Problem Set 4 (PHY231H) - Due: Dec 2nd, 2010

1. A certain compound in solution has an initial concentration

$$c(x, t = 0) = \frac{n}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{x^2}{2\sigma^2}}. \tag{1}$$

Assuming the concentration follows a diffusion equation

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \tag{2}$$

and a "guess" solution for spreading of the compound molecules,

$$c(x, t) = \frac{n}{\sqrt{2\pi r^2(t)}} \mathrm{e}^{-\frac{x^2}{2r^2(t)}} \tag{3}$$

obtain the equation satisfied by $r(t)$. With the initial condition $r^2(t = 0) = \sigma^2$, solve this equation, and plot the resulting concentration profile schematically as a function of time. (Hint: Maybe simpler to solve the equation for $r^2(t)$.)

2. A certain compound in solution has an initial periodic (wavelike) concentration profile

$$c(x, t = 0) = \sin(k\ x) \tag{4}$$

where $\lambda = 2\pi/k$ is the period of this pattern. Assuming the concentration follows a diffusion equation

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \tag{5}$$

and a "guess" solution

$$c(x, t) = f(t) \sin(k\ x) \tag{6}$$

for the concentration at a later time, obtain the equation satisfied by $f(t)$. With the initial condition $f(t = 0) = 1$, solve this equation, and plot the resulting concentration profile schematically as a function of time. What happens to the time for slow relaxation of this concentration if we make λ larger and larger?