Frequency and time... dispersion-cancellation, etc.

(AYA: An old experiment of mine whose interpretation helps illustrate this collapse-vs-correlation business, and which will serve as a segue into time & phase msmt)

- Dispersion cancellation in an HOM interferometer
  - (more "collapse versus correlations")
  - (useful for time measurements)

- What are time measurements?
  - (no time operator)
  - (indirect measurements)
  - (energy-time "uncertainty relation")

- States of an electromagnetic mode
  - (number-phase "uncertainty relation")
  - (homodyne measurements, et cetera)
  - Phase of a single photon...
Entangled photon pairs  
(spontaneous parametric down-conversion)

- A pump photon is spontaneously converted into two lower frequency photons in a material with a nonzero $\chi^{(2)}$

The time-reverse of second-harmonic generation.  
A purely quantum process (cf. parametric amplification)  
Each energy is uncertain, yet their sum is precisely defined.  
(For a continuous-wave pump!)  
Each emission time is uncertain, yet they are simultaneous.  
(What does this remind you of?)
Hong-Ou-Mandel interferometer

Remember: if you detect only one photon, the other photon "knows" where yours came from. Hence there is no interference (each detector sees 1/2 of the photons, irrespective of any phases or path-length differences).

But: if you detect both photons, there is no way to tell whether both were reflected or both were transmitted. \( r^2 + t^2 = (i^2 + 1^2)/2 = 0. \) 
(any lossless symmetric beam splitter has a \( \pi/2 \) phase shift between \( r \) and \( t \).)

CAVEAT: there must be no way to tell which occurred. If the paths aren't aligned right, no interference occurs. If one photon reaches the beam splitter before the other, no interference occurs.

How long is a photon?
In every experiment to date, the width of this feature is limited only by the bandwidth of the photons; in other words, the photons are as tightly correlated as they could possibly be given their own uncertainty in time ($\Delta t > 1/2\Delta \omega$).

$$|\Psi\rangle = \int d\omega' f(\omega')|\omega_0 + \omega'\rangle_s |\omega_0 - \omega'\rangle_i$$

Instead of an amplitude for each frequency component of each beam, there is an amplitude for each frequency-correlated pair of photons. Energy-entangled state.
What's the speed of a photon?

Silly questions about group velocity, phase velocity, "collective" nature of the index of refraction, precursors, et cetera.
More serious questions about how to quantize electromagnetic fields in a dispersive medium.
Longstanding debate about "superluminal" tunneling.

\[ |\Psi\rangle = \int d\omega' f(\omega') |\omega_0 + \omega'\rangle_s |\omega_0 - \omega'\rangle_i \]
Problem with propagation measurements

Quadratic term leads to group-velocity dispersion, broadening, chirp.
But wait!

\[ |\Psi\rangle = \int d\omega' f(\omega') |\omega_0 + \omega'\rangle_s |\omega_0 - \omega'\rangle_i \]

Detection probability is (assumed to be) proportional to photon number \( \hat{n} \equiv a^\dagger a \).

\( a(\omega) \) is the operator equivalent of the classical field amplitude \( \overline{E}(\omega) \), which is the Fourier transform of \( E(t) \).

Similarly, the quantum field amplitude \( E^+(t) \) is a Fourier transform of \( a(\omega) \):

\[ E^+(t) = E_0 \int_0^\infty \frac{a(\omega)}{\sqrt{2\pi}} e^{-i\omega t} \]

Probability is the square of amplitude: \( P(\omega) = \langle a^\dagger(\omega)a(\omega) \rangle \) and \( P(t) = \langle E^-(t)E^+(t) \rangle \).

As \( a(\omega) \) can be interpreted as giving the amplitude to detect a photon of frequency \( \omega \), so \( E^+(t) \) can be interpreted as giving the amplitude to detect a photon at time \( t \).

The amplitude to detect two photons is given by \( aa \); note that the probability \( \langle n|a^\dagger a^\dagger aa|n\rangle = n(n-1) \), not \( n^2 \): you can’t detect two photons if \( n = 1 \).
So, what can we predict?

\[ |\Psi\rangle = \int d\omega' f(\omega')|\omega_0 + \omega\rangle_s |\omega_0 - \omega\rangle_i \]

Detectors are "infinitely slow" (ns = 10^6 fs);...

Coinc. probability is an integral over all times at which D1 and D2 could fire:

\[ \int_0^T dt_1 \int_0^T dt_2 \langle \Psi | E_1^- (t_1) E_2^- (t_2) E_1^+ (t_1) E_2^+ (t_2) |\Psi \rangle \]

UGH! Each E is an integral over its own frequency! But... if T goes to infinity, life simplifies: We have only to integrate \( a^\dagger a^\dagger a a \) over all \( \omega_1, \omega_2 \).

The physical meaning: calculate the probability for each pair of frequencies which might reach the two detectors, and then integrate.

Why? No interference between paths leading to different frequencies at the detectors, because in principle one could go back and measure how much energy had been absorbed.

Note: it took a long time-integral to enforce this. If the detector had been open only for 1 fs, it would be impossible to tell what frequency it had seen.
No, we're not done yet...

The probability of detecting a given frequency pair:

\[ \langle \Psi | a_1^{\dagger} (\omega_1) a_2^{\dagger} (\omega_2) a_1 (\omega_1) a_2 (\omega_2) | \Psi \rangle = \sum_n \langle \Psi | a_1^{\dagger} (\omega_1) a_2^{\dagger} (\omega_2) | n \rangle \langle n | a_1 (\omega_1) a_2 (\omega_2) | \Psi \rangle \]

(one can always insert a complete set of states.)

