From Tunneling Atoms to Thermalizing Photons to Peering at Binary Stars: a few examples of how coherence enables quantum measurement science

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DRAMATIS PERSONÆ
Toronto quantum optics & cold atoms group:
 photons: Hugo Ferretti Edwin Tham Noah Lupu-Gladstein
 atoms: Ramon Ramos David Spierings Isabelle Racicot
 Atom-Photon Interfaces: Josiah Sinclair Shaun Pepper Alex Bruening
 Theory: Aharon Brodutch TBD: Arthur Pang


Some helpful theorists:
Daniel James, Pete Turner, Robin Blume-Kohout, Chris Fuchs, Howard Wiseman, János Bergou, John Sipe, Paul Brumer, Michael Spanner…
Quantum Games with Photons & Atoms

Quantum Data Compression, weak measurement, Q tomo, Q metrology,…

Electromagnetically-induced transparency, Rydberg atoms, quantum light-matter interfaces

Bose-Einstein condensation & weak measurements of tunneling times

Crossing the beams!
Quantum Games with Photons & Atoms

Crossing the beams!

Quantum Data Compression, weak measurement, Q tomo, Q metrology,…

Positions available for postdocs and graduate students
(Full funding guaranteed for students accepted to PhD programme in Toronto)
Imaging as a Quantum State Discrimination problem
  • Using coherence in the image plane to evade “Rayleigh’s curse”  (theory: Mankei Tsang et al.)

Improving thermometry by making use of coherence
  • (theory: Terry Rudolph et al.)

Using atomic coherence to study tunneling times, and play other fun games
  • The Larmor clock
  • Weak measurements
  • Experimental progress (Fabry-Perot for atoms?)
Mini-Pão 1: Better imaging as an optimal quantum discrimination problem
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Better imaging as an optimal quantum discrimination problem
Toy problem: imaging a binary star
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As we all know, if objects separated by less than ~width $\sigma$ of the PSR (diffraction limit), we can’t “resolve” them

... of course, that’s not to say that with enough data, we can’t tell there are two objects there, and where they are...
Toy problem: imaging a binary star

How well can we estimate the separation $s$ of two objects, for $s < \text{width } \sigma$ of PSR, given $N$ photons?

$\sigma / \sqrt{N}$ for $N$ photons would seem reasonable?
No such luck!

$\sigma / \sqrt{N}$ is indeed how well you can find the centre of one object.

But two closely separated gaussians just look like a slightly broader gaussian – the problem is to estimate the width, which proves much harder.
How well can you estimate a width?

\[ V = \langle x^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i^2 \]

Uncertainty \( \Delta V \) can be calculated from \( \Delta V^2 = \overline{V^2} - \overline{V}^2 \).

\[ V^2 = \frac{1}{N^2} \sum_i \sum_j x_i^2 x_j^2 \]

For \( i \neq j \), \( x_i^2 x_j^2 = \sigma^4 \)

For \( i = j \), \( x_i^2 x_j^2 = 3\sigma^4 \)

\[ \overline{V^2} = \frac{1}{N^2} \left\{ N^2 \sigma^4 + 2N \sigma^4 \right\} = \sigma^4 + \frac{2}{N} \sigma^4 \]

\[ \overline{V}^2 = \sigma^4 \]

\[ \Delta V = \sqrt{\frac{2}{N} \sigma^4} \]
How well can you estimate a separation?

\[ \Delta V = \sqrt{\frac{2}{N} \sigma^4} \]

\[ V = \sigma^2 + s^2 \]

\[ \Delta s = \Delta V / \frac{dV}{ds} = \frac{\sigma^2 \sqrt{2/N}}{2s} = \frac{\sigma^2}{s} \sqrt{\frac{1}{2N}} \]

The uncertainty in \( s \) does not merely remain large (\( \sigma / \sqrt{N} \)) as \( s \to 0 \); it actually diverges as \( 1/s \)!
The Fisher information drops to 0 — the error of any unbiased estimator of $s$ goes to infinity.
Where is the missing information?
The classical F.I. considers only *intensity*…

Our problem is to distinguish various possible states of $N$ photons – e.g., them in two spots separated by $s$ or two spots separated by $s+\delta$.

