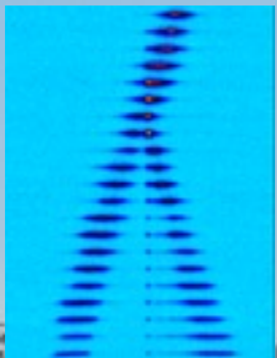
From Tunneling Atoms to Thermalizing Photons to Peering at Binary Stars:

a few examples of how coherence enables quantum measurement science



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Quantum Optics VIII
Maresias, Brasil
October 2016

DRAMATIS PERSONÆ

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Atoms: **Ramon Ramos** **David Spierings** Isabelle Racicot

Atom-Photon Interfaces: Josiah Sinclair Shaun Pepper Alex Bruening

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Some helpful theorists:

Daniel James, Pete Turner, Robin Blume-Kohout, Chris Fuchs, Howard Wiseman, János Bergou, John Sipe, Paul Brumer, Michael Spanner...

CQIQC

25 years cifar Canadian Institute for Advanced Research

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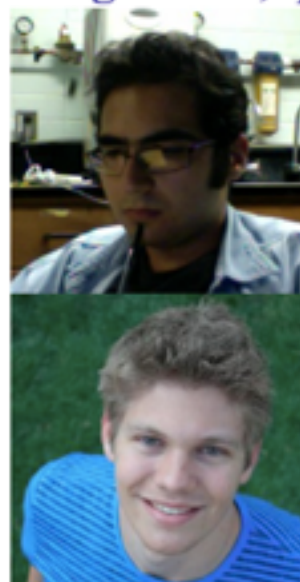
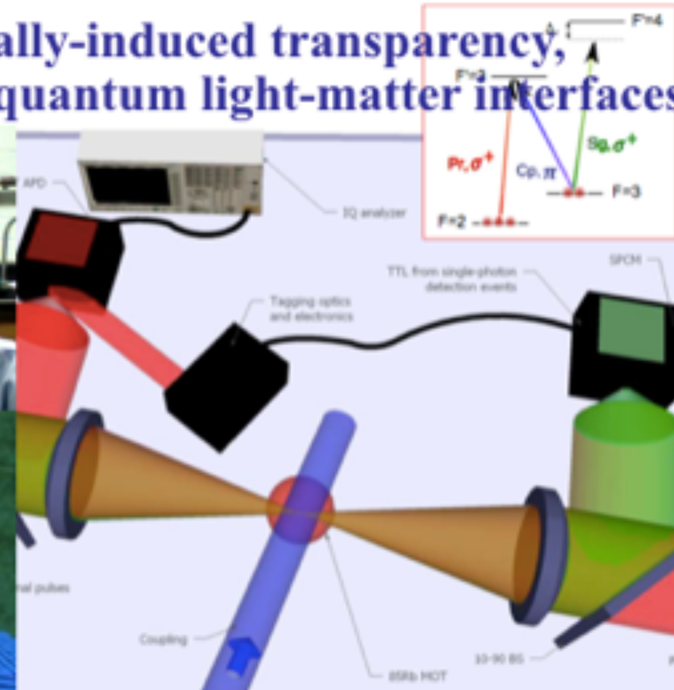
Quantum Games with Photons &Atoms

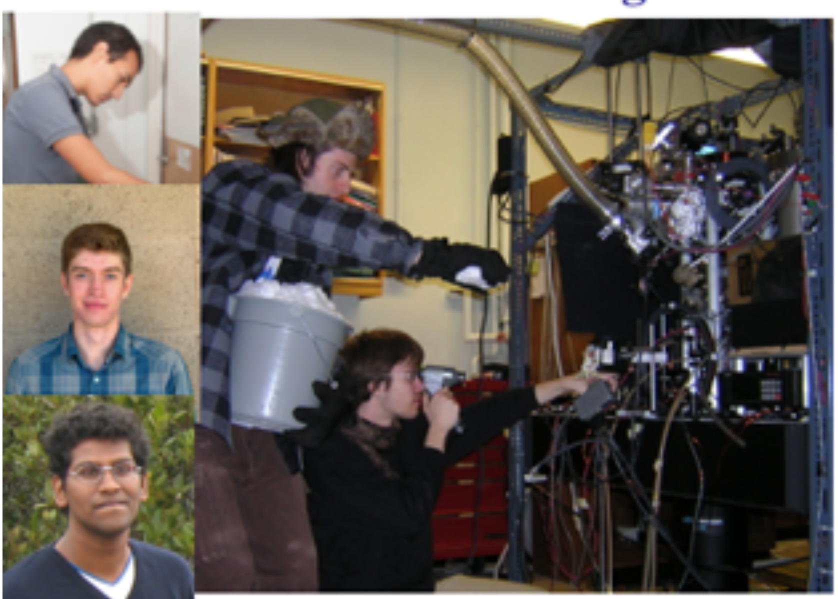
Crossing the beams!

Electromagnetically-induced transparency, Rydberg atoms, quantum light-matter interfaces





Bose-Einstein condensation & weak measurements of tunneling times



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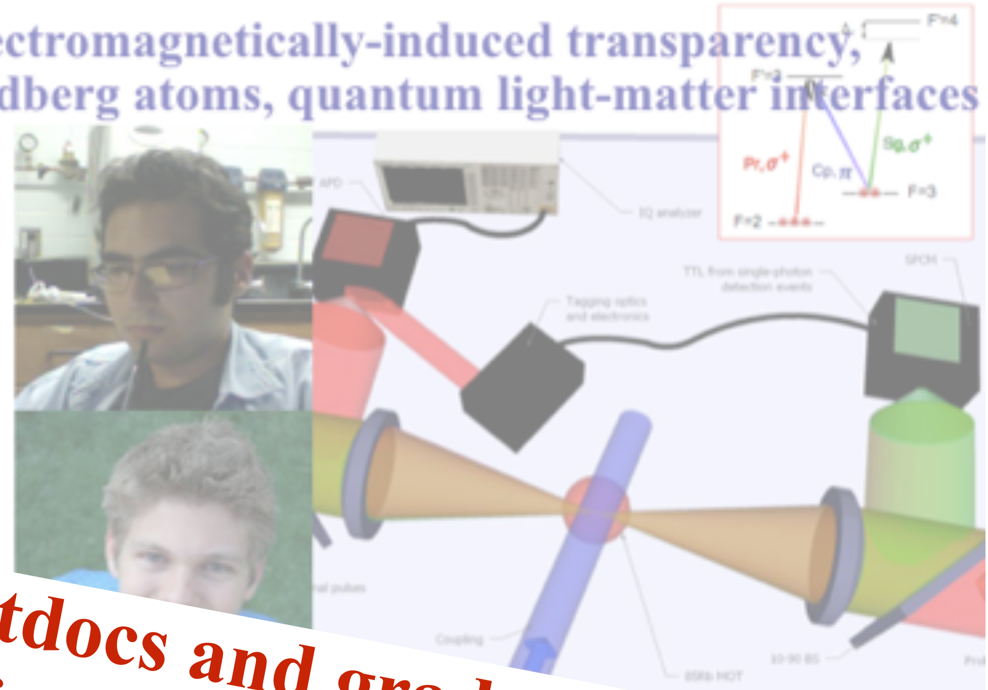

Positions available for postdocs and graduate students (Full funding guaranteed for students accepted to PhD programme in Toronto)

Quantum Data Compression,
weak measurement, Q tomo,
Q metrology,…

Menu: a few small snacks on a theme

Imaging as a Quantum State Discrimination problem

- **Using coherence in the image plane to evade "Rayleigh's curse" (theory: Mankei Tsang *et al.*)**

Improving thermometry by making use of coherence

- **(theory: Terry Rudolph *et al.*)**

Using atomic coherence to study tunneling times, and play other fun games

- **The Larmor clock**
- **Weak measurements**
- **Experimental progress (Fabry-Perot for atoms?)**

Mini-Pão 1: Better imaging as an optimal quantum discrimination problem

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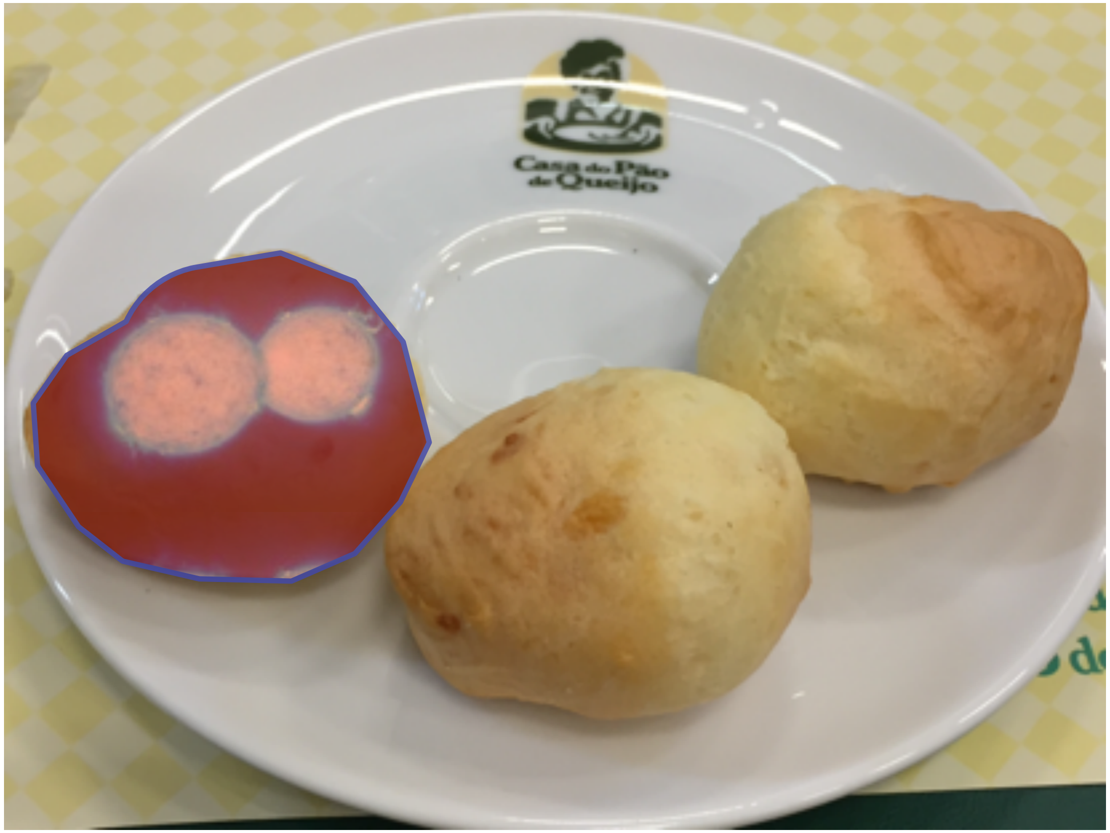
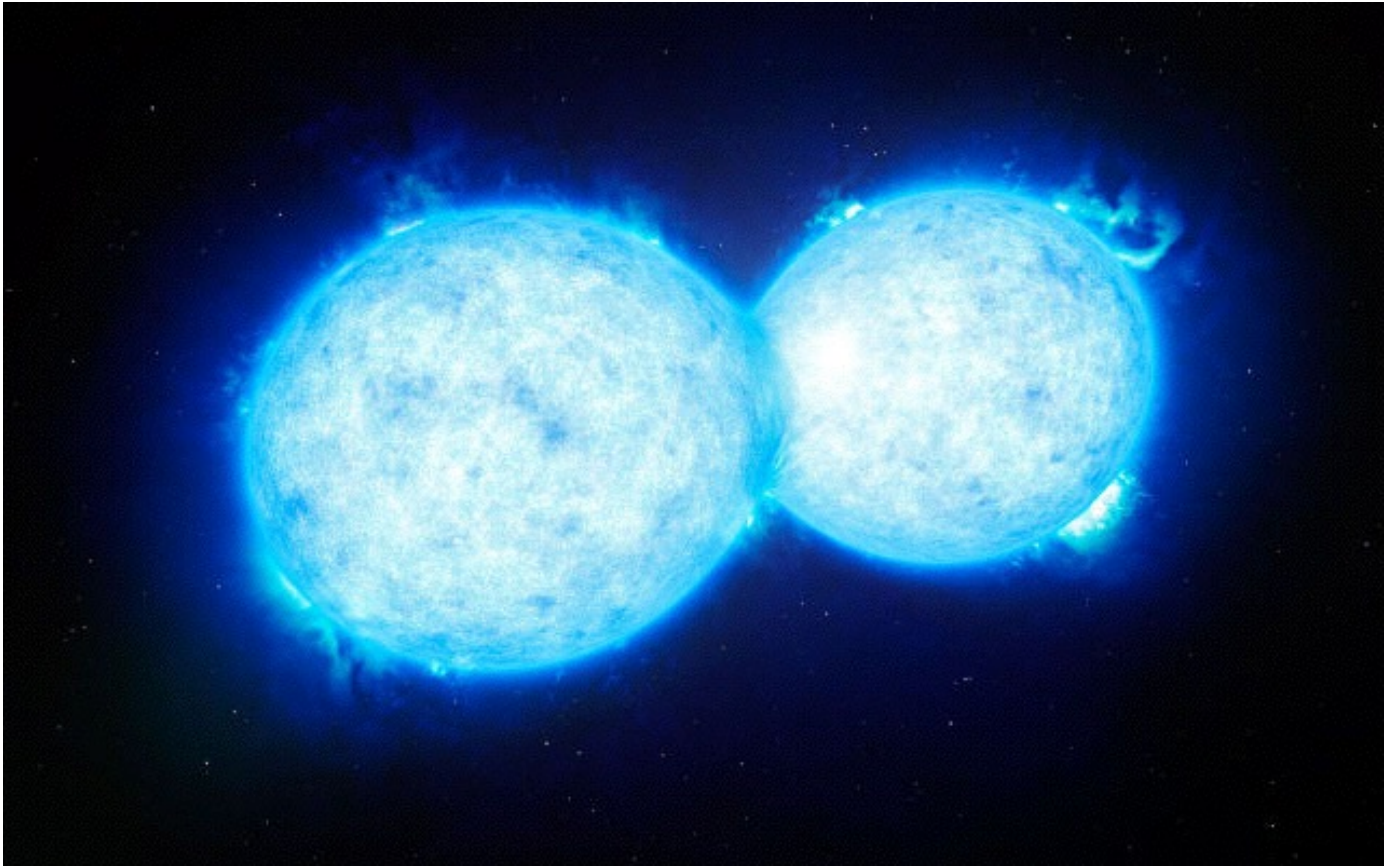
Better imaging as an optimal quantum discrimination problem



