

Earth's changing mass distribution: Making the most of GRACE

Frederik J Simons

F. A. Dahlen, Jessica C. Hawthorne, Dong V. Wang

Princeton University

Mark A. Wieczorek

IPG Paris

Lei Wang

Ohio State University

Shin-Chan Han

NASA Goddard

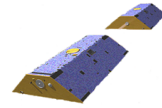
Chris T. Harig, Kevin W. Lewis, Alain Plattner

Princeton University



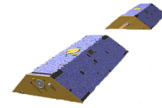
Outline

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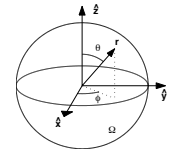


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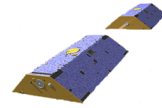


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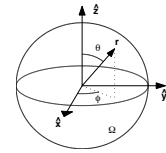


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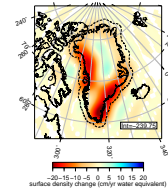
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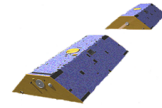


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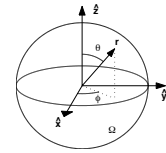


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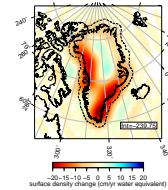
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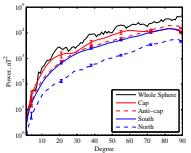
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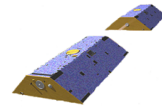


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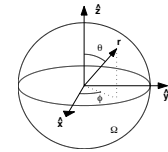


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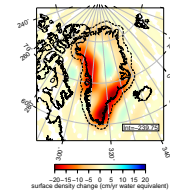
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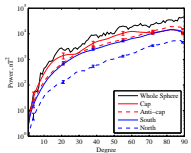
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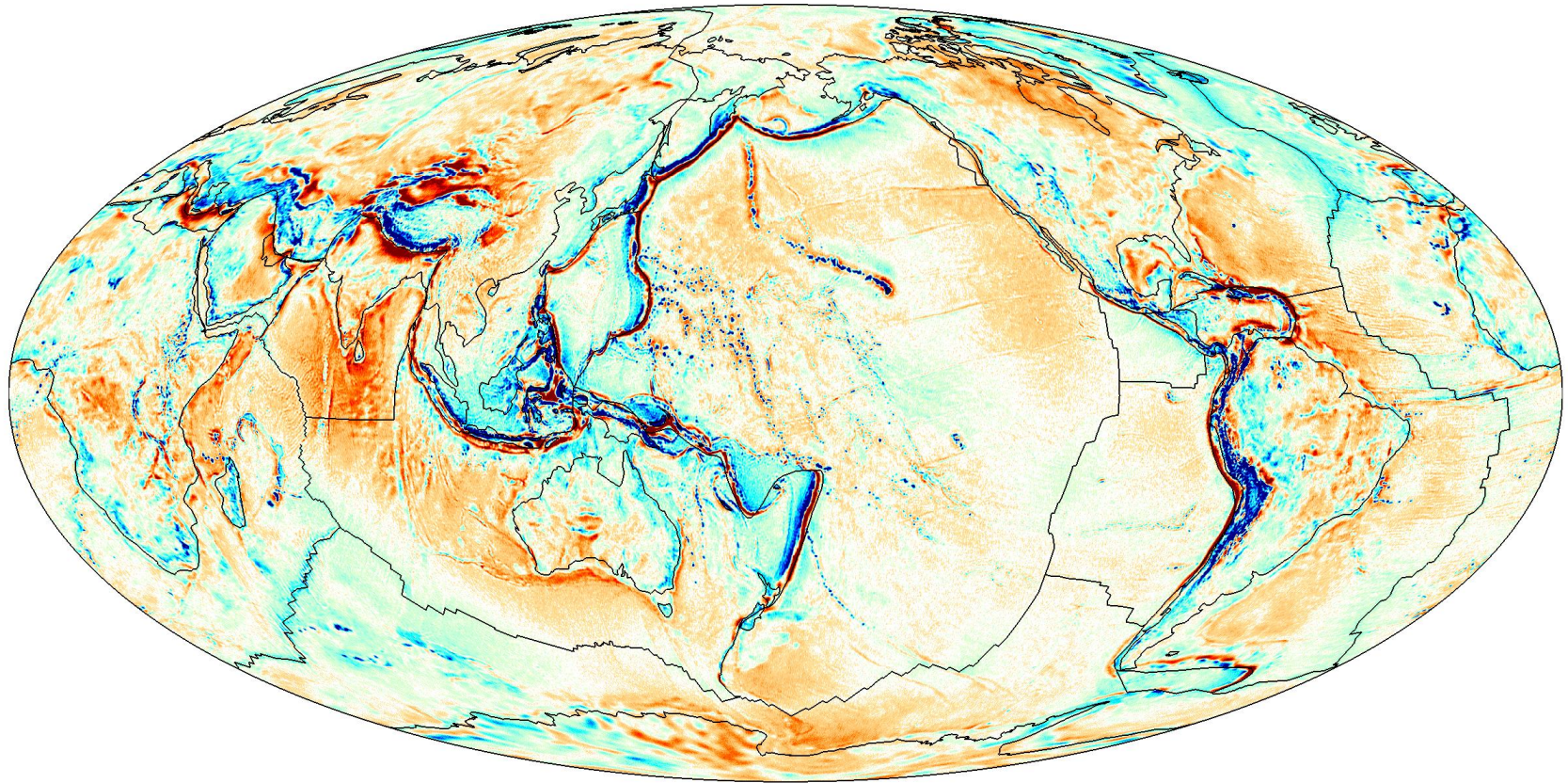


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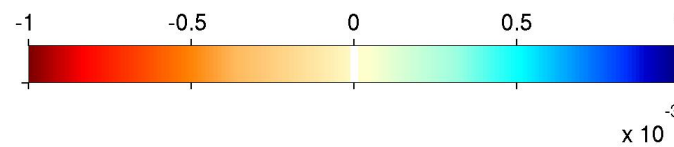


- *What have we learned? Conclusions & Outlook*
-

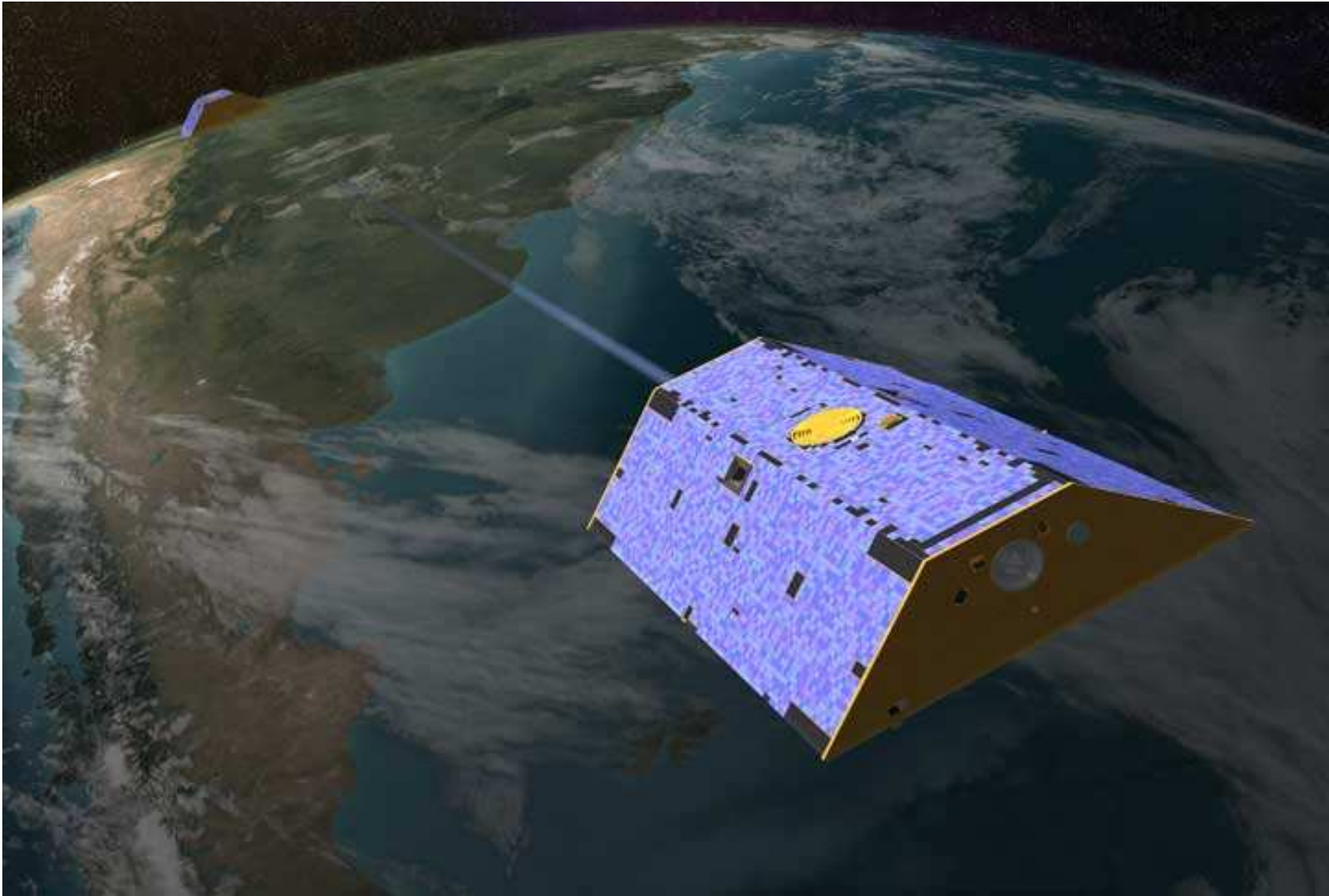
Earth's gravity field is highly variable...



EGM2008 free-air anomaly to L = 1500 with respect to WGS84 [m/s^2]



...and it changes over time



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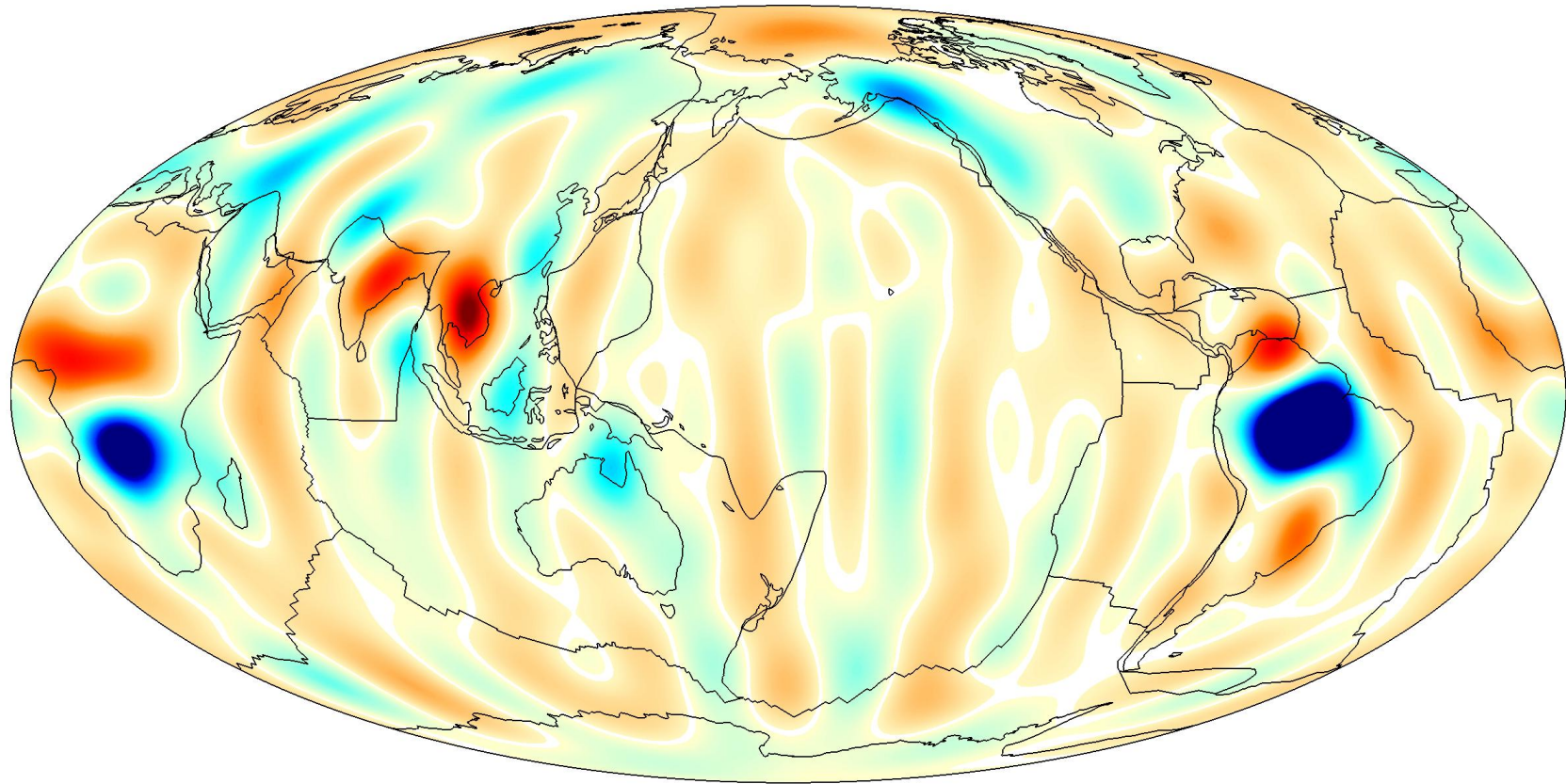
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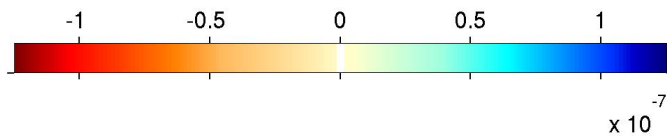
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 - **The question is, of course:**
with what *spatial, temporal, and spectral* resolution?
-

