Electron hydrodynamics

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Department of Physics Colloquium

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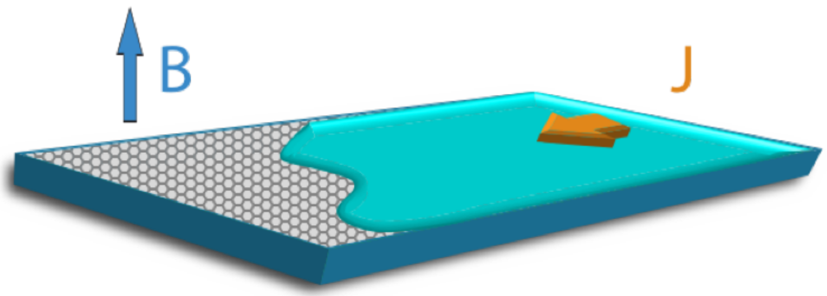

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GORDON AND BETTY MOORE FOUNDATION

Could it finally be true?

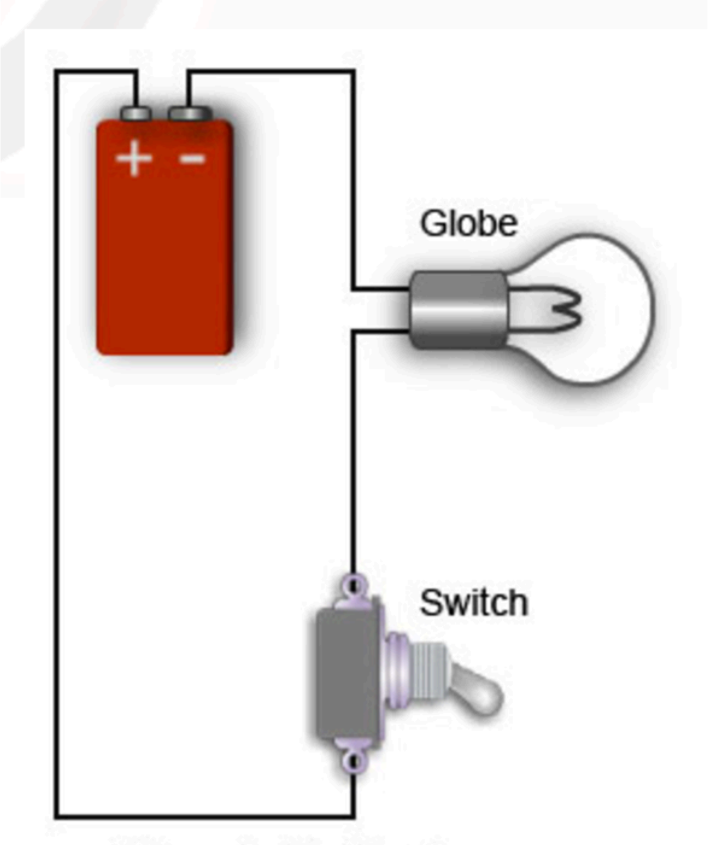

Figure 1: Electrical

?

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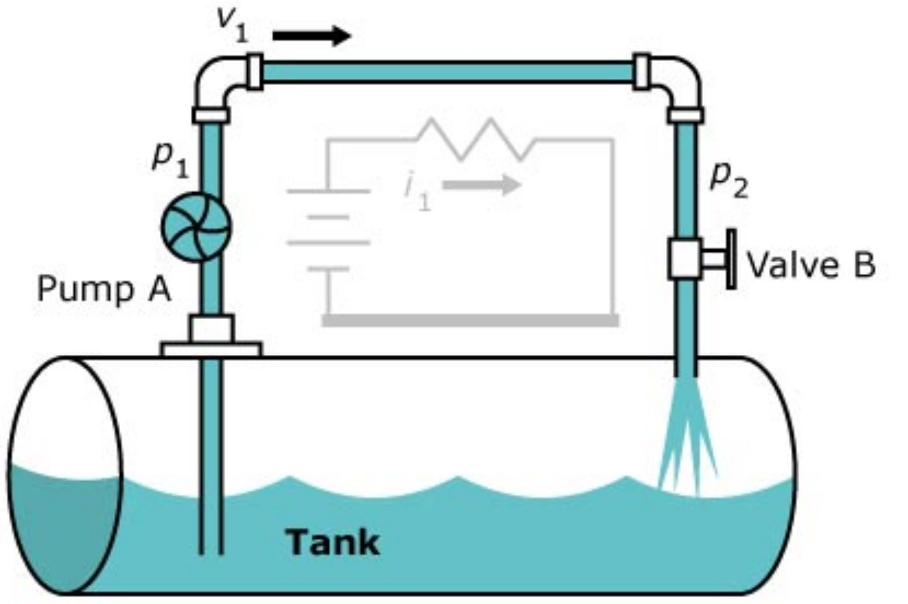
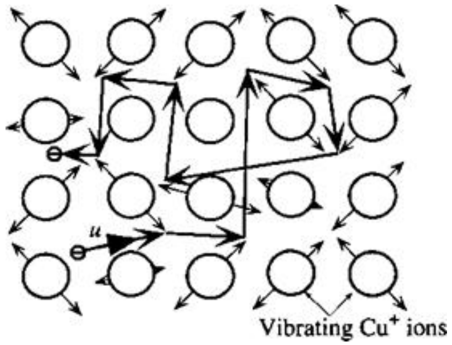


Figure 1—Water circulates endlessly around this closed loop much like elecrical current.

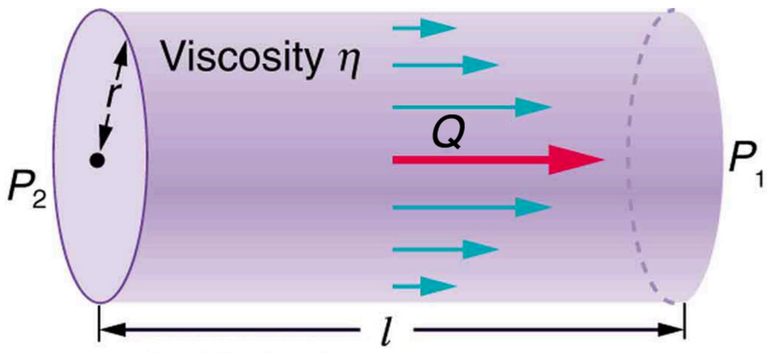
Electric conduction versus water flow

Metal



- Resistance arises through external scattering due to the lattice (impurities, phonons,…)

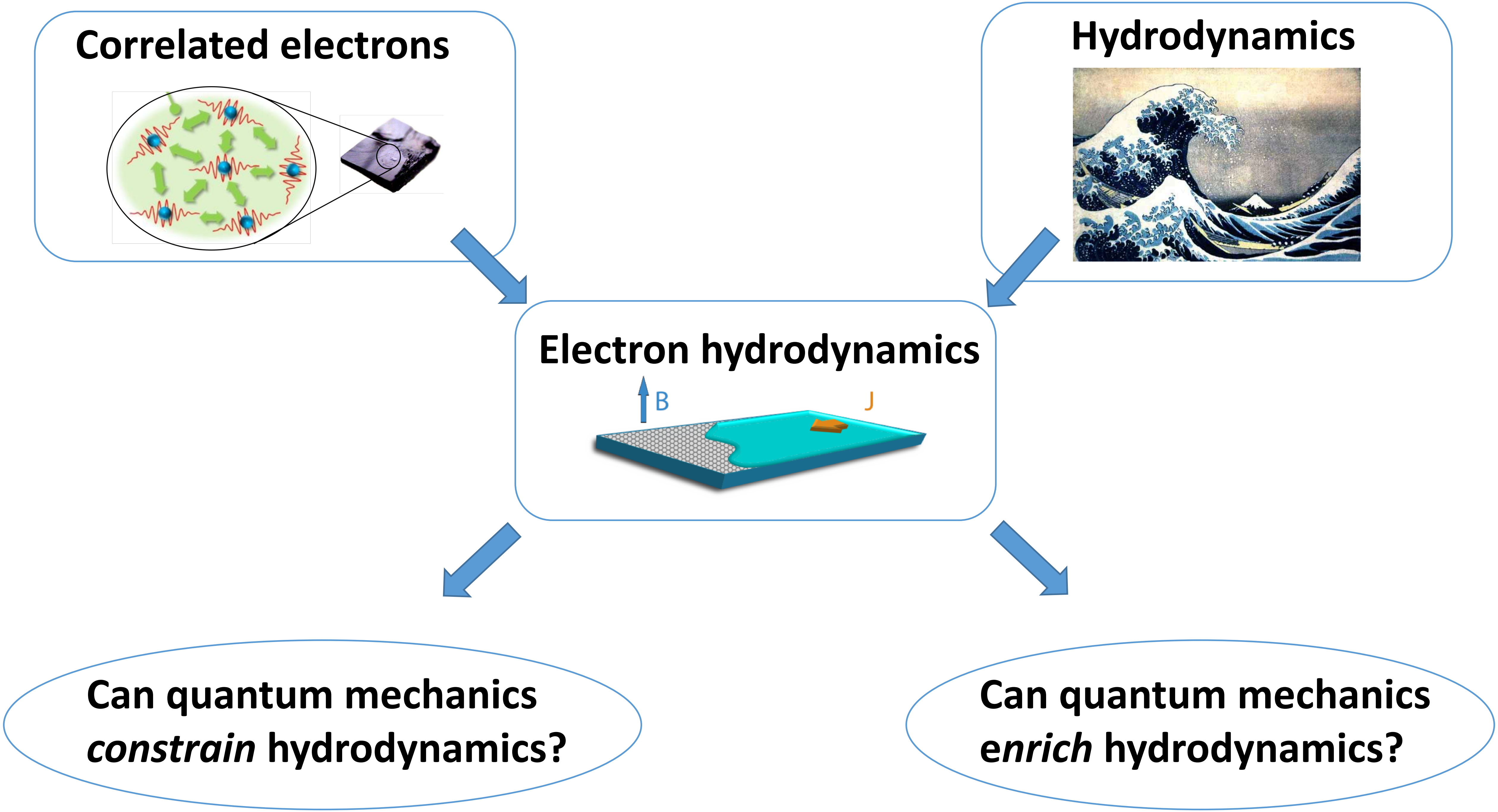
Water

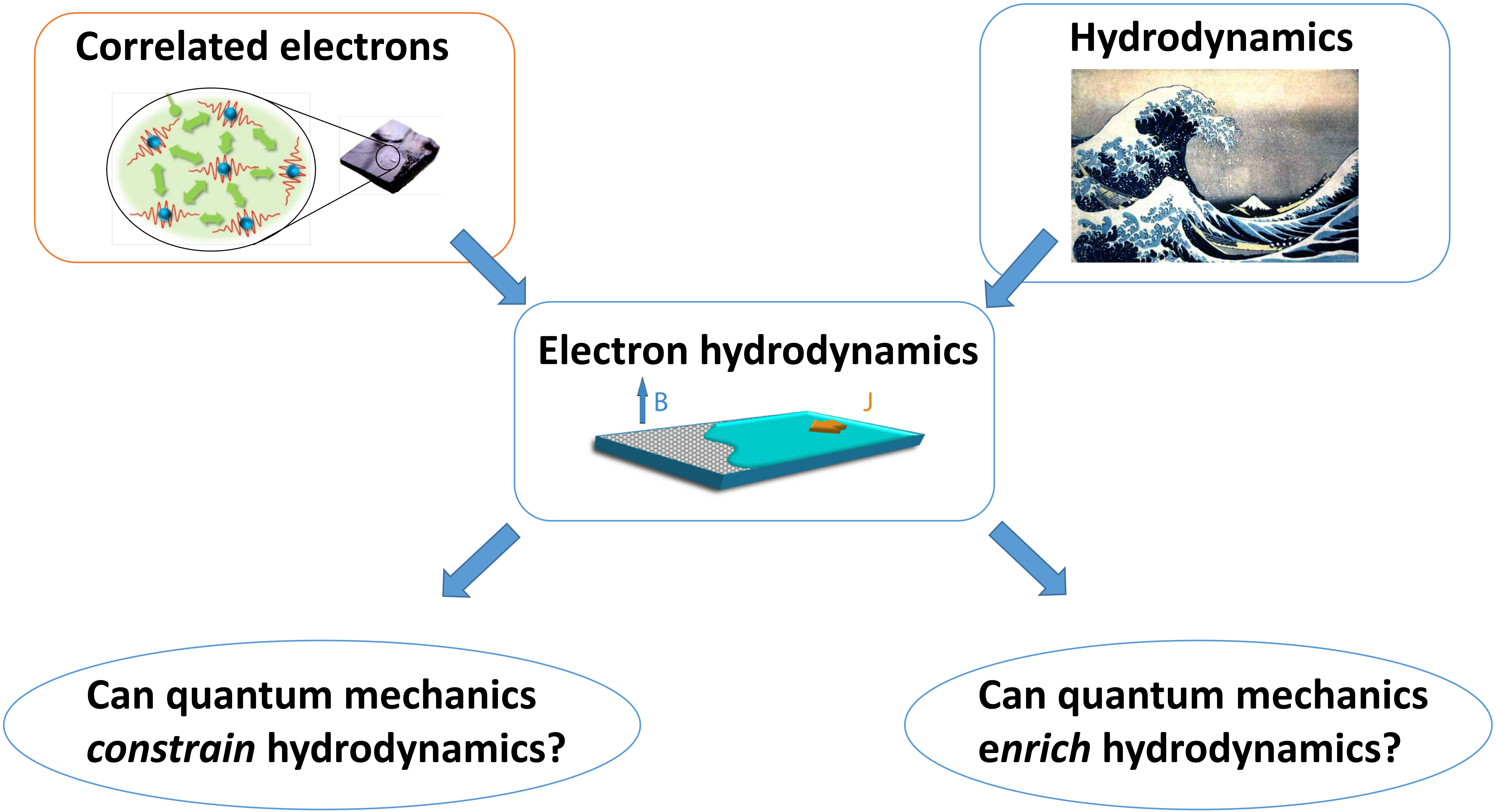


- Resistance arises through internal scattering (viscosity)

Outline

- Electron hydrodynamics – What and why?
- Viscous Fermi liquids
- Using magnetic fields to detect viscous effects
- Conclusion and outlook


Correlated electrons
Hydrodynamics
Electron hydrodynamics
B
J
Can quantum mechanics *constrain* hydrodynamics?
Can quantum mechanics *enrich* hydrodynamics?



