Collapse versus correlations, EPR, Bell Inequalities, Cloning

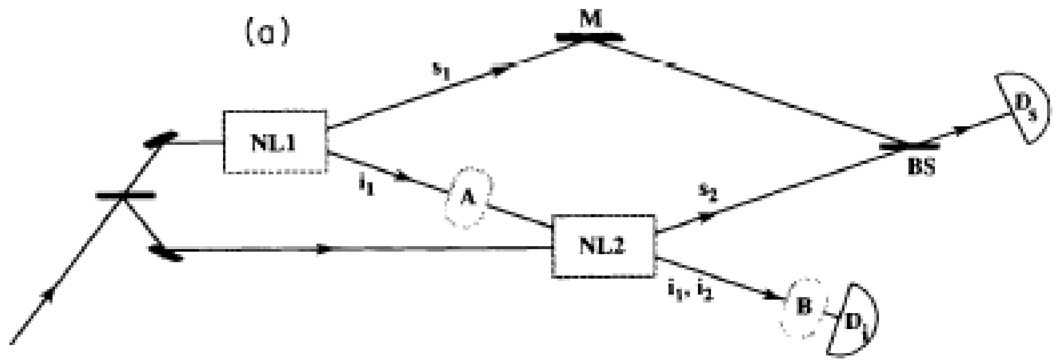
- The Quantum Eraser, continued
- Equivalence of the collapse picture and just blithely/blindly calculating correlations
- EPR & Bell
- No cloning

Lecture 4: 24 Jan 2012

--first--

- Three reading lists posted on web site
- First problem set posted on web site

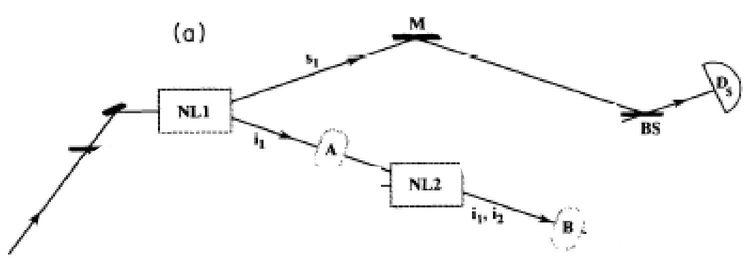
Second: any questions about the Zou-Wang-Mandel “induced coherence” experiment?

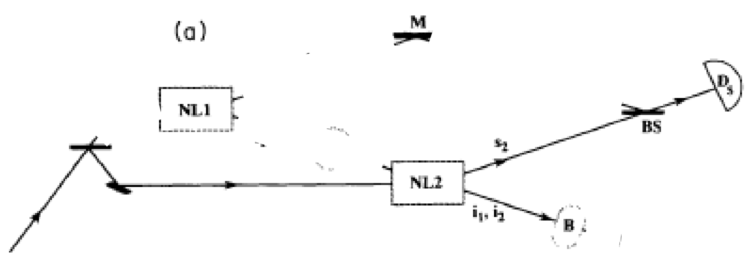


Zou, Wang, Mandel,
PRL 67, 318 (1991).

Like most experiments, this one can be confusing *if you think about it the wrong* way (e.g., semiclassically), but is simple & unambiguous if you remember the Feynman rules.

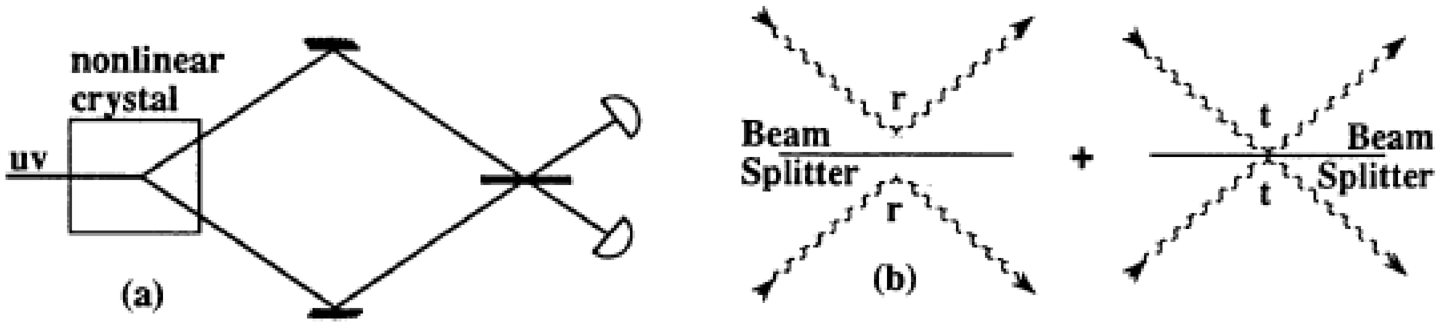
Final state: D2 fires and there is a photon at B (Di is irrelevant) – there are indeed two paths which lead to precisely this final state, with no distinguishing information left:





