Elastic Properties of Solids
(One or two weights)

This is a rare experiment where you will get points for breaking a sample!

The recommended textbooks and other resources:
*Physics for Scientists and Engineers* by R.A. Serway and J.W. Jewett, Volume I, Chapter 12.
*Physics for Scientists and Engineers with Modern Physics* by R.D. Knight, Volume I.

INTRODUCTION

The objective of this lab is to find the relationship between axial (normal) stress and strain for various materials. The Stress-Strain Apparatus stretches (and in some cases breaks) a test coupon while it measures the amount of stretch and force experienced by the test coupon. Software is used to generate a plot of stress versus strain, which allows Young’s Modulus, the elastic region, the plastic region, the yield point, and the break point to be ascertained.

The ratio of the force (F) applied to the cross-sectional area (A) of a material is called the tensile stress:

\[
\text{Stress} = \frac{F}{A} \quad \text{units: Newton per square meter or Pascal (Pa)}
\]

The ratio of the change in length (\(\Delta L\)) to the original length (\(L_o\)) of a material is called the tensile strain:

\[
\text{Strain} = \frac{\Delta L}{L_o} \quad \text{no units}
\]

![Stress-strain curve](image)

Figure 1 Stress-strain curve showing typical yield behaviour of a solid material

EXPERIMENT

The setup includes (see Fig. 2):
- Stress-Strain Apparatus AP-8214 assembled with the Rotary Motion Sensor CI-6538 and a three-step pulley
- Force Sensor CI-6746
- Force sensor attachment
- A set of three test coupons (sample stripes) of two different metals and plastic
- Calibration bar
- Tee handle with a socket for screw tightening
- Caliper
- Micrometer

In this experiment, a test coupon will be placed under the coupon clamps. A Rotary Motion Sensor is used to encode the translational motion from the micrometer, and one full turn of the micrometer (2π or 360 degrees) gives 1 mm of linear displacement. The lever arm translates the force on the coupon to the force sensor proportionally. Setup the apparatus as shown (without a test coupon) in Fig. 2, and connect the sensors to the interface. You need to check RMS orientation as follows. Start test recording and observe the values of “Angle” measurements. Turn the crank clockwise, and if the value of angle decreases, flip the polarity of the RMS sensor connection to PASCO Universal Interface.

**PROCEDURE**

**FIG. 2. Stress-Strain apparatus with a coupon installed**
Weight I
Apparatus Calibration

During the experiment, as the crank turns, force will be applied to the test coupon, causing it to stretch. However, this force will also cause the apparatus platform and the Force Sensor to bend. The displacement registered by the RMS will be the combination of the coupon stretching and the rest of the apparatus bending.

Regardless of how much the coupon stretches, the deformation of the rest of the apparatus is constant for a given force. One can measure this deformation directly by using the calibration bar (which does not stretch significantly) in place of a coupon as force is applied. The goal is to find the apparatus bending coefficient using the calibration bar, where the displacement is due only to bending of the apparatus. After recording the displacement where a coupon is tested, one can isolate the effect from bending of the apparatus using this coefficient, and find the displacement that is due only to stretching of the coupon.

Use the following steps to find the apparatus bending coefficient:

1. **Mount the Calibration Bar.** Remove the nuts and clips from the apparatus platform (Figure 2). Turn the crank to adjust the position of the bolts and slip the bolts through the holes in the calibration bar. *Do not replace the nuts when using the calibration bar.*

2. **Place the lever arm in the starting position.** Turn the crank counter-clockwise and pull the lever arm away from the Force Sensor (Figure 3).
3. **Set data recording conditions in Capstone.** It is most convenient to set automatic start and stop conditions for data recording in this experiment. Set the Recording Conditions (located in the same palette of the Record button) with measurement based conditions. For example, you can start recording when the force sensor reads values larger than 1.0 N, and stop recording once the value reaches 50.0 N, which is the maximum value that the force sensor can measure.

4. **Plot Displacement versus Force.** Press the Tare button on the Force Sensor. Use the Calculator tool to create a Displacement variable to be RMS Angle/2 π in units of mm. Start recording in Capstone. Turn the crank clockwise (when looking from the right-hand side) at a very slow but steady speed and observe the displacement vs force graph in Capstone. Once the force reaches the value specified (e.g. 50.0 N) in Recording Conditions, Capstone automatically stops recording. **Stop turning the crank once the force reaches 50.0 N as over-tightening can damage the apparatus!** You can rename the data run as "Calibration" in Data Summary.

