When Physical Intuition Fails

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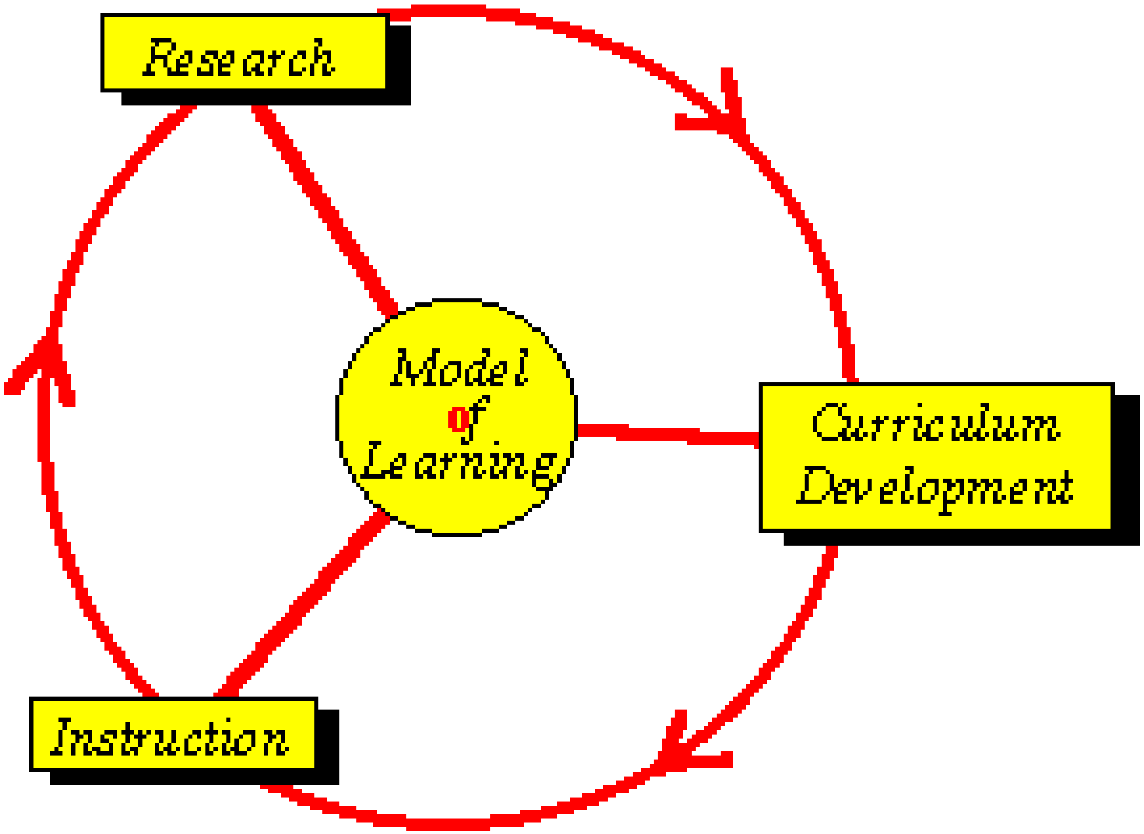
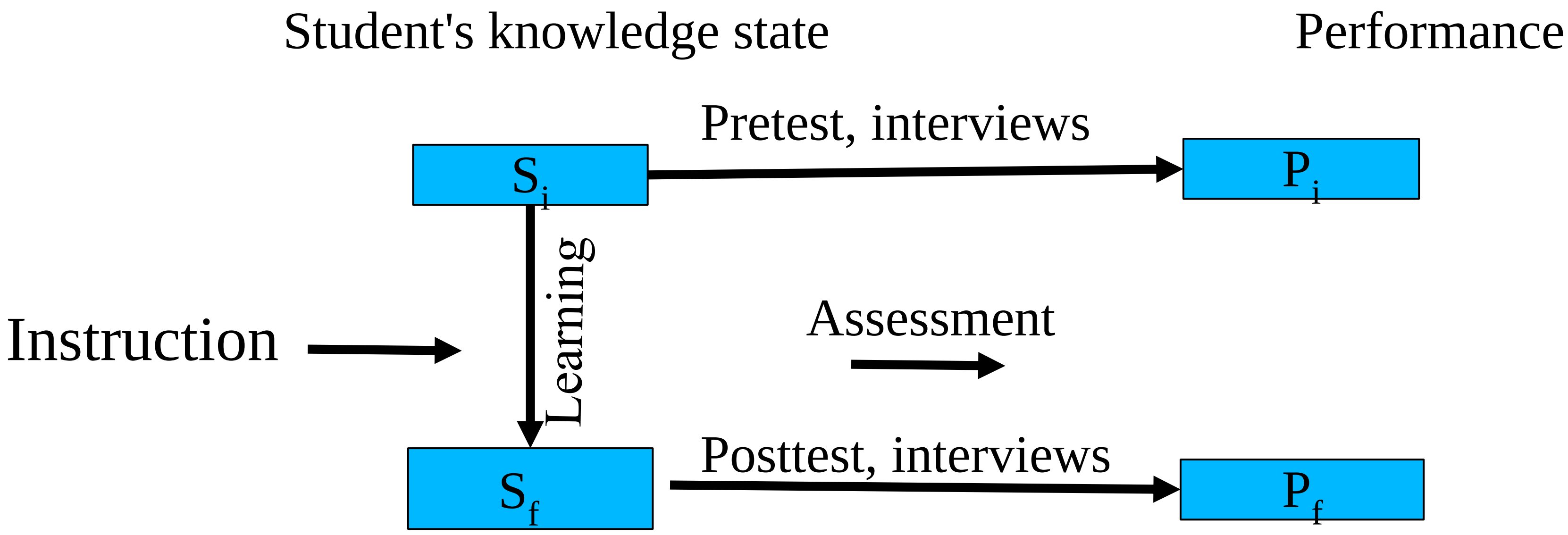
Physics Education Research