Toy problem: imaging a binary star



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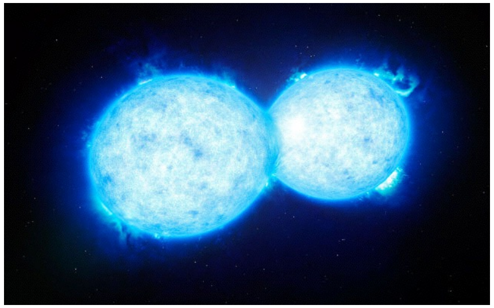

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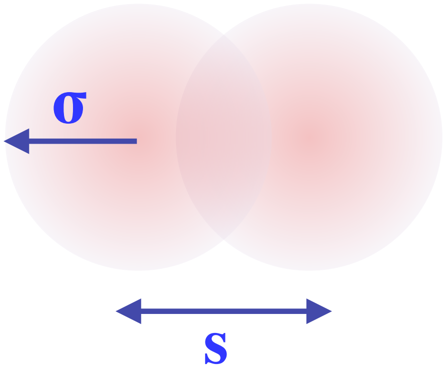
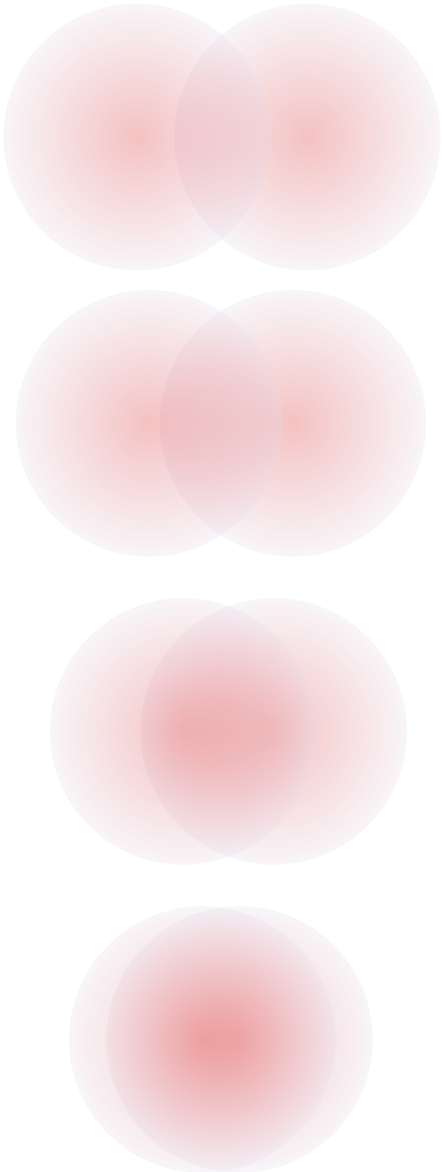


As we all know, if objects separated by less than ~width σ of the PSR (diffraction limit), we can't "resolve" them

… of course, that's not to say that with enough data, we can't tell there are two objects there, and where they are…

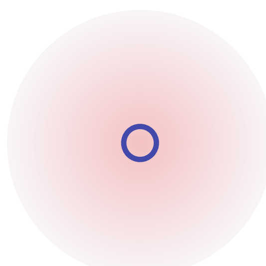
Toy problem: imaging a binary star



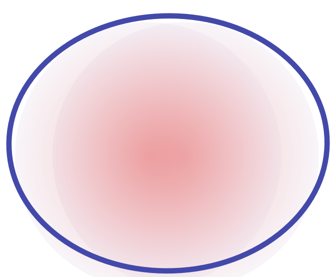
How well can we estimate the separation s of two objects, for s < width σ of PSR, given N photons?

σ / sqrt{N} for N photons would seem reasonable?

No such luck!



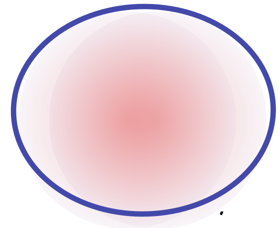
σ / sqrt{N} is indeed how well you can find the centre of *one* object.



But two closely separated gaussians just look like a slightly broader gaussian – the problem is to estimate the *width*, which proves much harder.

How well can you estimate a width?

$$V = \langle x^2 \rangle = \frac{1}{N}\Sigma_{(i=1)}^{N} x_i^2$$



Uncertainty ΔV can be calculated from $\Delta V^2 = \overline{V^2} - \overline{V}^2$.

$$V^2 = \frac{1}{N^2}\Sigma_i \Sigma_j x_i^2 x_j^2$$

For $i \neq j$, $\overline{x_i^2 x_j^2} = \sigma^4$

For $i = j$, $\overline{x_i^2 x_j^2} = 3\sigma^4$

$$\overline{V^2} = \frac{1}{N^2}\left\{N^2\sigma^4 + 2N\sigma^4\right\} = \sigma^4 + \frac{2}{N}\sigma^4$$

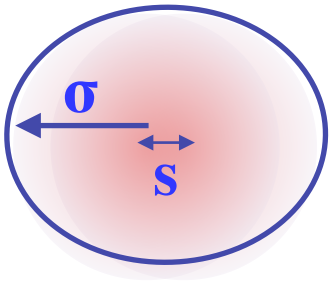
$$\overline{V}^2 = \sigma^4$$

$$\Delta V = \sqrt{\frac{2}{N}\sigma^4}$$

How well can you estimate a separation?

$$\Delta V = \sqrt{\frac{2}{N}\sigma^4}$$

$$V = \sigma^2 + s^2$$

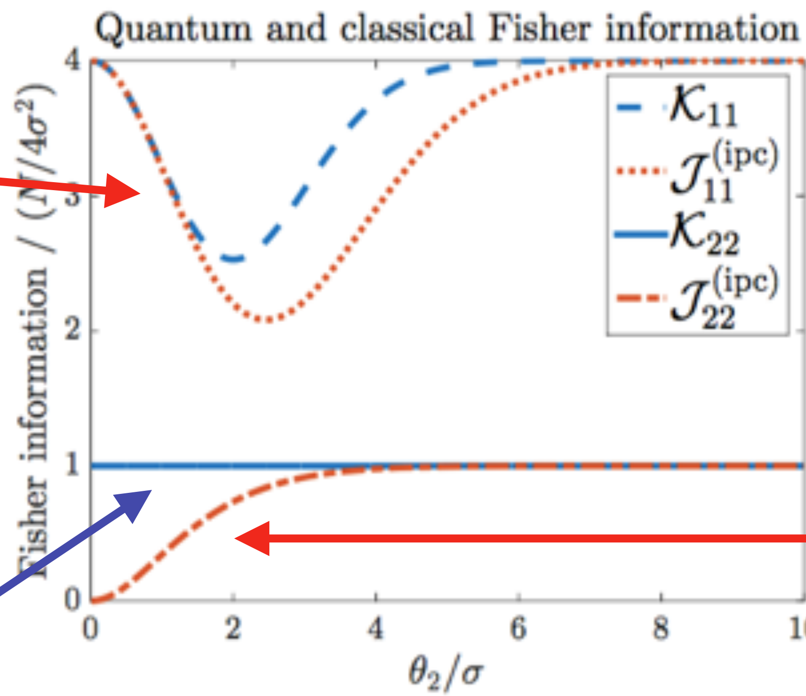


$$\Delta s = \Delta V / \frac{dV}{ds} = \frac{\sigma^2 \sqrt{2/N}}{2s} = \frac{\sigma^2}{s}\sqrt{\frac{1}{2N}}$$

The uncertainty in s does not merely *remain* large (σ/sqrt{N}) as s -> 0 ; it actually *diverges* as 1/s!

M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).

Information about *centroid*



Classical Fisher Information about *separation* (vanishes at small sep.)

Quantum Fisher Information about separation —- constant!!

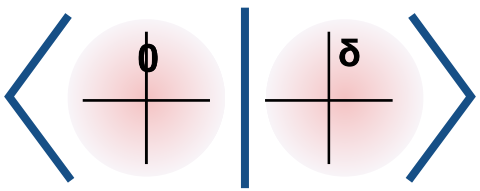
FIG. 2. Plots of Fisher information quantities versus the separation for a Gaussian point-spread function. $\mathcal{K}_{11}$ and $\mathcal{K}_{22}$ are the quantum values for the estimation of the centroid $\theta_1 = (X_1 + X_2)/2$ and the separation $\theta_2 = X_2 - X_1$, respectively, while $\mathcal{J}_{11}^{(\mathrm{ipc})}$ and $\mathcal{J}_{22}^{(\mathrm{ipc})}$ are the corresponding classical values for image-plane photon counting. The horizontal axis is normalized with respect to the point-spread function width σ, while the vertical axis is normalized with respect to $N/(4\sigma^2)$, the value of $\mathcal{K}_{22}$.

The Fisher information drops to 0 —
the error of any unbiased estimator of s goes to infinity.

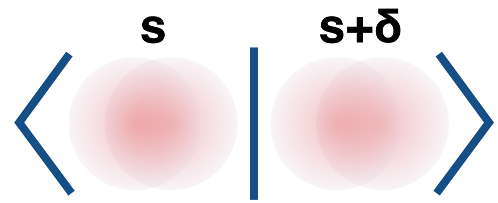
Where is the missing information? The classical F.I. considers only *intensity*…

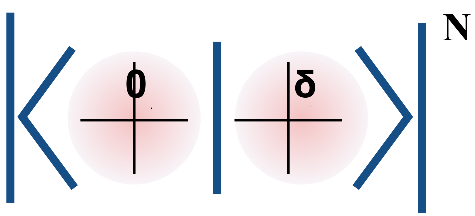
Our problem is to distinguish various possible states of N photons – e.g., them in two spots separated by s or two spots separated by s+δ.

Single spots are easy:

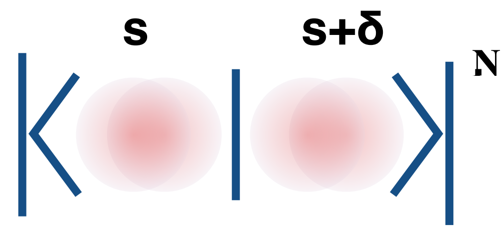


$\approx e^{-\delta^2}$
(almost identical)





$\approx e^{-N\delta^2}$
(nearly orthogonal, if delta > 1/sqrt{N})



For two incoherent sources, the 2-spot distinguishability is essentially the same as the 1-spot distinguishability… how to optimally distinguish?

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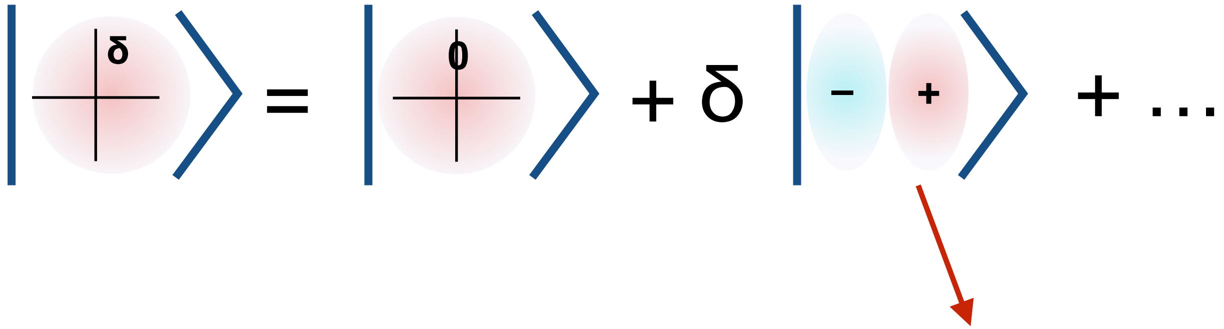
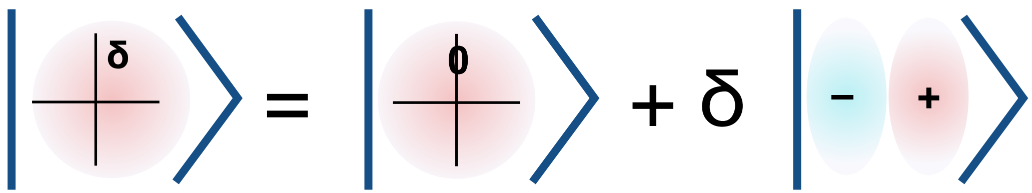
This becomes a quantum state discrimination problem



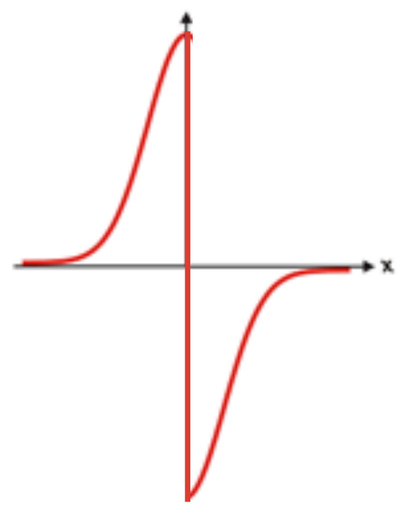
Project onto this TEM01 mode to determine δ (for small δ, *all* the info)

For two incoherent sources, the 2-spot distinguishability is essentially the same as the 1-spot distinguishability… how to optimally distinguish?

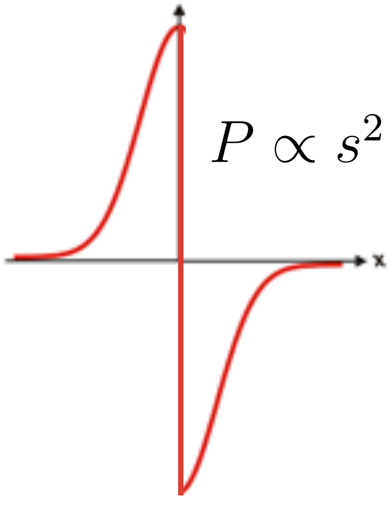
This becomes a quantum state discrimination problem



SPLICE:
Project onto any odd-parity mode, not necessarily TEM01 in particular —



Error analysis for odd-mode projection ("SPLICE")



$P \propto s^2 = \kappa s^2/\sigma^2$, with κ a numerical constant of order unity.

NOTE: For two incoherent spots displaced symmetrically from the origin, the overlap with the odd mode is exactly the same as for one spot or the other.

