



# Qu-Transitions



P. Coleman  
(CMT, Rutgers)

University Toronto  
Toronto, Sept 17  
2009.

Rutgers  
Center for Materials Theory

# Qu-Transitions

"Phase transitions in the quantum era"



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- The Qu-era: from classical to quantum.
- Classical criticality
- Heavy Fermion Quantum Criticality.
- New Approaches and Ideas
- Avoided Criticality

1758 in Paris: 72 years after “Principia”

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Classical revolution is still in full sway.

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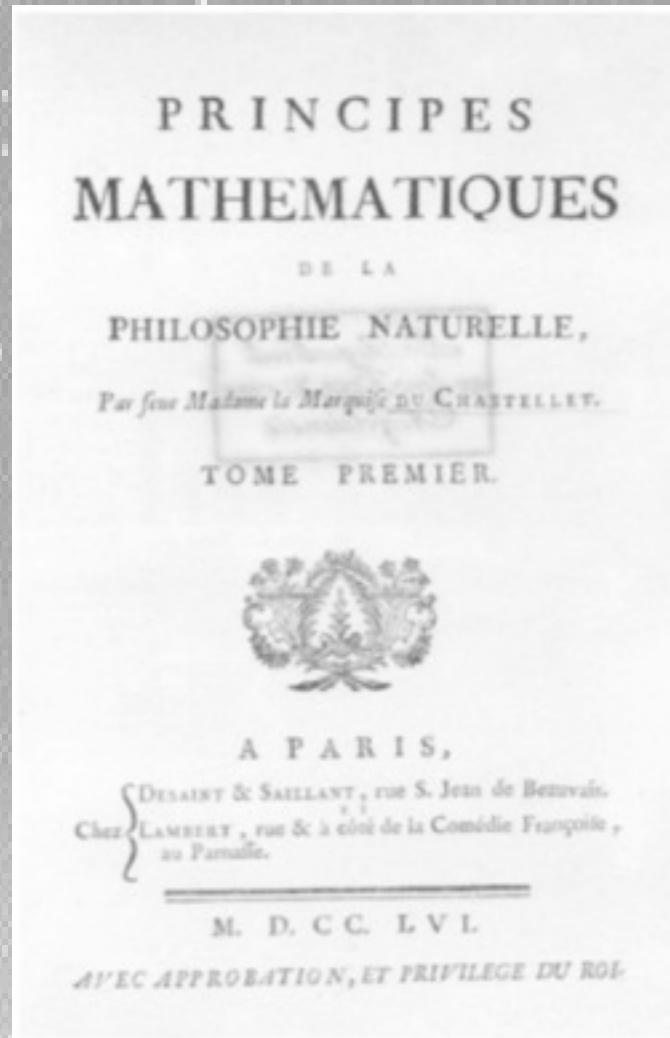
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PRINCIPES  
MATHEMATIQUES

DE LA

PHILOSOPHIE NATURELLE,

*Par feu Madame la Marquise du CHATELET.*

TOME PREMIER.



A PARIS,  
DESANT & SAILLANT, rue S. Jean de Beauvais.  
CHEZ LAMBERT, rue Sc à côté de la Comédie Françoise,  
au Parvis.

M. D. C. C. L. V. I.

AVEC APPROBATION, ET PRIVILEGE DU ROI.



Marquise Emilie du Châtelet  
(1707-1749)

Mathematical Physicist:  
Translator and interpreter of Principia.

"ce beau probleme astronomico-geometrique"

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Newton's Momentum

$$\sum_i m_i \vec{v}_i$$

Leibniz' "vis vivre"

$$\sum_i m_i (v_i)^2$$

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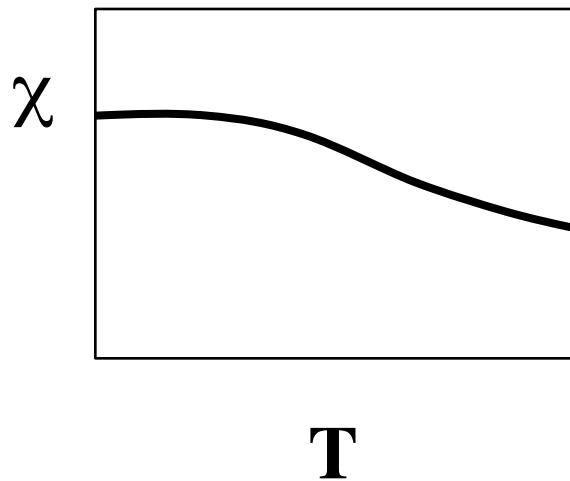
Resolution of the controversy (and the missing factor of a half) required a further 60-80 years.

108 years after Planck, many surprises later, the quantum era is in full sway.

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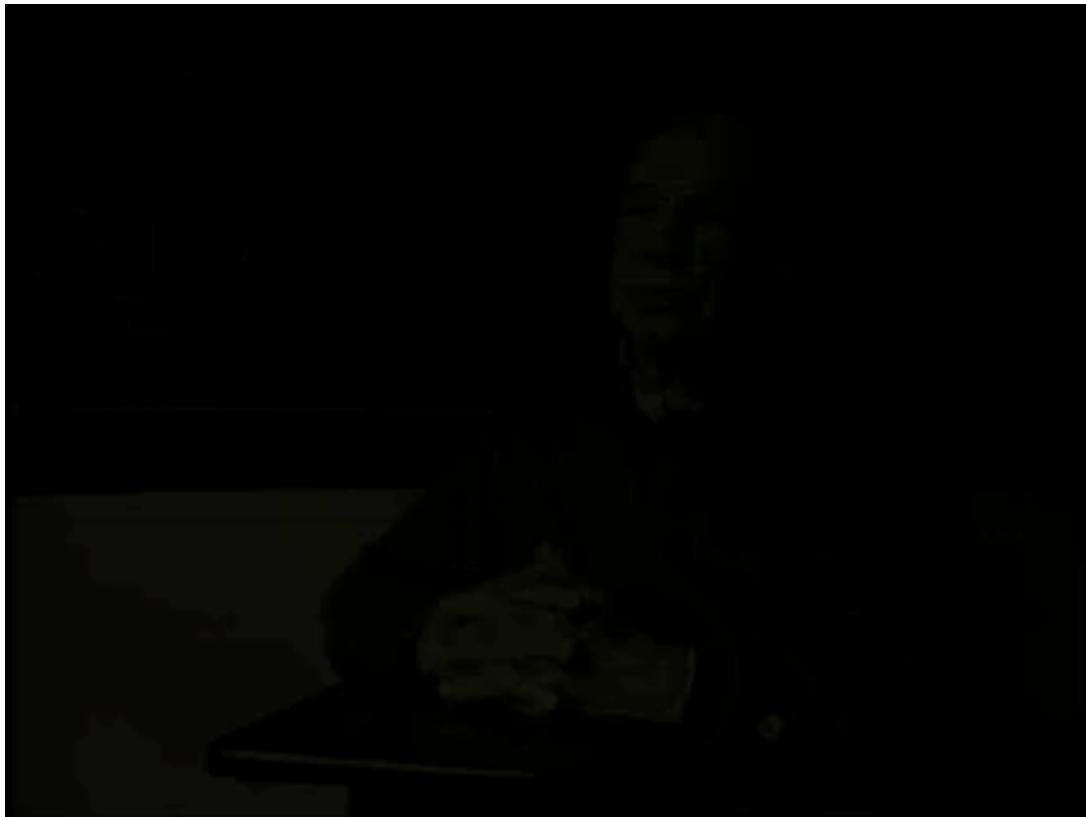


*“With a heavy heart, I have been converted to the idea that Fermi -Dirac, not Einstein-Bose is the correct statistics. I wish to write a short note on its application to paramagnetism.”*

*W. Pauli, in letter to Schrödinger, Dec 1926.*

108 years after Planck, many surprises later, the quantum era is in full sway.

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David Pines in  
[musicofthequantum.rutgers.edu](http://musicofthequantum.rutgers.edu)

# Quantum zero point fluctuations:

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# Quantum zero point fluctuations: major unsolved problem of the quantum era.

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- 73% of the mass of the cosmos is “Dark Energy”: an unidentified form of zero point energy, causing the expansion to accelerate.

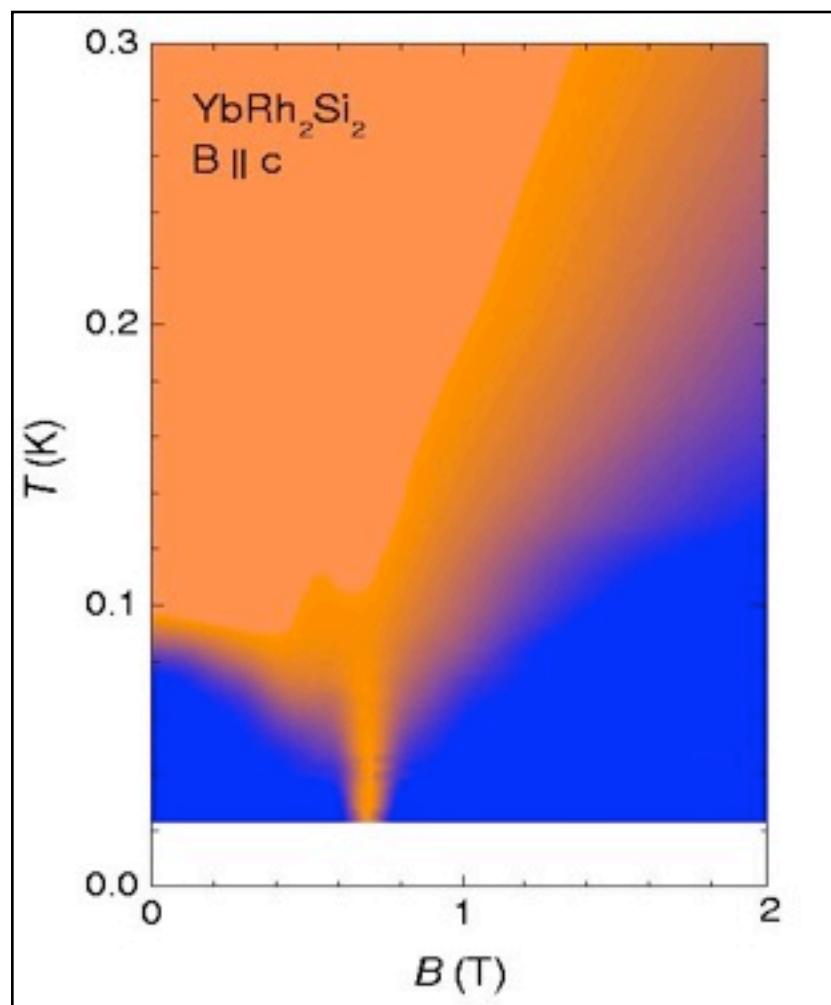
# Quantum zero point fluctuations: major unsolved problem of the quantum era.

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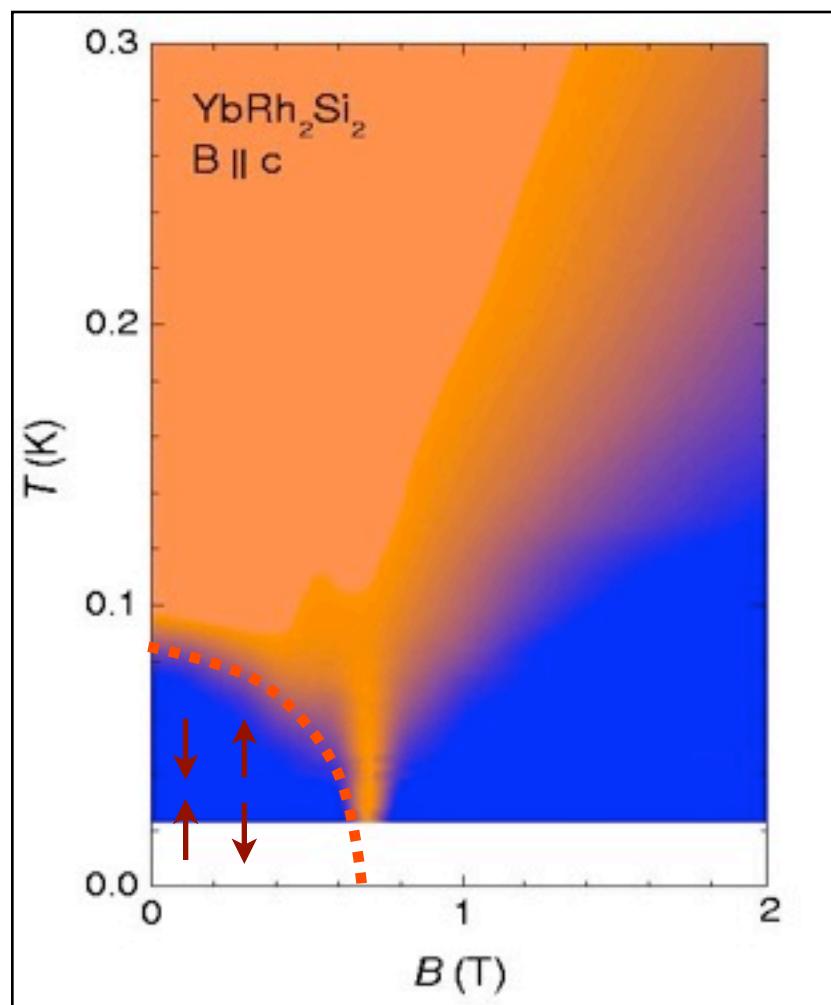


- Zero point fluctuations profoundly transform matter, endowing it with marked tendency to develop new forms of order.