- Investigate sources of students' difficulties in learning physics
- Devise, implement and assess curricula/pedagogies to reduce difficulties



Model of Learning

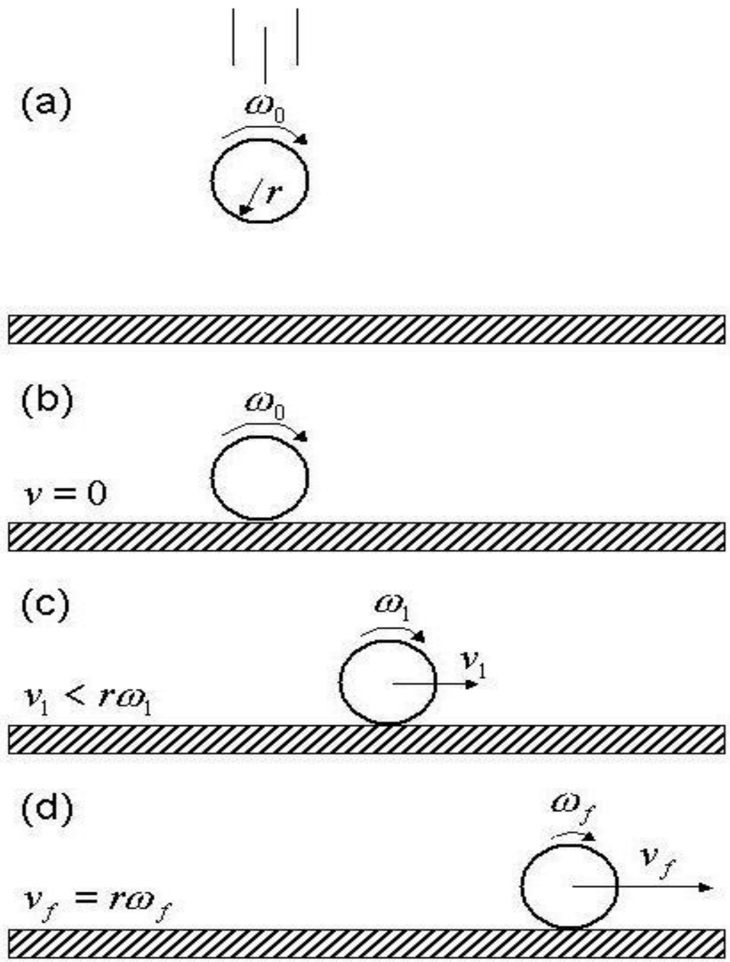


- Goal of instruction: Guide students from S_i -> S_f
- S_f depends on S_i and instructional design

When Physical Intuition Fails

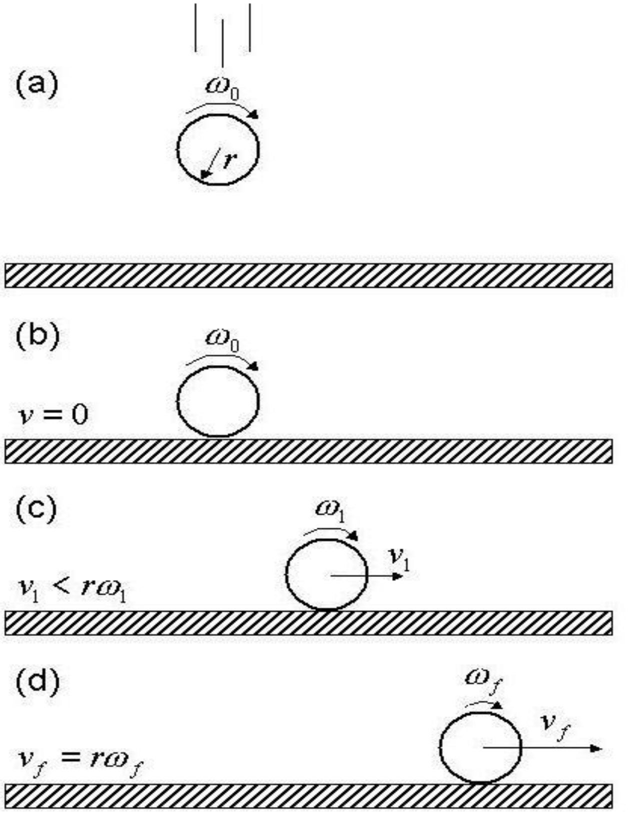
(Singh, Am. J. Phys., **70**(11), 1103-1109, 2002)

- 20 faculty and students were posed
  - a non-intuitive introductory physics problem