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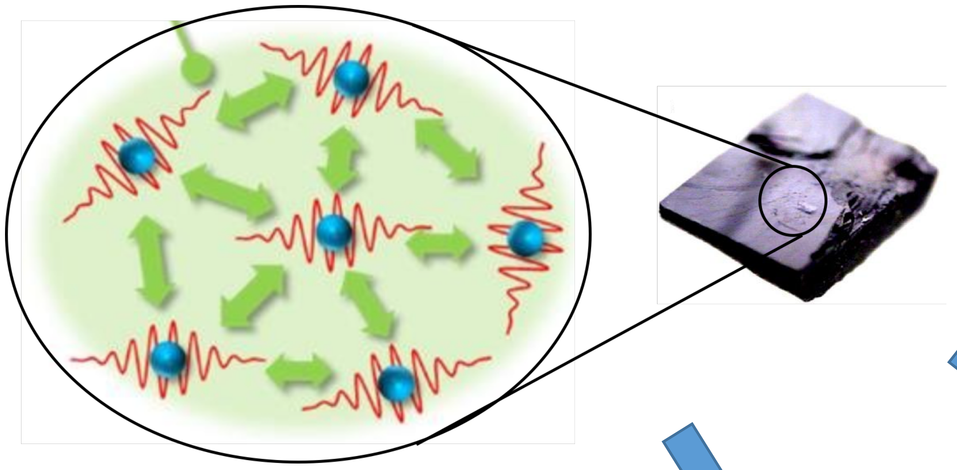
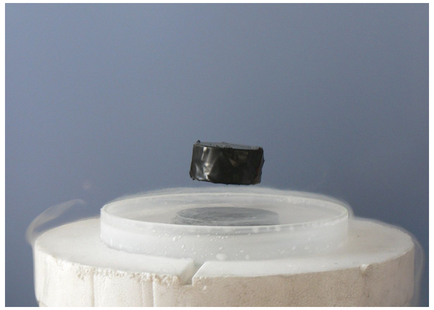
“More is different” [Anderson ‘72]



Spin liquids

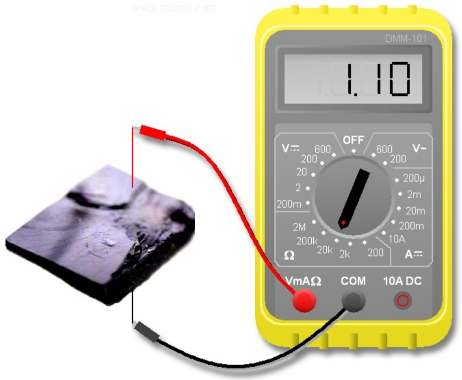
Topological phases



Superconductors

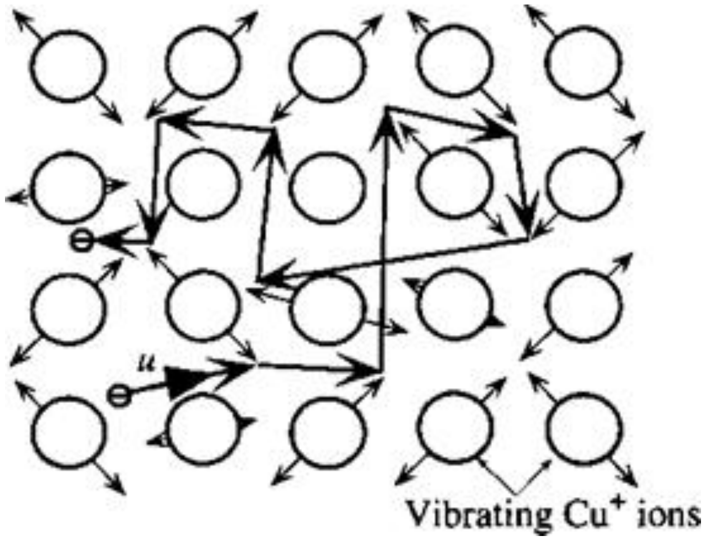
…

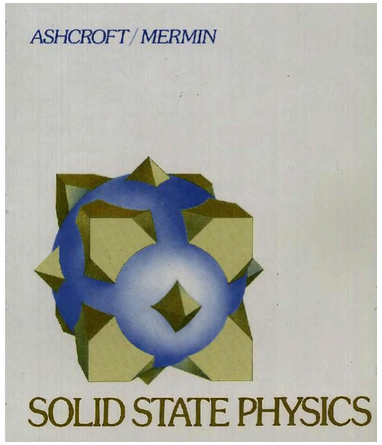
Conventional metallic transport [Drude 1900]



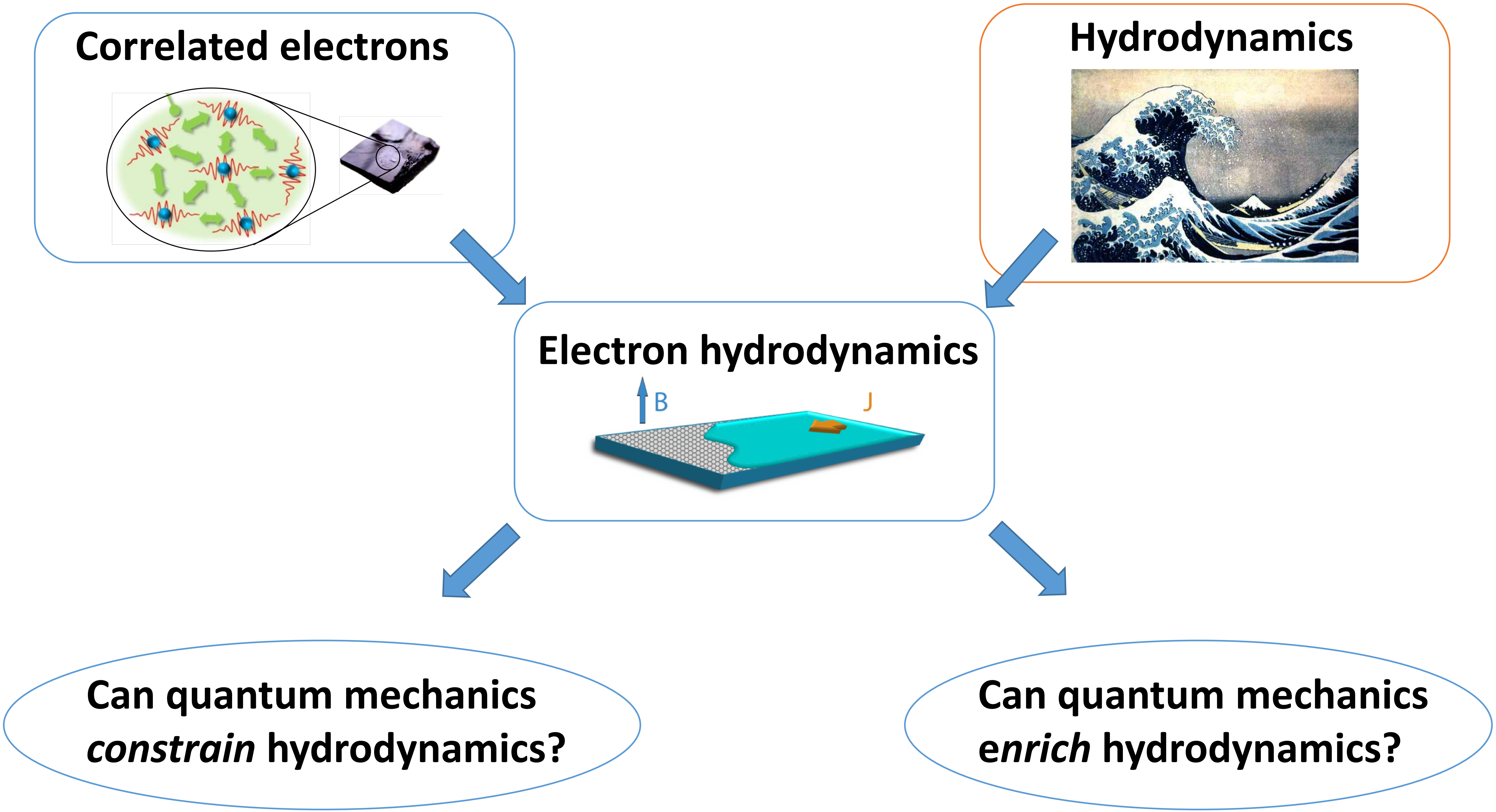
$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

Mean free time of electrons, set by defects and vibrations of the lattice





> Electron-electron scattering[5] plays a relatively minor role in the theory of conduction in solids, for reasons to be described in Chapter 17.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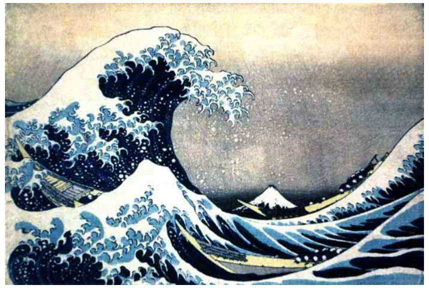
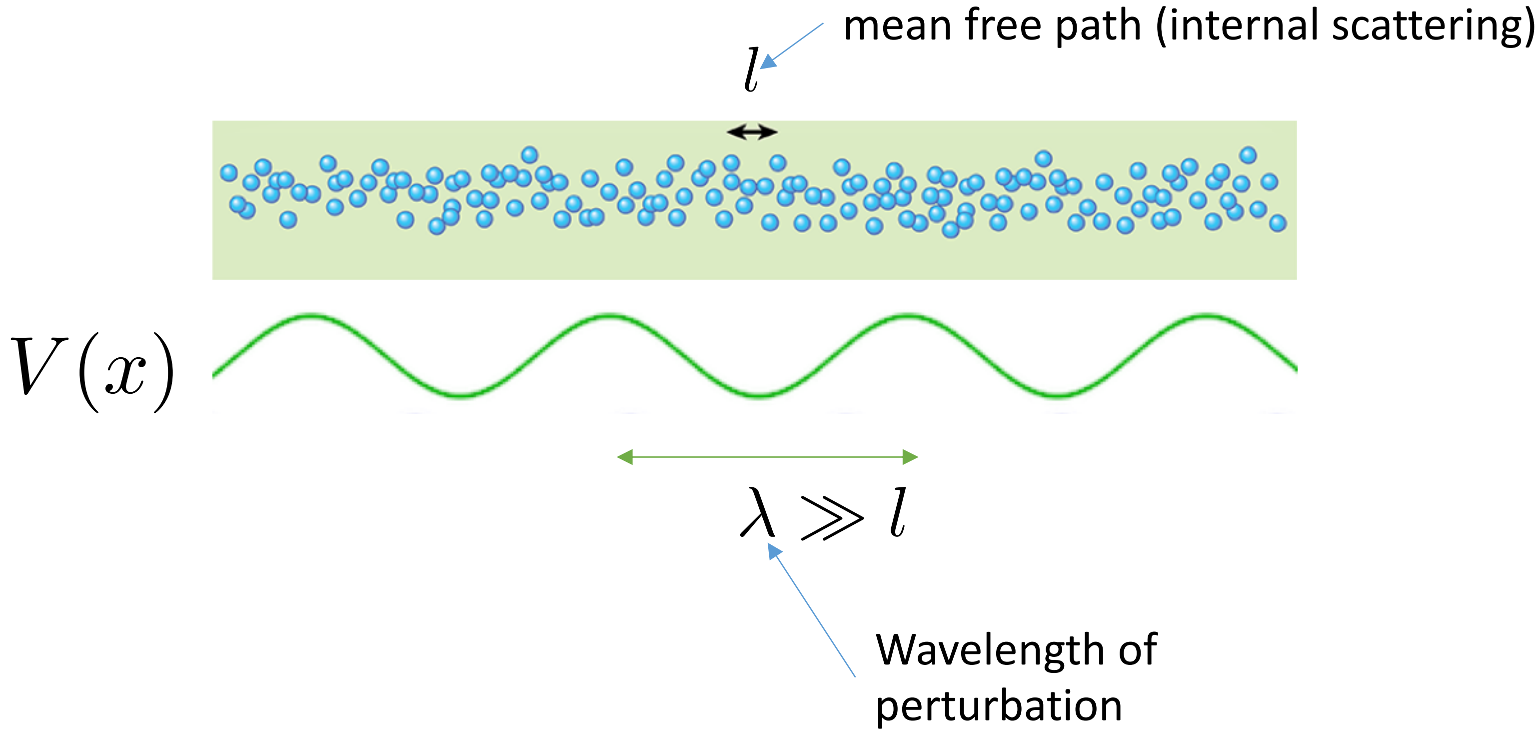
Hydrodynamics

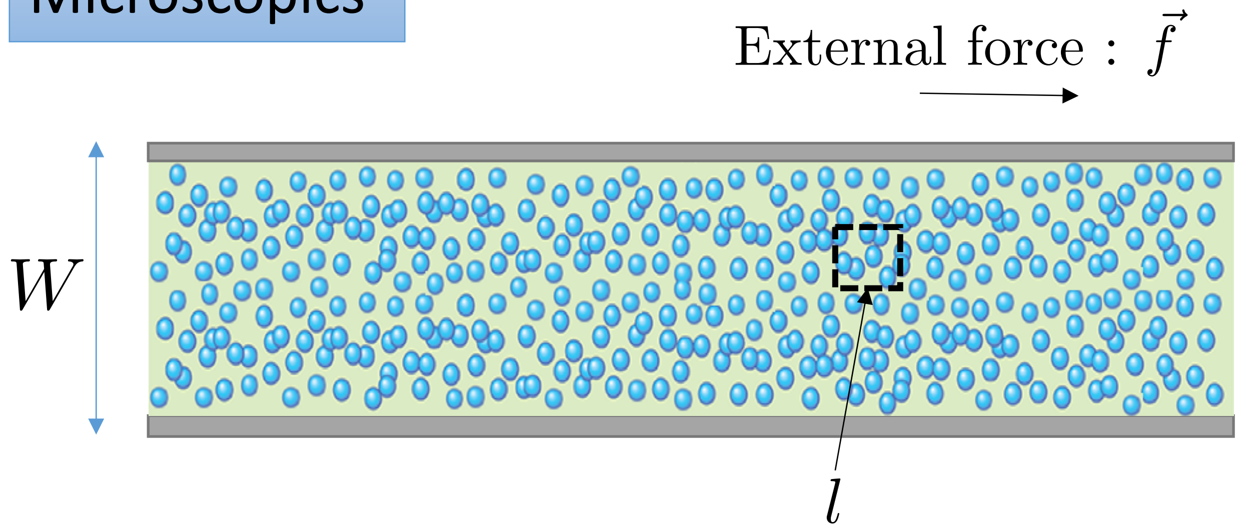


- Universal description of fluids based on conserved quantities: momentum, energy, charge,…
- Works at length/time scales much larger than the microscopic ones

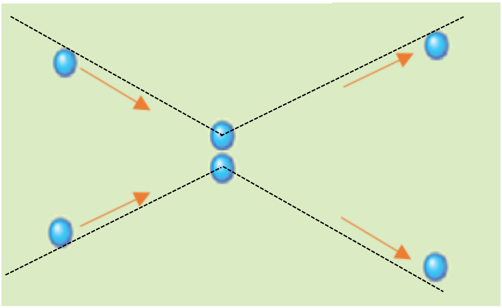


Viscous fluid description based on momentum conservation

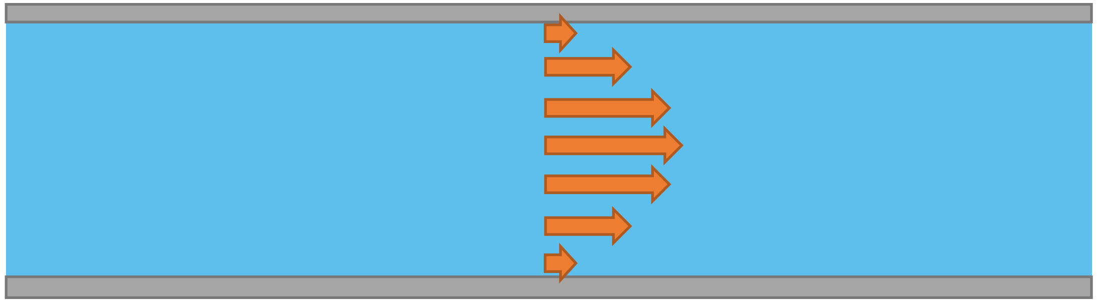
Microscopics



$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$

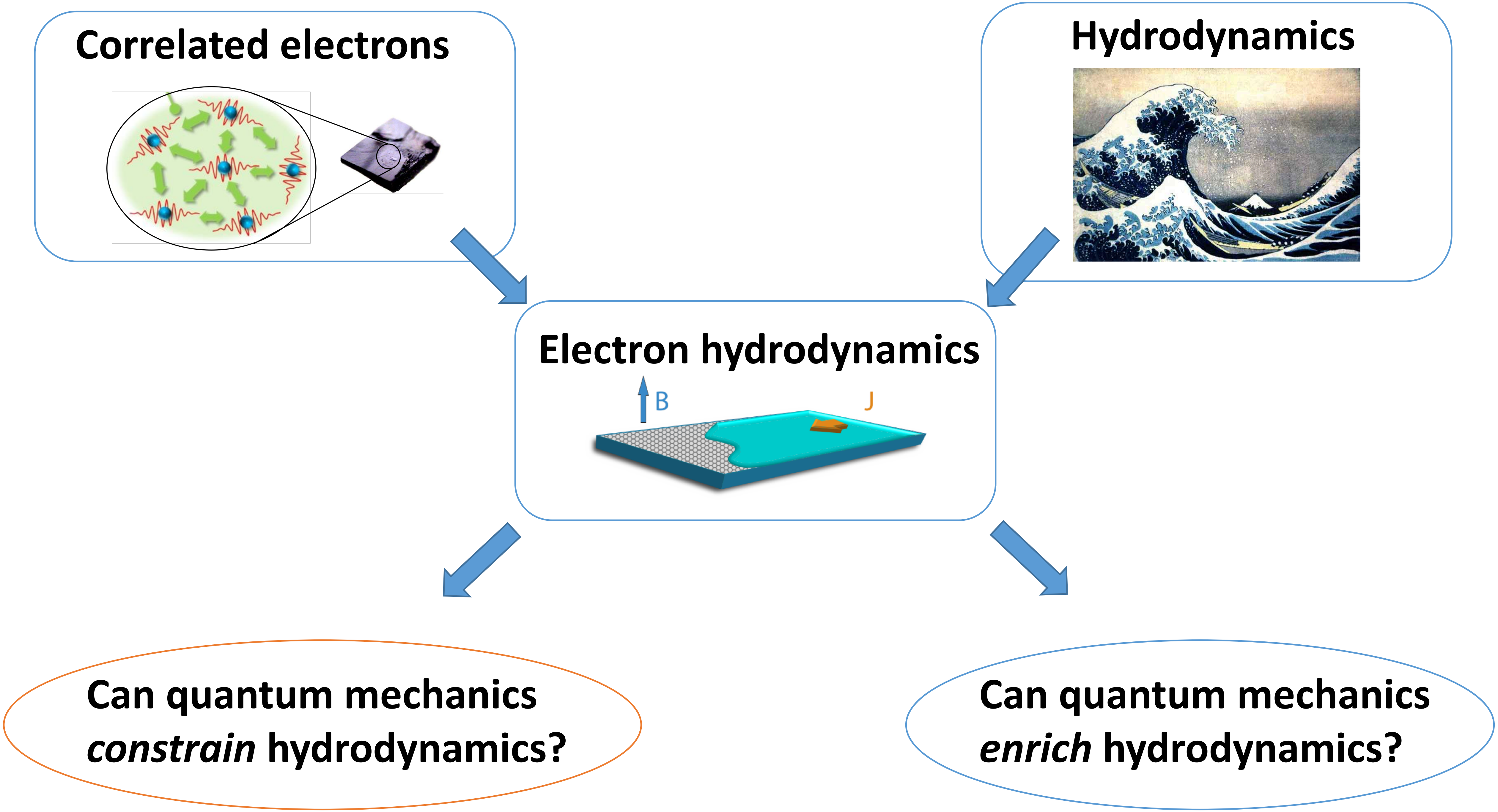


Hydrodynamics



Navier-Stokes

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} + \frac{\vec{f}}{m}$$



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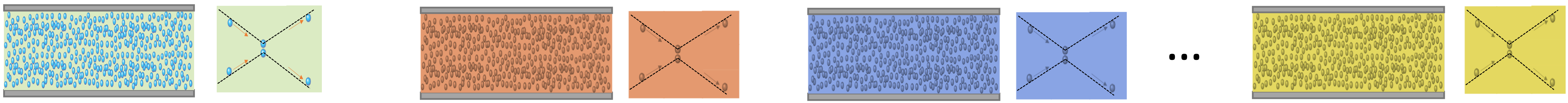
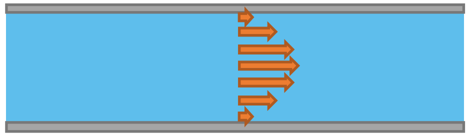
Can quantum mechanics *constrain* hydrodynamics?