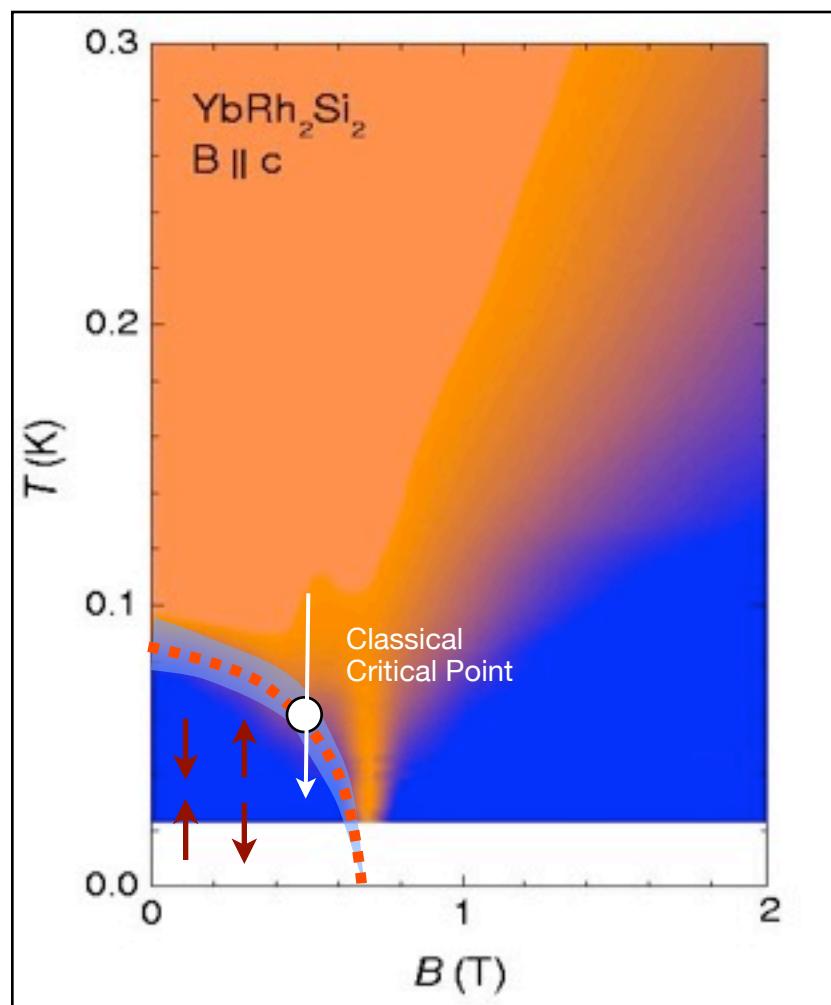
# Classical Criticality



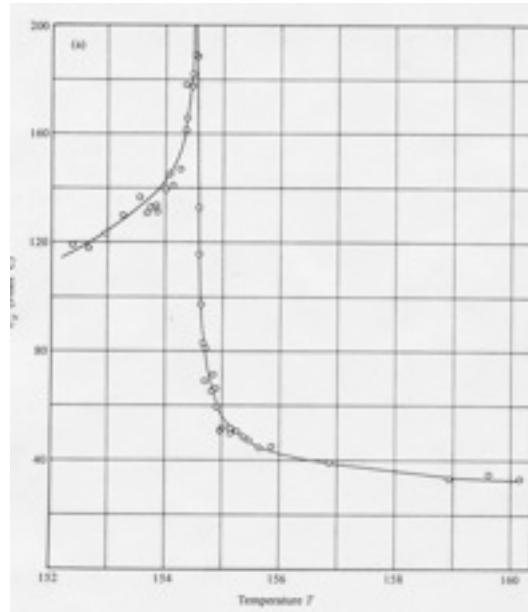
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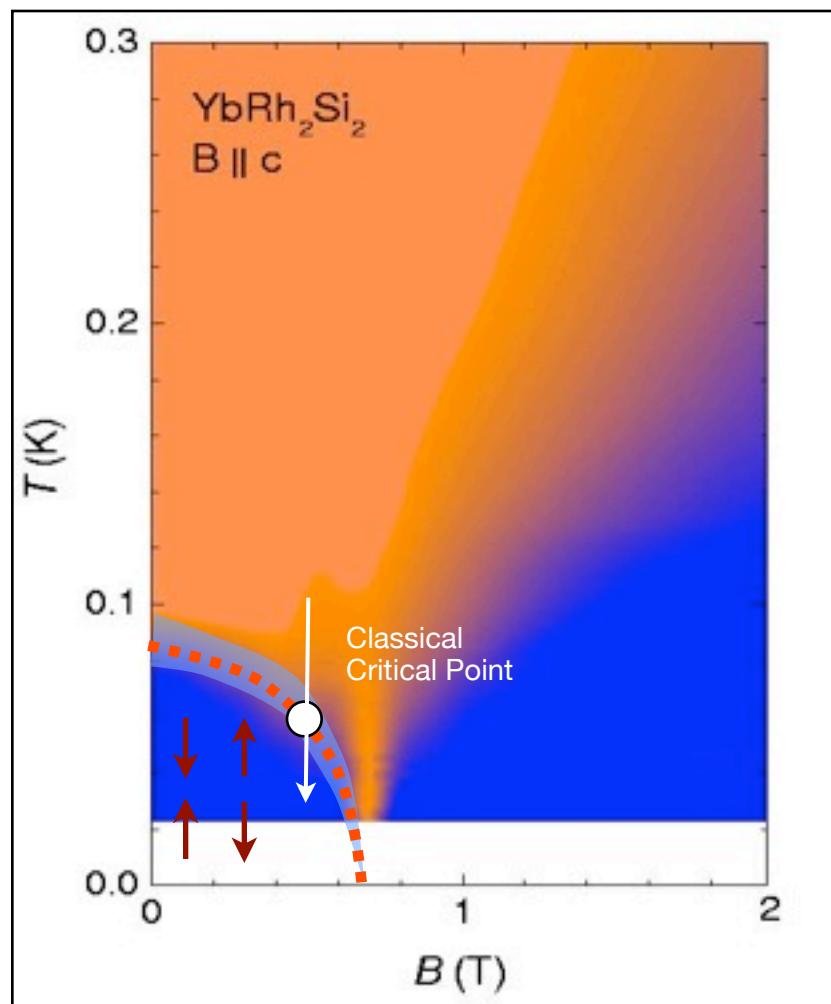
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Oxygen. (Voronel et al 1963).

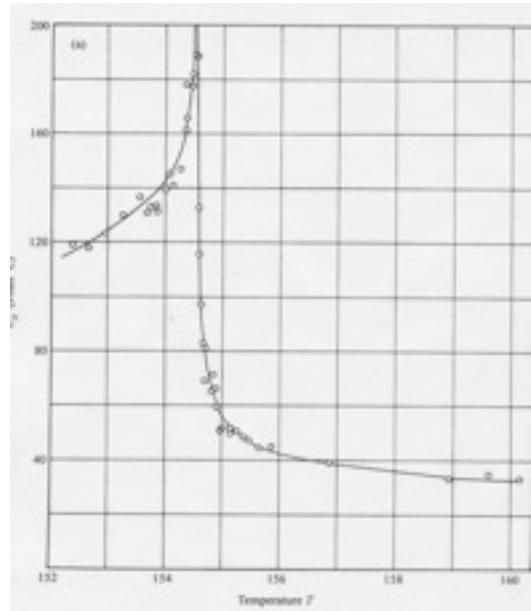


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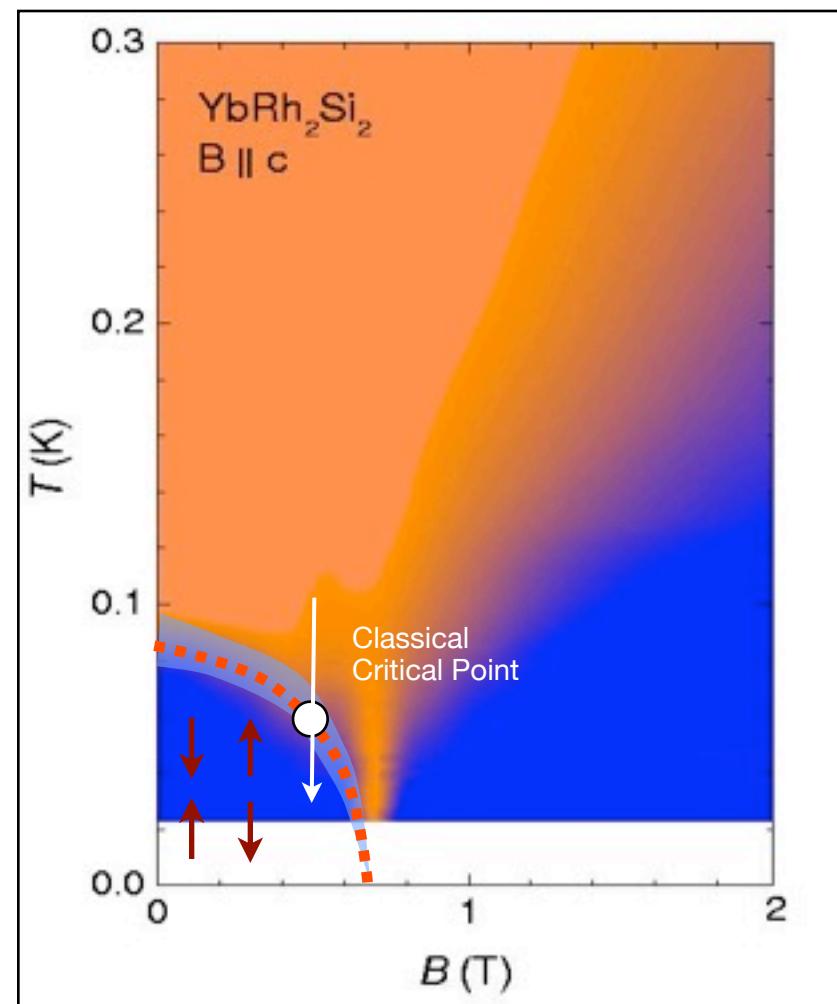
Michael Fisher



"New insights into physics often come from revisiting areas once thought to be closed." Michael Fisher.



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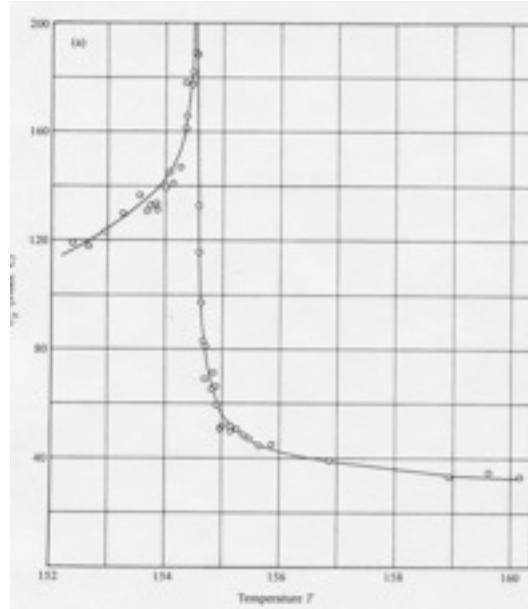


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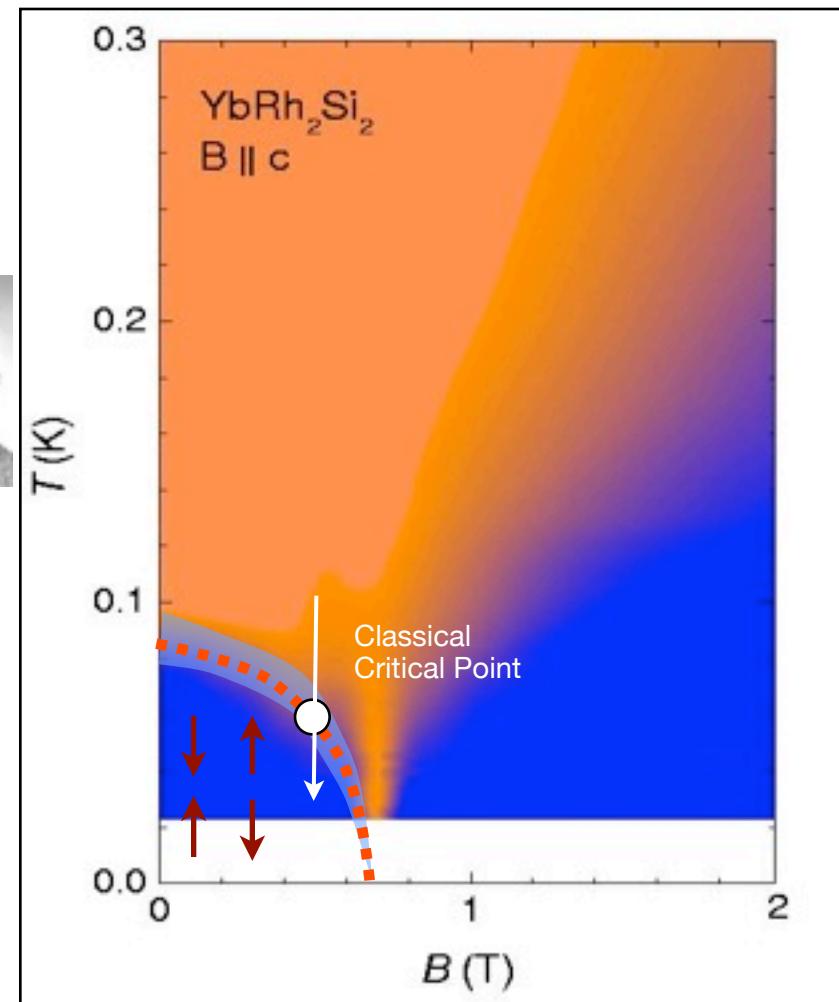
Michael Fisher    Leo Kadanoff    Ben Widom    Anatoly Larkin    Ken Wilson



