



Computed imaging: how physics connects external measurements to internal structure

P Scott Carney

Beckman Institute for Advanced Science and Technology Department of Electrical and Computer Engineering University of Illinois at Urbana-Champaign

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Prof. P. Scott Carney



<u>Post-Doctoral Fellows</u> Brynmor Davis

Graduate Students Jin Sun (PhD 2008)

http://optics.beckman.uiuc.edu



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Graduate StudentsPost-Doctoral FellowsTyler RalstonDan MarksAdam ZyskFellows







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- Sergey Bozhvolnyi (U Aalborg)





Inverse problems



RADAR



lliC=lliac Crest LatScrm=Lateral Sacrum Cx=Coccyx IshT=Ischial Tuberosity

X-ray projection

SPbR=Superior Pubic Ramus SymP=Symphysis Pubis IPbR=Inferior Pubic Ramus



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NMR spectroscopy



Optical Coherence Tomography *Spectral Domain





Inverse problems



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Inverse problems



Synthetic Aperture Radar *Rob Morrison



Magnetic Resonance Imaging *lan Atkinson





Optical Coherence Tomography *Spectral Domain









Physics $I(P) = I(P_0)e^{-\int_{P_0}^{P} \alpha(\mathbf{r})dl}$



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СТ

X-ray projection







Spectra to structure



NMR spectroscopy

Physics

 $\omega = 2B\mu/\hbar$









Echograms to structure



Maxwell Eq. – Reflectance – Stolt mapping





Optical Coherence Tomography: range finding with lowcoherence interferometry





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Signal processing: speckle reduction, numerical dispersion correction











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Blood vessel





Necrotic tissue



Luo W, Nguyen FT, Zysk AM, Ralston TS, Brockenbrough J, Marks DL, Oldenburg AL, Boppart SA, "Optical Biopsy of Lymph Node Morphology using Optical Coherence Tomography." Technology in Cancer Research and Treatment, 4 (5), 539-547, October 2005.





The problem









Beam Focusing



B. Hermann, *et.al.*, "Adaptive-optics ultrahigh-resolution optical coherence tomography," Opt. Let., vol. 29, pp. 2142-2144, 2004.

W. Drexler et. al., "In vivo ultrahigh-resolution optical coherence tomography," Opt. Let. Vol.24 No.17 pp. 1221-1223





Spectral domain OCT: all the data at once











- Linearize in the sample if possible
- Simulate and compare forward data to reality
- Apply appropriate approximations
- Invert, analytically if possible



























 $U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$

$$U_s(\mathbf{r}, \mathbf{r}_0, k) = \int d^3 r' G(\mathbf{r}', \mathbf{r}, k) \eta(\mathbf{r}') U_i(\mathbf{r}', \mathbf{r}_0, k)$$







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$$U_{i}(\mathbf{r}, \mathbf{r}_{0}, k)$$

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$$U_{i}\eta G$$

$$U_{i}\eta G$$

$$U_{i}\eta G$$





More math

$$g(\mathbf{r}, k) = \frac{1}{2\pi W_0^2(k)} e^{-r^2/2W_0^2(k)}$$
$$\alpha = \pi/NA \qquad W_0(k) = \alpha/k$$

$$\tilde{g}(\mathbf{q},k) = e^{-q^2 W_0^2/2} = e^{-q^2 \alpha^2/(2k^2)}$$



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$$G(\mathbf{r}',\mathbf{r},k) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{i}{2\pi} \int d^2q \ e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \frac{e^{-ik_z(\mathbf{q})(z-z')}}{k_z(\mathbf{q})}$$

 $\overline{k_z(\mathbf{q})} = \sqrt{k^2 - q^2}$





Forward simulation



T.S. Ralston, D.L. Marks, F. Kamalabadi, S.A. Boppart. "Deconvolution methods for mitigation of transverse blurring in optical coherence tomography." IEEE Trans. Image Proc. Special Issue on Molecular and Cellular Bioimaging, vol.14, no. 9, September 2005.

T.S. Ralston, D.L. Marks, P.S. Carney, S.A. Boppart, "Inverse scattering for optical coherence tomography," JOSA A, in press.





