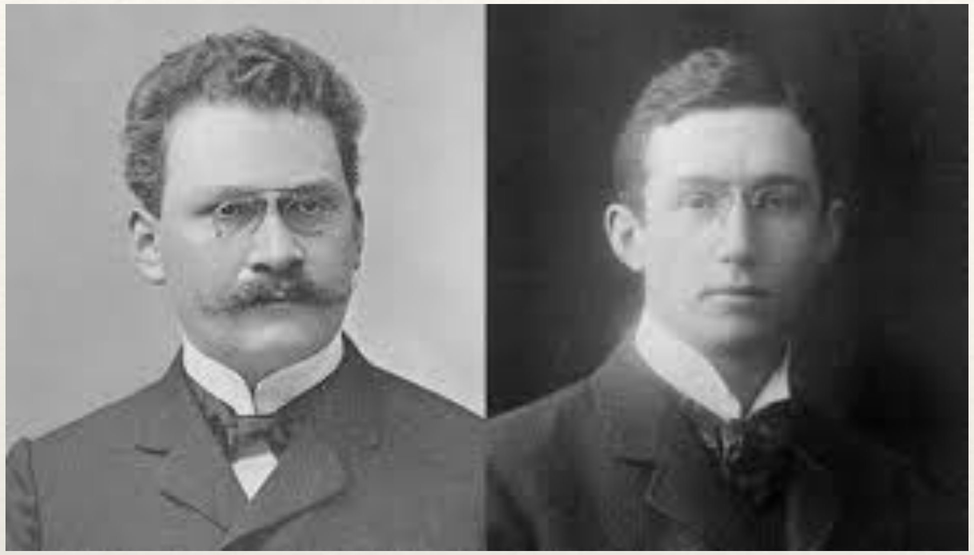
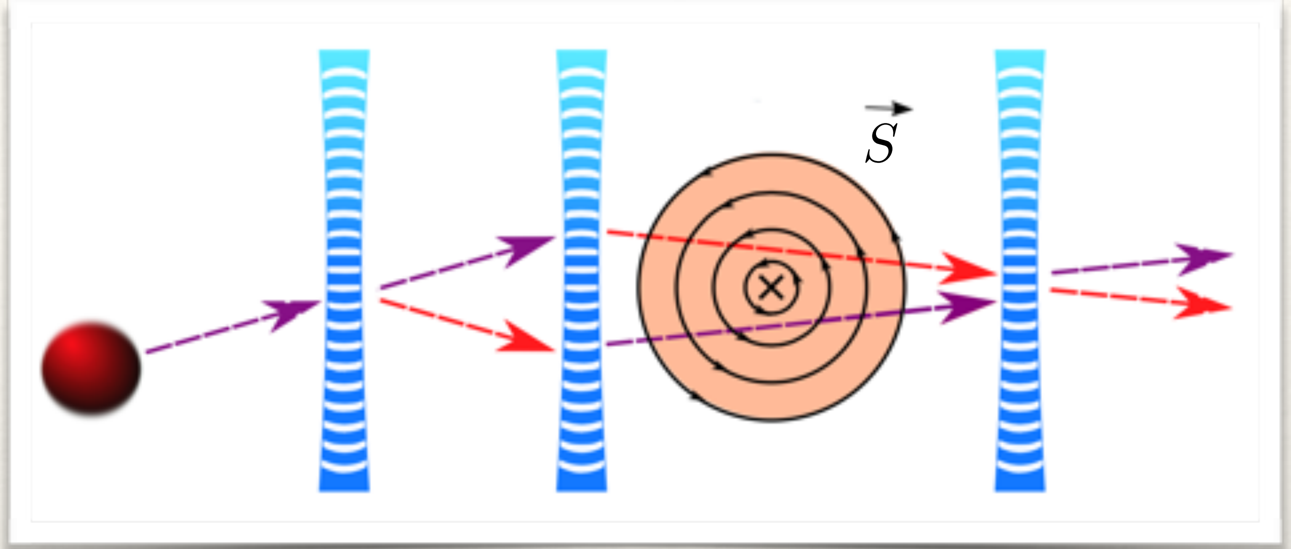




The Abraham-Minkowski controversy: an ultracold atom perspective

Duncan O'Dell,
McMaster University

University of Toronto, Physics Colloquium, October 5, 2017

Plan & Acknowledgements

Peter Milonni
Los Alamos / Rochester

Paul Lett
NIST

How Does The Photon Get Its Momentum?

physics colloquium, University of Toronto, 2005, URL http://www.physics.utoronto.ca/~colloq/2005_2006.html

Plan & Acknowledgements

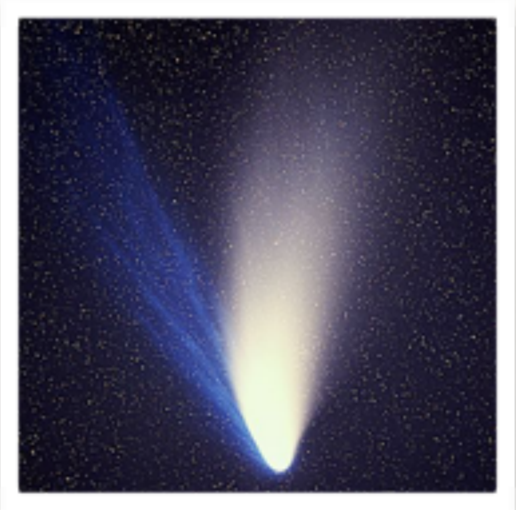
Dr. Nick Miladinovic
Ph.D. 2016

- Introduction to Abraham vs. Minkowski
- Two experiments (one done, one begging)
- Quantum matter: He-McKellar-Wilkens phase
- Proposal for optical HMW experiment

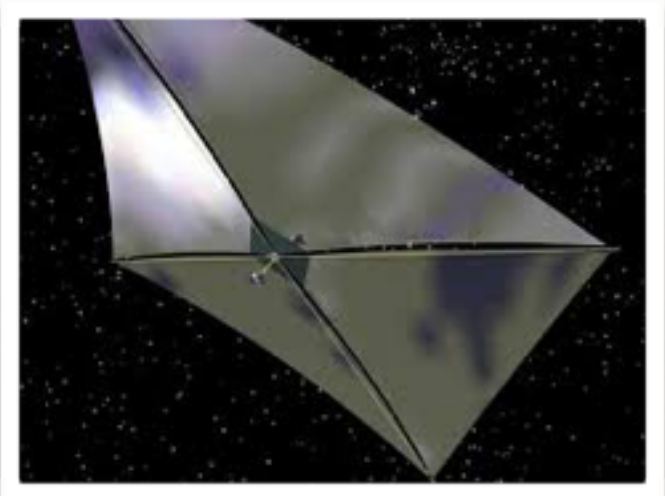
Optical forces on matter

Kepler (1619), Maxwell (1862), Lebedev (1900), Nichols & Hull (1901)