$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} + \frac{\vec{f}}{m}$$

Classically

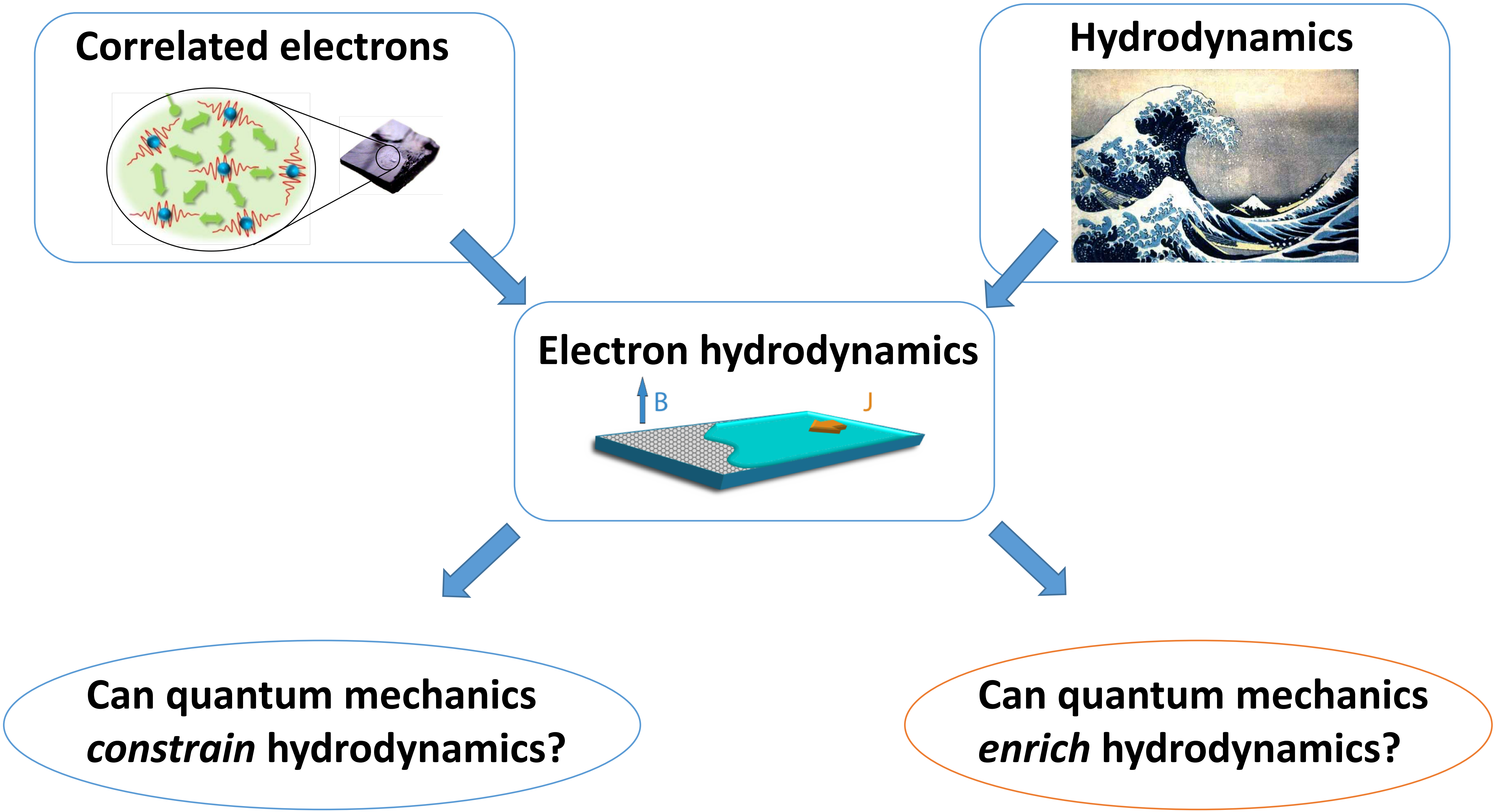
Viscosity can take any value

Quantum mechanically

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

[Kovtun, Son, Starinets, PRL 2005]

Conjectured fundamental bound based on holography.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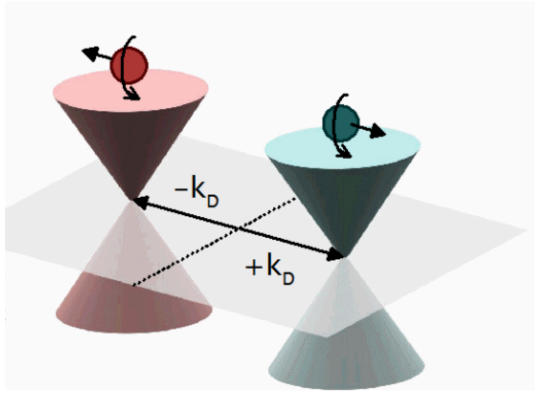
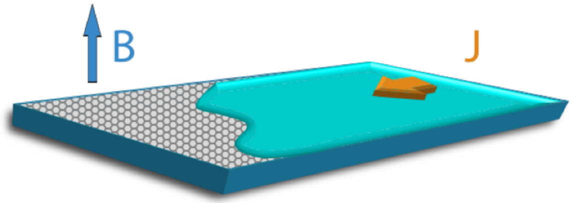
Can quantum mechanics *enrich* hydrodynamics?

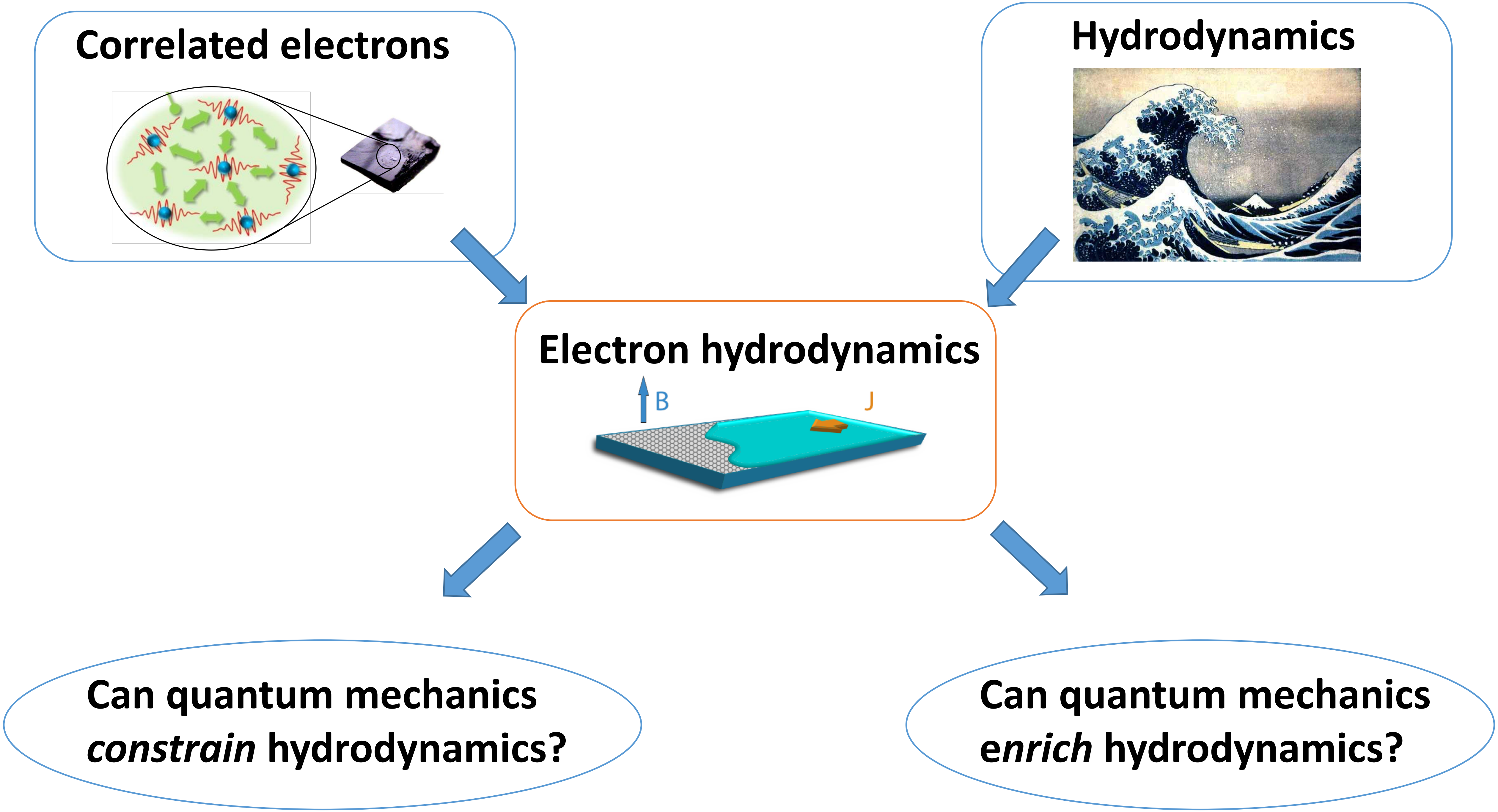
For electrons in a solid: ~~$H = \frac{P^2}{2m}$~~ → $H(\mathbf{k})\psi(\mathbf{k}) = E(\mathbf{k})\psi(\mathbf{k})$

Relativistic fluids: graphene, Weyl semimetals,…



Topological effects: Berry phase, Hall viscosity,…

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Outline

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- **Viscous Fermi liquids**
- Using magnetic fields to detect viscous effects
- Conclusion and outlook

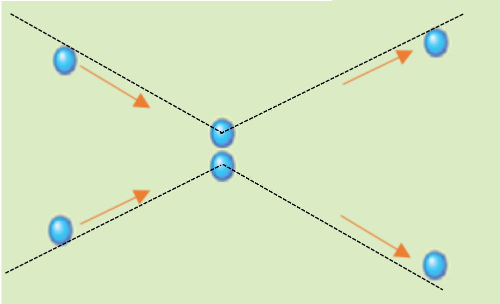
Hydrodynamic flow of electrons possible if…

$$l_{MC} \ll W \ll l_{MR}$$

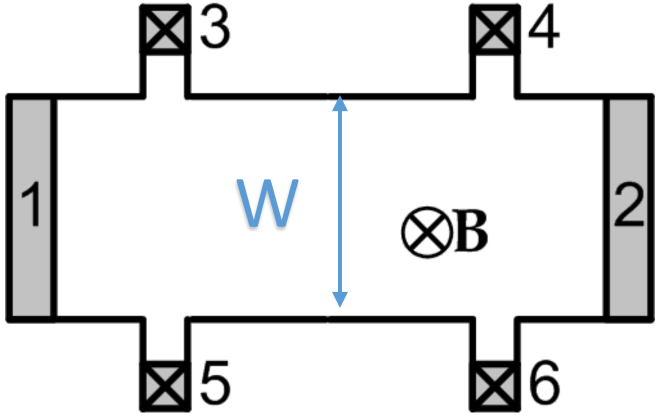
Momentum-conserving mean free path ("internal scattering"):

- e-e scattering

$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$



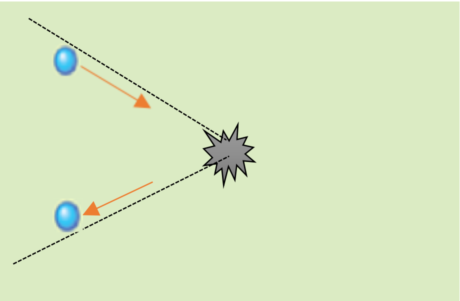
Sample size



Momentum-relaxing mean free path ("external scattering"):

- Impurities
- Phonons

$$\vec{v} \neq \vec{v}'$$



[Gurzhi 1963] [*Spivak, Kivelson 2006*] [Hruska, Spivak 2011] *[Andreev Kivelson Spivak 2011]*
[Torre,Tomadin,Geim,Spivak 2011] [Levitov Falkovich 2016] many more papers…

Which materials?

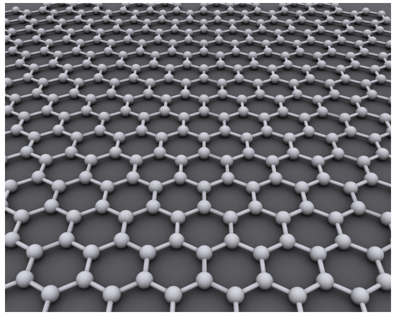
$$l_{MC} \ll W \ll l_{MR}$$

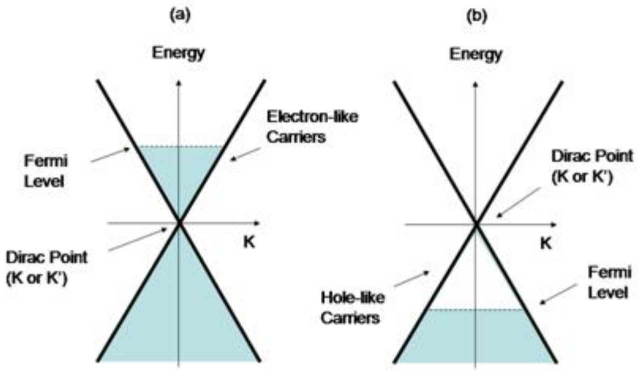
Strong interactions, not too low T — ~ 1um

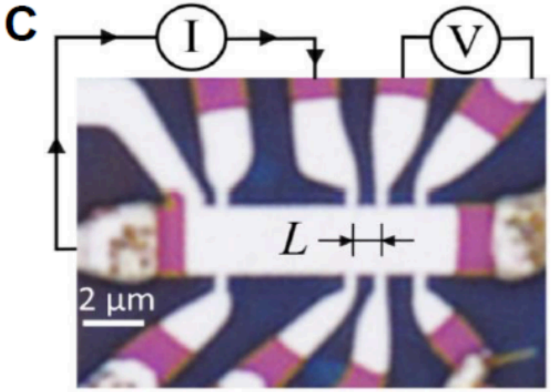
Mesoscopic samples — ~ 10 um

Clean samples, not too high T — ~ 100 um

- 2D electron gases [de Jong, Molenkamp] (1994)
- PdCoO2 (high mobility layered metal) (2016)
- **Graphene** [Bandurin et al] [Crossno et al] (2016)