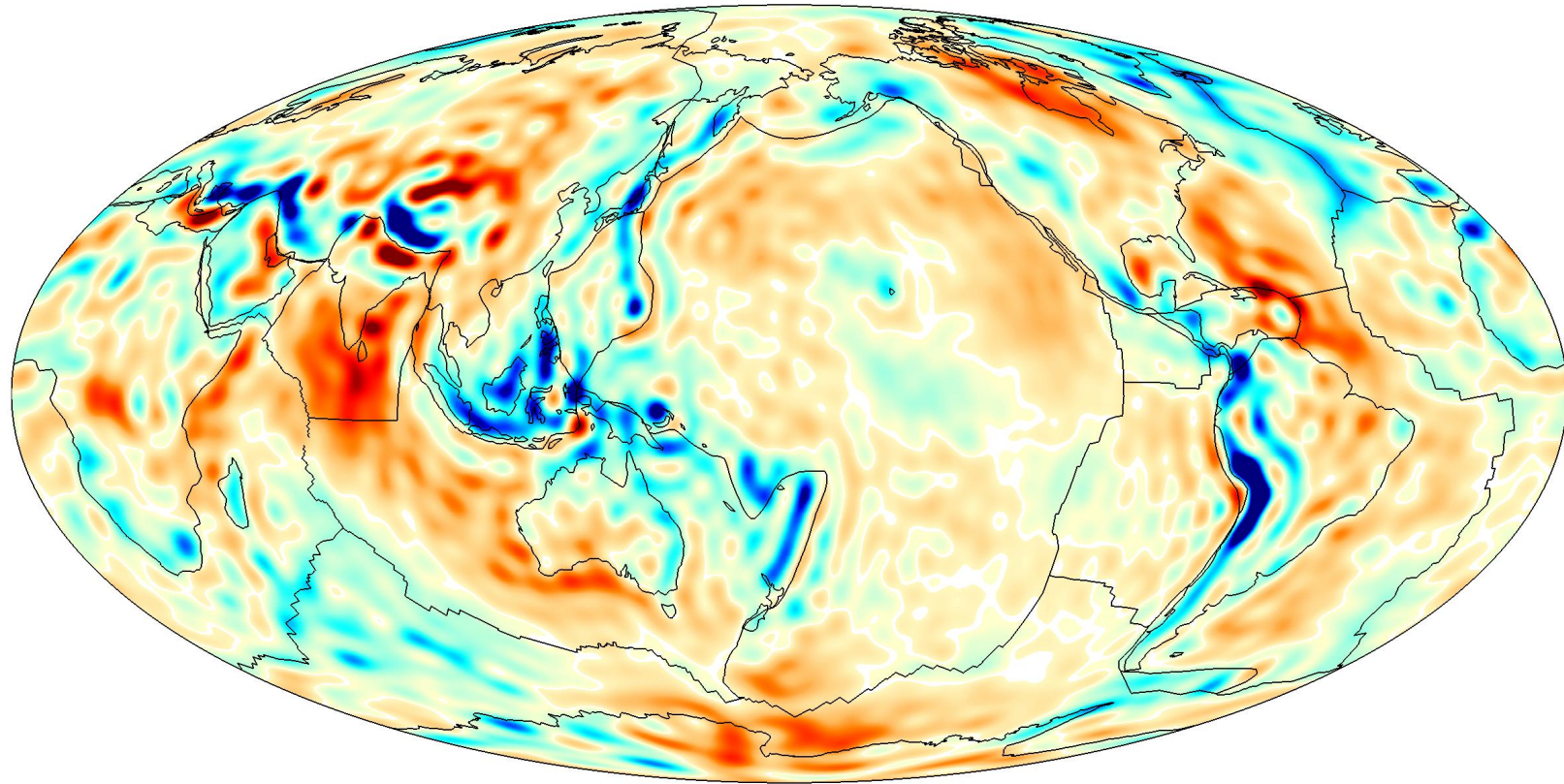
The hydrological signal is big and large



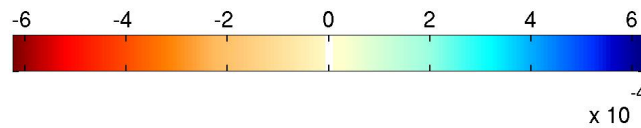
difference in free-air anomaly filtered to $L = 18$ [m/s²]
between October 2009 and April 2009



What lurks in the high-frequency “noise”? – 1

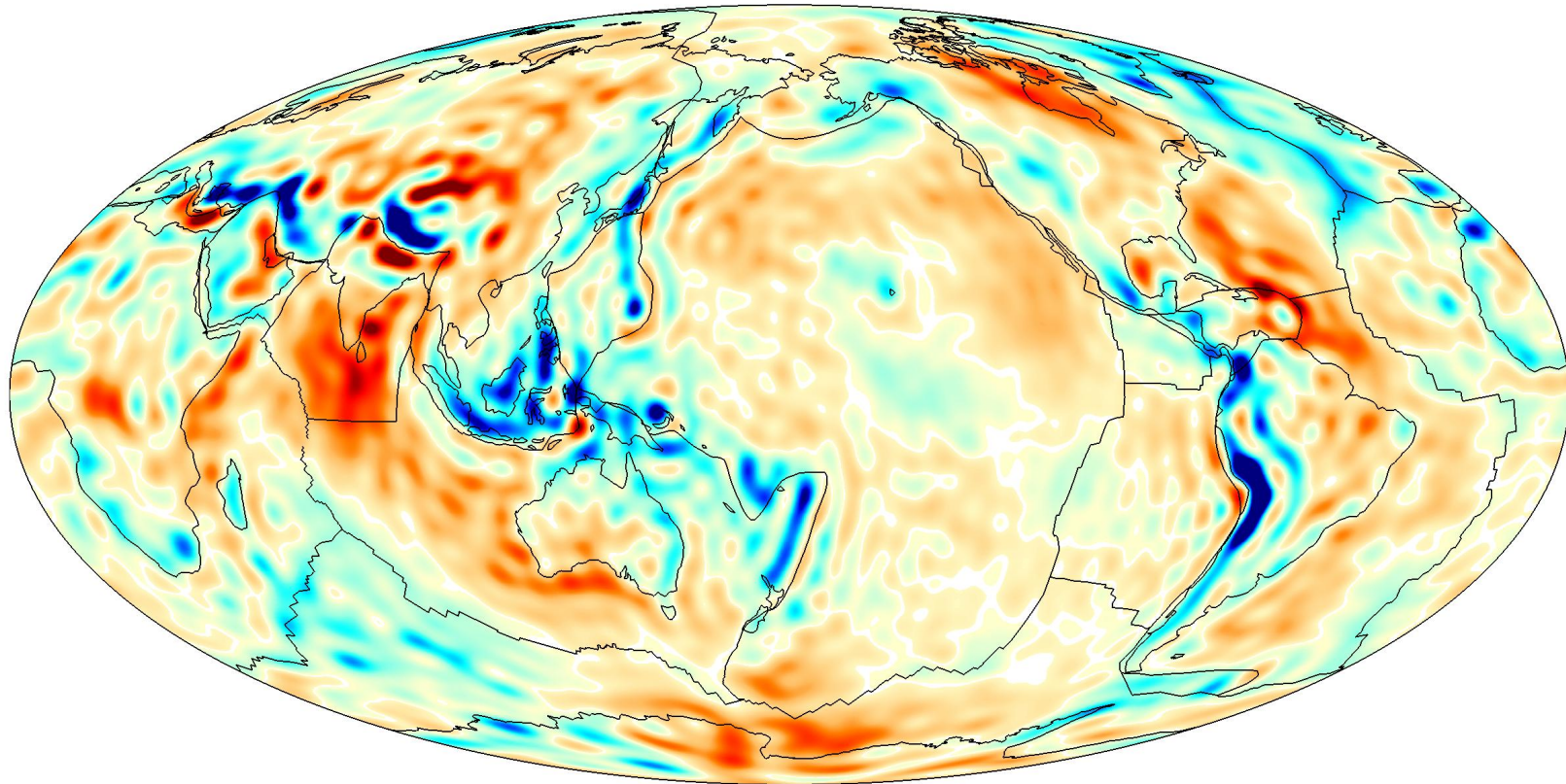


free-air anomaly filtered to $L = 58$ [m/s^2]
in December 2004

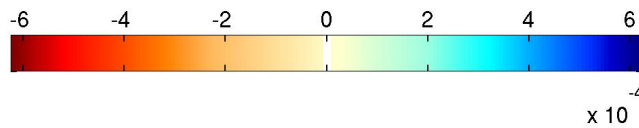


$\times 10^{-4}$

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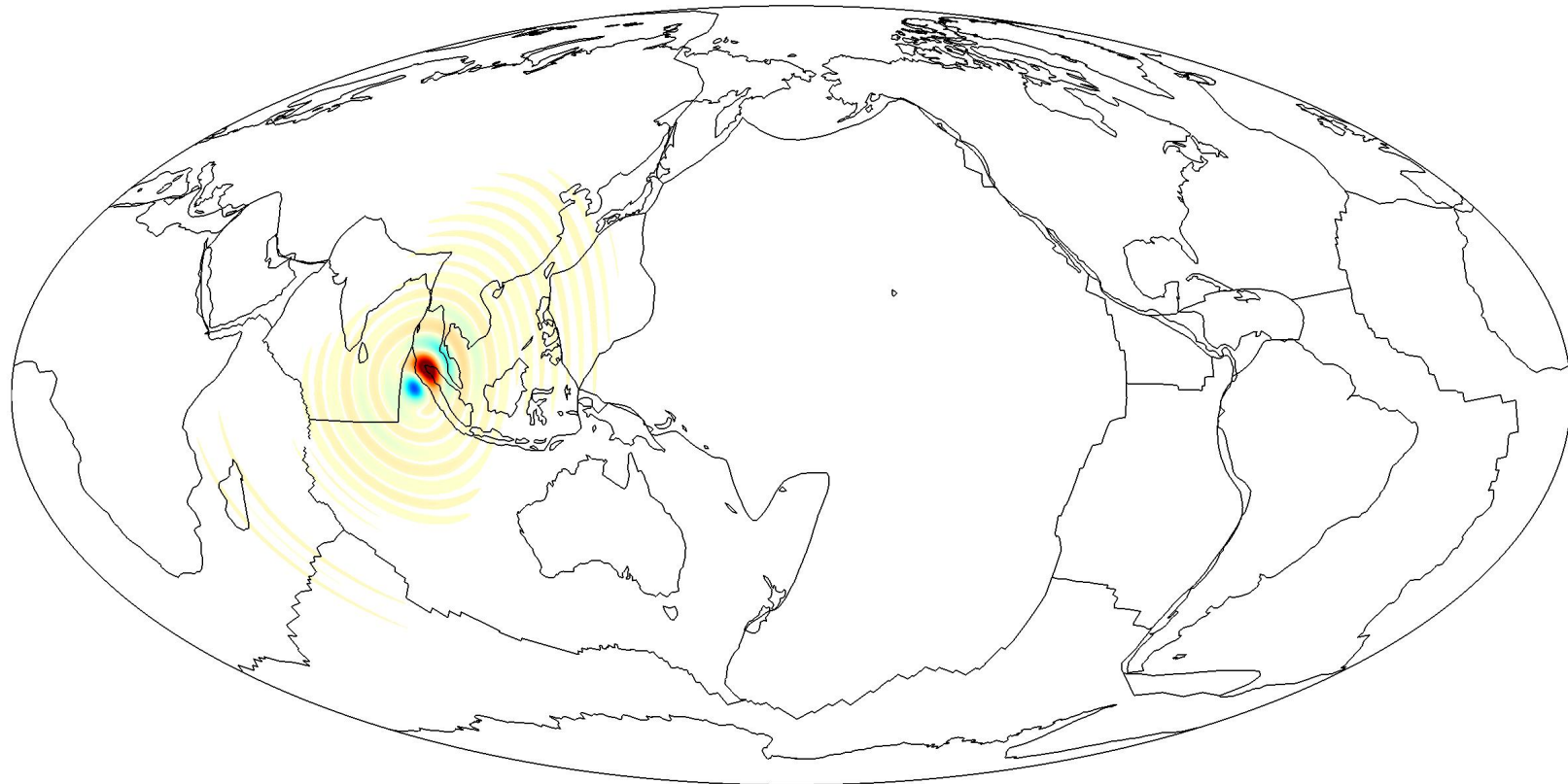


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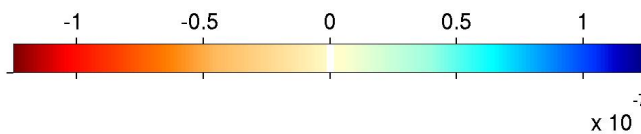


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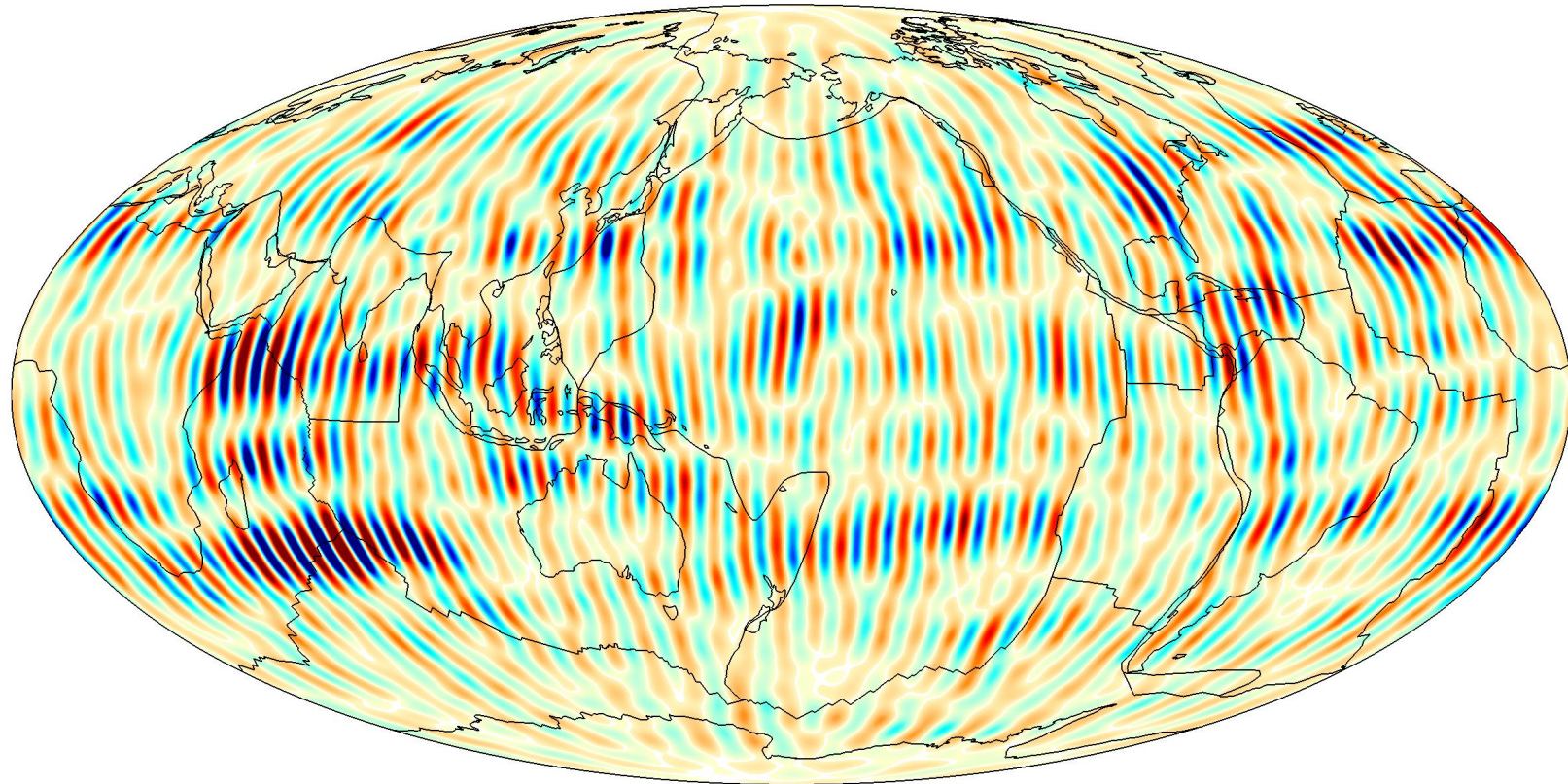
Earthquakes are small (even large ones)



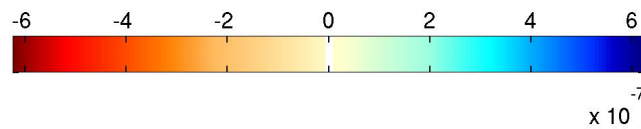
free-air anomaly to $L = 60$ predicted due to M122604A [m/s^2]