Expertise and Intuition

- Non-intuitive problem has two critical variables
  - How much friction
  - How long to start rolling
- Faculty
  - Difficulty solving non-intuitive problem on-the-spot
    - Often focused only on one variable
  - No difficulty with Ballistic pendulum which also involves two principles
- Students
  - Both equally difficult



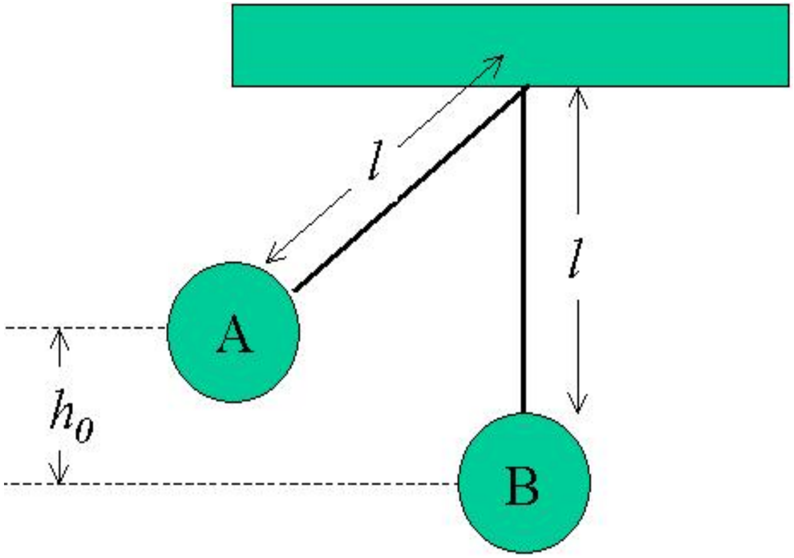
Energy & Momentum Question

(Singh & Rosengrant, Am. J. Phys., 71(6), 607-617, 2003)

- Two small spheres of putty, A and B, of equal mass hang from a ceiling on massless strings of equal length. Sphere A is raised to a height h_0 as shown below and released. It collides with sphere B (which is initially at rest); they stick and swing together to a maximum height h_f. Choose all of the following principles that must be invoked to find height h_f in terms of h_0?

(I) conservation of mechanical energy

(II) conservation of linear momentum



(a) (I) only 34% (b) (II) only 23% (c) both (I) and (II) 27%

(d) either (I) or (II) but not both 13% (e) none of the above 3%

Expertise and Intuition

- Perceived difficulty not only depends on inherent complexity of problem
  - Must assess difficulty of a problem from students' perspective
  - Experience, familiarity & intuition built
    - Crucial for optimal scaffolding

Improving Teaching and Learning of Quantum Mechanics

Chandralekha Singh

(AJP 2001, Physics Today, 2006 with Belloni+Christian, AJP and Phys. Rev. ST PER 2008, 2009, 2010, 2011, 2012, 2013)

OSP Simulations

(*M. Belloni & W. Christian*)

Investigation of Difficulties

Are misconceptions in advanced courses similar in nature to those for introductory courses?

- Can they be correlated with teaching style, place of study & textbook?
- Design and assess learning tools: Quantum Interactive Learning Tutorials (QuILTs) and peer-instruction tools
  - Based upon research on students’ difficulties and learning theory
  - Keep students actively engaged
  - Bridge gap between formalism and conceptual understanding/math-physics connection
  - Build on prior knowledge and help students build a robust knowledge structure
  - Exploit computer simulations to help build intuition

Investigation of Difficulties

- Question about whether $\mathbf{H}\psi = \mathbf{E}\psi$ is always true for all possible wave functions
  - 29% correct response
  - 39% incorrectly agree with the statement
  - Others who disagree incorrectly asserted that
    - it is a statement about measurement of energy so the state should collapse into an eigenstate of energy
    - True if energy is conserved

Time-dependence of Wave function

- Time-dependence of wave function for
  - Infinite square well initially in linear superposition of two stationary states

$$\psi = \sqrt{\frac{2}{7}}\phi_1 + \sqrt{\frac{5}{7}}\phi_2$$

  - 43% correct response
  - 31% incorrectly used common time-dependent phase factor

Question about measurement of an observable

Consider the following conversation between Andy and Caroline about the measurement of an observable Q for a system in state $|\Psi\rangle$ which is not an eigenstate of $\hat{Q}$.

Andy: When an operator $\hat{Q}$ corresponding to a physical observable Q acts on the state $|\Psi\rangle$, it corresponds to a measurement of that observable. Therefore, $\hat{Q}|\Psi\rangle = q_n|\Psi\rangle$ where q_n is the observed value.

Caroline: No. The measurement collapses the state so $\hat{Q}|\Psi\rangle = q_n|\Psi_n\rangle$ where Ψ_n on the right hand side of the equation is an eigenstate of $\hat{Q}$ with eigenvalue q_n. With whom do you agree?

A. Agree with Caroline only
B. Agree with Andy only
C. Agree with neither
D. Agree with both
E. The answer depends on the observable Q.

	A	B	C	D	E	blank
Q30	26%	13%	***46%***	13%	0%	1

Conclusions from surveys and interviews

- Advanced students also have many <u>common</u> difficulties and misconceptions
  - independent of background, teaching style & textbook
- Commonality of misconceptions originate from
  - inability to discriminate between related concepts
  - tendency to over-generalize
- Strikingly similar to "universal" nature of misconceptions in introductory physics

How to improve student understanding?