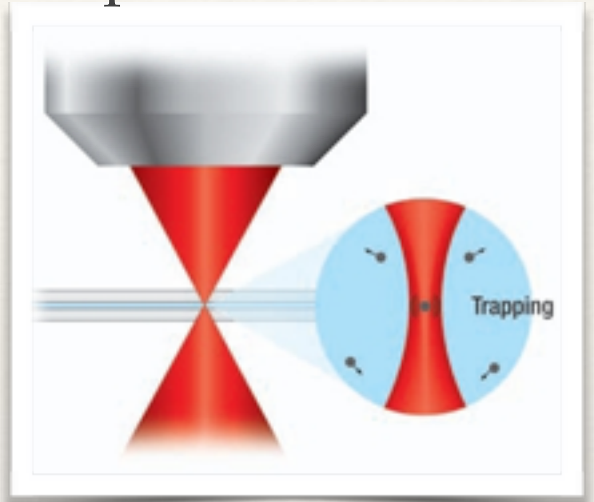
comet tails



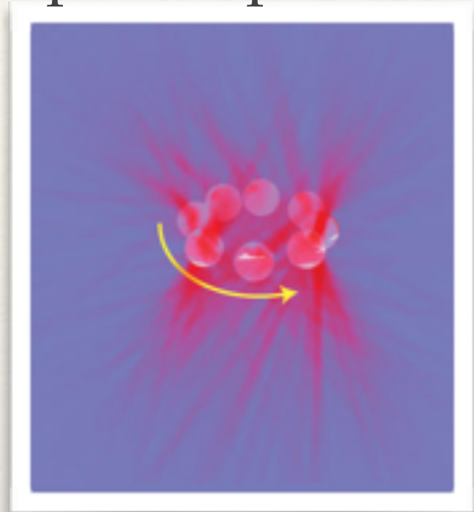
solar sails



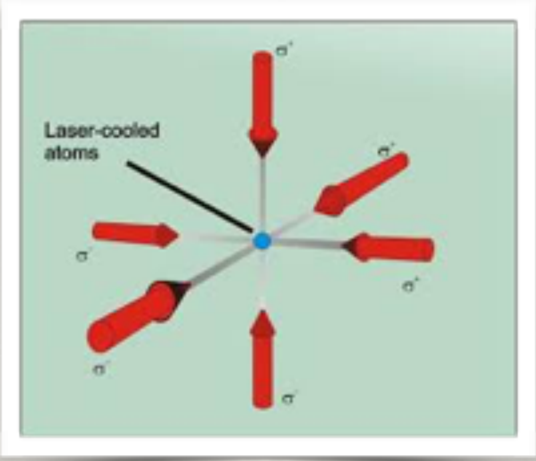
optical tweezers



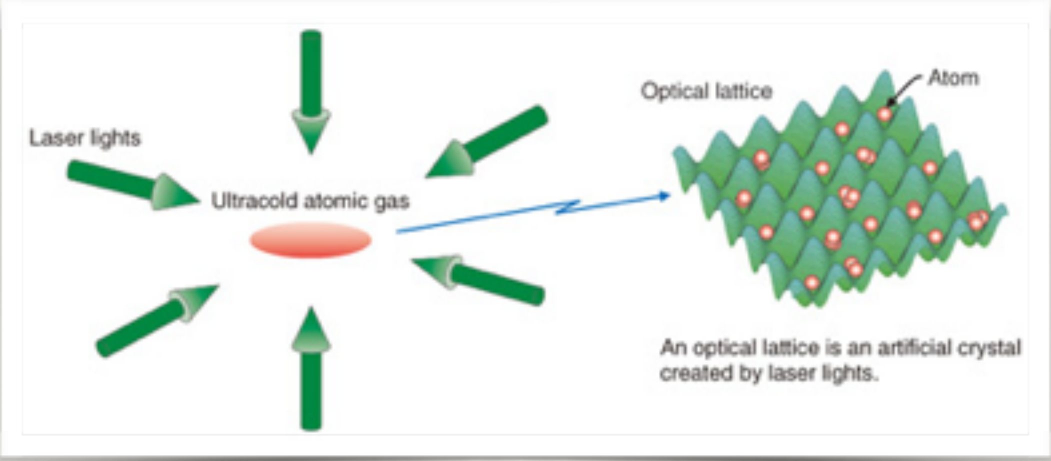
optical spanners



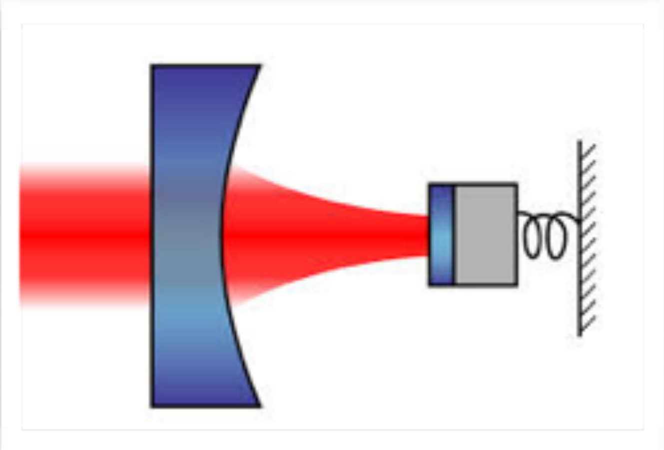
laser cooling



optical lattices



optomechanics



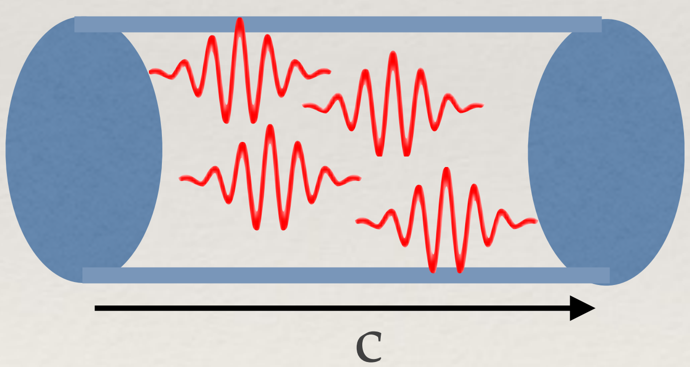
Momentum of light in vacuum

Energy flux in electromagnetic field:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \mathbf{E} \times \mathbf{B}/\mu_0$$

Poynting vector

Amount of energy in cylinder of unit area and length c is $|\mathbf{S}|$



Energy density in cylinder = $|\mathbf{S}|/c$

Momentum density = Energy density / c= $\mathbf{S}/c^2$

Momentum density: $$g = \mathbf{E} \times \mathbf{H}/c^2$$

In a medium? ***E, D, B, H…..***

Light in a medium: Abraham vs Minkowski

Max Abraham

1875-1922

Hermann Minkowski

1864-1909

MOMENTUM DENSITY of light in a medium:

$$\mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2 \qquad \mathbf{g}_M = \mathbf{D} \times \mathbf{B}$$

H. Minkowski, *Nachr. Ges. Wiss. Göttn. Math.-Phys. Kl.* **53**, (1908).
M. Abraham, Rend. Circ. Matem. Palermo **28**, 1 (1909).

Origin of controversy is the difficulty of separating electromagnetic field from matter.

N.B. other forms have been proposed, e.g. Einstein & Lieb $g = \epsilon_0 \mathbf{E} \times \mathbf{B}$

$$\mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2 \quad \longrightarrow \quad \mathbf{p} = \hbar \mathbf{k}_0/n$$

$$n = c/c'$$

refractive index

$$\mathbf{g}_M = \mathbf{D} \times \mathbf{B} \quad \longrightarrow \quad \mathbf{p} = n\hbar \mathbf{k}_0$$

Momentum is a classical effect but it is useful to consider a volume with a single photon:

$$\int \mathrm{d}V \; \frac{1}{2}(\mathbf{D}.\mathbf{E} + \mathbf{B}.\mathbf{H}) = \hbar\omega$$

Energy is on average shared equally between electric and magnetic parts:

$$\int \mathrm{d}V \; \frac{1}{2}\mathbf{B}.\mathbf{H} = \frac{\hbar\omega}{2} = \int \mathrm{d}V \; \frac{1}{2}\mathbf{D}.\mathbf{E}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad E, B \propto e^{i(nkx - \omega t)} \longrightarrow nkE \sim \omega B \longrightarrow E \sim \frac{c}{n} B = c' B$$

$$\int \mathrm{d}V \; \mathbf{B}.\mathbf{H} = \hbar\omega$$

divide by $n\,c$: $\int \mathrm{d}V \; \frac{\mathbf{B}.\mathbf{H}}{nc} = \frac{\hbar\omega}{nc}$

$$\longrightarrow \int \mathrm{d}V \; c' \frac{\mathbf{B}.\mathbf{H}}{c^2} = \int \mathrm{d}V \; \frac{\mathbf{E}.\mathbf{H}}{c^2} = \frac{\hbar k}{n}$$

$$\int \mathrm{d}V \; \mathbf{D}.\mathbf{E} = \hbar\omega$$

multiply by n/c : $\int \mathrm{d}V \; \frac{\mathbf{D}.\mathbf{E}}{c} n = n \frac{\hbar\omega}{c}$

$$\longrightarrow \int \mathrm{d}V \; \frac{\mathbf{D}.\mathbf{B}}{c} c = \int \mathrm{d}V \; \mathbf{D}.\mathbf{B} = n\hbar k$$

Light in a medium: Abraham vs Minkowski

Abraham

Momentum density:

$$\mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2$$

$$\mathbf{p} = \hbar \mathbf{k}_0 / n$$

"particle picture"

$$p = mc' = m\frac{c}{n}$$

Einstein: $E = mc^2$

$$\rightarrow p = \frac{E}{c^2}\frac{c}{n} = \frac{E}{nc}$$

Planck: $E = \hbar c k$

Minkowski

Momentum density:

$$\mathbf{g}_M = \mathbf{D} \times \mathbf{B}$$

$$\mathbf{p} = n\hbar \mathbf{k}_0$$

"wave picture"

de Broglie: $p = h/\lambda$

In medium: $\lambda \rightarrow \lambda/n$

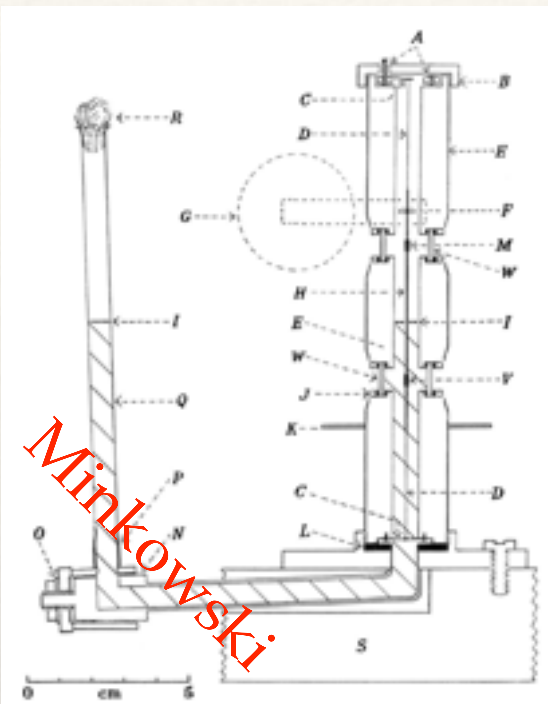
$k \rightarrow nk$

U. Leonhardt, Nature **444**, 823 (2006).

Large literature

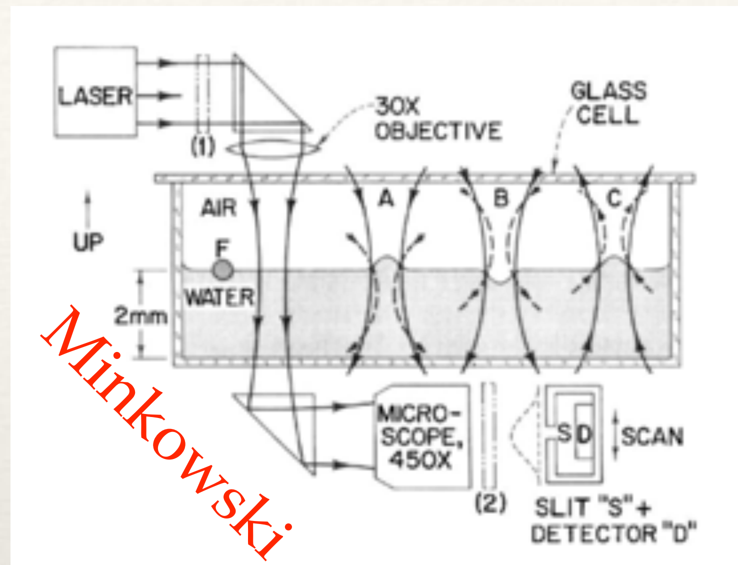
- …..
- Gordon, Phys. Rev. A **8**, 14 (1973)
- Walker, Lahoz & Walker, Can. J. Phys. **53**, 2577 (1975)
- Haugan & Kowalski, Phys. Rev. A **25**, 2102 (1982)
- Lembessis, Babiker, Baxter & Loudon, Phys. Rev. A **48**, 1594 (1993)
- Campbell et al, Phys. Rev. Lett. **94**, 170403 (2005)
- Loudon, Barnett & Baxter, Phys. Rev. A **71**, 063802 (2005)
- Leonhardt, Nature **444**, 823 (2006)
- Pfeifer, Nieminen, Heckenberg, & Rubinsztein-Dunlop, Rev. Mod. Phys. **79**, 1197 (2007)
- Mansuripur, Opt. Express **12**, 5375 (2004)
- She, Yu & Feng, Phys. Rev. Lett. 101, 243601 (2008).
- Mansuripur, Phys. Rev. Lett. **103**, 019301 (2009)
- Hinds & Barnett, Phys. Rev. Lett. **102**, 050403 (2009)
- Barnett & Loudon, Phil. Trans. R. Soc. A **368**, 927 (2010)
- Barnett, Phys. Rev. Lett. **104**, 070401 (2010)
- Zhang, Zhang, Wang & Liu, Phys. Rev. A **85**, 053604 (2012)
- …..

