Phase Diagram of QCD

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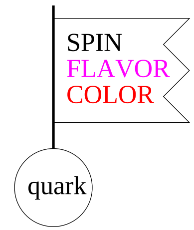

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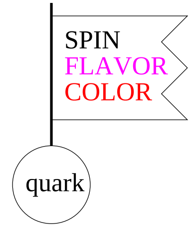

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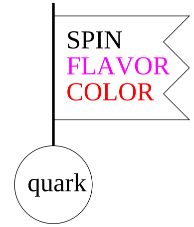

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- What force holds quarks together in hadrons? It weakens if the interaction is brief or if quarks are close to each other ($r \ll R_{\text{proton}} \sim 10^{-13}$ cm $\equiv 1$ fermi).

$$\alpha_{\text{strong}}(r) = \frac{Fr^2}{\hbar c} \stackrel{r\to 0}{\longrightarrow} 0$$

Compare to E.M.: $\alpha = 1/137$ (and only *grows* as $r \to 0$).
In contrast $\alpha_{\text{strong}}(1\text{fermi}) \sim 1$ and $\to 0$ as $r \to 0$.

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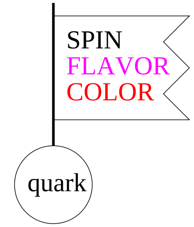

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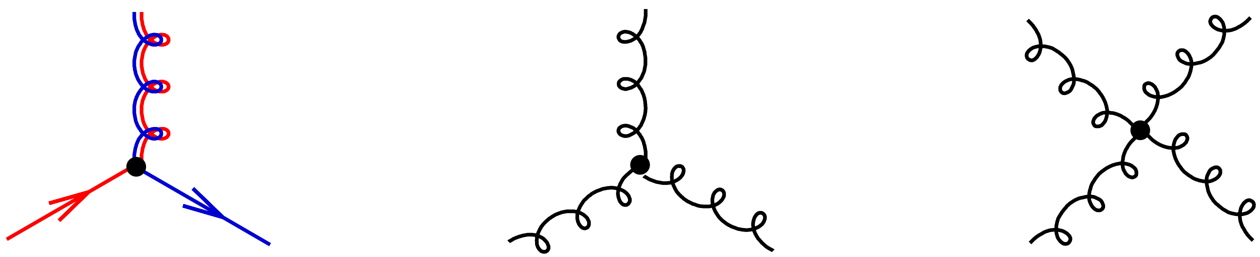
QCD – asymptotically free field theory

- Only one such class of theories: Non-abelian gauge theories. (QED is abelian gauge theory.)
- Bonus: it involves a hidden symmetry (in QED – U(1) – phase rotation). In QCD it is the color symmetry – SU(3).
- QCD is a quantum field theory:

$$S = \int d^4x \left[\sum_{f=1}^{N_f} \bar{q}_f \left(i\partial\!\!\!/ + gA\!\!\!/ - m_f \right) q_f \; - \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} \right];$$



- Predictive power

QCD thermodynamics

- Applications:
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  - Neutron stars (large density, low T)
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- QCD allows first principle calculations
- Questions: phases, phase diagram, as function of T, μ_B, ...
- Early expectations $\Rightarrow$

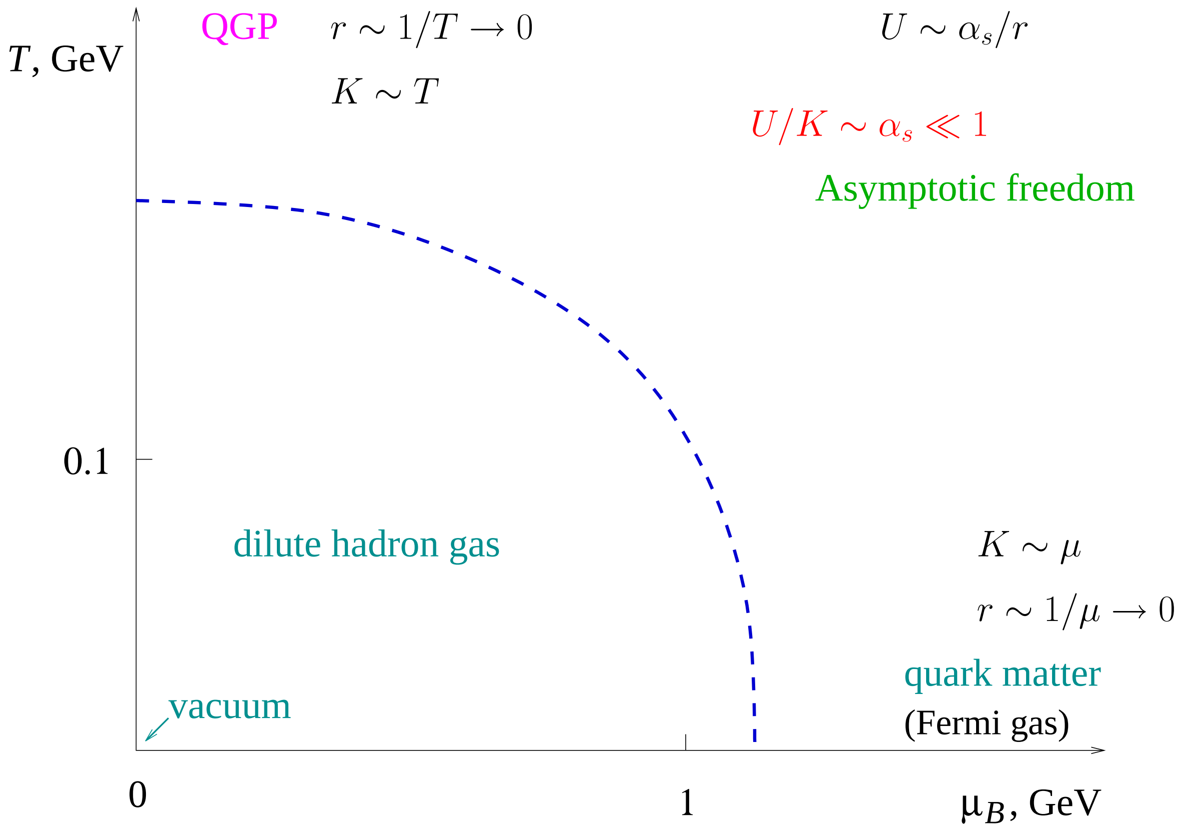
  Natural scale:

$$kT \sim \frac{\hbar c}{1\text{fm}^2} = 0.2\ \text{GeV}.$$

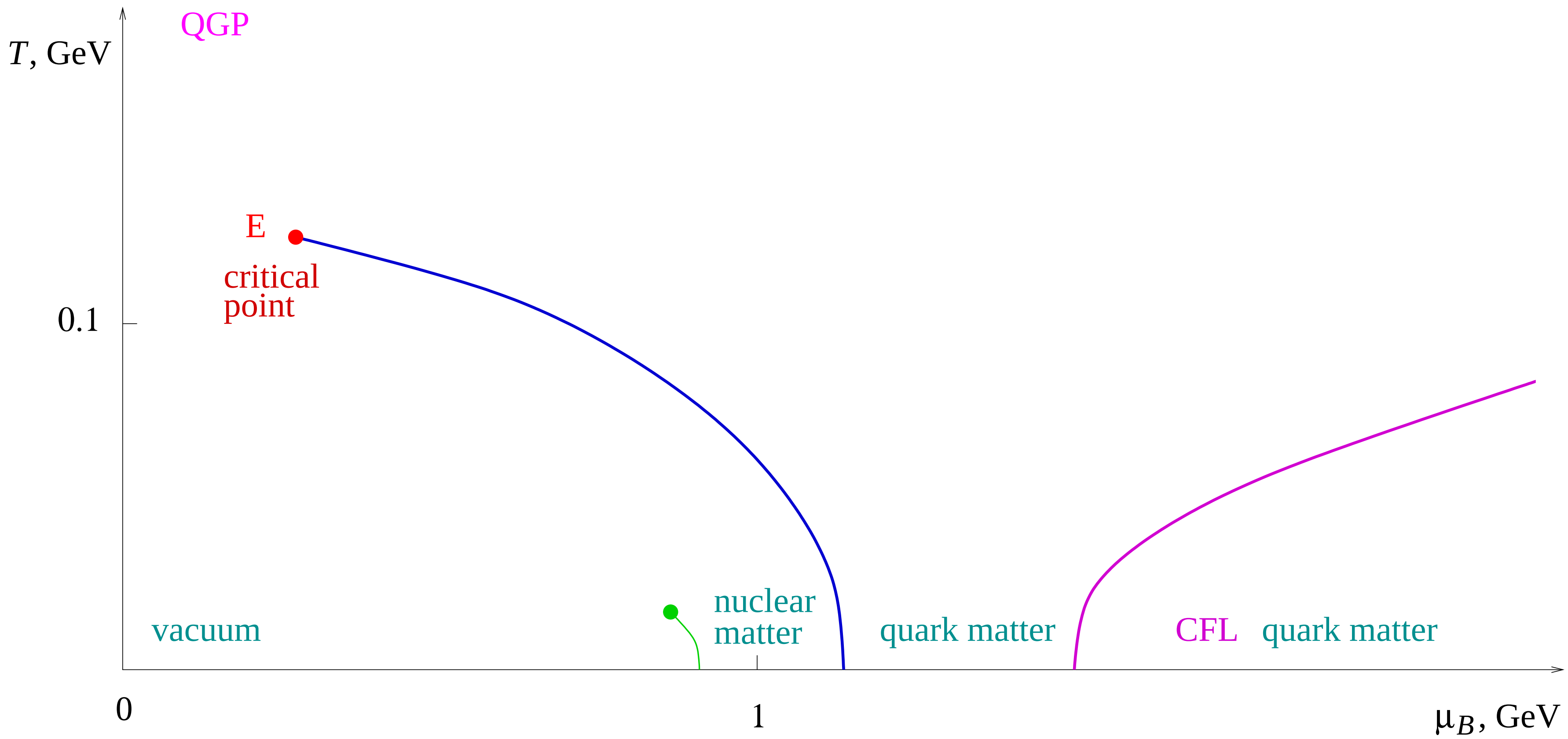
$$(T \sim 10^{12} K)$$

or

$$\rho_B \sim 1\text{fm}^{-3}.$$



Contemporary view



“Minimal” phase diagram

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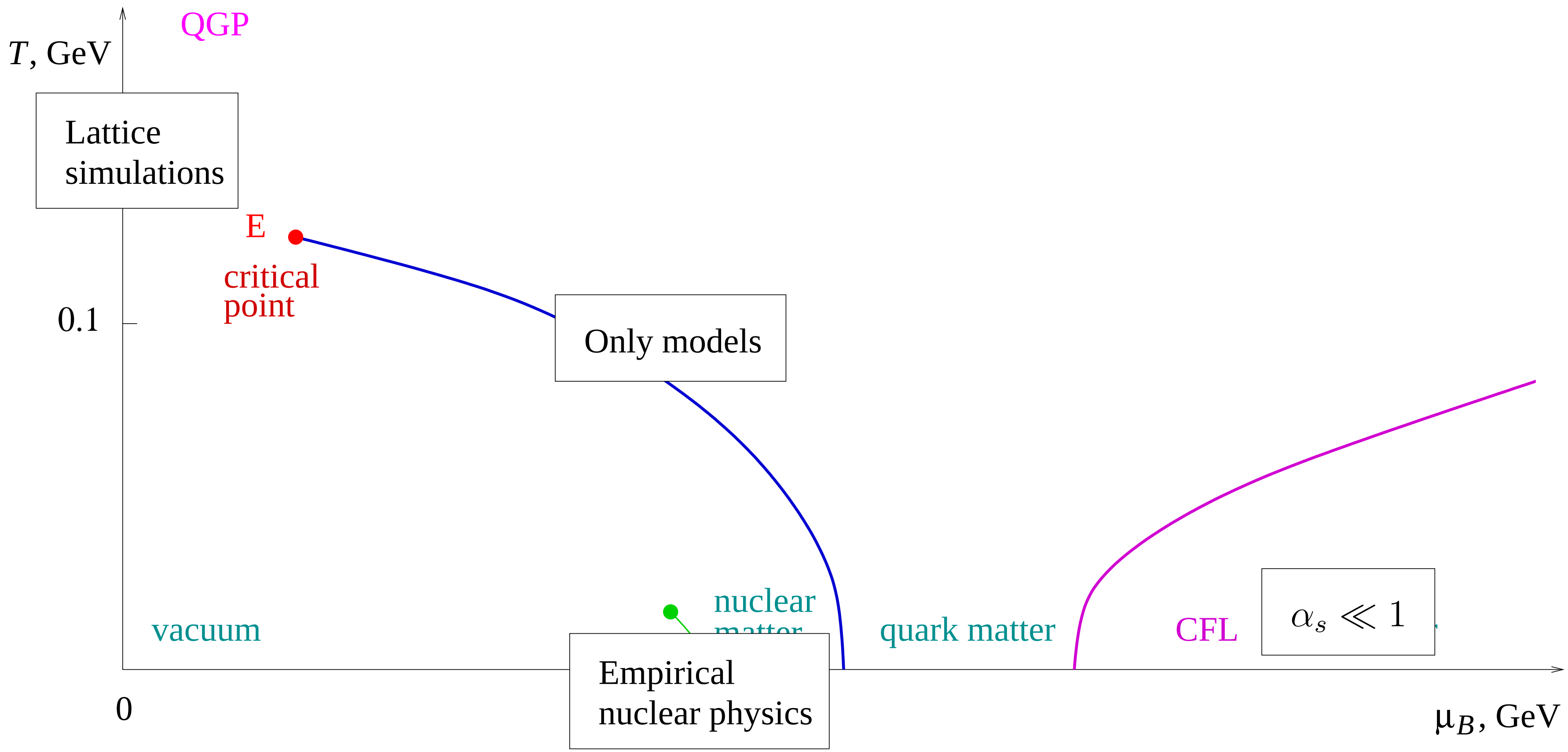