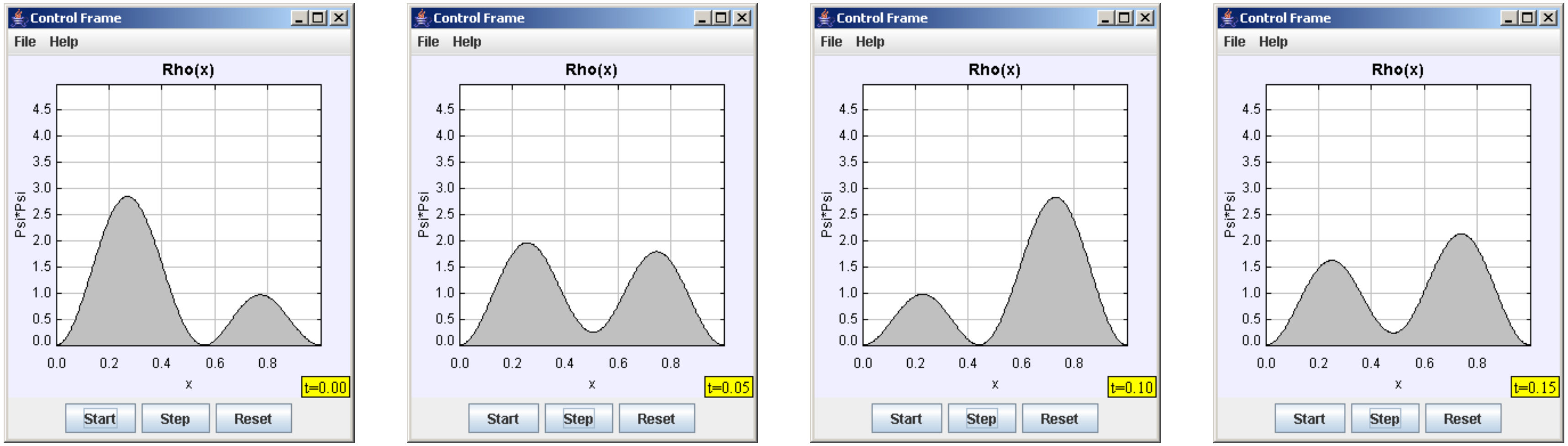
- Cognitive theory suggests
  - Learning is incremental and new knowledge builds on prior knowledge
    - Must know student's initial knowledge and build on it
  - Misconceptions and difficulties related to a particular topic can be classified into a few categories
    - **People's sense making shows patterns**
  - Students must construct their own understanding
    - Effective pedagogical strategies engage students in the learning process
  - Mental load during problem solving is subjective
    - Not only depends on inherent complexity of the problem but on the familiarity and intuition
      - Put yourself in students' shoes
    - Provide systematic tasks consistent with current knowledge

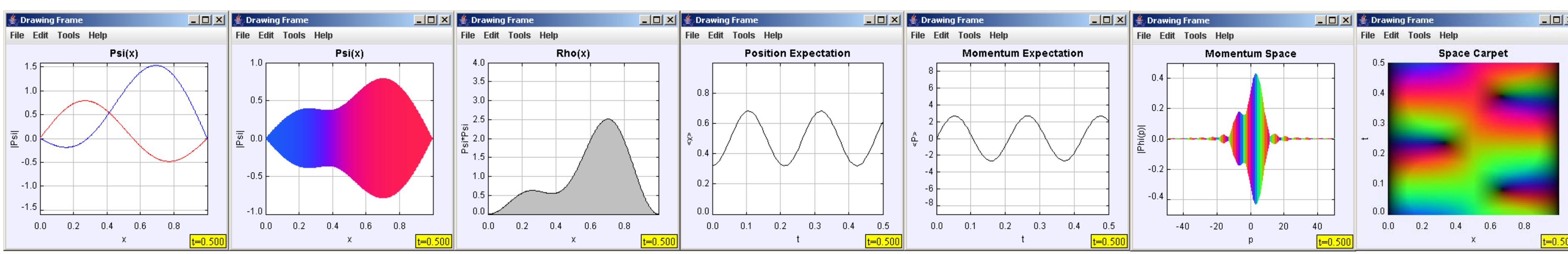
Quantum Interactive Learning Tutorials (QuILT)

- Based on findings of student difficulties in learning QM
  - Guided approach to learning that builds on students' prior knowledge
    - Hints and feedback is given as needed
  - bridge the gap between quantitative and conceptual aspects of QM
  - Keeps students actively engaged
  - Each tutorial comes with pre-test/post-test, often warm-up exercises and homework
  - Cyclic method of development and evaluation

Time-Evolution QuILT

3. Now open the simulation (double-click the green arrow) and choose the initial wave function $\Psi(x,0) = \sqrt{\frac{1}{8}}\psi_1(x) + \sqrt{\frac{7}{8}}\psi_2(x)$. Watch the time evolution of $|\Psi(x,t)|^2$. Is the time evolution of this wave function consistent with what you predicted earlier? Explain.





*Equal-mix superposition in the ISW shown with a variety of visualizations.

