$

we have a much simpler SU(2) dynamical symmetry; this is Kerman's quasispin symmetry.[18] Energy eigenstates would then be of the form

$$|n\rangle = (S_+)^n |0\rangle = \left[\textstyle\sum_j S^j_+\right]^n |0\rangle \,, \quad \text{with} \quad S_- |0\rangle = 0 \,. \tag{22}$$

In the pair-coupling model[19,20] it is supposed that, when the single particle energies are nondegenerate, the corresponding states are given approximately by the expression

$$|n(\alpha)\rangle \approx \left[\textstyle\sum_j \alpha_j S^j_+\right]^n |0\rangle \,, \tag{23}$$

where the $\{\alpha_j\}$ coefficients are determined variationally.

Pair-coupled states of the type given by Eq. (23) can be generated by projecting out the component of good particle number from a state of BCS form. Such states were proposed by Mottelson many years ago[21] and have been considered many times. What has been found recently[23] is that the BCS states and pair-coupled states have

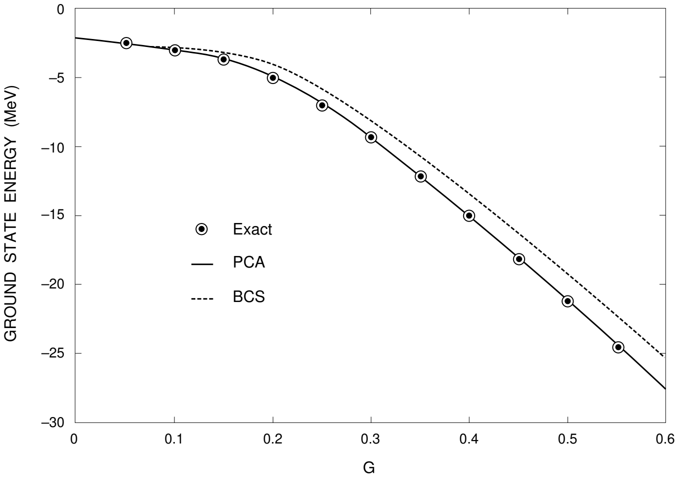

Fig. 6. Comparison of ground state energies, calculated in the BCS and pair coupling models, with exactly computed values.

a particularly simple expression in a coherent state representation which greatly facilitates their manipulation. It has been shown that a BCS state $|\Phi(\alpha)\rangle$ has coherent-state wave function

$$\varphi(z) = \langle 0|e^{S_-(\beta)}|\Phi(\alpha)\rangle = \prod_i (1+z_i)^{2s_i}\,, \tag{24}$$

and that a pair-coupled state $|n(\alpha)\rangle$ has a wave function which is a Schur function (well known in character theory)

$$\varphi_n(z) = \langle 0|e^{S_-(\beta)}|n(\alpha)\rangle = \begin{vmatrix} \phi_1 & 1 & 0 & 0 & \dots & 0 \\ \phi_2 & \phi_1 & 2 & 0 & \dots & 0 \\ \phi_3 & \phi_2 & \phi_1 & 3 & \dots & 0 \\ & & \dots & & & \\ \phi_n & \phi_{n-1} & \phi_{n-2} & \phi_{n-3} & \dots & \phi_1 \end{vmatrix}, \tag{25}$$

where

$$z_i = \alpha_i\beta_i\,, \quad \phi_m(z) = \sum_i 2s_i z_i^m\,. \tag{26}$$

Figs. 6 and 7 show results for the ground state energy and correlation function computed as a function of the pairing strength in the BCS and pair coupling approximations in comparison with exact values. The figures show the pair coupling approximation to be extraordinarily good.

The pair coupling approximation is of particular interest for our purposes because it is an example of a *quasi-dynamical symmetry*. We have seen that Kerman's SU(2) dynamical symmetry is broken when the single particle energies are nondegenerate. However, the structure of states continues to be of the same form, albeit with pair creation operators $\sum_j \alpha_j S_+^j$ which are weighted combinations of su(2) operators.