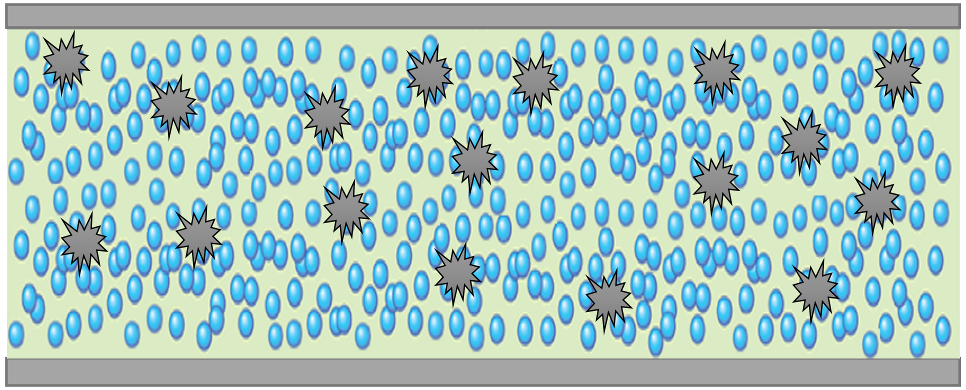


Bandurin et al, Science 351, 1055-1058 (2016)

Three regimes

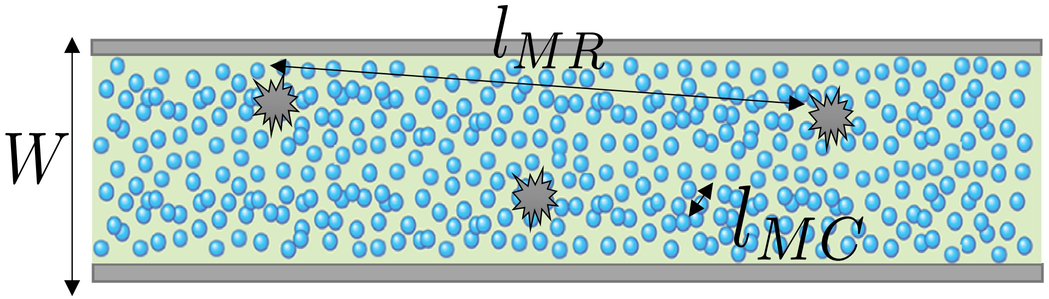
Ohmic

$$l_{MR}, l_{MC} \ll W$$



Viscous

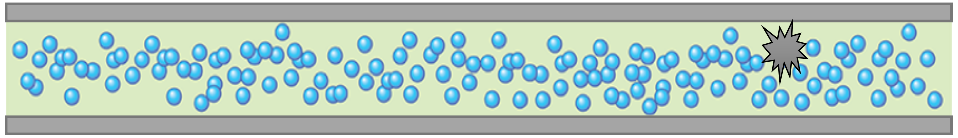
$$l_{MC} \ll W \ll l_{MR}$$



Diffuse-Ballistic

"Knudsen flow"

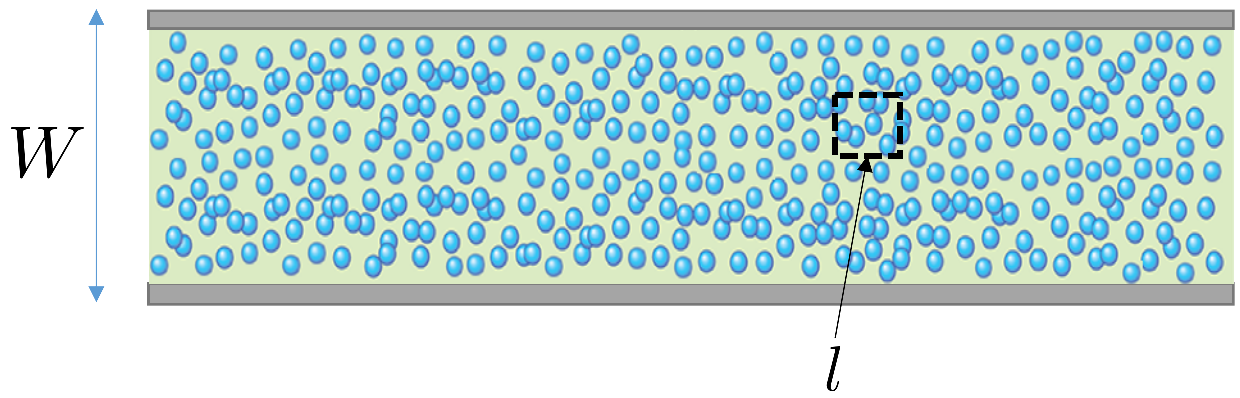
$$W \ll l_{MR}, l_{MC}$$



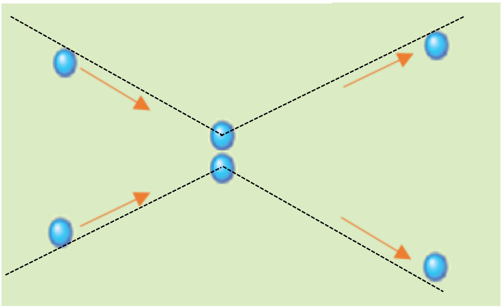
Viscous fluid

Microscopics

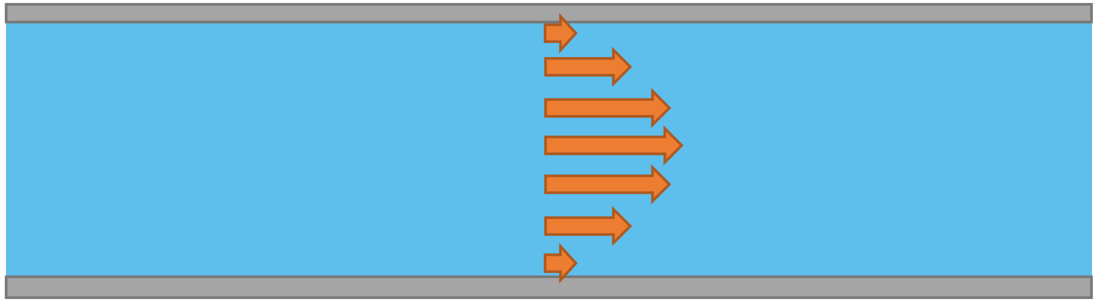
External force : $\vec{f}$



$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$



Hydrodynamics



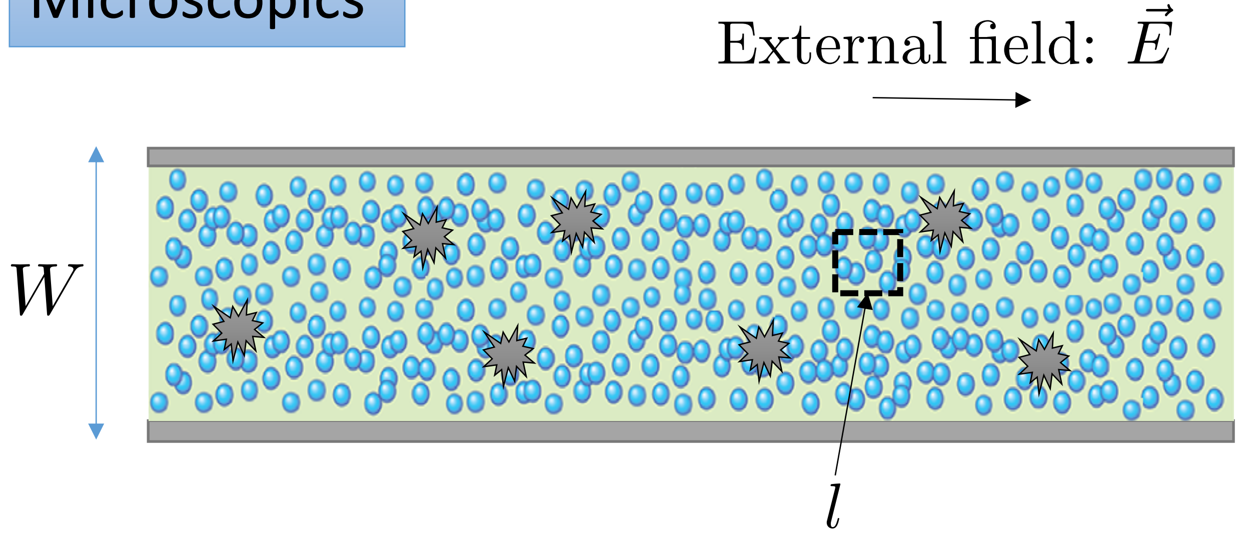
Navier-Stokes

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} + \frac{\vec{f}}{m}$$

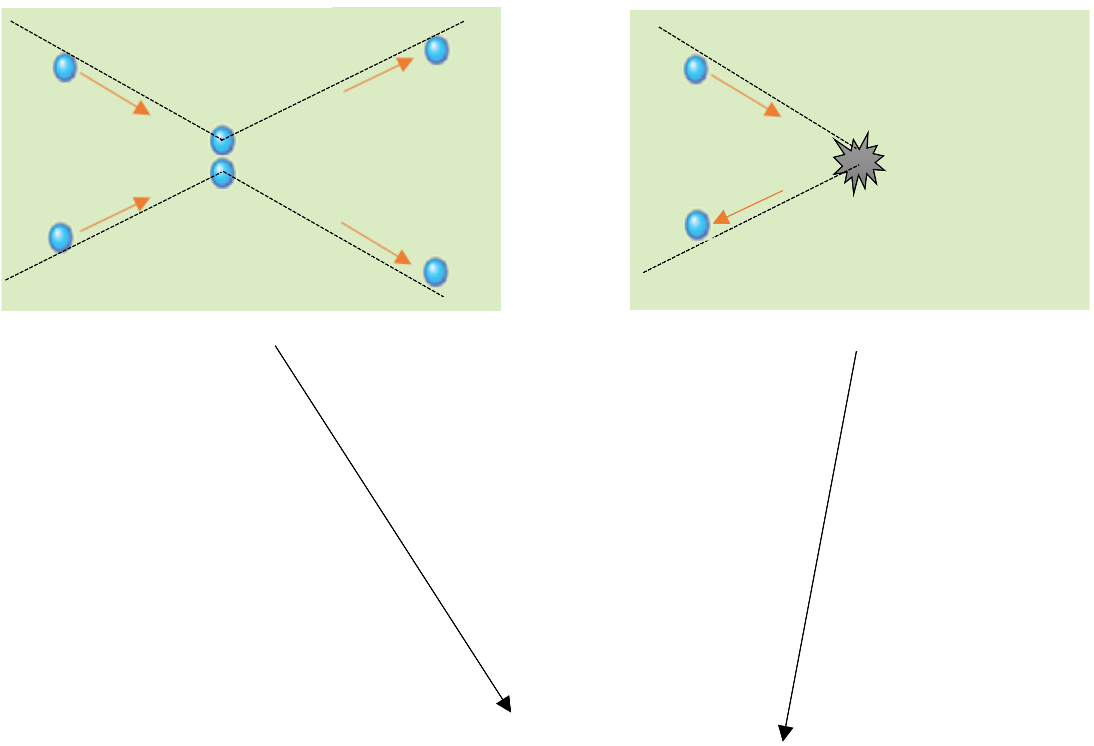
Viscous electronic fluid

$$\vec{j} = ne\vec{v}$$

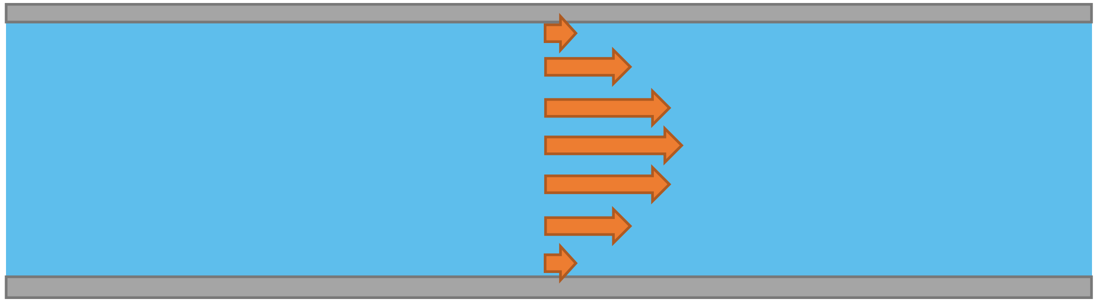
Microscopics



$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$ $\qquad$ $\vec{v} \neq \vec{v}'$



Hydrodynamics



$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e\vec{E}}{m}$$

Two regimes

$$\partial_t \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e\vec{E}}{m}$$

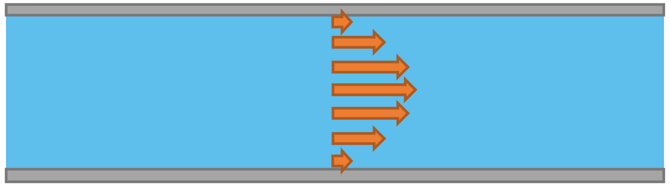
$$\vec{j} = ne\vec{v}$$

If term dominates

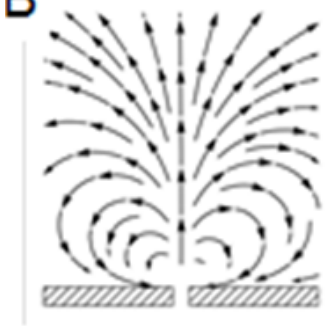
If term dominates

Viscous

Non-local relation between current and electric field!



B



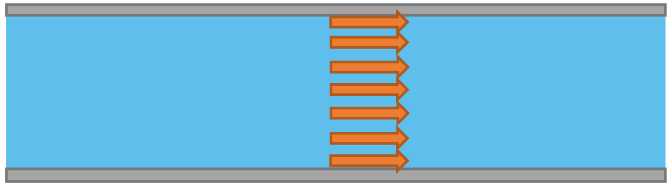
$$\boldsymbol{\eta} \nabla^2 \vec{j} = -\frac{e^2 n}{m} \vec{E}$$

$$\nabla \times \vec{j} \neq 0$$

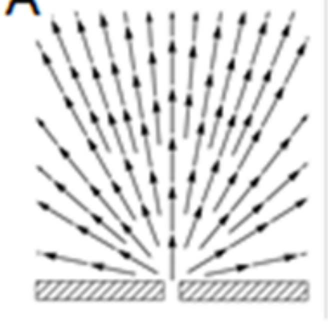
$$\bar{\rho} = \frac{m}{e^2 n} \eta \frac{12}{W^2}$$

Ohmic

$$\sigma = \frac{ne^2}{m} \tau_{MR}$$



A



$$\vec{j} = \sigma \vec{E}$$

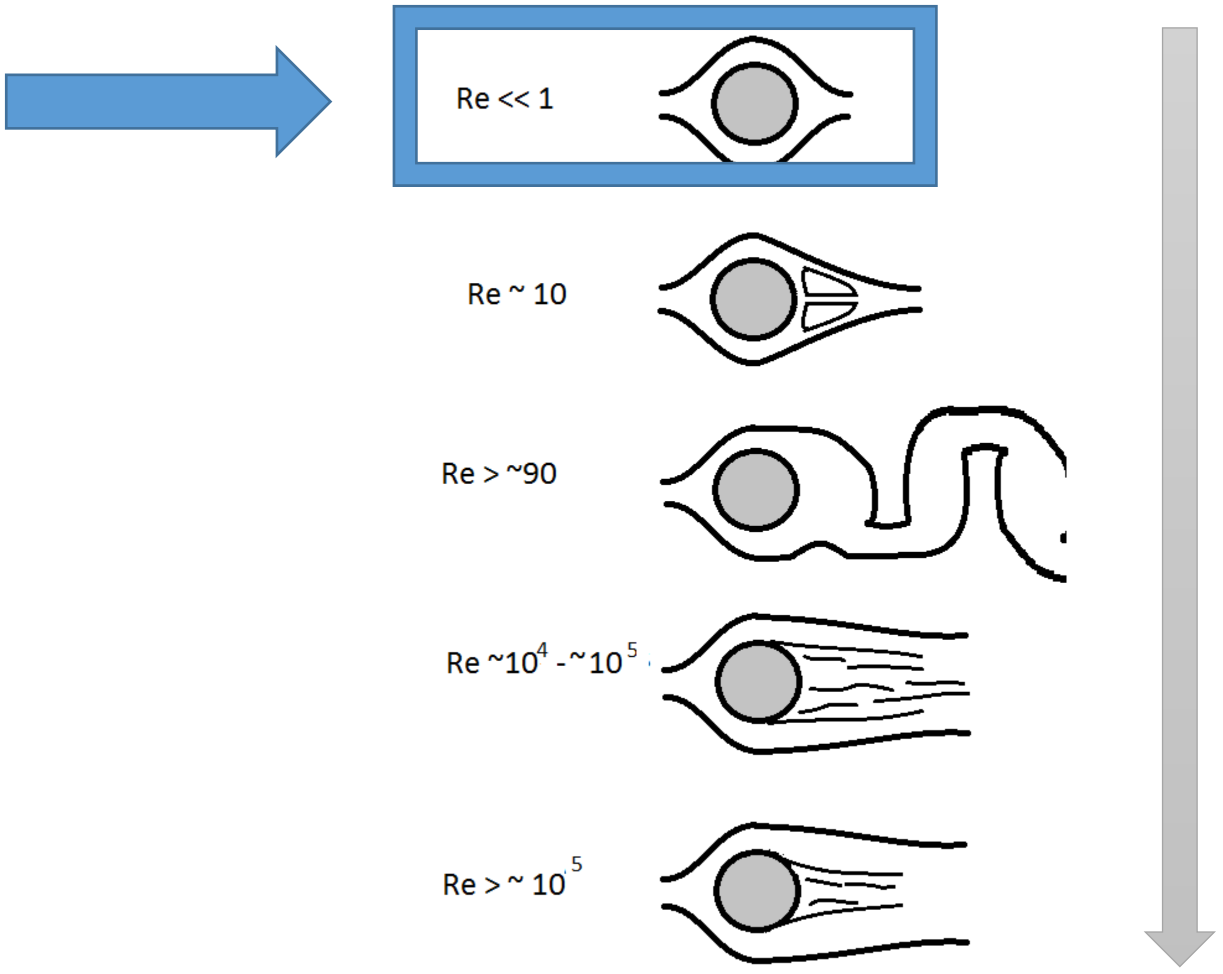
$$\nabla \times \vec{j} = 0$$

$$\bar{\rho} = \frac{m}{e^2 n} \frac{1}{\tau_{MR}}$$

Where did the convection term go?