Flow chart for computing the time evolution of a state

Given $\Psi(x,0)$ and the Hamiltonian $\hat{H}$ Find $\Psi(x,t)$

$\hat{H}\psi_m = E_m\psi_m$ Solve time-indep. Schröd. eq.

↓ Time dependence of stationary states

Discrete	Continuous
$\psi_m(x,t) = \exp(-iE_m t/\hbar)\,\psi_m(x)$	$\psi_k(x,t) = \exp(-iE(k)t/\hbar)\,\psi_k(x)$

↓ Expand $\Psi(x,0)$ in terms of stationary states ↓

Discrete	Continuous
$\Psi(x,0) = \sum_n c_n\psi_n(x)$	$\Psi(x,0) = \int C(k)\,\psi_k(x)\,dk$

↓ Find c_m or $c(k)$ using orthogonality ↓

Discrete	Continuous
$c_m = \int \psi_m^*(x)\,\Psi(x,0)\,dx$	$c(k) = \int \psi_k^*(x)\,\Psi(x,0)\,dx$

↓ General solution using superposition ↓

$$\Psi(x,t) = \sum_n c_n\,\psi_n(x,t) = \sum_n c_n \exp(-iE_n t/\hbar)\,\psi_n(x)$$

$$\Psi(x,t) = \int c(k)\psi_k(x,t)\,dk = \int c(k)\exp(-iE(k)t/\hbar)\,\psi_k(x)\,dk$$

Preliminary Evaluation

Tutorial	% Pretest Score	% Post-test Score
Time development of wave function	53	85
Uncertainty principle	42	83
Mach-Zender interferometer	48	83
Stern-Gerlach Experiment	55	86
Drawing Wavefunction	40	90
Measurement	67	90
Addition Angular Momentum	35	74

Peer Instruction Tools

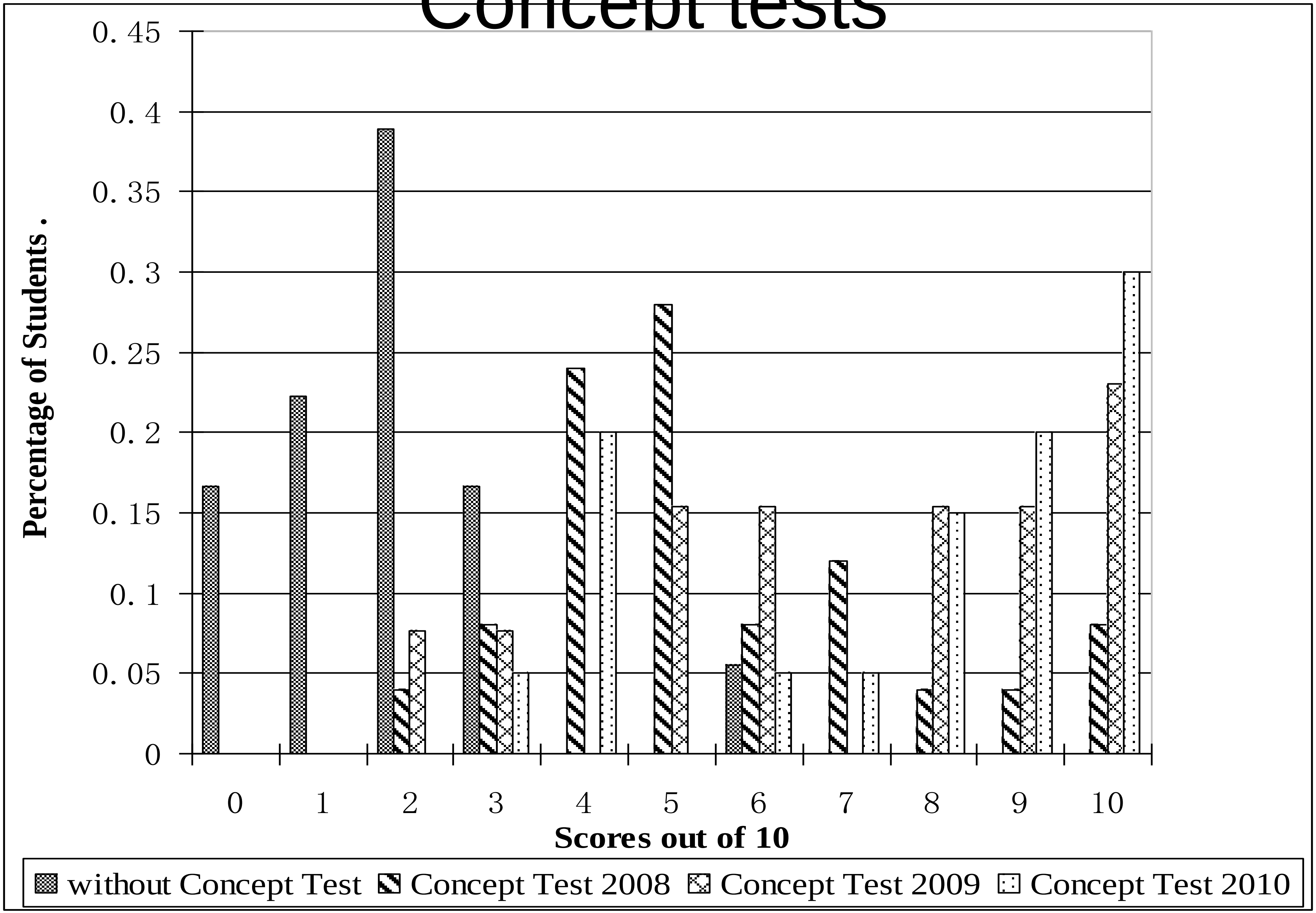
- We are developing peer-instruction tools for teaching quantum mechanics
  - Conceptests (~500 for full year)
  - JITT including reflective questions (can also be used as class discussion/homework questions)
- Conceptests can be integrated with lecture
    - Students discuss answers with each other before answering
  - Review questions at the beginning or end of lecture

