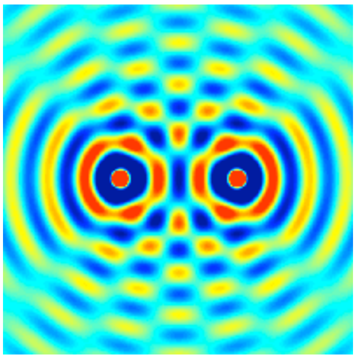
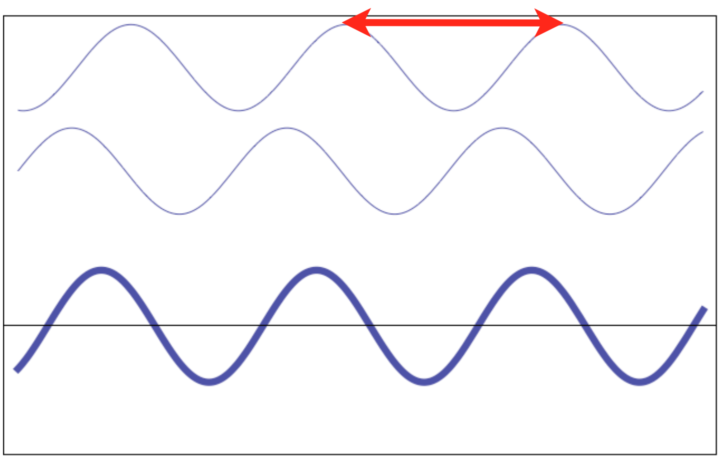
Waves and Interference



“Wavelength = distance between two peaks (or troughs)



Frequency = “how many waves per second”
= 1/ Period = 1/ “how many seconds between waves”

Velocity = distance / time
= wavelength / period
= wavelength * frequency

Units, dimensions, scaling

Linearity:

$$P = mv$$

$$(2m)\ v = 2P$$

$$m\ (2v) = 2P$$

Units / dimensions: meters, seconds, kilograms, meters / second (m/s)

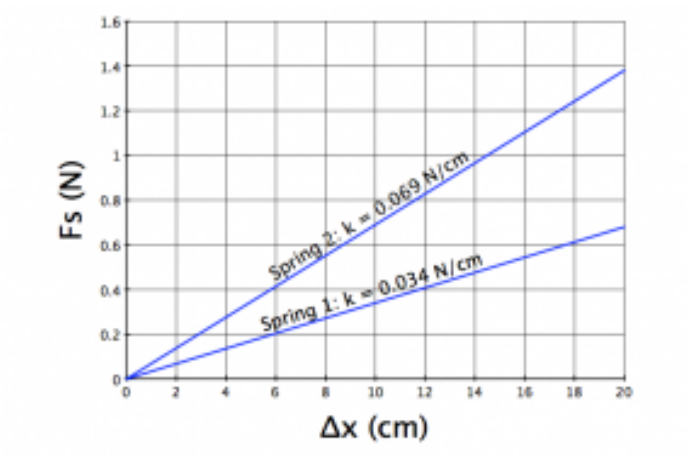
constant velocity = 1 m/s = (10 m) / (10 s)

Travel 10 times as long -> go 10 times as far

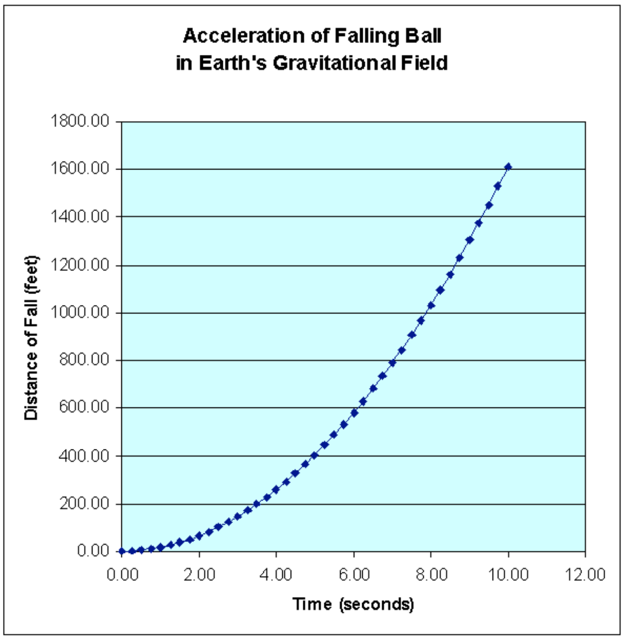
constant acceleration = 1 m/s² = (100 m) / (10 s)²