Single spots are easy:

$$\langle 0 | \delta \rangle \approx e^{-\delta^2}$$
(almost identical)

$$\langle 0^N | \delta^N \rangle \approx e^{-N\delta^2}$$
(nearly orthogonal, if $\delta > 1/\sqrt{N}$)

For two incoherent sources, the 2-spot distinguishability is essentially the same as the 1-spot distinguishability… how to optimally distinguish?
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This becomes a quantum state discrimination problem

\[ |\delta\rangle = |0\rangle + \delta |+\rangle + \ldots \]

Project onto this TEM01 mode to determine \(\delta\) (for small \(\delta\), all the info)
For two incoherent sources, the 2-spot distinguishability is essentially the same as the 1-spot distinguishability... how to optimally distinguish?

This becomes a quantum state discrimination problem

\[
\left| \delta \right\rangle = \left| \phi \right\rangle + \delta \left| - + \right\rangle
\]

SPLICE:
Project onto any odd-parity mode, not necessarily TEM01 in particular —
Error analysis for odd-mode projection (“SPLICE”)

\[ P \propto s^2 = \kappa s^2 / \sigma^2, \text{ with } \kappa \text{ a numerical constant of order unity.} \]

NOTE: For two incoherent spots displaced symmetrically from the origin, the overlap with the odd mode is exactly the same as for one spot or the other.

Signal (photon counts) \[ C = N \kappa s^2 / \sigma^2, \text{ with binomial fluctuations } \rightarrow \sqrt{C} \]

\[ \Delta s = \Delta C / \left( ds / dC \right) = \sqrt{N \kappa s^2 / \sigma^2} / 2N \kappa s / \sigma^2 = 1 / 2\sqrt{N \kappa} \]

INDEPENDENT OF \( s \) – for TEM01, retrieve \( \sigma / \sqrt{N} \)
Projecting a double-spot onto an odd-parity mode

Source of symmetric variable-separation pair of spots

heralded single photons
Fisher Information of various schemes to estimate separation of point sources

“IPC” (image-plane counting)

Tsang et al.’s “SPADE,” simplified for optimized behaviour at small separation

Our Superresolved Position Localisation by Inversion of Coherence about an Edge

In an experimentally simple setup, obtain about 64% of the full Fisher Information for small separations – crucially, independent of s.

Observed vs. actual separation

**SPLICE**
- Approx 1500 photons / image
- rms width of PSF $\sim 400\mu m$

**IPC**
- Approx 3000 photons / image
- rms width of PSF $\sim 400\mu m$

SD in inferred separation, vs. $s_{\text{actual}}$

“divergence” for IPC

near-quantum-limited for SPLICE

This is not an unbiased estimator!

Approx 1500 photons / image

Approx 3000 photons / image

But this “calibration curve”'s slope goes to 0 as s gets small; any attempt to invert it will cause the errors to diverge.

rms width of PSF ~ 400µm
CONCLUSION: We have shown that a simple phase-mask technique removes the 1/s catastrophe, and permits us to achieve near-quantum-limited resolution, providing an unbiased estimator with $\sigma/N^{1/2}$ resolution, yielding a quadratic-in-N advantage over even the best biased estimator possible with image-plane counting.

With about 1500 photons, SPLICE determined the separation 3 times more accurately than IPC could with about 3000 photons.
Mini-Pão 2: Thermometry as an optimal quantum discrimination problem
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Typical thermometry

Let energy of thermometer (small test object – in the extreme case, a single spin) equilibrate with that of sample… consider trying to distinguish two possible temperatures

At least for bosonic baths, a higher temperature means both a higher equilibrium state and a faster collision (equilibration) rate!