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e\vec{E}}{m}$$

Today:



$$Re = \frac{\vec{v} \cdot \nabla \vec{v}}{\eta \nabla^2 \vec{v}}$$

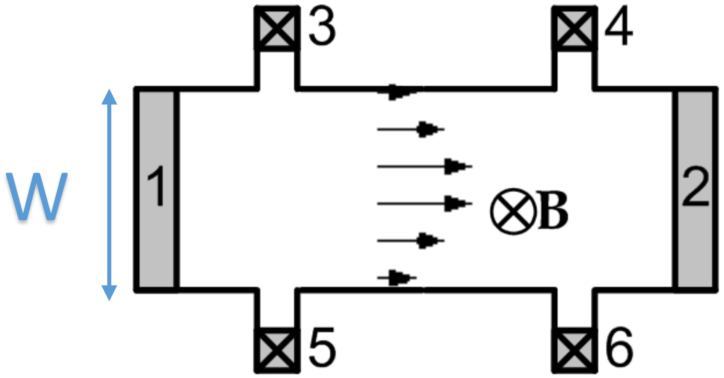
Outline

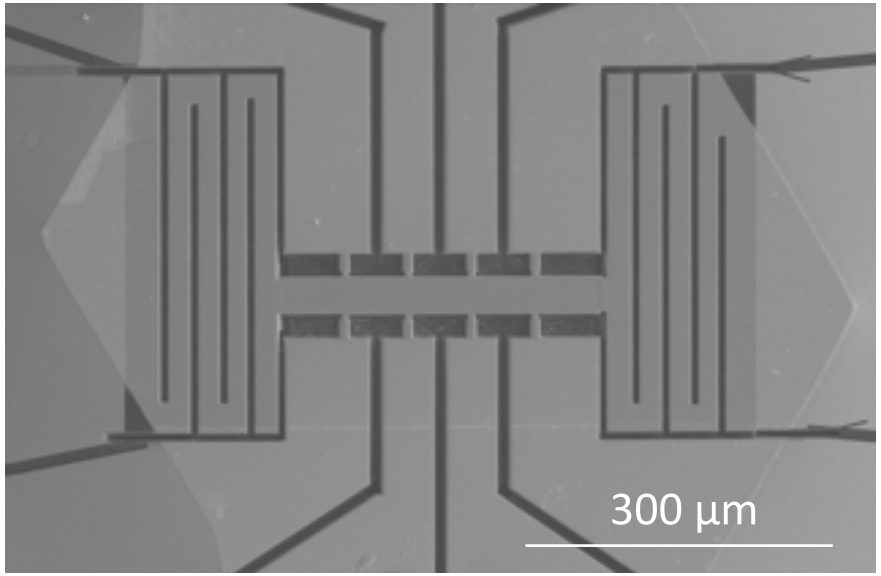
- Electron hydrodynamics – What and why?
- Electron hydrodynamics in good metals
- Using magnetic fields to detect hydro effects
- Conclusion and outlook

[TS et al, PRL '17]

How to identify hydro effects?

- Idea: Look at finite size corrections to transport coefficients
- Problem: how to distinguish from ballistic effects?
- Solution: use magnetic field





Navier-Stokes under magnetic field

$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m} \vec{E}$$

Add B field

$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m} (\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Lorentz force

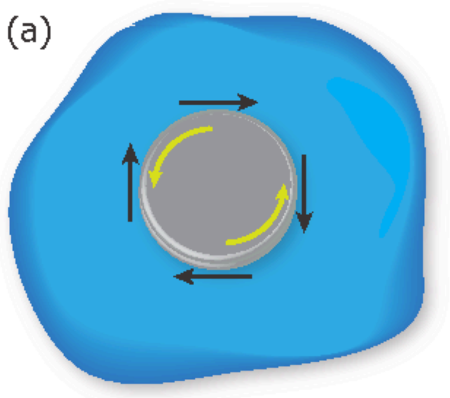
Hall viscosity

A tale of two viscosities

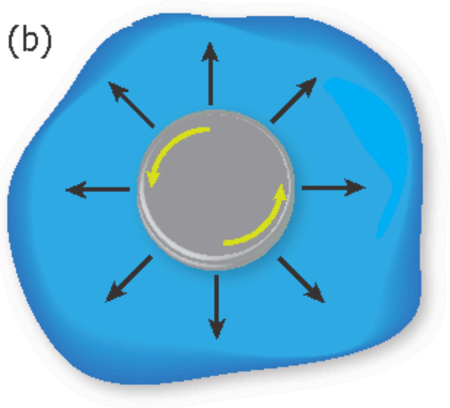
$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity

Hall viscosity



Probed by ρ_{xx}

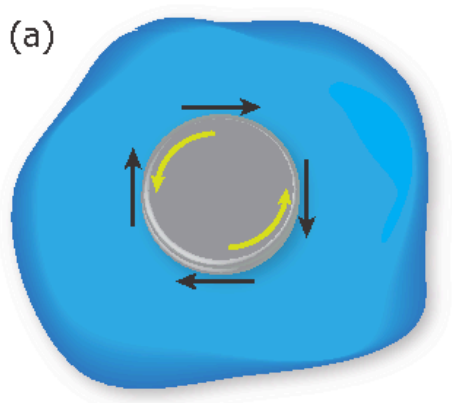


Probed by ρ_{xy}

A tale of two viscosities

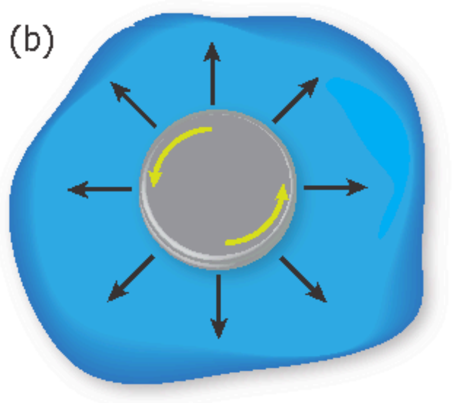
$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity



Probed by ρ_{xx}

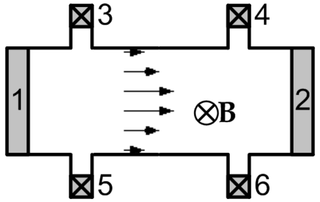
Hall viscosity



Probed by ρ_{xy}

Hydrodynamic solution

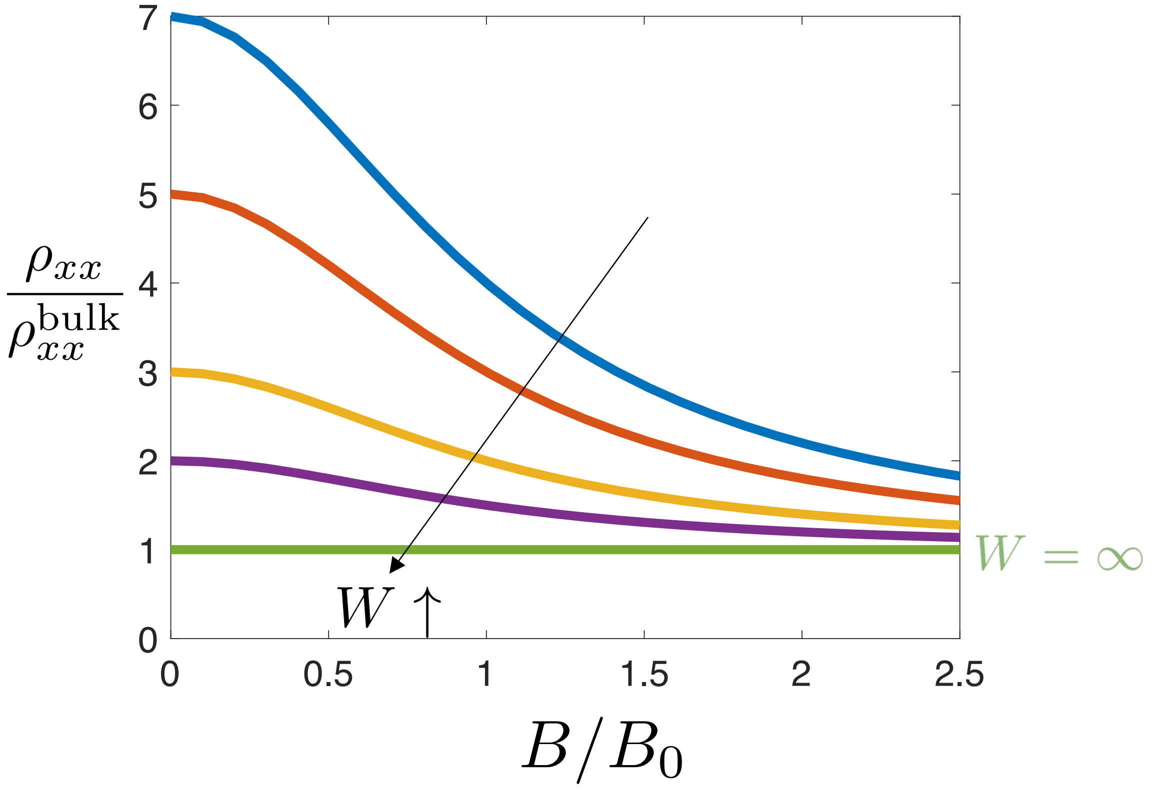
[TS et al, PRL '17]



$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} + \eta_{xy} \nabla^2 \vec{v} \times \vec{z} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) - \frac{1}{\tau_{MR}} \vec{v}$$

$$\rho_{xx} \simeq \rho_{xx}^{bulk} + \frac{m}{e^2 n} \eta_{xx} \frac{12}{W^2}$$

$$\rho_{xx}^{bulk} = \frac{m}{e^2 n} \frac{1}{\tau_{MR}}$$



Result from Boltzmann theory for a charged Fermi liquid:

$$\eta_{xx}(B) \sim \eta_{xx}(B=0) \frac{1}{1 + (B/B_0)^2}$$

Navier-Stokes is not enough

In theory:

$$l_{MC} \ll W \ll l_{MR}$$

Navier-Stokes

In practice:

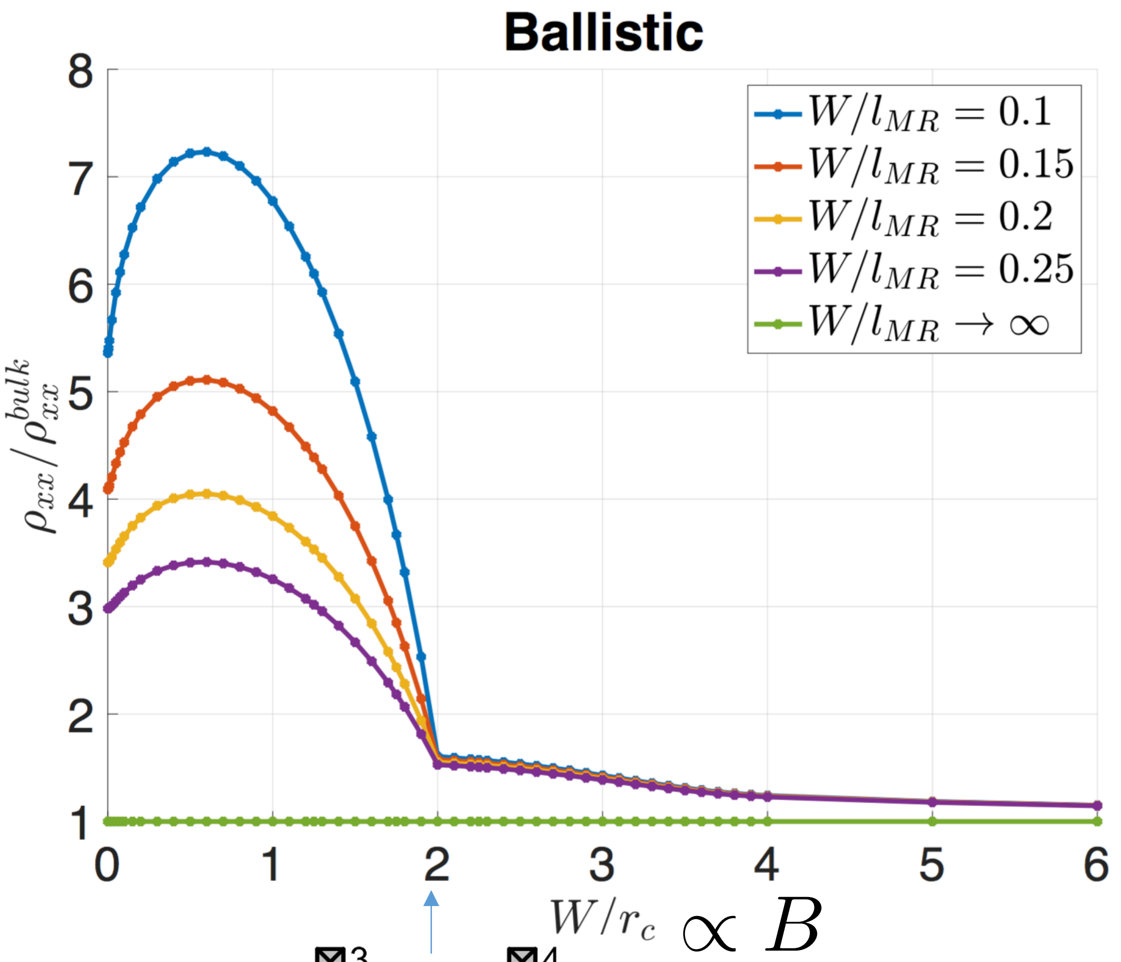
$$l_{MC} \lesssim W \lesssim l_{MR}$$

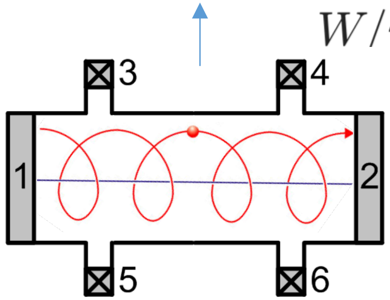
Kinetic theory

$$\partial_t f + \vec{v} \cdot \nabla_{\vec{r}} f + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) \cdot \nabla_{\vec{v}} f = -\frac{1}{\tau_{MR}} f - \frac{1}{\tau_{MC}} I[f]$$