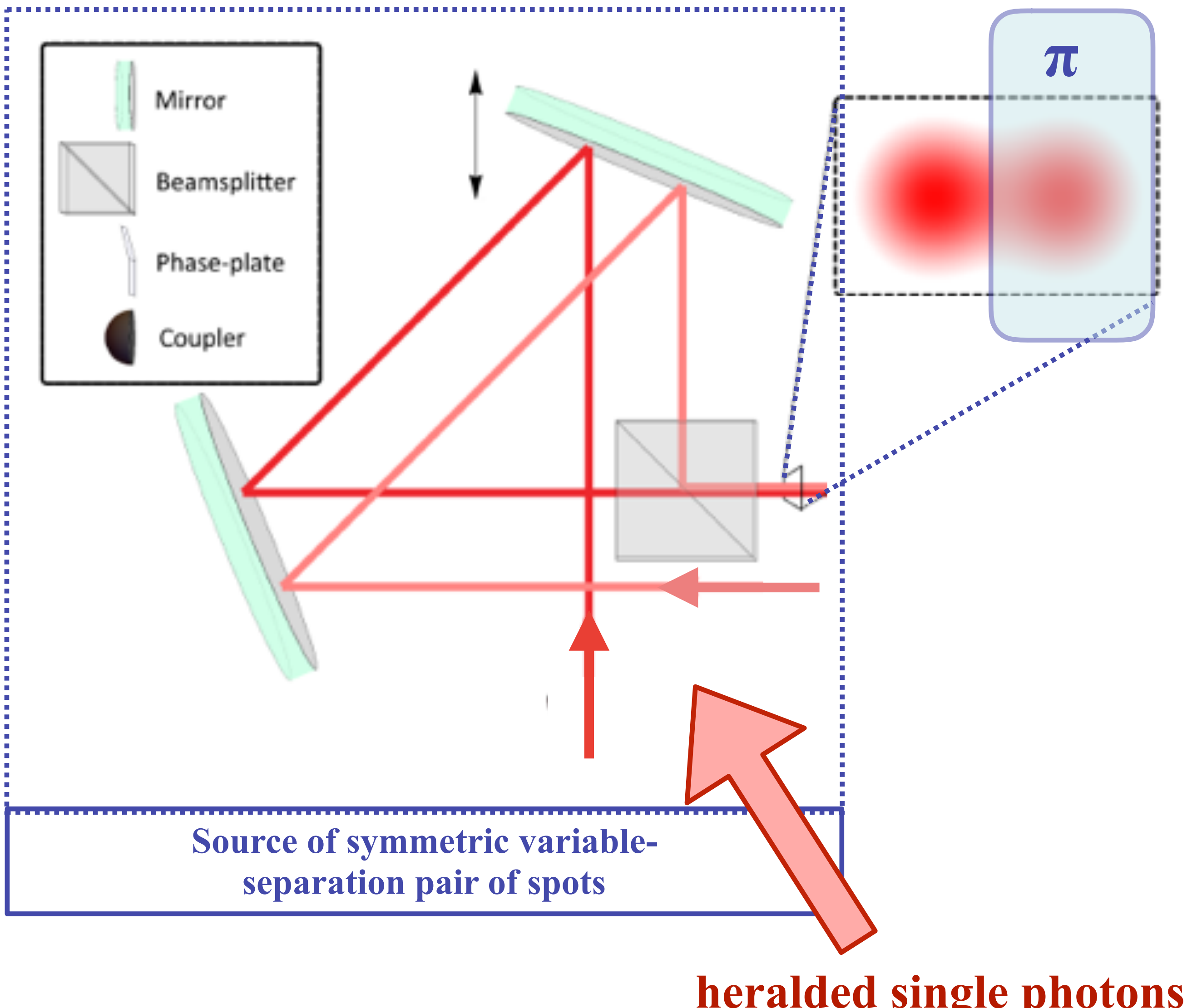
Signal (photon counts) $C = N\kappa s^2/\sigma^2$, with binomial fluctuations $\rightarrow \sqrt{C}$

$$\Delta s = \Delta C / \frac{dC}{ds} = \frac{\sqrt{N\kappa s^2/\sigma^2}}{2N\kappa s/\sigma^2}$$

$$= \frac{1}{2}\frac{\sigma}{\sqrt{N\kappa}}$$

INDEPENDENT OF s – for TEM01, retrieve σ/sqrt{N}

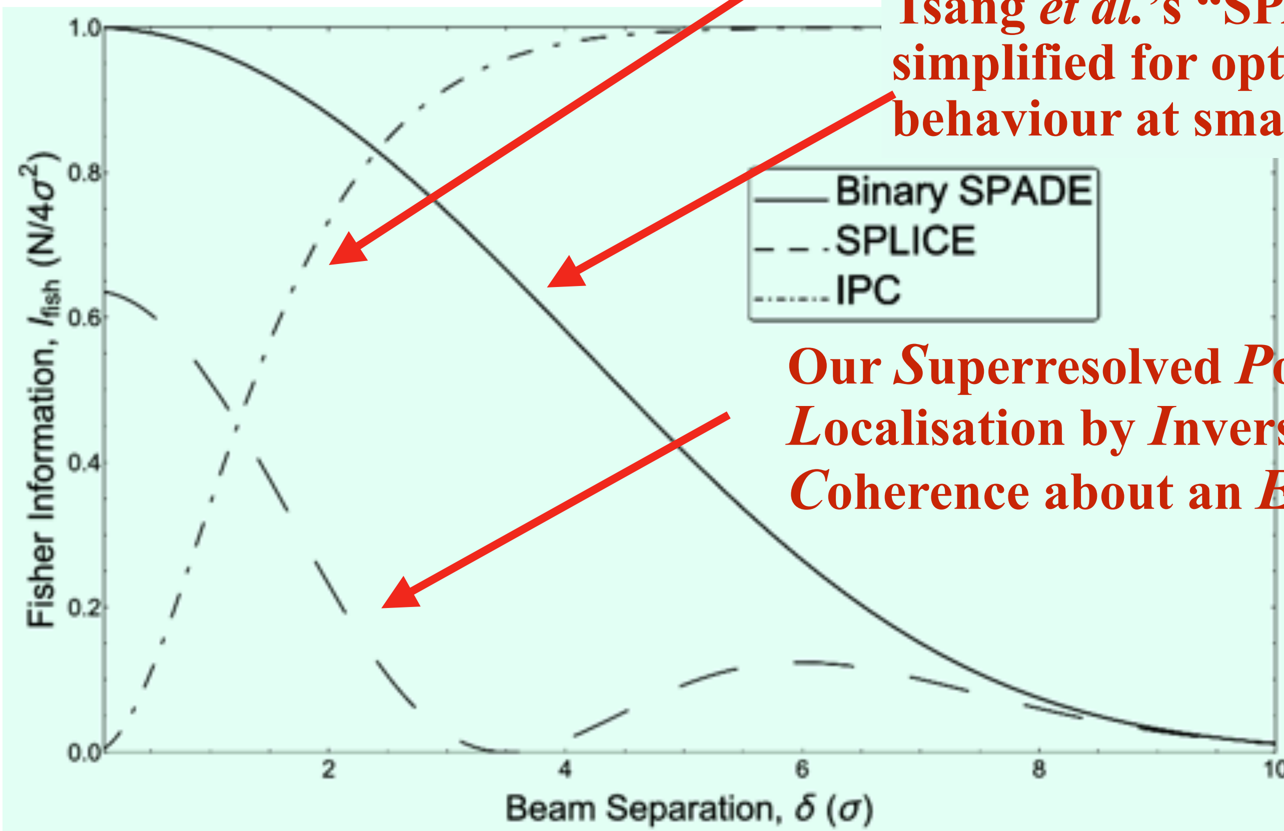
Projecting a double-spot onto an odd-parity mode



Fisher Information of various schemes to estimate separation of point sources

"IPC" (image-plane counting)

Tsang *et al.*'s "SPADE," simplified for optimized behaviour at small separation

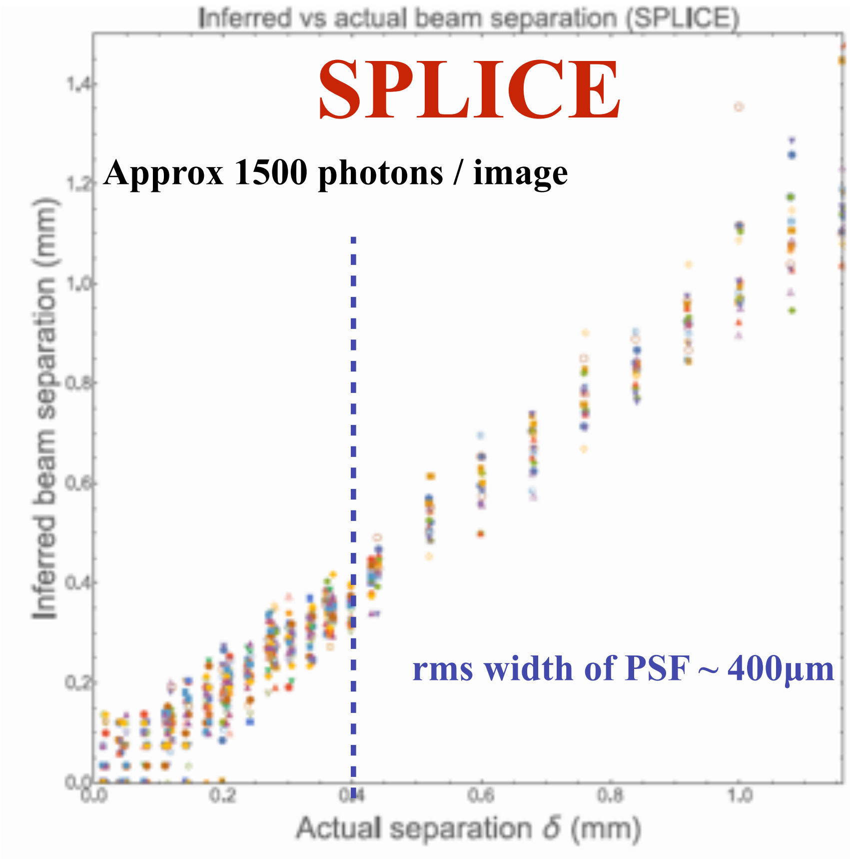


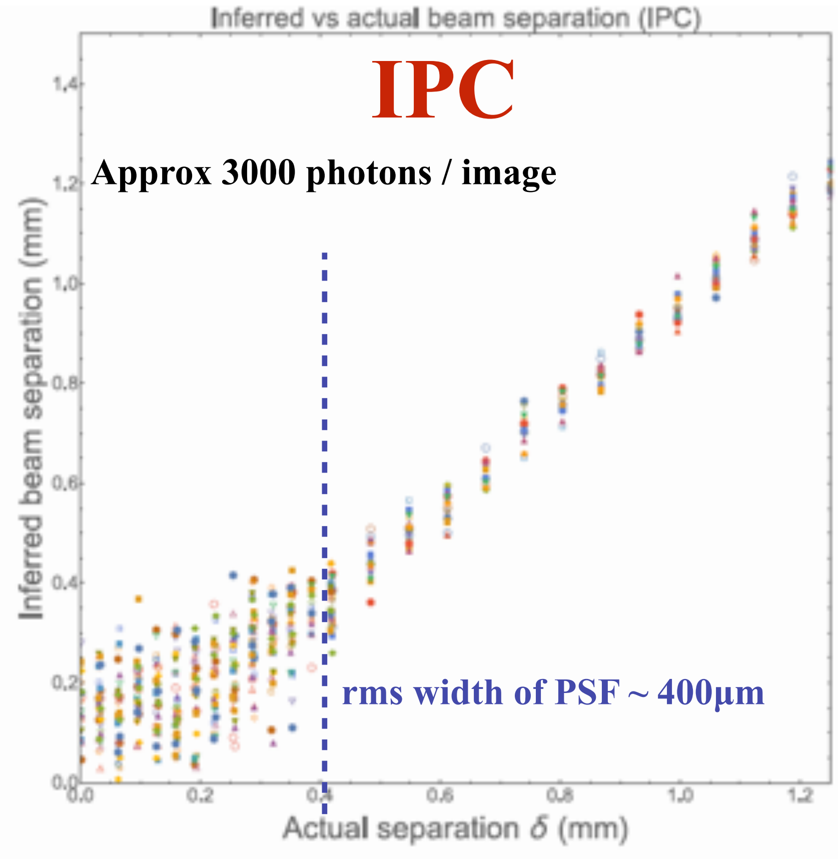
Our *S*uperresolved *P*osition *L*ocalisation by *I*nversion of *C*oherence about an *E*dge

In an experimentally simple setup, obtain about 64% of the full Fisher Information for small separations – crucially, independent of s.