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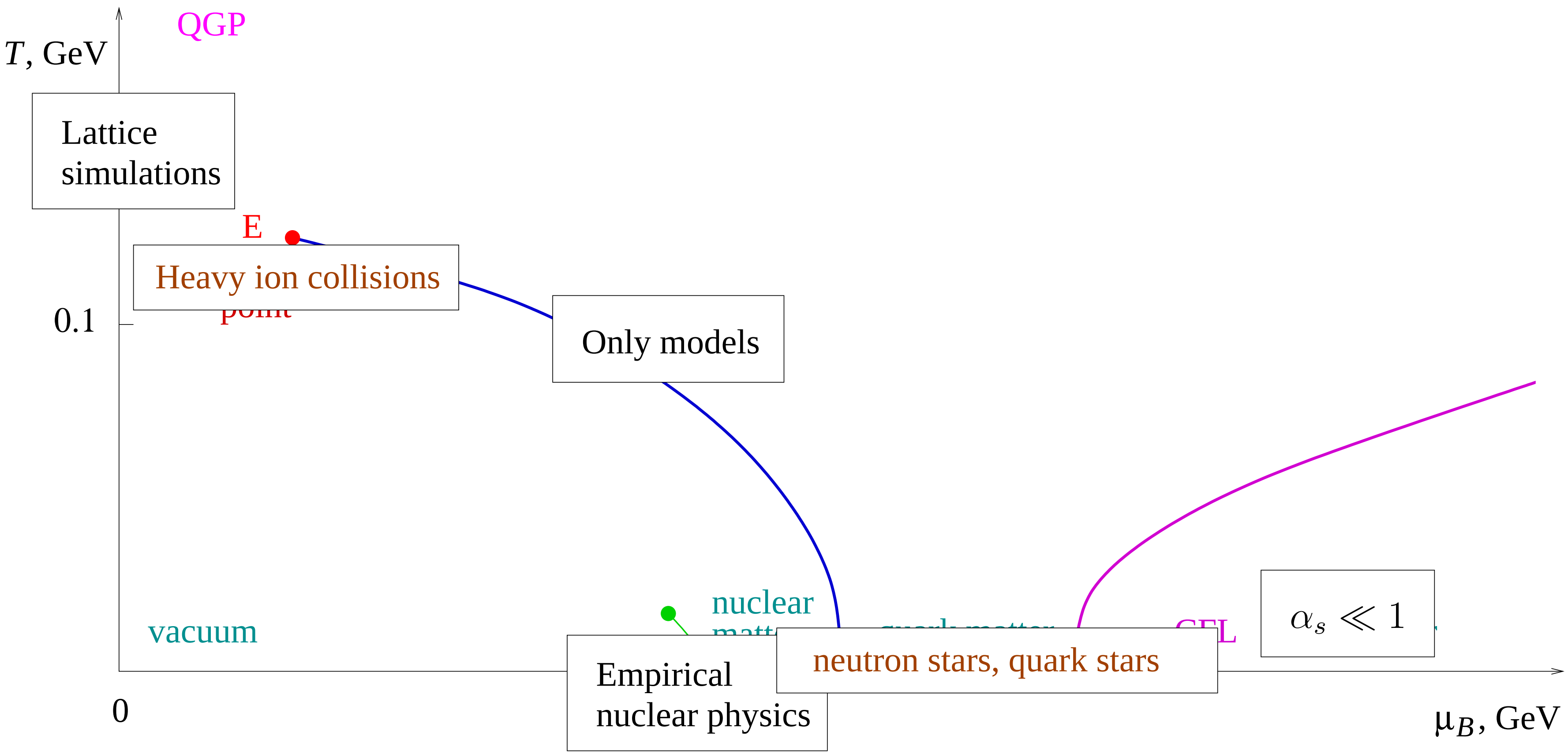

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Role of chiral symmetry ($m_q = 0$)

$q \equiv \begin{pmatrix} u \\ d \end{pmatrix}$ Chiral: $\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow e^{+i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, $\begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}} \begin{pmatrix} u_R \\ d_R \end{pmatrix}$

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Order parameter:
$\langle \bar{q}q \rangle = \langle q_L^\dagger q_R + \mathrm{h.c.} \rangle$

- non-zero in one phase (in vacuum),
- *exactly* zero in another phase — protected by symmetry!

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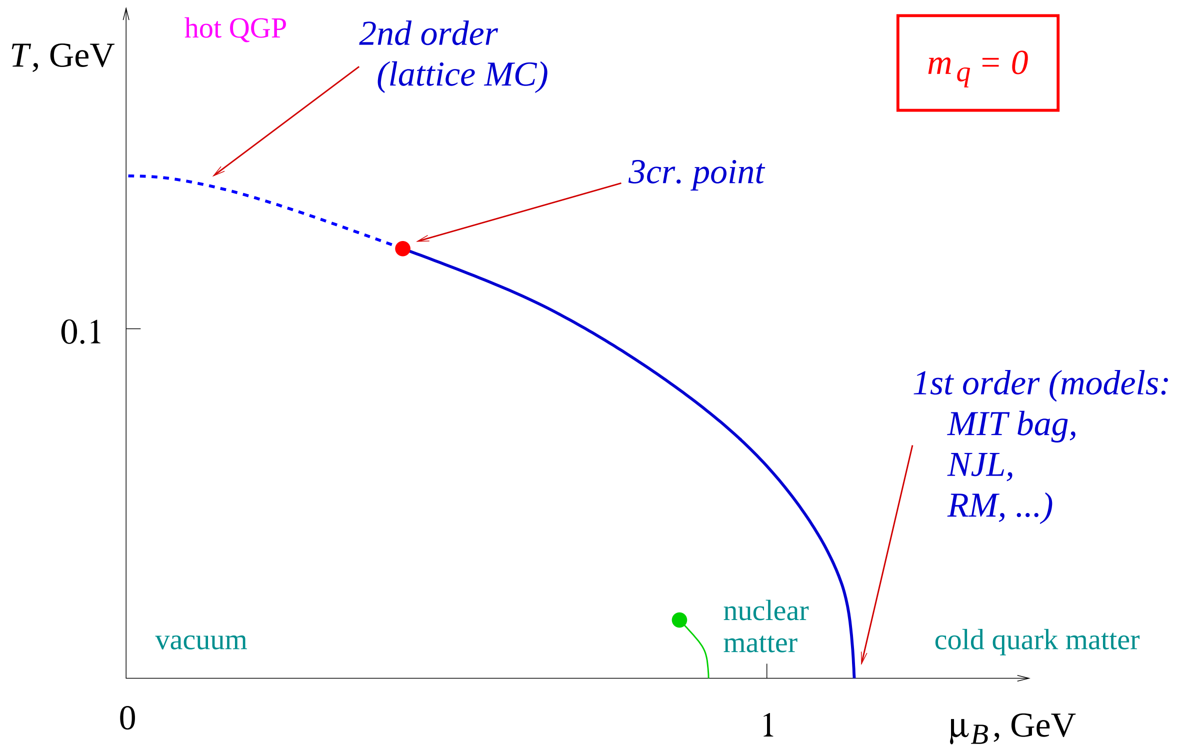
This requires singularity $\equiv$ phase transition