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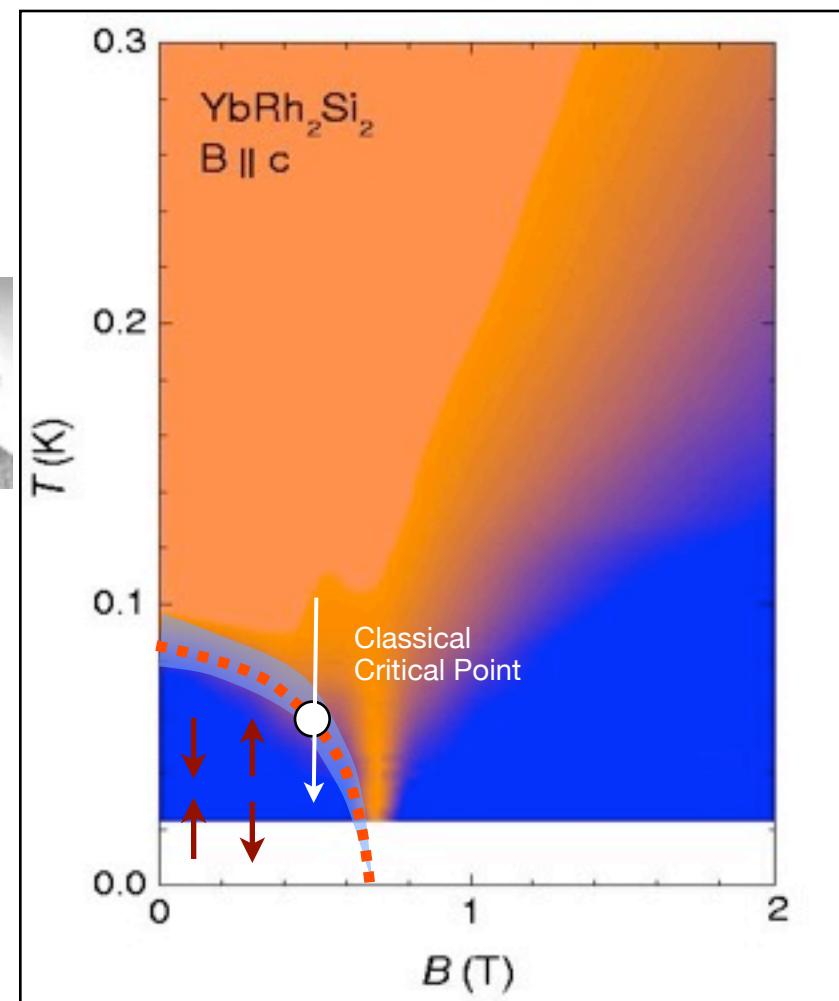
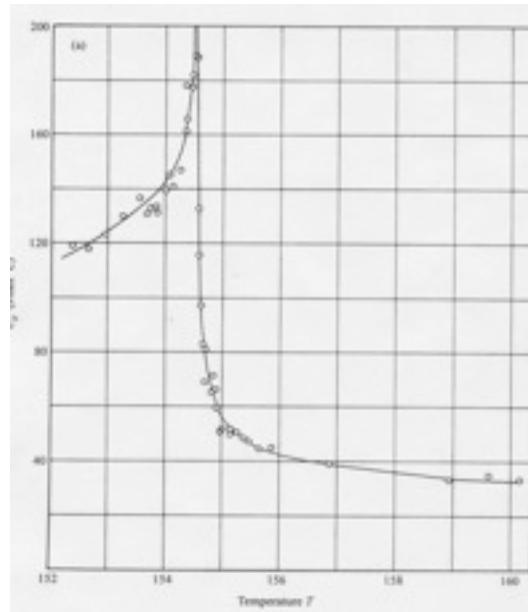


# Classical Criticality

Michael Fisher   Leo Kadanoff   Ben Widom   Anatoly Larkin   Ken Wilson



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"20th Century Revolution"

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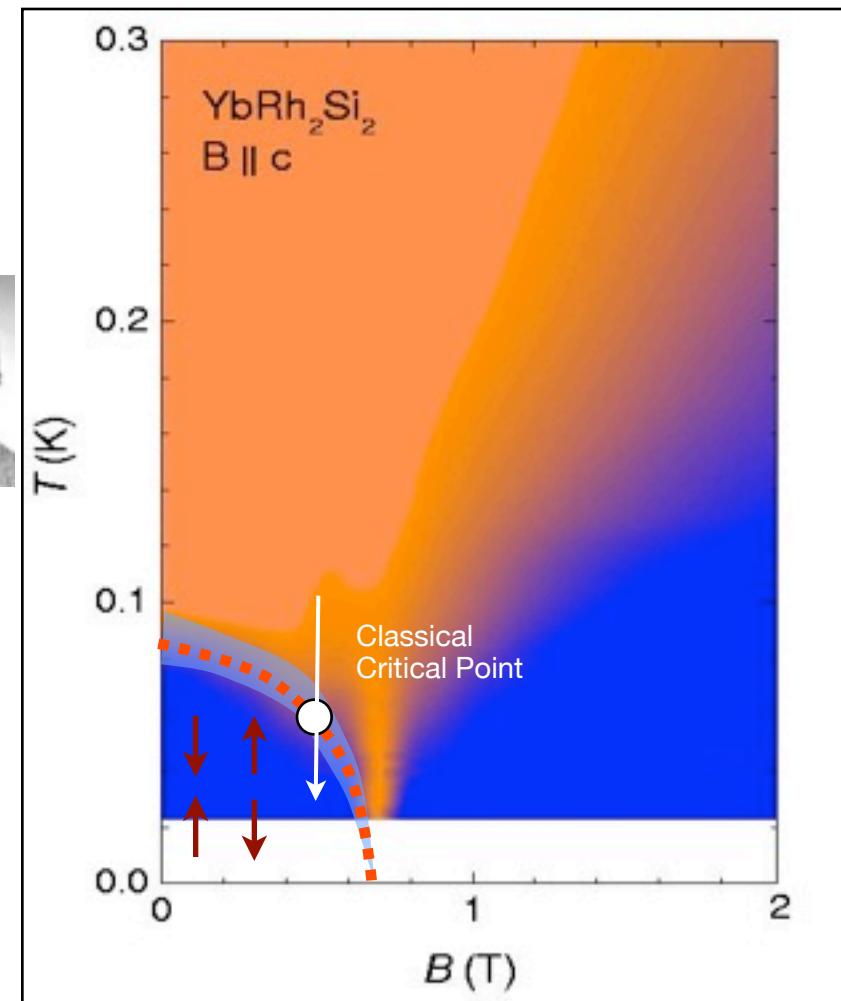
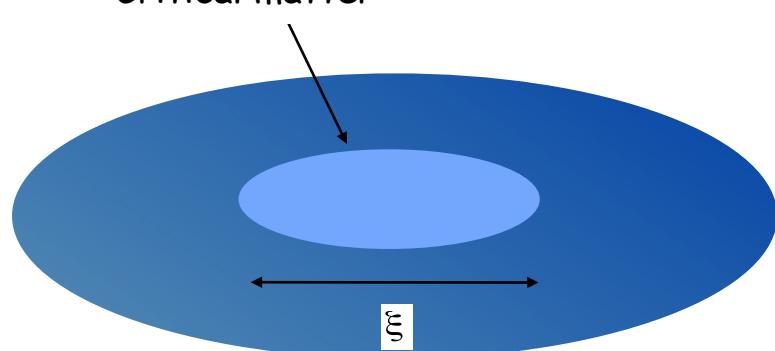
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$$\langle \psi(x)\psi(0) \rangle \sim \frac{1}{x^{d-2+\eta}}$$

Critical matter



"20th Century Revolution"

# Classical Criticality

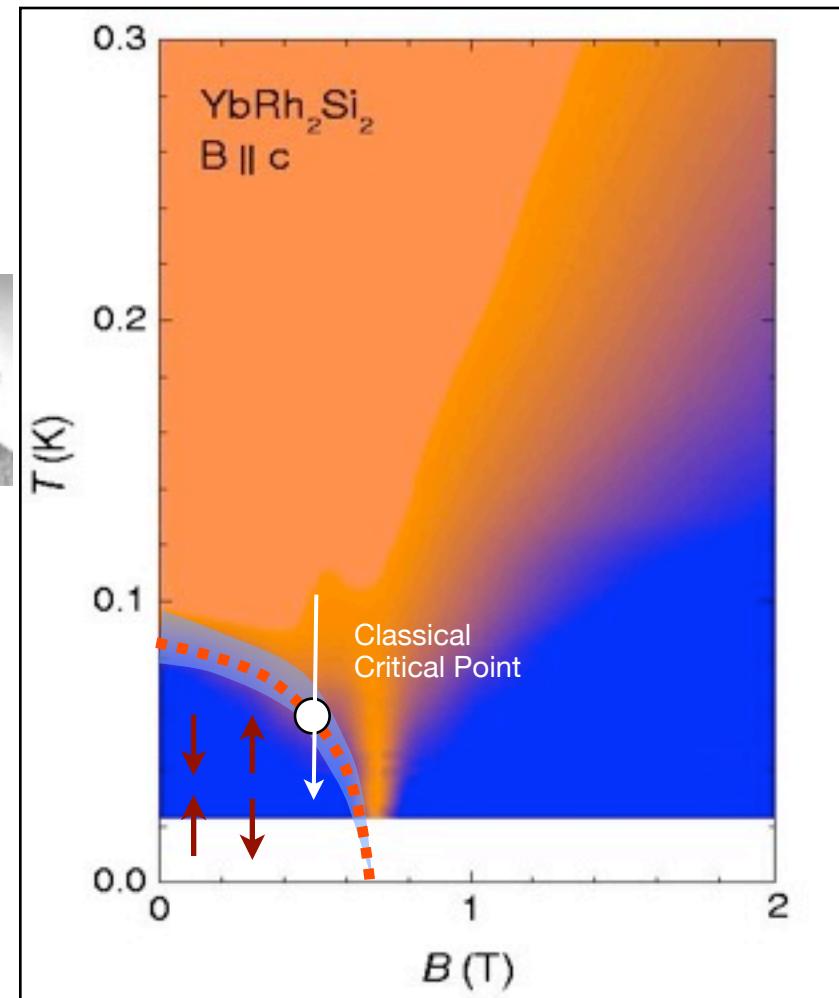
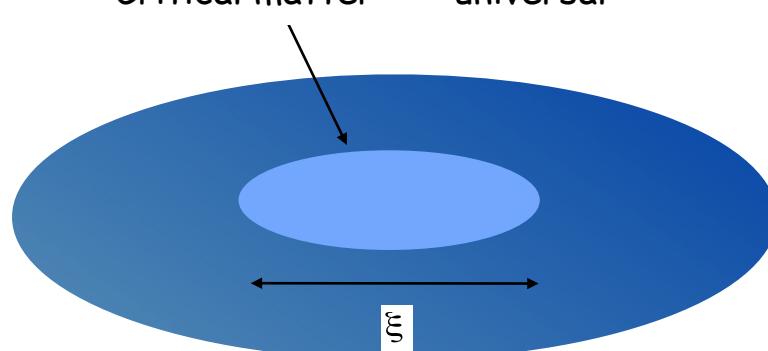
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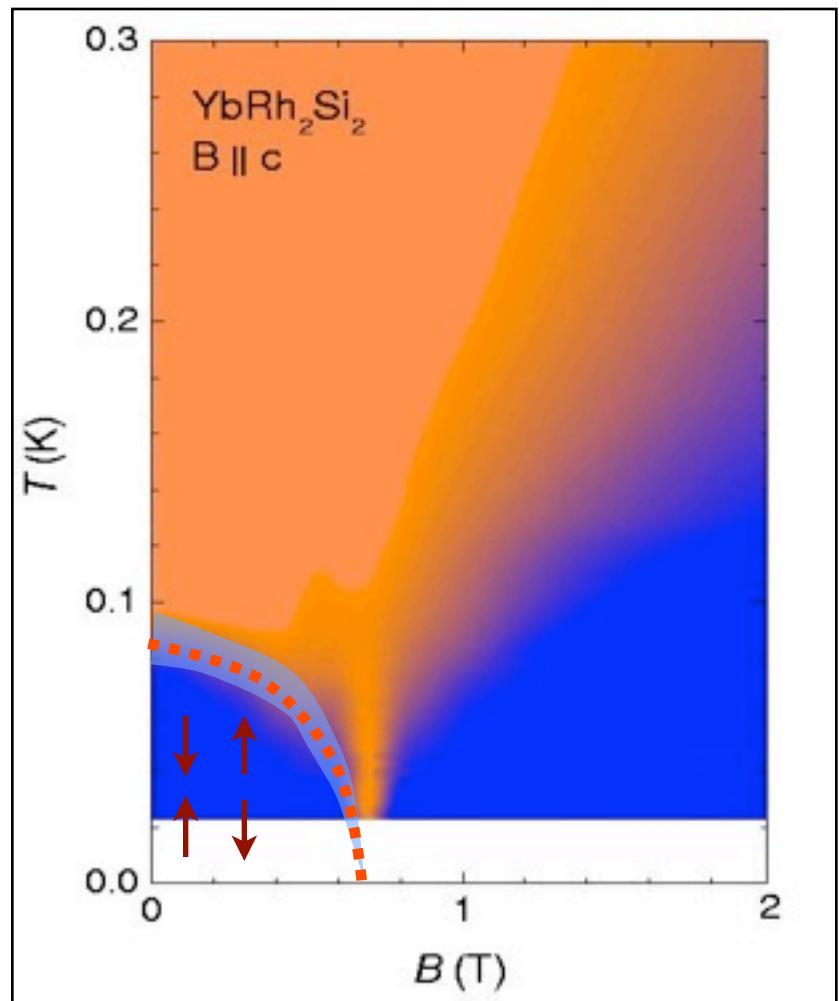
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Critical matter - universal