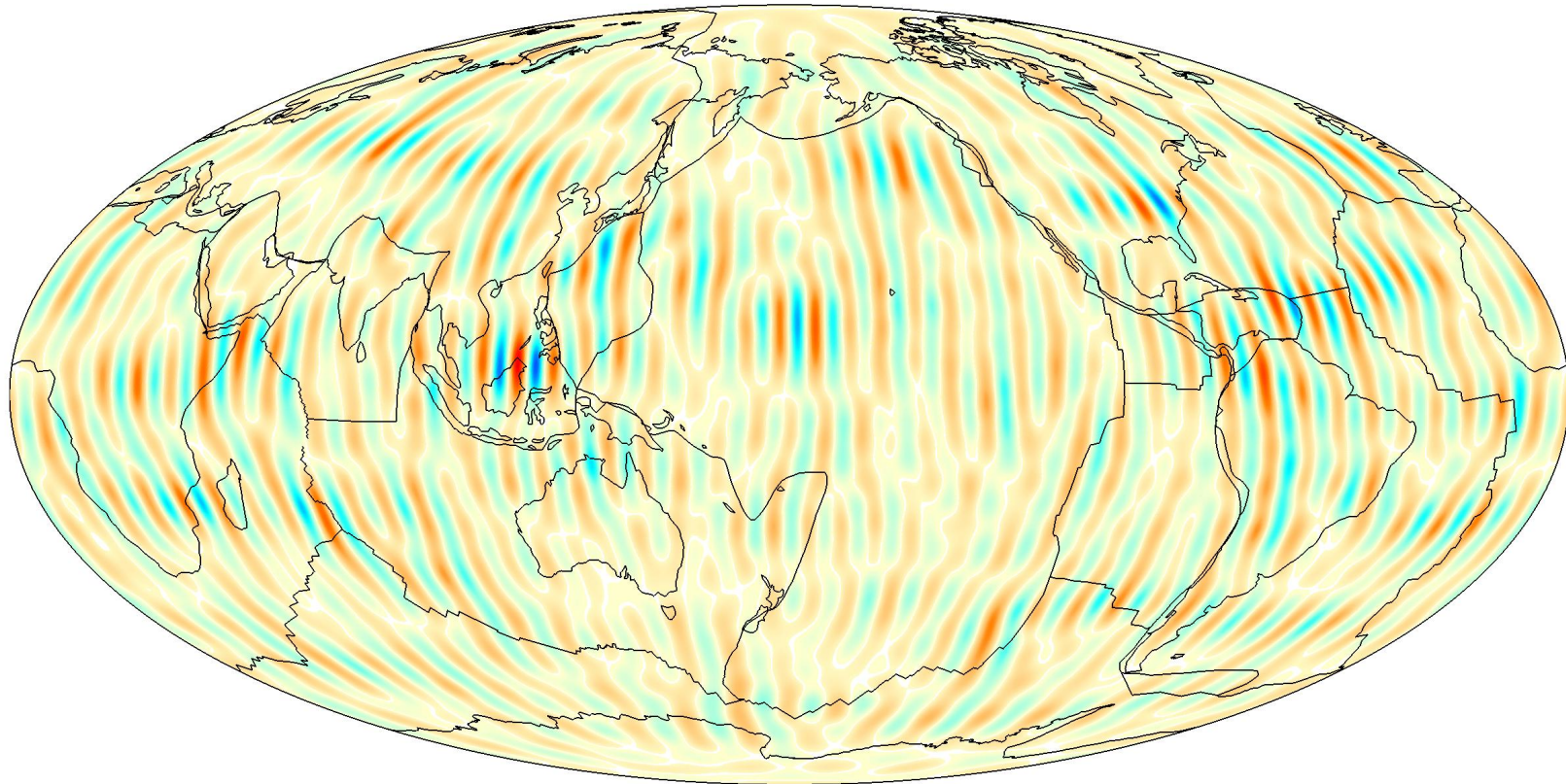
Difference Jan 2005 – Dec 2004



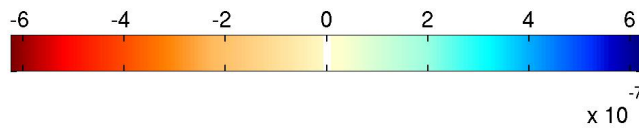
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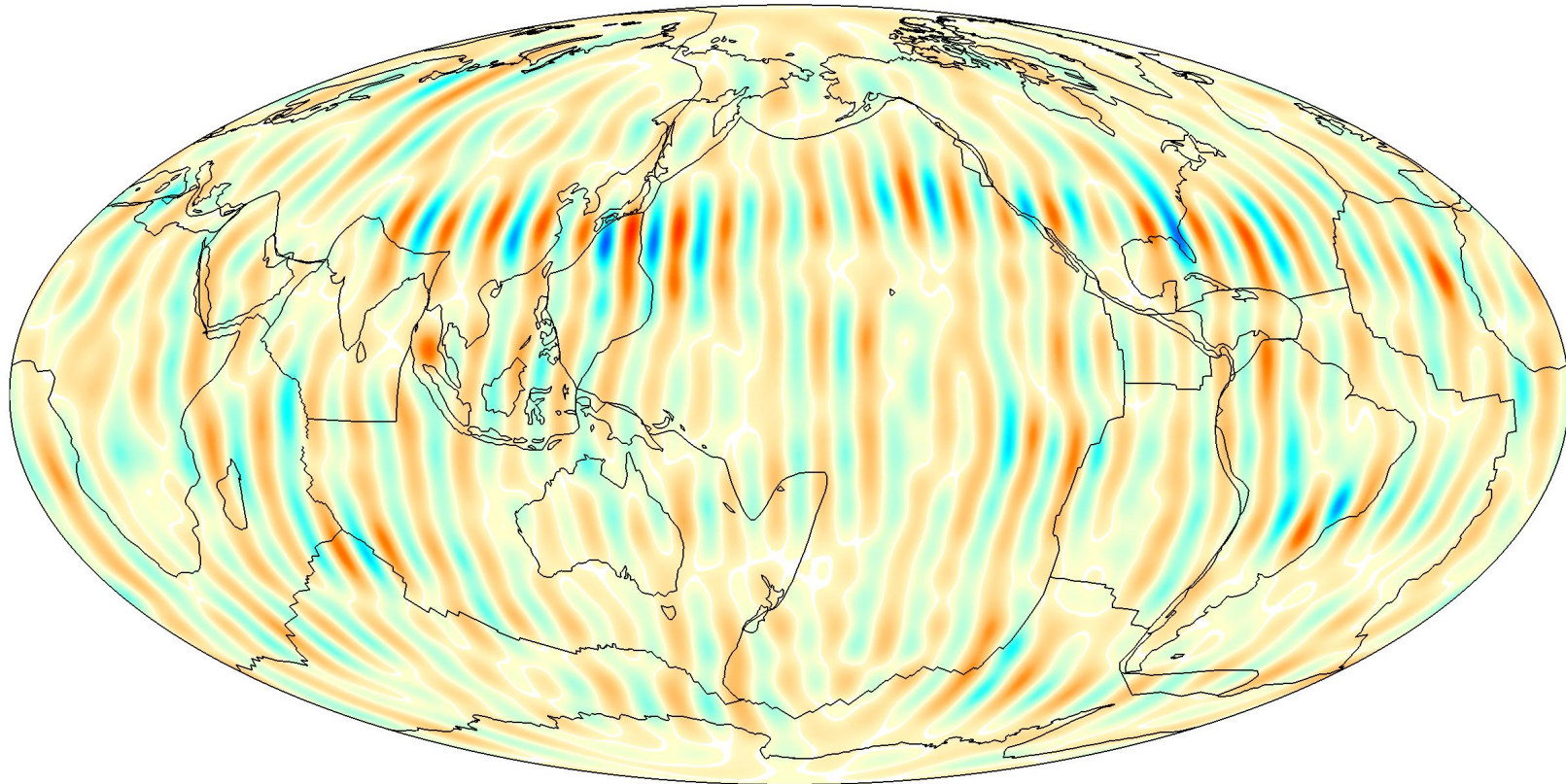
Filtered difference Jan 2005 – Dec 2004



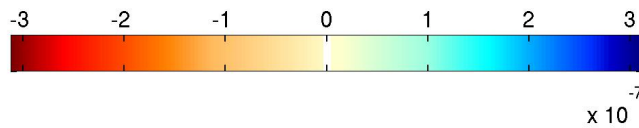
difference in free-air anomaly filtered to $L = 50 \text{ [m/s}^2]$
between December 2004 and January 2005



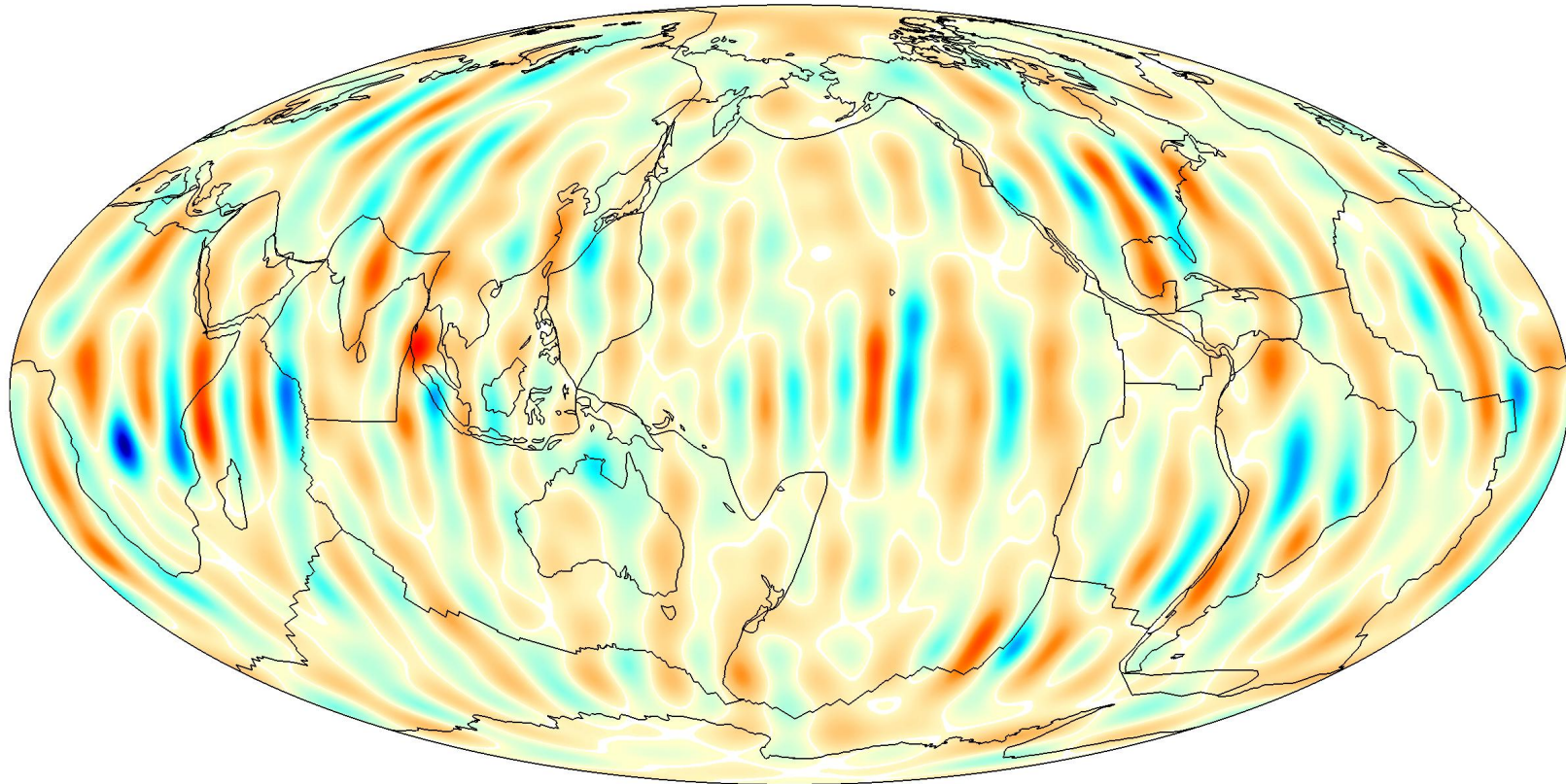
Filtered difference Jan 2005 – Dec 2004



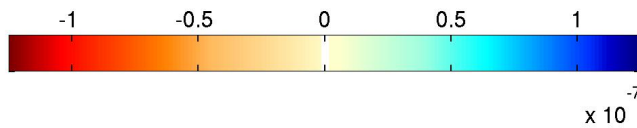
difference in free-air anomaly filtered to $L = 40$ [m/s^2]
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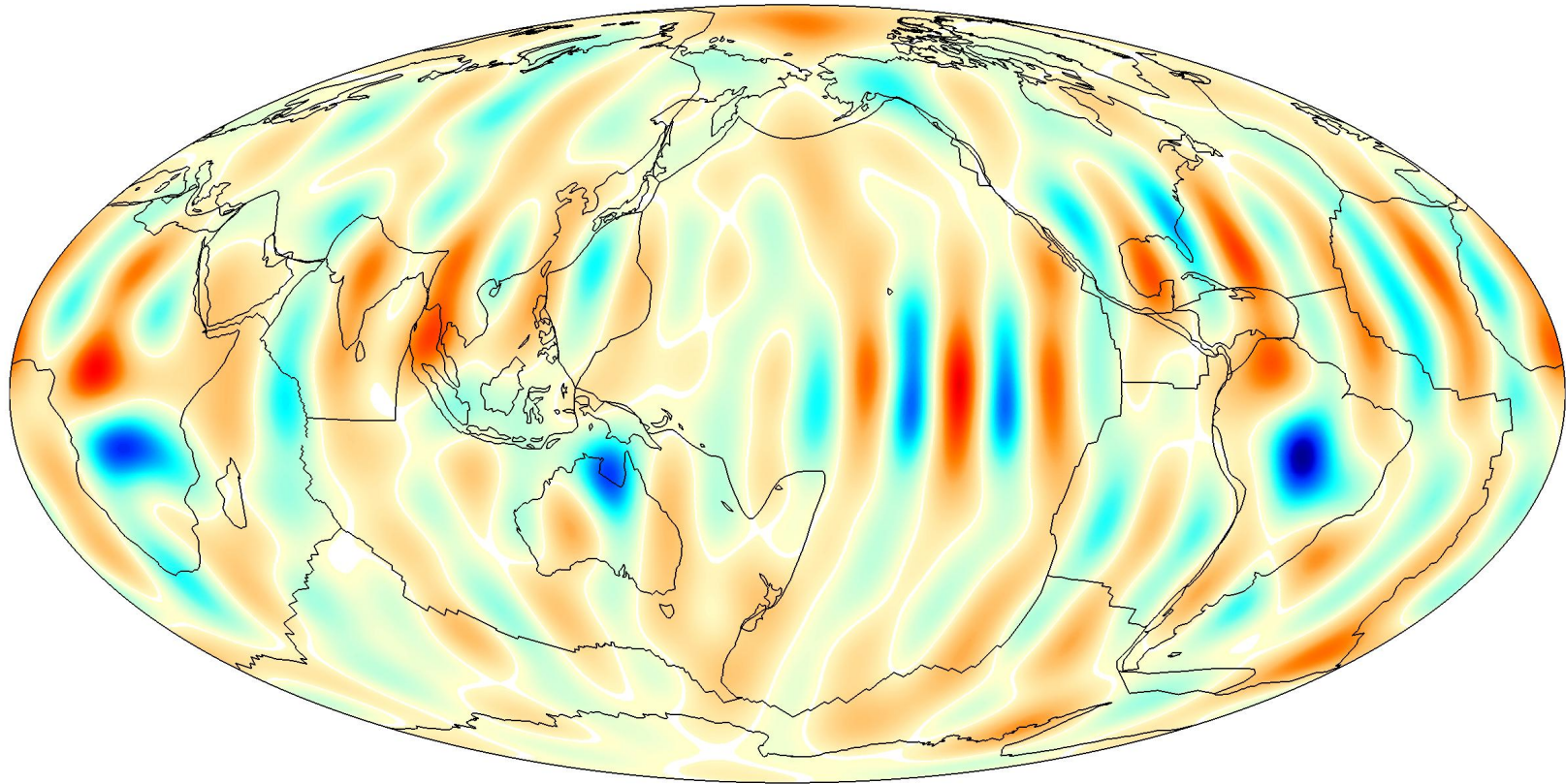
Filtered difference Jan 2005 – Dec 2004



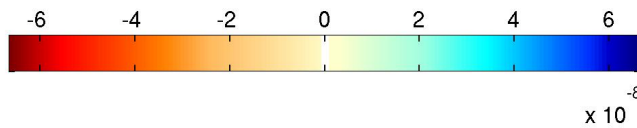
difference in free-air anomaly filtered to $L = 30$ [m/s^2]
between January 2005 and December 2004



Filtered difference Jan 2005 – Dec 2004



difference in free-air anomaly filtered to $L = 20$ [m/s^2]
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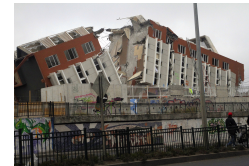
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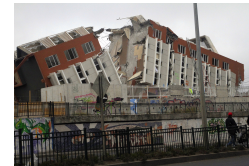
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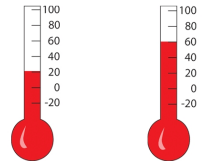
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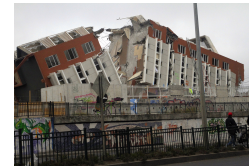
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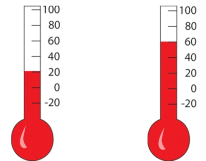
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Aware of the huge challenges to beat elevated noise levels at small spatial footprints, the community has developed a multitude of **filtering** methods to **enhance signal-to-noise ratios** and, in particular, to eliminate the prominent effect of the satellite orbits on the behavior of the solutions (**destriping**).