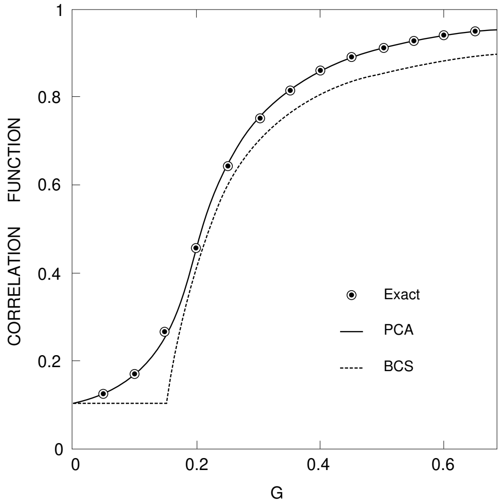

Fig. 7. Comparison of the correlation function, calculated in the BCS and pair coupling models, with exactly computed values.

7. Pairing in rotational nuclei

7.1. Pairing-plus-quadrupole model

To explore the effects of pairing in rotational nuclei, it is useful to consider the so-called pairing-plus-quadrupole model Hamiltonian

$$H = \sum \varepsilon_\nu a^\dagger_\nu a_\nu - \tfrac{1}{2}\chi Q \cdot Q - G S_+ S_- \,, \tag{27}$$

within the space of a single harmonic oscillator shell.

This is an old model which has only been solved approximately or in small spaces. The reason for its intransigence is the incompatible dynamical symmetries associated with its component parts. The $Q \cdot Q$ interaction has an Elliott su(3) dynamical symmetry,[22] whereas the pairing interaction $-GS_+S_-$ has a Kerman su(2) quasispin symmetry. These symmetries are incompatible in the sense that the smallest Lie algebra which contains both su(3) and su(2) is a large fermion-pair algebra. Moreover, using the fermion-pair algebra amounts to a large and impractical shell model calculation. However, we know that deformed nuclei exhibit simply rotational spectra even when pair correlations are important Thus, we are motivated to believe there must be a simple solution to the above problem in the adiabatic limit.

7.2. Quasi-dynamical symmetry models

We have learned that when a rotational band of states is observed (experimentally) to be adiabatic, we may expect its description in terms of mixed rotor bands to be simple, cf. Eq. (15), and to be such that the essential structure of the rotor band survives the mixing. This enables the effects of pairing to be calculated in a rotational SU(3) basis by carrying out the mixing in the intrinsic frame, thereby generating a single intrinsic state from which a whole band of rotational states can be projected. The analog of an intrinsic state in the SU(3) model is the highest weight state. Thus, for the pairing-plus-quadrupole model, it means diagonalizing the Hamiltonian in a basis restricted to highest weight su(3) states. Preliminary investigations, using the shell-model methods of Bahri and Draayer,[11] show that such a treatment would give rather good results. However, the calculations are not that easy and we would like an algebraic way of including pair correlations.

Coming at the problem from the other end, that is adding deformation degrees of freedom to the pair-coupling model, suggests that we consider pair-coupling in a deformed basis along the lines proposed by Sheikh.[24] This approach gives intrinsic wave functions for a rotor model of the form

$$|n+1;\mathrm{int}\rangle = \sum_{\mu\nu} \alpha_{\mu\nu} a^\dagger_\mu a^\dagger_\nu |n;\mathrm{int}\rangle \,. \tag{28}$$

In coming at the problem from this direction, one would be attempting to preserve the essential content of quasispin symmetry.

The obvious way to unify the above two approaches and perhaps the simplest overall solution to the problem is to recognize that rotational bands in the symplectic model are renormalized SU(3) bands and consequently they can be angular momentum projected from an intrinsic state. Similarly, pair-coupled states in spherical nuclei can be number projected, to a high degree of accuracy, from states of BCS form. This suggest that both rotational and pair correlations can be simultaneously handled by both angular momentum and number projecting states from a deformed intrinsic wave functions. However, we need not restrict to intrinsic states which are single Slater determinants or quasi-particle vacuum states as in the Hartree-Bogolyubov approximation.

8. Concluding remarks

The problem of including pair correlations in a microscopically realizable model is fundamentally difficult because of the absence of a simple dynamical symmetry. We have found that rotor and pair-coupling models separately have useful dynamical symmetries but they are incompatible and cannot easily be combined.

In analyzing the problem, the best we have been able to come up with so far are:

- Take limits (e.g. boson limits, group contractions, as in the IBM).
- Use quasi-dynamical symmetry ideas (e.g., adiabatic rotations and quasi su(2) as in a deformed pair-coupling model.).
- Follow the traditional Unified Model/Hartree-Bogolyubov approach of angular momentum and number projecting states from a deformed pair-correlated intrinsic state.

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