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The inverse problem

$$S(\mathbf{r}_{0},k) = A(k) \int_{z=0}^{z=0} d^{2}r \int d^{3}r' G(\mathbf{r}',\mathbf{r},k) g(\mathbf{r}'-\mathbf{r}_{0},k) g(\mathbf{r}-\mathbf{r}_{0},k) \eta(\mathbf{r}').$$







$$S(\mathbf{r}_0,k) = A(k) \int_{z=0} d^2r \int d^3r' G(\mathbf{r}',\mathbf{r},k) g(\mathbf{r}'-\mathbf{r}_0,k) g(\mathbf{r}-\mathbf{r}_0,k) \eta(\mathbf{r}').$$

$$\tilde{S}(\mathbf{Q},k) = i2\pi A(k) \int d^2q \int dz' \, \frac{1}{k_z(\mathbf{q})} e^{ik_z(\mathbf{q})(z'-z_0)} \, e^{ik_z(\mathbf{q}-\mathbf{Q})(z'-z_0)}$$

$$\times e^{\frac{-\alpha^2 Q^2}{4k^2}} e^{\frac{-\alpha^2 |\mathbf{q}-\mathbf{Q}/2|^2}{k^2}} \tilde{\eta}(\mathbf{Q}, z')$$







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$$\times e^{\frac{-\alpha^2 Q^2}{4k^2}} e^{\frac{-\alpha^2 |\mathbf{q} - \mathbf{Q}/2|^2}{k^2}} \tilde{\eta}(\mathbf{Q}, z')$$

asymptotic expansion







$$S(\mathbf{r}_{0},k) = A(k) \int_{z=0}^{z=0} d^{2}r \int d^{3}r' G(\mathbf{r}',\mathbf{r},k) g(\mathbf{r}'-\mathbf{r}_{0},k) g(\mathbf{r}-\mathbf{r}_{0},k) \eta(\mathbf{r}').$$

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asymptotic expansion

$$\tilde{S}(\mathbf{Q},k) = \frac{k^2}{\alpha^2} i2\pi^2 A(k) \frac{e^{-2ik_z(\mathbf{Q}/2)z_0}}{k_z(\mathbf{Q}/2)} e^{\frac{-\alpha^2 Q^2}{4k^2}} \tilde{\tilde{\eta}} \left[\mathbf{Q}, -2k_z(\mathbf{Q}/2)\right]$$







$$\tilde{S}(\mathbf{Q},k) = K(\mathbf{Q},k) \,\tilde{\tilde{\eta}} \left[\mathbf{Q}, -2k_z(\mathbf{Q}/2)\right]$$

Tikhonov solution:

$$\tilde{\tilde{\eta}}(\mathbf{Q},\beta) = \left[\frac{K^*(\mathbf{Q},k,\beta)\tilde{S}(\mathbf{Q},k)}{|K(\mathbf{Q},k,\beta)|^2 + 2Nk/k_z(\mathbf{Q}/2)}\right]_{k=\frac{1}{2}\sqrt{\beta^2 - Q^2}}$$

N is the noise floor







Polar vs. Hyperbolic Resampling



Radon SAR MRI CT



ISAM





















3D Rendered








3D Rendered







ISAM vs Histology









And now for something completely different

- Near-field
- Also scanning
- Also interferometric
- Harder









Betzig and Trautman, Science 257,189-195 (1992).



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 $1 \, \mu m$



PSTM holograms











PSTM/NSOM simple model



Monochromatic field ω

$$\omega_0 = ck_0$$

$$U_{\mathbf{q}}(\mathbf{r}) = \int \mathrm{d}^3 r' \; e^{\mathrm{i}\mathbf{k}(\mathbf{q})\cdot\mathbf{r}'} G(\mathbf{r},\mathbf{r}') V(\mathbf{r}')$$

$$K_{\mathbf{q}}(\mathbf{r};\mathbf{r}') = \int d^2 Q \ \lambda_{\mathbf{Q}}^{\mathbf{q}} g_{\mathbf{Q}}^{\mathbf{q}}(r) f_{\mathbf{Q}}^{\mathbf{q}*}(\mathbf{r}')$$
$$g_{\mathbf{Q}}^{\mathbf{q}}(r) = \frac{e^{i(\mathbf{Q}+\mathbf{q})\cdot\mathbf{r}}}{2\pi}$$

$$f_{\mathbf{Q}}^{\mathbf{q}*}(\mathbf{r}') = \frac{-\mathrm{i}e^{\mathrm{i}\mathbf{Q}\cdot\mathbf{r}'-\mathrm{i}|z'|k_{z}^{*}(\mathbf{Q}+\mathbf{q})-\mathrm{i}zk_{z}^{*}(\mathbf{q})}}{4\pi k_{z}^{*}(\mathbf{Q}+\mathbf{q})\lambda_{\mathbf{Q}}^{\mathbf{q}}}$$





