Ergodicity, Entanglement and Many-Body Localization in Quantum Dynamics

Ehud Altman – Berkeley

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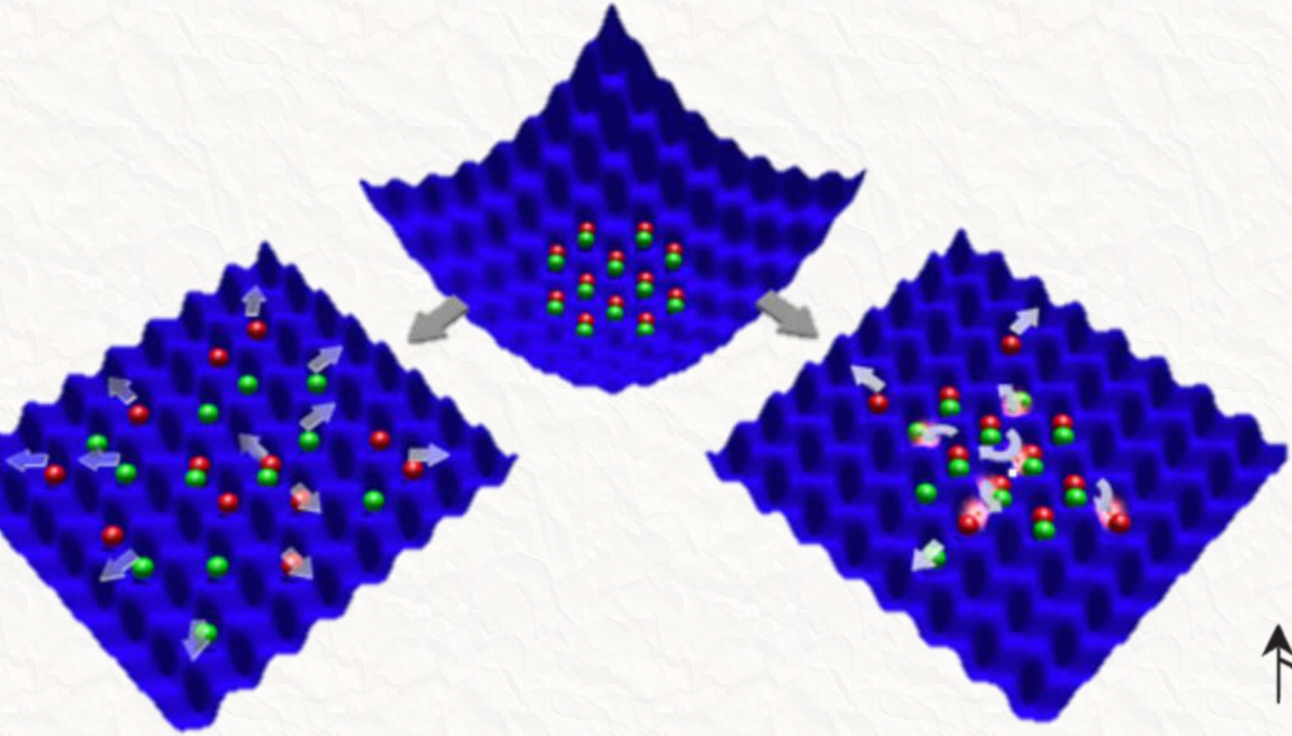
Others: D. Huse G. Refael, Y. Bahri, A. Vishwanath, E. Demler, V. Oganesyan, D. Pekker, Mark Fischer,
I. Bloch's exp group

ISRAEL SCIENCE FOUNDATION
erc
UQUAM ultracold quantum matter
BSF United States – Israel Binational Science Foundation

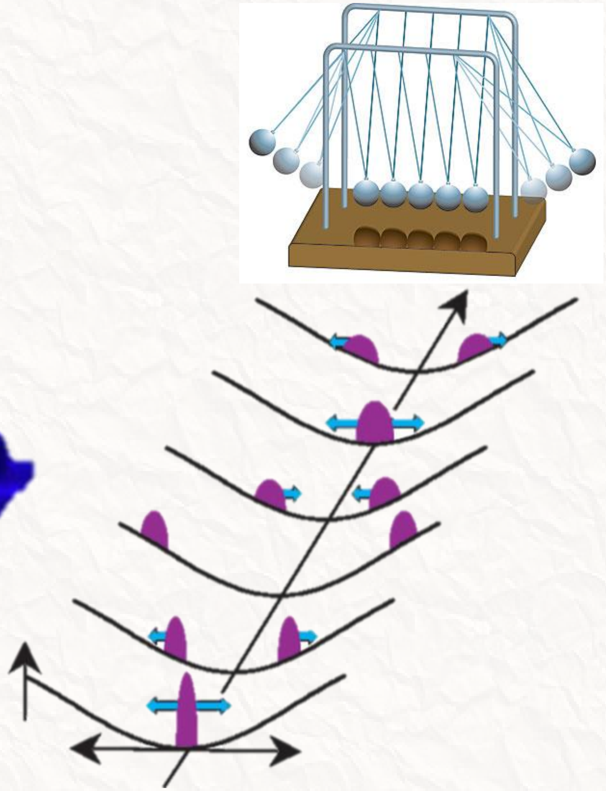
Experiments with ultra-cold atoms have put focus on fundamental questions in quantum dynamics

Can prepare precise initial states and
observe the ensuing unitary dynamics in real-time



Dynamics in optical lattices

I. Bloch group (2012)



Quantum integrability

D. Weiss group (2010)

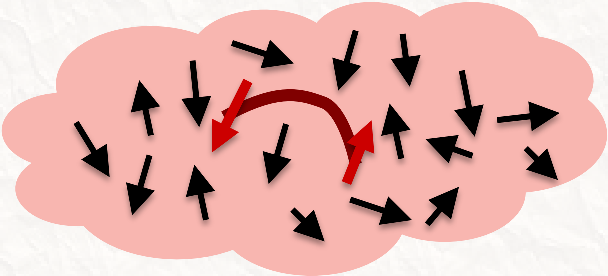
Questions

- What are the genuinely quantum effects in the dynamics of many quantum particles?
- Why does the macroscopic world appear classical? (How does classical hydrodynamics emerge from unitary quantum dynamics?)
- **Can a big system evade a classical/thermal fate?**
- How to characterize chaos in a many-body quantum system?

Ergodicity in quantum dynamics

Many-body time evolution often scrambles quantum correlations. Scrambling = quantum chaos (?)

Quantum information stored in local objects is rapidly lost as these get entangled with the rest of the system.



The only remaining structures of information are slow order parameter fields and conserved densities.

Classical hydrodynamic description (e.g. diffusion).

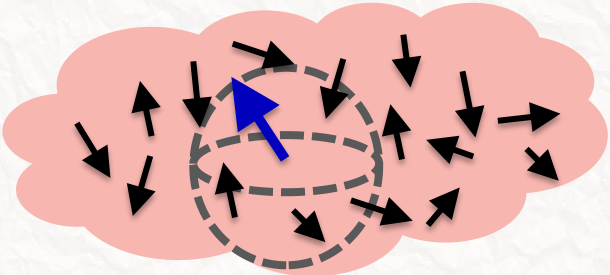
Two generic paradigms for closed system dynamics

Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

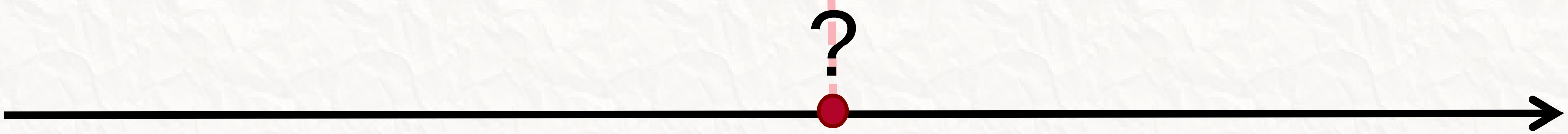
Classical hydrodynamics of remaining slow modes.

Many-body localization



Local quantum information persists indefinitely.

Need a fully **quantum** description of the long time dynamics!



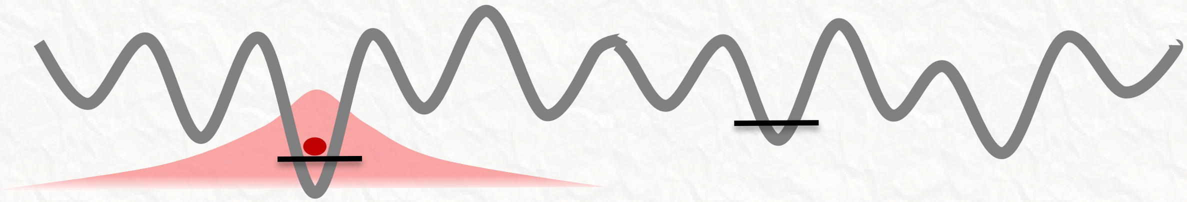
The MBL transition constitutes a sharp boundary between quantum and classical behavior at long times

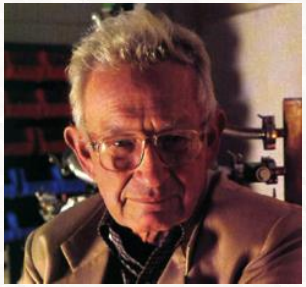
Outline of this talk

- What is MBL? Description of dynamics in the MBL state
- A theory of the MBL phase transition.
- Confronting theory with experiment
- Briefly: a new scheme for computing quantum thermalization dynamics.

Anderson localization

Single particle (Anderson 1958):

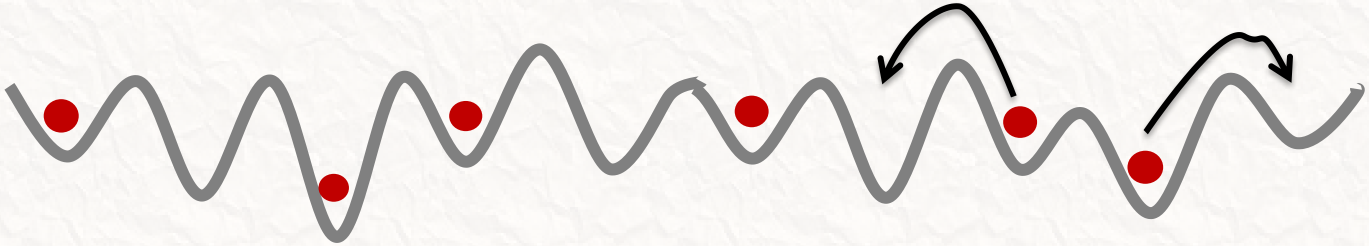




Vanishing probability of resonances.

Add interactions:

$$U c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta$$



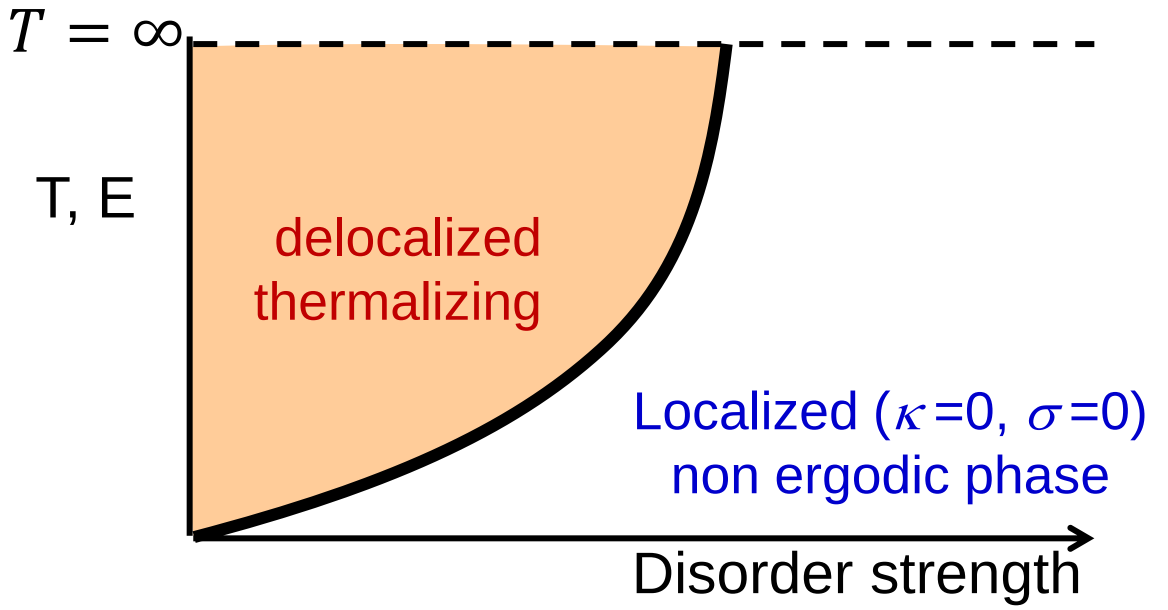
At high energies interaction connects between ~2^L localized states !

