Problem Set 1 (PHY231H) - Due: Oct 19th, 2010

1. We discussed in class that the number of heartbeats in the lifetime a mammal is (roughly) the same in a variety of mammals with lifespans ranging from the order of 1 year to over 100 years. Speculate on plausible explanations for this observation.

2. One of the themes of this course, which I mentioned in the introductory lecture is 'randomness'. Give two examples of events from your everyday experience where you think randomness plays an important role. Give two examples of biological processes where randomness is important.

3. Consider a damped harmonic oscillator described by the 'force' equation

$$m\frac{d^2x(t)}{dt^2} = -m\omega_0^2 x(t) - \gamma\frac{dx(t)}{dt} \tag{1}$$

Assume the initial conditions $x(0) = x_0$ and $\frac{dx}{dt}(t = 0) = 0$. Find the displacement $x(t)$ of this oscillator. Using this, find its energy as a function of time. Assuming units in which $m = 1$, $\omega_0 = 1$, $x_0 = 1$, plot the displacement and the energy for different values of damping: $\gamma = 0.1, 1, 10$. (You can use python to make the plots.)

4. We studied, in class, the energy absorption of the oscillator when an external periodic force of frequency ω is applied to it. In this case, the above differential equation changes to

$$m\frac{d^2x(t)}{dt^2} = -m\omega_0^2 x(t) - \gamma\frac{dx(t)}{dt} + f\cos(\omega t) \tag{2}$$

Obtain the steady state solution to this which does not depend on initial conditions. Using this find the energy absorbed by the system in each period $2\pi/\omega$ of the forcing. Let us choose units in which $m = 1$, $\omega_0 = 1$, and $f = 1$. Plot the energy consumed by the oscillator as a function of ω for $\gamma = 0.01, 0.1, 1, 10$. Comment on the qualitative changes in these plots as γ increases. (You can use python to make the plots.)