M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).
W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

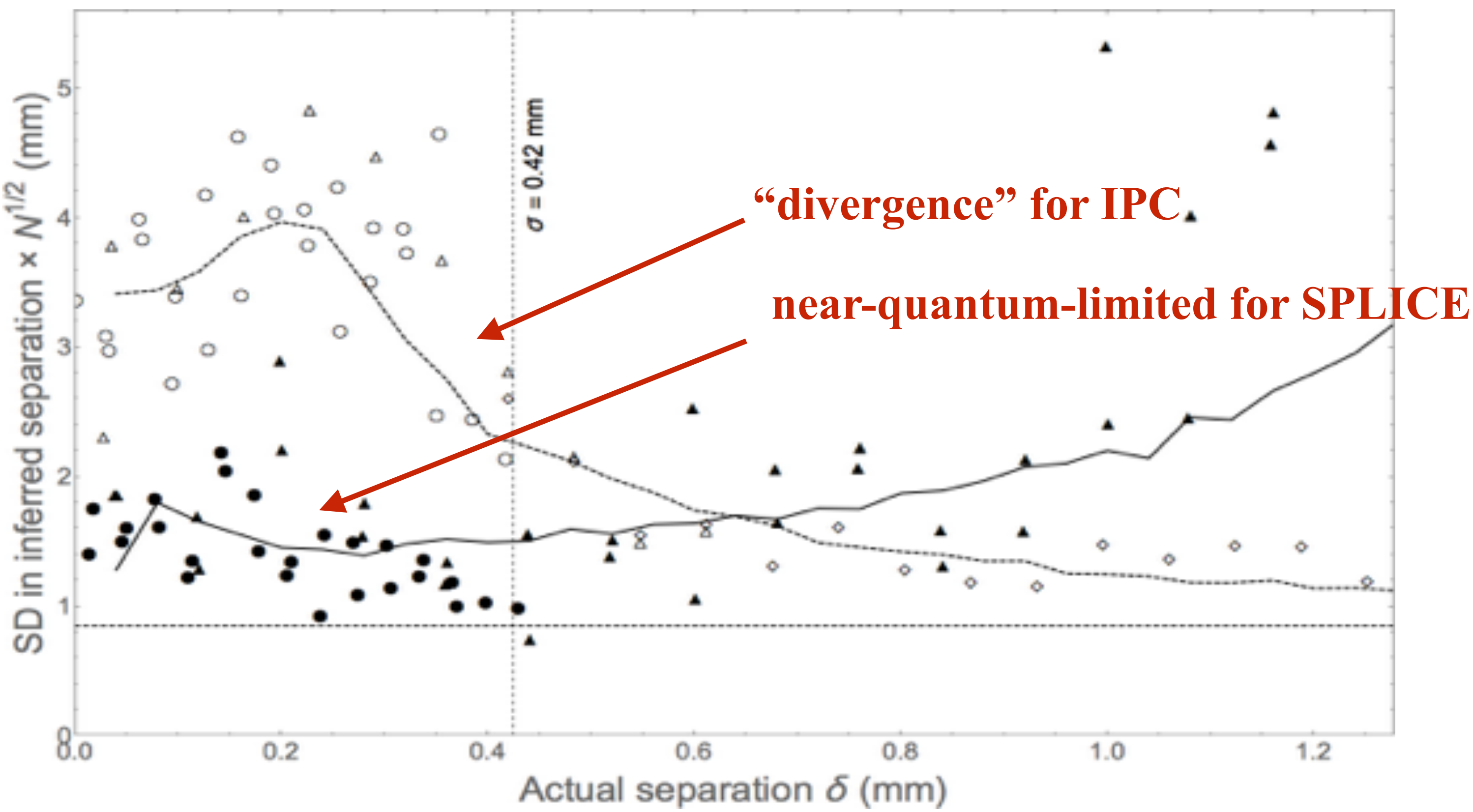
Observed vs. actual separation





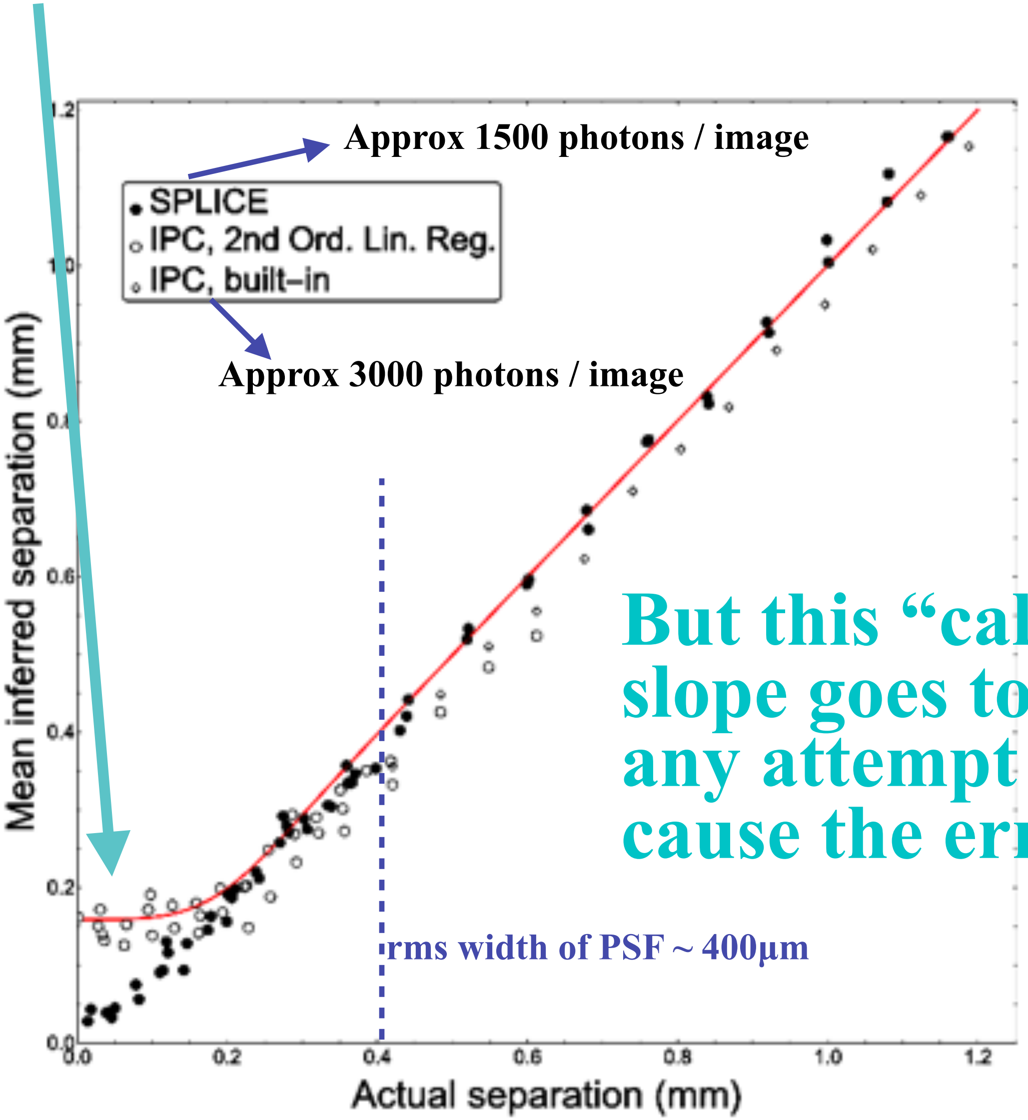
W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

SD in inferred separation, vs. s_{actual}



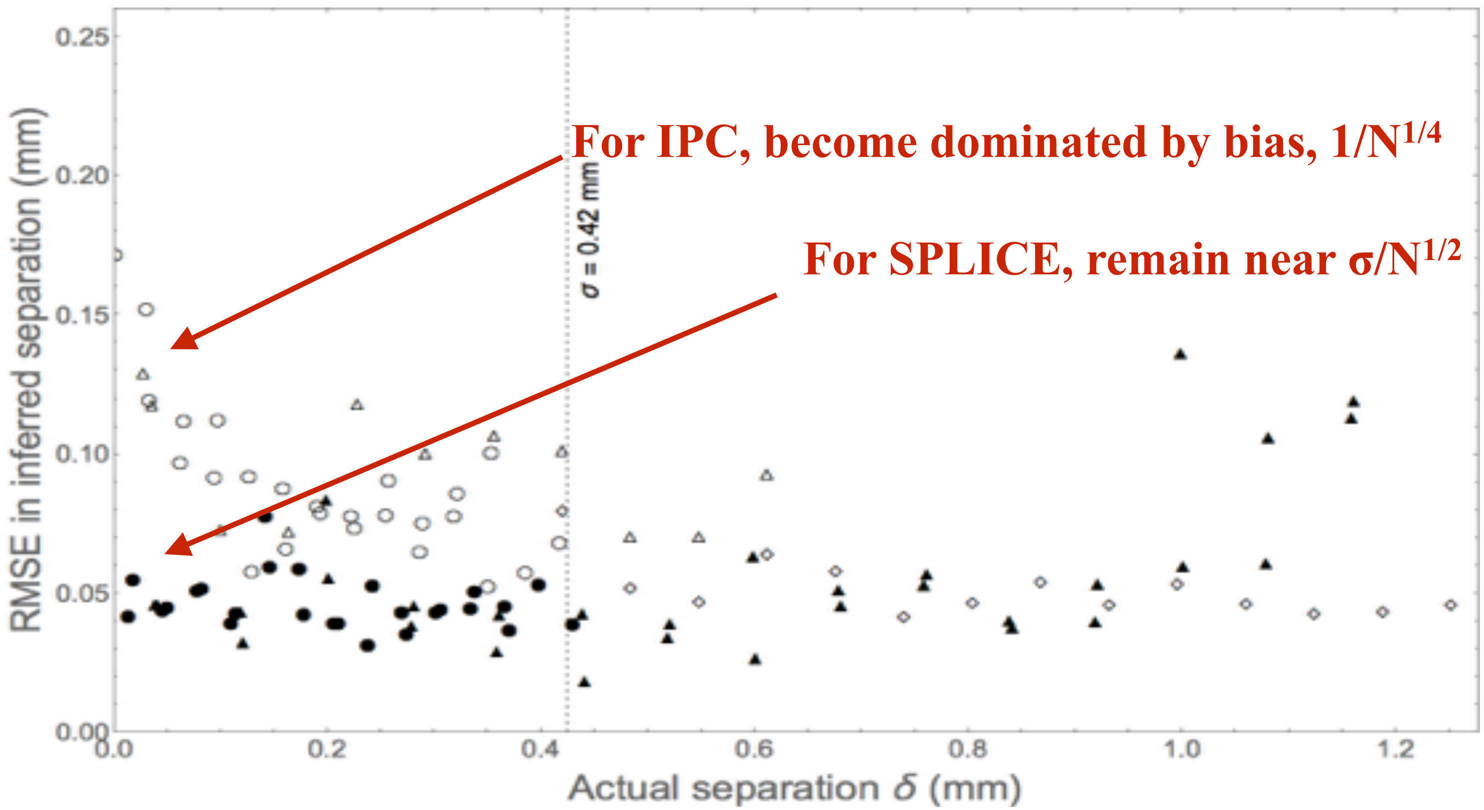
W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

This is not an unbiased estimator!



But this “calibration curve”s slope goes to 0 as s gets small; any attempt to invert it will cause the errors to diverge.

Total RMS error, *including* bias



CONCLUSION: We have shown that a simple phase-mask technique removes the 1/s catastrophe, and permits us to achieve near-quantum-limited resolution, providing an unbiased estimator with $\sigma/N^{1/2}$ resolution, yielding a quadratic-in-N advantage over even the best *biased* estimator possible with image-plane counting.

With about 1500 photons, SPLICE determined the separation 3 times more accurately than IPC could with about 3000 photons

W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

See also: T. Z. Sheng, K. Durak, and A. Ling, arXiv preprint arXiv:1605.07297 (2016); M. Paur et al. arXiv:1606.08332 (2016); F. Yang, A. I. Lvovsky et al arXiv:1606.02662 [physics.optics].

Mini-Pão 2:
Thermometry
as an optimal quantum discrimination problem

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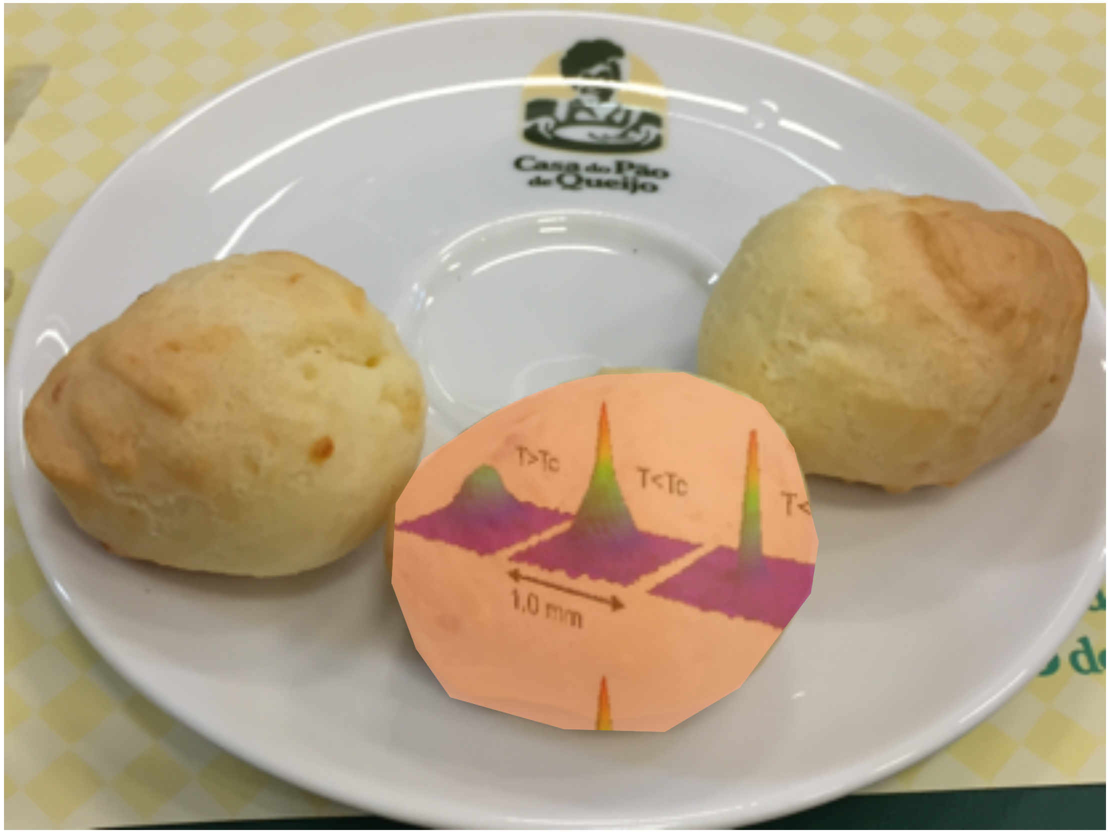
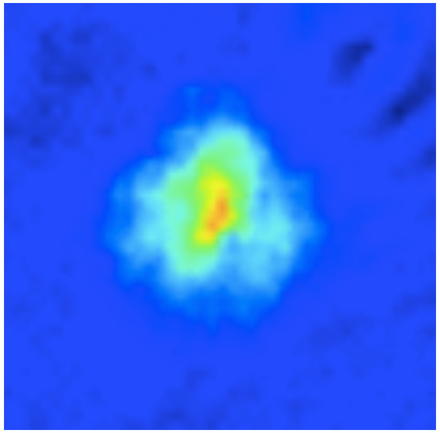
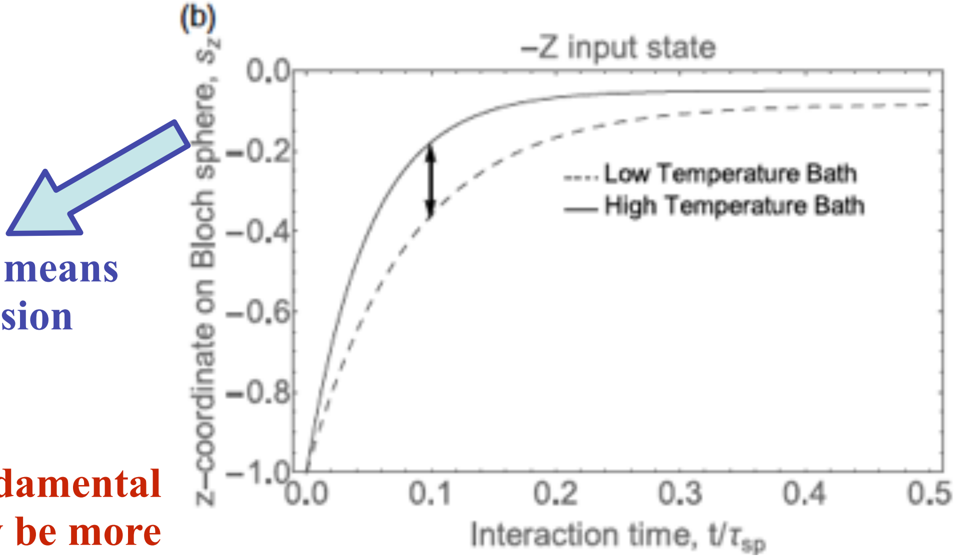