5. **Calculate the apparatus stretch coefficient.** Use the Data Highlight tool and Curve Fit tool on the Displacement vs Force graph and find the apparatus bending coefficient in appropriate unit.

**Software Setup**

Use the Calculator tool to calculate the stress and strain.

To find the stress, you need the applied force and cross-sectional area of the coupon under test. There is a mechanical advantage due to the lever for the measured force. Measure the lever arm for the applied force and measured force to find this mechanical advantage, so you can find actual applied force from measured force. Find the sample’s width and thickness using a caliper. You can use the following equations in the Calculator tool:

\[
\text{Stress} = \frac{\text{Applied force}}{\text{Coupon thickness} \times \text{Coupon width}}
\]

\[
\text{Applied force} = \frac{\text{Lever arm length}^2}{\text{Lever arm length}^1} \times \text{Measured force}
\]

You can input the thickness, width, lever arm lengths as parameters, and use your data for the variable of measured force.

Similarly, you can find the strain from the stretch data and coupon length measurements (for the narrow part). As the micrometer gives 1 mm of displacement for every full turn, you can find the displacement from the RMS angle data.

\[
\text{Strain} = \frac{\text{Calibrated stretch}}{\text{Coupon length}}
\]

\[
\text{Calibrated stretch} = \frac{\text{Measured stretch} - \text{Apparatus stretch coefficient} \times \text{Measured force}}{\text{RMS Angle in rad}/2 \pi}
\]

You can input the apparatus stretch coefficient you find from the calibration step above and the coupon length as parameters, and use your data for the variables of measured force and RMS angle.

**Data collection**
1. **Mount a coupon.** Remove the calibration bar and restore the clips and nuts. Place one end of the coupon under one of the clips. Start with one of the plastic coupons. Adjust the crank so that the opposite end of the coupon can slip easily under the other clip. Tighten both nuts with the wrench. With no force applied to the coupon, as little twist as possible should be visible in the coupon. *The clips should hold the coupon tightly enough that it will not slip when force is applied. However, over-tightening the nuts will damage the bolts. Use your best judgment. If in doubt, err on the side of under-tightening* (Figure 5).

![Figure 5: coupon attachment](image)

2. **Place the lever arm in the starting position.** Turn the crank counter-clockwise and pull the lever arm away from the Force Sensor (Figure 3).

3. **Collect Data.** Press the Tare button on the Force Sensor. Click the Start button. Wait until the digits of the experiment clock turn yellow. Turn the crank clockwise. Starting just before the lever arm comes into contact with the Force sensor, turn the crank very slowly. When finished collecting data, click Stop. (If the maximum force is reached, DataStudio will stop automatically.) If the coupon breaks, it should break in the middle. If the coupon breaks near the end, it was probably twisted slightly when it was mounted, resulting in a point of higher stress where it broke.

4. **Rename the data run to identify the coupon.** Use the same method used to rename the calibration data.

Test two other coupons made of different material or thickness. **As the supply for the coupons are limited, make sure you are familiar with the data collection process, and make all the necessary measurements before you put the coupons in the apparatus.** Enter the appropriate values for the calculation of Stress and Strain in the Calculator dialog for the new coupons. Continue with data collection.

**Data Analysis**

- On the Stress versus Strain graph, identify and record the elastic region, the plastic region, the yield point, and the break point (if available). To calculate Young’s modulus, select a data region covering the linear, lower left-hand part of the graph. (The very first part of the plot may not be linear. This nonlinearity likely is due to the straightening of bends and
twists in the coupon as force is first applied. Do not include this region in the selection.)
Apply a linear curve fit to the selected data. The slope of the line is Young's modulus in
units of MPa (or MN/m² or N/mm²). Record Young’s Modulus. Using data for Young’s
modulus for different materials from http://www.engineeringtoolbox.com/young-modulus-
d_417.html, and determine the substance the coupon was produced of.

QUESTIONS

1. Define mechanical stress in your own words and include a sketch. Discuss what is
   physically happening to a coupon when it is experiencing stress.