Previous experiments…



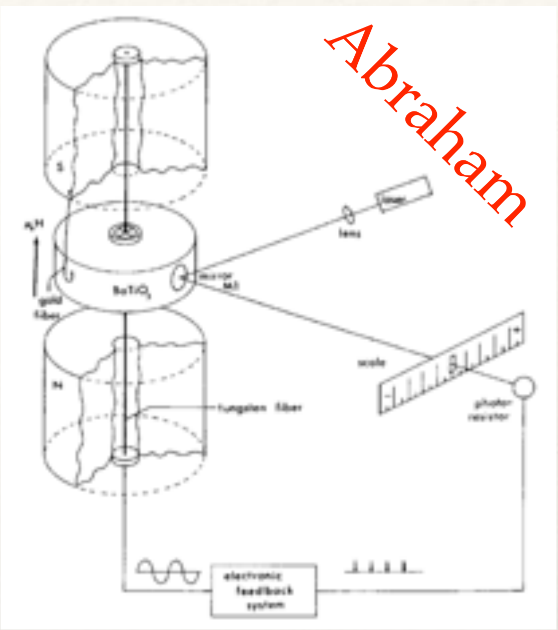
Recoil of mirror suspended in fluid

Jones and Richards, Proc. R. Soc. London, Ser. A **221**, 480 (1954)



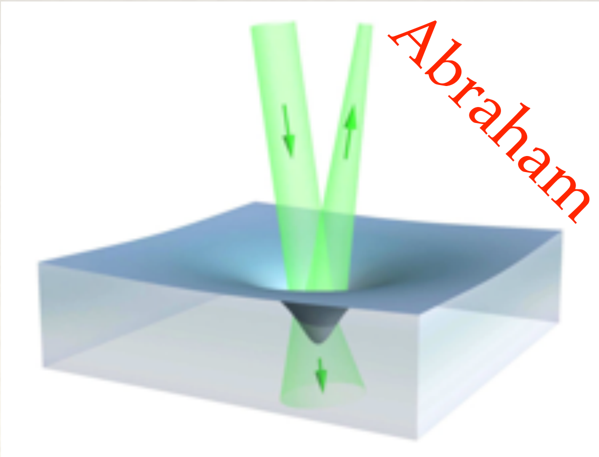
Liquid surface bulges out: Minkowski
Bulges in: Abraham

Ashkin and Dziedzic, Phys. Rev. Lett. **30**, 139 (1973)



Torque on disk: constant axial B-field, time varying radial E-field

Walker, Lahoz, and Walker, Can. J. Phys. **53**, 2577 (1975)

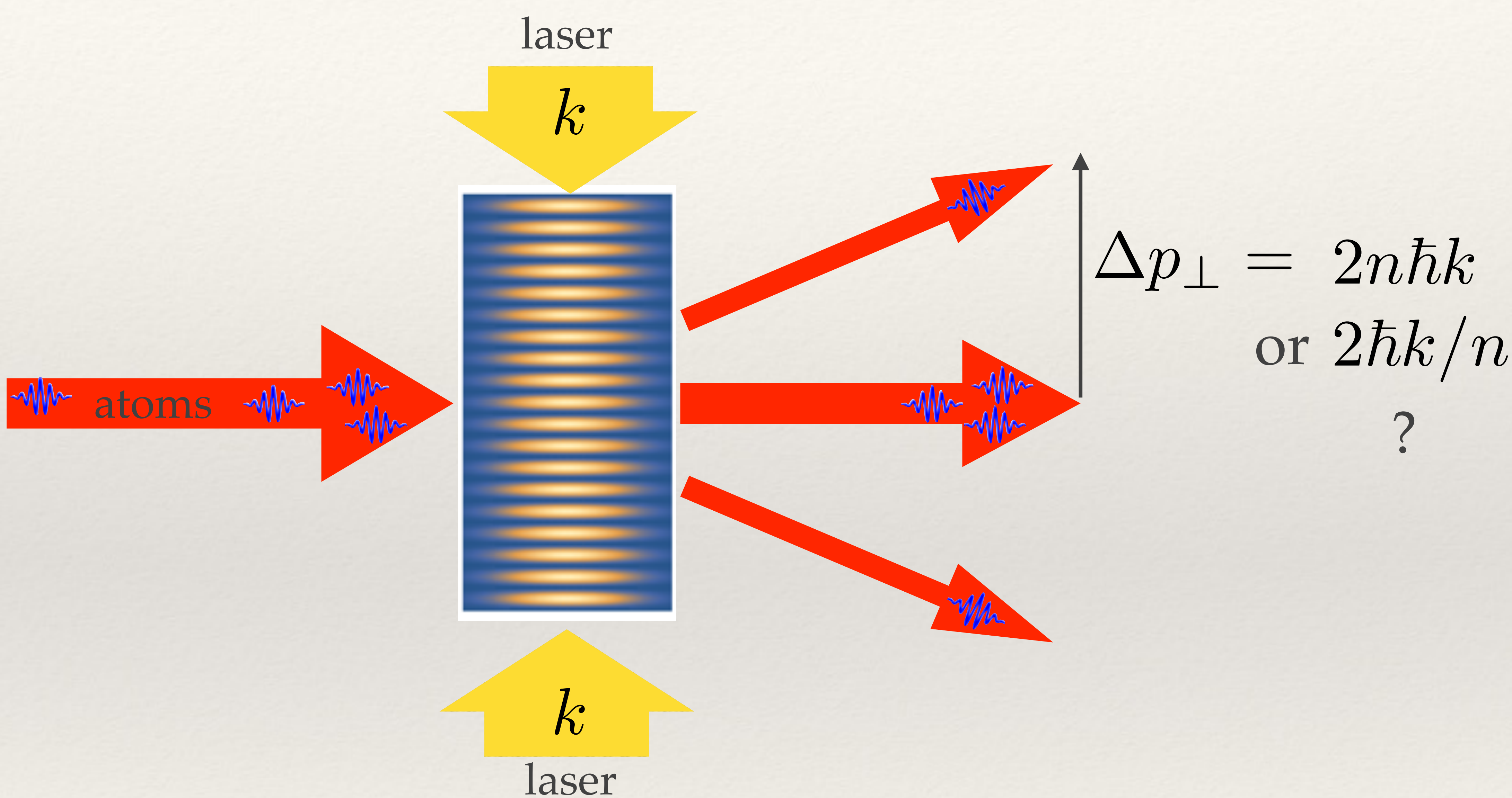


Zhang, She, Peng, and Leonhardt, New J. Phys. **17**, 053035 (2015)

Two atomic experiments

- Diffraction of atoms by light (Minkowski)
- Centre of mass motion of an atom in a light pulse (Abraham)

Kapitza-Dirac scattering of atoms by light



Standing wave of laser light diffracts atoms

P.L. Kapitza and P.A.M. Dirac, Proc. Cambridge Philos. Soc. **29**, 297 (1933) ;
(Stimulated Compton scattering of electrons light)

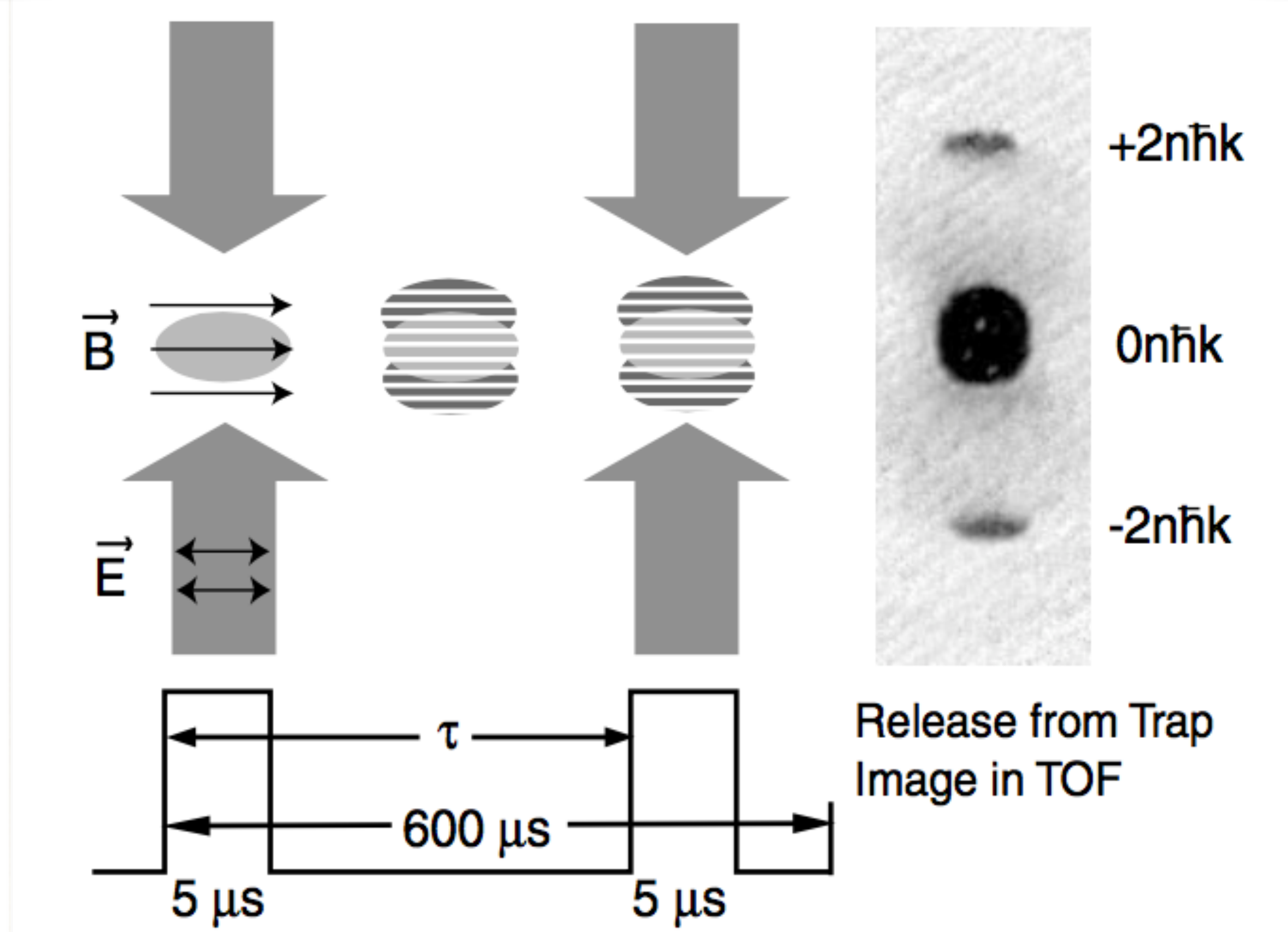
P.L. Gould, G.A. Ruff, and D.E. Pritchard, PRL **56**, 827 (1986)
(Diffraction of atoms by light)

Experiment: Kapitza-Dirac with a BEC

PRL 94, 170403 (2005) PHYSICAL REVIEW LETTERS week ending 6 MAY 2005

Photon Recoil Momentum in Dispersive Media

Gretchen K. Campbell, Aaron E. Leanhardt,* Jongchul Mun, Micah Boyd, Erik W. Streed, Wolfgang Ketterle, and David E. Pritchard†



Result seems to support Minkowski.