But we only started with 2 photons, so if we annihilate 2, there will be 0 left:

\[ = \langle \Psi | a_1^{\dagger} (\omega_1) a_2^{\dagger} (\omega_2) | 0 \rangle \langle 0 | a_1 (\omega_1) a_2 (\omega_2) | \Psi \rangle \]
\[ = \| \langle 0 | a_1 (\omega_1) a_2 (\omega_2) | \Psi \rangle \|^2 \] (the square of the two-photon amplitude)
\[ = \frac{1}{2} \langle 0 | [a_1 (\omega_1) a_s (\omega_2) e^{i \omega_1 \delta l / c + i k (\omega_2) d} - a_s (\omega_1) a_1 (\omega_2) e^{i \omega_2 \delta l / c + i k (\omega_1) d} ] | \Psi \rangle \| ^2 \]
\[ = \frac{1}{2} \delta (\omega_p - \omega_1 - \omega_2) f (\omega') e^{i \omega_0 \delta l / c + i k_0 d} [ e^{i \omega_1 \delta l / c + i d (\alpha')^2} - e^{- i \omega_1 \delta l / c + i d (\alpha' + \beta \omega')^2} ] \]

the phase difference is all that concerns us:

\[ 2i \omega' \delta l / c - 2i d \alpha \omega' \propto i \omega' (\delta l / c - \alpha d). \]

Note 0: all that math did nothing but get us back to Feynman’s rule!
Note 1: the GVD terms (quadratic in freq.) cancelled out.
Note 2: the pattern moves at the group velocity.
Note 3: the shape is the Fourier transform of f(\omega), like the pulse itself.
Nonlocal cancellation of dispersion

(Oh, and by the way...
yeah, single-photon pulses travel at the group velocity.)
REMEMBER: interference only occurs between two paths which yield the same $\omega_1$ and the same $\omega_2$.

If in the TT path, a "blue" photon is detected at D1 (and a "red" at D2), then I must compare with the phase of an RR path with the same outcome...

The two paths are indistinguishable even though in each case one of the pulses was broadened. Perfect interference occurs.

Note: this is only possible because we have assumed a cw (continuous) pump; otherwise, the time of detection of a blue photon would tell us whether it had travelled along R or T
After D1 fires, it projects the light in arm 2 into a superposition of two identical chirped wave packets—these two packets exhibit perfect interference.
Clock Synchronization with Dispersion Cancellation

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The dispersion cancellation feature of pulses which are entangled in frequency is employed to synchronize clocks of distant parties. The proposed protocol is insensitive to the pulse distortion caused by transit through a dispersive medium. Since there is cancellation to all orders, also the effects of slowly fluctuating dispersive media are compensated. The experimental setup can be realized with currently available technology, at least for a proof of principle.

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Demonstration of Dispersion-Canceled Quantum-Optical Coherence Tomography

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We present an experimental demonstration of quantum-optical coherence tomography. The technique makes use of an entangled twin-photon light source to carry out axial optical sectioning. It is compared to conventional optical coherence tomography. The immunity of the quantum version to dispersion, as well as a factor of 2 enhancement in resolution, is experimentally demonstrated.

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FIG. 4. QOCT and OCT normalized interferograms for a 90-µm fused-silica window buried beneath two cascaded 5-mm-thick windows of highly dispersive ZnSe. As shown at the
Classical Analogues of Two-Photon Quantum Interference

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Chirped-pulse interferometry (CPI) captures the metrological advantages of quantum Hong-Ou-Mandel (HOM) interferometry in a completely classical system. Modified HOM interferometers are the basis for a number of seminal quantum-interference effects. Here, the corresponding modifications to CPI allow for the first observation of classical analogues to the HOM peak and quantum beating. They also allow a new classical technique for generating phase super-resolution exhibiting a coherence length dramatically longer than that of the laser light, analogous to increased two-photon coherence lengths in entangled states.

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Quantum-optics experiments demonstrated a wide range of interference phenomena that had never before been seen in classical systems. Prominent examples include automatic dispersion and aberration cancellation [1–3], phase-insensitive interference [4], nonlocal interference [5,6], ghost imaging [7] and ghost diffraction [8], phase super-resolution [9–11], and phase supersensitivity [12–15]. Some of these phenomena form the basis for applications in quantum computing and metrology that promise to outperform their classical counterparts in terms of speed and precision, respectively. Recently, ghost imaging [16,17], automatic dispersion cancellation [18,19], phase super-resolution [20], and phase-insensitive interference [19] have been observed in classical optical systems exploiting correlation, but not entanglement. Chirped-pulse interferometry (CPI) [19] is a new, completely classical technique producing the same interferogram as a Hong-Ou-Mandel (HOM) interferometer [4] based on frequency-entangled photon pairs, but with vastly higher signal. It has cancellation, phase insensitivity, and robustness against loss, rendering it a promising tool for quantum metrology and imaging [1,28,29]. We have recently demonstrated chirped-pulse interferometry [19], a completely classical

FIG. 1 (color online). Two-photon interferometers and their
Observation of Nonlocal Modulation with Entangled Photons

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We demonstrate a new type of quantum mechanical correlation where phase modulators at distant locations, acting on the photons of an entangled pair, interfere to determine the apparent depth of modulation. When the modulators have the same phase, the modulation depth doubles; when oppositely phased, the modulators negate each other.

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Subjects: Quantum Physics (quant-ph)
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NOTE: the slow detectors were important!

Fast detector -> broad feature in the presence of dispersion (too much information)

Slow detector -> narrow feature in the presence of dispersion

FIG. 5. These eight graphs are of the coincidence rate $P_c$ in arbitrary units vs the optical delay $\tau$, integrated over several different hypothetical electronic gate windows. They are nu-