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 - Authors *disagree* on matters as fundamental as the **choice of basis** to represent the solution. Pixels? Mascons? Spherical harmonics?
How do these choices influence the results?
-

The problem – 1

The data **collected in** or **limited to** R are **signal plus noise**:

We may assume that $n(\mathbf{r})$ is **zero-mean** and **uncorrelated** with the signal,

and consider the **noise covariance**:

In other words: we've got **noisy** and **incomplete** data, on a sphere, Ω .

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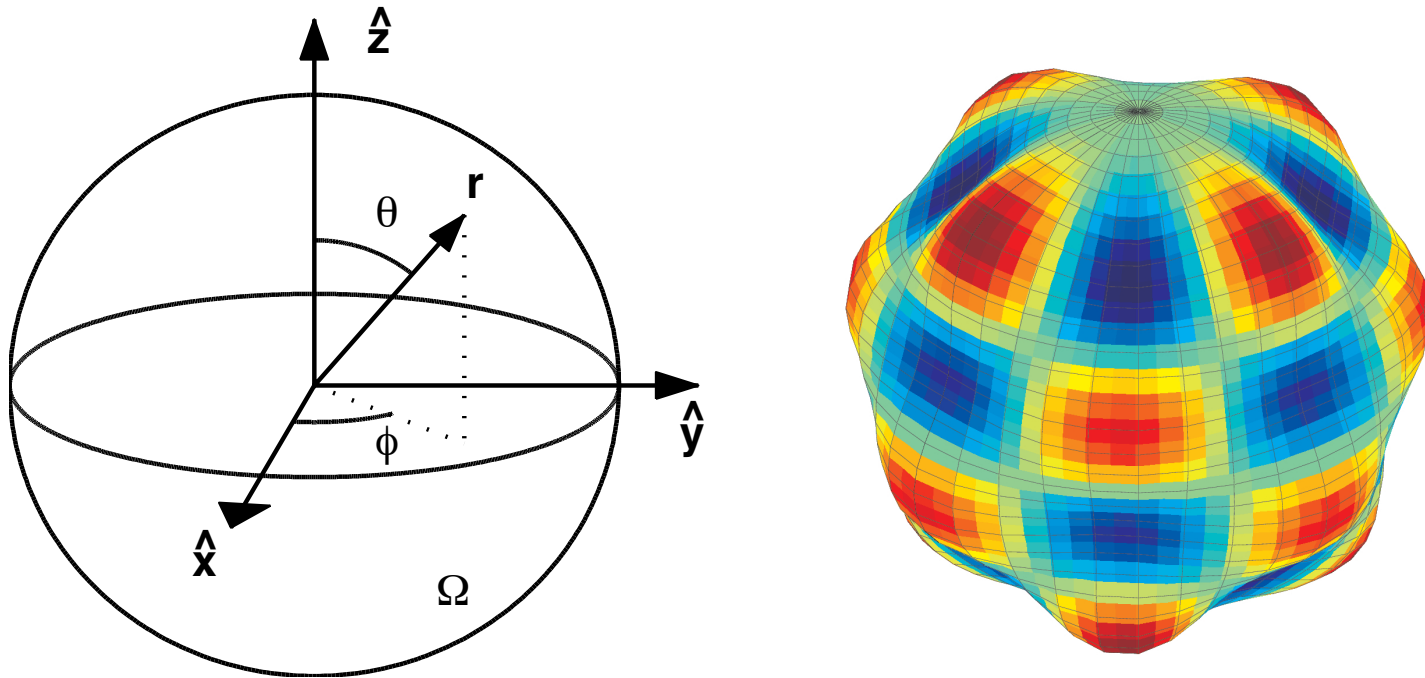
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To honor the spherical shape of the Earth,
we work in the **spherical-harmonic** basis.

Spherical harmonics

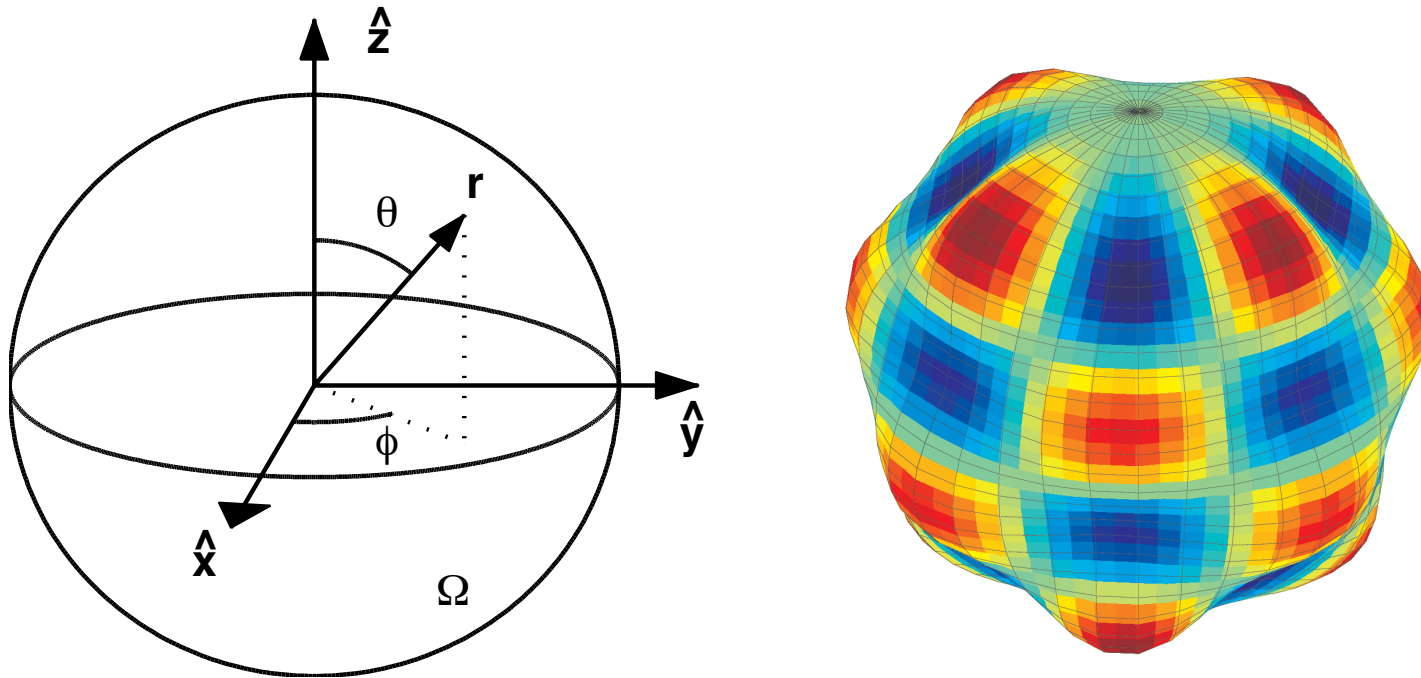
Scalar signals $s(\mathbf{r})$ modeled on a unit sphere Ω :



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$$\int_{\Omega} Y_{lm} Y_{l'm'} d\Omega = \delta_{ll'} \delta_{mm'} \quad \text{and} \quad s(\mathbf{r}) = \sum_{lm} s_{lm} Y_{lm}(\mathbf{r}).$$

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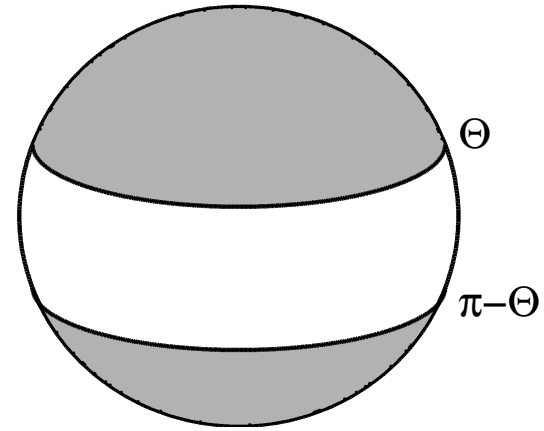
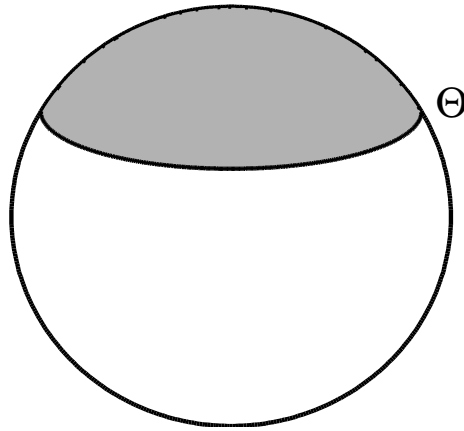
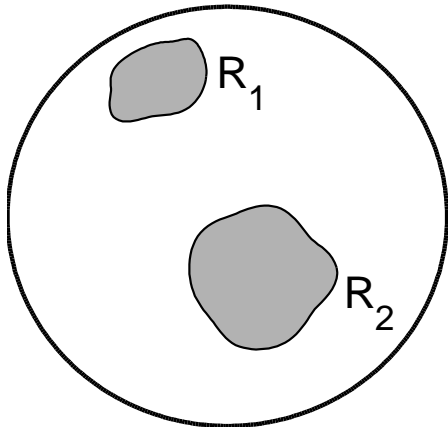
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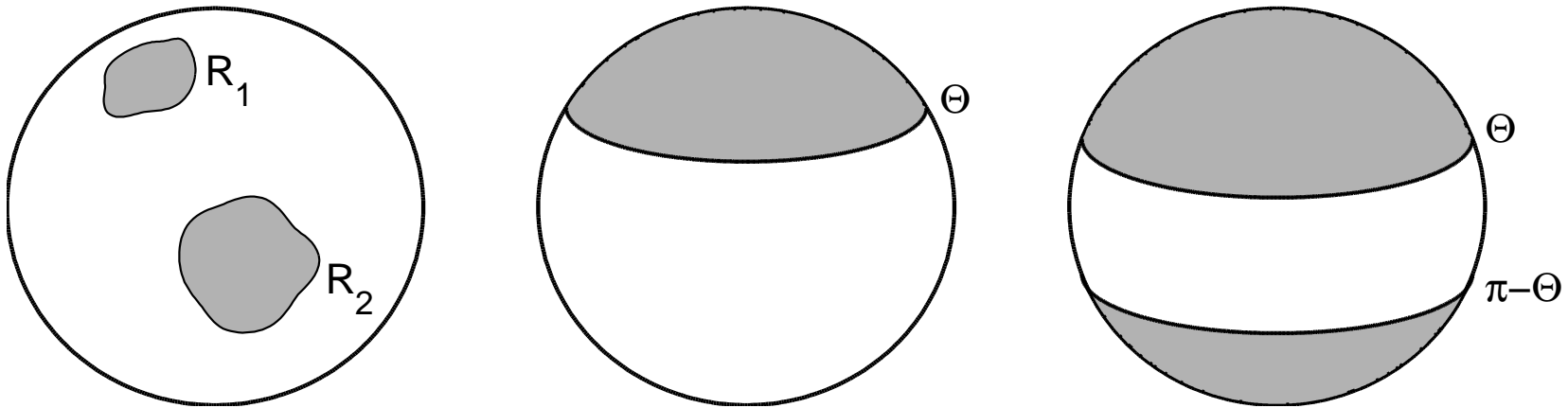
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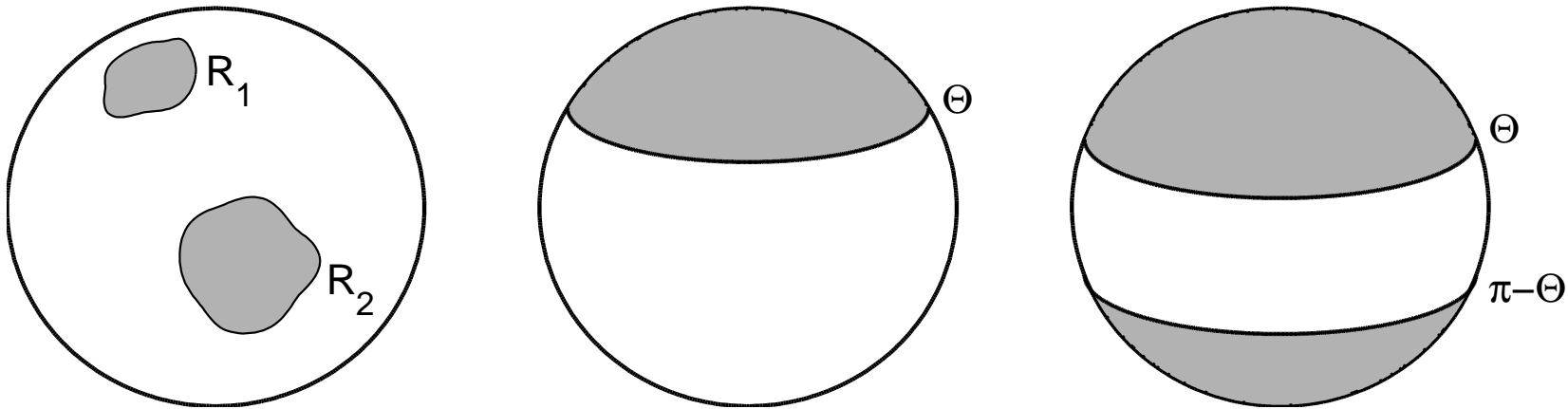


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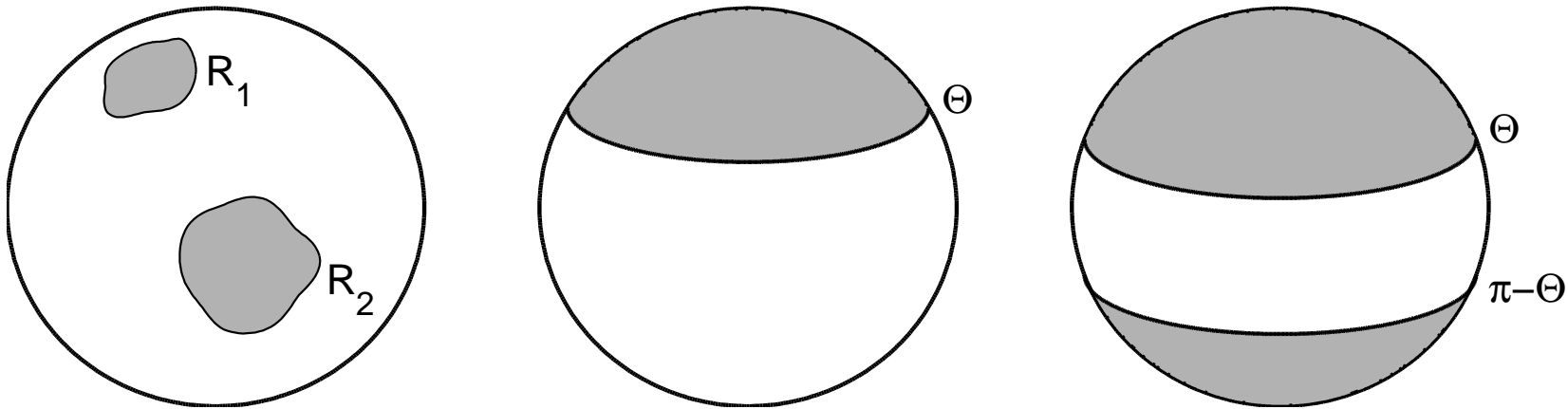
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These new, **doubly orthogonal**, functions are called **Slepian functions**, $g(\mathbf{r})$.