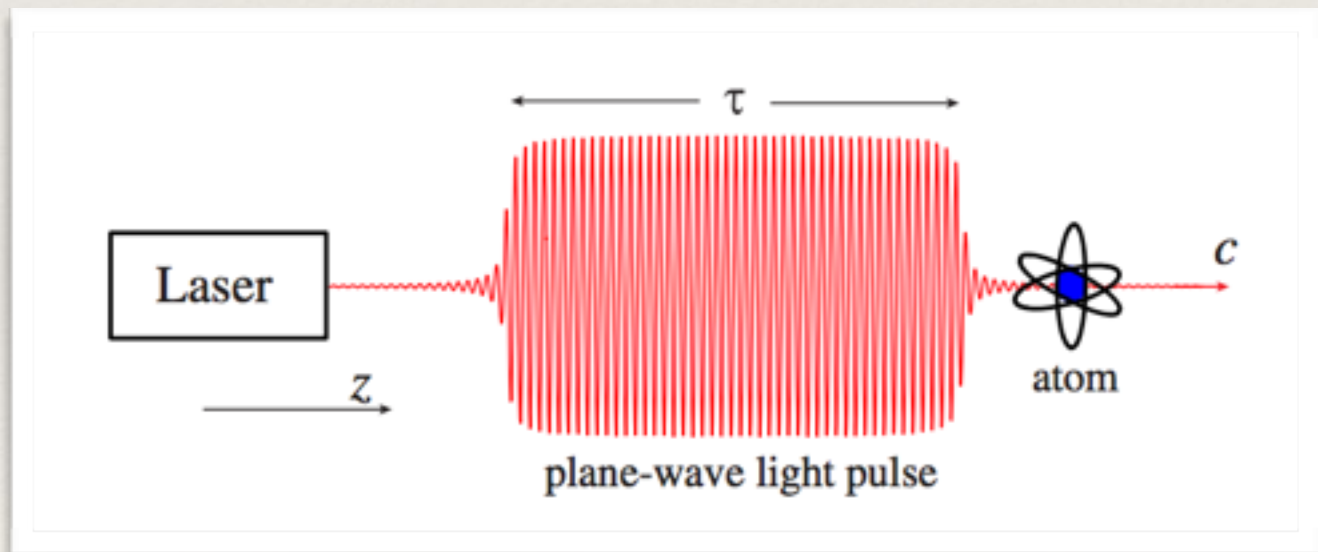
Force on an atom due to a light pulse

PRL 102, 050403 (2009) PHYSICAL REVIEW LETTERS week ending 6 FEBRUARY 2009

Momentum Exchange between Light and a Single Atom: Abraham or Minkowski?

E. A. Hinds
Centre for Cold Matter, Imperial College, Prince Consort Road, London SW7 2AZ, United Kingdom

Stephen M. Barnett
SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom
(Received 17 November 2008; published 3 February 2009)



Dielectric "medium" is a single atom:

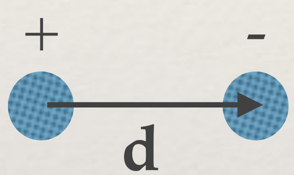
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\mathrm{atom}})$$
$$\mathbf{B} = \mu_0 \mathbf{H}$$

Which way does atom recoil?

Lorentz force on induced dipole

Lorentz force on a single charged particle: $\mathbf{F} = q\left(\mathbf{E} + \frac{d\mathbf{x}}{dt} \times \mathbf{B}\right)$

Lorentz force on a neutral dipole: $\mathbf{F} = q\left(\mathbf{E}(\mathbf{x}_1) - \mathbf{E}(\mathbf{x}_2) + \frac{d(\mathbf{x}_1 - \mathbf{x}_2)}{dt} \times \mathbf{B}\right)$



$$\approx q\left([(\mathbf{x}_1 - \mathbf{x}_2).\nabla]\mathbf{E}(\mathbf{x}) + \frac{d(\mathbf{x}_1 - \mathbf{x}_2)}{dt} \times \mathbf{B}\right)$$

$$= [\mathbf{d}.\nabla]\mathbf{E} + \dot{\mathbf{d}} \times \mathbf{B}$$

where: $\mathbf{d} = \alpha\mathbf{E} = q(\mathbf{x}_1 - \mathbf{x}_2)$

$$= \alpha\left([\mathbf{E}.\nabla]\mathbf{E} + \dot{\mathbf{E}} \times \mathbf{B}\right)$$

version 1

Using: $(\mathbf{E}.\nabla)\mathbf{E} = \frac{1}{2}\nabla E^2 - \mathbf{E} \times (\nabla \times \mathbf{E})$ and $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$:

$$\mathbf{F} = \alpha\left[\frac{1}{2}\nabla E^2 + \frac{d}{dt}(\mathbf{E} \times \mathbf{B})\right]$$

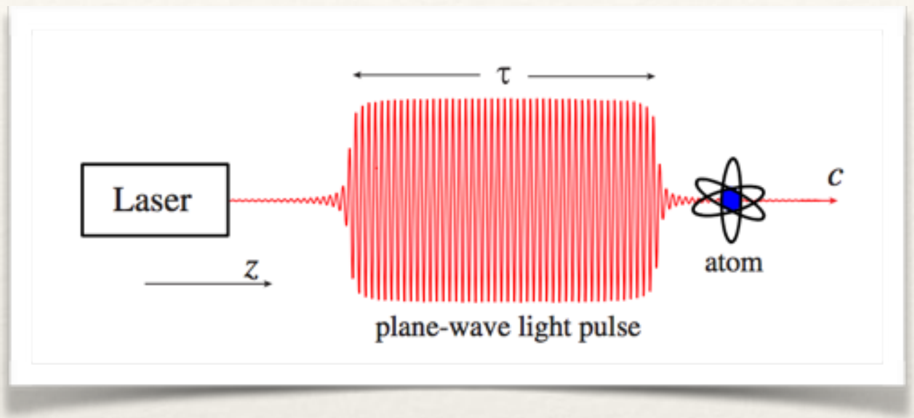
$\mu_0\mathbf{S}$

version 2

gradient force

Lorentz force acting on the internal current [J.P. Gordon, Phys. Rev. A **8**, 14 (1973)]

Force on an atom due to a light pulse



$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\text{atom}})$$

$$\mathbf{B} = \mu_0 \mathbf{H}$$

Lorentz force on induced dipole

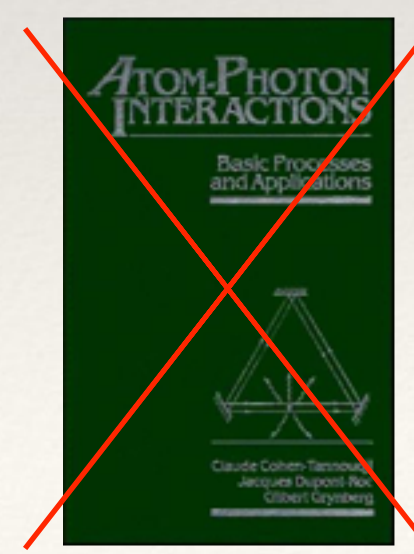
$$F_i = \mathbf{d} \cdot \frac{\partial}{\partial x_i}\mathbf{E} + \frac{\partial}{\partial t}(\mathbf{d} \times \mathbf{B})_i$$

gradient force + -2 x (gradient force) ! (Barnett & Hinds 2009)

$$\Delta P = \int F dt$$

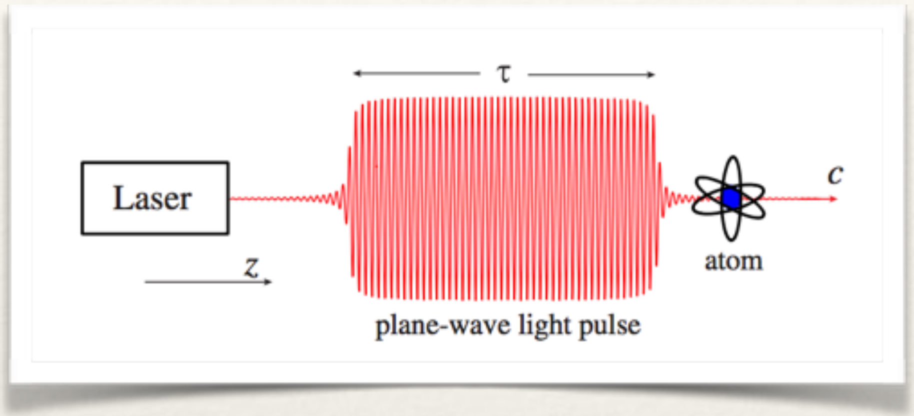
$\propto$ time gradient force acts

$\propto$ change in field strength



For static EM field configurations (typical cold atom expts.) the gradient force dominates, but not here.

Answer: Abraham!