Typical thermometry



Let energy of thermometer (small test object – in the extreme case, a single spin) equilibrate with that of sample…
consider trying to distinguish two possible temperatures



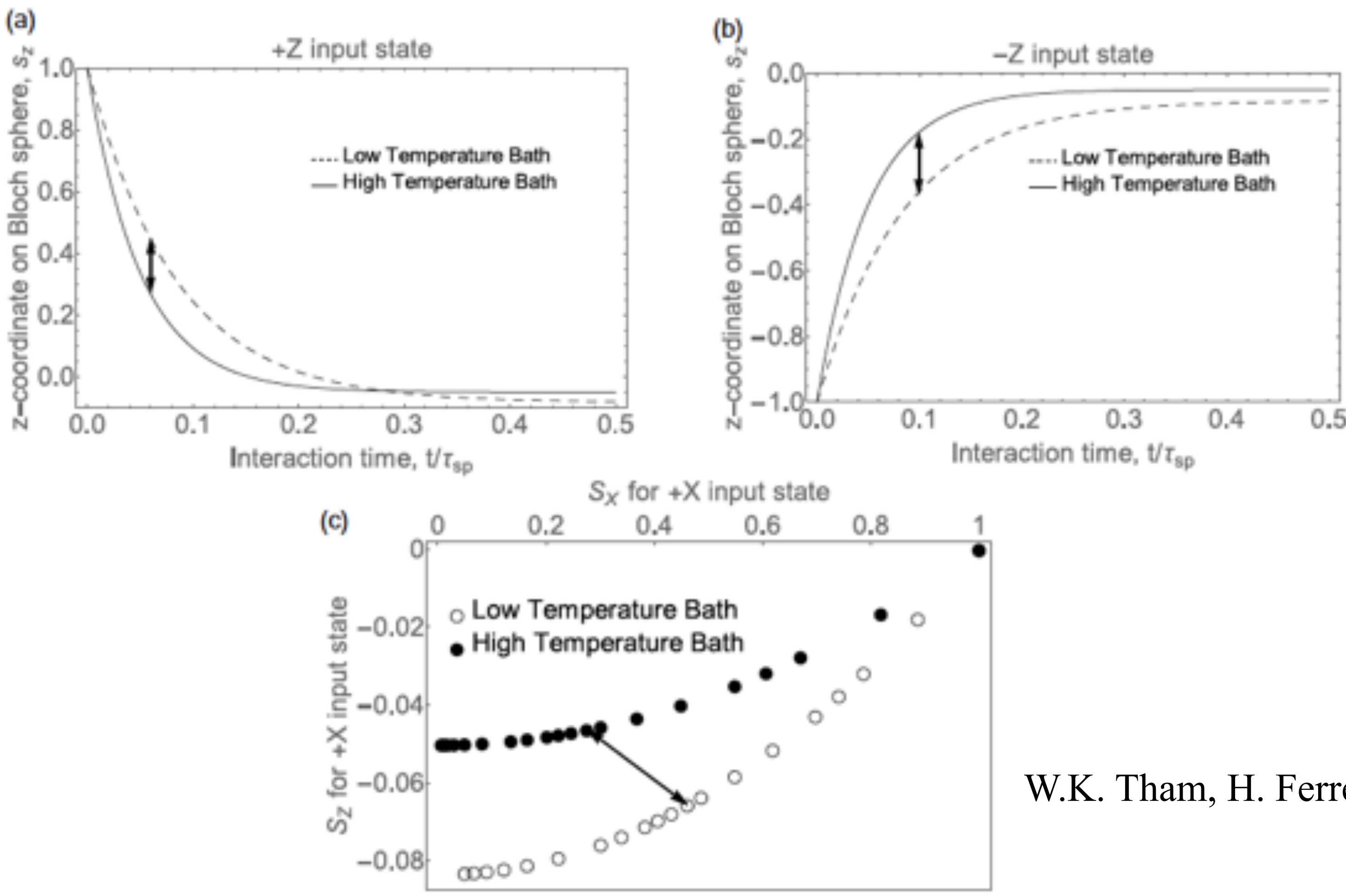
At least for bosonic baths, a higher temperature means both a higher equilibrium state and a faster collision (equilibration) rate!

For high enough temperatures (w.r.t. the fundamental energy scale of the thermometer), it may be more effective to probe the *rate* than the steady state:
as in this curve, maximum distinguishability achieved at finite times.

viz. Jevtic, Newman, Rudolph, & Stace, PRA 91, 012331 (2015)

But (for energy relaxation), $T_2 = 2T_1$

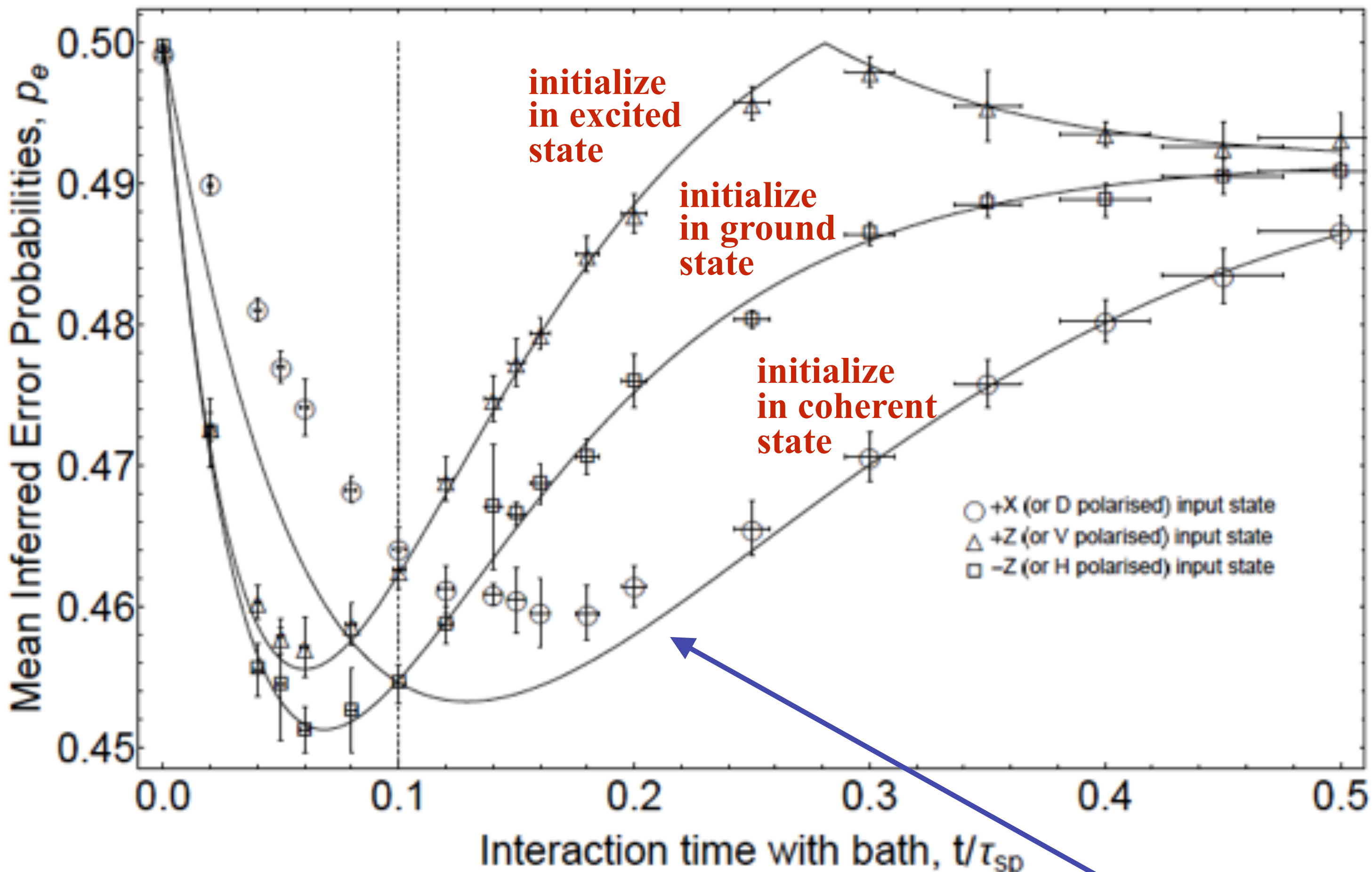
A thermometer which is initialized with *coherence* takes longer to reach this optimum time…



W.K. Tham, H. Ferretti, A.V. Sadashivan, & AMS, 1609.01589

Figure 1. **Bloch vector components vs interaction time.** Theoretically computed components of the Bloch vector after thermalizing for t seconds, (a) given a $+Z$ input state, (b) $-Z$ input state, and (c) $+X$ input state. For this latter case, both s_z and s_x are shown. The bath temperatures are $5.98\hbar\omega/k_B$ and $10\hbar\omega/k_B$. At $t = 0$ the state begins at the rightmost point of the bottom plot. Each subsequent timestep is shown as a pair of points, one each for high and low temperatures respectively. Arrows indicate where the greatest separation occurs.

Experimental results



NB: minimum error is always achieved at finite t, not asymptotically; at most times, minimum error is achieved by using a coherent input.

W.K. Tham, H. Ferretti,
A.V. Sadashivan, & AMS, 1609.01589

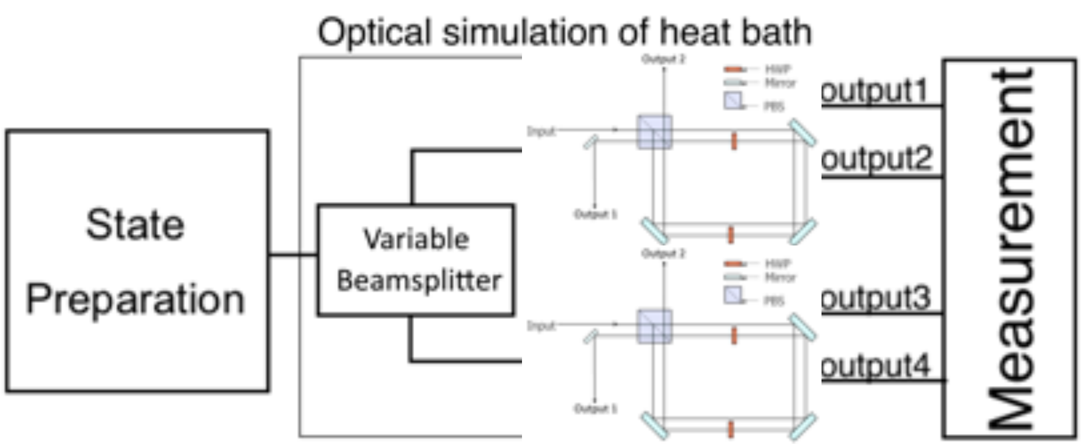
Thermometry (that's just the beginning)

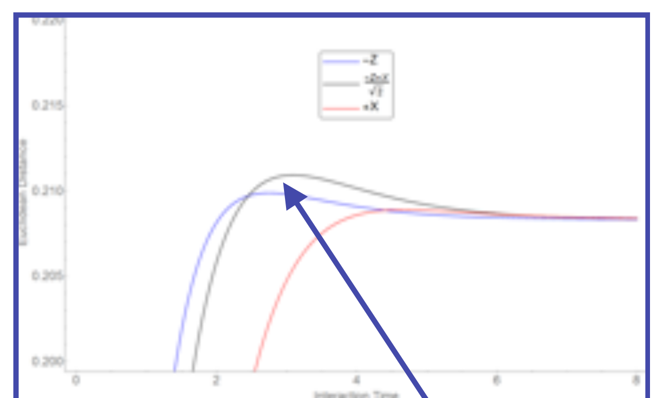
For more details, see W.K. Tham, H. Ferretti, A.V. Sadashivan, and A.M. Steinberg, 1609.01589;
for a simultaneous work,
Luca Mancino, Marco Sbroscia, Ilaria Gianani, Emanuele Roccia, and Marco Barbieri, 1609.01590.

Single-spin thermometer theory:
Jevtic, S., Newman, D., Rudolph, T. & Stace, T. Single-qubit thermometry. PRA 91, 012331 (2015)

Proposal for universal quantum-channel simulator:
Wang, D., Berry, D., de Oliveira, M. & Sanders, B., PRL 111, 130504 (2013).

See Marco Barbieri's paper above for interesting thoughts about free energy;
see ours for extensions to adaptive protocols for thermometry with a few qubits;
see Higgins, B., Doherty, A., Bartlett, S., Pryde, G. & Wiseman, H. PRA 83, 052314 (2011) for the relevant theory behind the latter.