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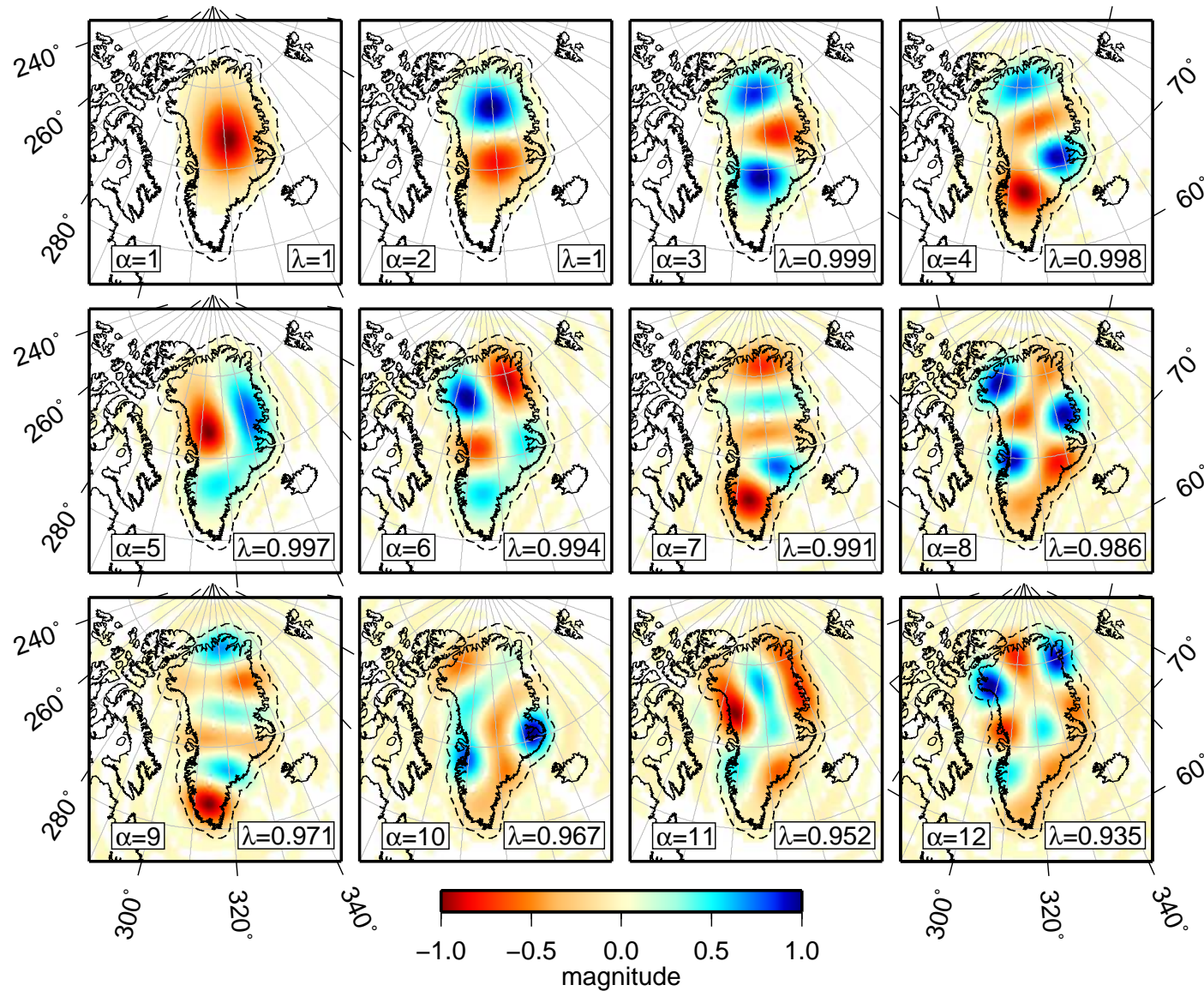
The **Shannon number**, or sum of the eigenvalues,

$$K = (L + 1)^2 \frac{A}{4\pi},$$

is the **effective dimension** of the space for which the bandlimited g are a **basis**.

*Voilà! We have concentrated a poorly localized basis of $(L + 1)^2$ functions, Y_{lm} , both *spatially* and *spectrally*, to a new basis with only about K functions, g .*

Slepian functions for Greenland, $L = 60$



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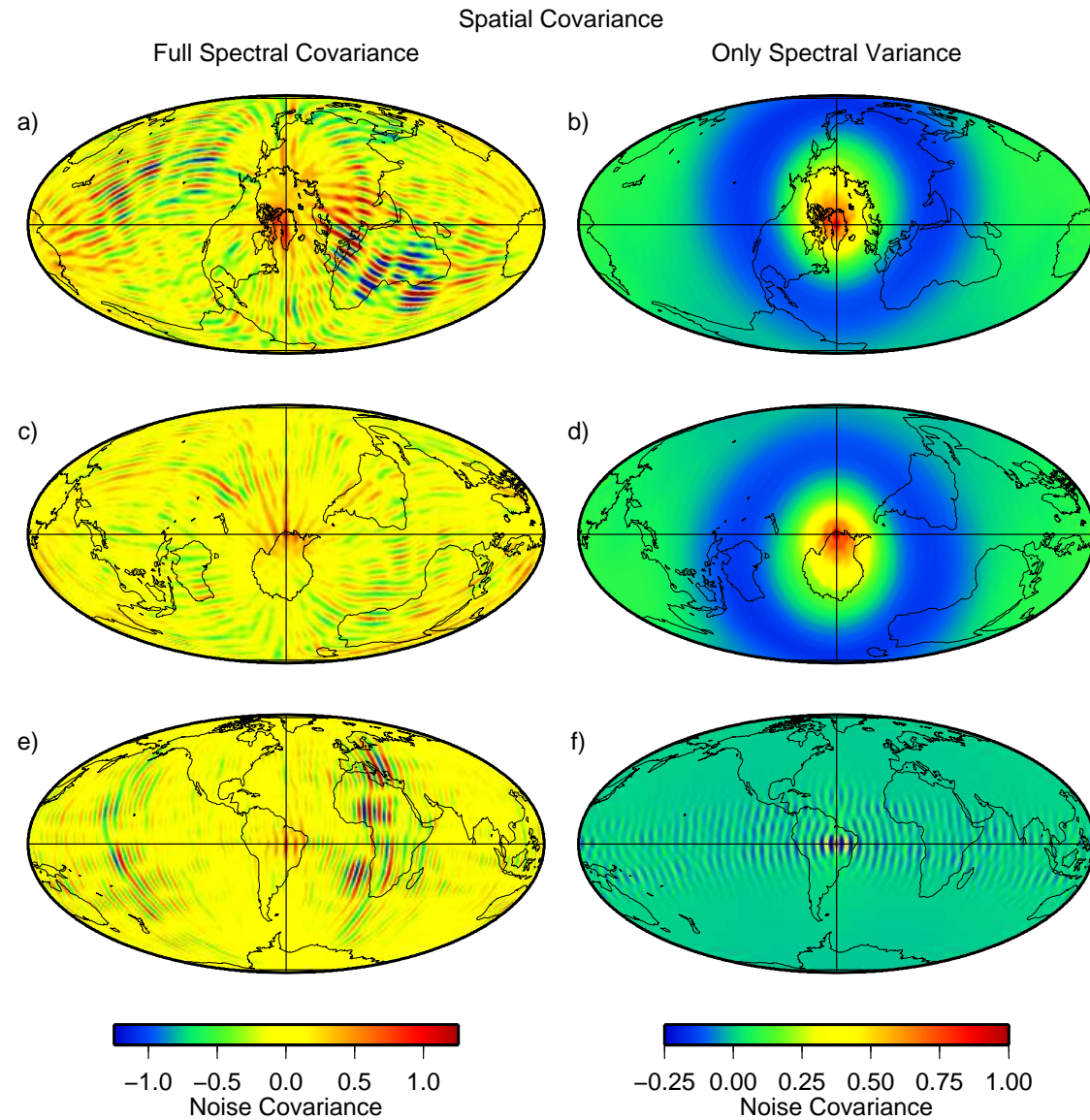
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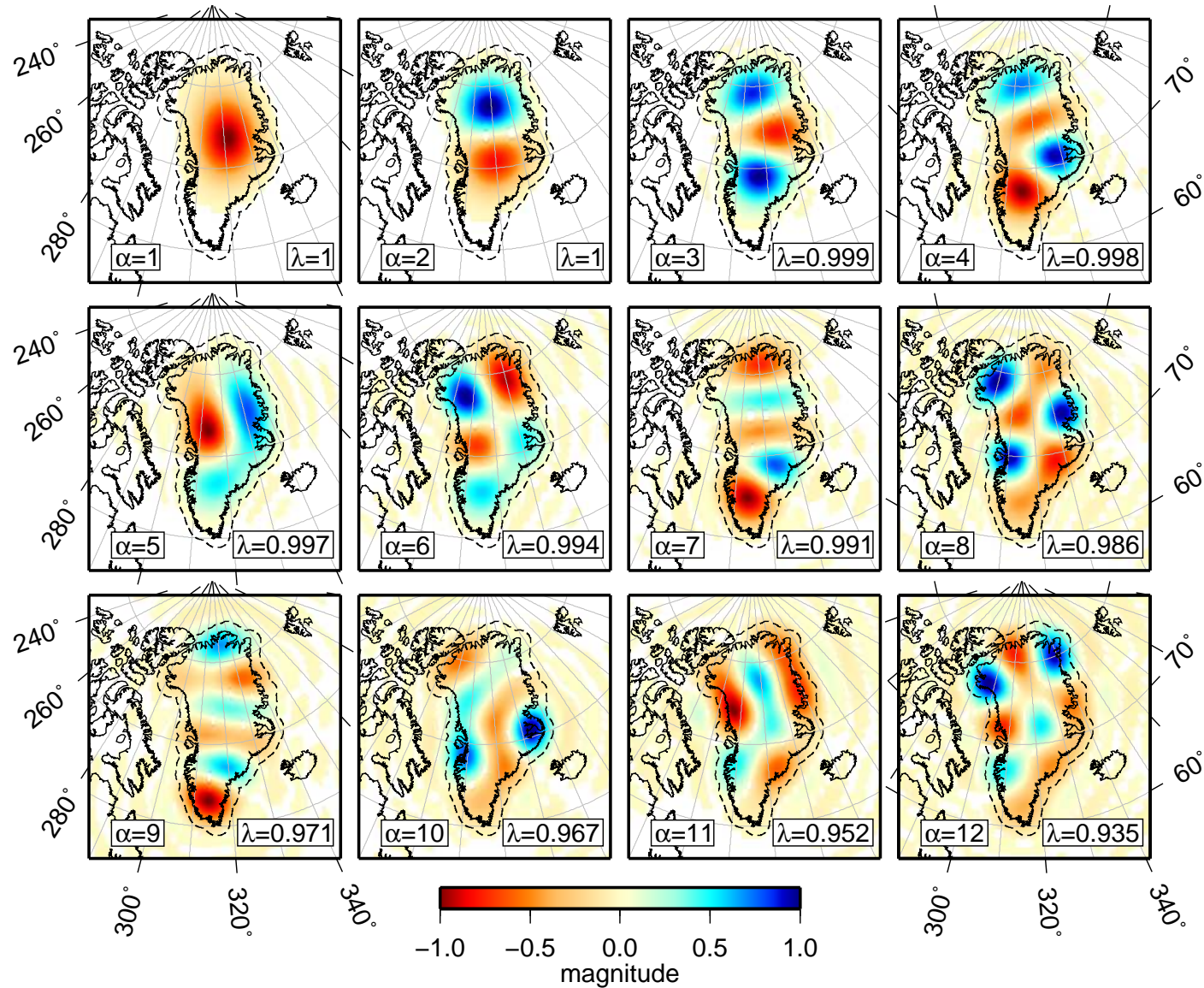
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 5. This is *very* different from most other approaches, though in spirit, it is *identical* to the stuff Slepian, Shannon and Wiener figured out in the 1950s.
-

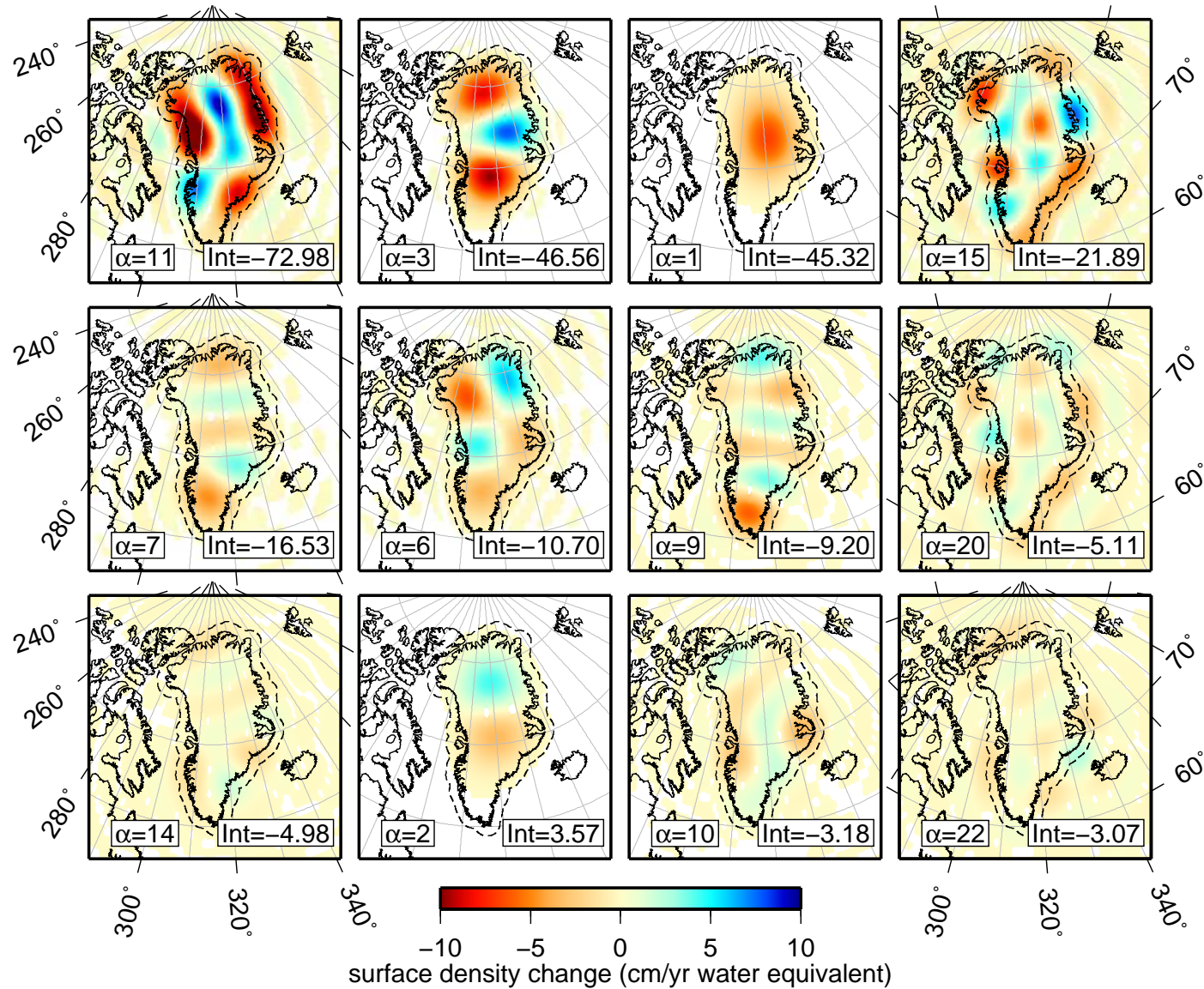
I. Look at the noise (in the pixel basis)