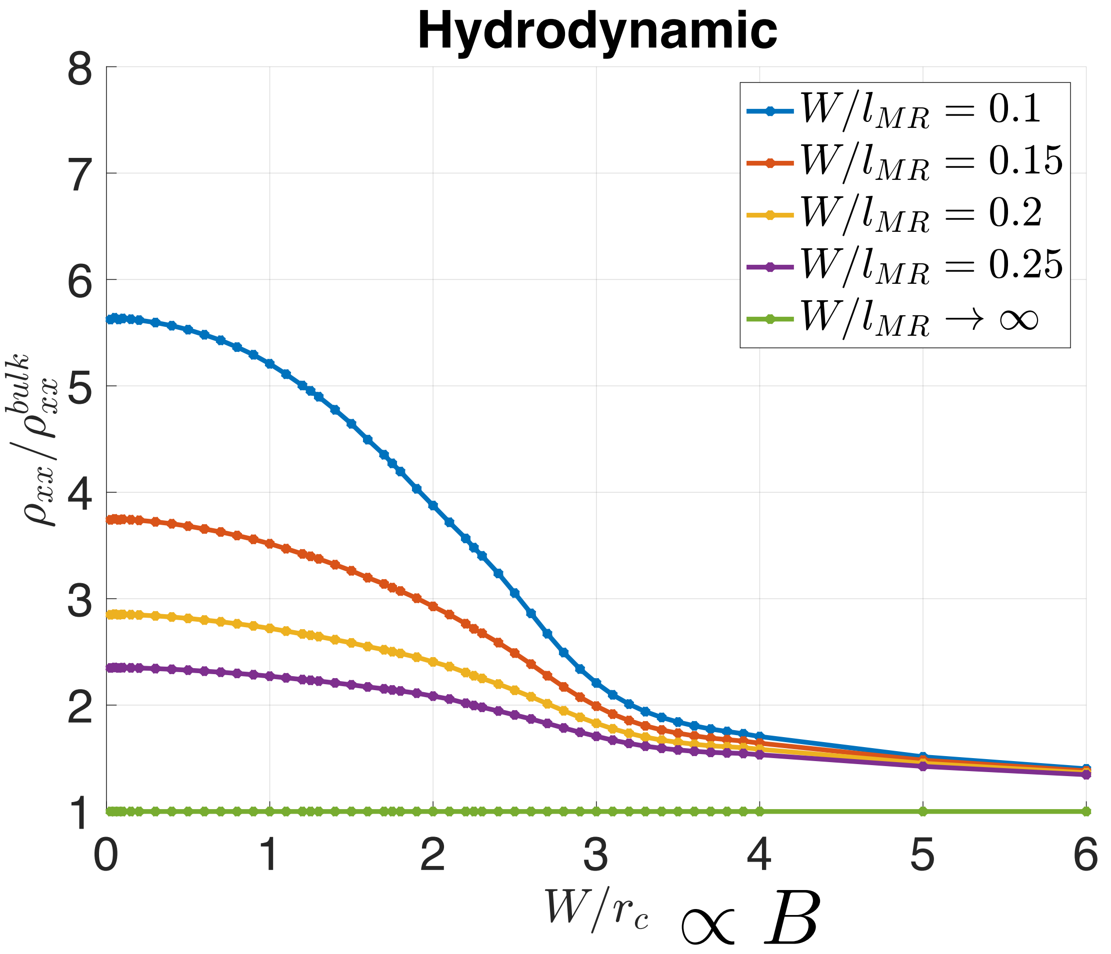
Results of Boltzmann-hydro: magnetoresistance

$W \ll l_{MR}, l_{MC}$



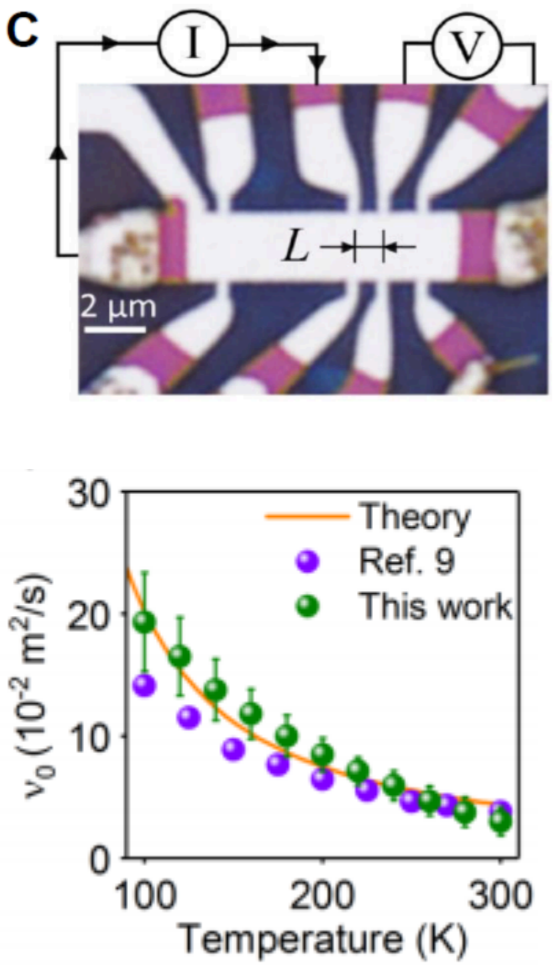


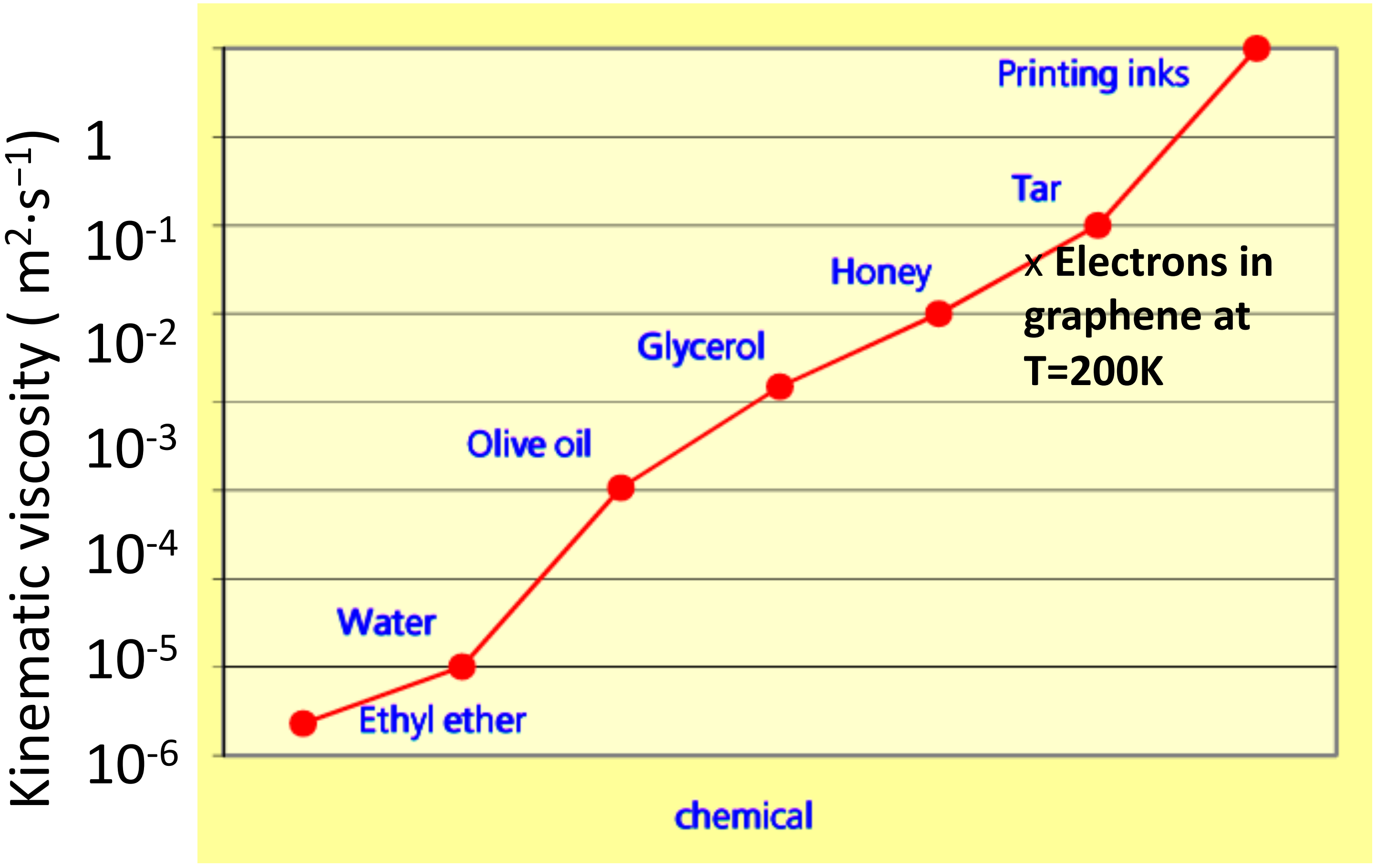
$l_{MC} \ll W \ll l_{MR}$



Measurement of shear viscosity in graphene

[Bandurin et al Science (2016)] [Berdyugin et al, Science 2018]

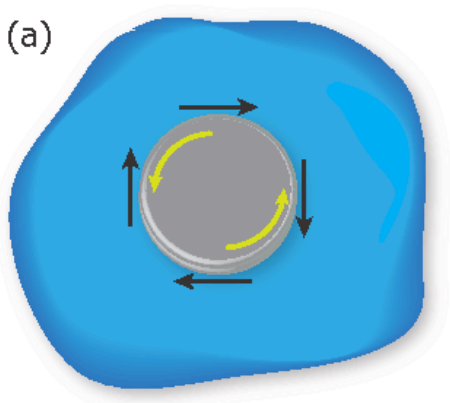




A tale of two viscosities

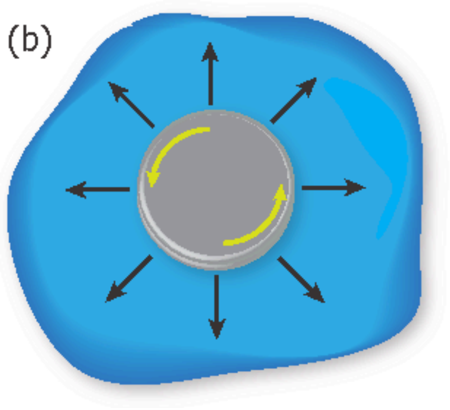
$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m} (\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity



Probed by ρ_{xx}

Hall viscosity

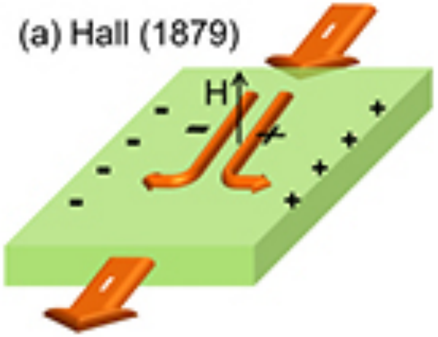


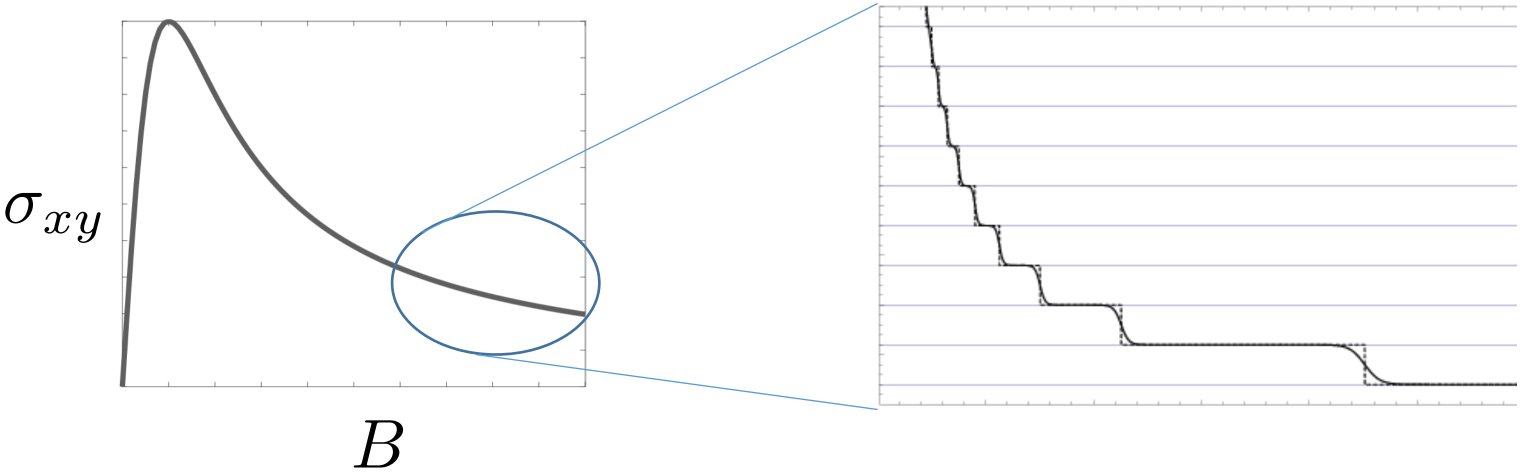
Probed by ρ_{xy}

Historical perspective on Hall effect

Classical Hall Effect [Hall 1879]

Quantum Hall Effect [von Klitzing 1980]

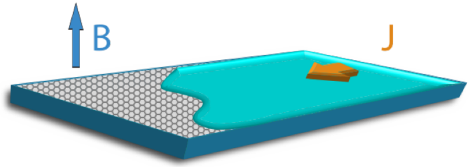


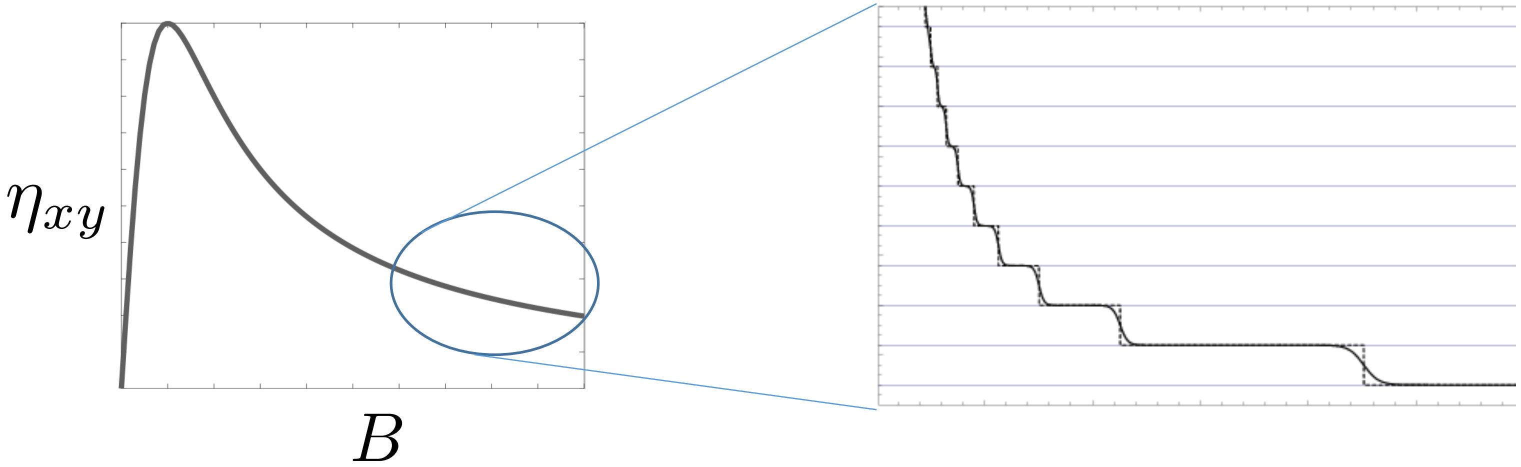


$$\sigma_{xy} = \nu \frac{e^2}{h}$$

Classical Viscous Hall Effect

Quantum Viscous Hall Effect

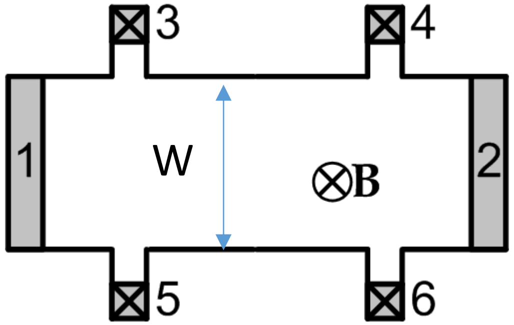




$$\eta_{xy} = \nu n \frac{\hbar}{4}$$

[TS et al, PRL ‘17]

Hall resistivity => Hall viscosity



$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} + \eta_{xy} \nabla^2 \vec{v} \times \vec{z} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B})$$

$$\rho_{xy} = \rho_{xy}^{bulk} \left(1 - \eta_{xy} \frac{12}{W^2} \frac{1}{\omega_c}\right)$$