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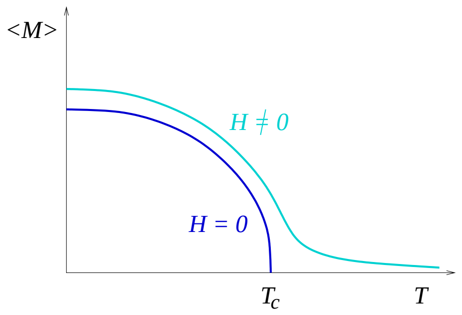
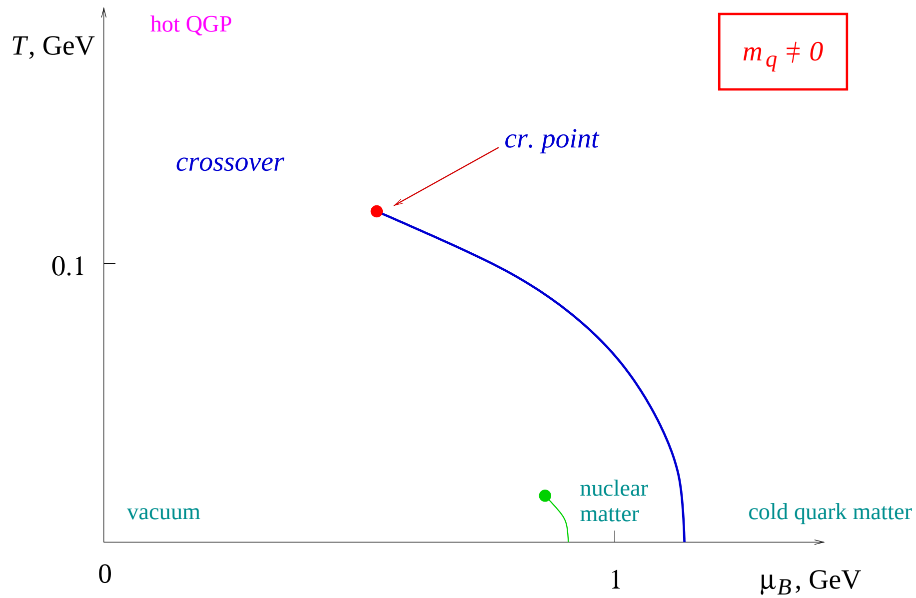
Compare ferromagnet at $H \neq 0$:



no phase transition, but crossover



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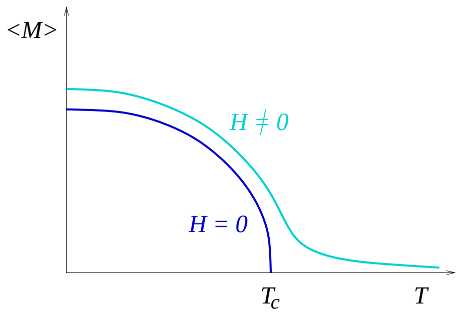

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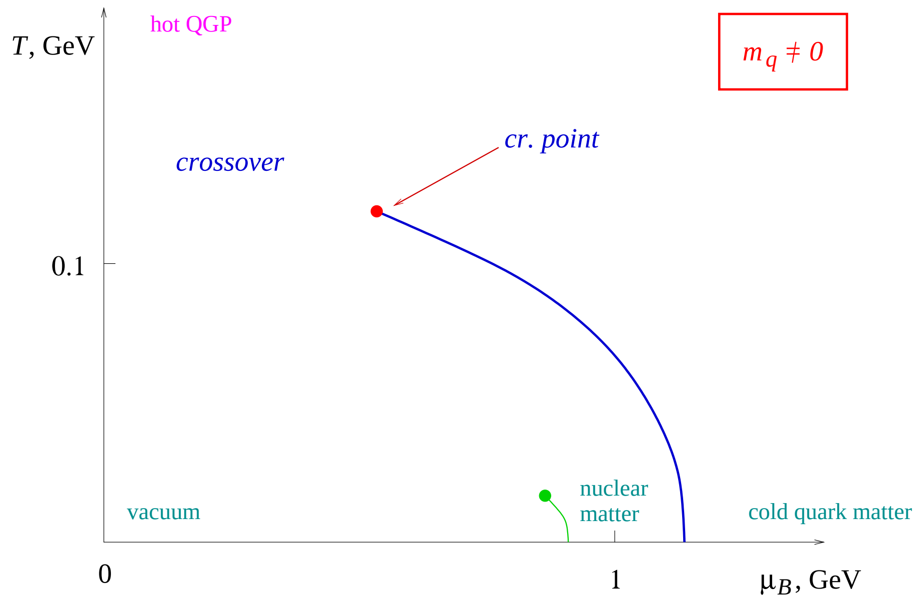

- Why all phases (can be) connected? Because $\nexists$ order parameters:
  no chiral symmetry ($m_q \neq 0$);
  ...
  No phase boundary does not mean same physics, of course – (compare water-vapor, gas-plasma, ...) Critical point in many liquids – critical opalescence

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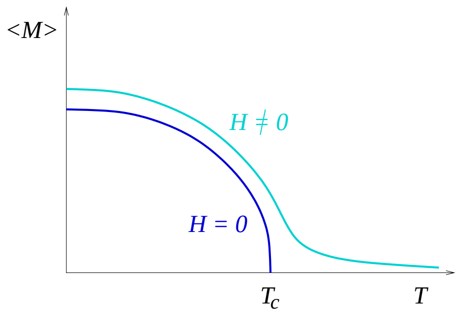