One example where *global* optimum requires coherence

Mini-Pão 3:
Towards weak measurements of atomic tunneling times

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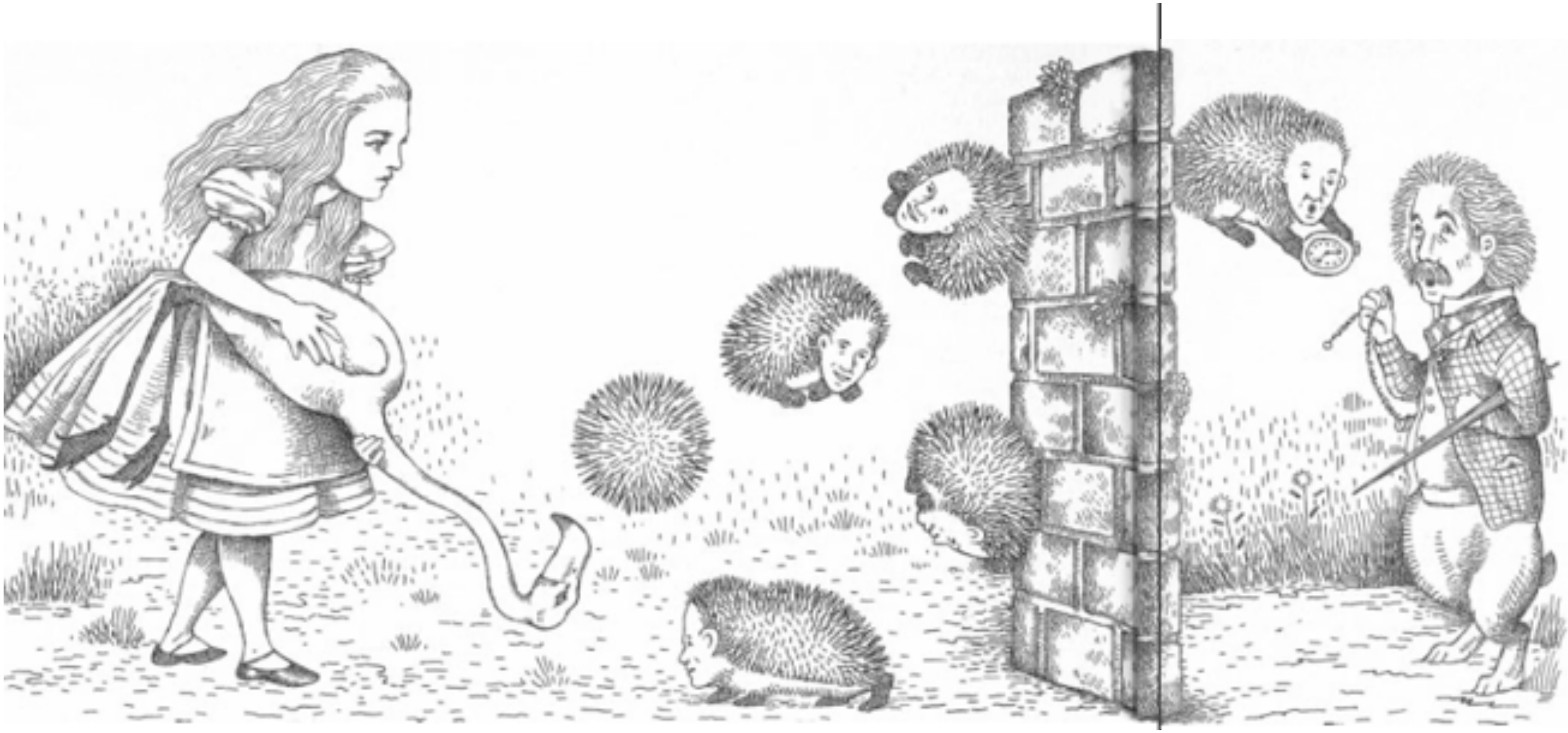
Towards weak measurements of atomic tunneling times

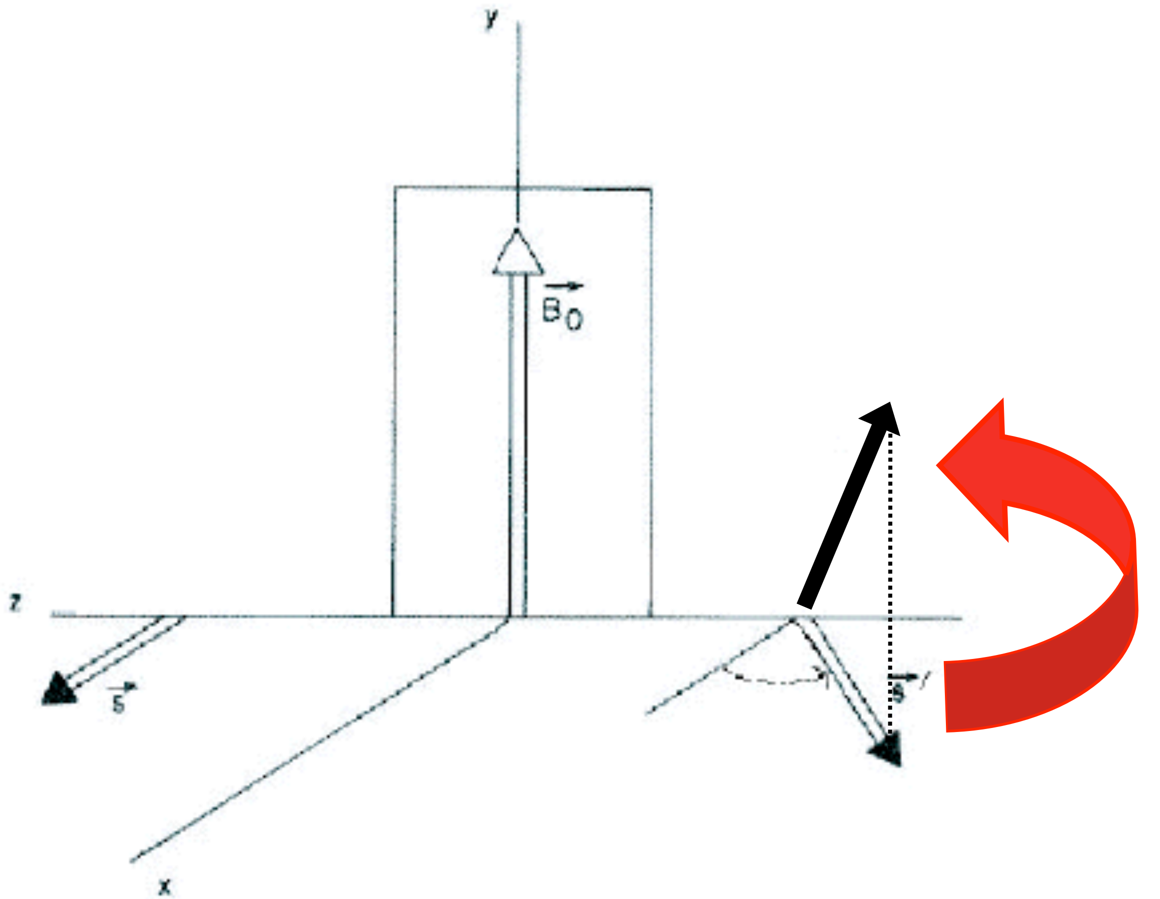
Watching a particle in a region it's "forbidden" to be in



How long has the transmitted particle spent in the region?

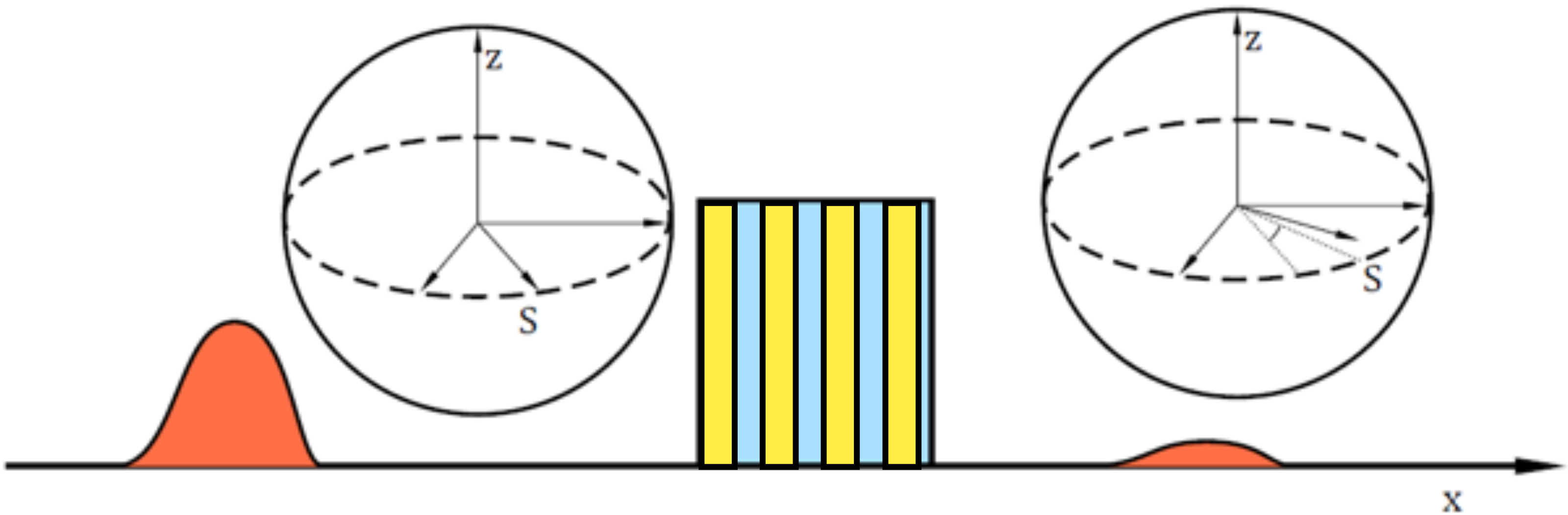
Need a clock...

"Larmor Clock" (Baz'; Rybachenko; Büttiker 1983)



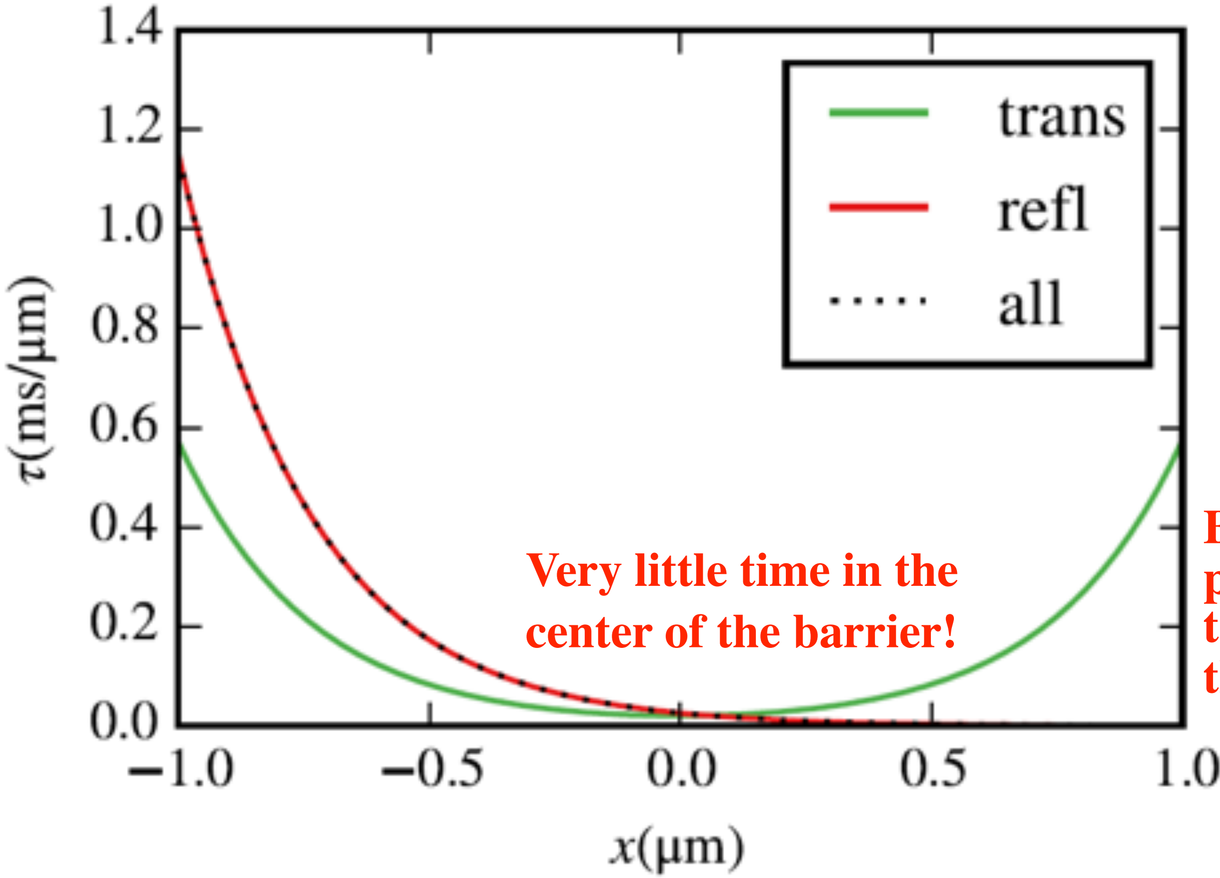
Two components mystified Büttiker;
Feynman approach led to complex times, which mystified every one;
It turns out these are weak values, whose Real and Imaginary parts are easily interpreted – but which hadn't been invented yet.

Local "Larmor Clock" – how much time spent in any given region?



- $\tau = \theta_{\text{rot}}/\omega_l$
  - In plane rotation measures the tunneling time
  - Spin aligns along z axis; back-action of the measurement.

Where does a particle spend time inside the barrier?