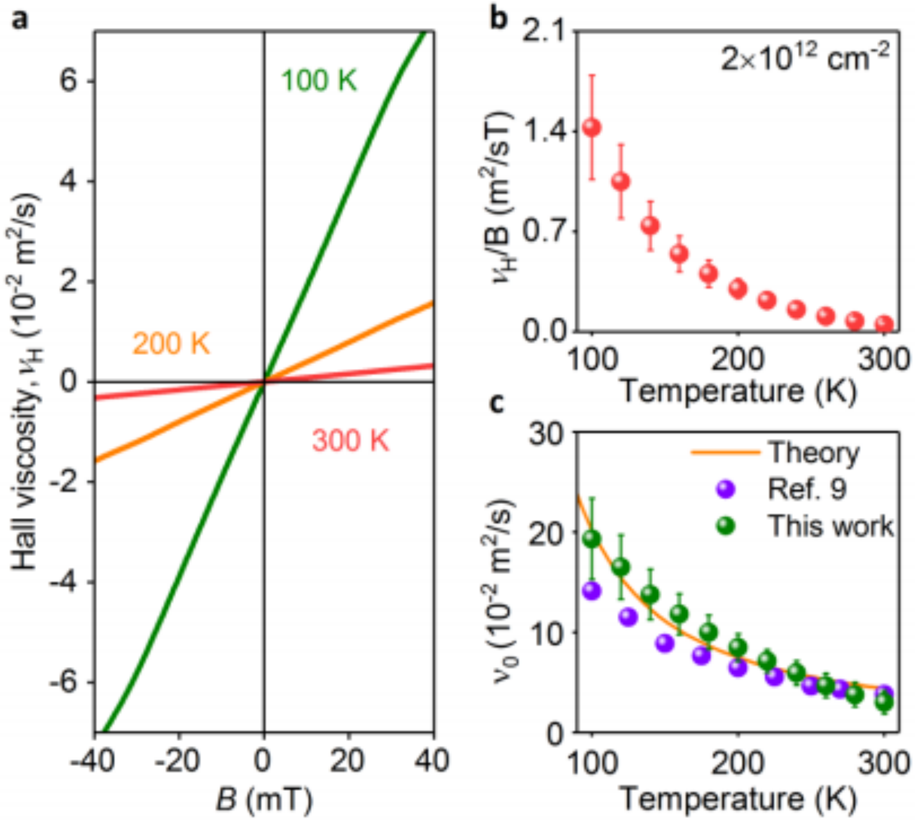
$$\rho_{xy}^{\text{bulk}} = \frac{B}{ne}$$

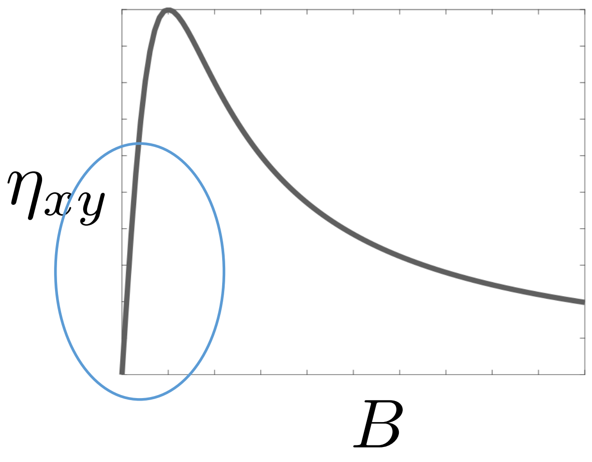
Hall viscosity can be measured by looking at finite-size effects in Hall resistivity

Measuring Hall Viscosity of Graphene's Electron Fluid Science 2018

A. I. Berdyugin, S. G. Xu, F. M. D. Pellegrino, R. Krishna Kumar, A. Principi, I. Torre, M. Ben Shalom, T. Taniguchi, K. Watanabe, I. V. Grigorieva, M. Polini, A. K. Geim, D. A. Bandurin

(Submitted on 5 Jun 2018)

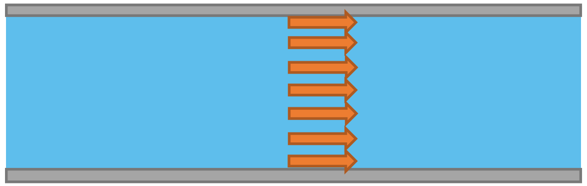




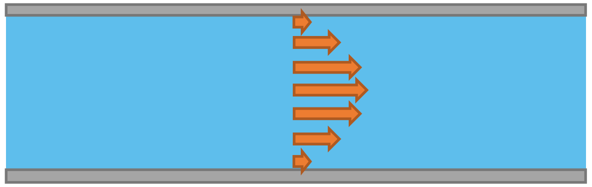
Last part of the talk: local properties

- So far we've only discussed quantities that are averaged over the cross section of the channel
- What about their spatial dependence? Any smoking gun features of hydro?
- Naïvely, yes:

Ohmic



Hydro

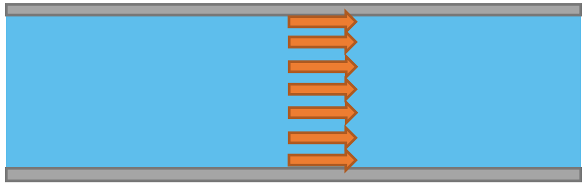


Last part of the talk: local properties

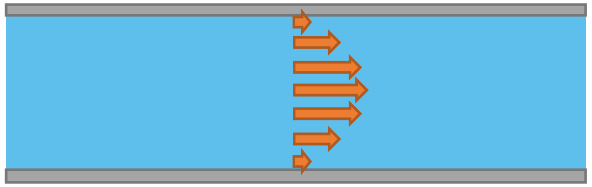
- So far we've only discussed quantities that are averaged over the cross section of the channel
- What about their spatial dependence? Any smoking gun features of hydro?
- Naïvely, yes:

But:

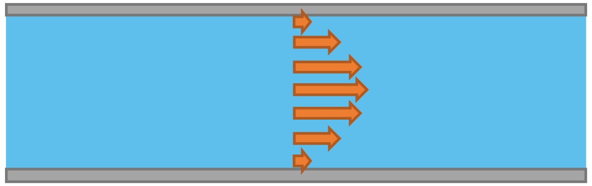
Ohmic



Hydro



Ballistic

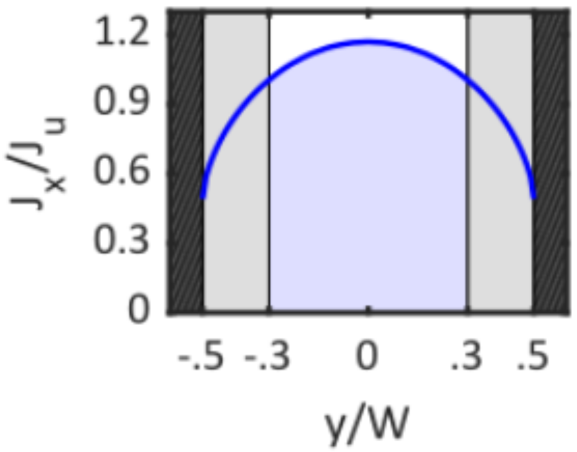


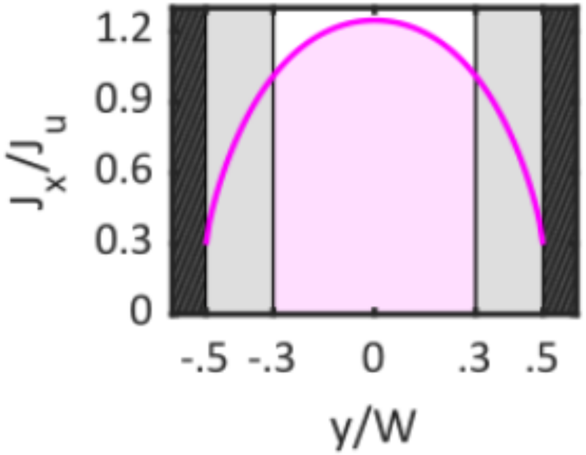
Magnetic fields are useful again:

Ballistic

Hydro

Current density



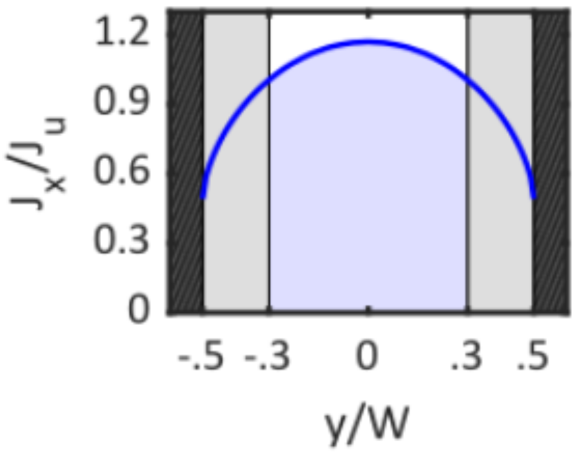


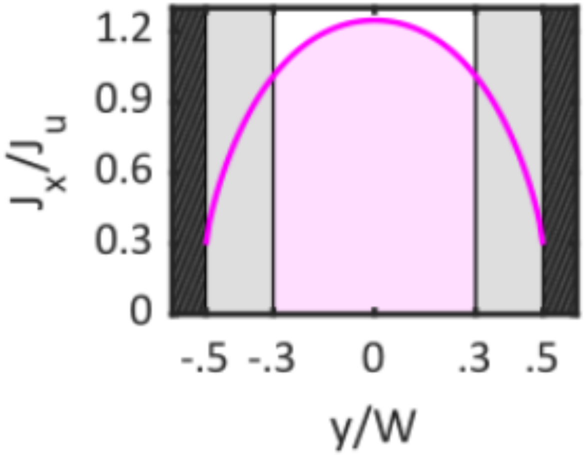
Magnetic fields are useful again:

Ballistic

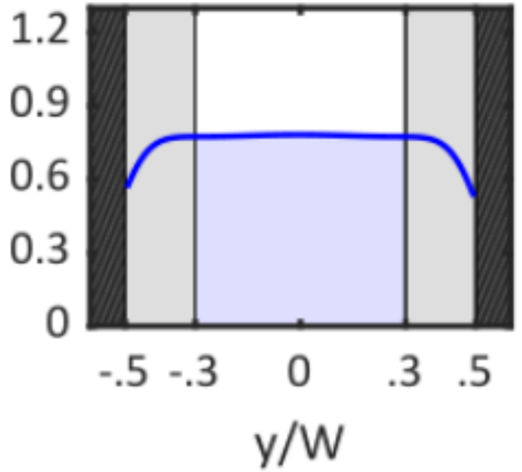
Hydro

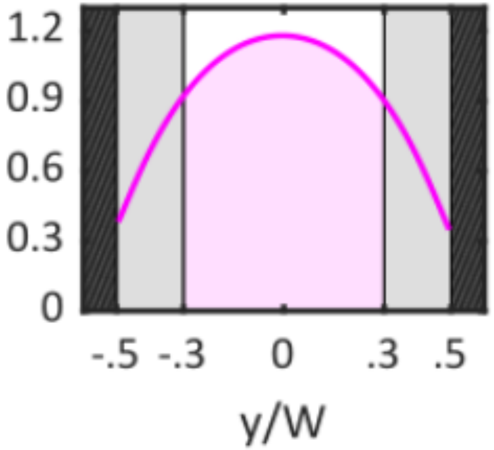
Current density





Hall electric field



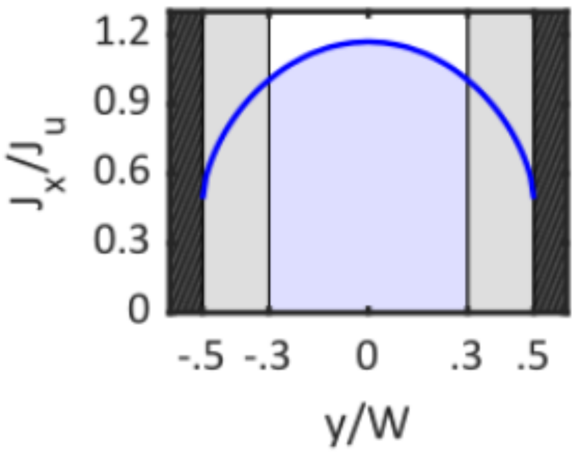


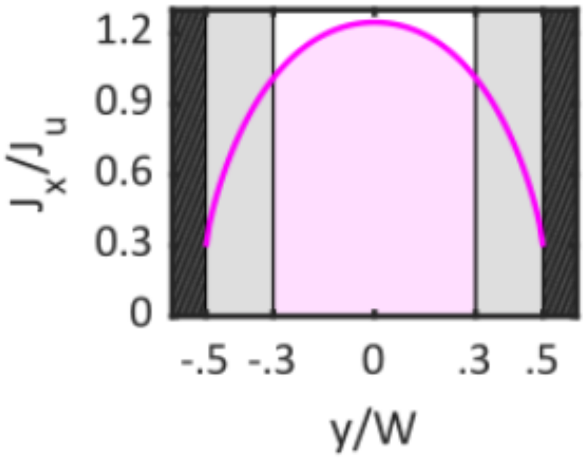
Magnetic fields are useful again:

Ballistic

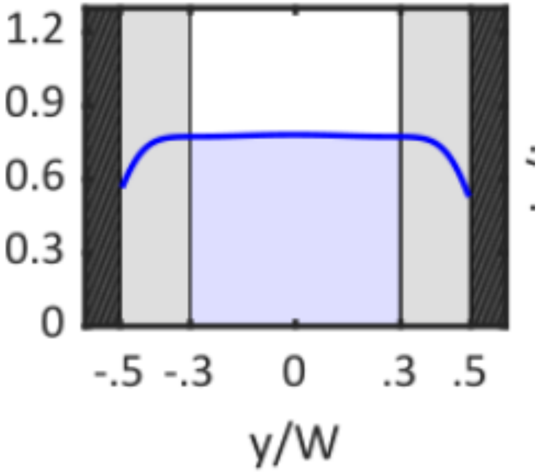
Hydro

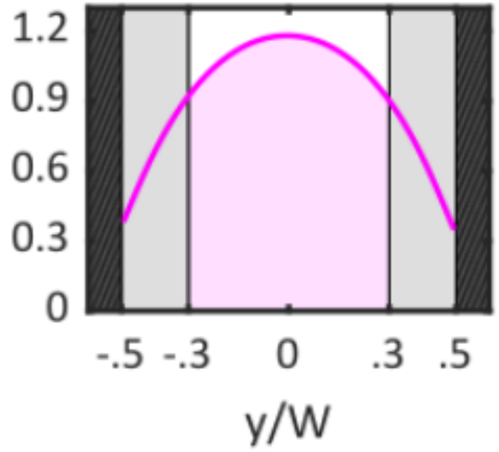
Current density





Hall electric field



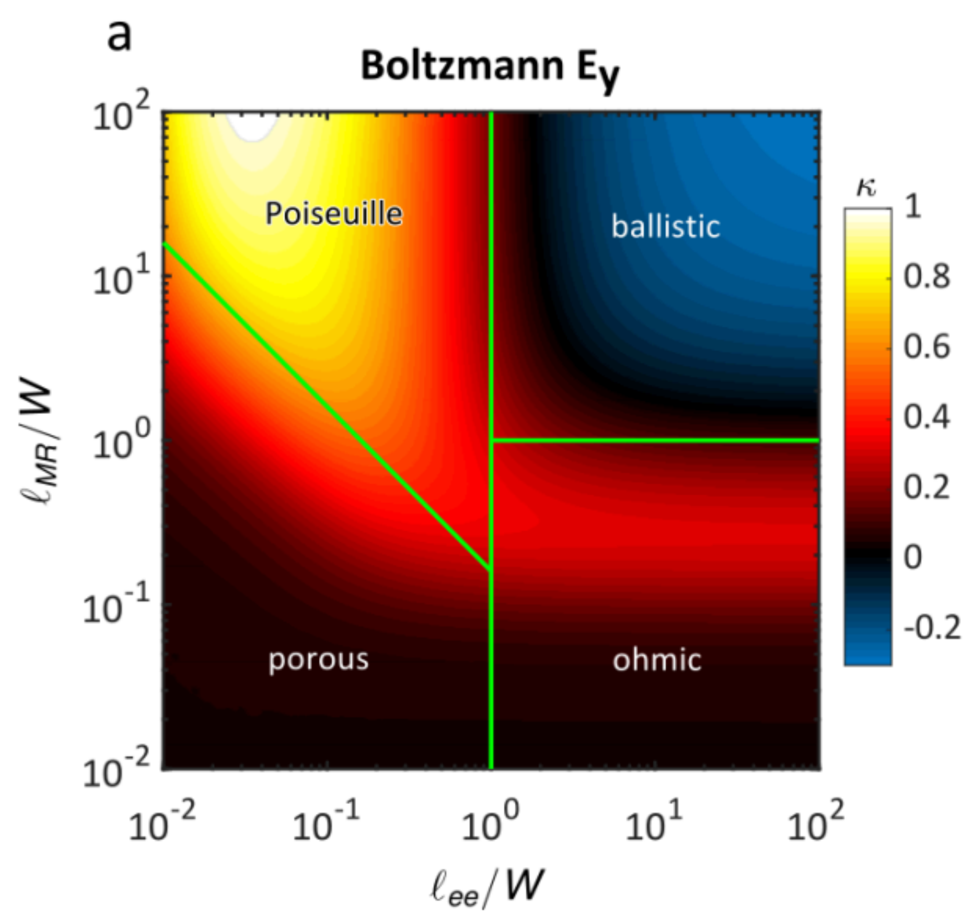


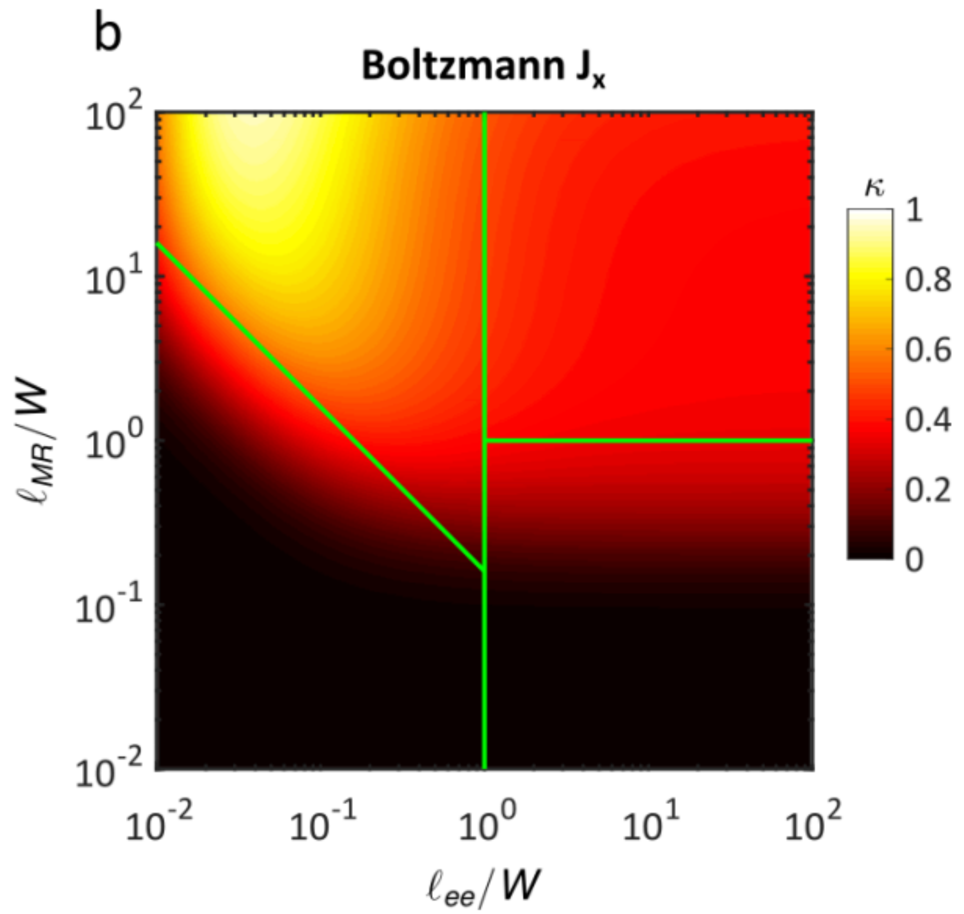
$$E_y = \frac{B}{ne}\left(j_x + \frac{1}{2} l_{ee}^2 \partial_y^2 j_x\right)$$

“Phase diagram” based on profile curvature κ

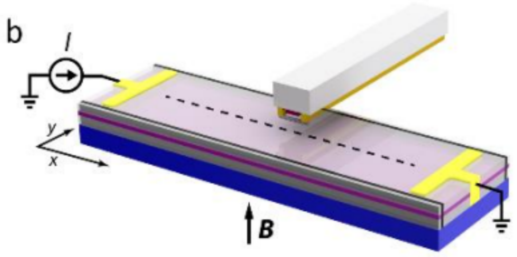
Hall electric field

Current



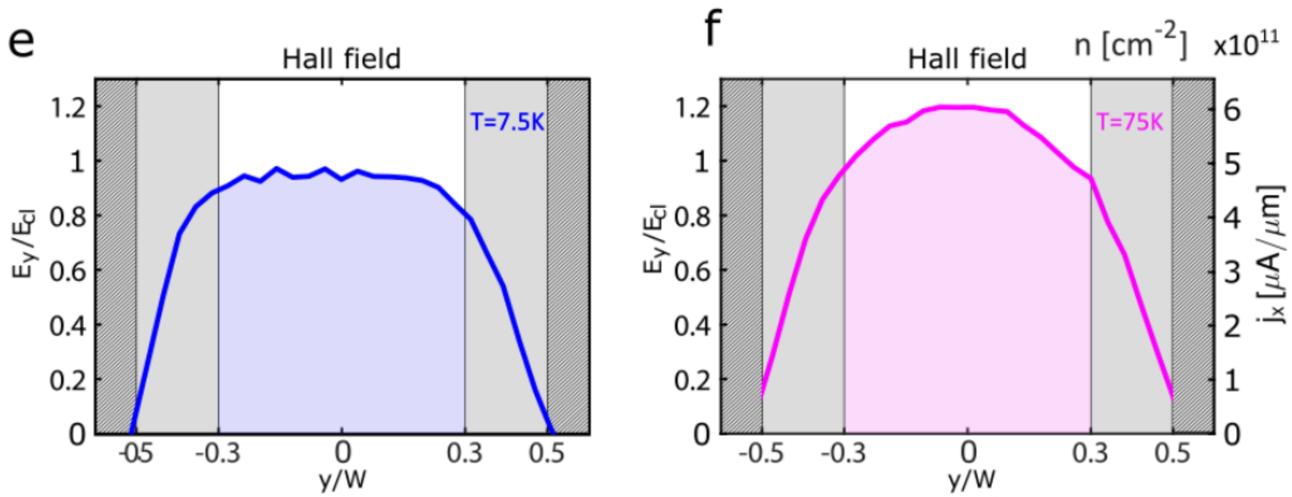


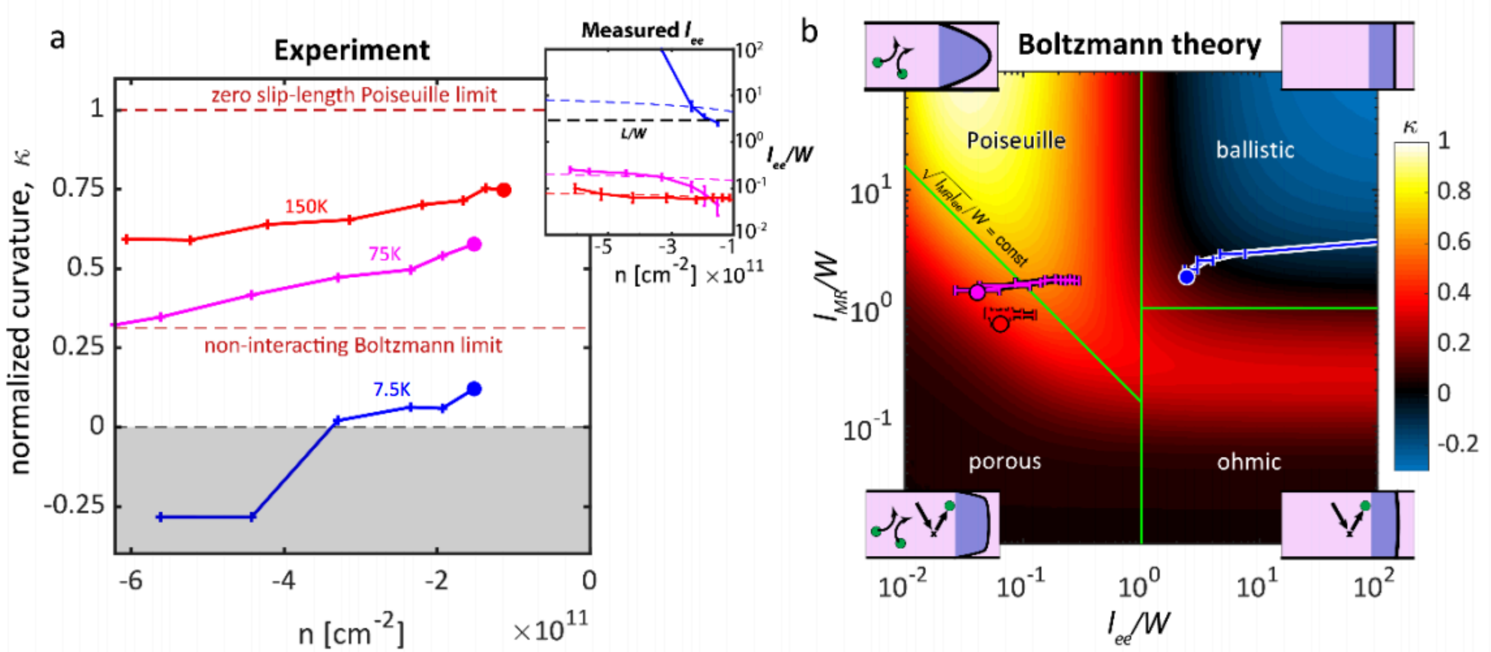
First visualization of hydrodynamic flow of electrons



Single-electron transistor

[J.A. Sulpizio[1†], L. Ella[1†], A. Rozen[1†], J. Birkbeck[2,3], D.J. Perello[2,3], D. Dutta[1], M. Ben-Shalom[2,3,4], T. Taniguchi[5], K. Watanabe[5], T. Holder[1], R. Queiroz[1], A. Stern[1], TS, A.K. Geim[2,3], and S. Ilani, **arXiv:1905.11662,** to appear in Nature (2019)]





References

[TS, Nandi, Schmidt, Mackenzie, and Moore, **Phys. Rev. Lett. 118, 226601**]

[Holder, Queiroz, TS, Silberstein, Rozen, Sulpizio, Ella, Ilani, Stern, **arXiv:1901.08546**]

[J.A. Sulpizio[1†], L. Ella[1†], A. Rozen[1†], J. Birkbeck[2,3], D.J. Perello[2,3], D. Dutta[1], M. Ben-Shalom[2,3,4], T. Taniguchi[5], K. Watanabe[5], T. Holder[1], R. Queiroz[1], A. Stern[1], TS, A.K. Geim[2,3], and S. Ilani, **arXiv:1905.11662**, to appear in Nature (2019)]

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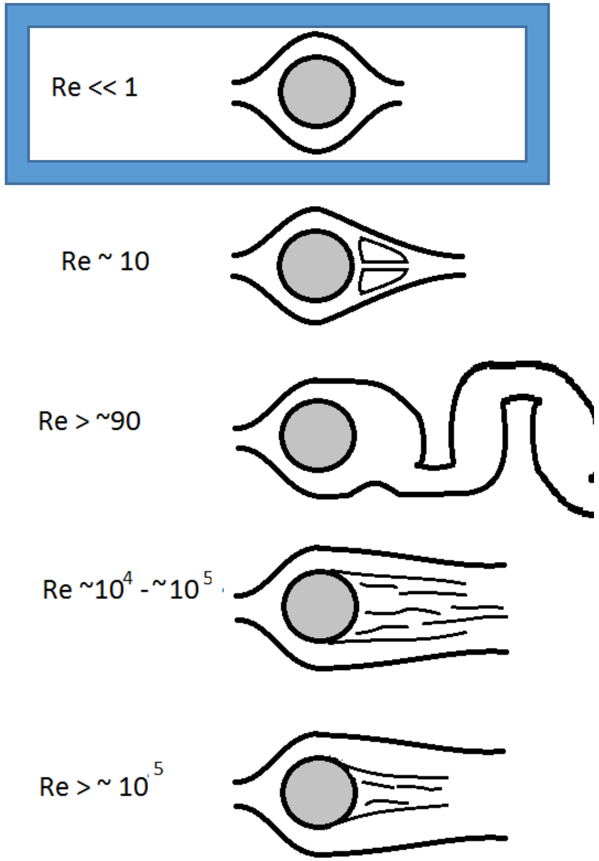
- Weizmann institute: Tobias Holder, Raquel Queiroz, Navot Silberstein, Asaf Rozen, Joseph A. Sulpizio, Lior Ella, Shahal Ilani, Ady Stern

Thanks for your attention!

Reynolds number?

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e\vec{E}}{m}$$

$$Re = \frac{v_{\text{drift}} W}{\eta} \leq \frac{v_{\text{drift}}}{v_F} \times \frac{l_{e-imp}}{l_{ee}} \ll 1$$

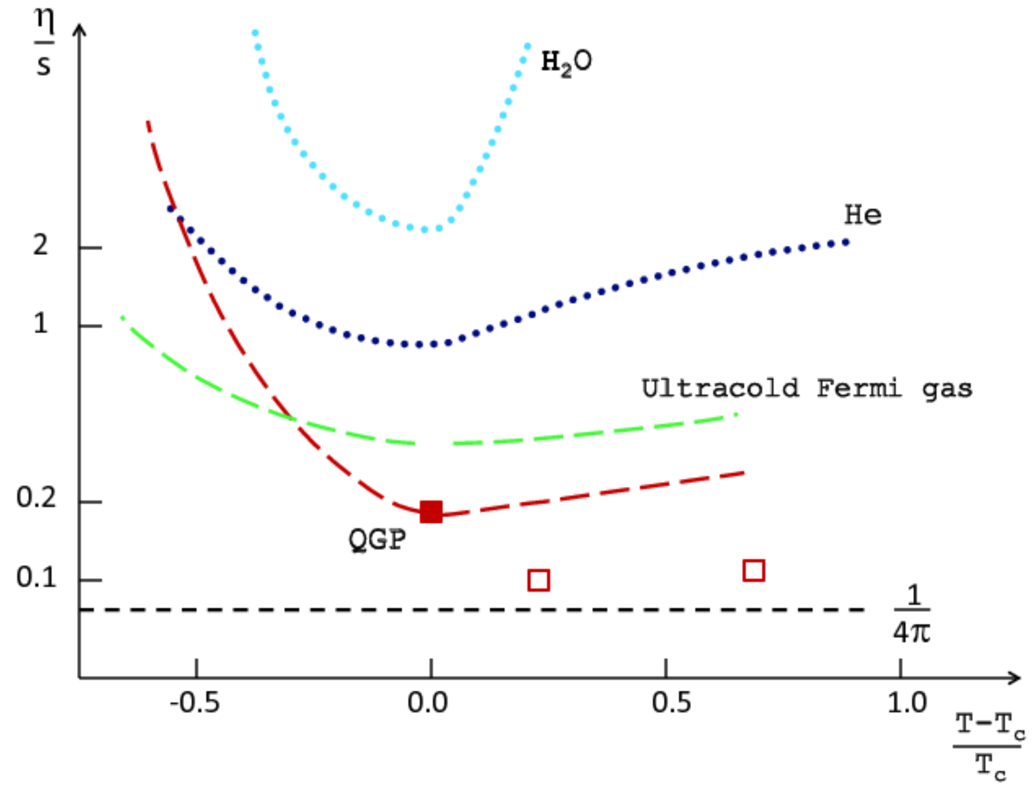


[Kovtun, Son, Starinets, PRL 2005]

Bound on viscosity

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

- Three arguments
  - Class of strongly interacting QFTs with gravity dual saturate this bound
  - Empirical evidence:
  - Extrapolation of kinetic theory



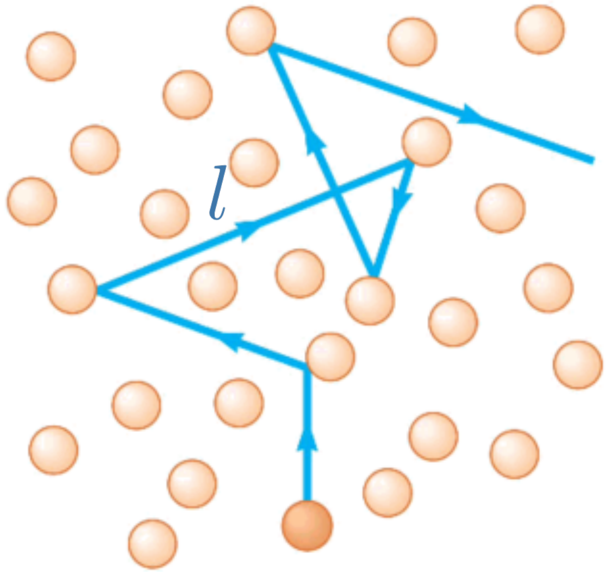
Bound on viscosity

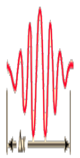
$$\frac{\eta}{s} \sim \frac{mvl}{k_B} \sim \frac{\hbar}{k_B}\frac{l}{l_{dB}}$$

$$\eta \sim nmvl$$

$$s \sim k_B n$$

$$p = mv \sim \frac{\hbar}{l_{dB}}$$





l_{dB}

Bound on viscosity

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\frac{l}{l_{dB}}$$