2. Define mechanical strain in your own words and include a sketch. Discuss what is
   physically happening to a coupon when it is experiencing strain.

3. Sketch a sample graph of stress v. strain from your data. Identify different regions. Discuss
   the relationship between stress and strain in these regions.

4. Does the coupon that can withstand the greatest force also experience the greatest stress?
   Explain.

5. Does the coupon that can withstand the greatest force also experience the greatest strain?
   Explain.

6. Do the graphs agree with Hooke’s Law? Explain.

Weight II
Shape and volume deformation measurement.

You will perform this part of the experiment using brass coupons of two different thicknesses:
0.003 in and 0.005 in.

For given four coupons, measure their thickness \( t_i \), width \( w_i \) and length \( L_i \). Compare your
results with the coupon specifications. The default value for \( L_i \) is 75 mm. Calculate the volume
\( V_i \) of the piece of the coupon that will be stretched in the experiment. Record the four obtained
values in your notebook.

Study the relationship between the Young’s modulus and the thickness of a sample. Do it in
two steps for each thickness of brass coupons.

- In first step, stretch two coupons of different thickness exactly as in the Exercises II.
- Then, stress the other two coupons below the breaking point, but still in elastic and
  plastic regions.

For the coupons that have not been broken, measure their average thickness \( t_f \), average width
\( w_f \) and calculate the \( L_i + \Delta L \), obtained from your graph. Find the volume of the coupon been
stretched \( V_f \). Compare the results with those obtained before stretching the coupon. Make
notes on what quantities have been changed due to stress and whether the shape of the coupon
has also been changed.
QUESTIONS

1. Compare the brass coupons of different thicknesses with respect to their Young’s Modulus and yield points. Include an explanation at the atomic level.

2. If a coupon is stretched beyond its yield limit, but not beyond its break point, will its Young’s Modulus change? Explain.

3. Describe what would happen to the Stress-Strain graph if a coupon were heated. Explain.

**Poisson’s ratio.**

In this exercise, use the coupons that were not broken in exercise III. As it is mentioned in Introduction, the deformation in one dimension causes the change in sizes in other dimensions of a solid rod. E.g. the elongation of a rod fixed between two holders can result in contraction of the rod in the perpendicular direction. Figure 8, taken from http://www.e-sunbear.com/biomech_05.html, demonstrates the width contraction accompanying the length increase.

The coupons in this experiment are cut of sheet metal and have a shape that is very different from the uniform rod or a wire. Therefore, the exact solutions for uniform rods are not applicable to the coupons. However, it is still possible to estimate the other elastic modulus called *Poisson’s ratio*. If the force is applied to the coupons as in exercises I – III, it can be considered a force that causes the *axial strain* (e.g. force $F$ in Fig. 8), which will be denoted as $\varepsilon_{\text{axial}}$ (positive for axial tension, negative for axial compression). This strain is accompanied by the transverse strain $\varepsilon_{\text{trans}}$ (negative for axial tension (stretching), positive for axial compression). The Poisson’s ratio is given by

$$
\nu = \frac{d\epsilon_{\text{trans}}}{d\epsilon_{\text{axial}}}
$$

Replacing the differential values of strains by the finite changes in the values of length (the axial strain), width and thickness (the transverse strain), derive a formula for the Poisson’s ration for the coupons. Estimate the Poisson’s ratio for brass coupons.

1. Is the result positive or negative?
2. How can you explain that the Poisson’s ratio can be of both signs?
3. Most materials have Poisson's ratio values ranging between 0.0 and 0.5. A perfectly incompressible material deformed elastically at small strains would have a Poisson's ratio of exactly 0.5. Why does it equal to 0.5?
4. Explain the following idea of an author of the article posted here

**FIG.: 8.** Width contraction as a result of elongation under axial stress
“You might think that the way to measure the elastic modulus of a material would be to apply a small stress (to be sure to remain in the linear-elastic region of the stress-strain curve), measure the strain, and divide one by the other. In reality, moduli measured as slopes of stress-strain curves are inaccurate, often by a factor of 2 or more, because of contributions to the strain from material creep or deflection of the test machine. Accurate moduli are measured dynamically: by exciting the natural vibrations of a beam or wire, or by measuring the velocity of longitudinal or shear sound waves in the material”.

http://www.grantadesign.com/education/sciencenote.htm