Phase Diagram of QCD

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QCD – the theory of strong interactions

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  changing color has no observable consequence
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What force holds quarks together in hadrons? It weakens if the interaction is brief or if quarks are close to each other \( (r \ll R_{\text{proton}} \sim 10^{-13} \text{ cm} \equiv 1 \text{ fermi}) \).

\[
\alpha_{\text{strong}}(r) = \frac{F r^2}{\hbar c} \rightarrow 0 \quad r \rightarrow 0
\]

Compare to E.M.: \( \alpha = 1/137 \) (and only grows as \( r \rightarrow 0 \)). In contrast \( \alpha_{\text{strong}}(1 \text{ fermi}) \sim 1 \) and \( \rightarrow 0 \) as \( r \rightarrow 0 \).
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QCD – asymptotically free field theory

- Only one such class of theories: Non-abelian gauge theories. (QED is abelian gauge theory.)
- Bonus: it involves a hidden symmetry (in QED – U(1) – phase rotation). In QCD it is the color symmetry – SU(3).
- QCD is a quantum field theory:

\[
S = \int d^4 x \left[ \sum_{f=1}^{N_f} \bar{q}_f \left( i\gamma^\mu \frac{\partial}{\partial x^\mu} + g A^\mu - m_f \right) q_f - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} \right];
\]

- Predictive power
QCD thermodynamics

Applications:
- Neutron stars (large density, low $T$)
- Heavy-ion collisions (large $T$, large density)

QCD allows first principle calculations
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Questions: phases, phase diagram, as function of $T$, $\mu_B$, ... 

Early expectations $\Rightarrow$

Natural scale:

\[ kT \sim \frac{\hbar c}{1\text{fm}^2} = 0.2 \text{ GeV}. \]

\[ (T \sim 10^{12} \text{ K}) \]

or

\[ \rho_B \sim 1\text{fm}^{-3}. \]

\[ \begin{align*}
T, \text{GeV} & \quad r \sim 1/T \to 0 \\
K & \sim T \\
U & \sim \alpha_s/r \\
U/K & \sim \alpha_s \ll 1 \quad \text{Asymptotic freedom}
\end{align*} \]

QGP

$\begin{align*}
U & \sim \alpha_s/r \\
K & \sim \mu \\
r & \sim 1/\mu \to 0 \\
\mu_B, \text{GeV} & \quad \text{quark matter (Fermi gas)}
\end{align*} \]
Contemporary view

“Minimal” phase diagram
Contemporary view

“Minimal” phase diagram
Contemporary view

QGP

Lattice simulations

Heavy ion collisions

E critical point

Vacuum

0.1

T, GeV

0

μ_B, GeV

Only models

Empirical nuclear physics

Quark matter

Nuclear matter

Neutron stars, quark stars

α_s ≪ 1

“Minimal” phase diagram
Role of chiral symmetry ($m_q = 0$)

$q \equiv \left( \begin{array}{c} u \\ d \end{array} \right)$  Chiral: \[
\left( \begin{array}{c} u_L \\ d_L \end{array} \right) \rightarrow e^{+i\alpha \cdot \tau} \left( \begin{array}{c} u_L \\ d_L \end{array} \right), \quad \left( \begin{array}{c} u_R \\ d_R \end{array} \right) \rightarrow e^{-i\alpha \cdot \tau} \left( \begin{array}{c} u_R \\ d_R \end{array} \right)
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Order parameter:

\[
\langle \bar{q}q \rangle = \langle q_L^\dagger q_R + \text{h.c.} \rangle
\]

- non-zero in one phase (in vacuum),
- exactly zero in another phase — protected by symmetry!

Compare ferromagnet: for $T < T_c$ $\langle M \rangle \neq 0$, while for $T > T_c$ $\langle M \rangle = 0$. 

Phase Diagram of QCD – p.6/16
Role of chiral symmetry \((m_q = 0)\)

\[ q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{Chiral:} \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow e^{+i\alpha \cdot \tau} \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow e^{-i\alpha \cdot \tau} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \]

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Compare ferromagnet: for \(T < T_c \langle M \rangle \neq 0\), while for \(T > T_c \langle M \rangle = 0\).
This requires singularity \(\equiv\) phase transition
Compare ferromagnet at $H \neq 0$:

no phase transition, but crossover
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Why all phases (can be) connected? Because $\not\exists$ order parameters:

no chiral symmetry ($m_q \neq 0$);

No phase boundary does not mean same physics, of course – (compare water-vapor, gas-plasma, . . .) Critical point in many liquids – critical opalescence
Compare ferromagnet at $H \neq 0$:

no phase transition, but crossover

(Bielefeld group, lattice Monte Carlo)
Large $\mu$, color superconductivity and CFL

- Asymptotic freedom $\Rightarrow \alpha_s(\mu) \to 0$.

- Quarks of “different color” (color antisymmetric state) attract. Fermi sphere is unstable towards condensation of quark pairs (Cooper).

- The simplest and favorable is (for 2 flavors) $u_R d_R$.

- Does not break chiral symmetry (unlike $\bar{q} q = q_L^\dagger q_R + h.c.$).
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- However, for 3 flavors, analogous SU(3)$_{\text{chiral}}$ is broken. $X = u_R d_R + s_R u_R + d_R s_R$ is not flavor singlet.
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- However, for 3 flavors, analogous $\text{SU}(3)_{\text{chiral}}$ is broken. $X = u_Rd_R + s_Ru_R + d_RS_R$ is not flavor singlet.

- But flavor transformation on $X$ can be undone by a color transformation, which is “invisible”. However, there is also $Y = u_Ld_L + \ldots$, and although it can also be undone by a color rotation...

- the simultaneous transformations of $q_L$ and $q_R$ do change the ground state if they are not equal. This is the broken chiral symmetry.

Ground state locks color and flavor: CFL (Alford, Rajagopal, Wilczek)
Phase Diagram of QCD

- QGP
- Vacuum
- Nuclear matter
- Quark matter
- CFL
- Critical point E
Location of the CP (theory)
Event
Heavy-ion collisions and the phase diagram

(from Braun-Munzinger, Redlich, Stachel)
Locating the QCD critical point experimentally

Energy scan.

Signatures: event-by-event fluctuations.

Susceptibilities diverge $\Rightarrow$ fluctuations grow towards the critical point.
Near the critical point (for CERES acceptance):

\[ \sim 2\% \times \left( \frac{G}{300 \text{ MeV}} \right)^2 \left( \frac{\xi_\sigma}{3 \text{ fm}} \right)^2 \]

\( (\xi_\sigma = 1/m_\sigma) \)

Signal one is looking for: non-monotonic dependence on \( \sqrt{s} \).
Summary and conclusions

Phase diagram of QCD is a challenging, long-standing problem.

$\alpha_s \ll 1$ calculations can address regions of $T, \mu \gg 1 \text{fm}^{-1}$. Many interesting phenomena.

The most interesting region is still out of reach of controllable calculations.

Existence of the critical point on the QCD phase diagram is suggested by many theoretical approaches, including, recently, lattice QCD.

Heavy ion experiments can discover the critical point – by measuring fluctuation observables as a function of collision parameters.

Search is underway.

Needed:

Theory: a reliable (systematically improvable) method to study $T\mu$ plane and locate CP.

Experiment: scan of the QCD phase diagram — $\sqrt{s}$ scan.