Practice problems (PHY231H)

1. We studied, in class, the motion of an oscillator when an external periodic force of single frequency ω is applied to it. Assume an external force such that the oscillator equation becomes

$$\frac{d^2 s}{dt^2} = -\omega_0^2 s - \gamma \frac{ds}{dt} + f_0 \cos(\omega t) + f_1 \cos(2\omega t) \tag{1}$$

Obtain the steady state solution to this which does not depend on initial conditions.

2. We wrote down the wave equation in class for the displacement $s(x,t)$:

$$\frac{\partial^2 s}{\partial t^2} = v^2 \frac{\partial^2 s}{\partial x^2} \tag{2}$$

Assume a solution of the form $A\cos(x - ct)$ and plug it into the above equation. For what values of A and c is this a solution? Assume another solution of the form $A(x + ct)^2$ and see for what values of A and c it satisfies the above differential equation.

3. You are given a pendulum consisting of a point mass m hanging from a string of length L. The acceleration due to gravity is g. Let ω_0 be the pendulum natural (angular) frequency. If you double m, what is the new angular frequency? If you double the string length, find the new angular frequency.

4. Consider a weakly damped oscillator described by the equation

$$\frac{d^2 s}{dt^2} = -\omega_0^2 s - \gamma \frac{ds}{dt} \tag{3}$$

with $\gamma < \omega_0$. The particle starts off with some velocity and begins making oscillations. Roughly how many periods will such a particle oscillate before it loses most of its energy?

5. You are given two oscillators, with close by natural frequencies ω_1 and ω_2, with $\omega_2 > \omega_1$. Both oscillators have the same mass, and the same damping γ which appears in Eq.1. If γ is really tiny, the two oscillators will respond very differently to an external force $f_0 \cos \omega t$ since they will show resonance when $\omega = \omega_1$ in one case and $\omega = \omega_2$ in the other case. Roughly how large must γ become beyond which the energy absorption of the two oscillators becomes very similar?