- Let us denote $\frac{T}{\mu} \equiv c^2$ for reasons which will become clear momentarily.

$$\therefore \frac{\lambda_0}{T_0} = \sqrt{\frac{T}{\mu}} = c \quad \therefore \boxed{\lambda_0 = c T_0}$$

Wave eqn:

$$\boxed{\frac{\partial^2 s}{\partial t^2} = c^2 \frac{\partial^2 s}{\partial x^2}}$$

- Consider $s(x,t) = s(x - V_0 t) \equiv s(W)$

$W = x - V_0 t$

$$\therefore \frac{\partial s}{\partial x} = \frac{ds}{dW} \cdot \underbrace{\left(\frac{\partial W}{\partial x}\right)}_{=1} = \frac{ds}{dW} \quad \therefore \boxed{\frac{\partial^2 s}{\partial x^2} = \frac{d^2 s}{dW^2}}$$

$$\frac{\partial s}{\partial t} = \frac{ds}{dW} \cdot \underbrace{\frac{\partial W}{\partial t}}_{=-V_0} = -V_0 \frac{ds}{dW}$$

$$\therefore \frac{\partial^2 s}{\partial t^2} = V_0^2 \frac{d^2 s}{dW^2}$$

$\therefore$ Substituting into the above wave eqn,

$$V_0^2 \cancel{\frac{d^2 s}{dW^2}} = c^2 \cancel{\frac{d^2 s}{dW^2}}$$

$\therefore$ $\boxed{s(x - V_0 t) \text{ is a solution to the wave eqn if } V_0 = c}$

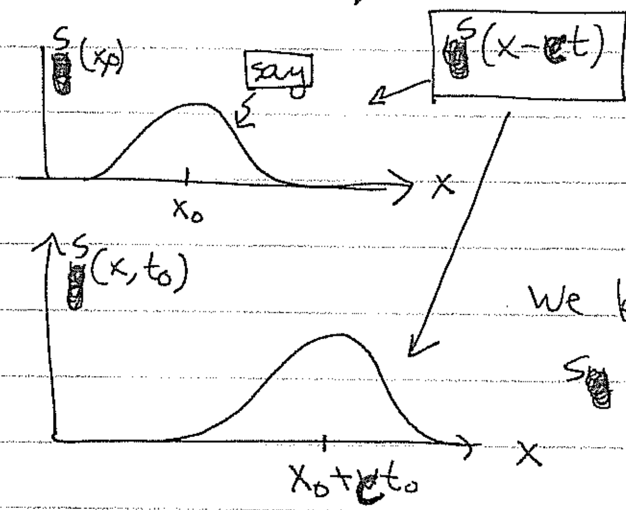
- But $s(x-ct)$ is a very general function. It could be $\cos(x-ct)$, or just $(x-ct)$ or $e^{(x-ct)}$ or anything!

- Check that $s(x+ct)$ is also similarly a general solution to the wave equ

Example

$t=0$

$t=t_0$



We know this because
$$s(x,t_0) = f(x-ct_0)$$
$$= s(x-ct_0, 0)$$

$\therefore$ Value of "s" at a given point is the same as it was at an earlier time (by t_0) at a space point to the left (by vt_0)

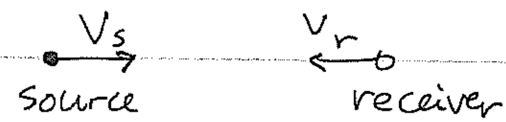
$\therefore$ $c \equiv$ "speed of the wave"

- Note: $\boxed{\lambda_0 = c T_0}$

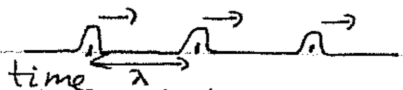
wavelength speed period

* <u>Doppler effect</u> :- (Just for this section, let speed of sound in air be called "c")

Consider a sound source moving with speed V_s towards a receiver, which is moving with speed V_r towards the source



View solutions to the wave eqn in the form of "pulses" :-

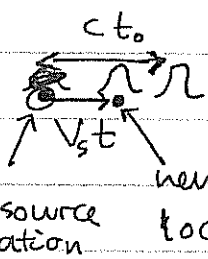


the time between these pulses is $\frac{1}{f_s}$ where f_s is the "source frequency". $f_s = \frac{c}{\lambda_0}$ where c = speed of sound in air.