the prototypical two-photon interference effect:

the Hong-Ou-Mandel interferometer



$t^2 + r^2 =$
$1/2 - 1/2 = 0$;
no coincidence
counts.

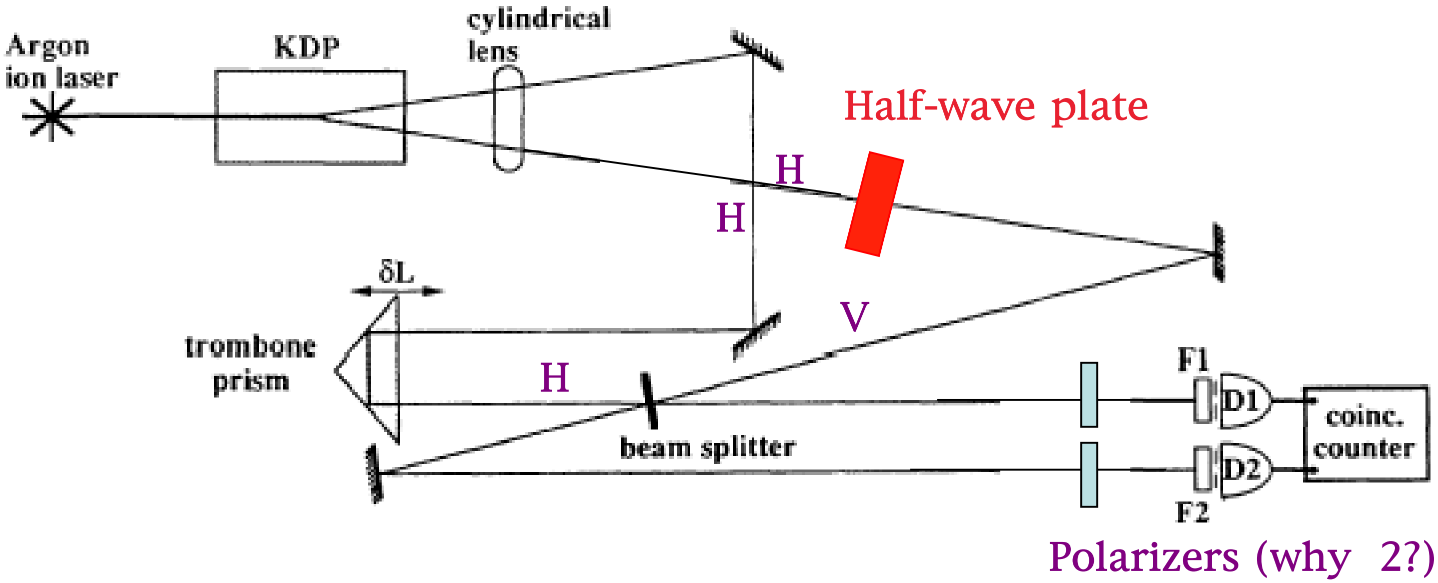
Remember: if you detect only one photon, the *other* photon "knows" where yours came from. Hence there is no interference (each detector sees 1/2 of the photons, irrespective of any phases or path-length differences).

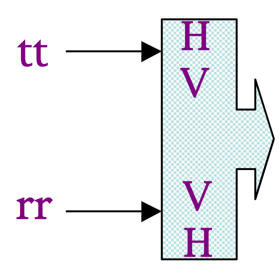
But: if you detect *both* photons, there is no way to tell whether both were reflected or both were transmitted. $r^2+t^2 = (i^2 + 1^2)/2 = 0$.
(any lossless symmetric beam splitter has a $\pi/2$ phase shift between r and t.)