Can localization survive ?

Many-Body Localization (MBL)

Basko, Aleiner, Altshuler (BAA) 2005; Gornyi, Mirlin, Polyakov 2005:
Insulating phase stable below a critical T or E; metal above it.

Mathematical proof for quantum spin chains: Imbrie 2014



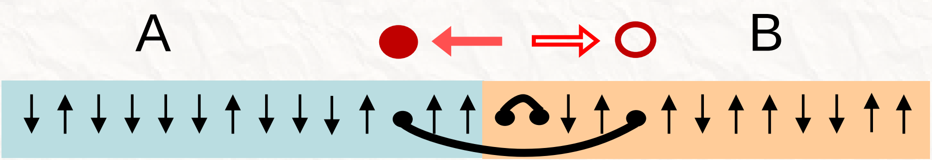
What can we say about static and dynamic properties of MBL states?

Entanglement entropy growth in time evolution

Znidaric, Prosen and Prelovsek (2008); Bardarson, Pollmann and Moore (2012)

$$H_0 = \sum_i h_i S_i^z + J_\perp \sum_i \left(S_i^+ S_{i+1}^- + \text{H.c.}\right) \qquad H_{\text{int}} = J_z \sum_i S_i^z S_{i+1}^z$$

Compute time evolution starting from a simple state in one dimension:



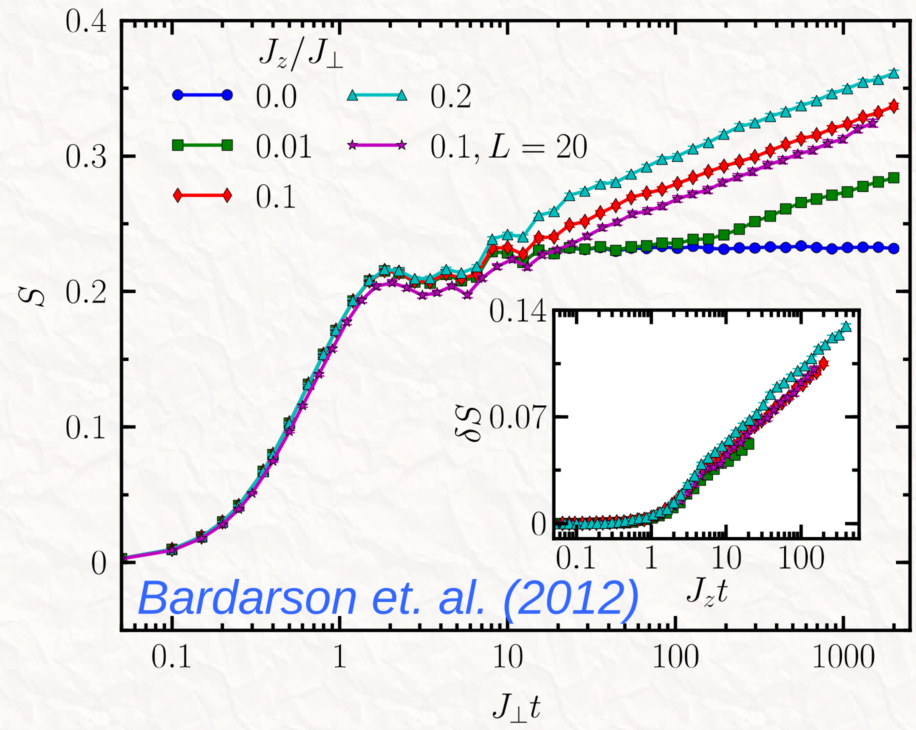
$$e^{-iHt}|\Psi_0\rangle$$

$$\Rightarrow \quad S_A(t) \sim \log t$$

Slow growth of entanglement entropy.

Low entanglement allows efficient encoding and computation.

S_A Saturates to a volume law in a finite subsystem but smaller than expected thermal entropy

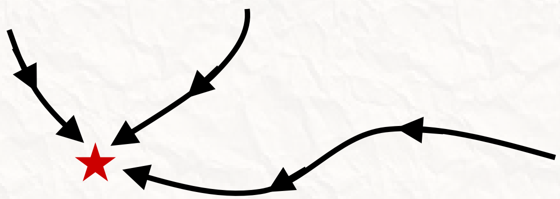


MBL phase is a stable RG fixed point

R. Vosk & EA, PRL 2013, 2014

$$H_{\mathrm{mic}} = \sum_i h_i \sigma_i^x + J_i^z \sigma_i^z \sigma_{i+1}^z + J_i^x \sigma_i^x \sigma_{i+1}^x$$

- Fixed point characterized by complete set of local integrals of motion:



$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \dots$$

Huse & Oganesyan (2013),
Serbyn etal. (2013)

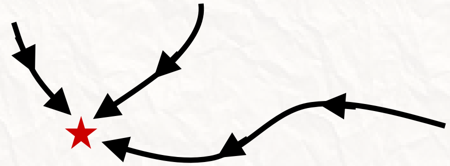
$$\tilde{\sigma}_i^x = Z \sigma_i^x + \text{exponential tail}$$

Note the analogy with Fermi-liquid theory!

$$H_{FL} = \sum_{k \sim k_F} \epsilon_k \hat{n}_k + \sum_{k,k'} f_{kk'} \hat{n}_k \hat{n}_{k'} \qquad [H_{FL}, \hat{n}_k] = 0$$

Effective model of the locaized phase

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$$

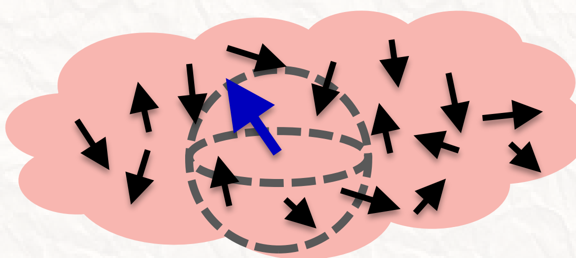


$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \ldots$$

Reveals surprisingly rich dynamics in MBL phase:

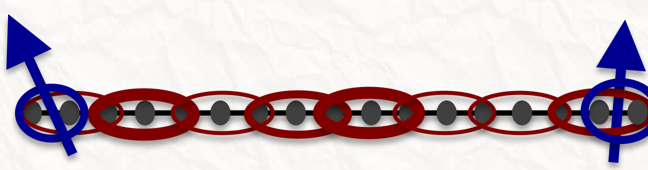
- Slow log(t) entanglement growth and anomalous relaxation
  Vosk and EA (2013), Serbyn (2013), Vasseur et al (2014), Serbyn et. al. (2014)

- Distinct localized phases
  (glass, paramagnetic, topological …)
  Huse et. al. 2013, Vosk and EA 2014, Pekker et. al. 2014



- Persistent quantum coherence, spin echos
  Bahri, Vosk, EA and Vishwanath 2013, Serbyn etal 2013

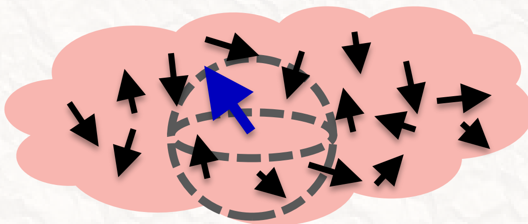
- Topologically protected edge states at high energies
  Bahri, Vosk, EA and Vishwanath 2013



But cannot address the MBL transition using this approach!

Theory of the transition from MBL to ergodic fluid

Many-body localized



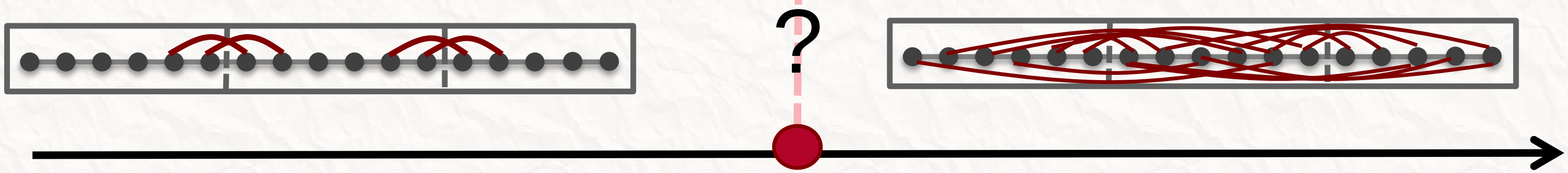
Thermalizing

Quantum coherent dynamics

"Classical" dynamics

Area law eigenstate entanglement

Volume-law eigenst. entanglement



The many-body localization transition:

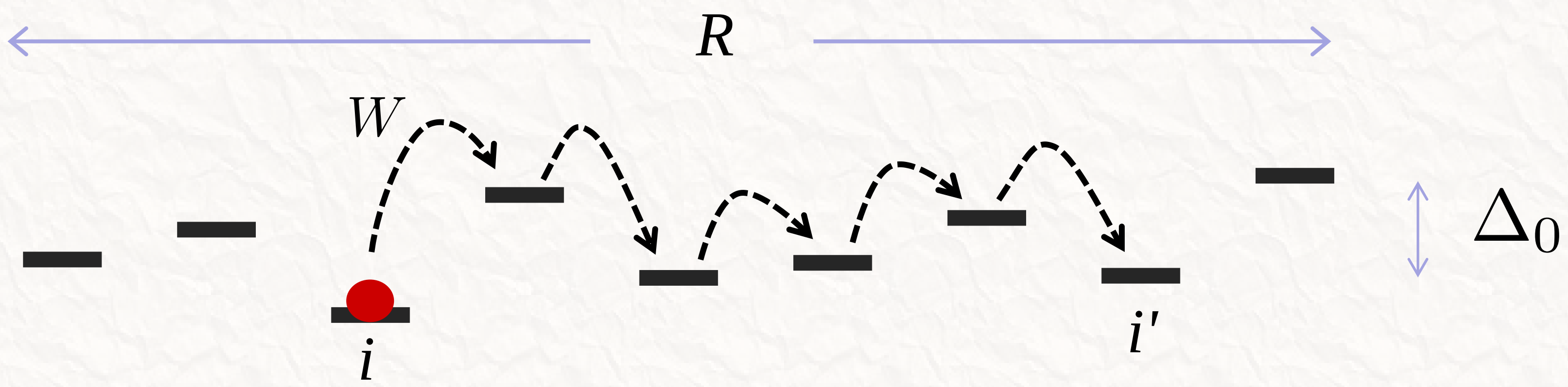
1. Sharp interface between quantum and classical worlds
2. Fundamental change in entanglement pattern.
   More radical than in any known transition.

R. Vosk, D. A. Huse and EA, PRX **5**, 031032 (2015)

Essence of many-body localization

Warm-up: essence of single particle localization (Anderson 58)

$$H = \sum_i V_i\, c_i^\dagger c_i - W \sum_{\langle ij \rangle} c_i^\dagger c_j + \text{H.c.} \qquad \langle V_i V_j \rangle = \Delta_0^2\, \delta_{ij}$$



Is there a likely resonance within a range *R* of site *i* ?