$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\rm atom})$$

$$\mathbf{B} = \mu_0 \mathbf{H}$$

Lorentz force on induced dipole

$$F_i = \mathbf{d} \cdot \frac{\partial}{\partial x_i}\mathbf{E} + \frac{\partial}{\partial t}(\mathbf{d} \times \mathbf{B})_i$$

$$\mathbf{g}_M = [\epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\rm atom})] \times \mu_0 \mathbf{H} = \mathbf{g}_A + \mathbf{d} \times \mathbf{B}\ \delta(\mathbf{r} - \mathbf{r}_{\rm atom})$$

$$\underbrace{\int \mathbf{g}_M \mathrm{d}^3 r}_{\boxed{\mathbf{p} = n\hbar\mathbf{k}_0}} = \underbrace{\int \mathbf{g}_A \mathrm{d}^3 r}_{\boxed{\mathbf{p} = \hbar\mathbf{k}_0/n}} + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\rm atom})$$

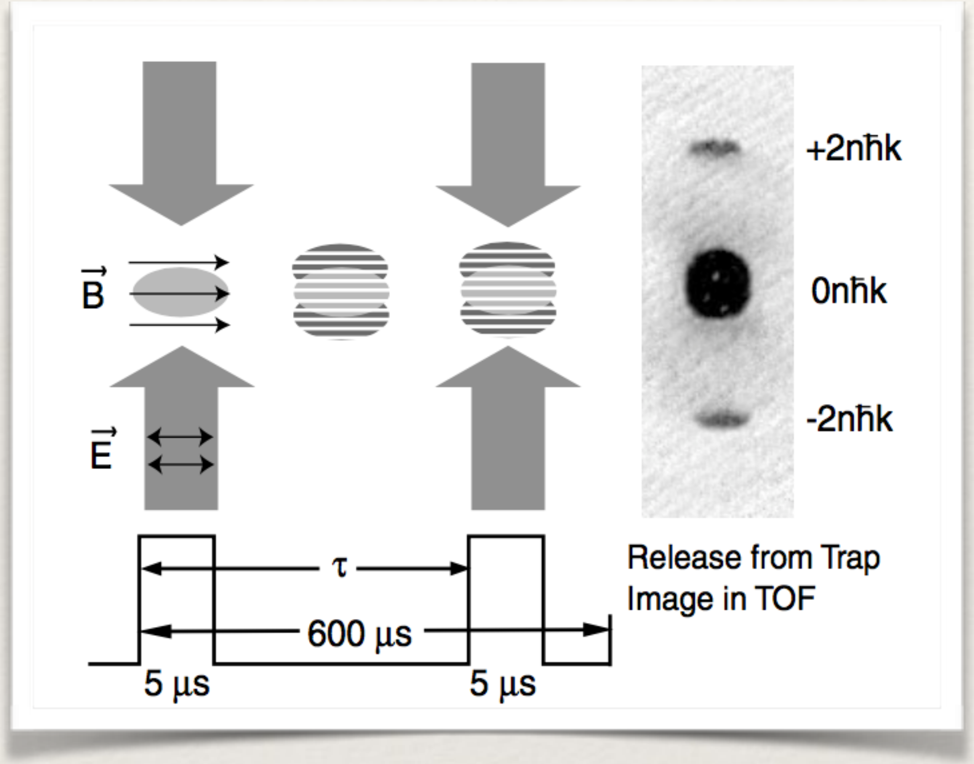
$\Rightarrow$ $g_M > g_A$

$\Rightarrow$ **d**x**B** term is responsible for difference between Abraham and Minkowski

$\Rightarrow$ Lorentz force predicts **Abraham**. Check: total momentum (atom+field) is conserved; if **d**x**B** term dropped from Lorentz force on atom then momentum is added to light

Implications for Campbell et al.

$$\int \mathbf{g}_M \mathrm{d}^3 r = \int \mathbf{g}_A \mathrm{d}^3 r + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$$



Assume linear response: $\mathbf{d} = \alpha \mathbf{E}$

$$\mathbf{d} \times \mathbf{B} = \alpha \mathbf{E} \times \mathbf{B}$$
$$= \alpha \mu_0 \mathbf{S}$$

where $\mathbf{S} = (\mathbf{E} \times \mathbf{B})/\mu_0$

Poynting vector

In the experiment by Campbell et al. $\mathbf{S} = 0$ and so $\mathbf{g}_M = \mathbf{g}_A$

Abraham and Minkowski momenta are the same!

canonical vs kinetic momentum

PRL **104,** 070401 (2010) PHYSICAL REVIEW LETTERS week ending 19 FEBRUARY 2010

Resolution of the Abraham-Minkowski Dilemma

Stephen M. Barnett
Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, United Kingdom
(Received 7 October 2009; published 17 February 2010)

The dilemma of identifying the correct form for the momentum of light in a medium has run for a century and has been informed by many distinguished contributions, both theoretical and experimental. We show that *both* the Abraham and Minkowski forms of the momentum density are correct, with the former being the kinetic momentum and the latter the canonical momentum. This identification allows us to explain why the experiments supporting each of the rival momenta gave the results that they did. The inclusion of dispersion and absorption provides an interesting subtlety, but does not change our conclusion.

$$\mathbf{p}_{\mathrm{atom}}^{M} + \int \mathbf{g}_M \mathrm{d}^3 r = \mathbf{p}_{\mathrm{atom}}^{A} + \int \mathbf{g}_A \mathrm{d}^3 r$$

total momentum conserved

$$\mathbf{p}_{\mathrm{atom}} + \int \mathbf{g}_M \mathrm{d}^3 r = M\dot{\mathbf{r}}_{\mathrm{atom}} + \int \mathbf{g}_A \mathrm{d}^3 r$$

Blount, Bell Telephone Laboratories technical memorandum 38139–9 unpublished (1971)

Lembessis, Babiker, Baxter and Loudon, Phys. Rev. A. **48**, 1594 (1993)

Lagrangian: kinetic vs. canonical momentum

charged particle in EM field: $$L_{\text{charge}} = \frac{1}{2} m\dot{x}^2 + q\dot{\mathbf{x}} \cdot \mathbf{A} - q\phi$$

$$\longrightarrow \quad p_i = \frac{\partial L}{\partial \dot{x}_i} = m\dot{x}_i + qA_i$$

neutral electric dipole: $$L_{\text{dipole}} = \frac{1}{2} m\dot{x}^2 + \mathbf{d} \cdot (\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B})$$

where: $V_{\text{dipole}} = -\mathbf{d} \cdot \mathbf{E}$ $\qquad$ $\mathbf{E}_{\text{motion}} = \mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}$

Wilkens, Phys. Rev. Lett. **72**, 5 (1994); Wei, Han, & Wei, Phys. Rev. Lett. **75**, 2071 (1995).

$$\longrightarrow \quad \frac{\partial L}{\partial \dot{\mathbf{x}}} = \underbrace{\boxed{\mathbf{p}_{\text{atom}}}}_{\text{canonical}} = \underbrace{\boxed{M\dot{\mathbf{r}}_{\text{atom}}}}_{\text{kinetic}} - \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\text{atom}})$$

Map between charge and dipole

$$L_{\text{charge}} = \frac{1}{2}m\dot{x}^2 + q\dot{\mathbf{x}}\cdot\mathbf{A} - q\phi$$ **charged particle**

$$L_{\text{dipole}} = \frac{1}{2}m\dot{x}^2 + \mathbf{d}\cdot(\mathbf{E} + \dot{\mathbf{x}}\times\mathbf{B})$$ **neutral electric dipole**

$$\boxed{\mathbf{B}\times\mathbf{d} \leftrightarrow (q\mathbf{A})_{\text{eff}} \qquad \mathbf{d}\cdot\mathbf{E} \leftrightarrow -(q\phi)_{\text{eff}}}$$

Define effective fields:

$$\mathbf{E}_{\text{eff}} \equiv -\nabla\phi_{\text{eff}} - \frac{\partial\mathbf{A}_{\text{eff}}}{\partial t} = \frac{1}{q}\left[\nabla(\mathbf{d}\cdot\mathbf{E}) - \frac{\partial}{\partial t}(\mathbf{B}\times\mathbf{d})\right]$$

$$\mathbf{B}_{\text{eff}} \equiv \nabla\times\mathbf{A}_{\text{eff}} = \frac{1}{q}\nabla\times(\mathbf{B}\times\mathbf{d})$$

Force on atom in a plane wave

Assume plane wave laser beam: $\nabla \times (\mathbf{B} \times \mathbf{d}) = 0 \longrightarrow \mathbf{B}_{\text{eff}} = 0$

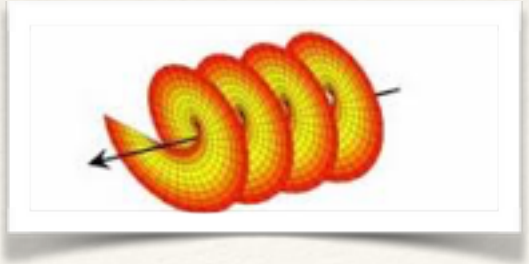
$$\mathbf{E}_{\text{eff}} \equiv -\nabla\phi_{\text{eff}} - \frac{\partial \mathbf{A}_{\text{eff}}}{\partial t} = \frac{1}{q}\left[\nabla(\mathbf{d}\cdot\mathbf{E}) - \frac{\partial}{\partial t}(\mathbf{B}\times\mathbf{d})\right]$$

$$\longrightarrow \quad \mathbf{F}_{\text{atom}} = q\mathbf{E}_{\text{eff}}$$

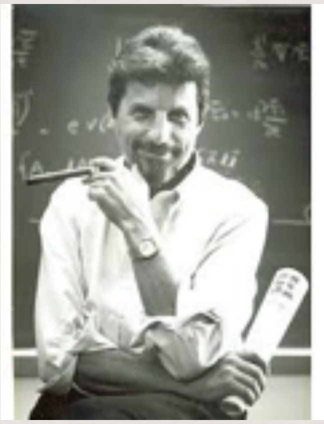
Assume linear response: $\mathbf{d} = \alpha\mathbf{E}$

$$\longrightarrow \quad \mathbf{F} = q\mathbf{E}_{\text{eff}} = \nabla\left(\frac{\alpha}{2}E^2\right) + \alpha\frac{\partial}{\partial t}(\mathbf{E}\times\mathbf{B})$$

Note that for structured light: $\nabla \times (\mathbf{B} \times \mathbf{d}) \neq 0$



Quantum Matter: He-McKellar-Wilkens phase



Aharonov-Bohm effect Phys. Rev. A **115**, 485 (1959)