CAVEAT: there must *be* no way to tell which occurred.
If the paths aren't aligned right, no interference occurs.
If one photon reaches the beam splitter before the other, no interference occurs.

Dirac: two photons never interfere with each other; each photon interferes only with itself.
Mandel (after Feynman): *or*, one photon *pair* can interfere with *it*self!

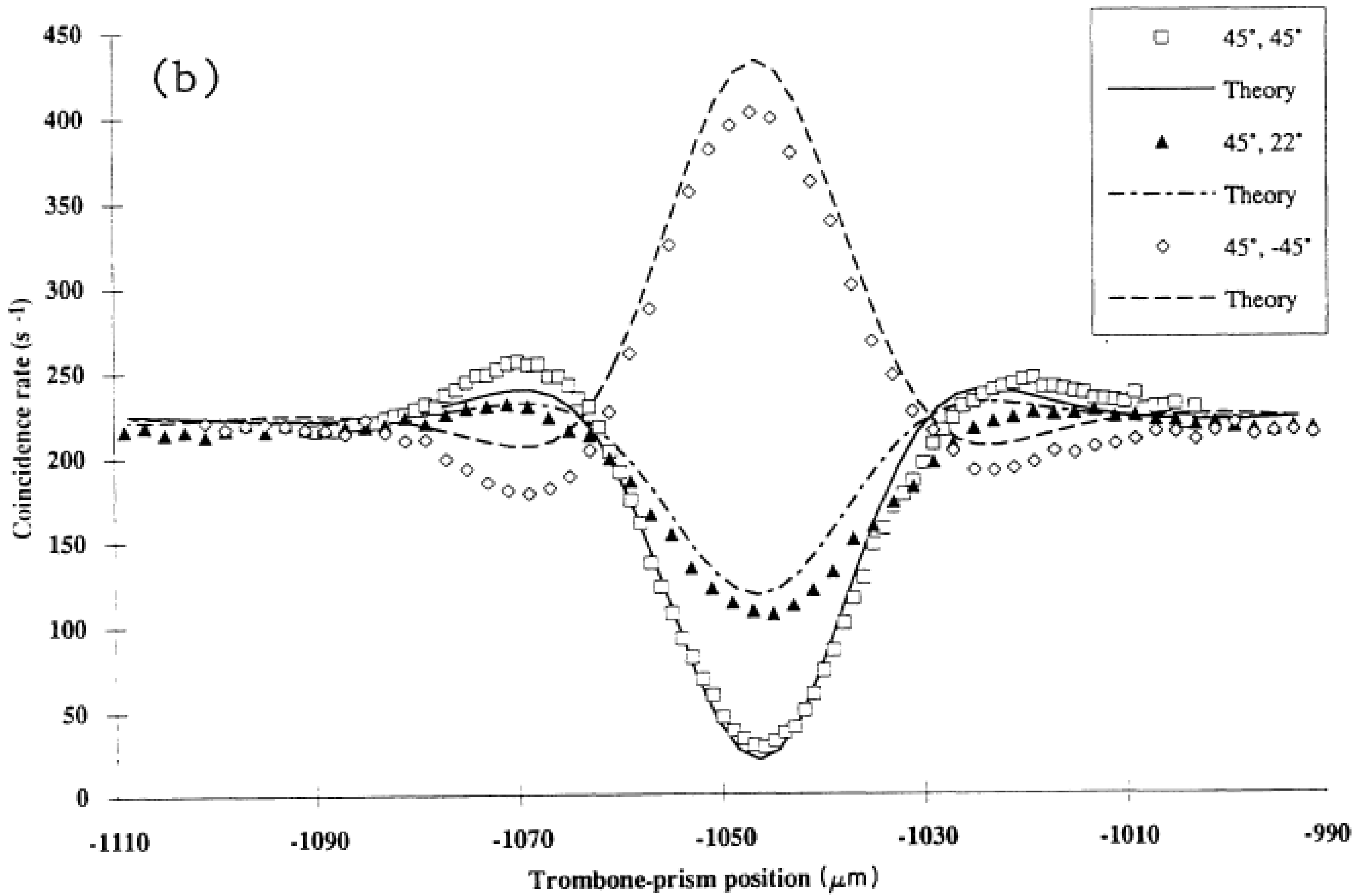
The polarisation quantum eraser





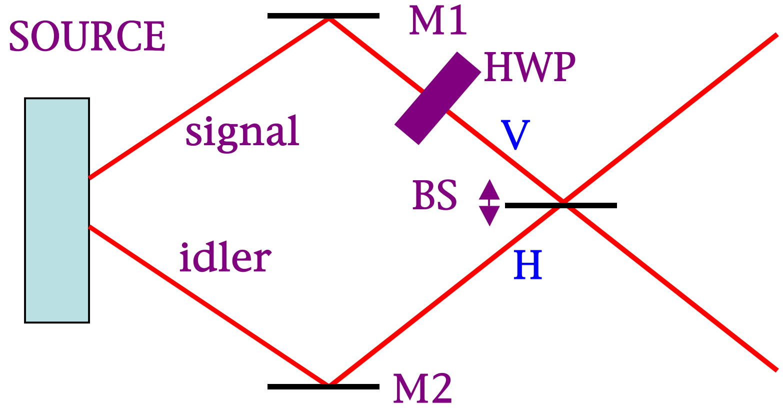