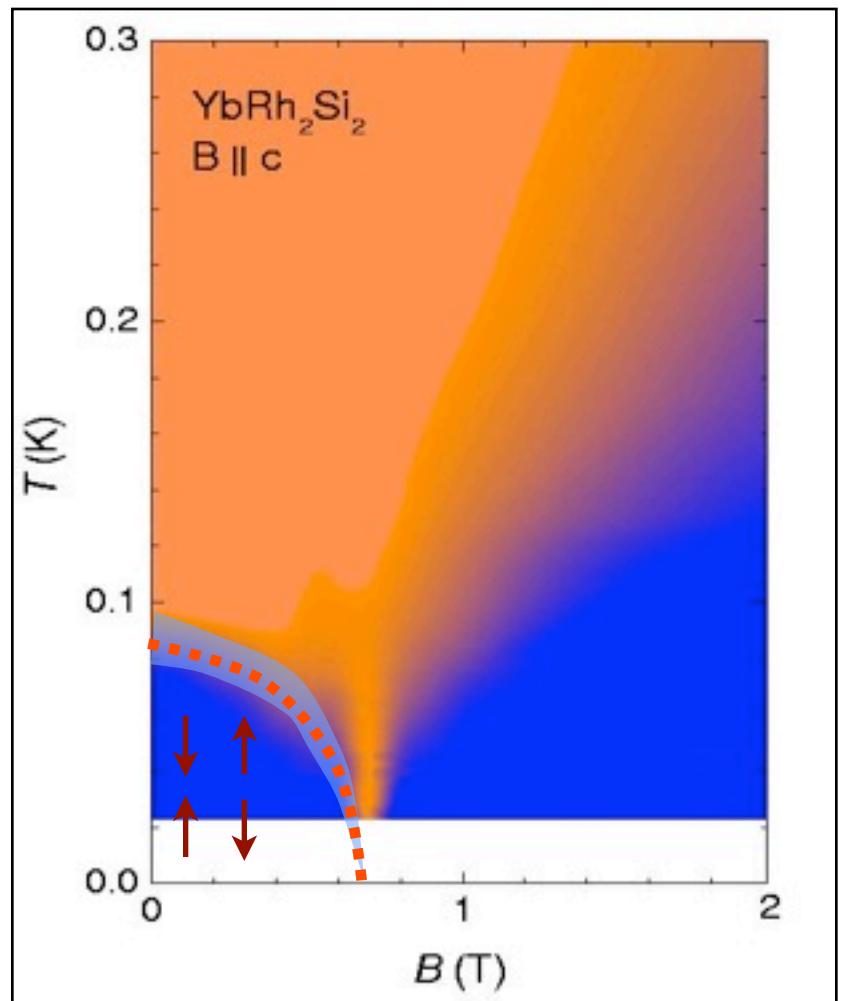
"20th Century Revolution"

# Quantum Phase-Transition



# Quantum Phase-Transition

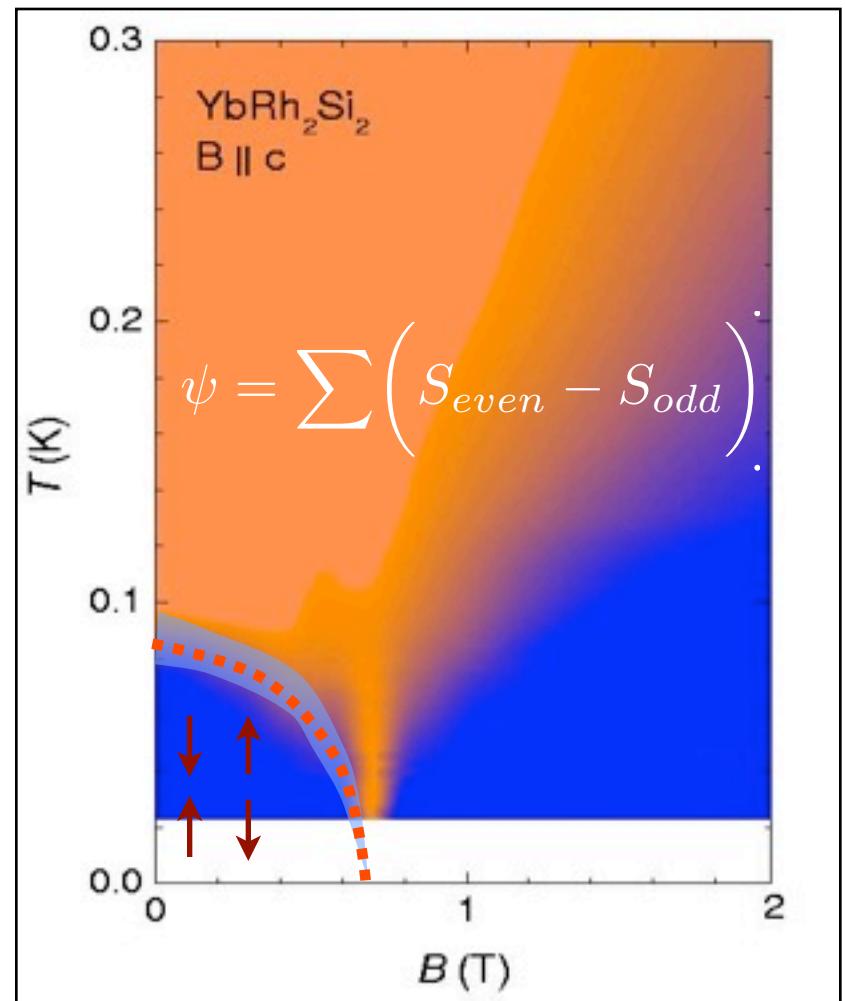
Phase transition  
driven by zero point motion.



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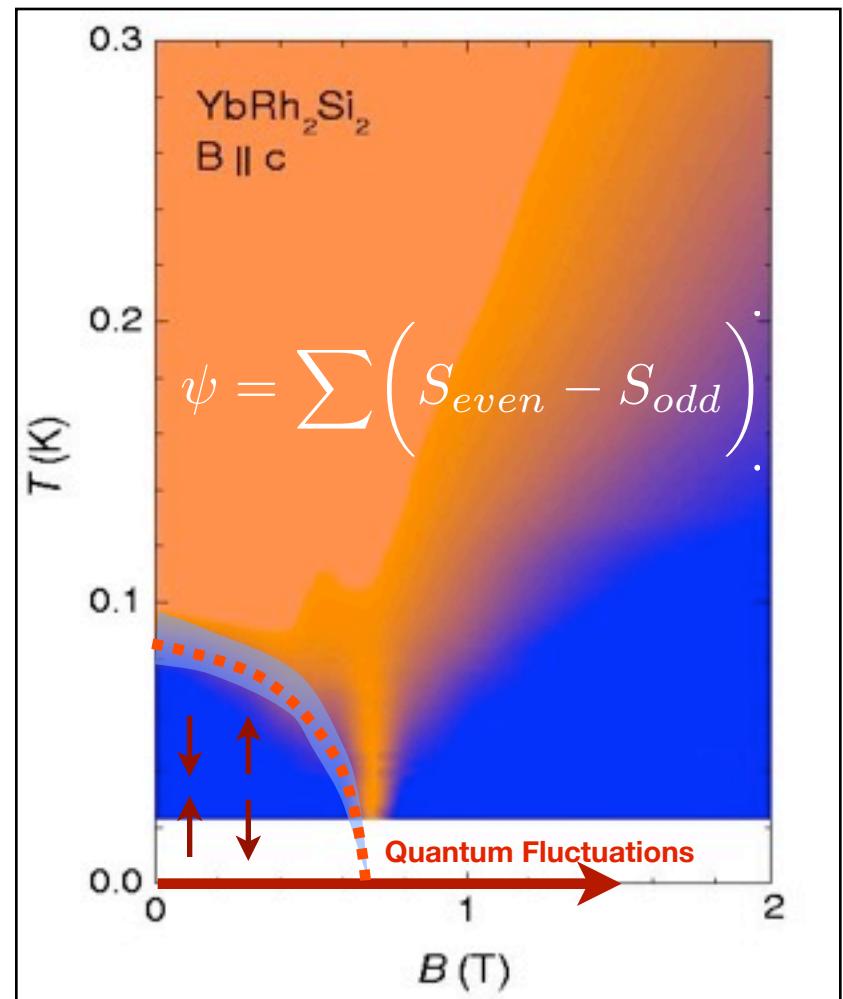
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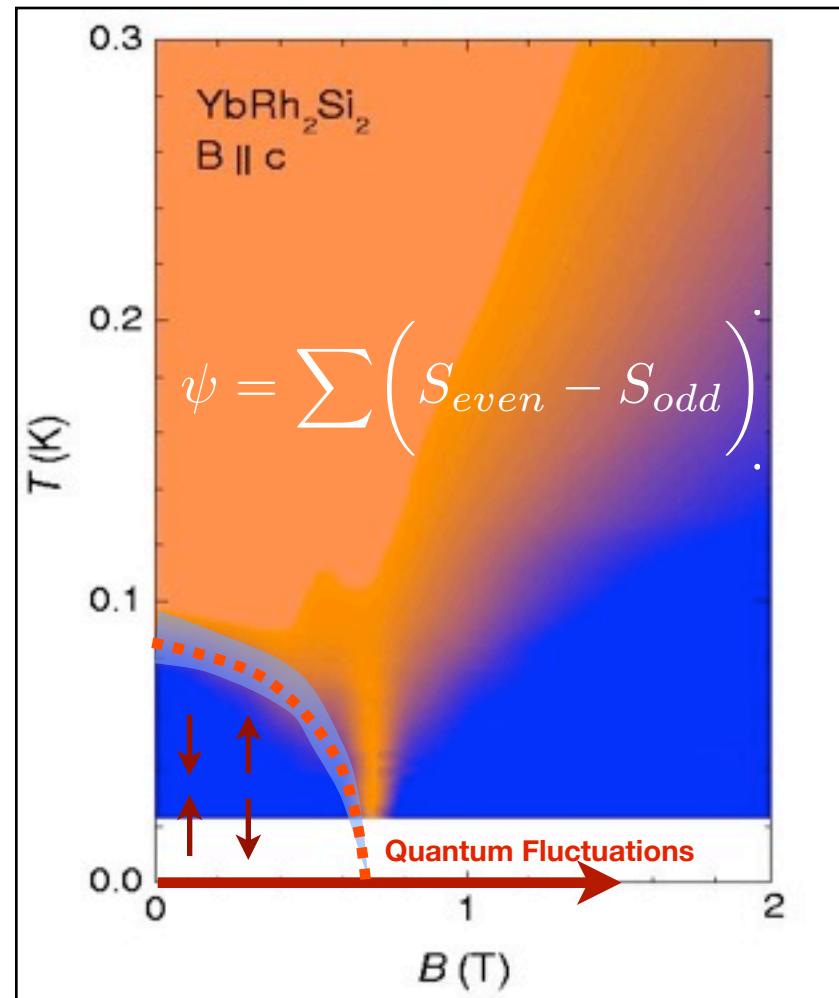
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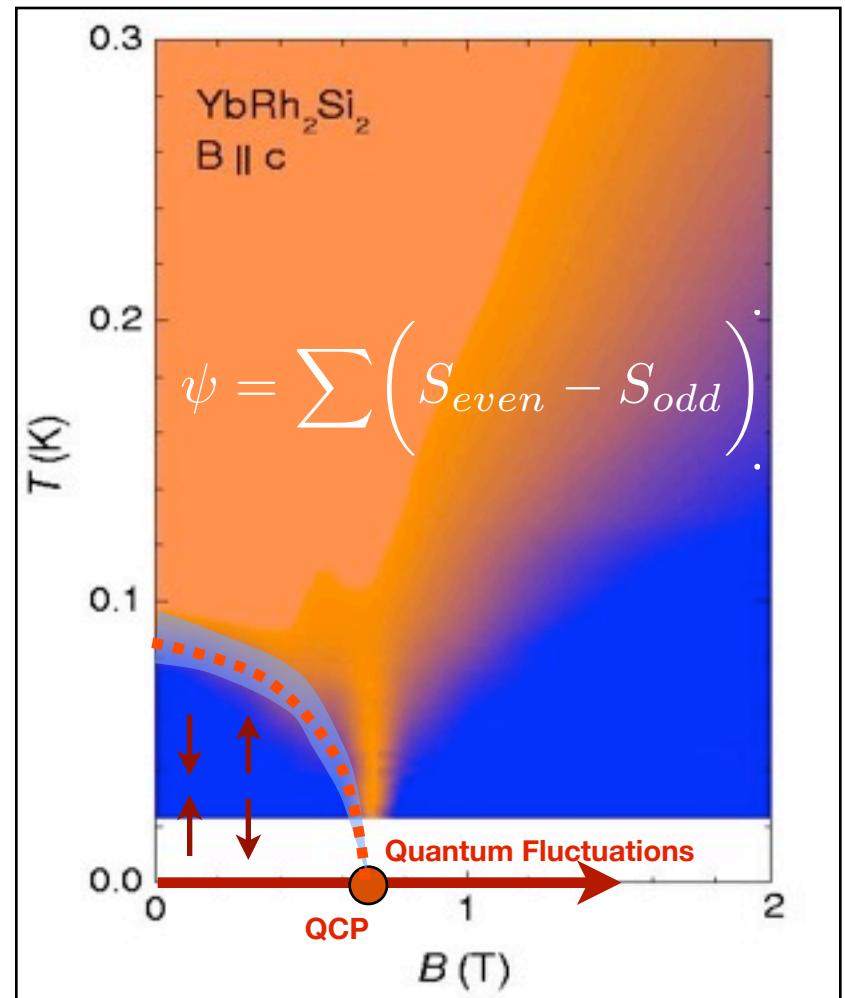