Site nearest in energy within this range: $\Delta(R) \sim \Delta_0 / R^d$

Matrix element for hopping to this range: $J(R) \sim W \left(\frac{W\, d}{\Delta_0} \right)^R$

Resonance condition $J(R) > \Delta(R)$ satisfied only if $W d > \Delta_0$

Essence of many-body localization

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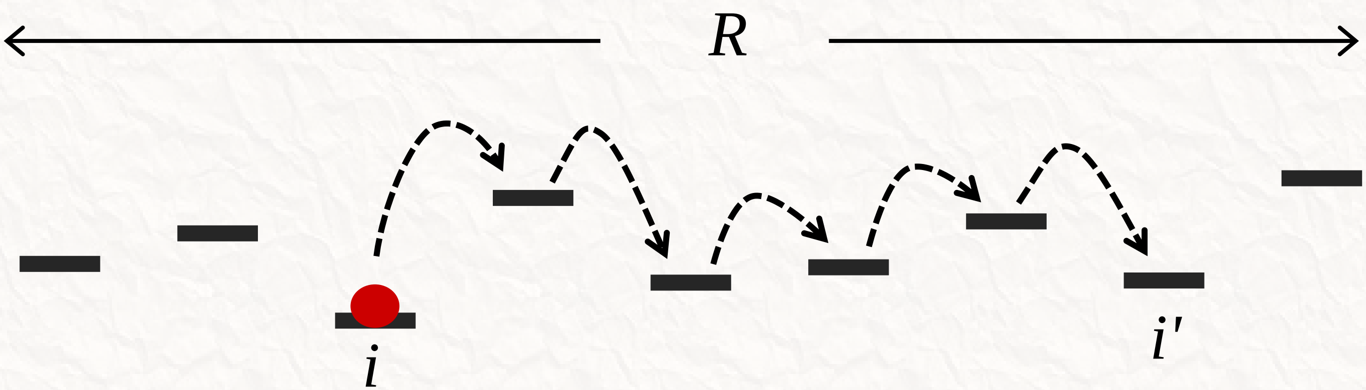
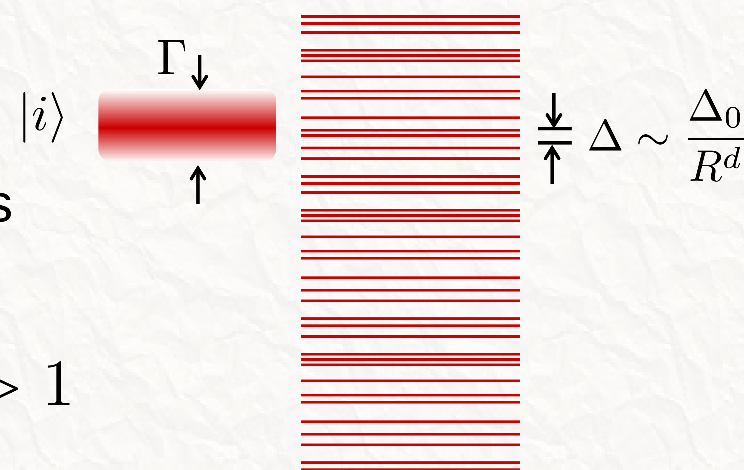


Can view delocalization as a decay of state i into a continuum:

$$\Gamma(R) \sim J(R)^2 \Delta(R)^{-1}$$



Condition for the surrounding states to serve as an effective bath:

$$\frac{\Gamma(R)}{\Delta(R)} = \left(\frac{J(R)}{\Delta(R)} \right)^2 > 1$$

Rough criterion for MBL (T=∞)

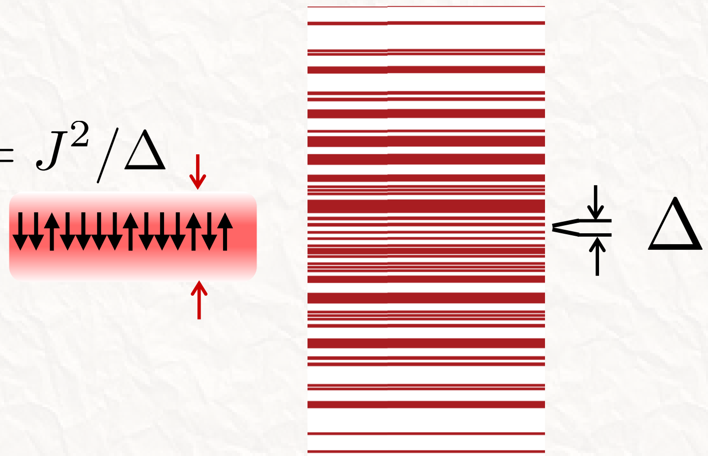
$$H = \sum_i V_i S_i^z + \sum_{\langle ij \rangle} J^z S_i^z S_j^z + J^x S_i^x S_j^x$$

Matrix element to move between typical configurations of L spins:

↓↓↑↓↓↓↓↑↓↓↓↑↓↑ → ↑↓↑↓↑↑↑↓↑↑↓↓↑↑

$$J(L) \sim J^z \left(J^x/\Delta_0\right)^L \equiv J^z e^{-L/\xi_*}$$

$$\Gamma = J^2/\Delta$$



$$\Delta(L) \sim \frac{\Delta_0}{2^L} = \Delta_0 e^{-L\,\ln 2}$$

$$g(L) \equiv \frac{\Gamma(L)}{\Delta(L)} = \left(\frac{J(L)}{\Delta(L)}\right)^2$$

<u>Delocalized phase:</u> $g(L) \gg 1$

Resonance condition = condition for the system to serve as it's own bath:

Rough criterion for MBL (T=∞)

$$H = \sum_i V_i S_i^z + \sum_{\langle ij \rangle} J^z S_i^z S_j^z + J^x S_i^x S_j^x$$

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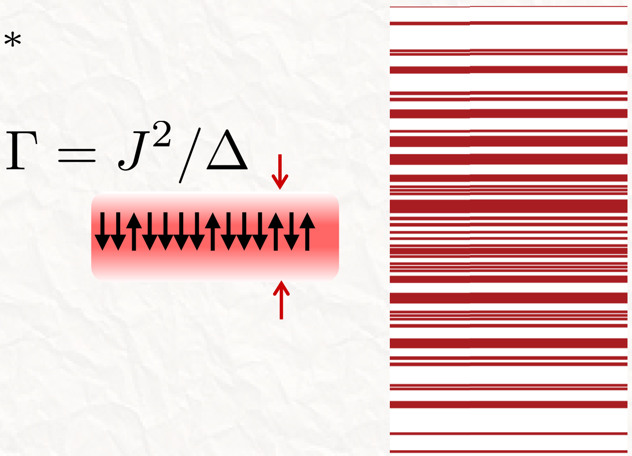

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$$g(L) \equiv \frac{\Gamma(L)}{\Delta(L)} = \left(\frac{J(L)}{\Delta(L)}\right)^2$$

Localized phase: $g(L) < 1$

requires $\xi_* < 1/\ln 2$

Does it mean non-diverging localization length and 1st order transition? **NO!**

Toy model of the critical point

(Zhang, Zhao, Huse PRB 2016)

We want a thermal system of length L:

$$g(L) \gg 1$$

Now consider 3 subsystems of length L/3.

Must they all individually have g(L/3)>>1?

No! The minimal configuration should be something like this:

$g(L/3) \gg 1$	$g(L/3) \ll 1$	$g(L/3) \gg 1$

The thermal sides are then just able to thermalize the middle.

Now apply this reasoning to each of the two thermal sides to get:

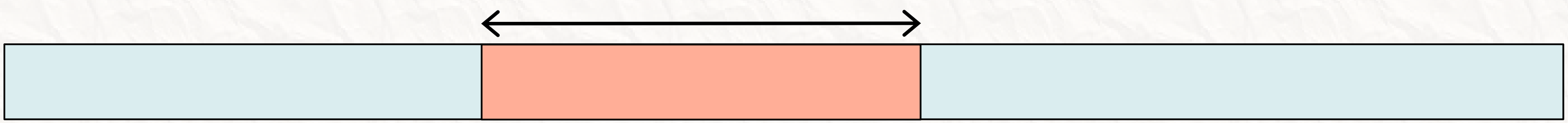
And iterate:

Toy model of the critical point

Critical system is a Cantor set of bare thermal regions
with fractal dimension: $d_f \approx \ln 2 / \ln 3$
This should be just enough to thermalize the whole system!

Fluctuation in the tuning parameter (bare disorder)
resulting in a critical bubble

ξ

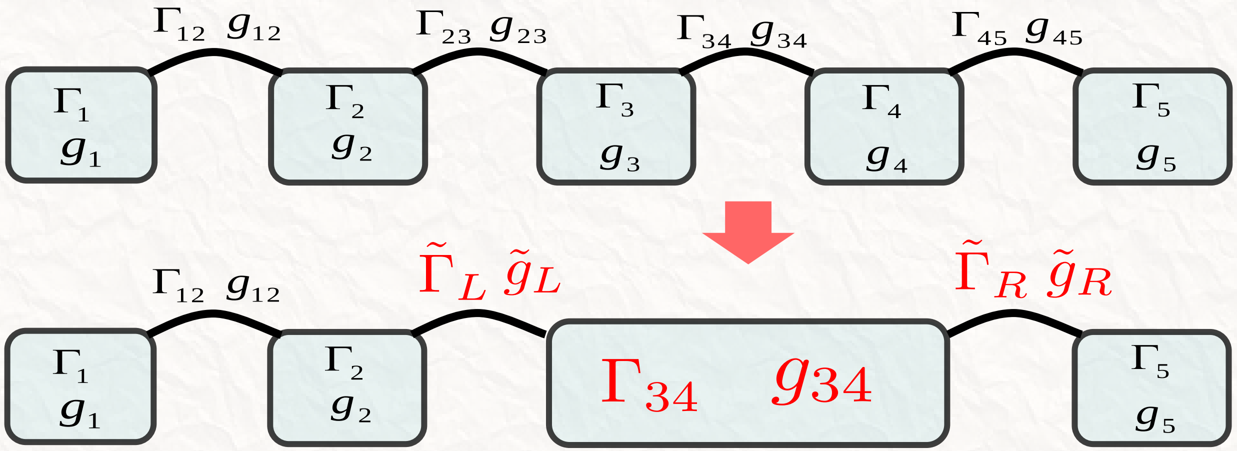


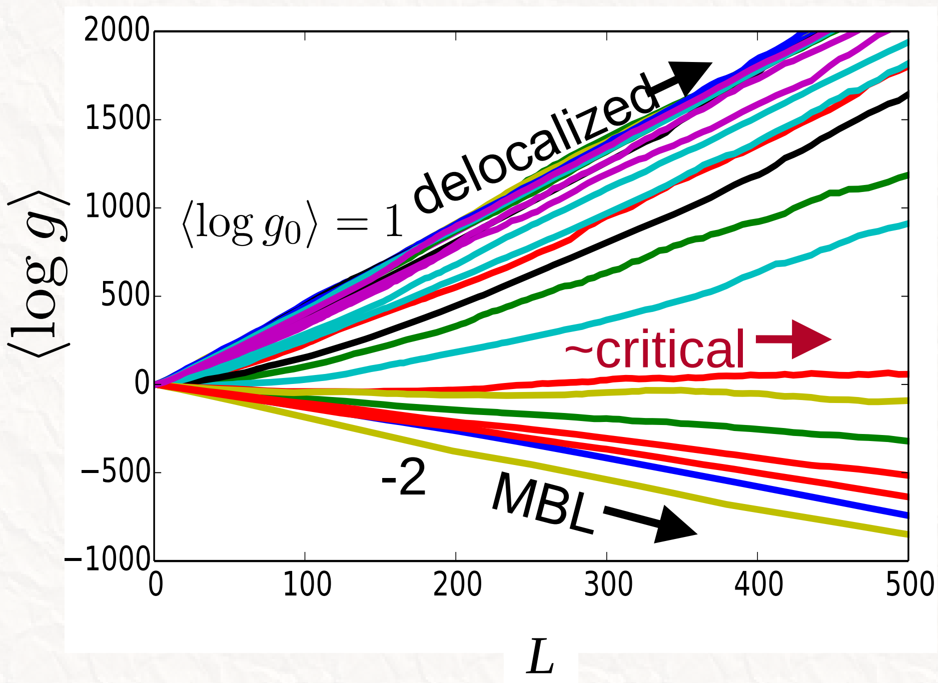
$$\delta\Delta \sim \frac{1}{\sqrt{N_{\text{special}}}} \sim \frac{1}{\xi^{d_f/2}}$$

$\Rightarrow$ $\xi \sim (\Delta - \Delta_c)^{-2/d_f}$ $\qquad$ $\nu = 2/d_f \approx 3.2$