distinguishable;
no interference.

And coming back again!



Simple collapse picture

|HV> – |VH>



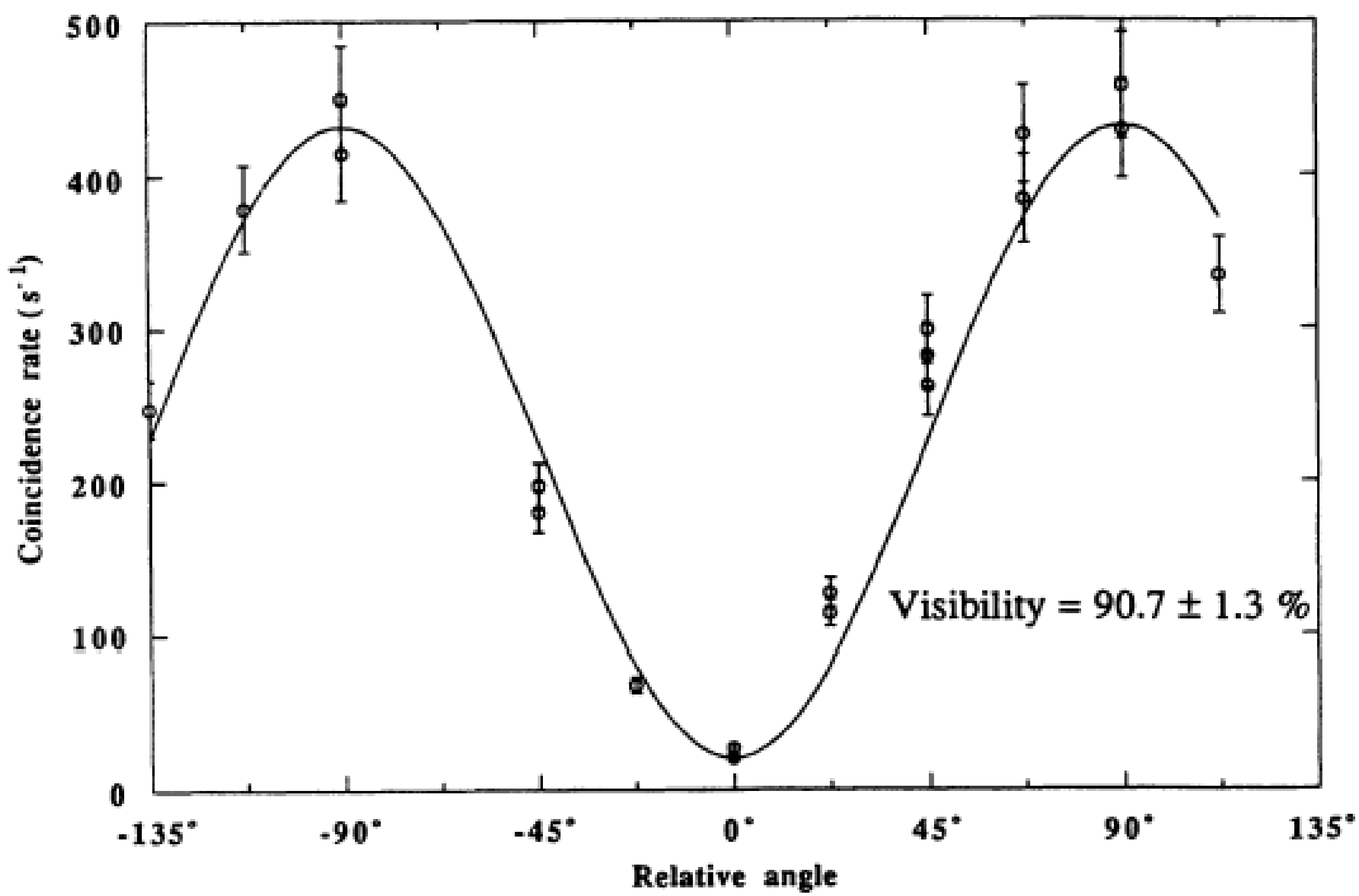
Suppose I detect a photon at θ here. This collapses my photon into H cos θ + V sin θ.
This means an amplitude of cos θ that the *other* photon was V, and of sin θ that it was H.
Being careful with reflection phase shifts, this collapses the other output port into V cos θ - H sin θ, which of course is just $(\theta + \pi/2)$.