What happens when the time  
and length scale of zero point  
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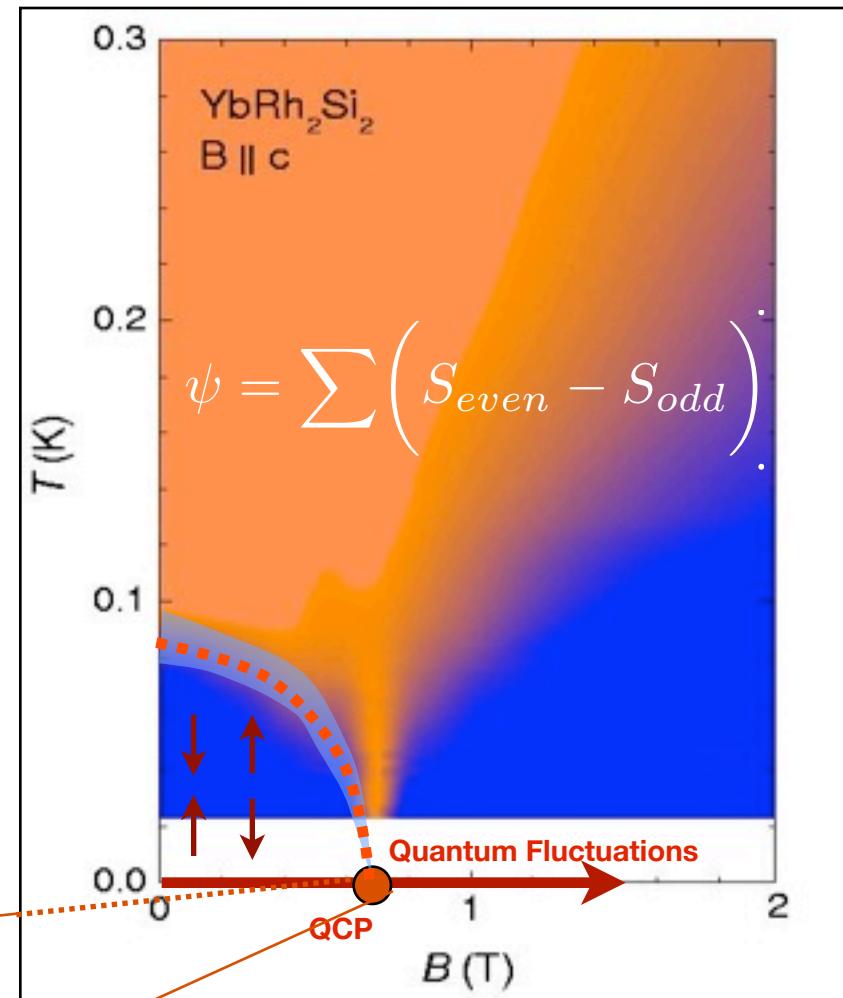
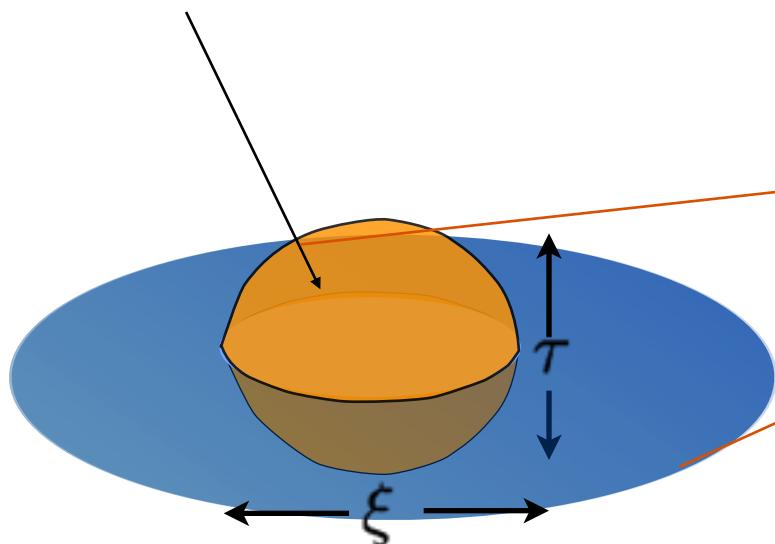
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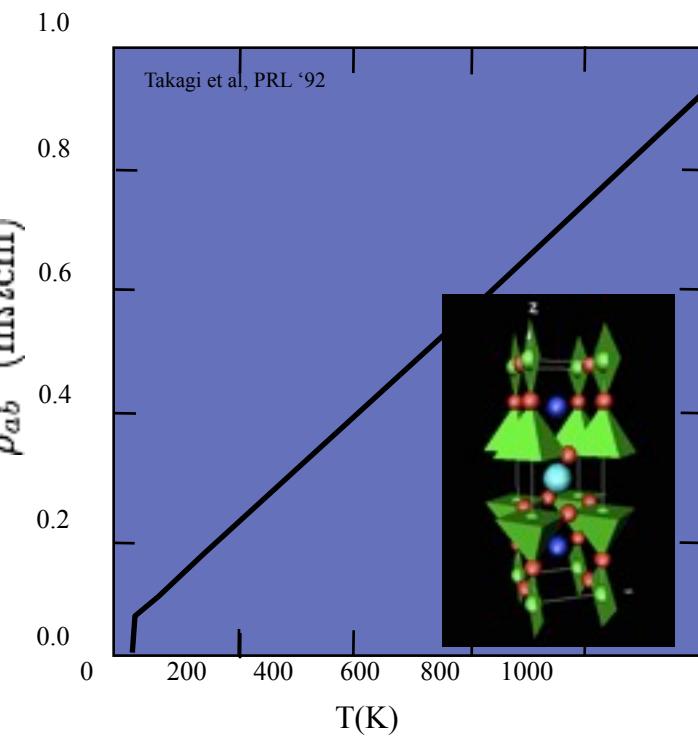
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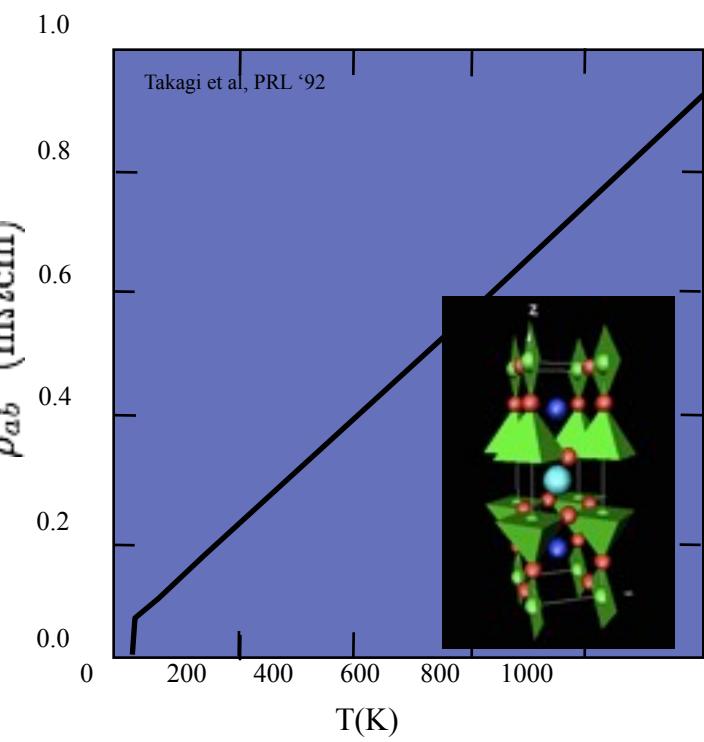
Quantum Critical matter



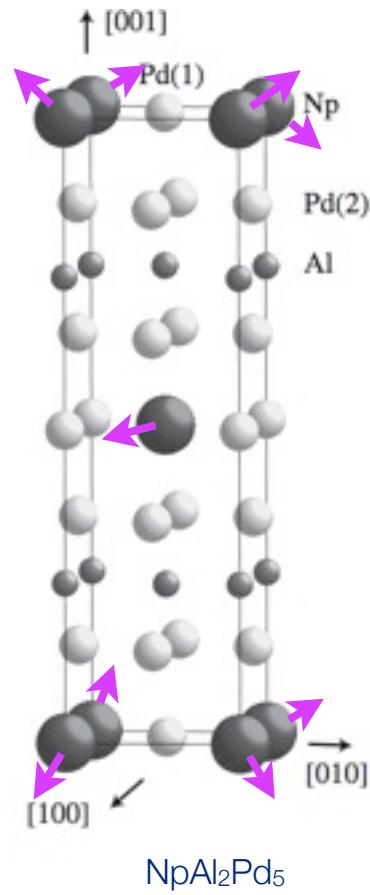
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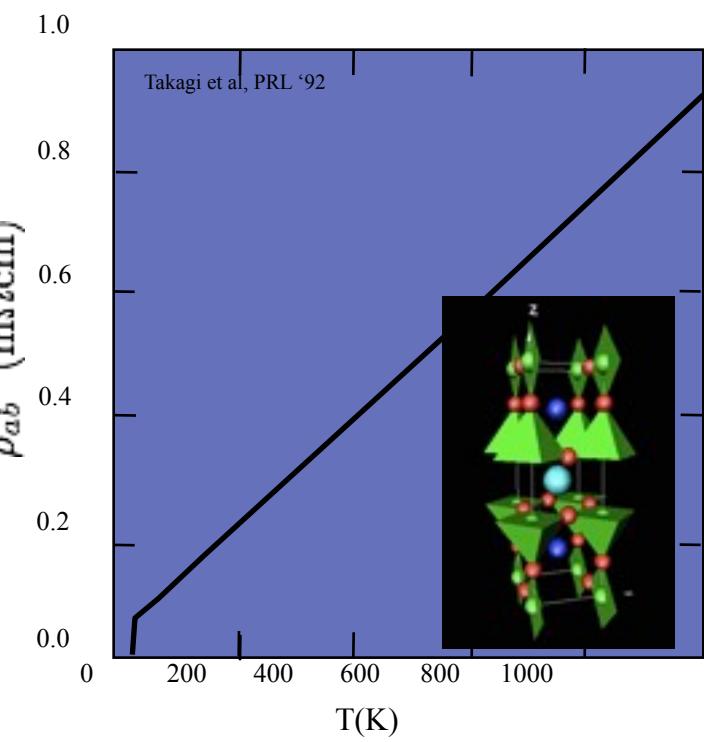
Cuprates  
Tc=11-92K