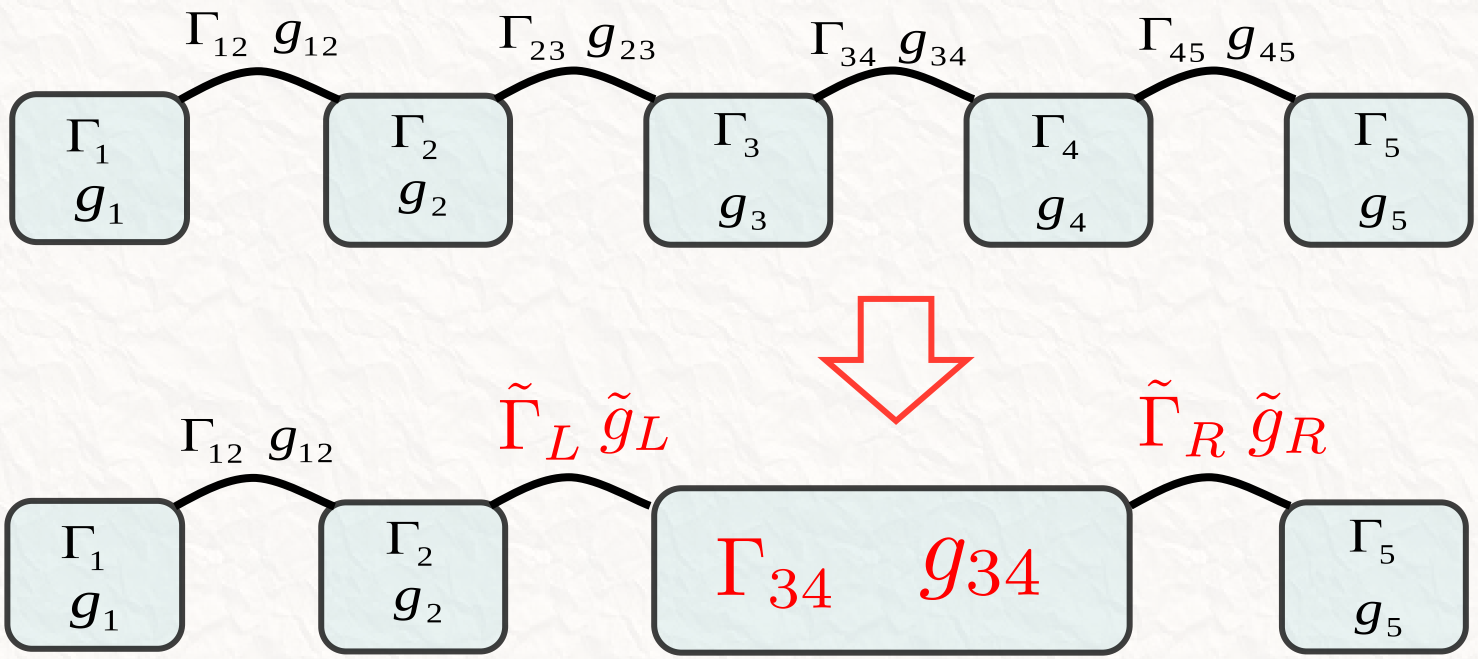
The Idea can be formulated precisely as RG flow

R. Vosk, D. Huse and E.A. arXiv:1412.3117





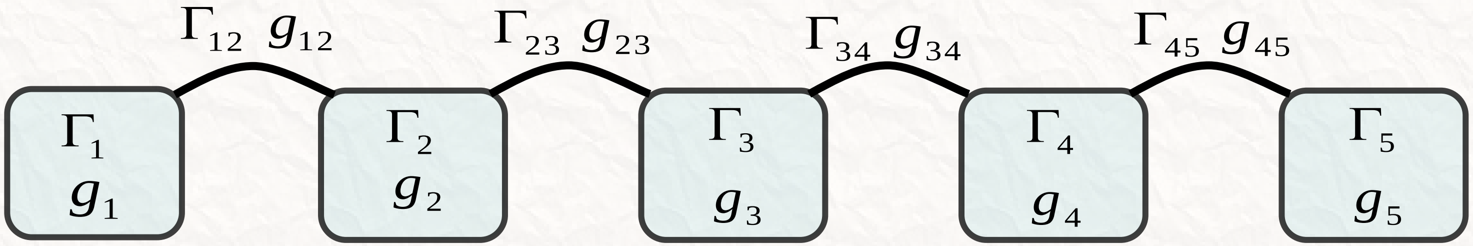
Schematics of the RG

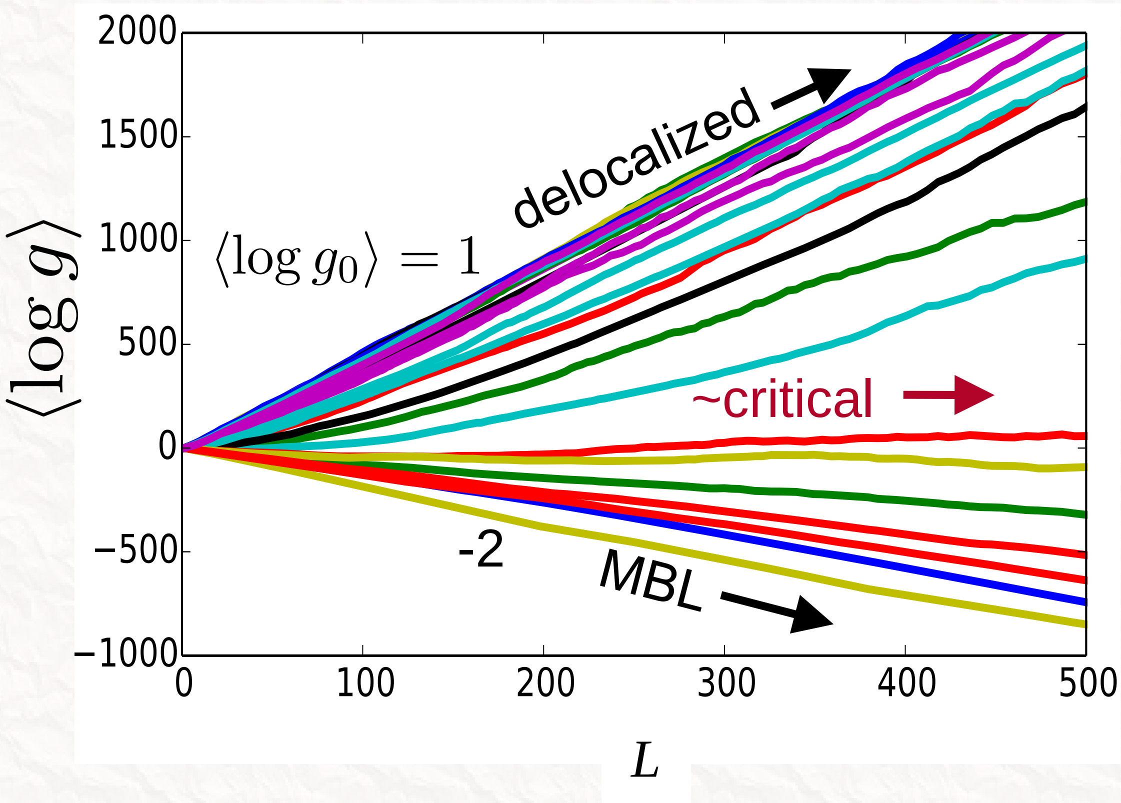


Join blocks which exchange information on the fastest scale.
Then compute renormalized couplings to the left and right.

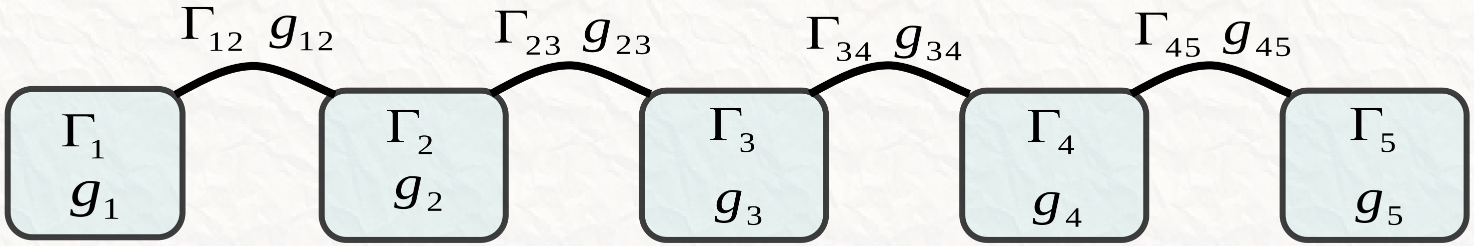
Computing the flow will tell us whether we end up with one big thermalizing matrix (g>>1) or a big insulator (g<<1) at large scales

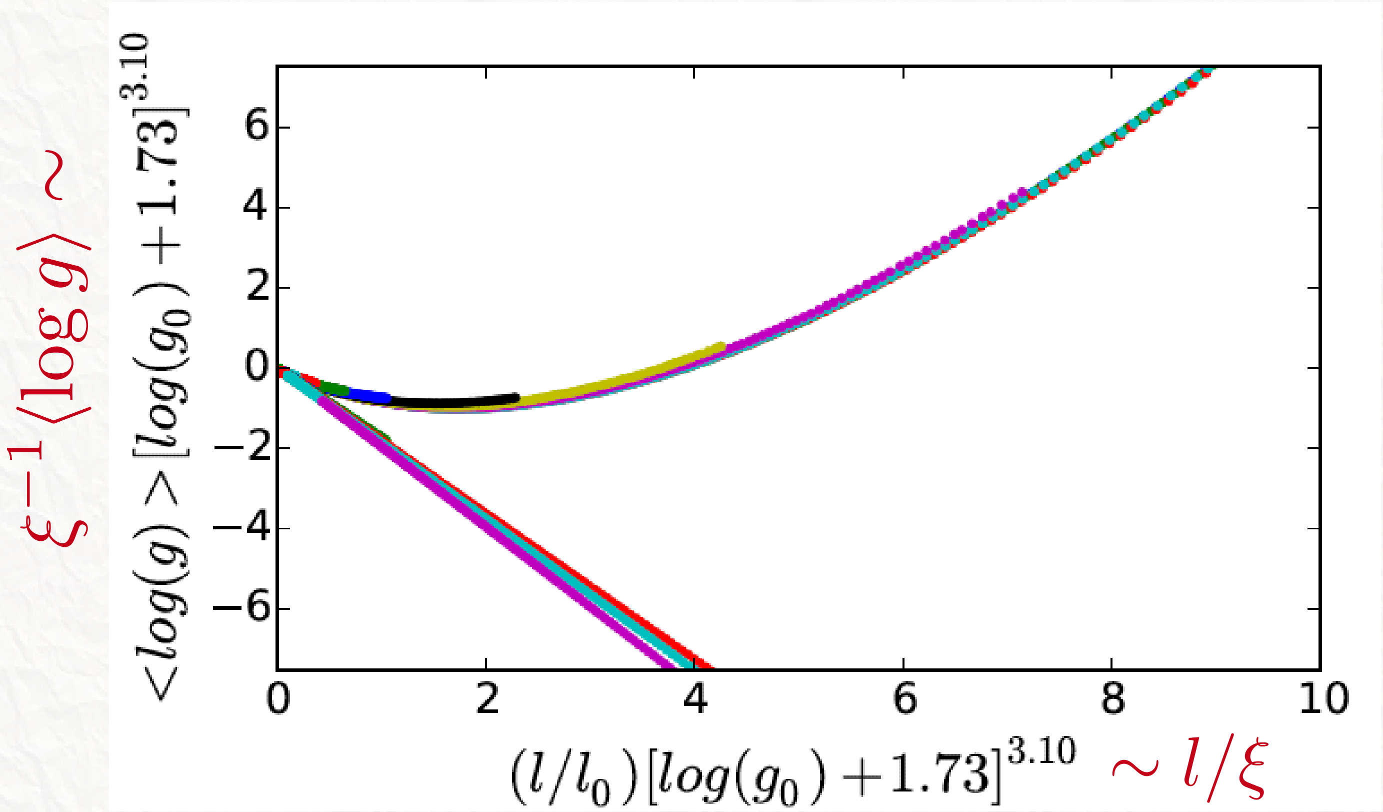
Outcome of the RG flow





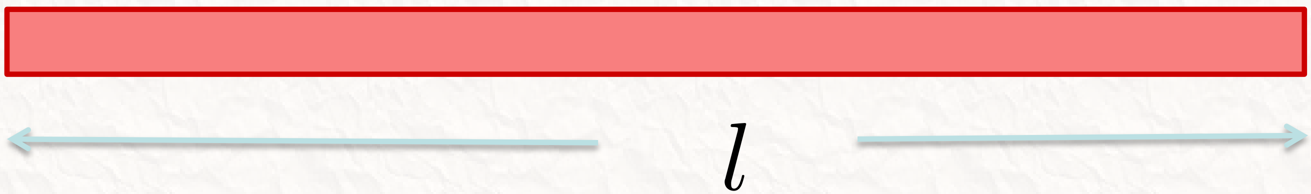
Outcome of the RG flow





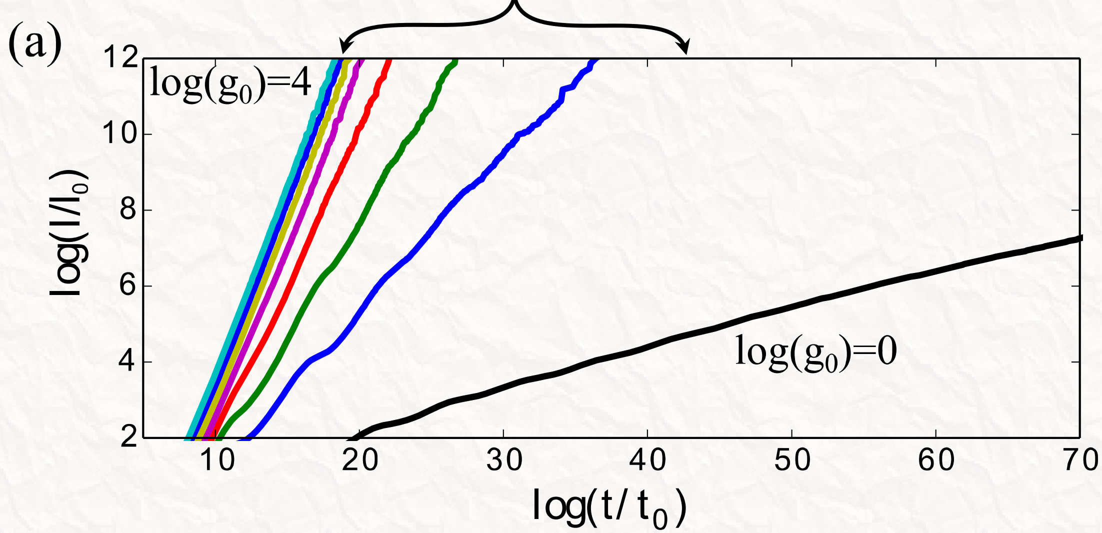
RG result 1: Dynamical scaling for transport