Here I'm left with a photon 90^0 away from whatever I detected. Now I just have linear optics to think about.

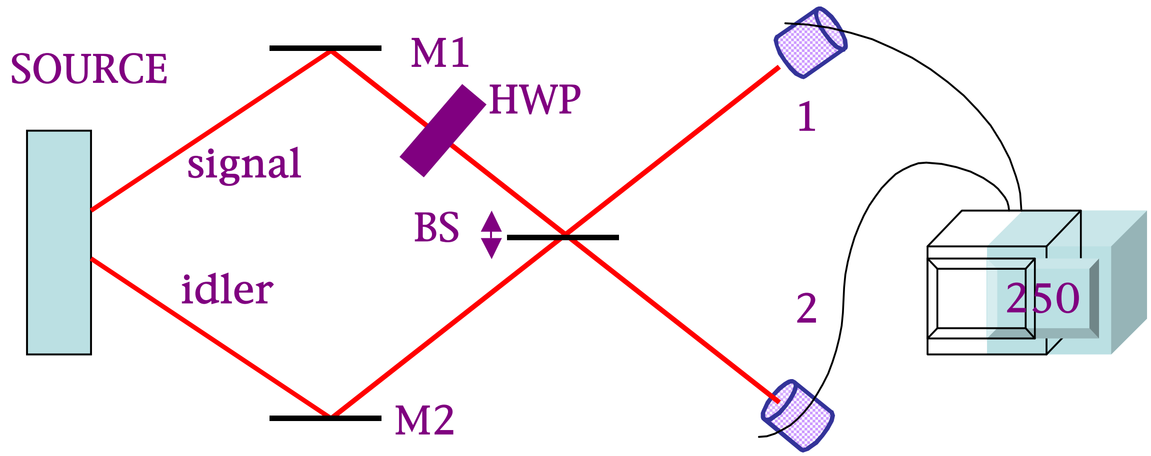
Of course I get sinusoidal variation as I rotate this polarizer.

"...and experiment is for those who don't trust their calculations."

Polarisation-dependence of rate at centre of H-O-M dip...



But did I *need* to invoke collapse?
(and if so, which photon did the work?)