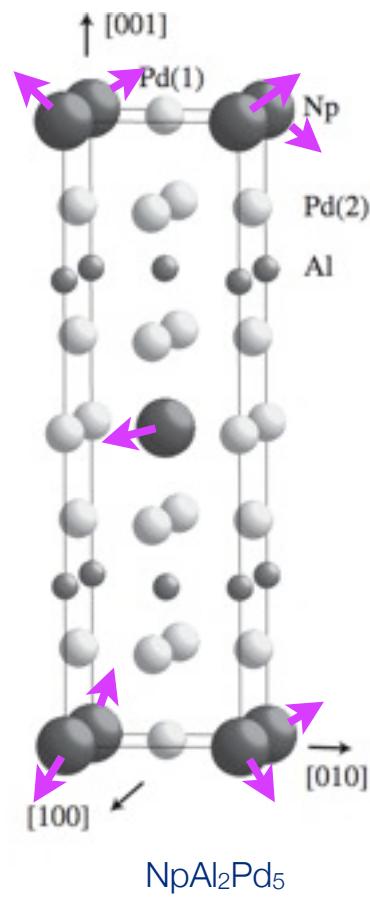
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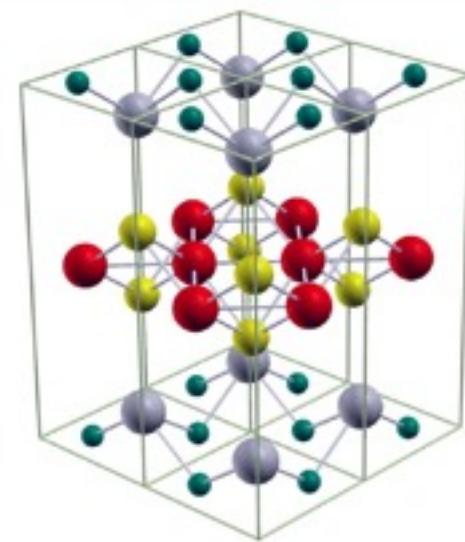
HF 115s  
Tc=0.2 -18.5 K



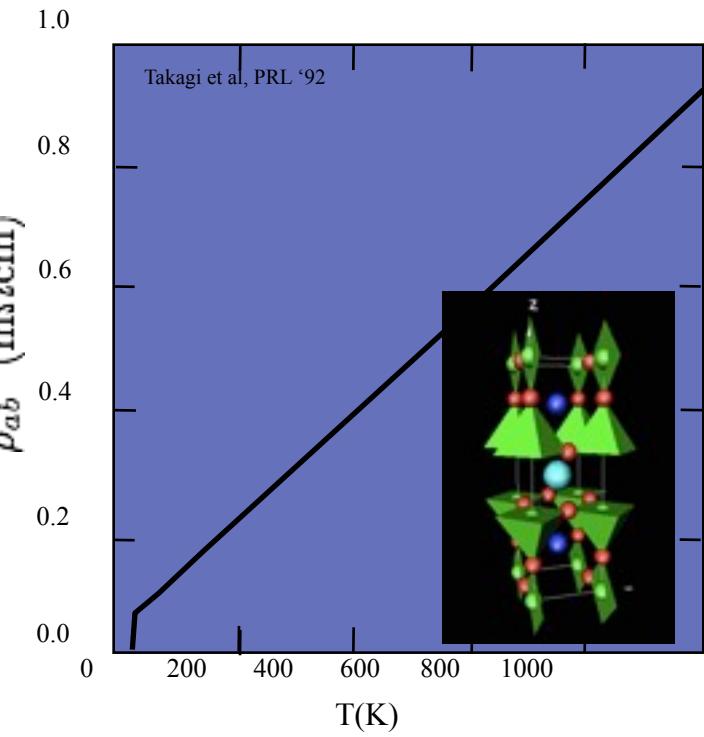
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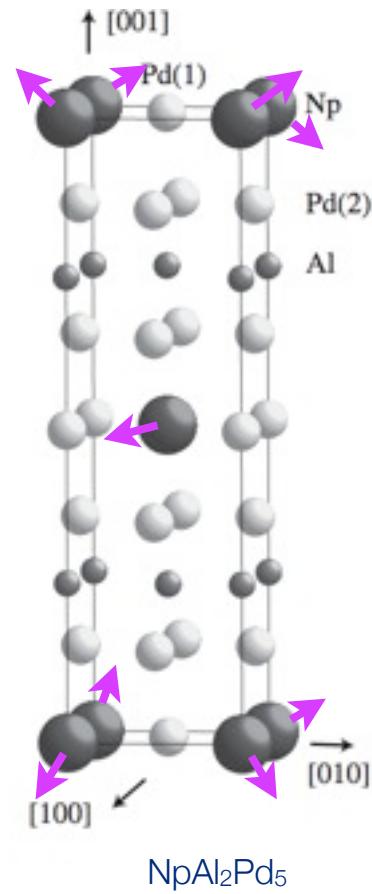
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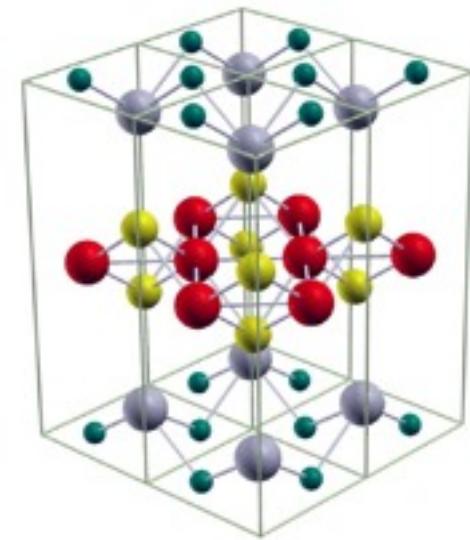
Iron based SC  
Tc= 6 - 53 ++ ? K



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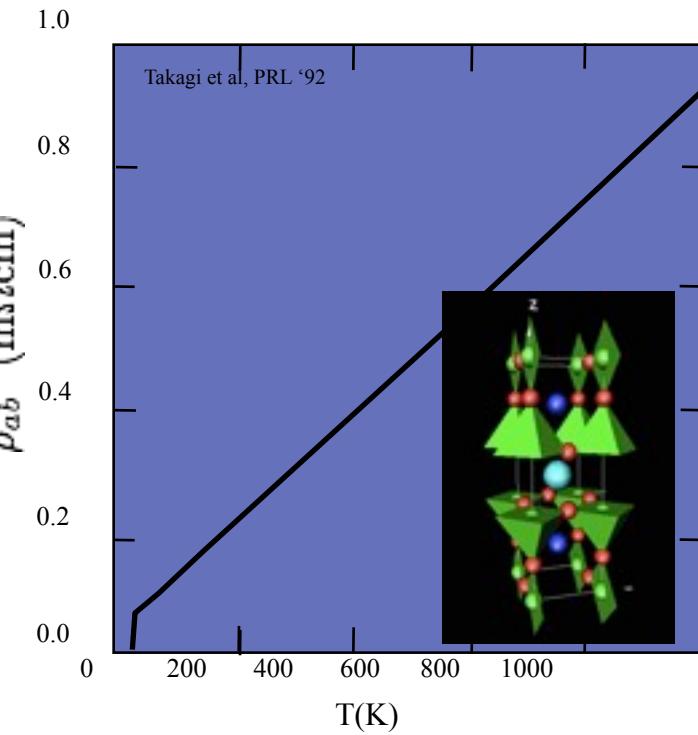


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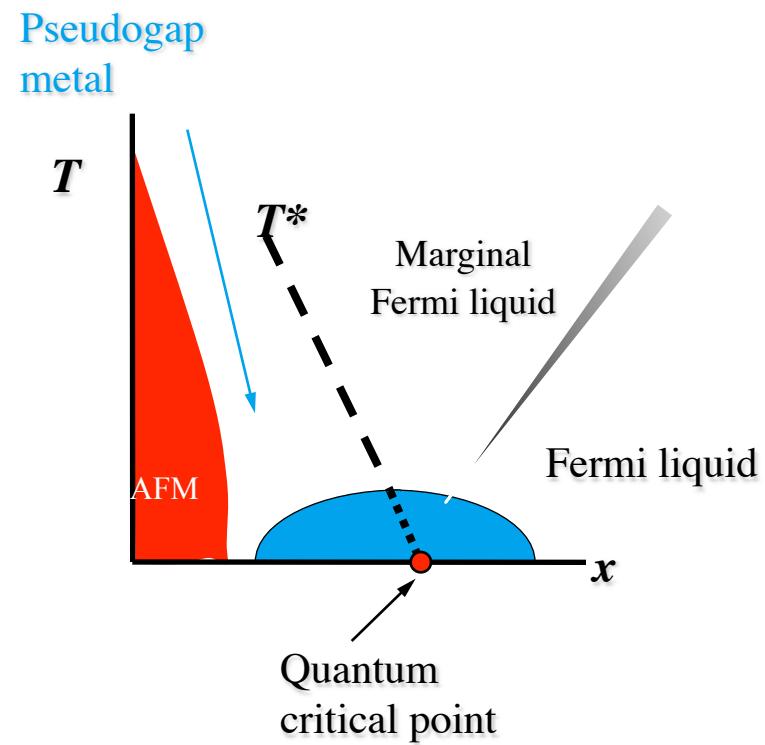


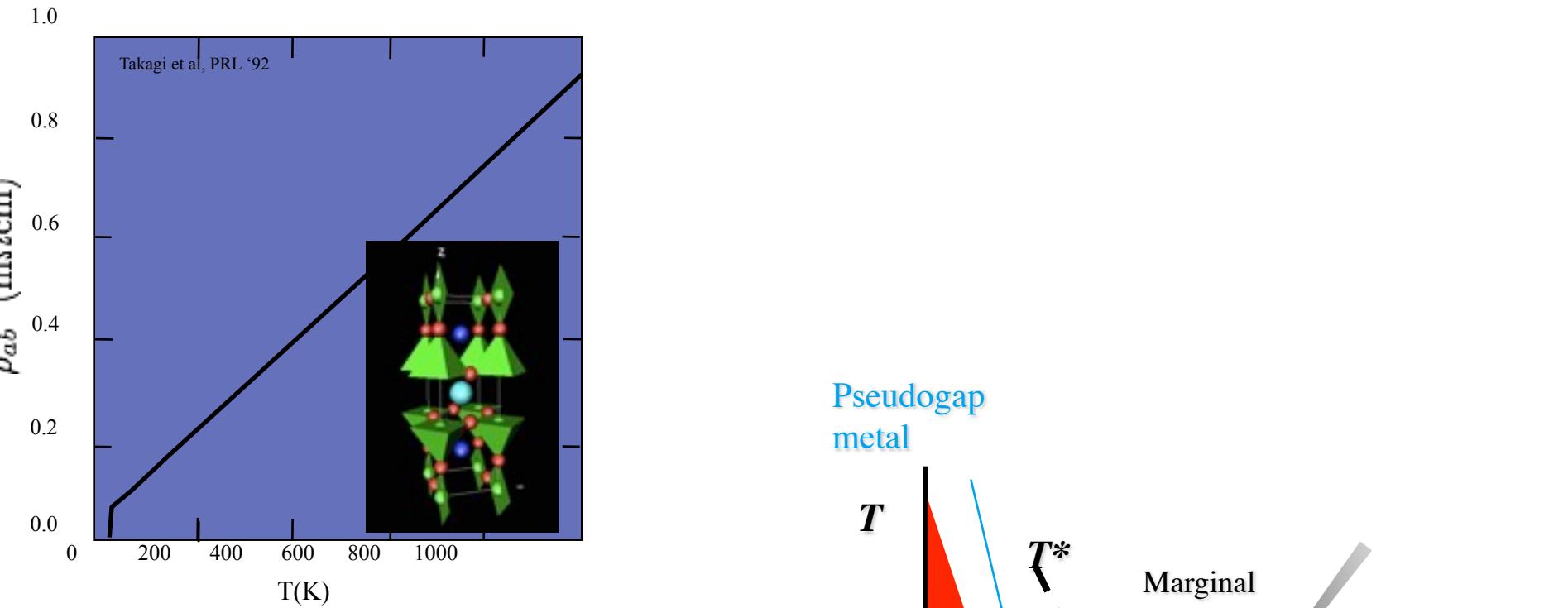
Iron based SC  
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What kind of electronic fluid yields  
high Tc superconductivity?