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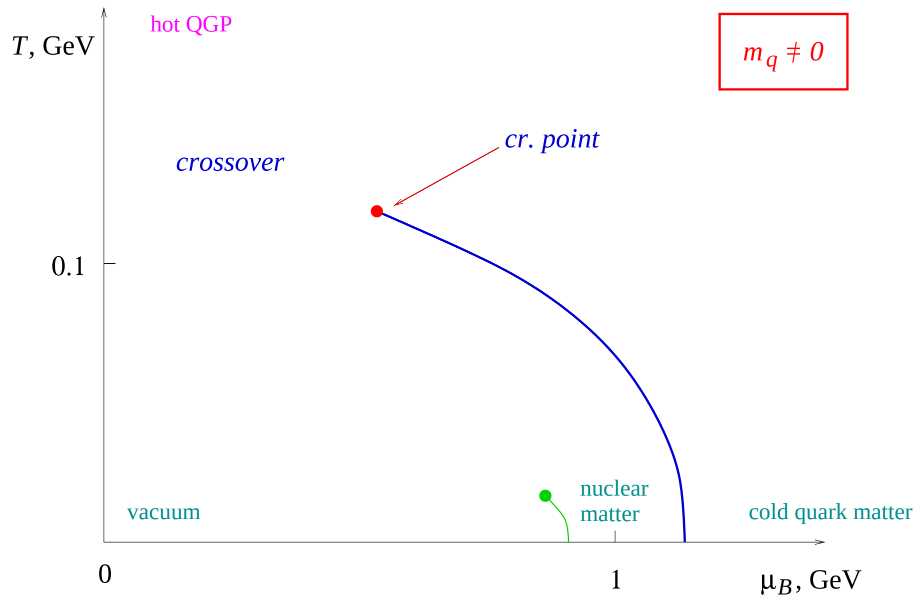
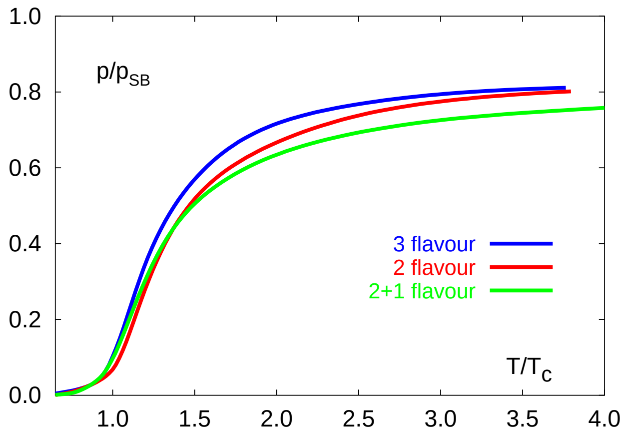
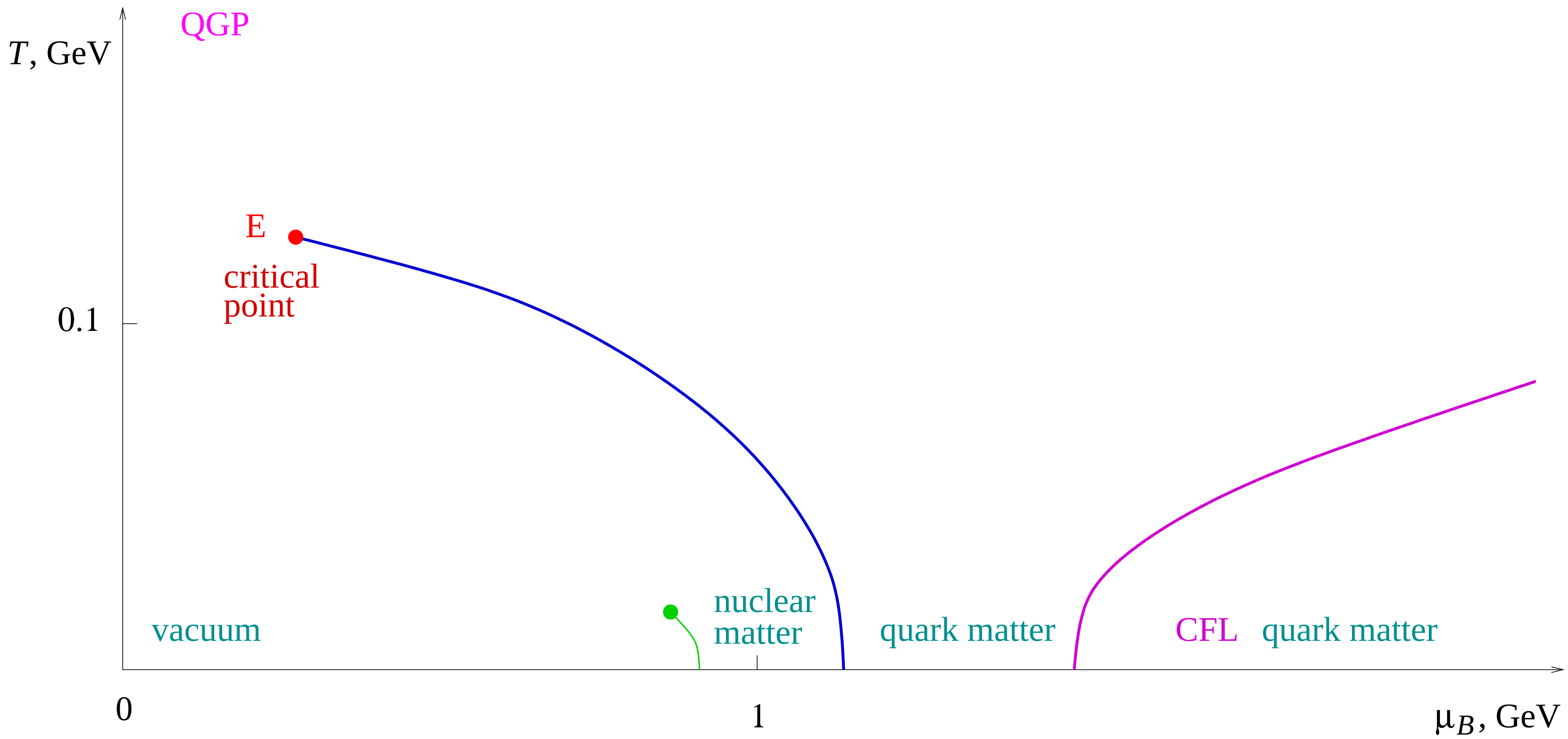




(Bielefeld group, lattice Monte Carlo)

Phase Diagram of QCD



Large μ, color superconductivity and CFL

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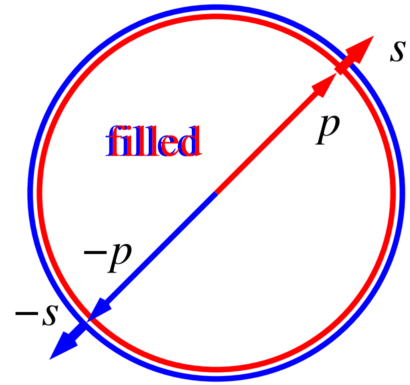
- Quarks of "different color" (color antisymmetric state) attract. Fermi sphere is unstable towards condensation of quark pairs (Cooper).



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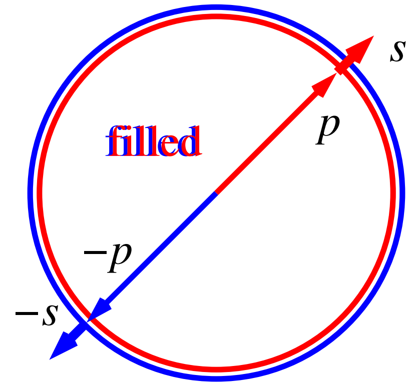

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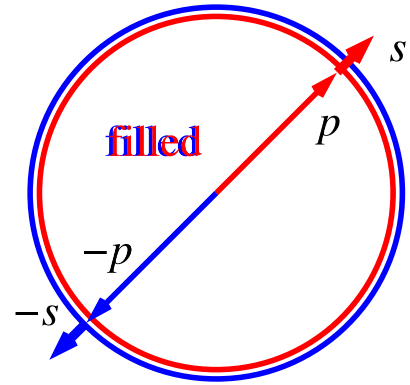