Travel 10 times as long -> go 10² = 100 times as far

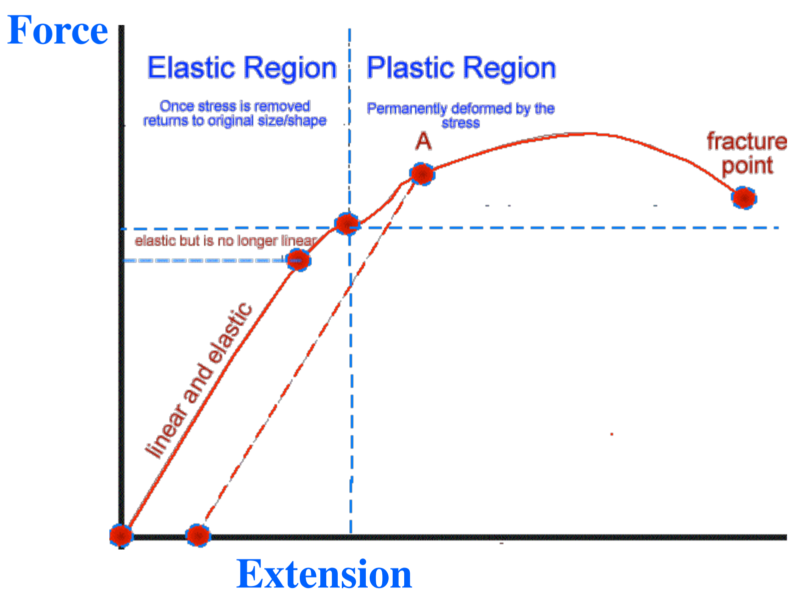
"Linearity" & dimensional analysis



Something nonlinear



More realistic "springs"
(rubber bands, etc): LINEAR vs NONLINEAR



Velocity = distance / time
= wavelength / period
= wavelength * frequency

Frequency = N Hertz or N “cycles per second”:
units of “per second” (1 / time)

Velocity = m / s
= meters * (1/seconds)
= wavelength * frequency

If you triple the velocity but leave the frequency constant, what happens to the wavelength?

It’s moving faster, yet no more waves are passing you? They must be further apart (3 *).

$\underbrace{\text{Velocity}}_{*3} = \underbrace{\text{meters}}_{?} / \underbrace{\text{second}}_{*1}$

Double the force / acceleration:

instead of 10 m / s^2, go 20 m/s^2

Distance (meters) = accel (m/s^2) * time-squared (s^2)

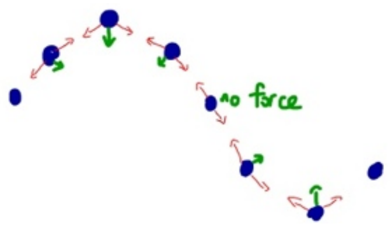
If I could quadruple the force of gravity, how much further would you fall in 3 seconds?

If I could quadruple the force of gravity, how much more or less time would it take you to fall 10 meters?