Relation between transport time $\tau_{\rm tr}$ and length l of blocks:

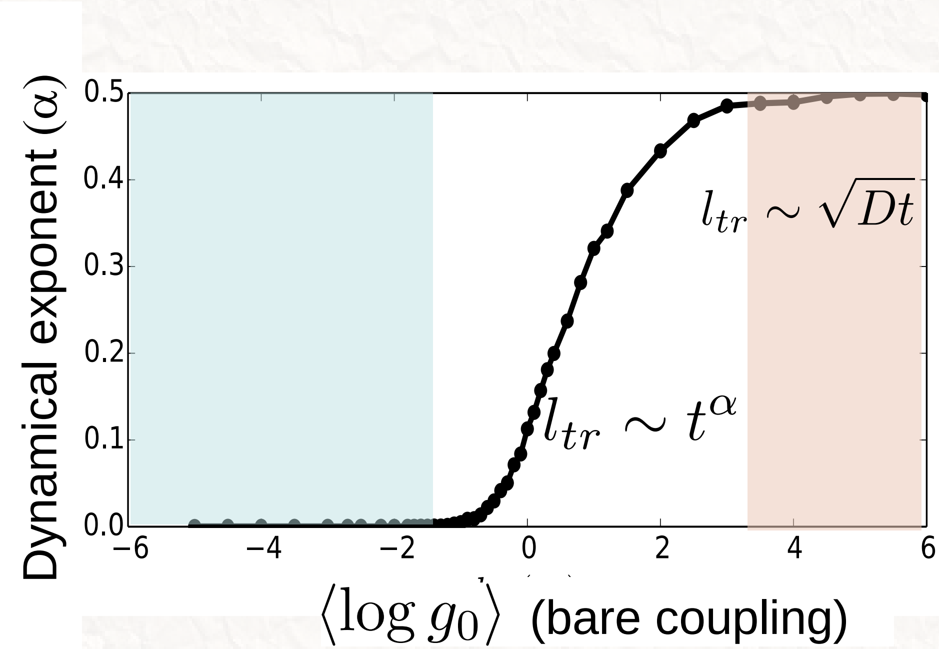


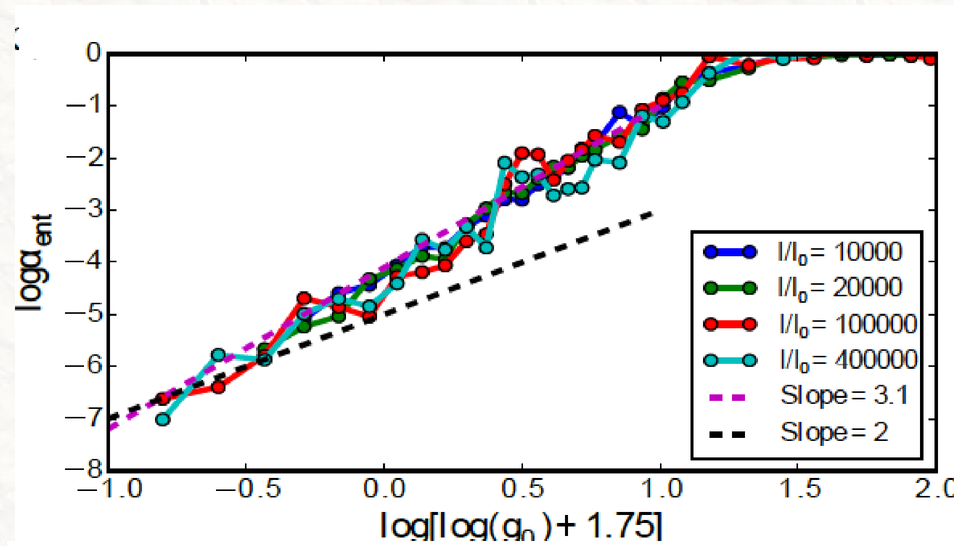
Diffusion: $\tau_{tr} = l^2$ $\qquad$ $l_{tr} = (D\tau)^{1/2}$ (1/2 crossed out) ← $\alpha < \frac{1}{2}$

Delocalized, but not diffusion



Sub-diffusive behavior in the ergodic phase

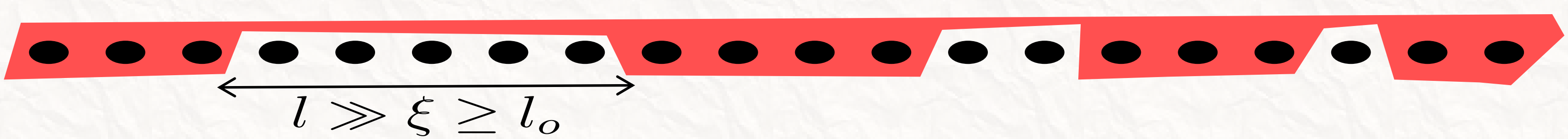




$$\alpha \sim 1/\xi$$

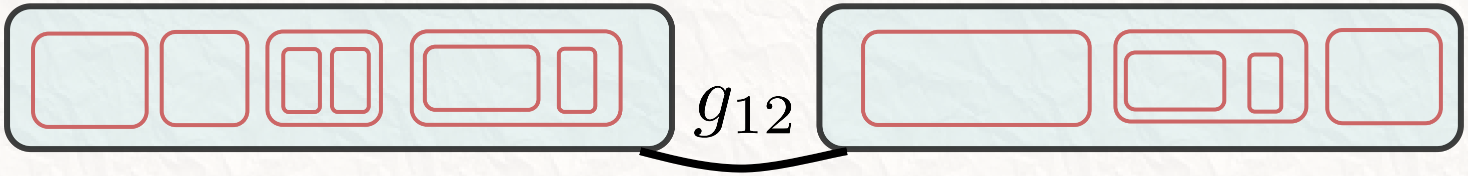
Seen also in numerical studies: Bar-Lev et.al 2014; Agarwal et.al 2014

Result of Griffiths effects. long insulating inclusions inside the metal are exponentially rare but give exponentially large contribution to the transport time.

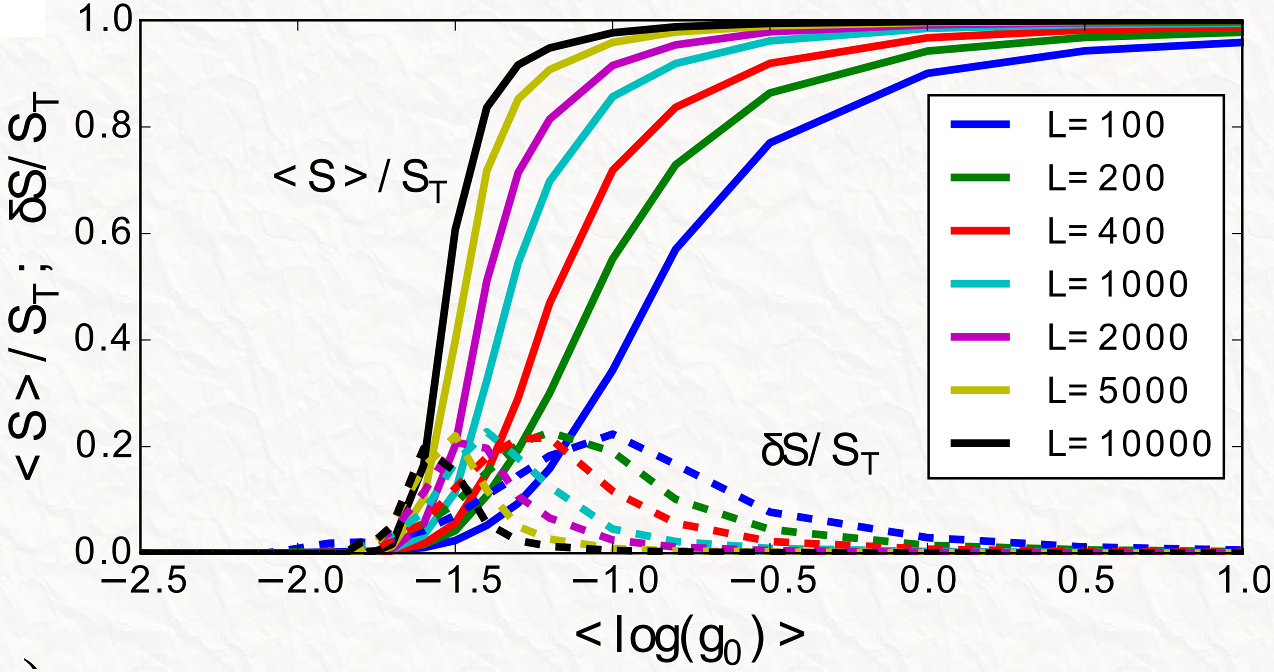


Relaxation with slow power-law tails

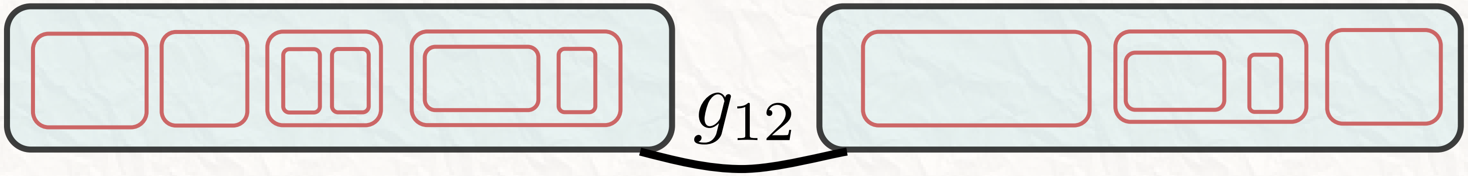
RG result - eigenstate entropy



$$S_E(L/2) \sim \log_2\left[g(L)+1\right]$$



RG result II – eigenstate entropy



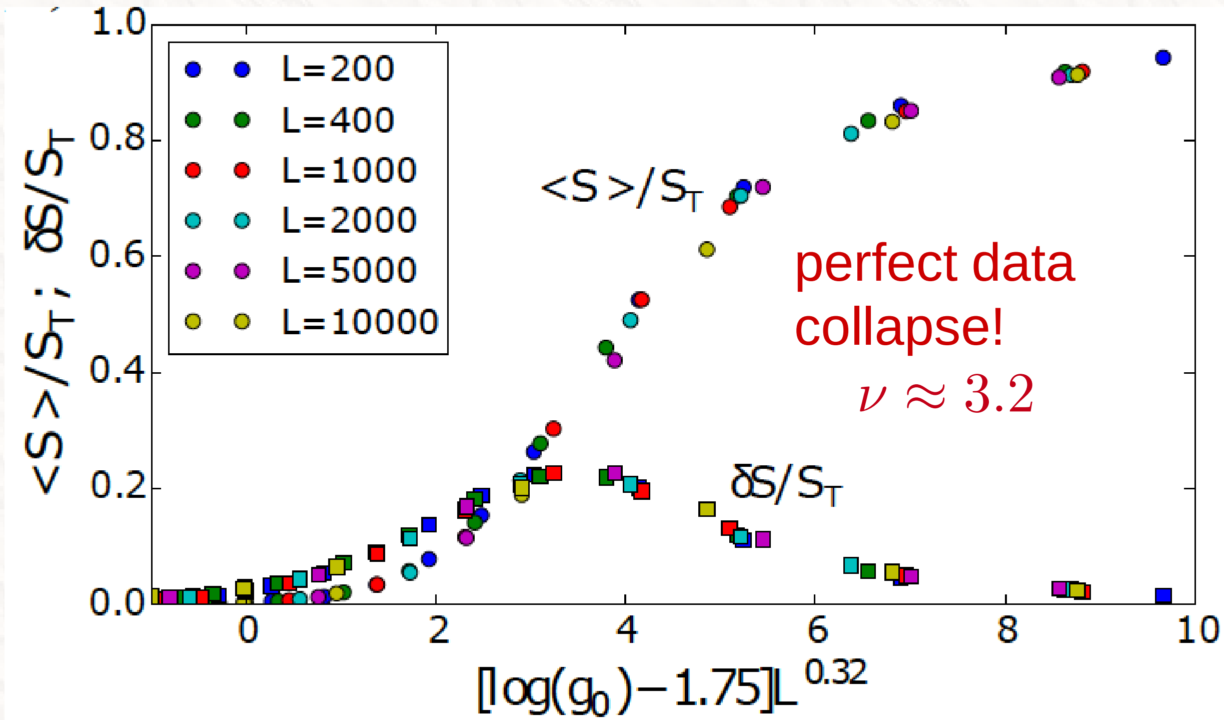
$$S_E(L/2) \sim \log_2\left[g(L) + 1\right]$$

