Basic Math Review I

* Basic derivatives :-

① $\frac{d}{dx} x^n = n x^{n-1}$

② $\frac{d}{dx} \sin(xa) = (\cos(xa)) \cdot a$

③ $\frac{d}{dx} \cos(xa) = -(\sin(xa)) \cdot a$

④ $\frac{d}{dx} e^{ax} = a e^{ax}$

⑤ $\frac{d}{dx} \ln x = \frac{1}{x}$

Chain rules :

[1] $\frac{d}{dx}(f \cdot g) = f \frac{dg}{dx} + g \frac{df}{dx}$

[2] $\frac{d}{dx} f(u(x)) = \frac{df}{du} \cdot \frac{du}{dx}$

Derivative is the slope of function at a point

* Basic integrals :-

① $\int dx \cdot x^n = \frac{x^{n+1}}{n+1} + \text{const.}$

② $\int dx \sin(xa) = \frac{-\cos(xa)}{a} + \text{const.}$

③ $\int dx \cos(xa) = \frac{\sin(xa)}{a} + \text{const.}$

④ $\int dx \, e^{ax} = \frac{e^{ax}}{a} + \text{const.}$

⑤ $\int dx \cdot \frac{1}{x} = \ln x + \text{const}$

Chain rule

$\int dx \, f \cdot \frac{dg}{dx} = fg - \int g \frac{df}{dx} \cdot dx$

Definite integral

If $\int f(x) dx = F(x) + c$

$\int_a^b f(x) dx = F(b) - F(a)$

Integrals are areas of the function

Differential Eqns : Math review II

* Simple differential equations:-

(1) $$\frac{dy}{dx} = x^n \Rightarrow y = \frac{x^{n+1}}{n+1} + \text{const}$$

(2) $$\frac{dy}{dx} = \sin ax \Rightarrow y = \frac{-\cos ax}{a} + \text{const}$$

(3) $$\frac{dy}{dx} = \cos ax \Rightarrow y = \frac{\sin ax}{a} + \text{const}$$

(4) $$\frac{dy}{dx} = e^{ax} \Rightarrow y = \frac{e^{ax}}{a} + \text{const}$$

(5) $$\frac{dy}{dx} = \frac{1}{x} \Rightarrow y = \ln x + \text{const}$$

(6) $$\frac{d^2y}{dx^2} = \sin x \Rightarrow \frac{dy}{dx} = -\cos x + \overset{a}{\cancel{\text{const}}} \Rightarrow y = -\sin x + \underbrace{ax + b}_{\text{2 constants}}$$

* Slightly (but not much) more complicated diff. eqns :-

(1) $$\frac{dy}{dx} = \pm ay \Rightarrow y = Be^{\pm ax}$$

(2) $$\frac{d^2y}{dx^2} = a^2 y \Rightarrow y = Be^{ax} + Ce^{-ax}$$

(3) $$\frac{d^2y}{dx^2} = -k^2 y \Rightarrow y = Be^{ikx} + Ce^{-ikx}$$

(4) $$\frac{d^2y}{dx^2} = -k^2 y \Rightarrow y = A\sin kx + D\cos kx$$

* General 1st order eqn:

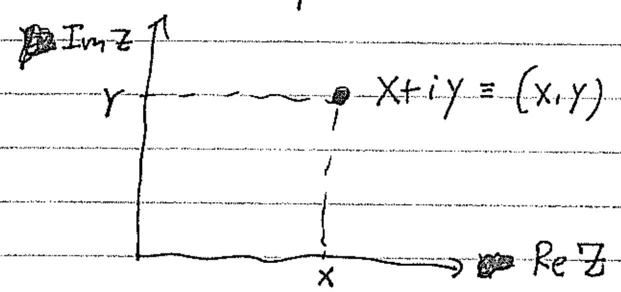
$$\frac{dy}{dx} + p(x)\cdot y = q(x) \Rightarrow y = \frac{\int u(x)\,q(x)\,dx + c}{u(x)}$$

w/ $u(x) = \exp\left(\int p(x)\,dx\right)$ = "integrating factor"

Complex numbers : Math Review III

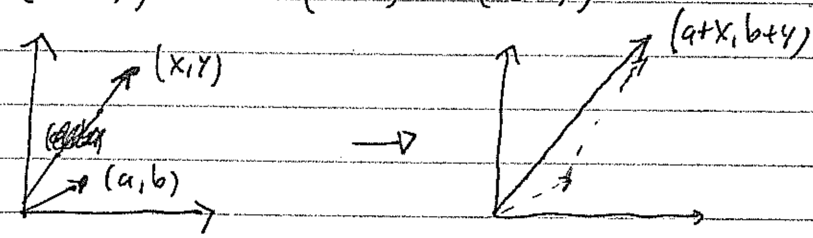
* $\boxed{z^2+1=0 \Rightarrow z = i \quad , \quad \text{ie}: \; i^2=-1}$ → Definition & why "i" arises in doing algebra.

* Geometric interpretation :



* Adding/Multiplying complex numbers :

[1] $(a+ib)+(x+iy) = (a+x)+i(b+y)$



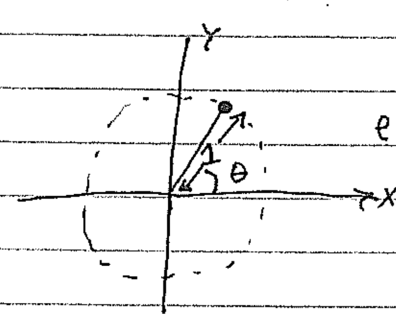
LIKE VECTOR ADDITION

[2] $(a+ib)(x+iy) = (ax-by)+i(ay+bx)$: weird [not a "vector" process]

* Euler identity :

$e^{i\theta} = \cos\theta + i\sin\theta$

Geometric



$e^{i\theta}$ traces out unit circle in the XY plane

* <u>Derivation of Euler relation</u> :-

1 Let $f(x) = \cos x + i \sin x$;

$$\therefore \frac{d}{dx}\left[f(x)\, e^{-ix}\right] = \left(\frac{df}{dx}\right) e^{-ix} + f(x) \cdot (-i)\, e^{-ix}$$

$$= (-\sin x + i\cos x)\, e^{-ix} - i\,(\cos x + i \sin x)\, e^{-ix}$$

$$= 0$$

$$\therefore \quad f(x)\, e^{-ix} = \text{constant}$$

But $f(0)\, e^{-i0} = 1 \Rightarrow \boxed{f(x)\, e^{-ix} = 1}$

Now $f(x)\, \cancel{e^{-ix}} \cdot \cancel{e^{+ix}} = e^{ix} \Rightarrow$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

2 Let $g(x) = e^{ix}$

$$\therefore \frac{dg}{dx} = i e^{ix} = ig$$

But $\frac{d}{dx}(\cos x + i \sin x) = (-\sin x + i \cos x) = i(\cos x + i \sin x)$

$\therefore \quad \boxed{g = \cos x + i \sin x}$ $\quad$ ($\because$ they agree at $x = 0$)