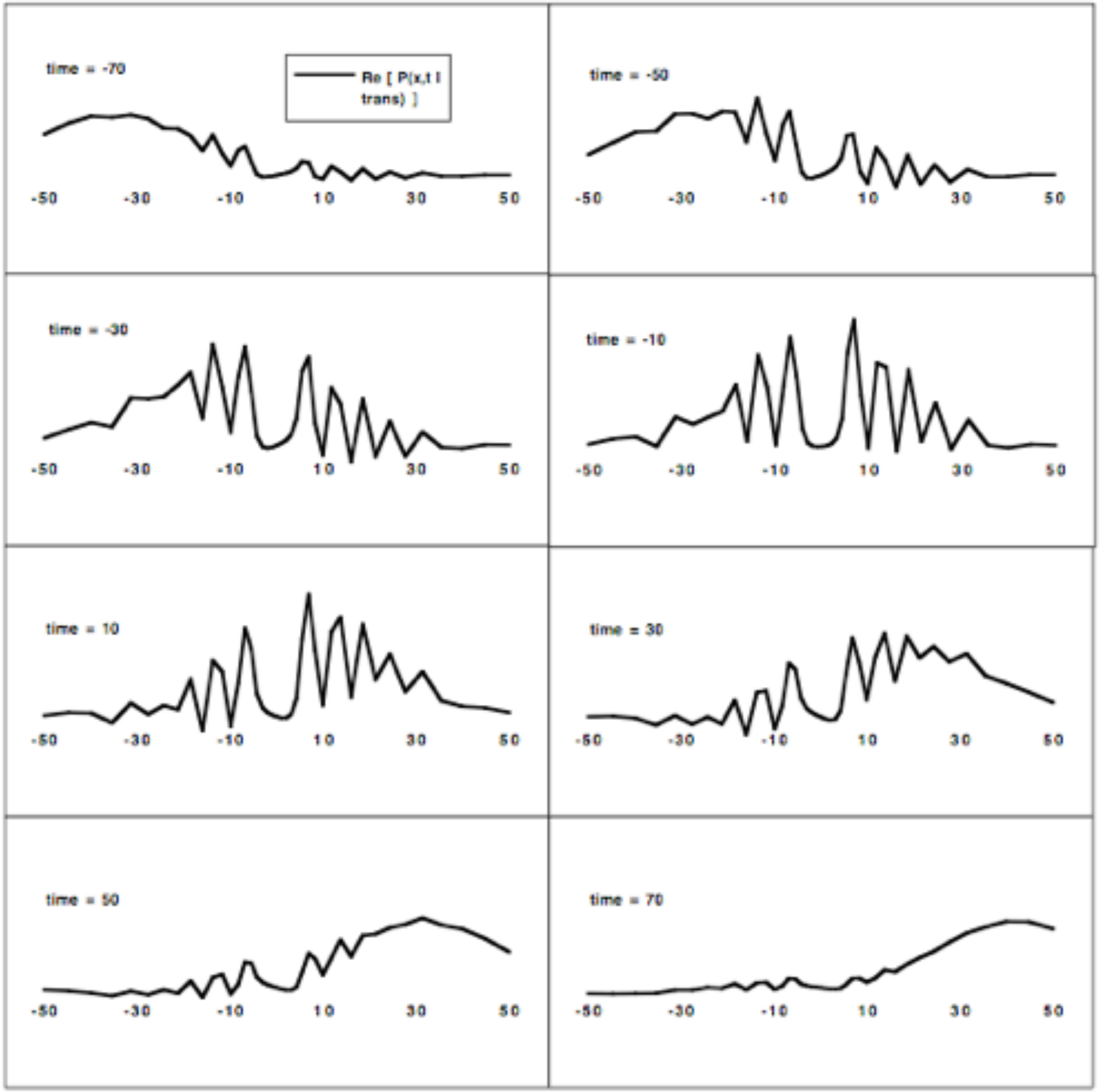


Very little time in the center of the barrier!

But – unlike the reflected particles – the transmitted ones "see" the region near the exit!

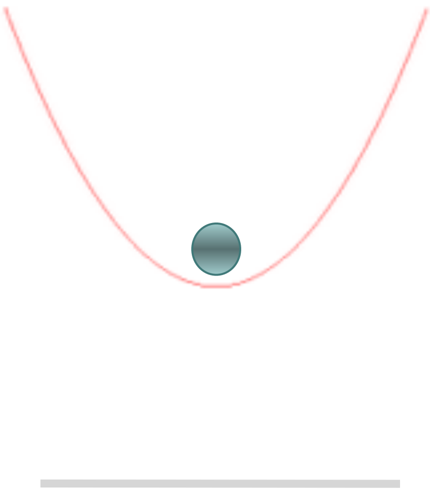
AMS, Phys. Rev. Lett., 74(13), 2405–2409, *Phys. Rev. A, 52*(1), 32–42.

Conditional-probability "movie" of tunneling



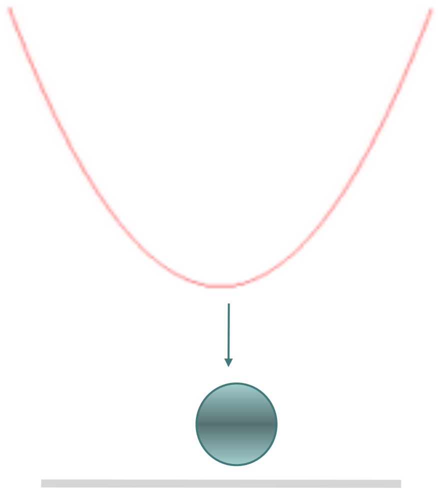
One possible experimental sequence

- BEC in magnetic trap



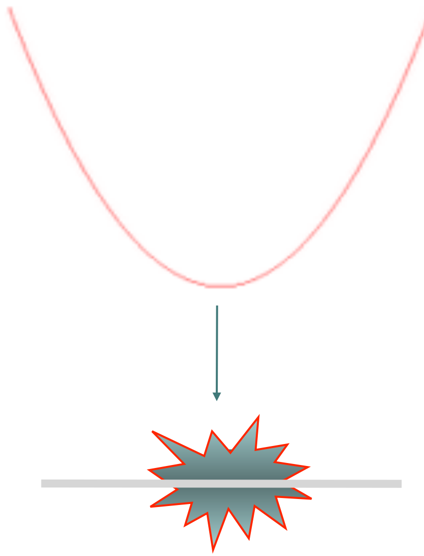
One possible experimental sequence

- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms

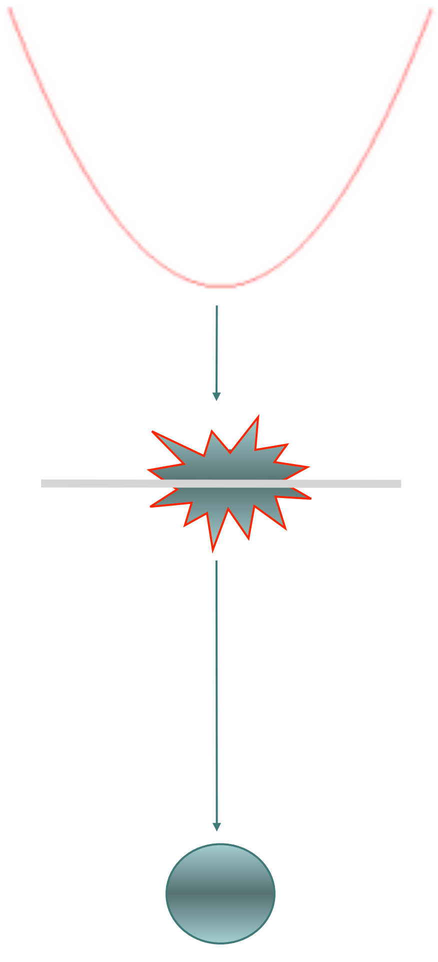


One possible experimental sequence

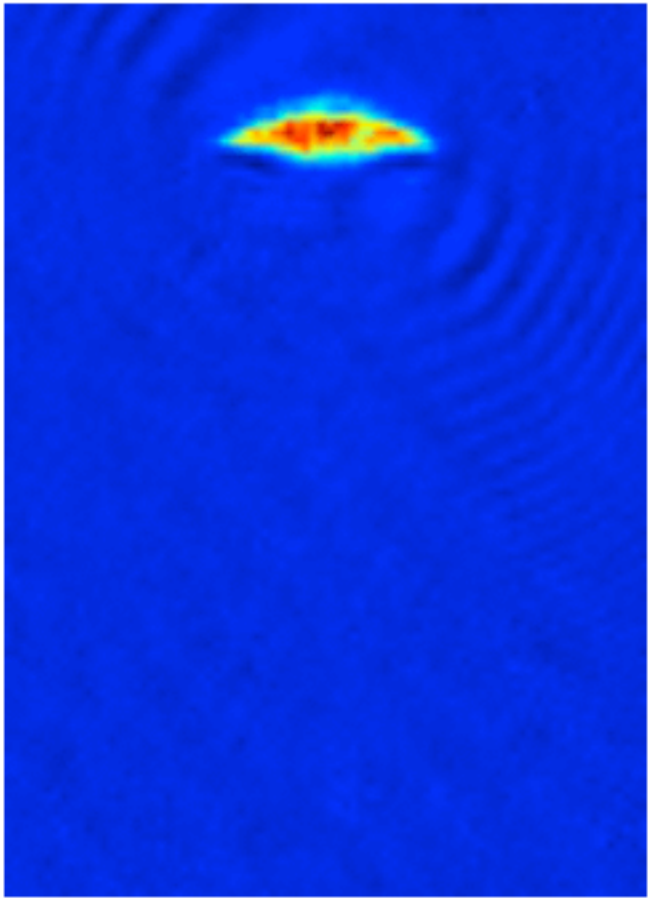
- BEC in magnetic trap
- Turn off trap, free expansion of condensate for 5 ms
- Interaction with barrier



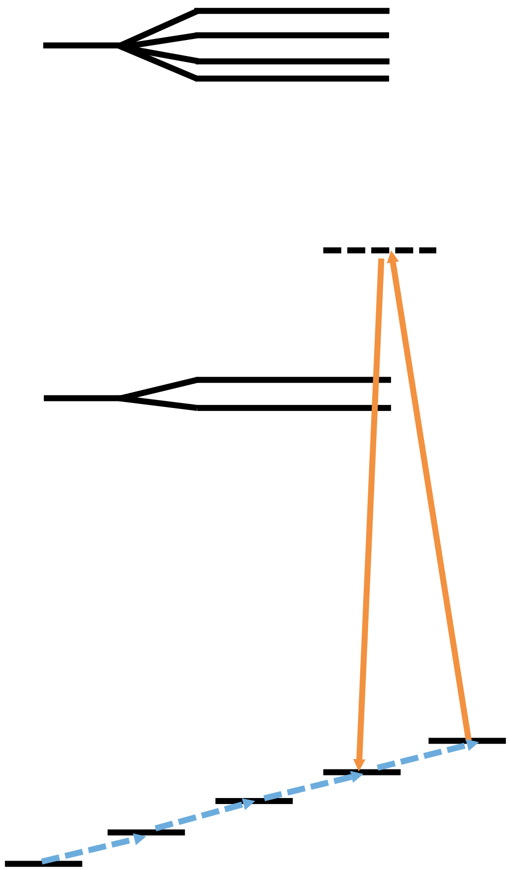
One possible experimental sequence

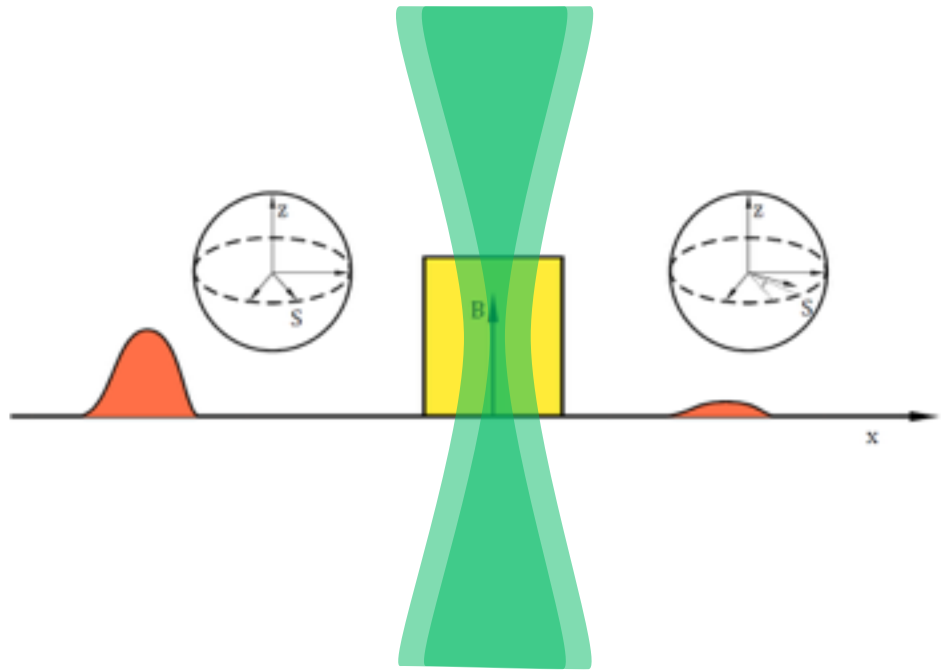


Atoms spilling *around* an optical "ReST" trap



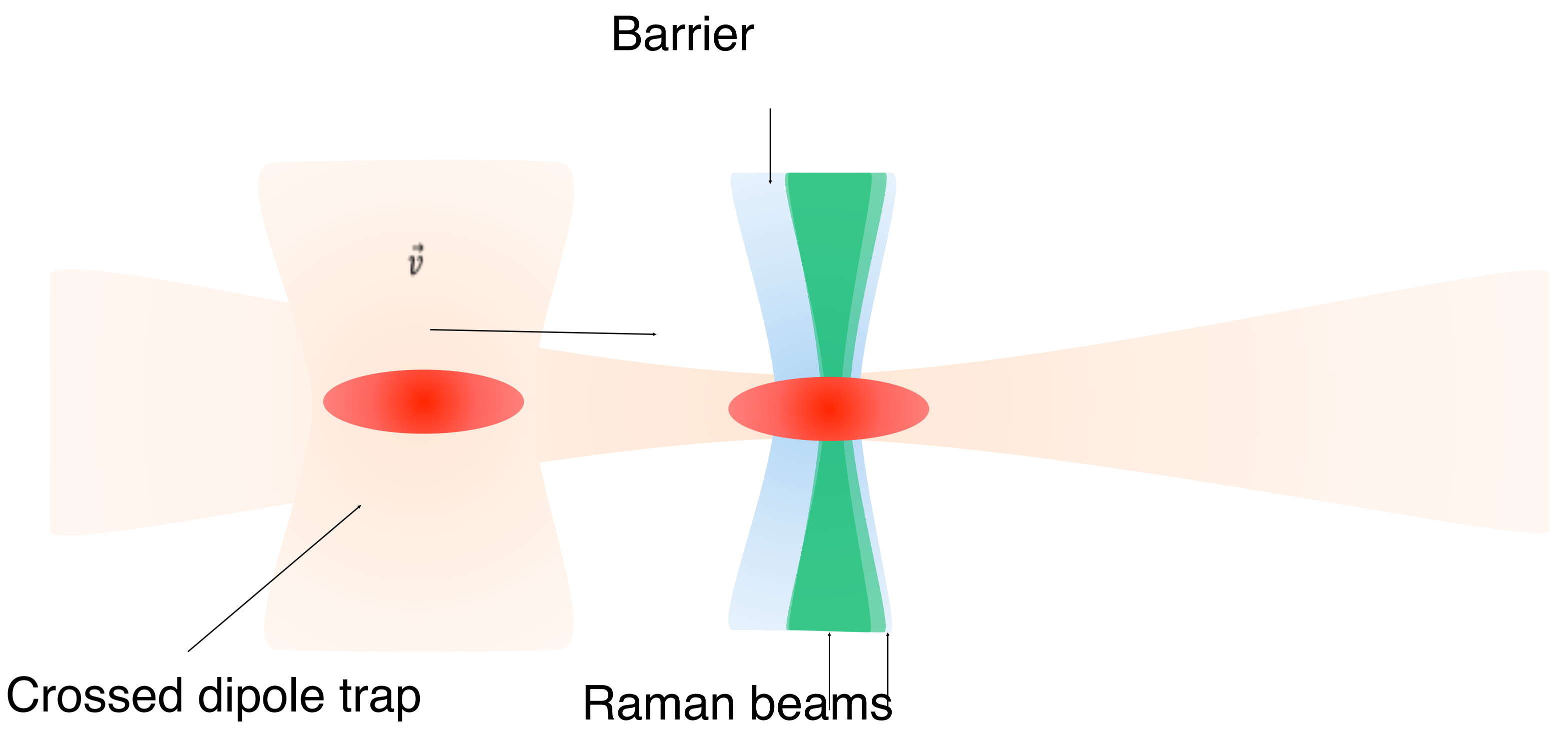
Localized (fictitious) magnetic field
(Raman coupling of two ground states)