Peer Instruction for Quantum Mechanics

Peer Instruction tools have been designed based upon research on student difficulties and cognitive issues in learning quantum mechanics

- Often several conceptests are related
  - Sometimes deal with common difficulties
  - help students develop a coherent knowledge structure related to a particular QM concept
  - Math/physics connection
  - Some are abstract while others deal with concrete applications and manifestations
    - Different representations of knowledge are used

Infinite Square Well: with and without Concept tests



Quantum Measurement Conceptests/QuILT

- Student Performance
  - Traditional instruction: 26%
  - ConcepTest Only: 68%
  - ConcepTest & QuILT: 90%
- Student performance significantly better after Conceptests
  - Even better after both tools

Quantum Mechanics Survey

-31 item research-based conceptual multiple-choice survey

- Connection between math/physics (quantitative and conceptual aspects) & knowledge structure

-only focuses on QM in one spatial dimension

-validity and reliability studies conducted

Scores for Different Groups on QMS: 10 Universities, 14 Classes

- Developed and validated Quantum Mechanics Survey (AJP, 2012)
  - Without research-based learning tools
    - 165 undergraduates: 39%
    - 33 first year graduate students at the end of a full year quantum mechanics: 52%
  - With research-based learning tools (QuILTs and peer instruction tools)
    - 28 undergraduates at the end of the first semester QM: 72%
    - 26 undergraduates at the end of the second semester QM: 69%

Reflection on their mistakes by Undergraduates in QM

- Metacognition/reflection is a sign of expertise
  - Experts automatically reflect upon problem solving process, learn from their mistakes and organize, repair and extend their knowledge
- QM undergraduates
  - Do they voluntarily reflect upon & learn from what they did incorrectly the first time?
  - Do they check their work with instructor's solution provided automatically?
  - Do they perform better if asked the same question a second time?

QM Experiment: Setup

- 14 students, upper-level Quantum Mechanics
- 4 problems given on 2 midterms
- Relevant material covered in lecture, homework and text via “standard” teaching approach
  - 3 problems selected by difficulty: students struggled with these on the midterms (roughly 50% combined average)
  - 1 selected because it was easy

- These four problems were repeated as part of the final exam
  - Hypothesis: Students who successfully learn from errors on their midterm will improve on the final
  - Graded on rubric

Results

Physics scores						
Problem	1	2	3	4	All	1,2,3
Midterm mean	69	60	43	93	66	57
Final exam mean	58	54	46	80	60	53

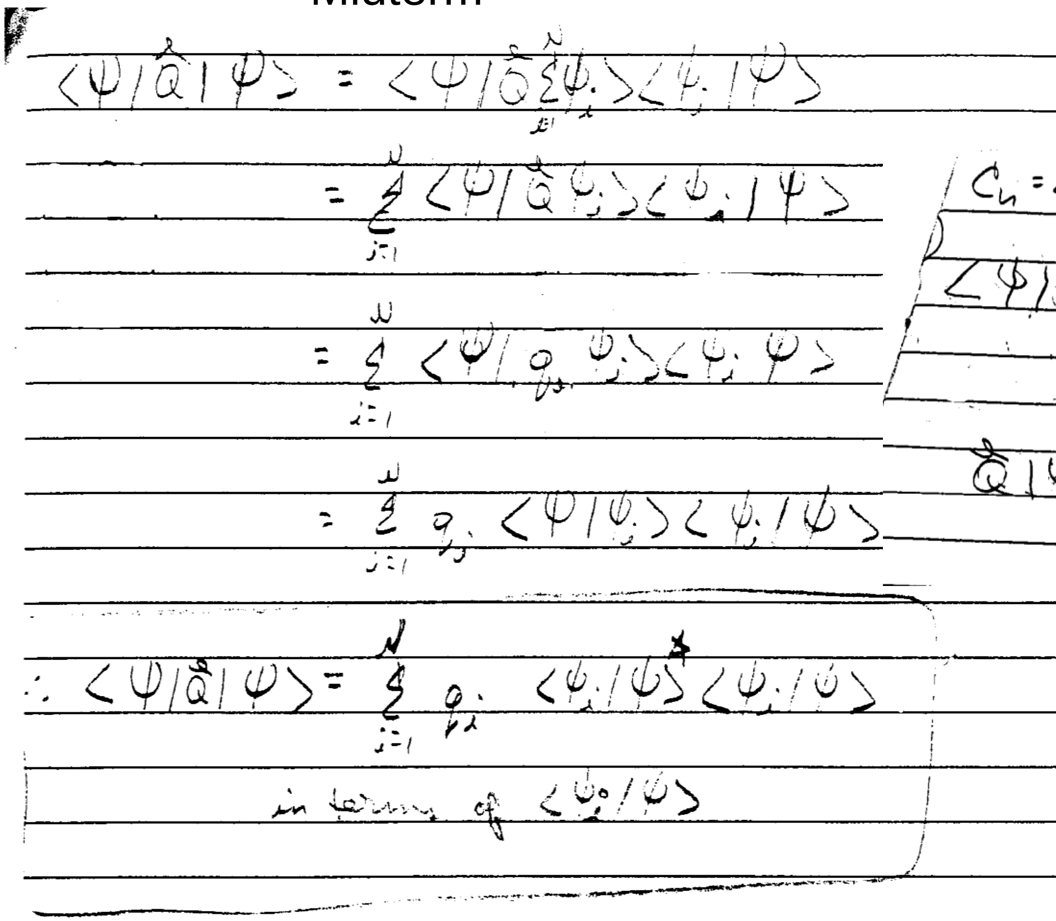
All problems (physics only):	Good to good**	Good to bad	Bad to good	Bad to bad
# of instances*	15	6	5	16
% of instances*	36%	14%	12%	38%