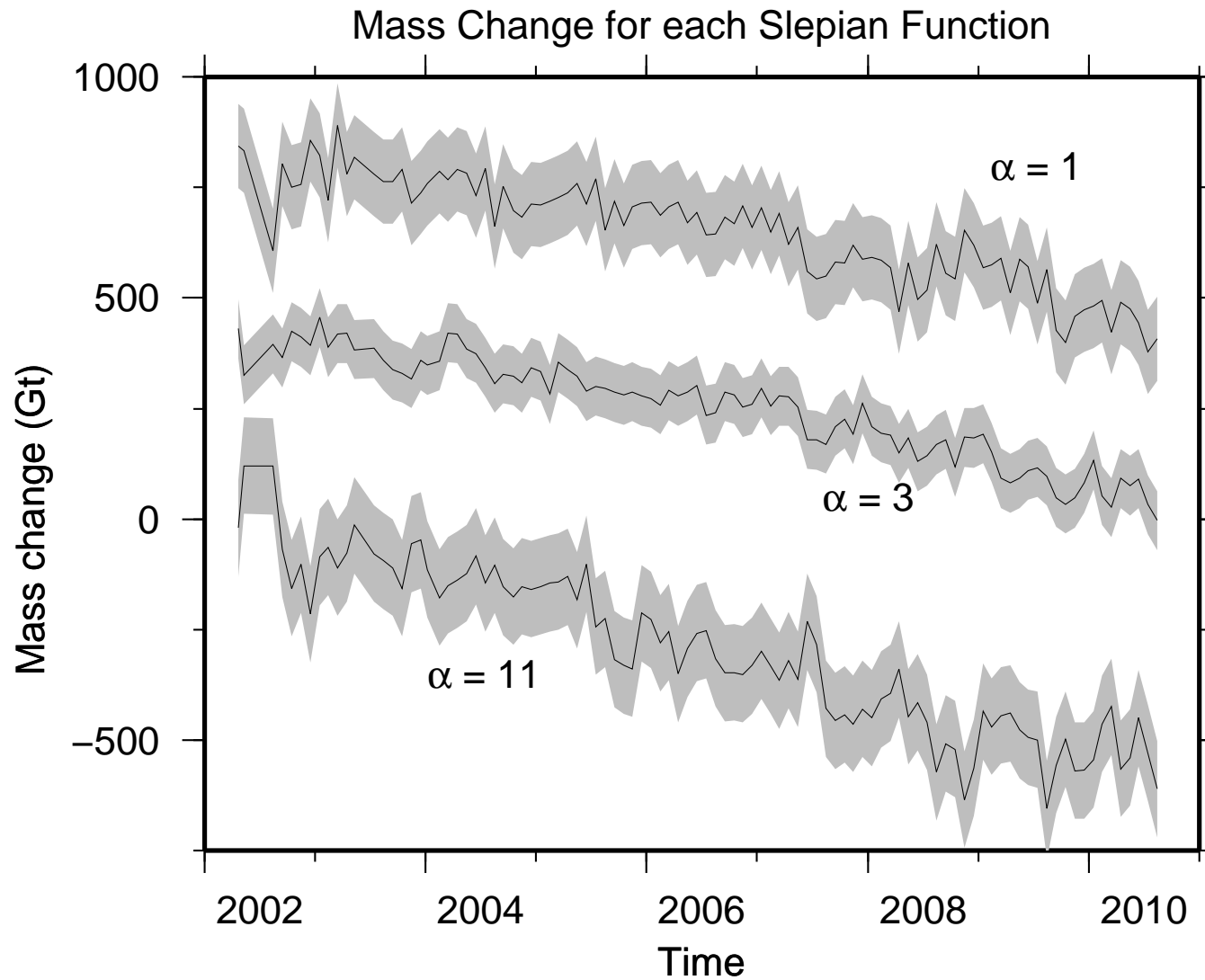
II. Construct an appropriate basis



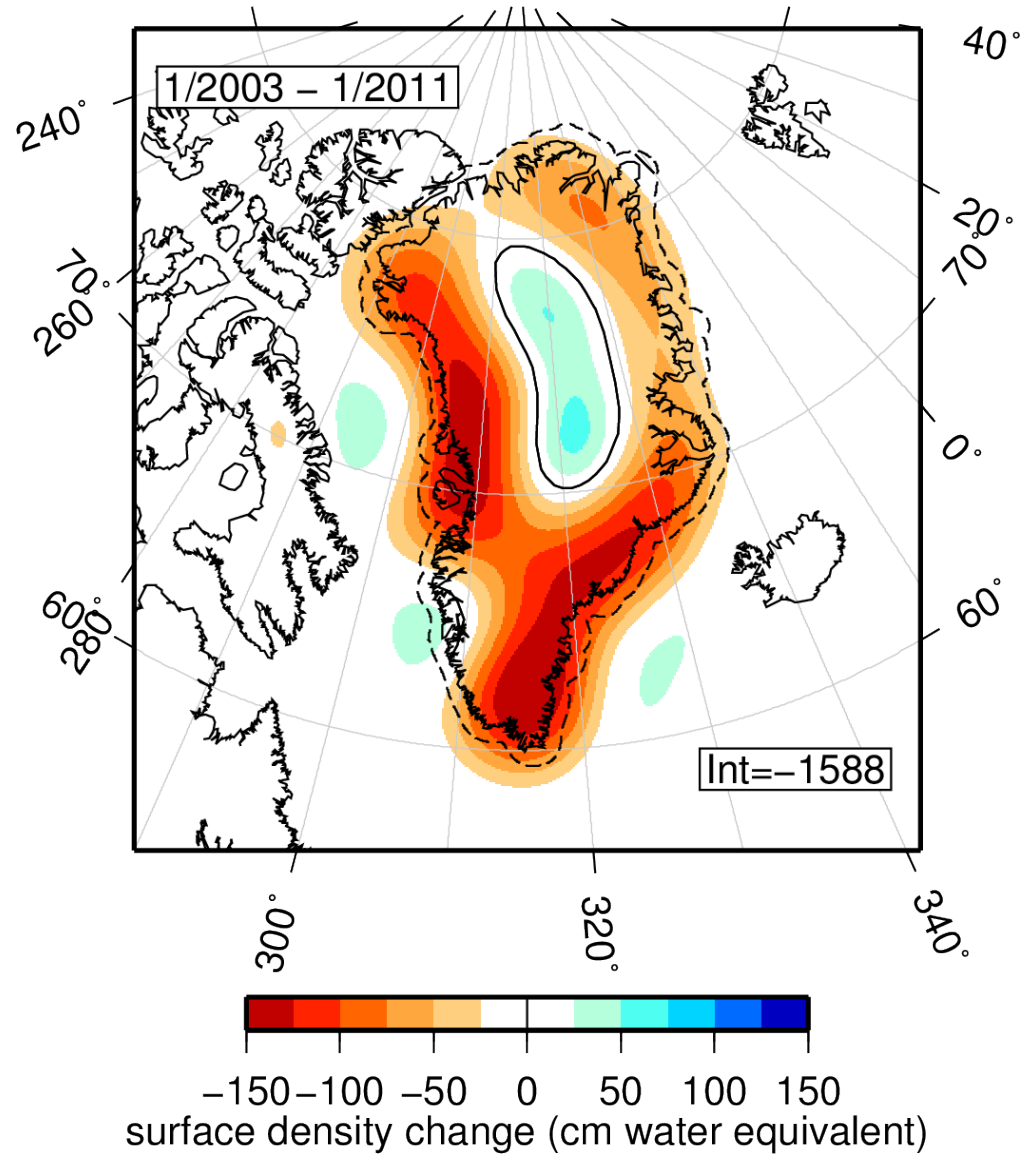
III. Project the signal onto the new basis



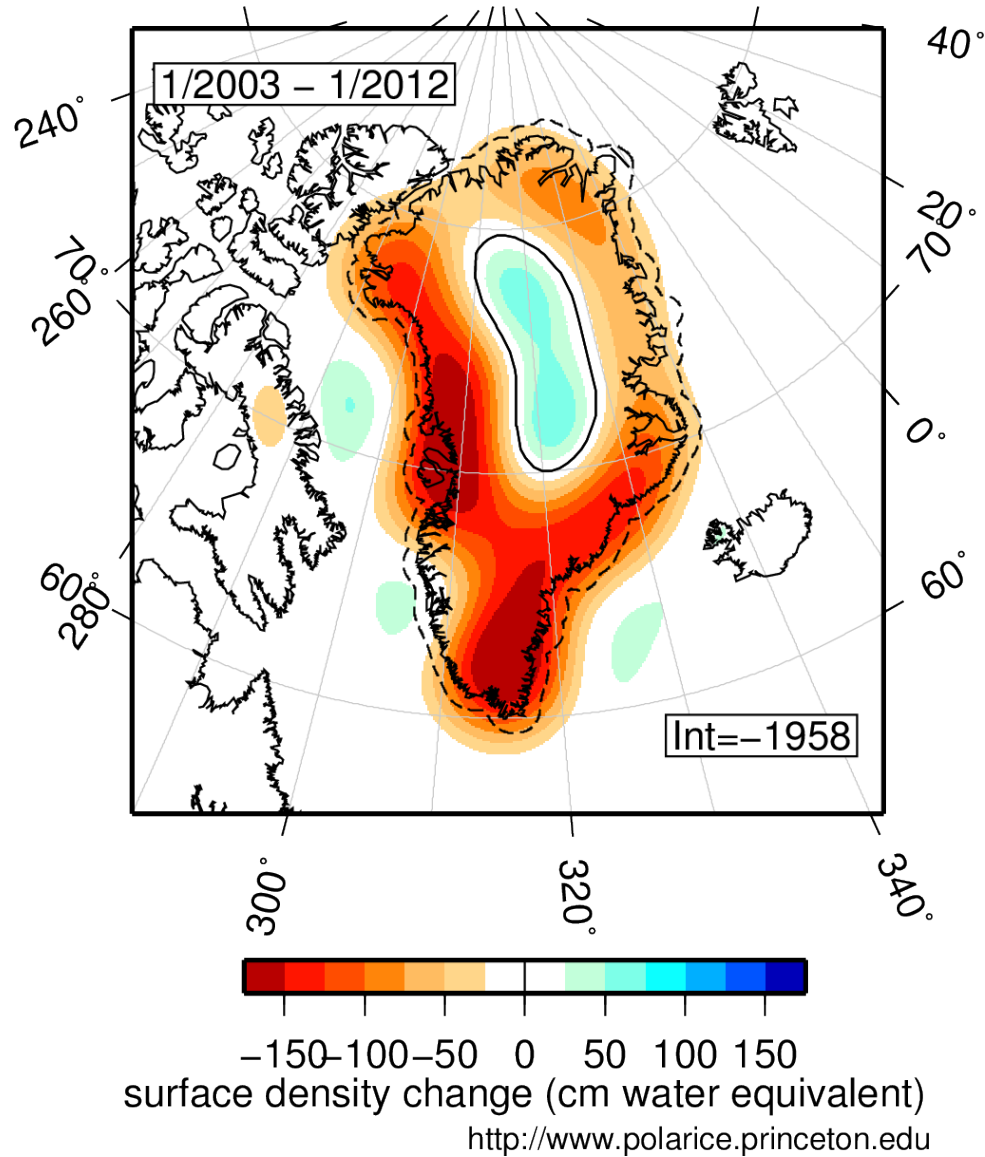
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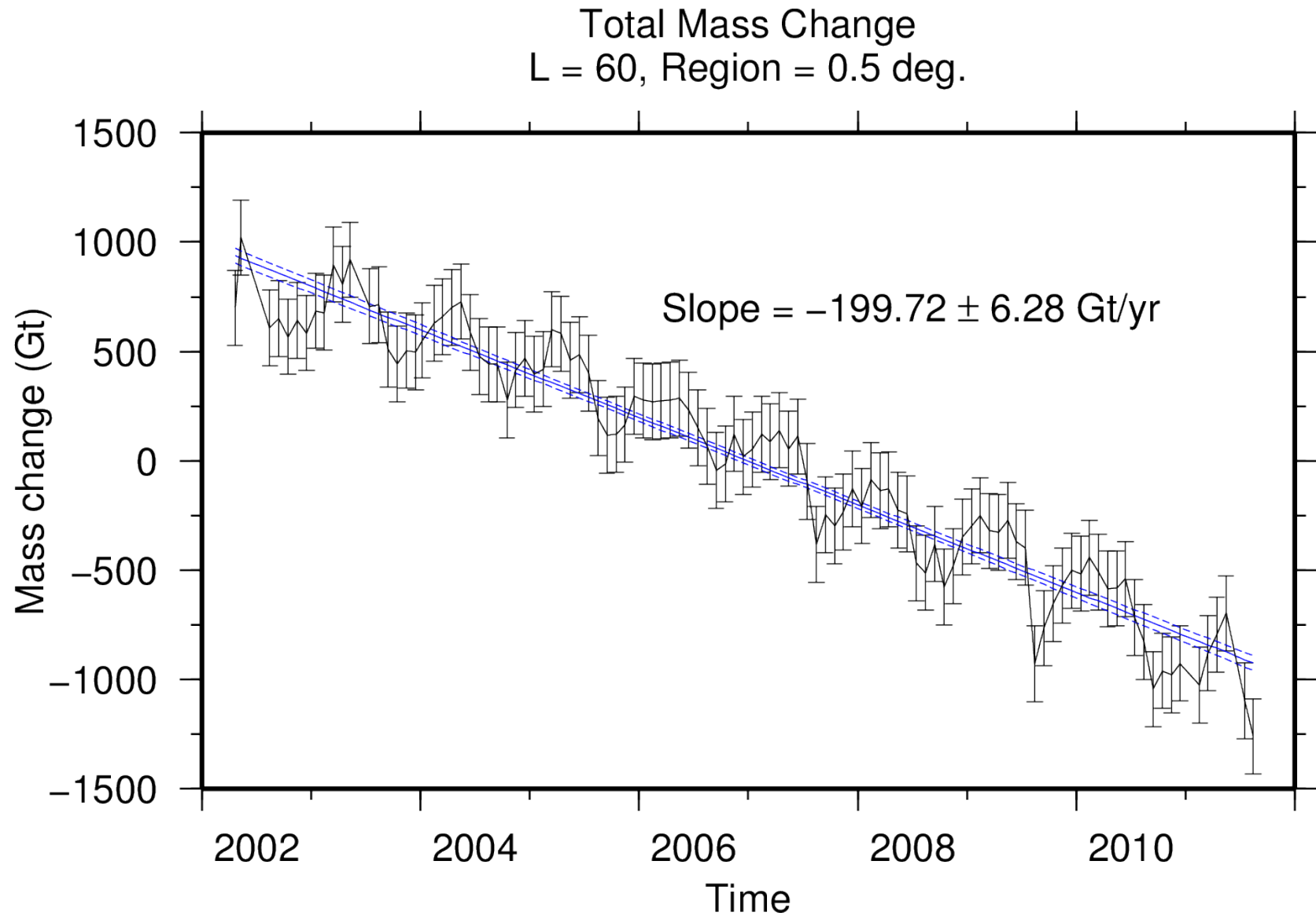
IV. Construct the final total estimate