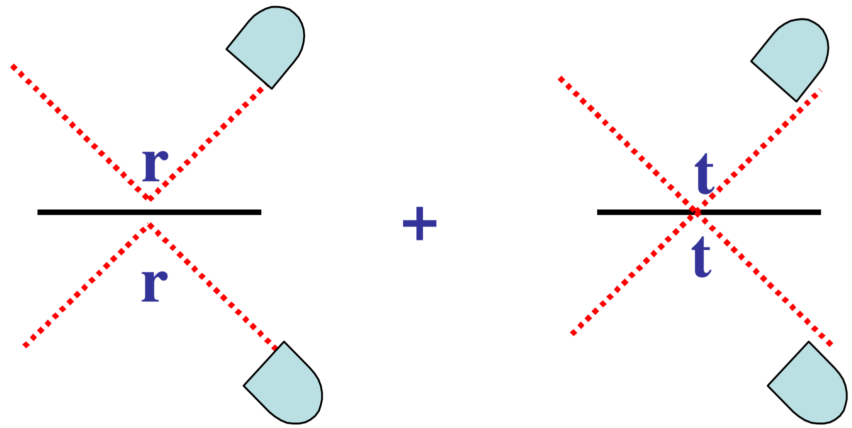
Vs Hi $\longrightarrow$ (V2 + i V1) (H1 + i H2)
= 1H 2V - 1V 2H + i [1H 1V + 2H 2V]

In coincidence, only see |HV> - |VH> .... that famous EPR-entangled state.
Of course we see nonlocal correlations between the polarisations.

These joint-detection probabilities can be calculated directly, without collapse; add the amplitudes from HV and VH: $P(\theta_1,\theta_2) = |\cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2)|^2$
$= \sin^2(\theta_1 - \theta_2)$.

This *is* the Bell-Inequality experiment done by Shih&Alley and Ou&Mandel.

Hong-Ou-Mandel Interference as a Bell-state filter



$r^2+t^2 = 0$; total destructive interf. (if photons indistinguishable).

If the photons begin in a symmetric state, no coincidences.

{Exchange effect; cf. behaviour of fermions in analogous setup!}

The only *antisymmetric* state is the singlet state |HV> – |VH>, in which each photon is unpolarized but the two are orthogonal. Nothing else gets transmitted.

This interferometer is a "Bell-state filter," used for quantum teleportation and other applications.

More Bohr-Einstein debates

Einstein:
I can't believe God plays dice with the universe.

Bohr:
Albert, stop telling God what to do.

Einstein, Podolsky, & Rosen (1935)

Source

Alice **Bob**

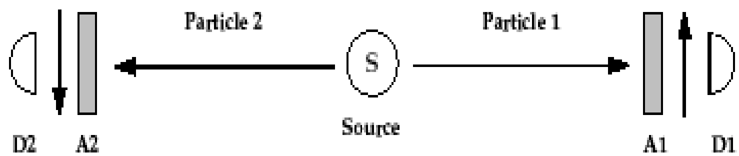


FIG. 1. Bohm's version of the EPR Gedankenexperiment

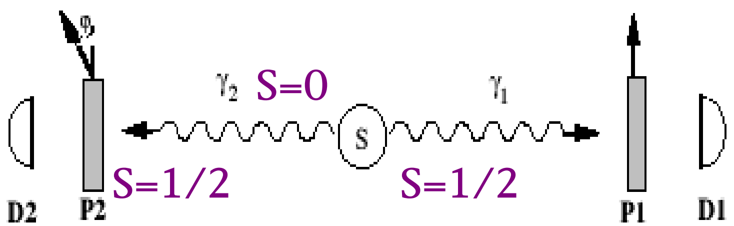


FIG. 2. Optical version of EPR experiment

2 particles emitted together at the same time with opposite speeds.

If Alice measures her particle's position, she knows Bob's. But if she measures her particle's momentum, she knows Bob's.

Did her measurement "affect" Bob's particle instantaneously?

Spooky action at a distance

Or did Bob's particle already have both?

Hidden variables (QM "incomplete")

Schrödinger 1935:

"entanglement"

"Verschränkung" (SP?)

$$|\psi\rangle = |B\rangle_L |W\rangle_R + |W\rangle_L |B\rangle_R$$

(In-)compatible observables

We all know you can't know both X & P because $[X,P] = ih$. (Broken hbar in this font, and I'm too lazy to do this in LaTeX right now, but you're all too well educated to be confused by this...

$[X_1,P_1] = ih$ $[X_1,P_2] = 0$

$[X_2,P_2] = ih$ $[X_2,P_1] = 0$

$$
\begin{aligned}
[X_1-X_2 , P_1+P_2] &= [X_1,P_1] + [X_1,P_2] - [X_2,P_1] - [X_2,P_2] \\
&= ih + 0 - 0 - ih \\
&= 0
\end{aligned}
$$

$X_1 - X_2$ and $P_1 + P_2$ are compatible, as are $X_1 + X_2$ and $P_1 - P_2$; on the other hand, $X_1 + X_2$ and $P_1 + P_2$ are incompatible as are $X_1 - X_2$ and $P_1 - P_2$.