Near critical point expect distribution of S to scale:

$$P(S, L, g_0) = \frac{1}{L}\tilde{P}\left[\frac{S}{L}, \frac{L}{\xi(g_0)}\right]$$

In particular all moments:

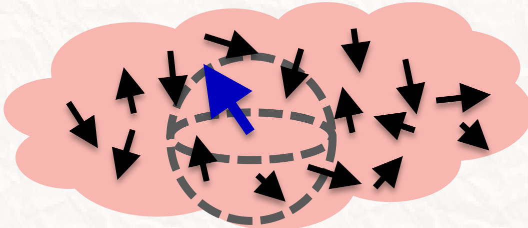
$$\mu S(L, g_0) = L f_\mu\left[\frac{L}{\xi(g_0)}\right]$$



- Universal jump to full thermal entropy ➡ Direct transition to thermal state

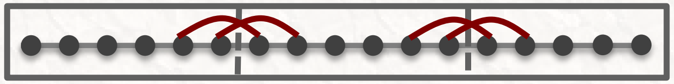
Summary of this part

Many-body localized



Quantum coherent dynamics

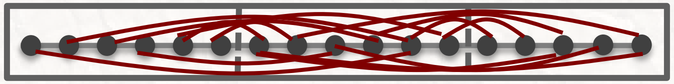
Area law entanglement

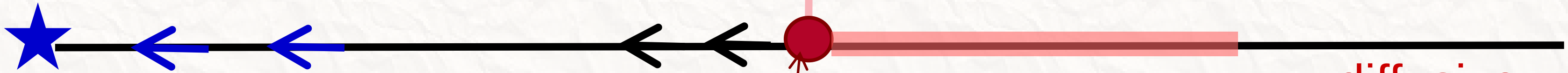


Thermalizing

“Classical” dynamics

Volume law entanglement





Localized fixed-point

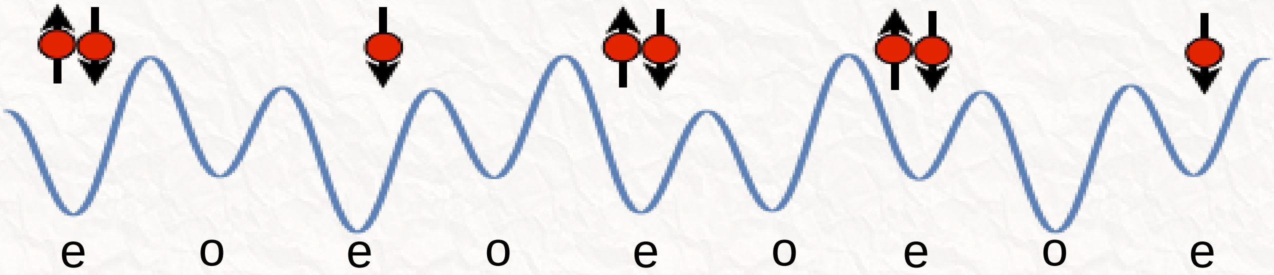
S_A broadly distributed at crit. point

More microscopic description?

Theoretical/numerical approaches for quantum thermalization?

Experimental observation of MBL: fermions in a quasi-random optical lattice

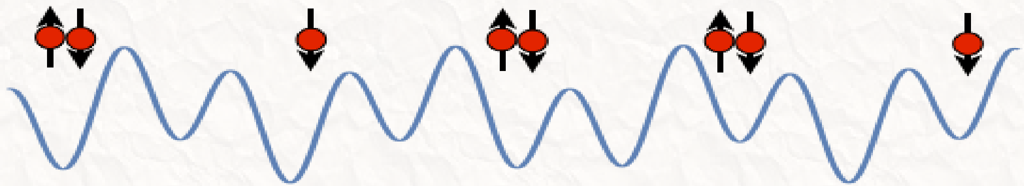
Science 349, 842 (2015)



With:
Immanuel Bloch’s group (LMU)
Mark Fischer and Ronen Vosk (WIS)

Quantum quench protocol

1. Fermions in optical lattice prepared in period-2 density modulation (particles only on even sites)

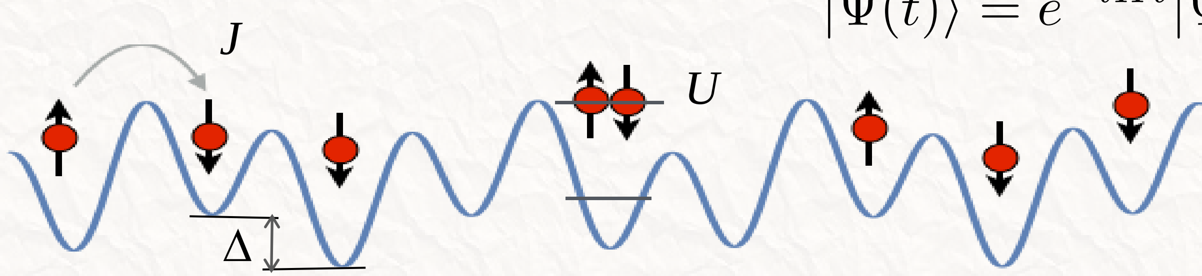


1. Evolve the state with the 1d lattice Hamiltonian:

$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \varphi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}.$$

Incommensurate potential

$$|\Psi(t)\rangle = e^{-i\hat{H}t}|\Psi(0)\rangle$$



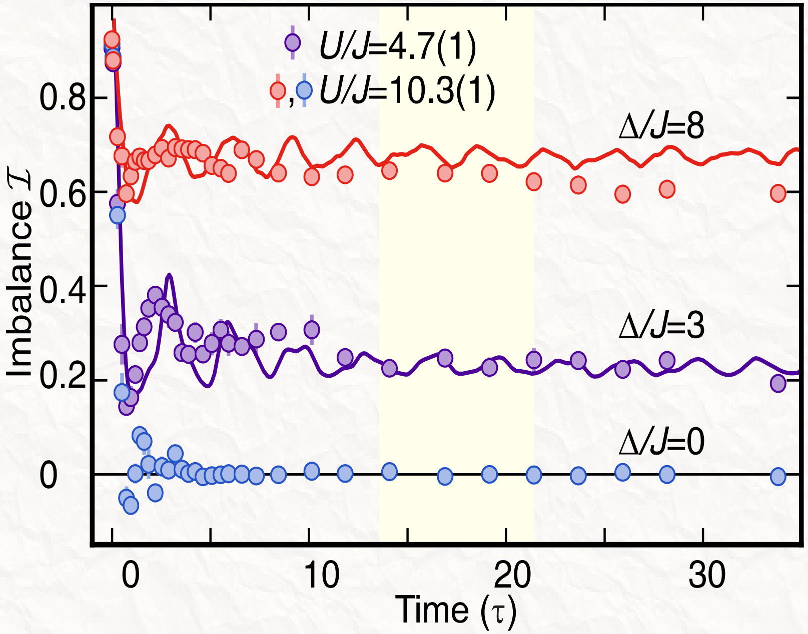
Numerics suggest that this model shows generic MBL (Iyer et. al. PRB 2013)

What to measure?

Imbalance between even and odd sites (density modulation):

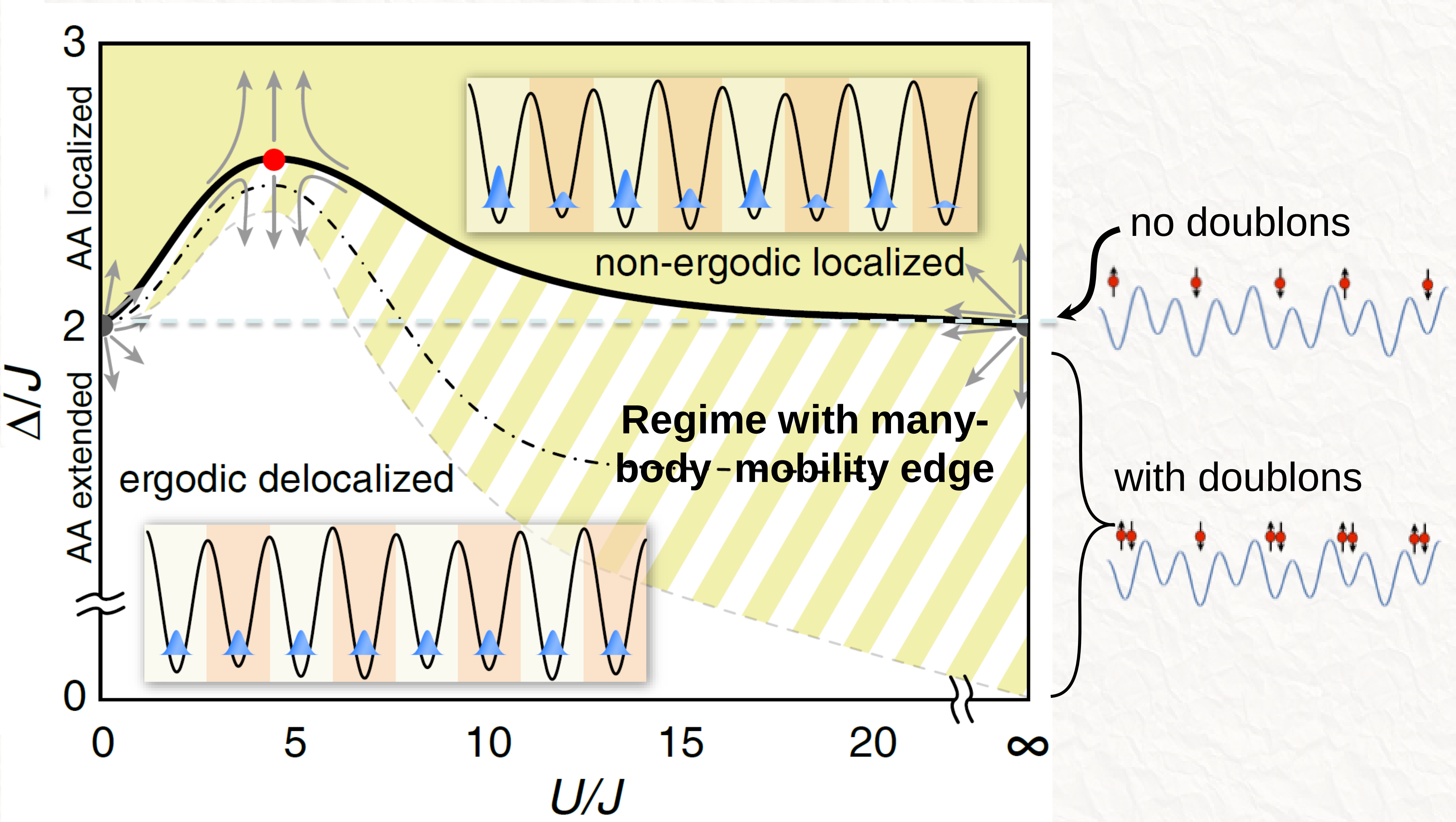
$$\mathcal{I} = \frac{1}{N}\sum_{j=1}^{L}(-1)^j\langle n_j\rangle = \frac{\langle N_e - N_o\rangle}{N_e + N_o}$$

Incomplete relaxation is direct evidence for ergodicity breaking!