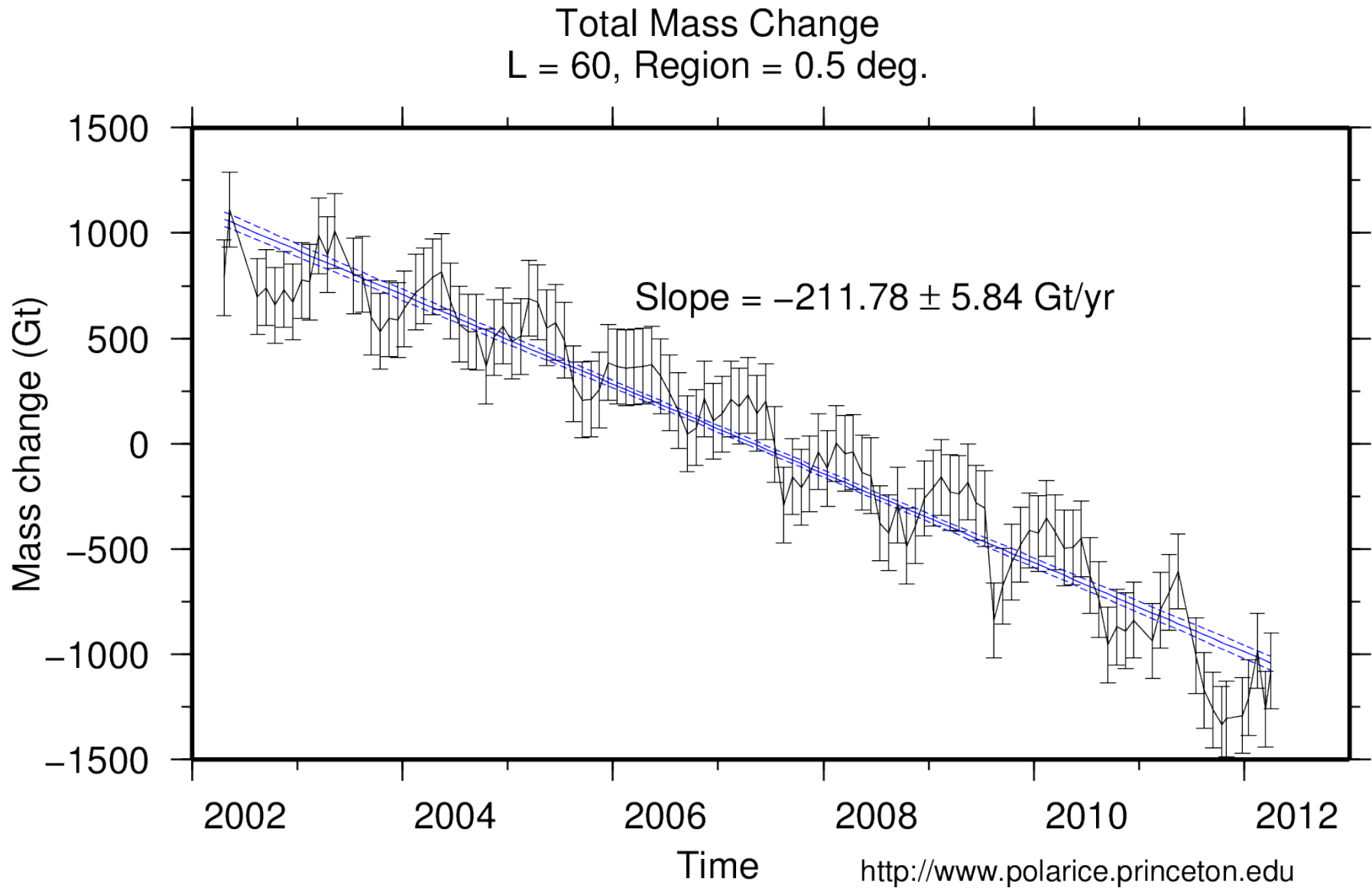
IV. Construct the final total estimate **UPDATE**



IV. Invert for the total budget (if you must)



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 - Maps of the time-averaged mass loss show a marked concentration at the **outlet glaciers**. Observed rates compare well with GPS surveys.
-

Common problems

Planetary gravity/magnetic field:

Problem 1

Given $d(\mathbf{r})$ and $\langle n(\mathbf{r})n(\mathbf{r}') \rangle$, estimate the signal $s(\mathbf{r})$ at **source level**:

$$\hat{s}(\mathbf{r}) = \sum_{\alpha=1}^J \hat{s}_{\alpha} g_{\alpha}(\mathbf{r}), \quad (1)$$

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Also: Cosmic Microwave Background radiation:

Problem 2

Given $d(\mathbf{r})$ and $\langle n(\mathbf{r})n(\mathbf{r}') \rangle$, and assuming the universe behaves as

$$\langle s_{lm} \rangle = 0 \quad \text{and} \quad \langle s_{lm} s_{l'm'}^* \rangle = S_l \delta_{ll'} \delta_{mm'}, \quad (2)$$

estimate the **power spectral density** S_l , for $0 \leq l < \infty$.

Quadratic spectral estimators

Whole-sphere ... *unattainable*

$$\hat{S}_l^{\text{WS}} = \frac{1}{2l+1} \sum_m \left| \int_{\Omega} d(\mathbf{r}) Y_{lm}^*(\mathbf{r}) d\Omega \right|^2 - \text{noise correction.} \quad (3)$$

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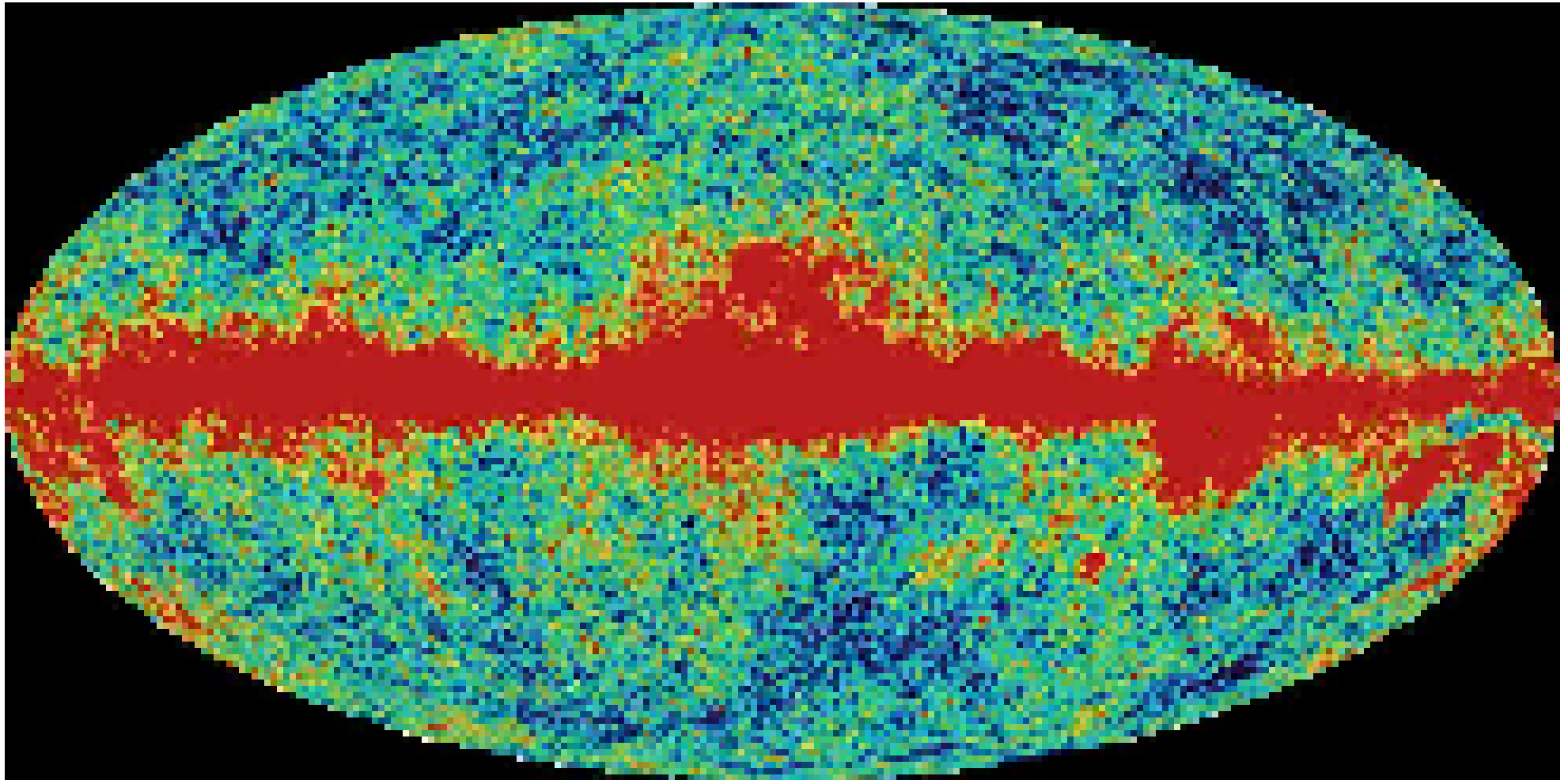
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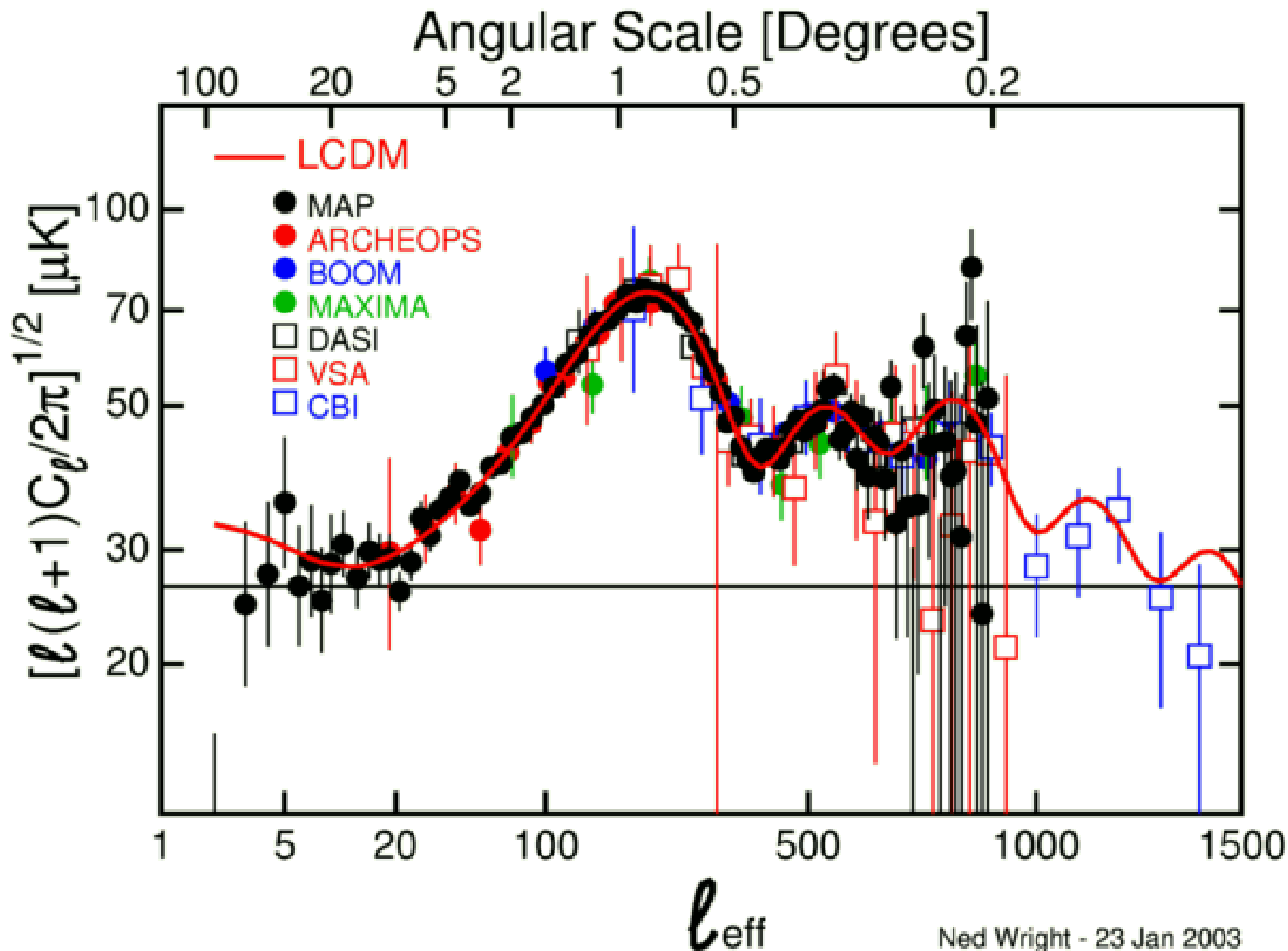
Multiple-taper ... *bandlimited bias, lower variance, easily implemented*

$$\hat{S}_l^{\text{MT}} = \frac{1}{N} \sum_{\alpha} \lambda_{\alpha} \hat{S}_l^{\alpha}. \quad (6)$$