You can know the position *difference* and the momentum *sum*, without nevertheless knowing either position or either momentum!

The EPR state: $\Psi = \delta(x_1-x_2)$ clearly has $x_1 - x_2 =0$, and it's easy enough to take the derivatives and see that $P_1 + P_2 =0$ as well.

Bell points out that its Wigner function is $W(x_1, x_2, p_1,p_2) = \delta(x_1 - x_2)\delta(p_1+p_2)$.
{and that this amounts to a hidden-variable model; see the references!}

Hidden variables?

Einstein seems to have thought the particles "knew" what they were going to do, even if we didn't: QM not wrong but "incomplete".

John Bell's example, "Bertlmann's socks":

Bell's Theorem

Forget Quantum Mechanics.

Suppose you've got two particles, and A & B can choose what to measure on each of them – "color" or "dirtiness", for example. For each measurement, they either get "1" or "0". If there are "hidden variables," then A's choice doesn't affect B, and vice versa – from this alone, you can prove something.

Independence: $P(A\&B) = P(A) \bullet P(B)$

Correlation due only to a common cause:

$$P(A\&B \mid \lambda) = P(A \mid \lambda) \bullet P(B \mid \lambda);$$

note that the full $P(A\&B) = \Sigma\, P(A\&B \mid \lambda)\, P(\lambda) \neq P(A) \bullet P(B)$ in general.

Bell's version of Einstein locality:

if A controls parameter setting a and B controls parameter setting b, then

$P(A\&B \mid a,b) = \Sigma\, P(A\&B \mid a,b,\lambda)\, P(\lambda)$ [for some unknown $P(\lambda)$, of course],

but $\quad P(A\&B \mid a,b,\lambda) = P(A \mid a, \lambda) \bullet P(B \mid b, \lambda);$

B cannot depend on a, and A cannot depend on b (although A & B may both depend on the common cause λ).

The content of Bell's Theorem: this already leads to a contradiction with QM!

Reading about EPR-Bell

First off, I've already recommended Bell's book of reprints (*Speakable and unspeakble in quantum mechanics*), as well as Wheeler & Zurek's collection *Quantum Theory and Measurement*. These are wonderful sources. But here are some specific articles:

The EPR "paradox" was published in Einstein, Podolsky, & Rosen, PR 47, 777 (1935).

Bell's theorem was published in Physics 1, 195 (1965); the "Bertlmann's socks" version appears both in his book and in Journal de Physique 42, C2-41 (1981).
His claim that the original EPR state *cannot* violate a Bell inequality appears in the book and in "EPR correlations and EPW distributions," in *New Techniques and Ideas in Quantum Measurement Theory* (Ann. NY Acad. Sci, 1986). *{What about the Franson exp't, then?!}*

The first *testable* form of Bell's inequalities was derived in Clauser, Horne, Shimony, and Holt, PRL 25, 880 (1969);
and a form closer to the one I hand-wave here appears in Clauser & Horne, PRD 10, 526 (1974).
(I learned this proof from Philippe Eberhard, and I believe it's the one orginally due to Stapp, as you can read about in the Clauser-Shimony review below.)

A nice review of the both the theory (various idealized and less-idealized forms of the inequalities) and the early experiments is in Clauser & Shimony, Rep. Prog. Phys. 41, 1881 (1978), including the pioneering experiment by Freedman & Clauser, PRL 28, 938 (1972).

The later experiments by Aspect are often considered to have been the most conclusive, and appeared in Aspect, Grangier, & Roger, PRL 47, 460 (1981) and Aspect, Dalibard, & Roger, PRL 49, 1804 (1982).

Many more generalized Bell-inequality experiments have been done since, and some but not all are referred to in the review articles listed on the course web page.

Some recent ones include Salart, Baas, Branciard, Gisin, & Zbinden, Nature 454, 861 (2008); Rowe, Kielpinski, Meyer, Sackett, Itano, Monroe, & Wineland, Nature 409, 791 (2001); etc.