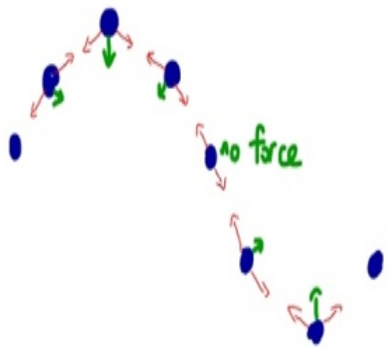
Distance (meters) = accel (m/s^2) * time-squared (s^2)

fix distance *4 /4

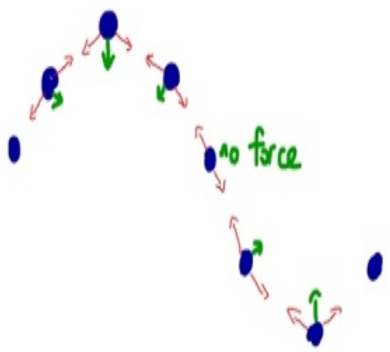
If time-squared 4* smaller, time is 2* smaller.



Suppose I stretch the string twice as far. LINEARITY: twice the force.



Has twice as far to go; feels twice the force; how much longer does it take to get back to where it started?



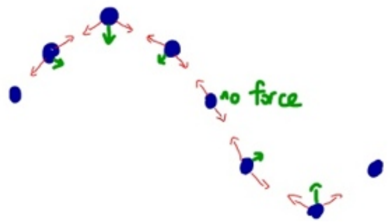
Distance (meters) = accel (m/s^2) * time-squared (s^2)

***2 *2 ?**

-> the frequency is independent of the amplitude

What if I keep the same shape, but
increase the tension in the rope?

Bigger force, but the same distance to go -> shorter time
(higher frequency)



Suppose I made a shorter wavelength.

Recall, a net force on a given bead
because of *difference* between beads to L & R;
shorter wavelength = bigger change *per unit distance*
-> bigger force

Half the wavelength -> 4* the force -> 4* the acceleration

Multiply (m/s^2) by 4 by dividing (s^2) by 4 or (s) by 2;
halve the period, or double the frequency.

Conclusion (waves in strings)

If you halve the wavelength, you double the frequency: velocity = wavelength * frequency is constant.

Velocity of sound (in string)

Tightening the string increases the force and hence the velocity (and since wavelength is set by the length of the string, this increases the frequency)

Back to Maxwell

$\underset{\sim}{E}$ acts on charges

↓

they move (current, I)

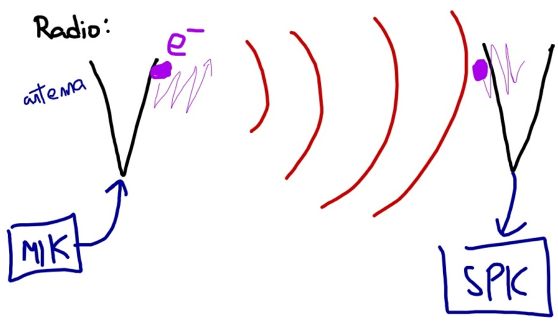
↓

this creates mag. field $\underset{\sim}{B}$

↓

changing $\underset{\sim}{B}$ creates $\underset{\sim}{E}$

self-sustaining cycle



How big is the "force"? (What is the frequency?

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

These equations allowed him to figure out how quickly a variation in E (with a given lengthscale) would "relax" (and then oscillate).

As with strings, he found the frequency was proportional to one over the wavelength.

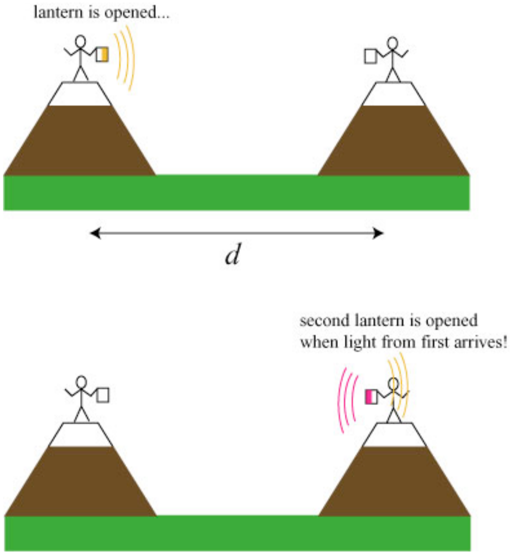
What do we conclude?