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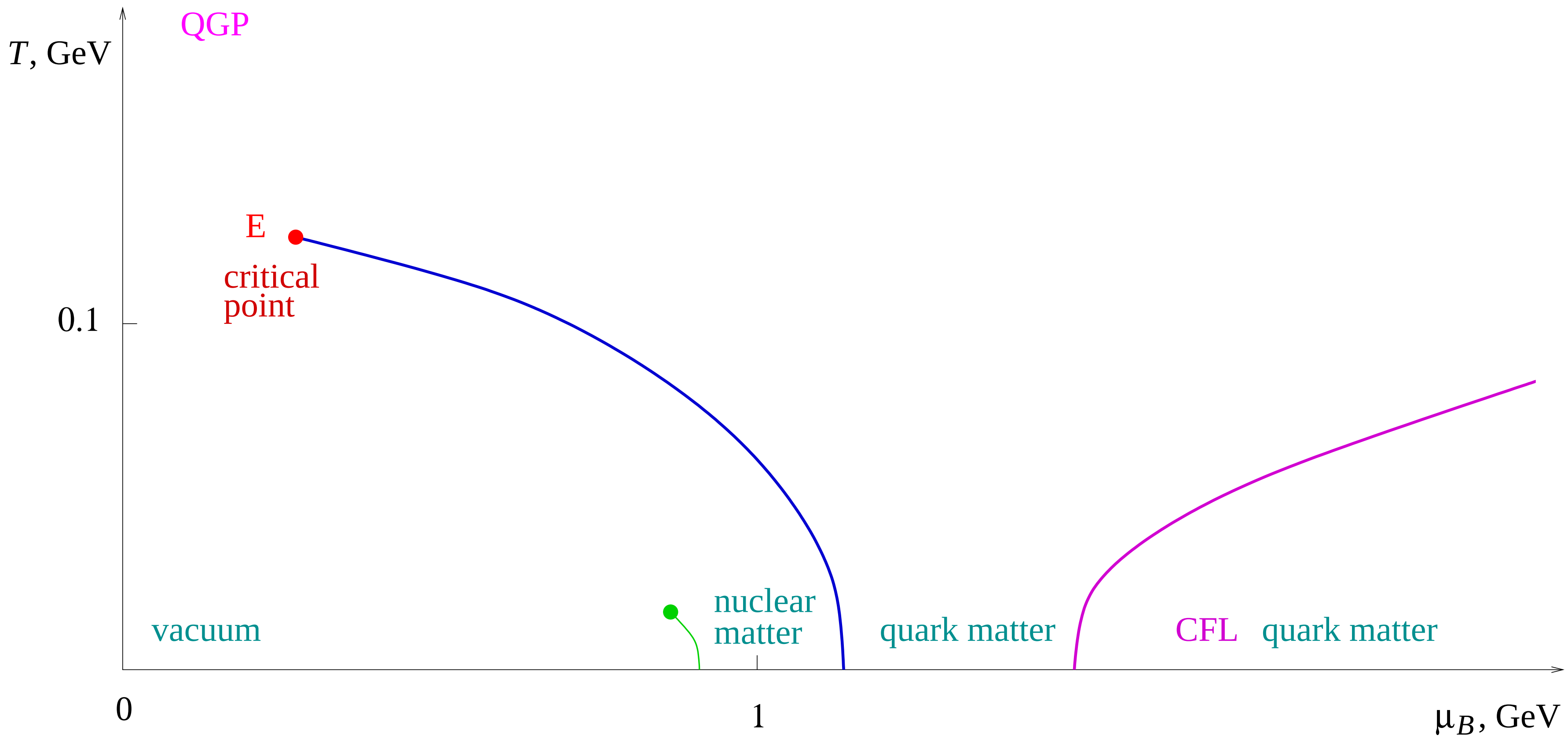
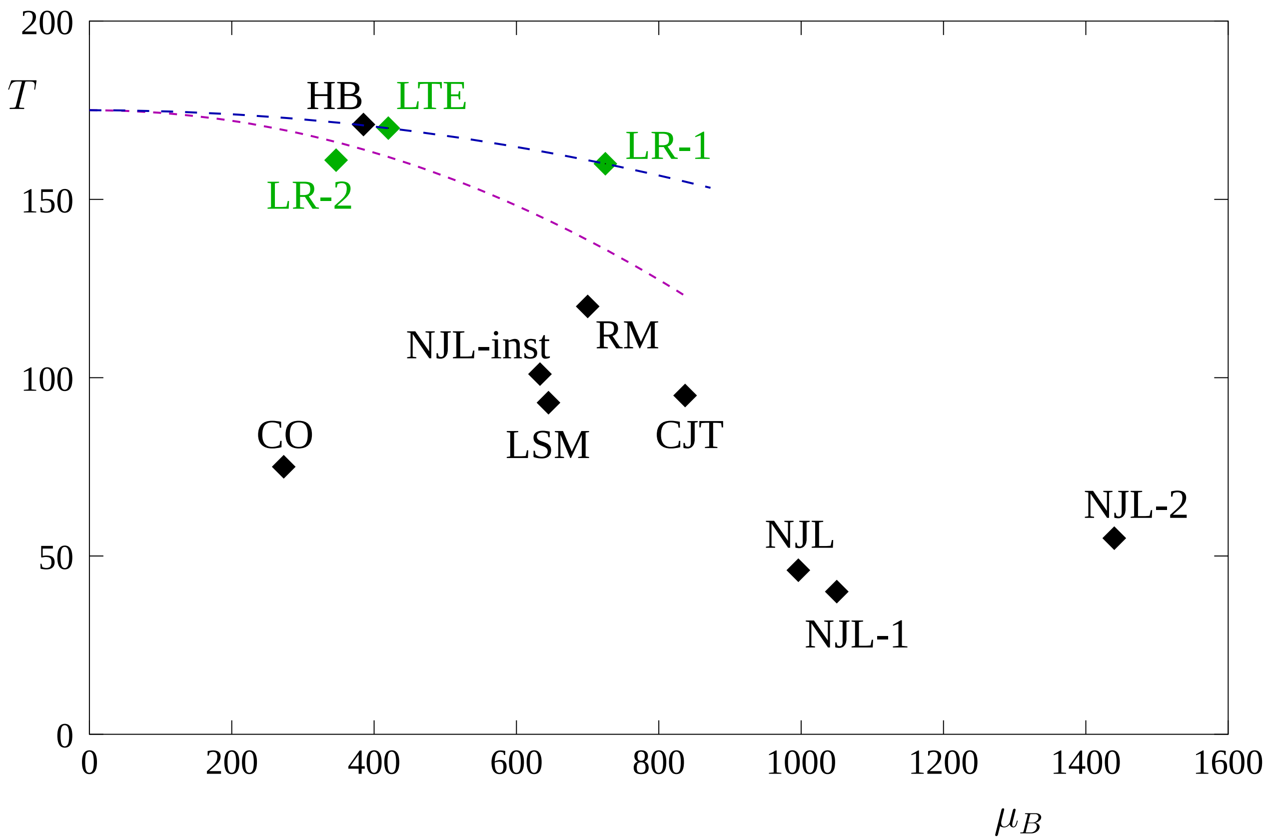
- But flavor transformation on X can be undone by a color transformation, which is "invisible". However, there is also $Y = u_L d_L + \ldots$, and although it can also be undone by a color rotation...
- the simultaneous transformations of q_L and q_R do change the ground state if they are not equal. This is the broken chiral symmetry.