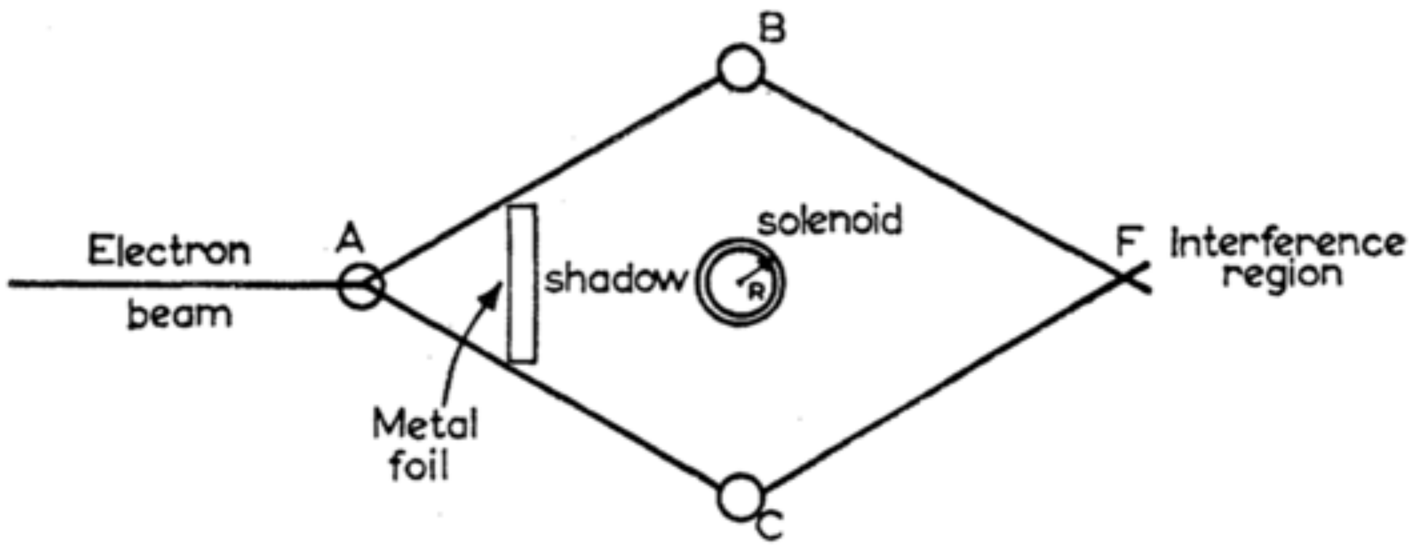


FIG. 2. Schematic experiment to demonstrate interference with time-independent vector potential.

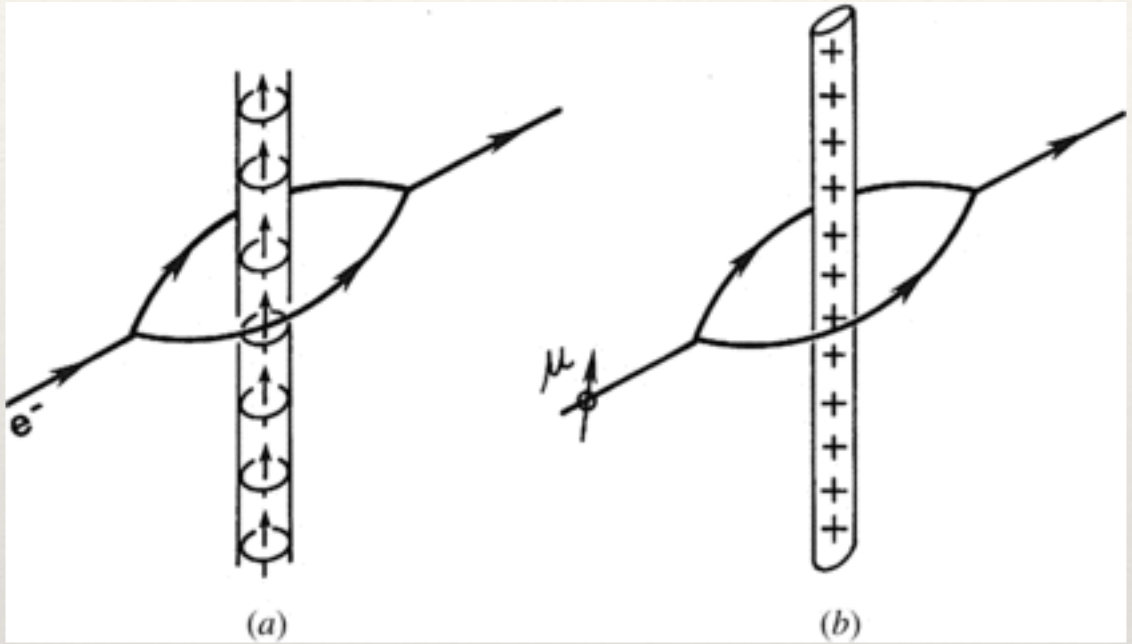
Measured: Chambers, Phys. Rev. Lett. **5,** 3 (1960).

Force on electron is zero: $\mathbf{B} = \nabla \times \mathbf{A} = 0$ outside: $\mathbf{A}_{\text{solenoid}} \propto \frac{1}{r}\hat{\phi}$

Phase of electron ψ is not zero: $\phi_{\text{AB}} = (q/\hbar) \oint \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r}$

The Aharonov-Casher phase

Neutral **magnetic dipole** moving in purely radial electric field



AB effect AC effect

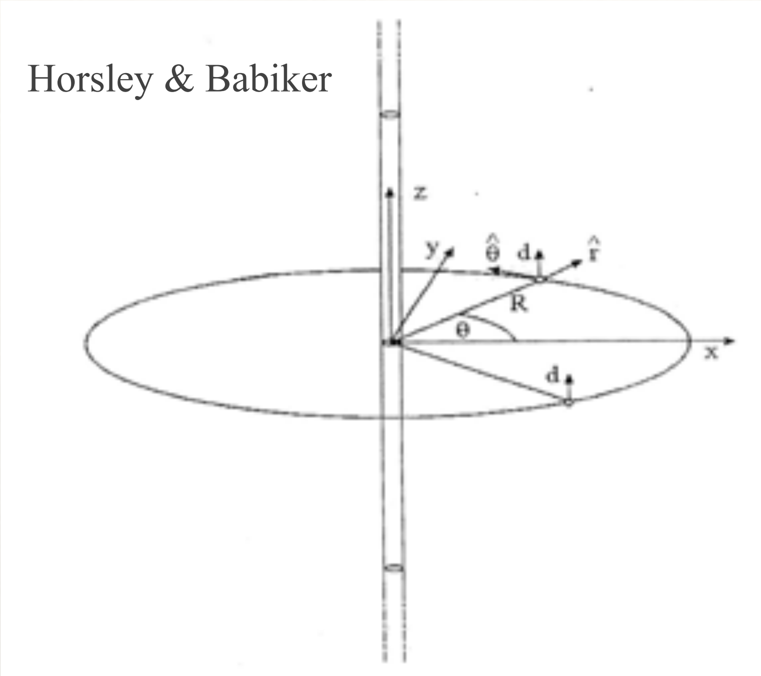
$$\phi_{\mathrm{AC}} = (\hbar c^2)^{-1} \oint [\mathbf{E}(\mathbf{r}) \times \mu] \cdot d\mathbf{r}$$

Aharonov & Casher Phys. Rev. Lett. **53**, 319 (1984).

Measured: Cimmino et al, Phys. Rev. Lett. **63,** 380 (1989)

The He-McKeller-Wilkens phase

Neutral **electric dipole** moving in a STATIC radial magnetic field



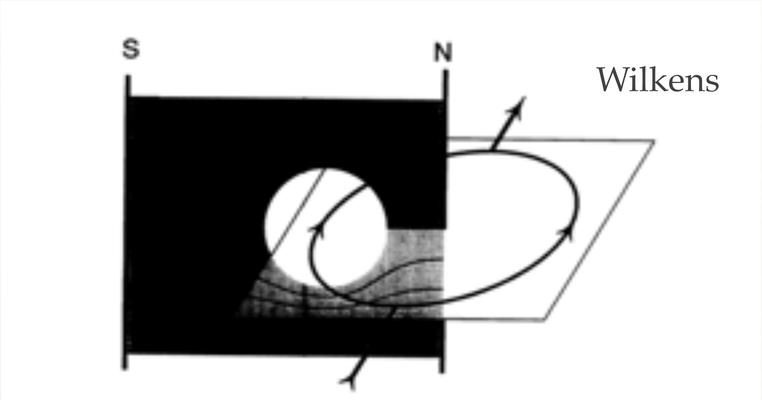


FIG. 1. Geometry of the interferometric experiment. Shown is the "Dirac sheet" with the two vertical edges effectively realizing two oppositely charged lines of magnetic monopoles. The "Dirac strings" (flux lines) which connect the two edges leave a hole for the interferometric path. The interferometric plane is orientated perpendicular to the magnet poles and the electric dipole moment of the interfering particles.