Velocity = frequency * wavelength
is proportional to (1/wavelength) * wavelength = constant.

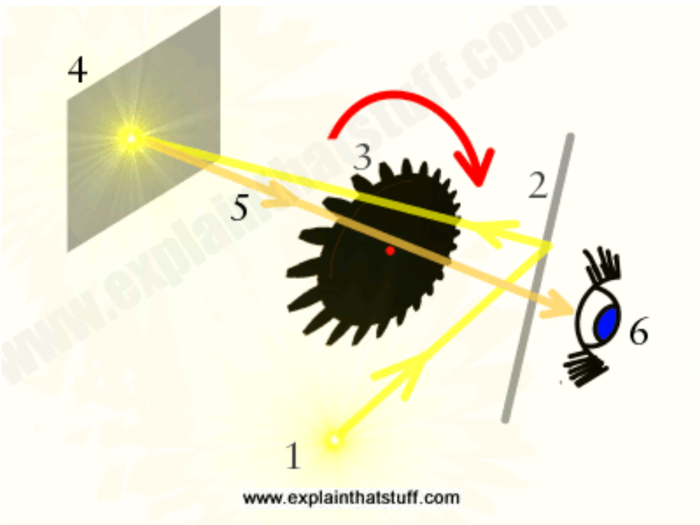
And he could calculate this constant from the known electrical and magnetic constants measured by Oersted, Ampere, Faraday, ...

Maxwell predicted that em waves could be produced, and would travel at a velocity of 300,000 km/s.

But wait!

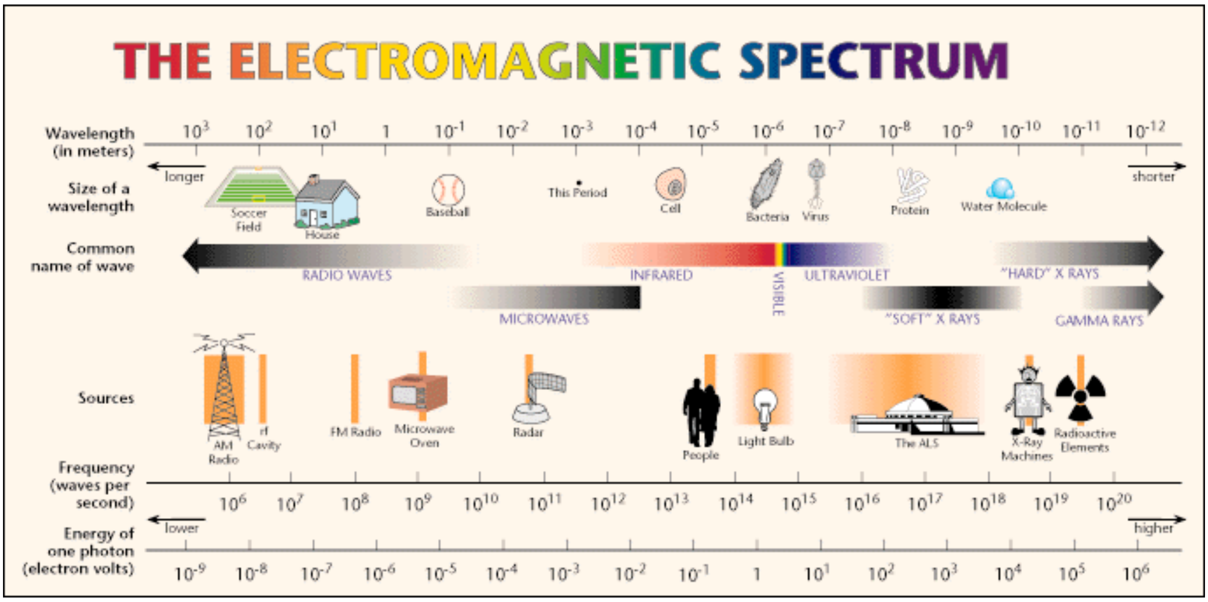


(...actually more like this)



Conclusion: Maxwell invented light

Light is an em wave



(So: are electric & magnetic fields "real," or are they just helpful ways of thinking about what forces things *would feel* in the right places?)