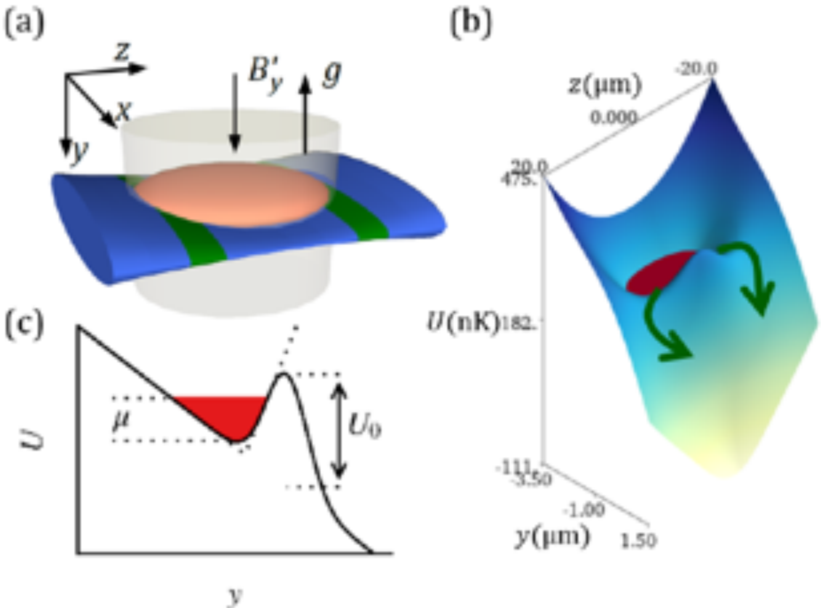


Raman beams

Experimental sequence: current plan

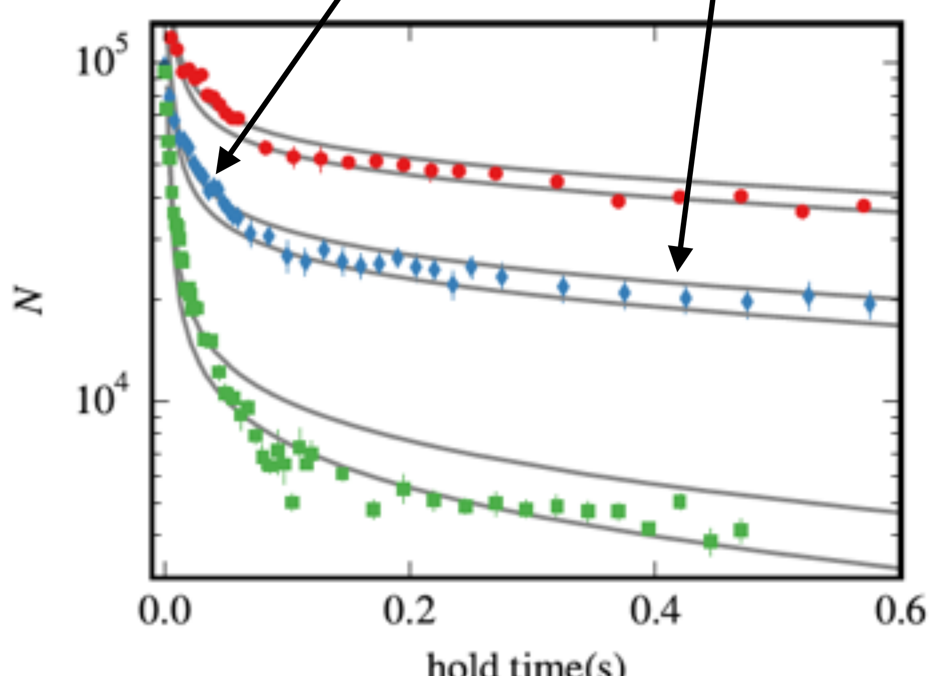


Our first observation of single-barrier tunneling



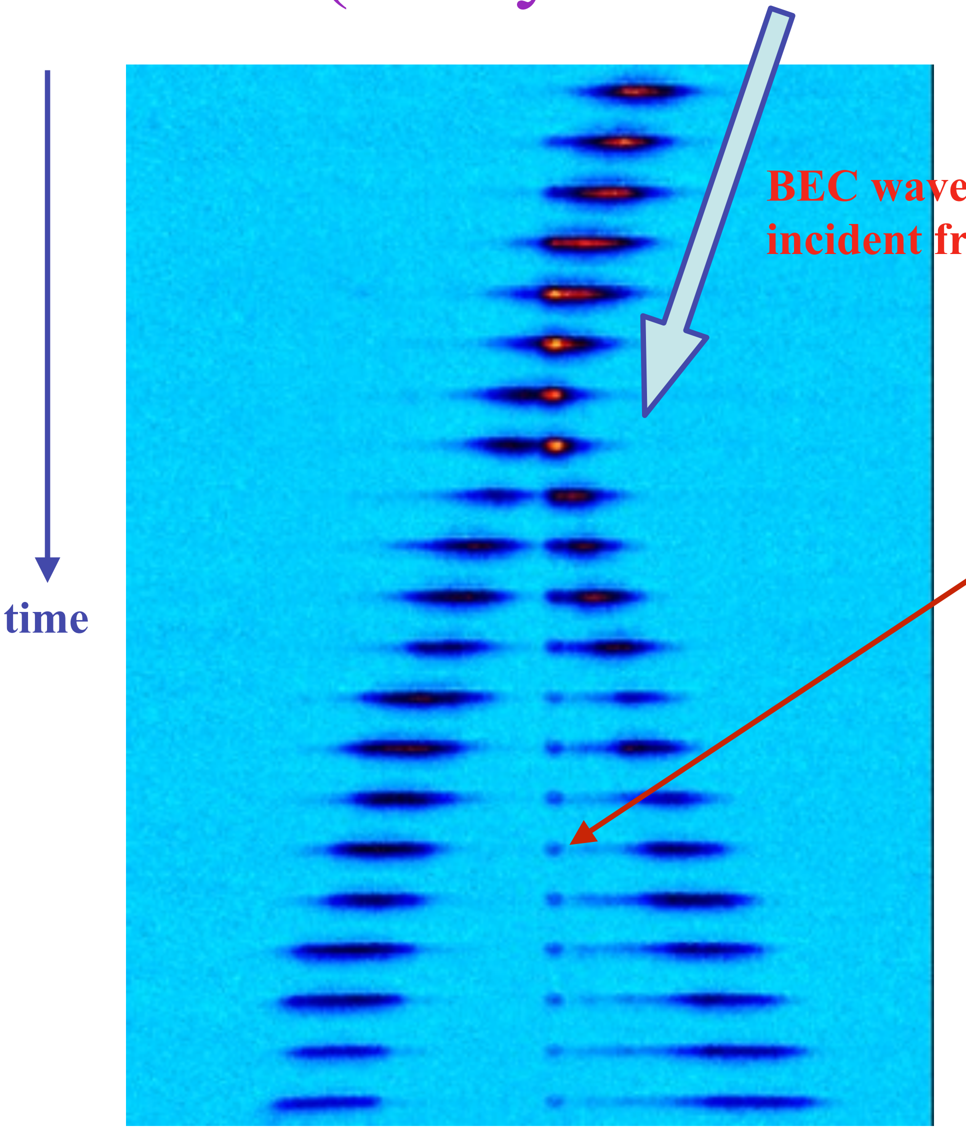
Time-dep tunneling rate due to mean-field interaction (nonlinearity)

Background decay rate



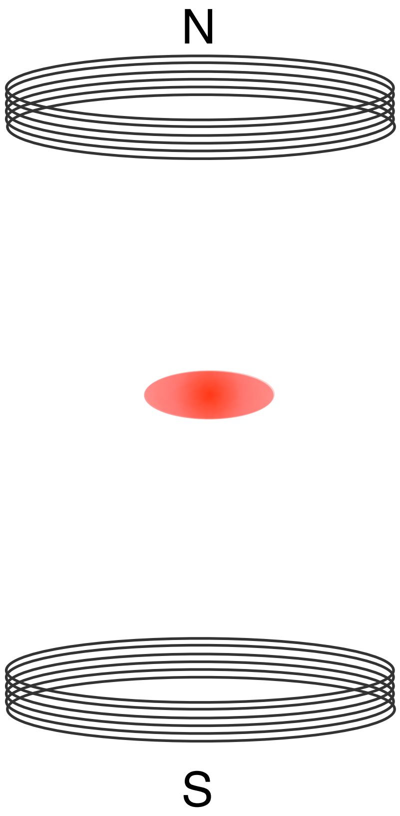
S. Potnis, R. Ramos, K. Maeda, L.D. Carr, AMS, 1604.06388;
see also earlier work, e.g.,
R. Chang, *et al.*, PRL 112, 170404 (2014)

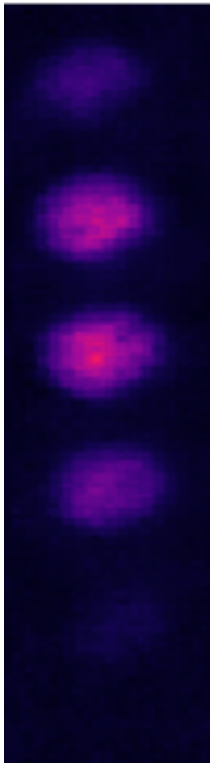
Preliminary evidence of tunneling through a *double* barrier (Fabry-Perot cavity for atoms)



A narrow frequency component of the BEC remains trapped in the FP cavity –
– we don't yet know for how long!

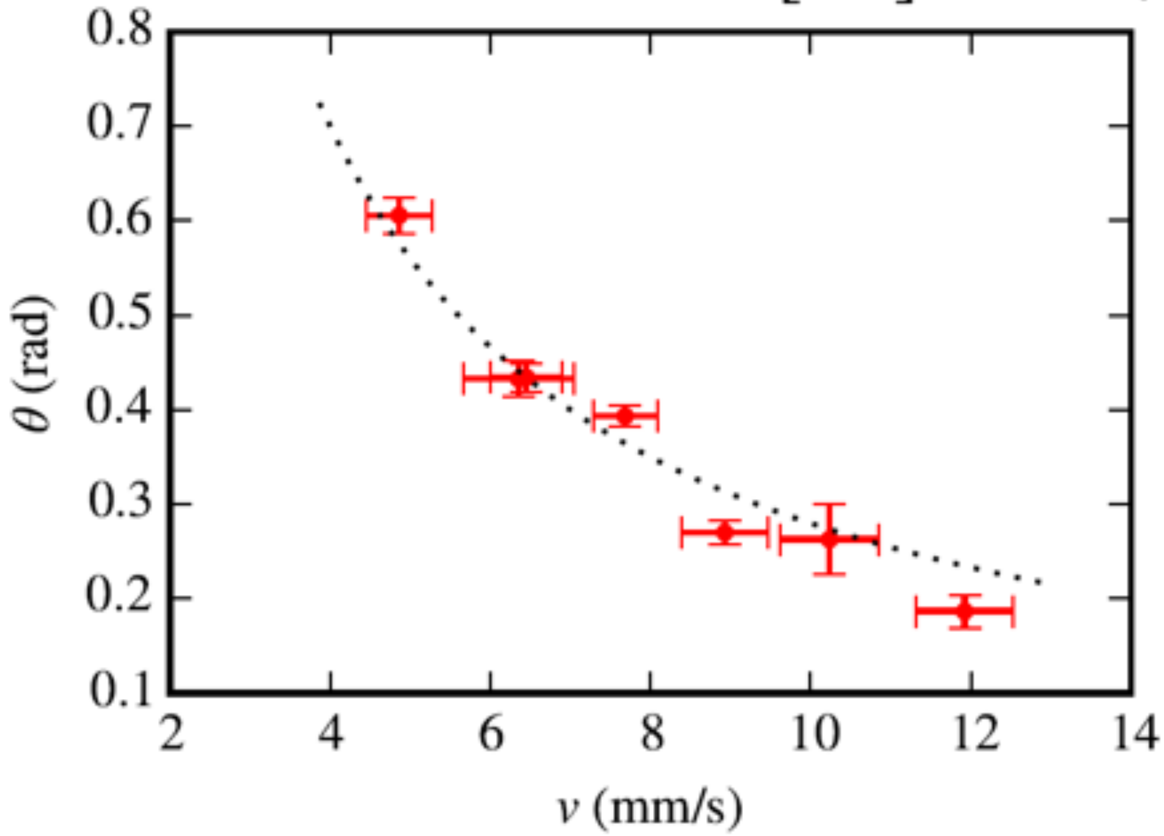
Stern-Gerlach measurement





Calibration of Larmor clock for free propagation

$\tau[\text{ms}] = 1.9(2) \times \theta[\text{rad}]$



(A [very low-precision] confirmation that : t = L / v !)

Summary

- **Even in the image plane, much (even most) of the information may be in the optical phase and not the intensity – a new route to super-resolution, requiring no structured illumination!**

W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

- **To build optimal thermometers at the mesoscopic scale, one should use coherence relaxation as well as energy relaxation.**

Such thermalization processes can be simulated using a universal optical quantum simulator.

Adaptive techniques will be useful for building finite many-spin thermometers.

W.K. Tham, H. Ferretti, A.V. Sadashivan, AMS, 1609.01589

- **After talking about it for 20 years, we are getting close to being able to probe atoms while they tunnel through an optical barrier, using weak measurement to ask "where they were" before being transmitted!**

We have preliminary evidence that our Fabry-Perot cavity for ultracold Rubidium atoms is working.

In progress – for previous work, see e.g. S. Potnis, R. Ramos, K. Maeda, L.D. Carr, AMS, 1604.06388; R. Chang, S. Potnis, R. Ramos, C. Zhuang, M. Hallaji, A. Hayat, F. Duque-Gomez, J. Sipe, AMS, PRL 112, 170404 (2014)