  Ground state *locks color and flavor*: CFL (Alford, Rajagopal, Wilczek)

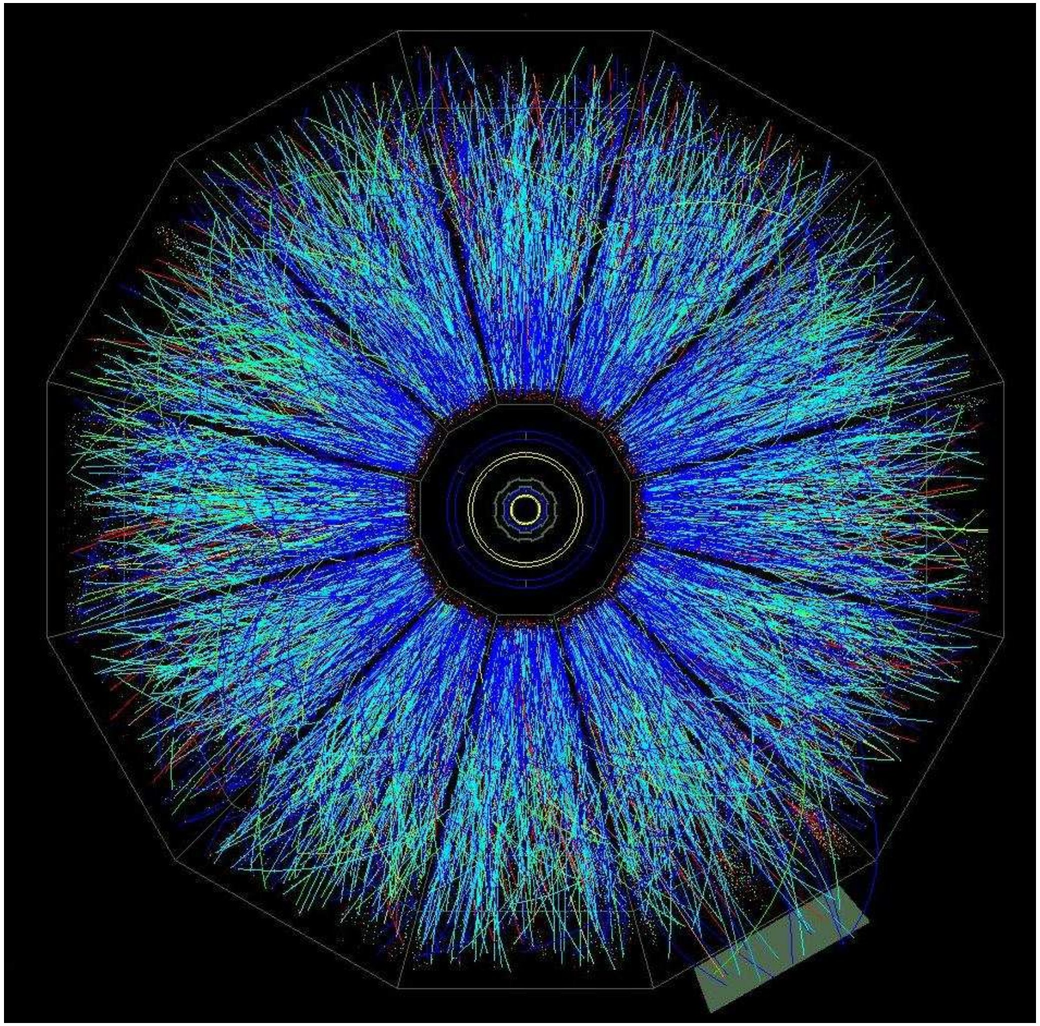
Phase Diagram of QCD



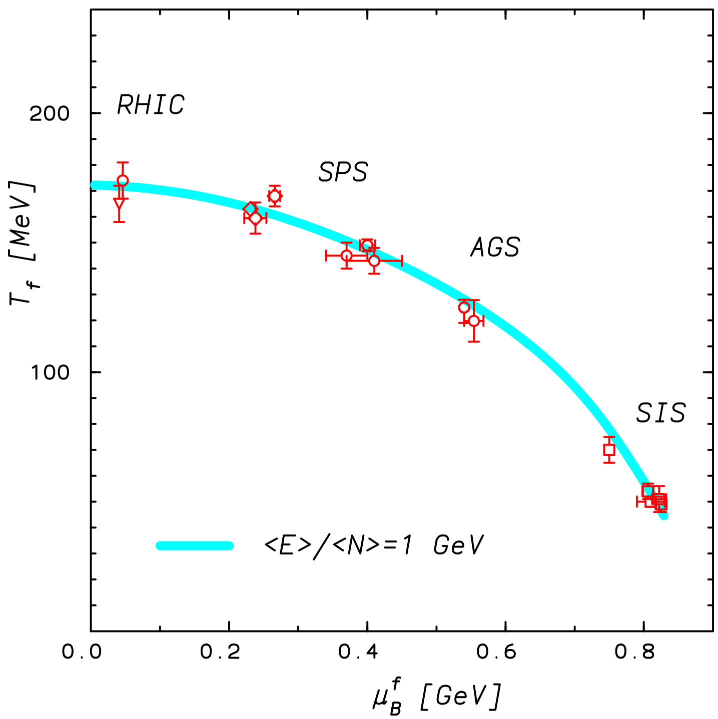
Location of the CP (theory)



