* Beats & Interference :-

Consider two sound waves of slightly different frequencies but moving with the same speed in air.

$$\left.\begin{aligned} f_1 \lambda_1 &= v \qquad f_1 \neq f_2 \\ f_2 \lambda_2 &= v \end{aligned}\right\} \quad \text{Equivalently} \quad \begin{aligned} \omega_1 / k_1 &= v \\ \omega_2 / k_2 &= v \end{aligned} \qquad \because \omega = 2\pi f, \quad k = \frac{2\pi}{\lambda}$$

we can ask what happens when both sources are on. The total wave is given by

$$S(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

where A is the amplitude which is assumed same for both. Now, we can add these using

$$\cos\theta + \cos\varphi = \cos\left[\left(\frac{\theta+\varphi}{2}\right) + \left(\frac{\theta-\varphi}{2}\right)\right] + \cos\left[\left(\frac{\theta+\varphi}{2}\right) - \left(\frac{\theta-\varphi}{2}\right)\right]$$

$$\therefore \boxed{\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)}$$

$$\therefore S(x,t) = 2A\cos\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right) * \cos\left(\frac{k_1-k_2}{2}x - \frac{\omega_1-\omega_2}{2}t\right)$$

If we sit at a given point, say $x=0$, & receive this signal, we will find

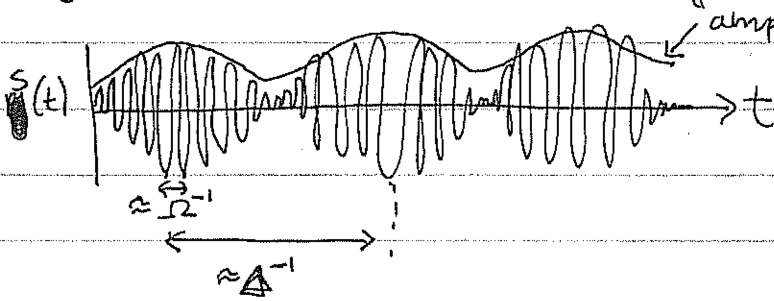
$$S(0,t) = 2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)\cos\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$$

If $\frac{\omega_1+\omega_2}{2} = \Omega$ & $\frac{\omega_1-\omega_2}{2} = \Delta$;

$$\boxed{S(0,t) = 2A\cos(\Delta t)\cos(\Omega t)}$$

The intensity of sound heard is proportional to

$S^2(0,t) = 4A^2\cos^2(\Delta t)\cos^2(\Omega t)$



If $\Delta \ll \Omega$, the intensity we hear will actually be the <u>short time average of $S^2(0,t)$</u> which is just $4A^2\cos^2(\Delta t)\left(\frac{1}{2}\right) = 2A^2\cos^2\Delta t = A^2(1+\cos 2\Delta t)$

$\boxed{\begin{array}{l}\text{Will appear as sound of frequency} \approx \Omega \\ \text{whose amplitude is varying with frequency} \approx \Delta \\ \text{\& intensity varies with freq. } 2\Delta \equiv |\omega_1-\omega_2|\end{array}}$