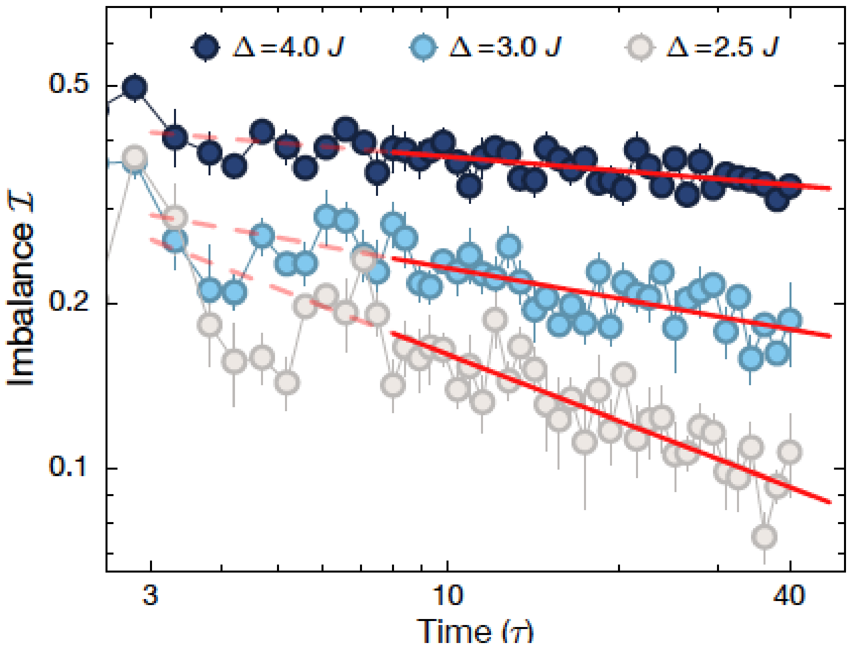
If the system is localized, the density wave operator has finite overlap with an integral of motion and therefore cannot relax fully.

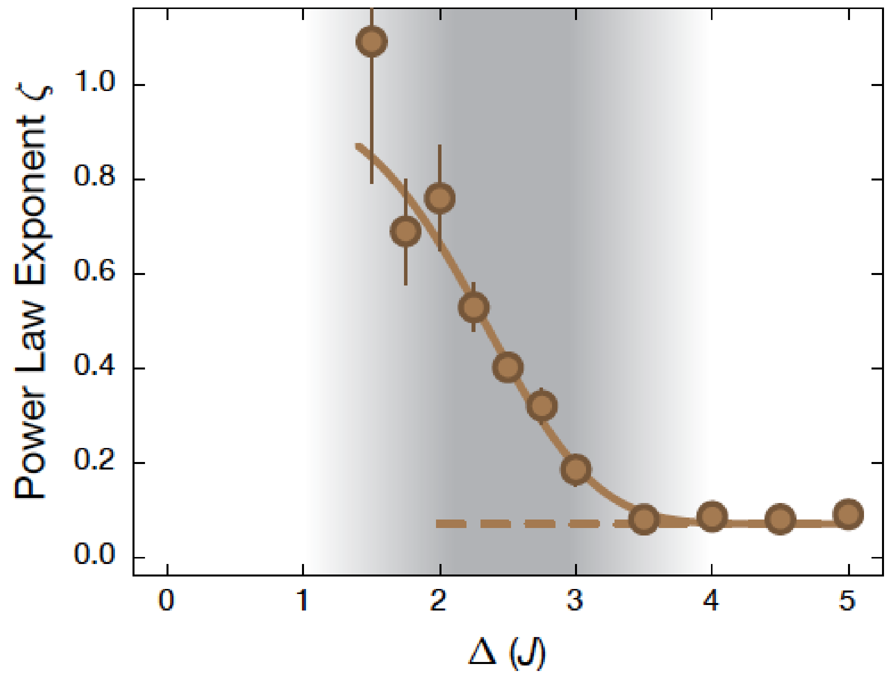
Phase diagram



Focus on the relaxation near the MBL transition

Observe critical slowing down on the thermal side of the transition.
Can be interpreted as Griffith physics by fitting power law relaxation (?)





Bordia et. al (I. Bloch's group), to be published

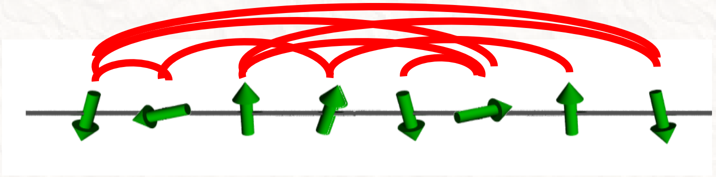
Can we compute thermalizing dynamics?
Approach MBL transition controllably from the thermal side?

Can we compute dynamics of a thermalizing system?

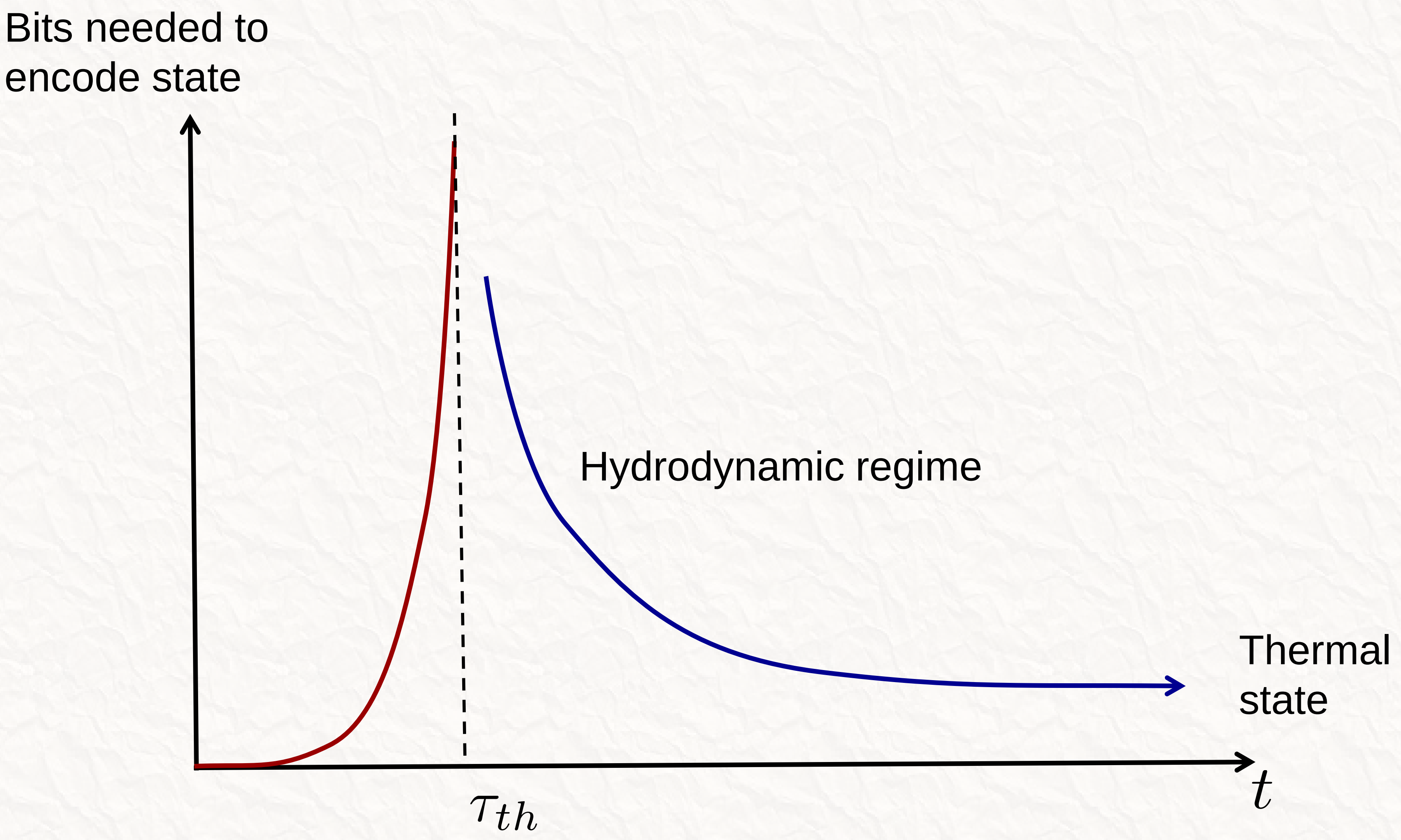
Eyal Leviatan, Jens Bardarson, Frank Pollmann, David Huse and EA
arXiv:1702.08894

Problem with DMRG (or MPS) calculation:
linear growth of entanglement entropy
(Flow of quantum info. From local operators to increasingly non-local ones)

Apparent paradox:
We expect to obtain emergent classical dynamics after thermalization time of order 1, long range entanglement should not matter



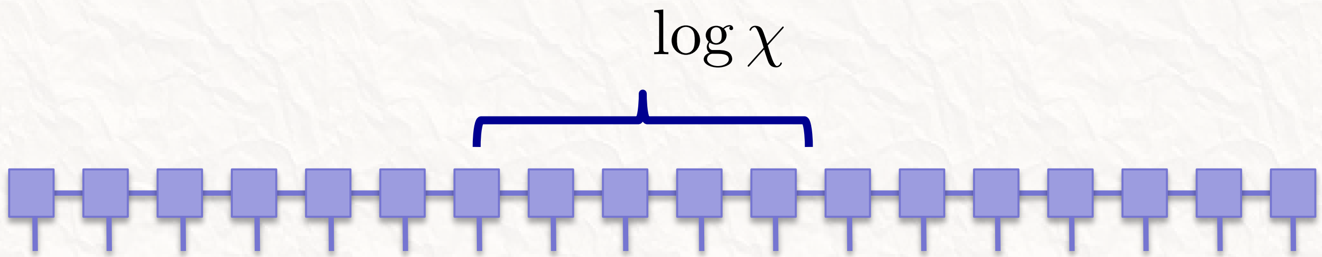
“Information paradox”



Idea: use the time-dependent variational principle (TDVP) to compute a thermalizing system

Variational manifold: MPS states with fixed bond dimension
[Haegeman et. al. PRL 2011]

$$|\psi\rangle = \sum_{\sigma_1\cdots\sigma_N} A^1_{\sigma_1}\cdots A^N_{\sigma_N}\,|\sigma_1\cdots\sigma_N\rangle \qquad \dim A^i_{\sigma_i} = \chi$$



Variational manifold defines a classical Lagrangian:

$$\mathcal{L}[\alpha,\dot\alpha] = \langle\psi[\alpha]\,|\,i\partial_t\,|\,\psi[\alpha]\rangle - \langle\psi[\alpha]\,|\,H\,|\,\psi[\alpha]\rangle$$

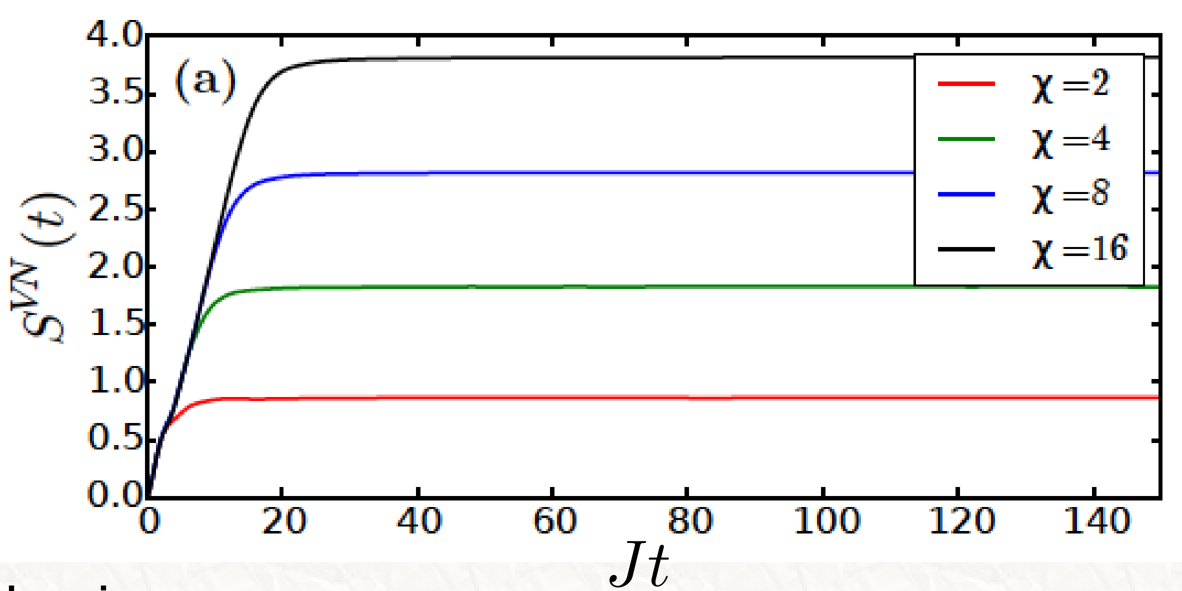
Euler-Lagrange equations generate a classical trajectory in the variational manifold:

$$\frac{\partial\mathcal{L}}{\partial\alpha_i} - \frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\dot\alpha_i} = 0$$