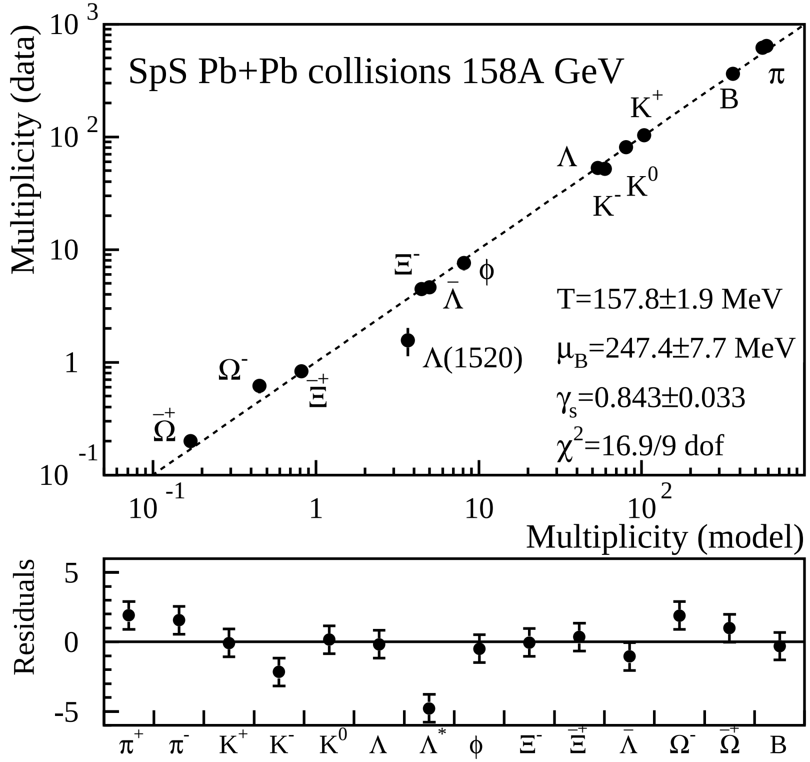
Event



Heavy-ion collisions and the phase diagram

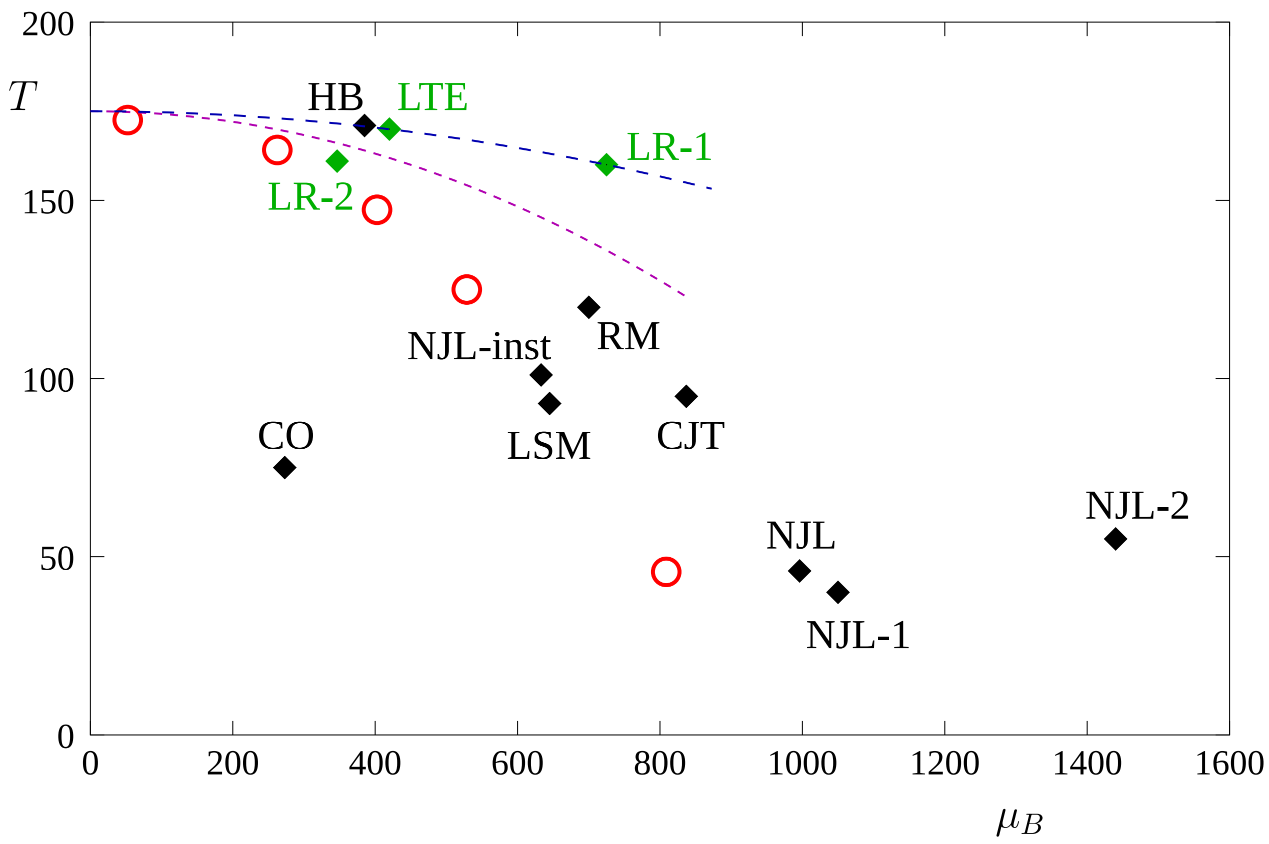


(from Braun-Munzinger, Redlich, Stachel)



(from Becattini et al)

Locating the QCD critical point experimentally

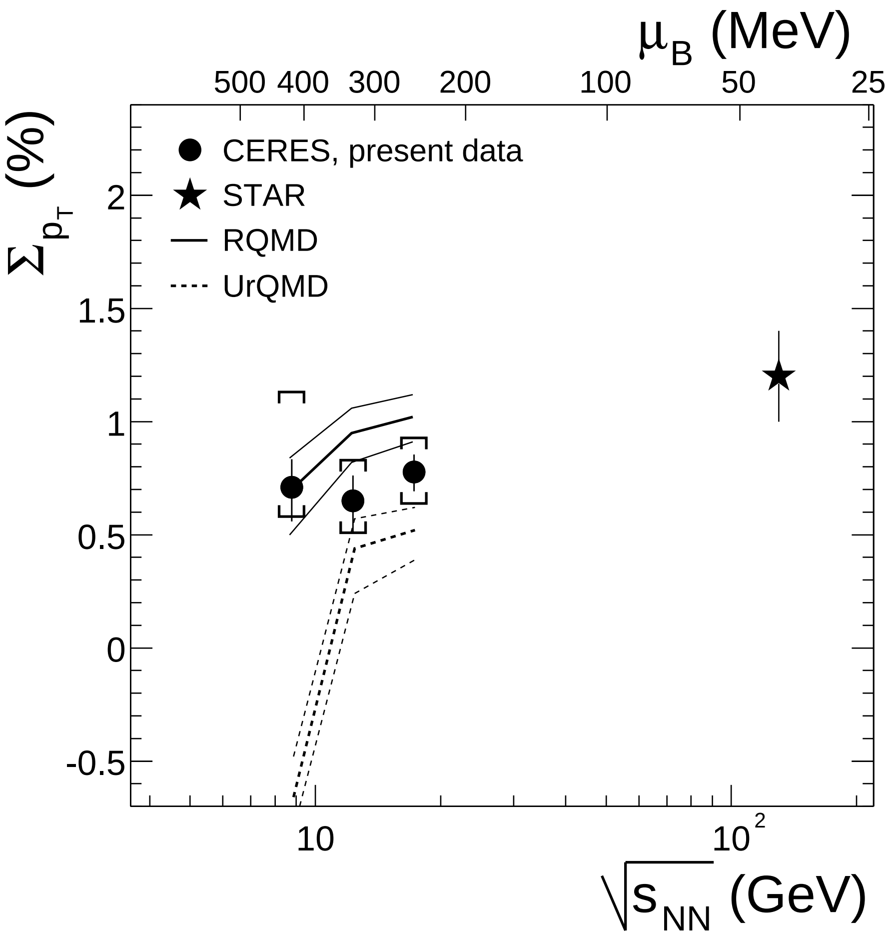


Energy scan.

Signatures: event-by-event fluctuations.

Susceptibilities diverge ⇒ fluctuations grow towards the critical point.

Data (example): p_T fluctuations (CERES)



Near the critical point (for CERES acceptance):

$$\sim 2\% \times \left(\frac{G}{300\ \mathrm{MeV}}\right)^2 \left(\frac{\xi_\sigma}{3\ \mathrm{fm}}\right)^2$$

($\xi_\sigma = 1/m_\sigma$)

Signal one is looking for:
non-monotonic dependence on $\sqrt{s}$.

Summary and conclusions

- Phase diagram of QCD is a challenging, long-standing problem.
- $\alpha_s \ll 1$ calculations can address regions of T, $\mu \gg 1$ fm^{-1}. Many interesting phenomena.
- The most interesting region is still out of reach of controllable calculations.
- Existence of the critical point on the QCD phase diagram is suggested by many theoretical approaches, including, recently, lattice QCD.
- Heavy ion experiments can discover the critical point – by measuring fluctuation observables as a function of collision parameters.

  Search is underway.
- Needed:

  Theory: a reliable (systematically improvable) method to study $T\mu$ plane and locate CP.

  Experiment: scan of the QCD phase diagram — $\sqrt{s}$ scan.