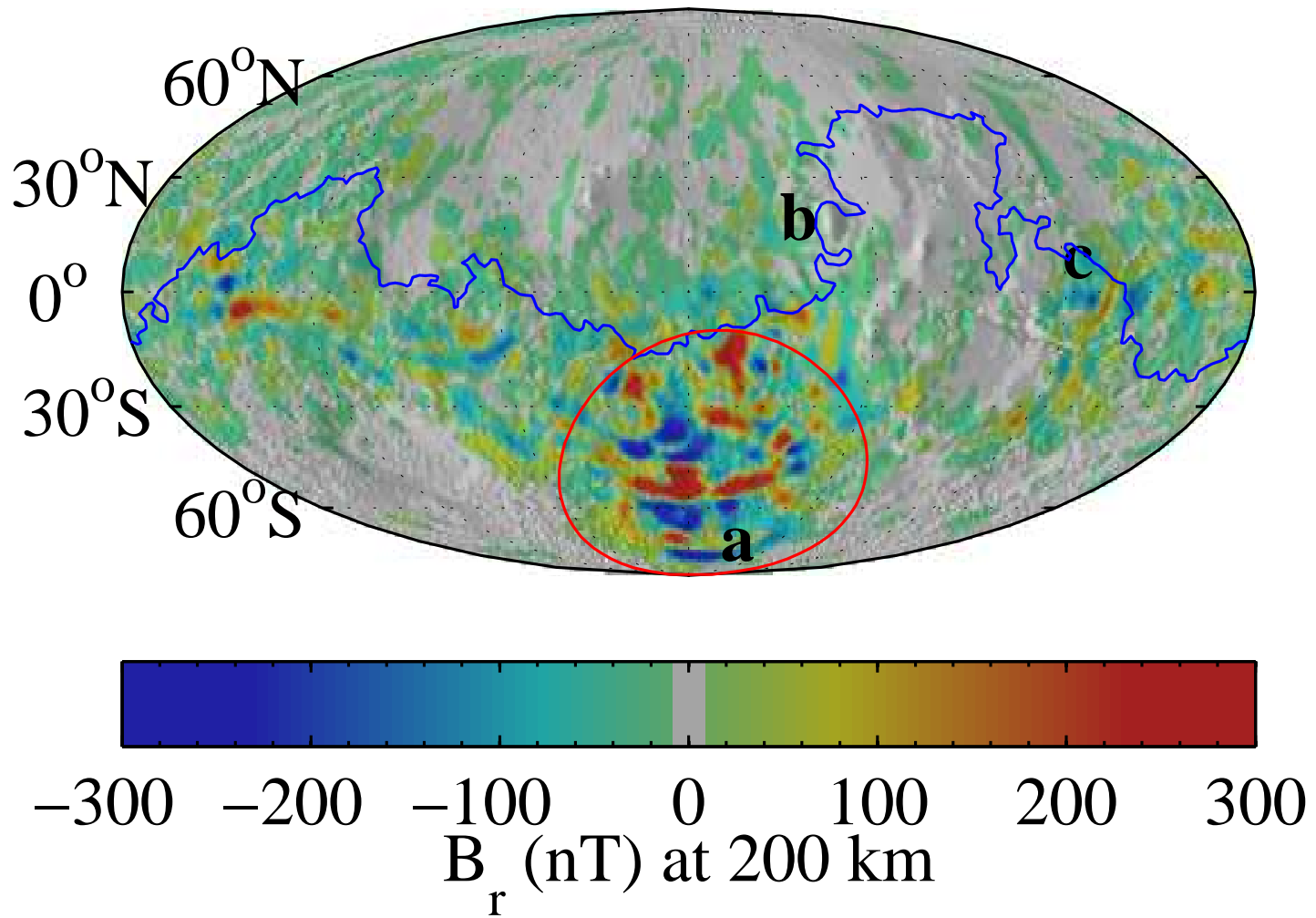
WMAP & Cosmic Background Radiation — 1



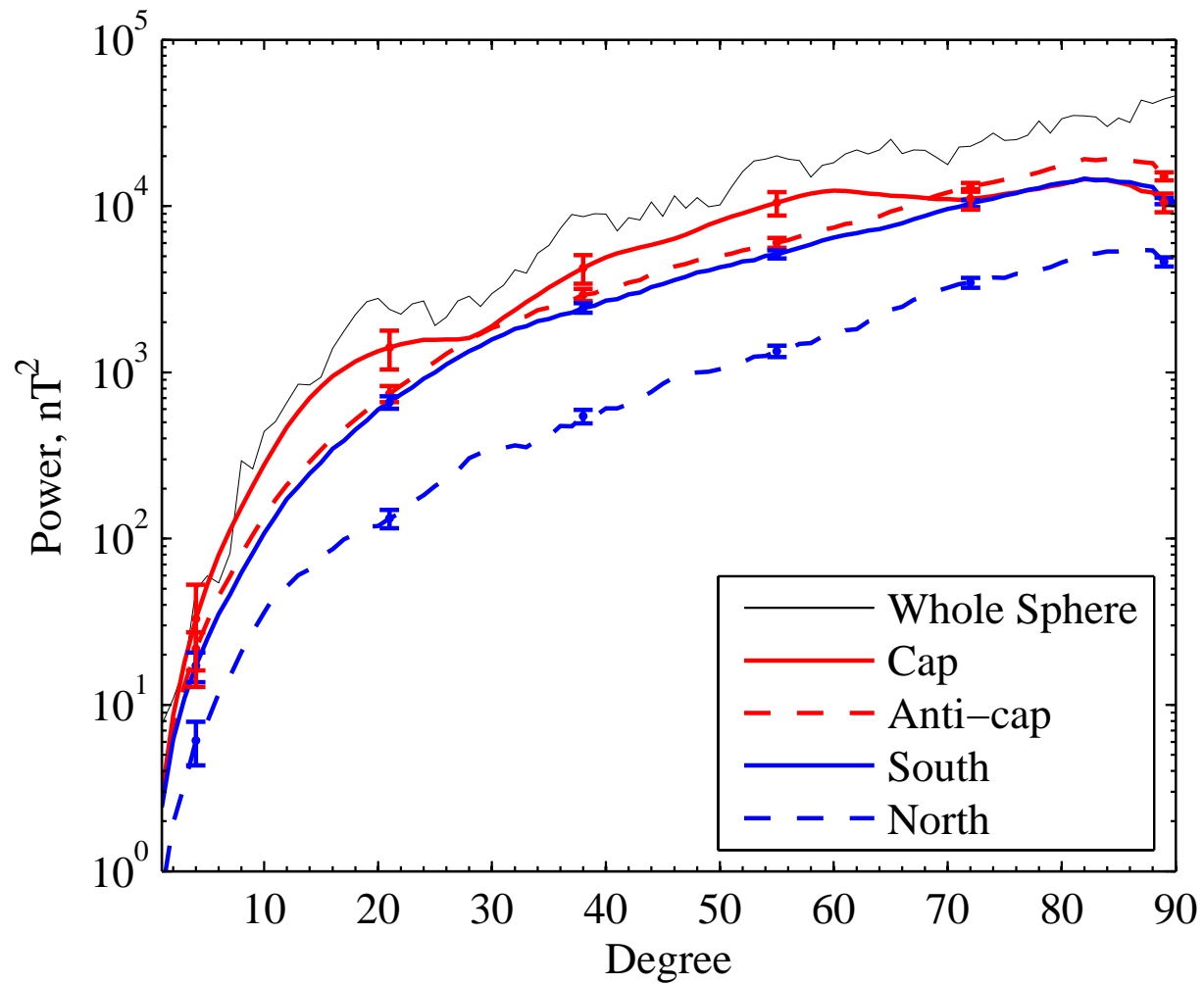
WMAP & Cosmic Background Radiation — 2



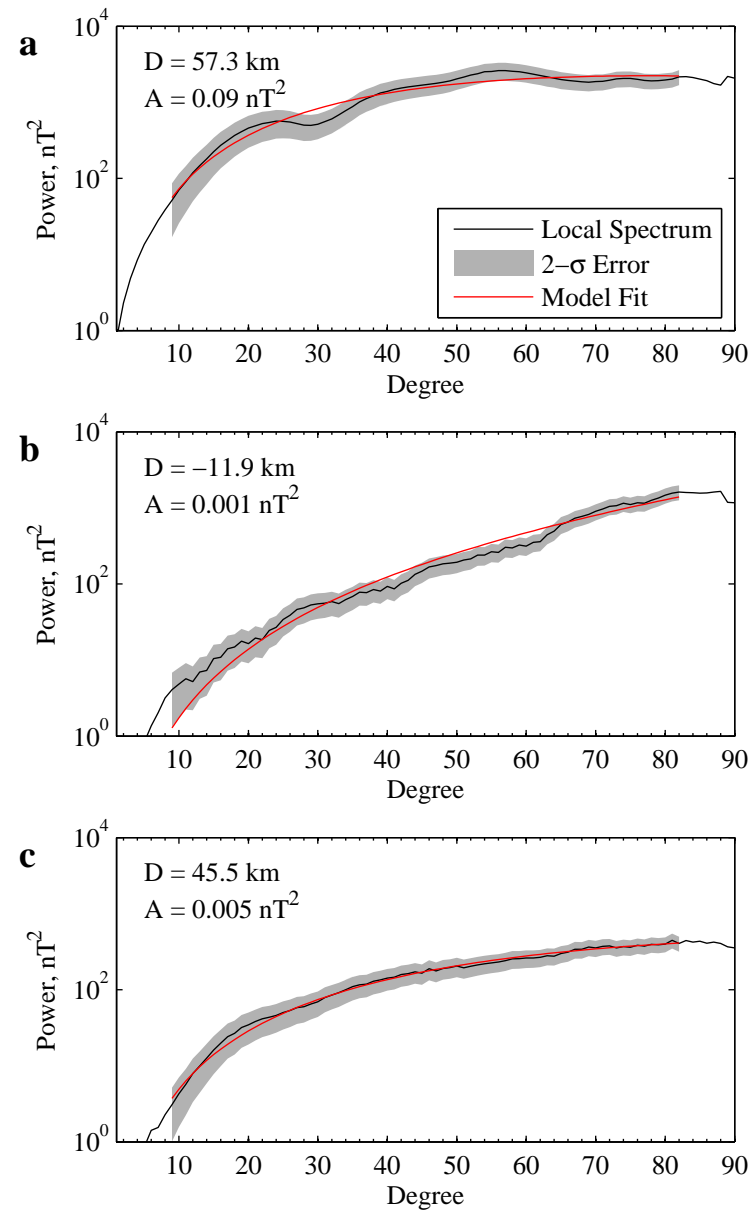
Mars' radial magnetic field



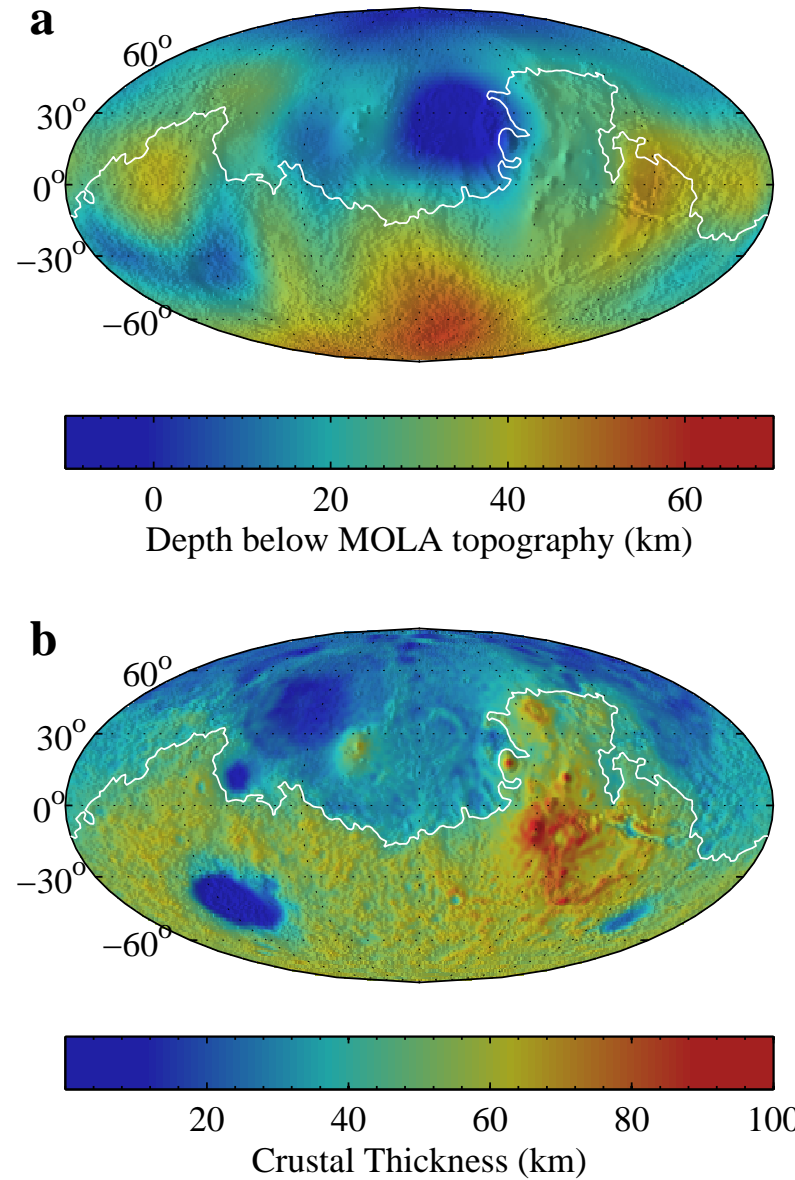
Mars' magnetic field: Power spectral density



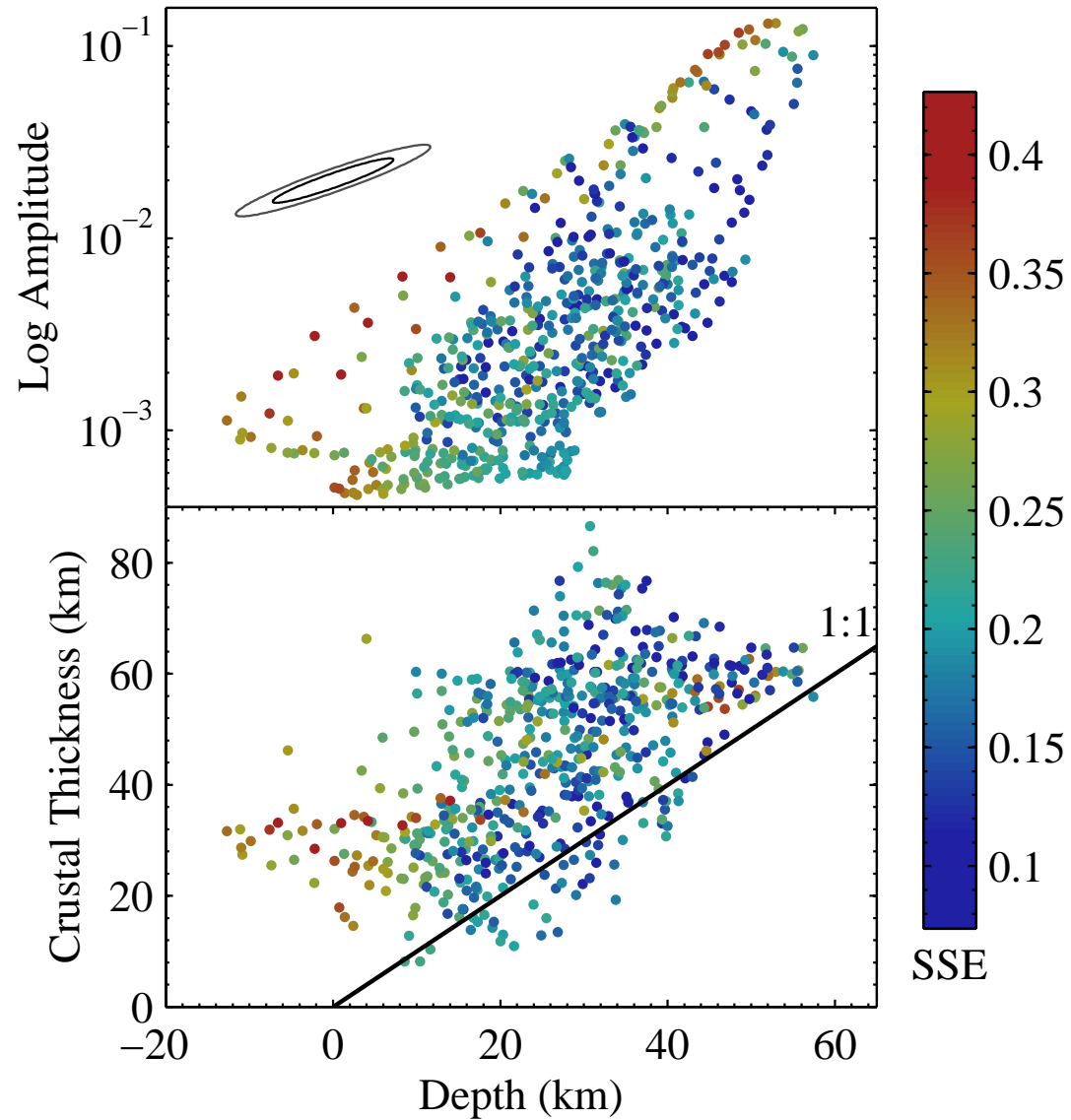
Mars: Magnetic strength vs decorrelation depth



Mars: Decorrelation depth vs crustal thickness



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 - The **correlation** of magnetic **source depths and strengths** with independent **crustal thickness** estimates indicates that a significant fraction of the martian crustal column may contribute to the observed field, as would be consistent with an **intrusive magmatic origin**.
-