Look at these pulses sent out by the source at times $0, \frac{1}{f_s}, \frac{2}{f_s}, \ldots$

$t=0$

$t = t_0 = \frac{1}{f_s}$



next pulse is emitted when 1st puls has travelled distance $\frac{c}{f_s} = ct_0$ but by then the source itself has moved by $V_s t_0 = V_s \cdot \frac{1}{f_s}$

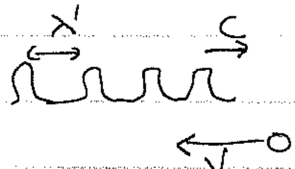
$\therefore$ Distance between 2 consecutive pulses $\equiv \lambda' = (c - V_s) \cdot \frac{1}{f}$

$\therefore$ "new wavelength" $\lambda' = (c - V_s) \cdot \frac{1}{f_s} = \left(\frac{c}{f_s}\right)\left(1 - \frac{V_s}{c}\right)$

$$\lambda' = \lambda_0\left(1 - \frac{V_s}{c}\right)$$

$\therefore$ Wavelength has decreased due to sound source moving in the direction of receiver

Next consider the receiver:-



The receiver sees pulses spaced apart by $\lambda' = \lambda_0\left(1 - \frac{V_s}{c}\right)$ and is whizzing past these at speed $(V_r + c)$ effectively.

The time interval between successive pulses as perceived by the receiver is thus

$$T_r = \frac{\lambda'}{V_r + c} = \lambda_0\left(1 - \frac{V_s}{c}\right) \cdot \frac{1}{(V_r + c)}$$

$$\frac{1}{f_r} = \frac{1}{f_s}\left(\frac{c - V_s}{c + V_r}\right)$$

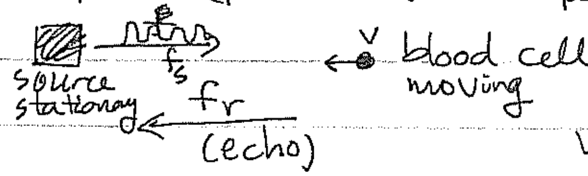
$\therefore$ Frequency as perceived by receiver is

$$\boxed{f_r = f_s \frac{(c + V_r)}{(c - V_s)}}$$

Wavelength as perceived by receiver is

$$\boxed{\lambda_r = \lambda' = \lambda_0\left(1 - \frac{V_s}{c}\right)}$$

- Knowing the frequencies f_s & f_r and one of the velocities tells us the other velocity. This is used in Doppler ultrasound, where a stationary source is used to bounce waves off a moving object (eg: blood cells) to measure the object speed (eg: blood flow speed)



echo freq. is

$$\boxed{f_r = f_s\left(\frac{c + V}{c - V}\right)}$$

We know f_s, f_r, c (c = speed in tissue

- From a wave point of view, the change in frequency is simply because what I call "x" as a stationary source is called "(x − $v_r t$)" by a receiver moving towards me (source).

$A\cos(k_0 x - \omega_0 t)$ $\quad k_0 = \frac{2\pi}{\lambda_0}$; $\omega_0 = \frac{2\pi}{T_0} \equiv 2\pi f_s$

transforms into

$$A\cos\left(k_0(x - v_r t) - \omega_0 t\right) = A\cos\left(k_0 x - \underbrace{(\omega_0 + k_0 v_r)}_{\omega_r} t\right)$$

$$\therefore \quad \omega_r = \omega_0 + k_0 v_r = \omega_0 + \left(\frac{\omega_0}{c}\right) v_r = \omega_0\left(1 + \frac{v_r}{c}\right)$$

$$\therefore \quad \boxed{f_r = f_s\left(1 + \frac{v_r}{c}\right)}$$

- Doppler effect is used by bats, for example, who use echo[location] to determine ~~to~~ the location & speed of their prey (insects).