- No improvement!

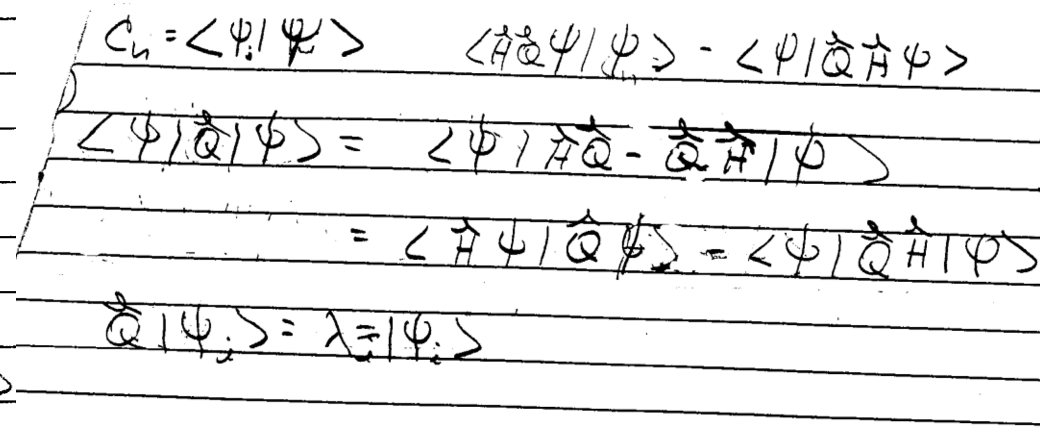
* "Instance" = 1 attempt on 1 problem (problem 4 not included - too easy)
** "Good" = at least 60% score, "Bad" < 60%

Performance on the same question on midterm and final exams :Student X

Midterm



Final



Interview Results

- 6 student interviewed when in QM II (asked to solve same problems and about their problem solving approaches and attitudes):
  - Significantly worse performance in interview 2 months later
  - Discussions about final exam during Interviews suggest that some students did selective studying/memorizing during midterm but could not do so during final when there was too much material
  - Some students explicitly said they do not like to look at the instructor solutions to midterm exams
    - because they don’t expect those questions repeated in final exam
    - because it pains them to realize that they have done poorly

Lessons Learned from Written Responses

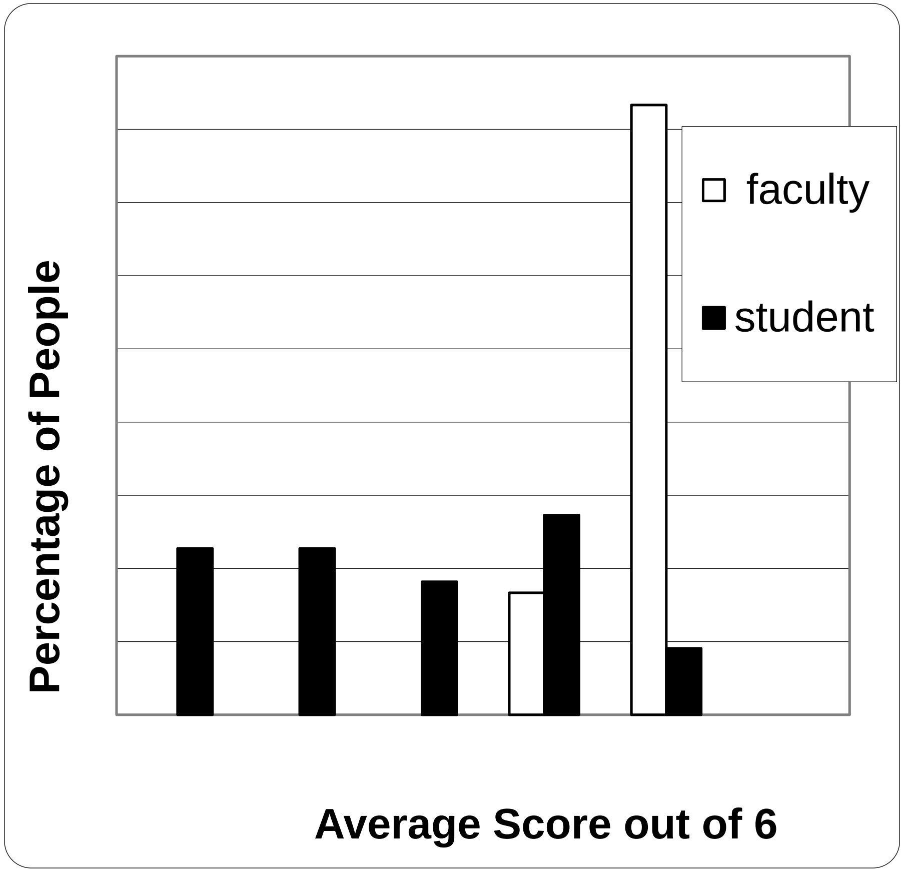
- Many students in QM do not necessarily learn from their mistakes
- They do not automatically use their mistakes as an opportunity for repairing, extending and organizing their knowledge
- They are not automatically doing self-monitoring
  - Some perform poorly both times
  - Some do well on midterm but regress on final