Calculation of the pseudo-inverse

$$M_{\mathbf{q}\mathbf{q}'}(\mathbf{Q})\delta^{(2)}(\mathbf{Q}-\mathbf{Q}') = \langle f_{\mathbf{Q}}^{\mathbf{q}}|f_{\mathbf{Q}'}^{\mathbf{q}'}\rangle\lambda_{\mathbf{Q}}^{\mathbf{q}}\lambda_{\mathbf{Q}'}^{\mathbf{q}'}$$

 $M(\mathbf{Q})\mathbf{c}_{\ell}(\mathbf{Q}) = \Lambda_{\mathbf{Q}\ell}\mathbf{c}_{\ell}(\mathbf{Q})$

$$\psi_{\mathbf{Q}\ell}(\mathbf{r}') = \Lambda_{\mathbf{Q}\ell}^{-1} \sum_{\mathbf{q}} c_{\ell\mathbf{q}}(\mathbf{Q}) \lambda_{\mathbf{Q}}^{\mathbf{q}} f_{\mathbf{Q}}^{\mathbf{q}}(\mathbf{r}')$$

$$\phi_{\mathbf{Q}\ell\mathbf{q}'}(\mathbf{r}) = c_{\ell\mathbf{q}'}(\mathbf{Q})g_{\mathbf{Q}}^{\mathbf{q}'}(\mathbf{r})$$

$$K^{+}(\mathbf{r}';\mathbf{r},\mathbf{q}) = \int d^{2}Q \sum_{\ell} \Lambda_{\mathbf{Q}\ell}^{-1} \psi_{\mathbf{Q}\ell}(\mathbf{r}') \phi_{\mathbf{Q}\ell\mathbf{q}}^{*}(\mathbf{r})$$







And the results are ...





terrible.







Fix the physics: Multiple interactions with the probe



tip corrections

Conclusions

- Inverse scattering and computed imaging extend the utility and scope of data collection methods
- Physics makes it go.
- ISAM is easy and much better that OCT
- NFOT turns holograms into images and 3-d is next.

(End of the standard menu, begin epicurean options)

Cross-validation

Autocorrelation mitigation

CS LETTERS / Vol. 32, No. 11 / June 1, 2007

d performance in the spectrometer [2]. ter it is assumed that a single measureh with τ_0 set such that the real and cons can just be separated. A quantitative he object is made from these data using d ISAM, and the resulting autocorrelaare investigated.

 $S(\mathbf{r}_{\parallel},k)=S_{sr}(\mathbf{r}_{\parallel},kc/n)$ can be defined by in temporal to spatial spectra according sion relation $k=n\omega/c$. In samples with arying background properties, a more tion can be employed and dispersion digitally [10,15]. Autocorrelation and age artifacts arise because S_{sr} is not diole, and S is therefore calculated from

the two-dimensional Fourier transform d by a tilde) with respect to \mathbf{r}_{\parallel} , the most I model [11] can be written as

$$,k) = \int \widetilde{L}(\mathbf{Q}_{\parallel},k,z)e^{i2k_{z}z}\widetilde{\eta}(\mathbf{Q}_{\parallel},z)\mathrm{d}z\,, \qquad (4)$$

 ${}^{2}-Q_{\parallel}^{2}$)^{0.5} is the axial component of the η is the object susceptibility, and \tilde{L} is dethe specific instrumentation used. When ence in the factor \tilde{L} is neglected and it is $k_{z}=k$ (an extreme form of the paraxial n), the conventional OCT model is replicit the factor and axial effects are the factor is. OCT reconstruction may be

Fig. 1. (a) The x-z plane of the object and (b) the resulting OCT and (c) ISAM images with $c \tau_0 = -1$ mm. The structures in (a) have been broadened for display. In (b) and (c) the contribution of R_{sr} is shown in blue, R_{sr}^* (conjugate image) in green, and R_{ss} (autocorrelation) in red. The color scale is clipped at 10% of the maximum signal so that low-level de-

Autocorrelation mitigation ECE Illinois

En face Comparison

Unprocessed data

Reconstruction

Better to be lucky than good

General result for vector fields, high NA:

$$\tilde{S}(\mathbf{Q}_{\parallel},k) = \int dz \; \tilde{h}_{\alpha\beta}(\mathbf{Q}_{\parallel},z_0-z;k) \tilde{\eta}_{\alpha\beta}(\mathbf{Q}_{\parallel},z)$$

fields. red by alities mmond nuproxi-ISAM appli-. Fur-5] and l adds so exusing [1] of s then he rene deted in mulaproces in-a-2