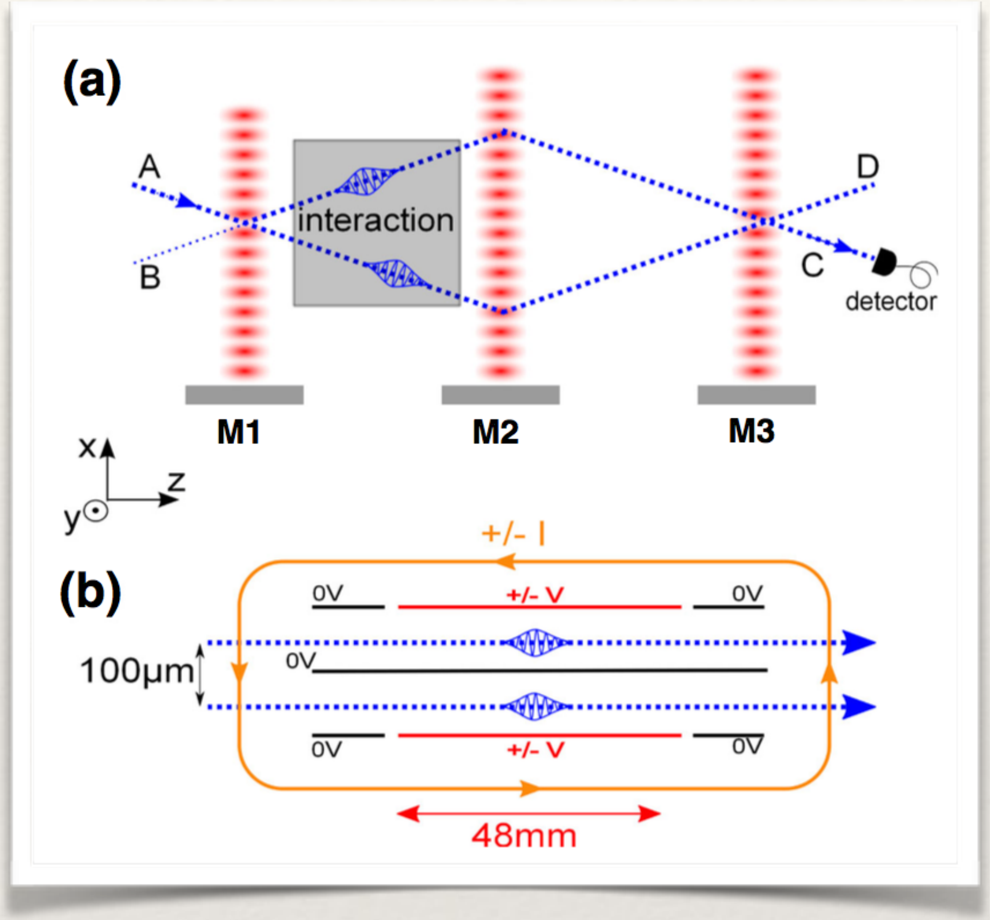
$$\boxed{\phi_{\mathrm{HMW}} = \hbar^{-1} \oint [\mathbf{B}(\mathbf{r}) \times \mathbf{d}] \cdot d\mathbf{r}}$$

Same "dxB" origin as Abraham vs Minkowski!

He & McKellar, Phys. Rev. A **47**, 3424 (1993), Wilkens, Phys. Rev. Lett. **72**, 5 (1994), Wei, Han, & Wei, Phys. Rev. Lett. 75, 2071 (1995), Horsley & Babiker Phys.Rev. Lett. **95**, 010495 (2005)…

Seems to require a straight line of magnetic monopoles…

Observation of the He-McKellar-Wilkens phase



Observed for static electric and magnetic fields by S. Lepoutre, A. Gauguet, G. Trénec, M. Buchner & J. Vigué, Phys. Rev. Lett. **109**, 120401 (2012).

^{7}Li

capacitor ~ 800 V

B~ 20 mT

φ_{HMW} ~ 40 mrad

Geometric phase: depends only on path taken, not on speed

Note: 1 mrad corresponds to $\lambda/10{,}000$ = 1-10 femtometers!

Optical HMW phase?

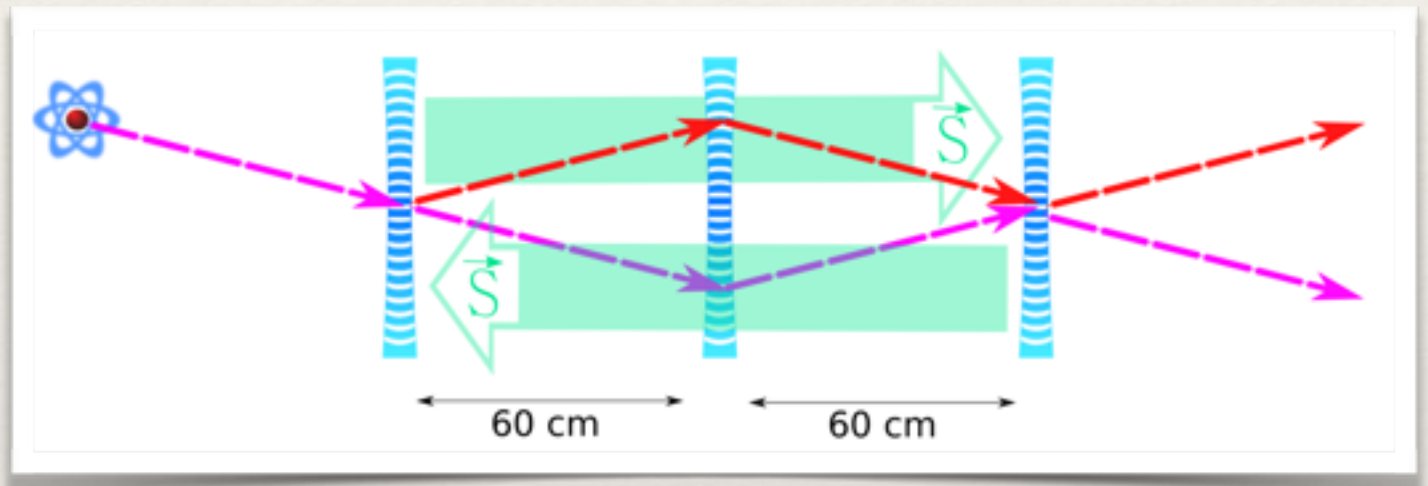
$$\psi \propto e^{(i/\hbar)\int L dt} = e^{i[\phi_{\rm dyn}(t_{\rm int}) + \phi^{\rm optical}_{\rm HMW}]}$$

$$\phi_{\rm dyn}(t_{\rm int}) = \frac{1}{\hbar}\int_0^{t_{\rm int}} \left(\frac{mv^2}{2} + \mathbf{d}\cdot\mathbf{E}\right) dt$$

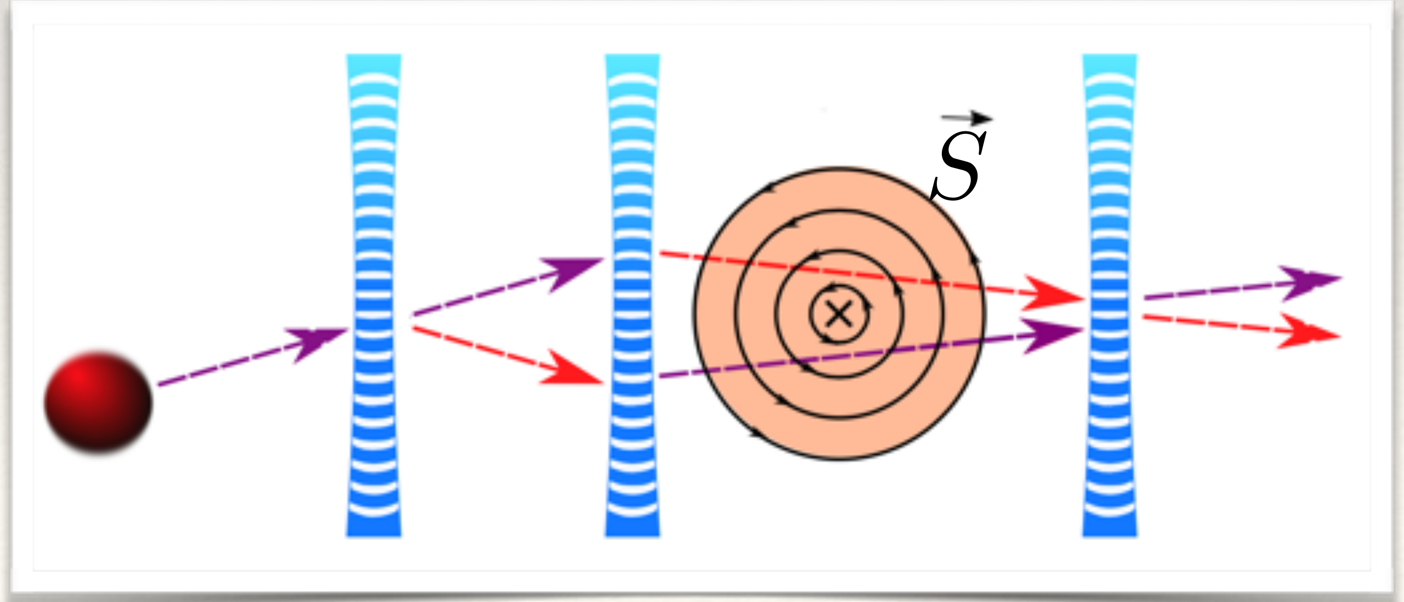
$$\phi^{\rm optical}_{\rm HMW} = \frac{\alpha\mu_0}{\hbar}\oint \mathbf{S}(\mathbf{r})\cdot d\mathbf{r}$$

Atom interferometer

Two gaussian beams



One Laguerre-Gauss beam



Note: this is different to the NIST experiments where an LG-beam + G-beam generates a rotation in a BEC: Andersen et al, PRL 97, 170406 (2006).

Estimation of magnitude of optical HMW phase

$$\phi_{\mathrm{HMW}} = \hbar^{-1} \oint [\mathbf{B}(\mathbf{r}) \times \mathbf{d}] \cdot d\mathbf{r} \quad \sim \frac{B\,d\,l}{\hbar} \quad = \frac{E\,d\,l}{\hbar c}$$

l = length of interaction region ~1m

$B = E/c$

$$d\,E = V_{\mathrm{dip}} \sim \frac{\hbar\Omega^2}{\delta_L} = \hbar\left(\frac{\Omega}{\Gamma}\right)^2 \frac{\Gamma}{\delta_L}\Gamma$$

$$= \hbar \frac{I}{2I_{\mathrm{sat}}} \frac{\Gamma}{\delta_L}\Gamma$$

Ω = Rabi frequency

$\delta_L = \omega_L - \omega_a$

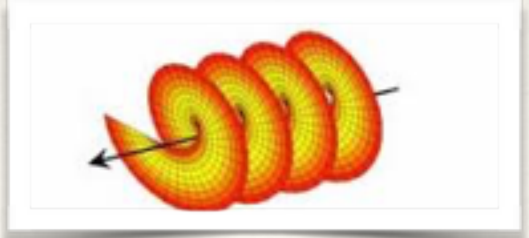
Γ = decay rate $\sim 10^6\ s^{-1}$

$$\phi_{\mathrm{HMW}} = \frac{I}{2I_{\mathrm{sat}}} \frac{\Gamma}{\delta_L} \frac{\Gamma l}{c} = \frac{I}{2I_{\mathrm{sat}}} \frac{\Gamma}{\delta_L} \frac{1}{300}$$