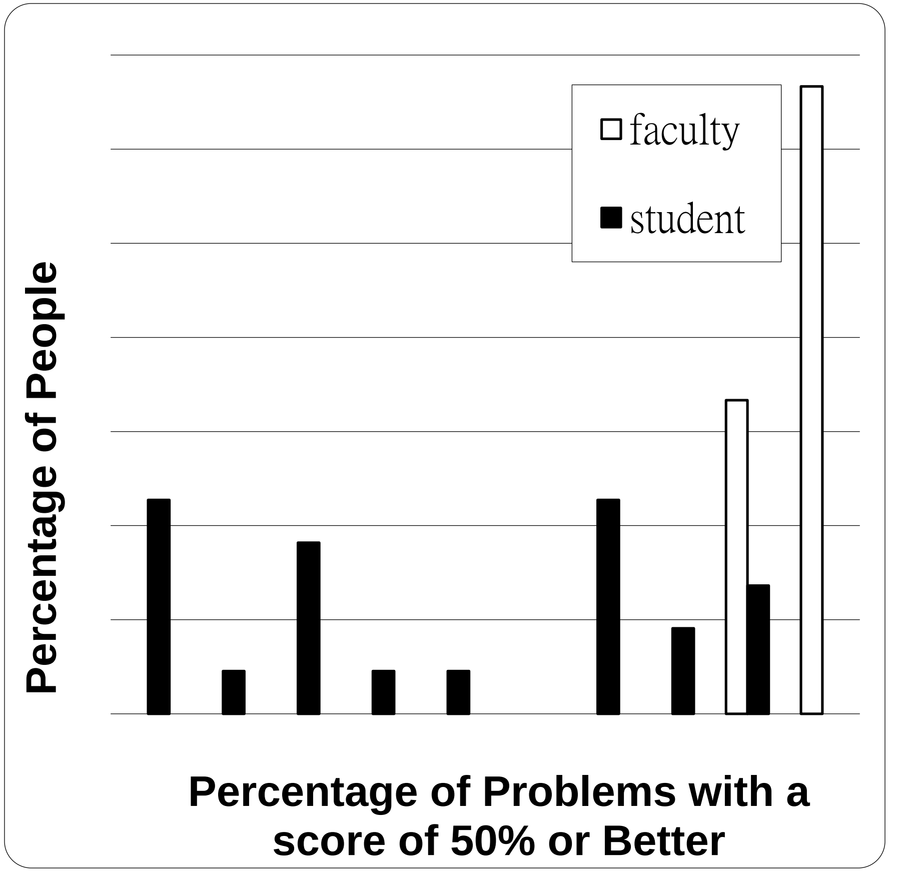
Categorization of QM Problems

(With Shih-Yin Lin, EJP, 2010)

- Categorization task (grouping together problem based upon similarity of solution) can be used to
  - Measure of expertise (Chi et. al. 1981)
    - Since experts have well-organized knowledge hierarchy
      - may categorize problems differently than students
- How is Categorization of introductory problems different from categorization of QM problems

Performance – Faculty vs. Student





Wide distribution of student performance in QM very similar to the distribution of intro. Students' categorization in intro physics (Mason and Singh, PRST PER)

Summary

- Research shows that advanced undergraduate and graduate students in quantum mechanics courses have
  - Common difficulties which are universal in nature similar to those documented for introductory students
    - Independent of school, teaching style and textbook
    - Distribution of students' expertise
  - Do not automatically take the time to learn from their mistakes and do not take the opportunity to organize and extend their knowledge
    - Need explicit guidance in developing self-monitoring skills
  - Develop & assess research-based QuILTs and peer instruction tools

Summary of Learning Tools

- Quantum Interactive Learning Tutorials (QuILTs) and Peer Instruction Tools
  - Based upon research on student difficulties/iterative development and evaluation based upon faculty/student feedback
  - Bridge the gap between conceptual and quantitative aspects
  - Exploit computer simulations to enhance learning
  - Students can work on them in class and QuILTs also as part of homework (self-paced)
  - Research-based QuILTs & Peer Instruction tools help students acquire <u>usable</u> knowledge by
    - Accounting for cognitive issues in learning physics
    - Keeping students actively involved in learning process
    - explicitly emphasizing and rewarding development of reasoning skills