Results: evolution of the local spin

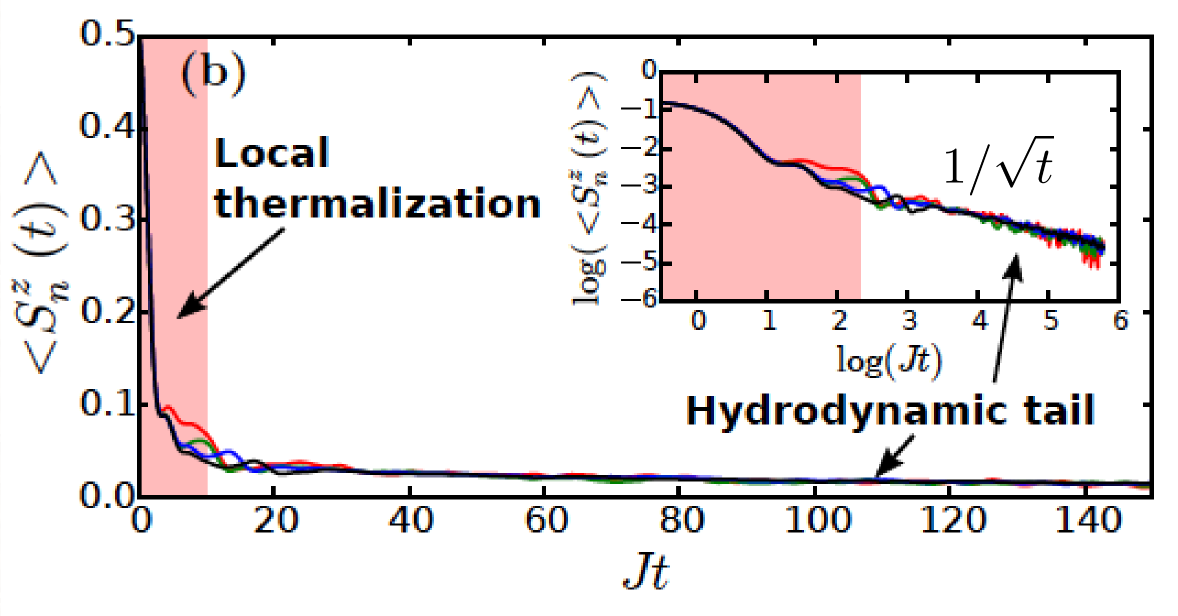
Entanglement growth
(unentangled initial states)

Saturation value $\log \chi$ depends on bond dim.



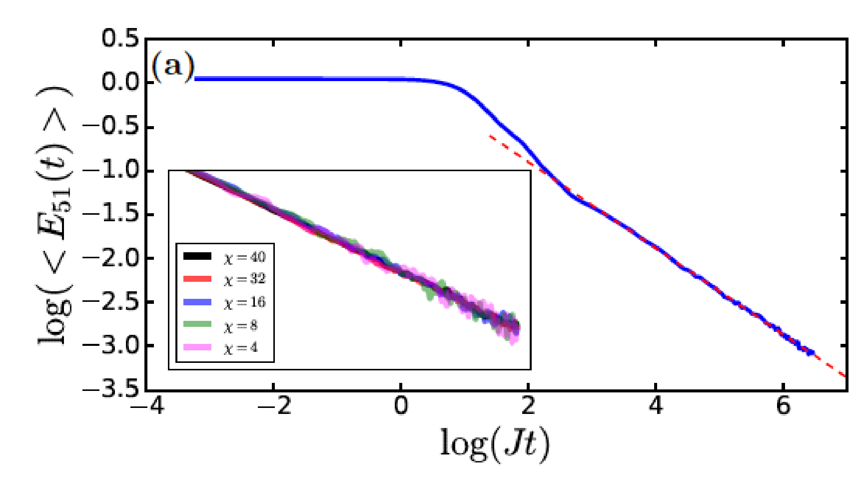
Relaxation of the perturbed spin
(same initial conditions)

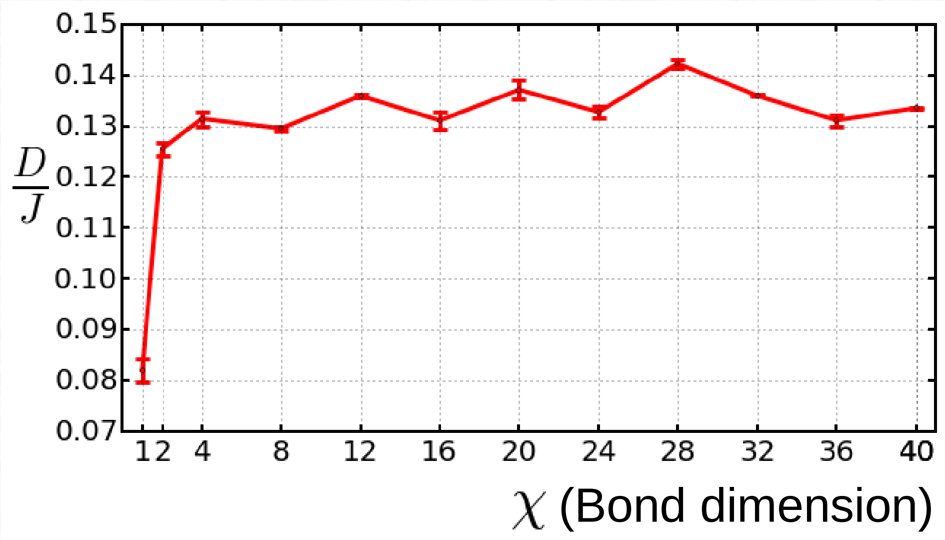
Obtain hydrodynamic tail.
Almost independent of χ



Extract energy diffusion constant

$$H = \sum_i J S_i^z S_{i+1}^z - h_\perp S_i^x - h_{||} S_i^z$$





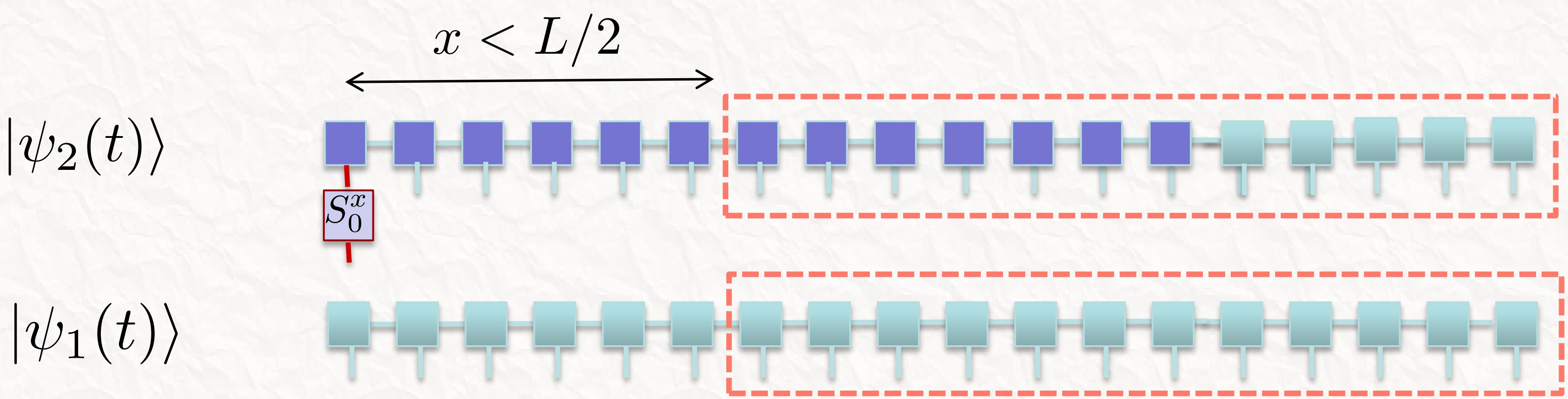
Diffusion coefficient converges to ~5% accuracy for bond dimension > 2.

The method captures the emergent hydrodynamic behavior.

Can it capture the characteristics of quantum chaos?

Computing a diagnostic of chaos

Classical systems: $$||\alpha_1(t) - \alpha_2(t)||^2 \sim e^{\lambda_L t}$$

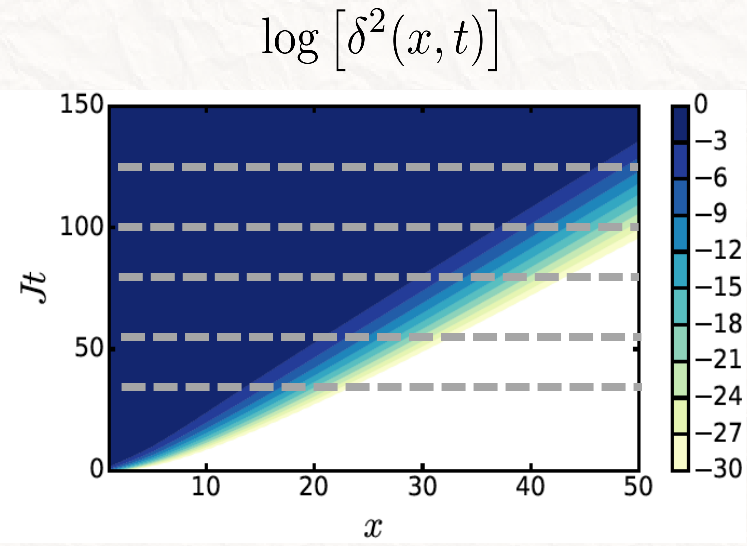


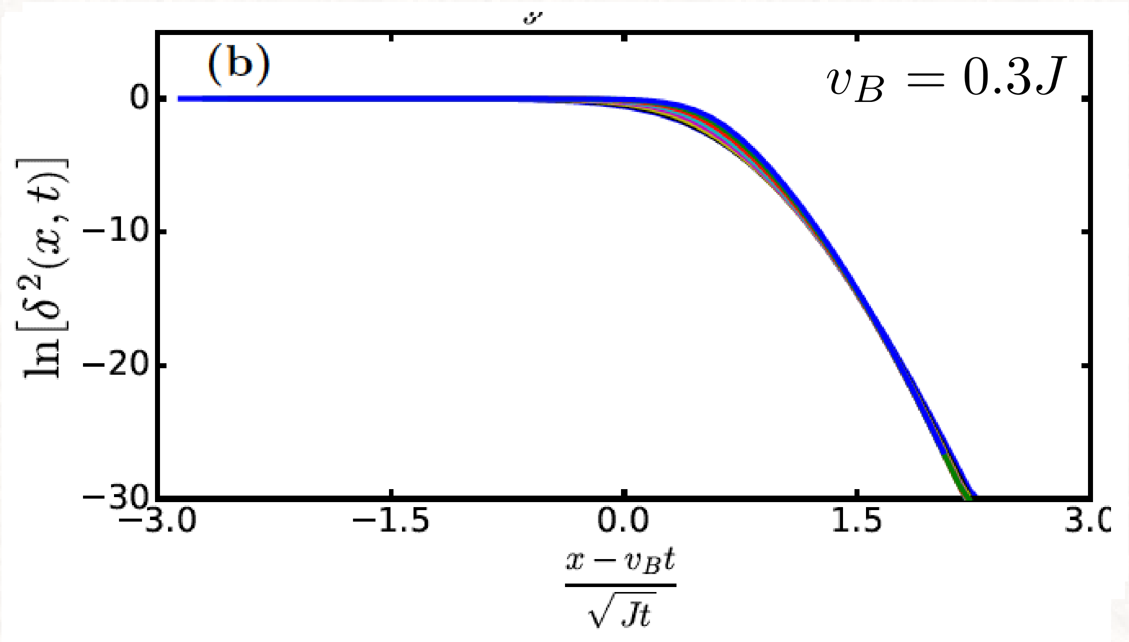
The perturbation is a single-site unitary applied at the left edge.

$$\mathrm{tr}\left[(\rho_1(x,t) - \rho_2(x,t))^2\right]$$

normalized measure: $$\delta^2(x,t) = \frac{\mathrm{tr}[(\rho_1^R(x,t) - \rho_2^R(x,t))^2]}{\mathrm{tr}[\rho_1^R(x,t)^2] + \mathrm{tr}[\rho_2^R(x,t)^2]}$$

Propagation of chaotic front





Front propagates balistically, but at the same time broadens diffusively:

$$\delta x = \sqrt{D_B t}$$

Agrees with a model of random time dependent unitaries
(von Keyserlingk et. al. arXiv:1705.08910)

Outlook – many open questions

- How to develop a controlled approach to the many-body localization transition?

- Emerging understanding of MBL in 1d?
  Is there MBL in higher dimensions?

- First experiments on MBL and critical slowing down.
  How to control dissipation in cold atomic systems ?
  Can we observe signatures of MBL in solid state systems?

- A new computational approach to capture emergent hydrodynamics and quantum chaos in 1d.
  Higher dimension?