1. Basic illustration of a coherent microscope. A source an interferometer where one arm produces a reference field

 $\tilde{h}_{\alpha\beta}(-\mathbf{Q}_{\parallel}, -z; k) =$ $-4\pi^2 k\mu_r \left|P(k)\right|^2$

$$\times \int d^2 q_{\parallel} \frac{\hat{F}_{\alpha}\left(\frac{\mathbf{q}_{\parallel}}{k}\right)}{\sqrt{k_z(\mathbf{q}_{\parallel})}} \frac{\hat{G}_{\beta}\left(\frac{\mathbf{Q}_{\parallel}-\mathbf{q}_{\parallel}}{k}\right)}{\sqrt{k_z(\mathbf{Q}_{\parallel}-\mathbf{q}_{\parallel})}}$$

 $\times e^{i\left[k_z(\mathbf{q}_{\parallel})+k_z(\mathbf{Q}_{\parallel}-\mathbf{q}_{\parallel})\right]z}$

Stationary phase (far from focus)

$$\tilde{h}_{\alpha\beta}(-\mathbf{Q}_{\parallel},-z;k)\sim$$

$$\frac{i4\pi^{3}k}{z}\mu_{r}\left|P(k)\right|^{2} e^{i2k_{z}(\mathbf{Q}_{\parallel}/2)z}\hat{F}_{\alpha}\left(\frac{\mathbf{Q}_{\parallel}}{2k}\right)\hat{G}_{\beta}\left(\frac{\mathbf{Q}_{\parallel}}{2k}\right)$$

Peaked aperture functions (near focus)

$$h_{\alpha\beta}(-\mathbf{Q}_{\parallel},-z;k) \sim -4\pi^{2}k\mu_{r} \left|P(k)\right|^{2} \frac{e^{i\left[k_{z}(\bar{\mathbf{q}}_{\parallel})+k_{z}(\mathbf{Q}_{\parallel}-\bar{\mathbf{q}}_{\parallel})\right]z}}{\sqrt{k_{z}(\bar{\mathbf{q}}_{\parallel})k_{z}(\mathbf{Q}_{\parallel}-\bar{\mathbf{q}}_{\parallel})}} \int d^{2}q_{\parallel} \hat{F}_{\alpha}\left(\frac{\mathbf{q}_{\parallel}}{k}\right) \hat{G}_{\beta}\left(\frac{\mathbf{Q}_{\parallel}-\mathbf{q}_{\parallel}}{k}\right)$$

$$ar{\mathbf{q}}_{\parallel} = rac{\mathbf{Q}_{\parallel} \varsigma_F^2}{\varsigma_F^2 + \varsigma_G^2}$$

Connection to SAR

Figure 3. An illustration of the differences between the data acquisition geometries in SAR and ISAM. SAR involves a one-dimensional scan track, while ISAM scans over a plane. Unlike SAR beams, ISAM fields include a region within the object that is in focus. Note that the same aperture is assumed for both transmission and reflection in SAR; similarly the source is imaged onto the detector by the reference arm in ISAM (see Fig. 1). The spectral bands and scan lengths also vary greatly between SAR and ISAM.

Figure 9. Raw strip-map radar image of a 1:32 scale model of a F14 fighter aircraft before Stolt Fourier resampling (a), and after Stolt Fourier resampling (b).

From Gregory L. Charvat, Lincoln Labs.

Other bits of magic

- Fast spectral domain acquisition
- Phase drift correction
- Dispersion compensation
- Spectral reweighting

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Unstable phase in adjacent planes

Phase fronts

in frame

across frames

Phase reference

Phase correction using reference coverslip

Ralston TS, Marks DL, Carney PS, Boppart SA. "Phase stability technique for inverse scattering in optical coherence tomography." IEEE International Symposium on Biomedical Imaging, Arlington, VA, April 7, 2006.

Cross-section from a 3D scan

Cross-section from a 3D scan

Cross-section from a 3D scan

Spectrum correction in a ECE Illinois

No spectrum correction

With spectrum correction

Drift (thermal)

Distorted point response

Distorted point response

Simulation adjusting the assumed NA

Simulation adjusting the assumed focal plane

2D projection of 3D scattering

Constant phase surfaces for a point scatterer

$$k_z(\mathbf{q}) = \sqrt{k^2 - q^2}$$

High density sample

2 micron TiO_2 suspended in silicone, SD-OCT













Low Density Experiment



SD-OCT data

Unfiltered reconstruction





Human Tissue – Adipose/Cancer^{CE} Illinois