Odd history (brief recap)

c.1700:
Huygens thought light was a wave
Newton thought this was ridiculous
(moves in straight lines, not like sound)

1807: Young observes two-slit interference
Conservatives still not convinced.
1818: Fresnel explains diffraction using wave theory

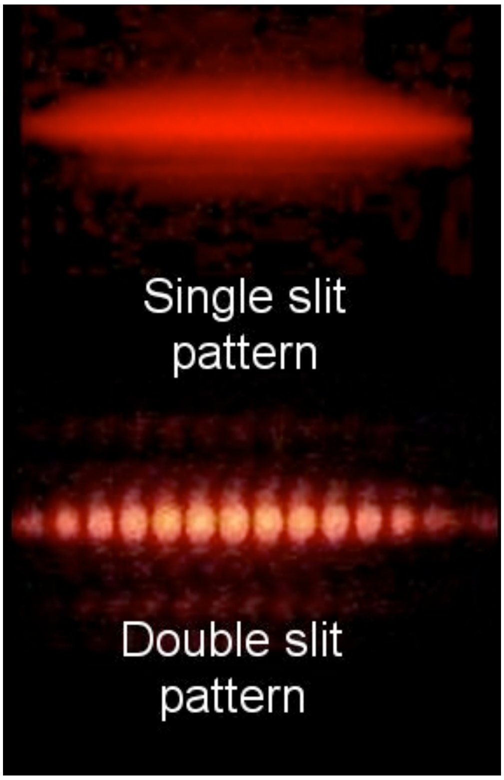
Poisson points out that if Fresnel were right, constructive interference would make bright spots at the centres of shadows

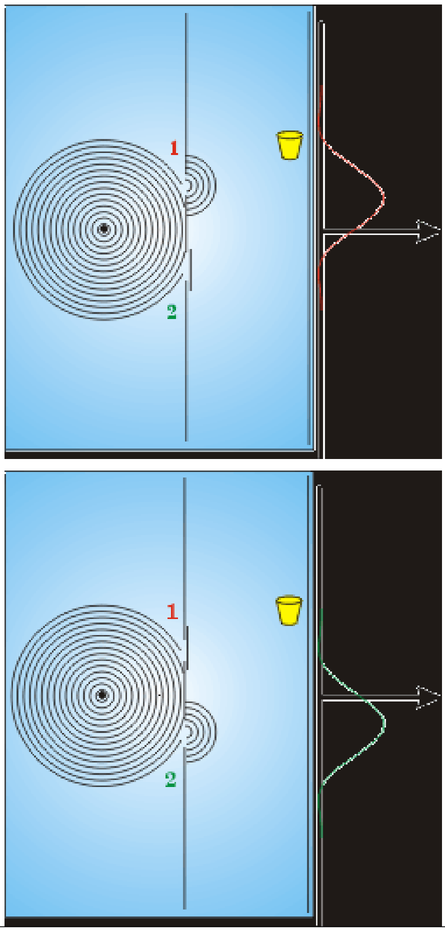
Arago checks – there *are* bright spots at the centres of shadows!

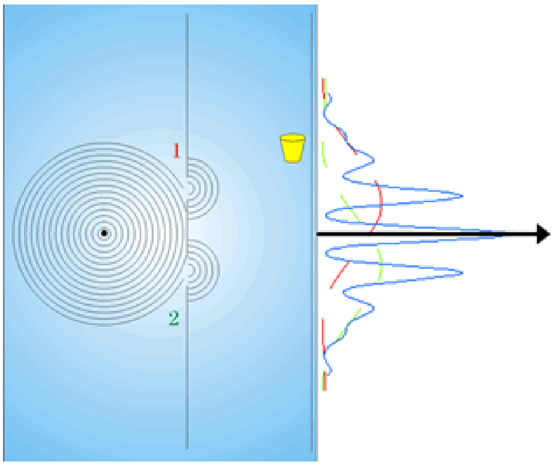
Why hadn't any one noticed?!