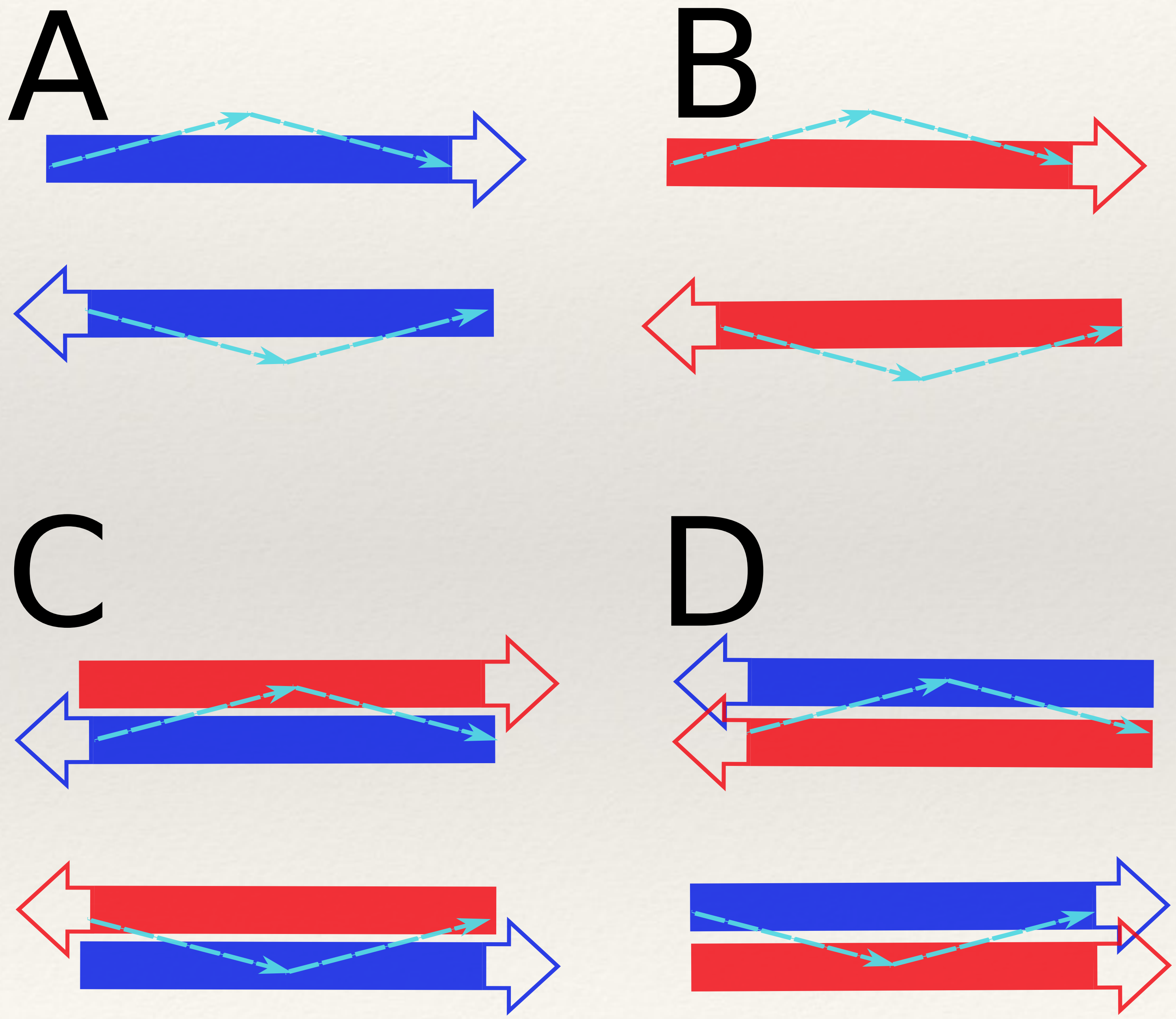
$\rightarrow$ can achieve mrad

Summary

- Abraham and Minkowski were both right: kinetic vs canonical momentum.
- Difference depends on Poynting vector.
- Abraham-Minkowski is classical physics....but connected to a (quantum) geometric phase: optical version of the He-McKellar-Wilkens phase.
- An atomic interferometer might be able to measure this phase.
- More exotic behaviour when $\nabla \times (\mathbf{B} \times \mathbf{d}) \neq 0$ ......



Laser configurations



Canonical vs kinetic momentum

$$\mathbf{p}_{\mathrm{atom}}^{M} + \int \mathbf{g}_M \mathrm{d}^3 r = \mathbf{p}_{\mathrm{atom}}^{A} + \int \mathbf{g}_A \mathrm{d}^3 r$$

total momentum conserved

$$\int \mathbf{g}_M \mathrm{d}^3 r = \int \mathbf{g}_A \mathrm{d}^3 r + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$$

where: $\underbrace{\boxed{\mathbf{p}_{\mathrm{atom}}}}_{\text{canonical}} = \underbrace{\boxed{M\dot{\mathbf{r}}_{\mathrm{atom}}}}_{\text{kinetic}} - \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$

$$\mathbf{p}_{\mathrm{atom}} + \int \mathbf{g}_M \mathrm{d}^3 r = M\dot{\mathbf{r}}_{\mathrm{atom}} + \int \mathbf{g}_A \mathrm{d}^3 r$$

Lembessis, Babiker, Baxter and Loudon, Phys. Rev. A. **48**, 1594 (1993)

Canonical vs kinetic momentum

Momentum density: $\mathbf{g}_M = \mathbf{D} \times \mathbf{B}$ $\qquad \mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2$

For a dielectric "medium" consisting of a single atom:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\mathrm{atom}}) \qquad \mathbf{B} = \mu_0 \mathbf{H}$$

$$\Rightarrow \int \mathbf{g}_M \mathrm{d}^3 r = \int \mathbf{g}_A \mathrm{d}^3 r + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$$

Thus [Lembessis et al, Phys. Rev. A. **48**, 1594 (1993)] :

$$\mathbf{p}_{\mathrm{atom}} + \int \mathbf{g}_M \mathrm{d}^3 r = M\dot{\mathbf{r}}_{\mathrm{atom}} + \int \mathbf{g}_A \mathrm{d}^3 r$$

where: $\underbrace{\boxed{\mathbf{p}_{\mathrm{atom}}}}_{\text{canonical}} = \underbrace{\boxed{M\dot{\mathbf{r}}_{\mathrm{atom}}}}_{\text{kinetic}} - \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$

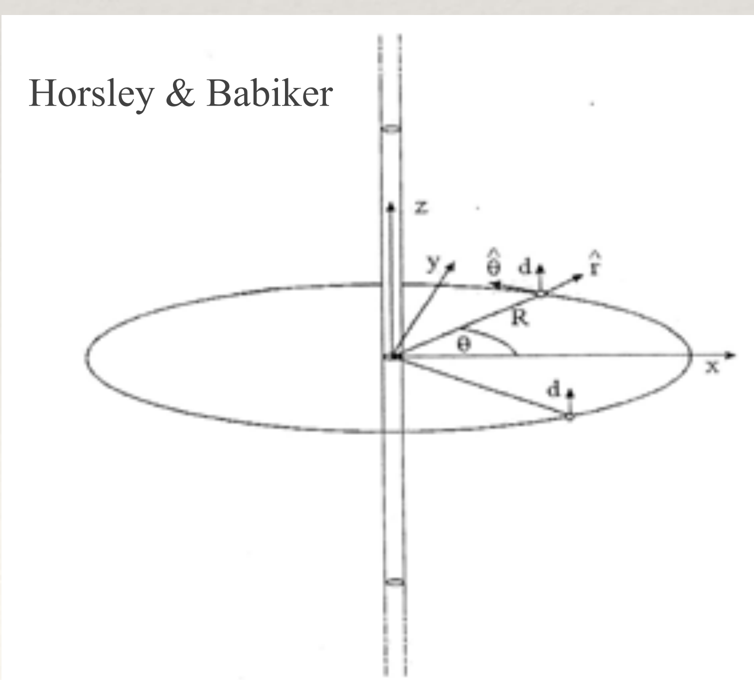
He-McKellar-Wilkens phase: electric dipole moving in B-field

Aharonov-Bohm phase: $\phi_{\mathrm{AB}} = (q/\hbar) \oint \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r}$

HMW phase: $\phi_{\mathrm{HMW}} = \hbar^{-1} \oint [\mathbf{B}(\mathbf{r}) \times \mathbf{d}] \cdot d\mathbf{r}$ (neutral electric dipole moving in a STATIC magnetic field)

He & McKellar, Phys. Rev. A **47**, 3424 (1993), Wilkens, Phys. Rev. Lett. **72**, 5 (1994), Wei, Han, & Wei, Phys. Rev. Lett. 75, 2071 (1995), Horsley & Babiker Phys.Rev. Lett. **95**, 010495 (2005)…

Seems to require a straight line of magnetic monopoles…



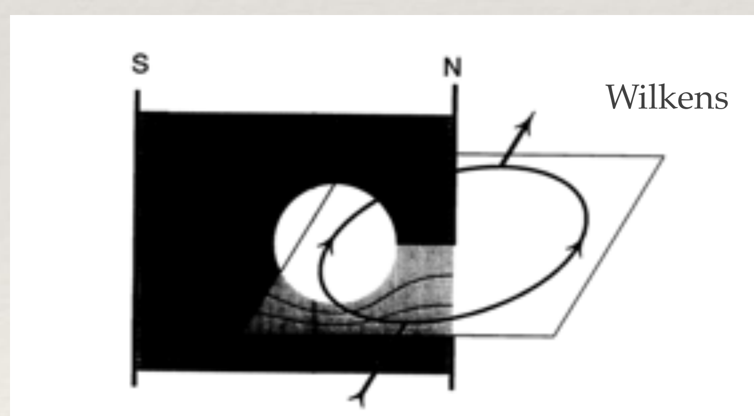


FIG. 1. Geometry of the interferometric experiment. Shown is the "Dirac sheet" with the two vertical edges effectively realizing two oppositely charged lines of magnetic monopoles. The "Dirac strings" (flux lines) which connect the two edges leave a hole for the interferometric path. The interferometric plane is orientated perpendicular to the magnet poles and the electric dipole moment of the interfering particles.

HMW is dual to the **Aharonov-Casher** phase:

$$\phi_{\mathrm{AC}} = (\hbar c^2)^{-1} \oint [\mathbf{E}(\mathbf{r}) \times \mu] \cdot d\mathbf{r}$$

Phys. Rev. Lett. **53**, 319 (1984).