For high enough temperatures (w.r.t. the fundamental energy scale of the thermometer), it may be more effective to probe the rate than the steady state: as in this curve, maximum distinguishability achieved at finite times.

But (for energy relaxation), $T_2 = 2T_1$

A thermometer which is initialized with coherence takes longer to reach this optimum time…

W.K. Tham, H. Ferretti, A.V. Sadashivan, & AMS, 1609.01589
Experimental results

NB: minimum error is always achieved at finite t, not asymptotically; at most times, minimum error is achieved by using a coherent input.

W.K. Tham, H. Ferretti, A.V. Sadashivan, & AMS, 1609.01589
Thermometry (that’s just the beginning)

For more details, see W.K. Tham, H. Ferretti, A.V. Sadashivan, and A.M. Steinberg, 1609.01589; for a simultaneous work, Luca Mancino, Marco Sbroscia, Ilaria Gianani, Emanuele Roccia, and Marco Barbieri, 1609.01590.

Single-spin thermometer theory:

Proposal for universal quantum-channel simulator:

See Marco Barbieri’s paper above for interesting thoughts about free energy; see ours for extensions to adaptive protocols for thermometry with a few qubits; see Higgins, B., Doherty, A., Bartlett, S., Pryde, G. & Wiseman, H. PRA 83, 052314 (2011) for the relevant theory behind the latter.

One example where global optimum requires coherence
Mini-Pão 3: Towards weak measurements of atomic tunneling times
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Watching a particle in a region it’s “forbidden” to be in

How long has the transmitted particle spent in the region?
Need a clock...
Two components mystified Büttiker; Feynman approach led to complex times, which mystified every one; It turns out these are weak values, whose Real and Imaginary parts are easily interpreted – but which hadn’t been invented yet.
Local “Larmor Clock” – how much time spent in any given region?

\[ \tau = \frac{\theta_{\text{rot}}}{\omega_l} \]

- In plane rotation measures the tunneling time
- Spin aligns along z axis; back-action of the measurement.
Where does a particle spend time inside the barrier?

Very little time in the center of the barrier!

But – unlike the reflected particles – the transmitted ones “see” the region near the exit!

Conditional-probability “movie” of tunneling
One possible experimental sequence

- BEC in magnetic trap
One possible experimental sequence

- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms
One possible experimental sequence

- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms
- Interaction with barrier
One possible experimental sequence
Atoms spilling *around* an optical “ReST” trap
Localized (fictitious) magnetic field (Raman coupling of two ground states)
Experimental sequence: current plan

Barrier

Crossed dipole trap

Raman beams
Our first observation of single-barrier tunneling

S. Potnis, R. Ramos, K. Maeda, L.D. Carr, AMS, 1604.06388; see also earlier work, e.g., R. Chang, et al., PRL 112, 170404 (2014)
Preliminary evidence of tunneling through a double barrier (Fabry-Perot cavity for atoms)

A narrow frequency component of the BEC remains trapped in the FP cavity – we don’t yet know for how long!
Stern-Gerlach measurement
Calibration of Larmor clock for free propagation

\[ \tau [\text{ms}] = 1.9(2) \times \theta [\text{rad}] \]

(A [very low-precision] confirmation that: \( t = \frac{L}{v} \)!)
Summary

• Even in the image plane, much (even most) of the information may be in the optical phase and not the intensity – a new route to super-resolution, requiring no structured illumination!


• To build optimal thermometers at the mesoscopic scale, one should use coherence relaxation as well as energy relaxation. Such thermalization processes can be simulated using a universal optical quantum simulator. Adaptive techniques will be useful for building finite many-spin thermometers.

  W.K. Tham, H. Ferretti, A.V. Sadashivan, AMS, 1609.01589

• After talking about it for 20 years, we are getting close to being able to probe atoms while they tunnel through an optical barrier, using weak measurement to ask “where they were” before being transmitted!

  We have preliminary evidence that our Fabry-Perot cavity for ultracold Rubidium atoms is working.