The wavelength is tiny (< 1/1000 of a mm).
This means lines *almost* straight,
and Arago spot very dim (unless object tiny),
and interference hard to see (unless slits tiny),
.... and people are just conservative.

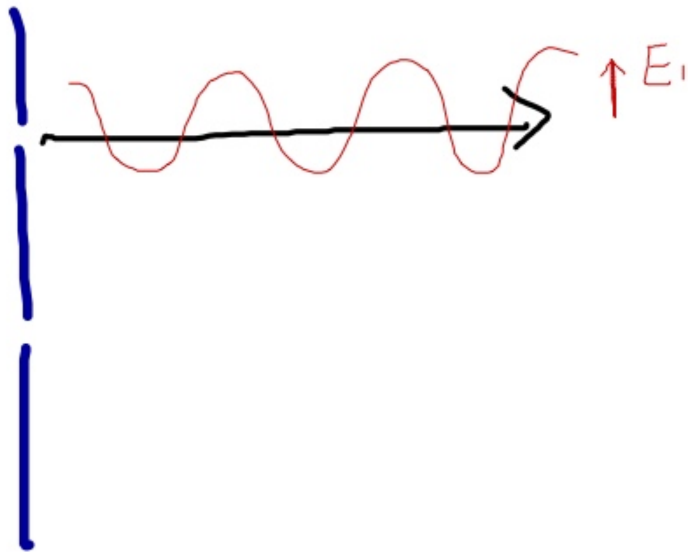
Also: what then is doing the waving?
(Until Maxwell, who would have guessed light had anything to do with electricity?)


Single slit
pattern
Double slit
pattern

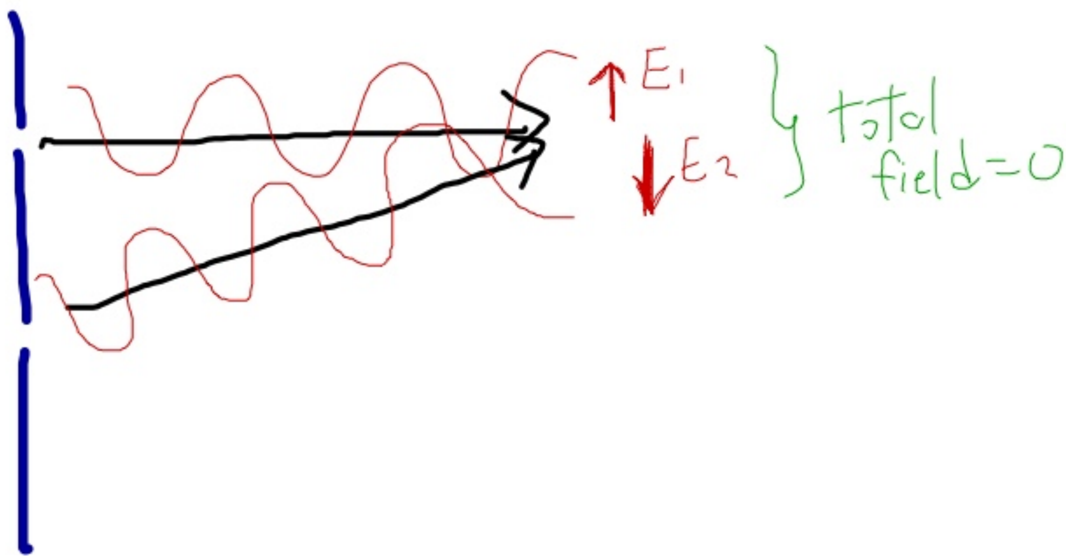

1
2
1
2


1
2

Light really is electric fields, which point places



Light really is electric fields, which point places



Interference is persuasive evidence that light is a wave.