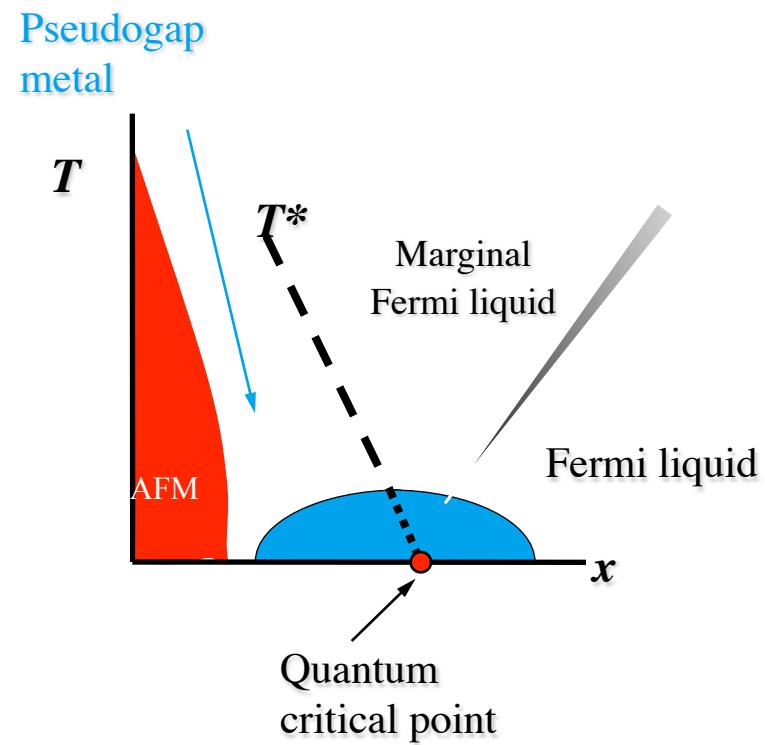
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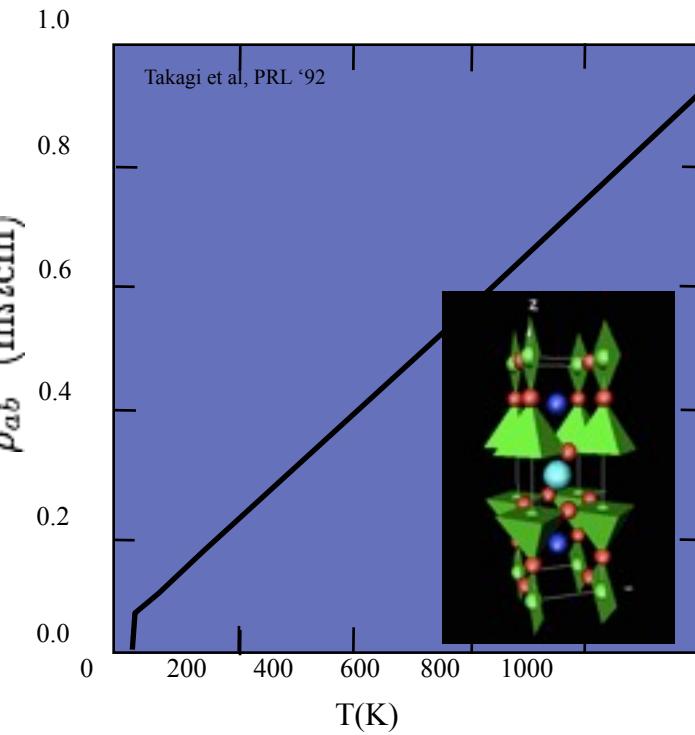




Cuprates  
 $T_c=11\text{-}92K$

“Avoided criticality”

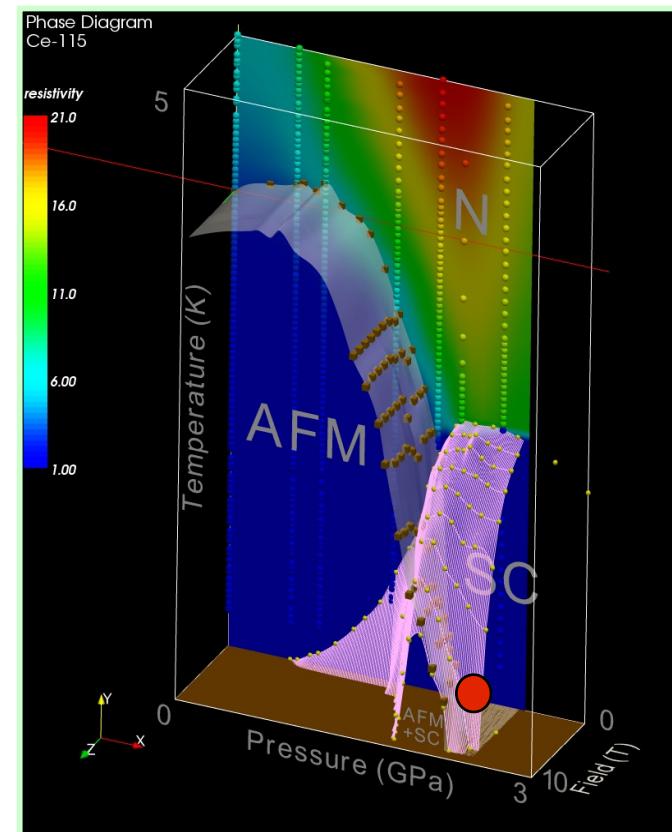




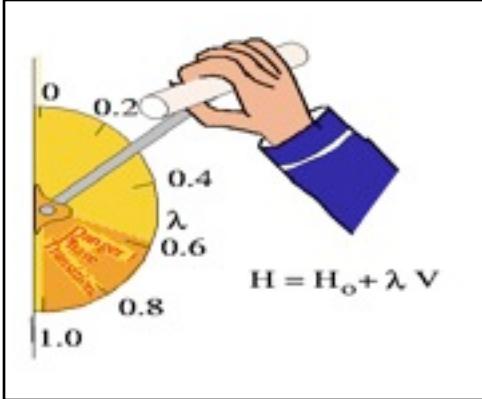
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## CeRhIn<sub>5</sub>

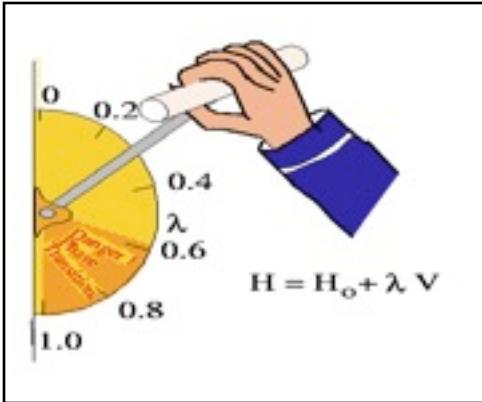


Tuson Park, (2007).



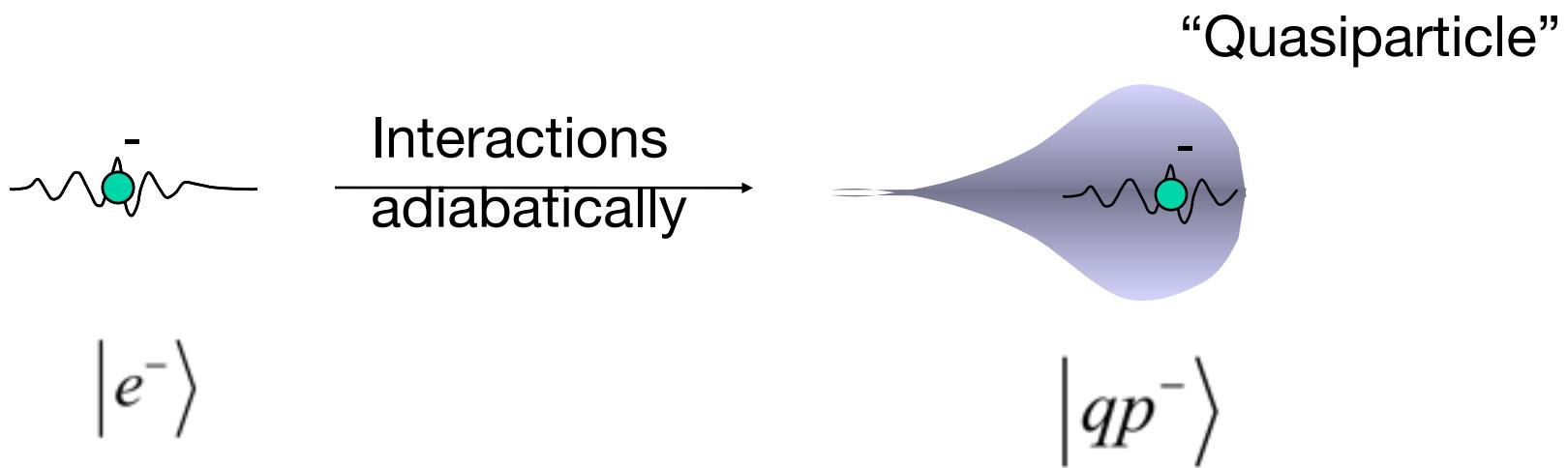
Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

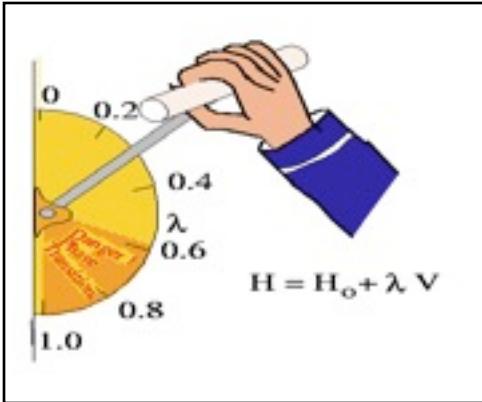
Landau, JETP 3, 920 (1957)



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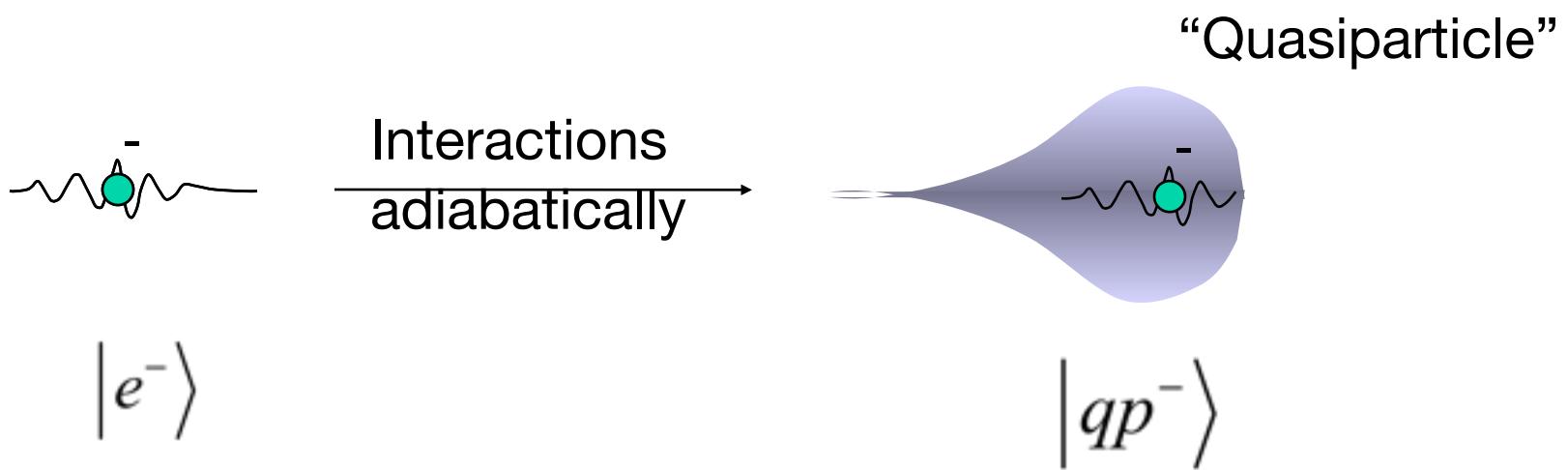
Landau, JETP 3, 920 (1957)



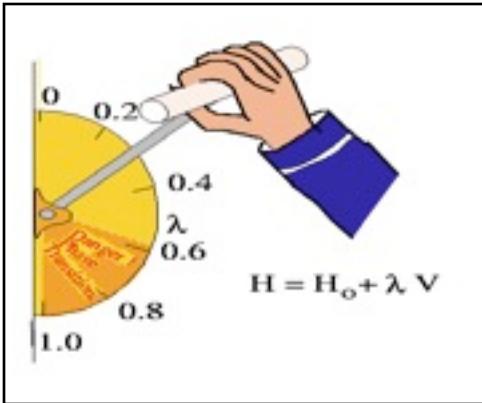


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Landau, JETP 3, 920 (1957)



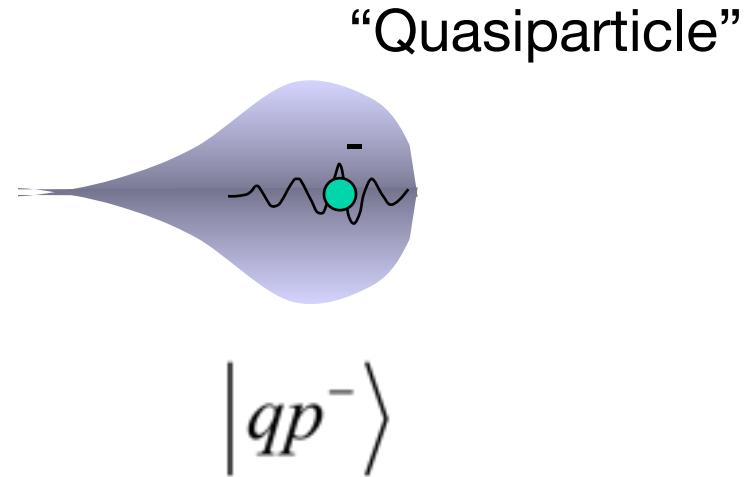
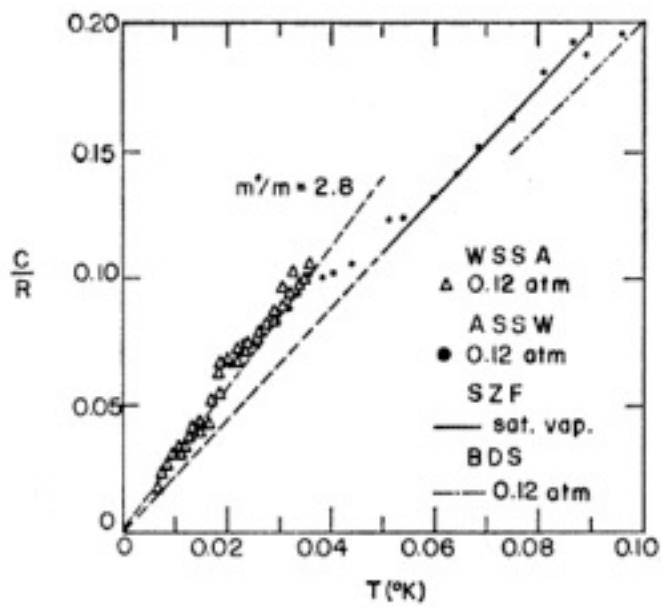
$$\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$$



Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

Landau, JETP 3, 920 (1957)

He-3 (1950/60s)  
(Fairbanks, many others)



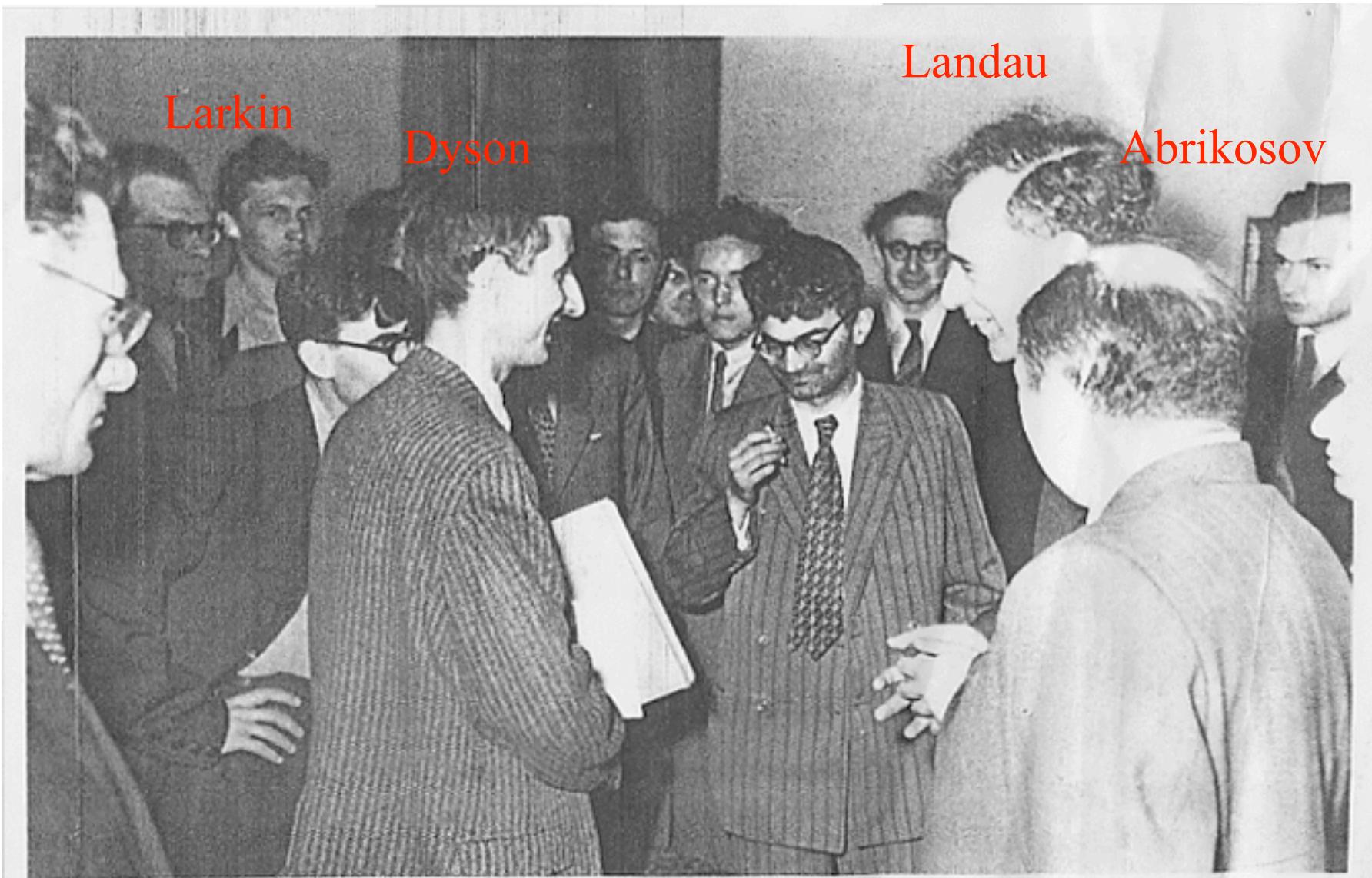
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# Landau's Question.



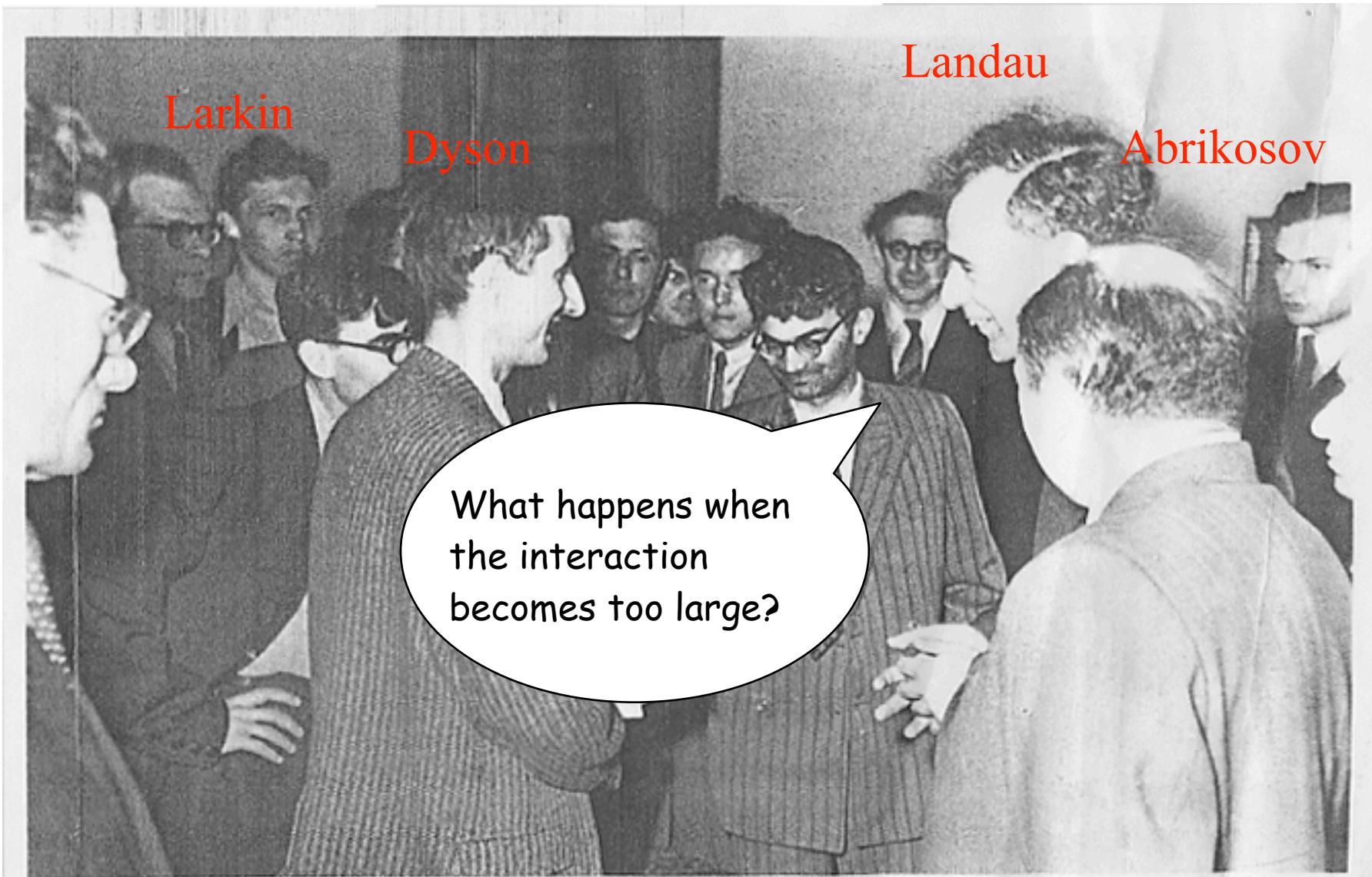
20. Moscow, 1956. Freeman Dyson (front, left), talking with I. Pomeranchuk and Lev Landau.

# Landau's Question.



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What happens when the interaction becomes too large?



Landau 1936



What happens when the interaction becomes too large?

“Electrons order”



Landau 1936



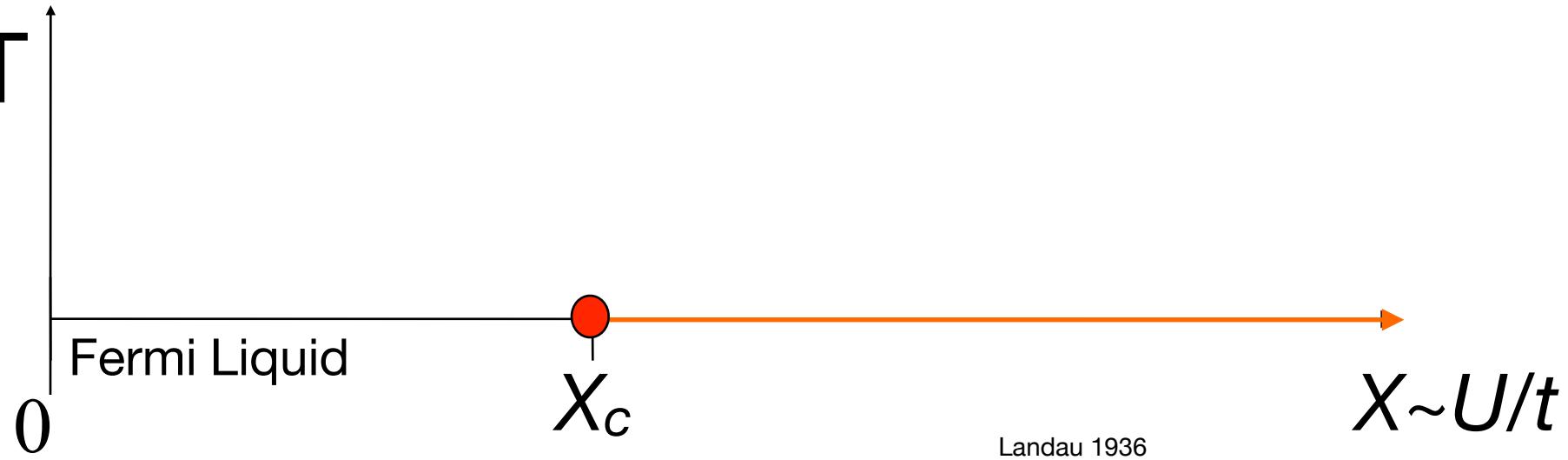
What happens when the interaction becomes too large?

“Electrons order”

Mott 1947



“Electrons localize”



What happens when the interaction becomes too large?

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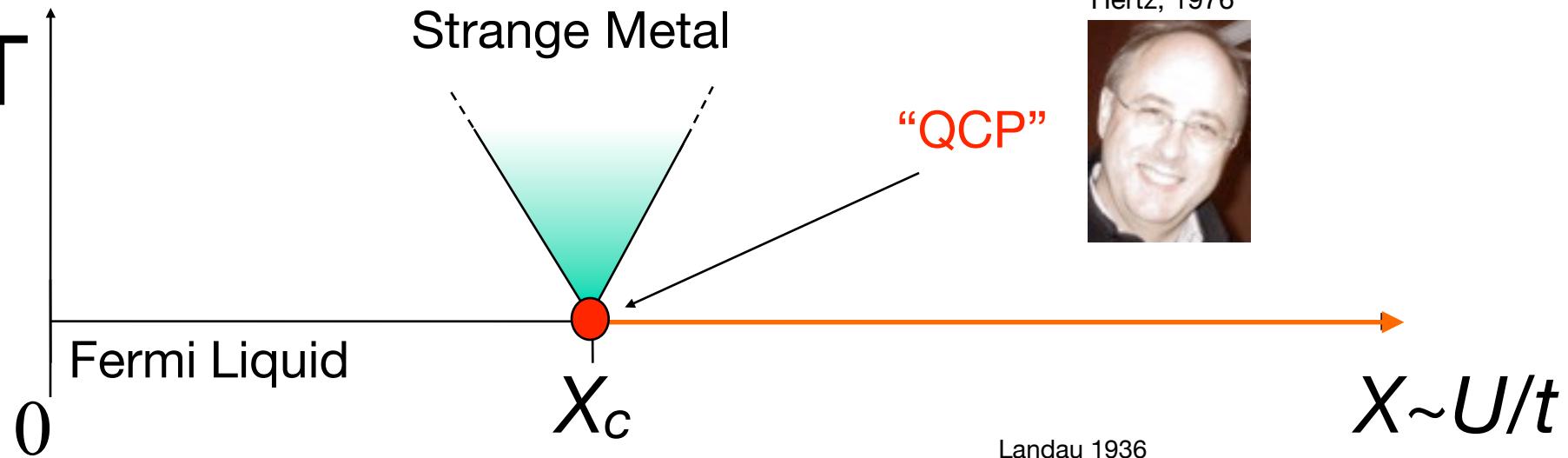


“Electrons localize”

Anderson 1961



“Local moments form”



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# Kondo effect

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(a digression)

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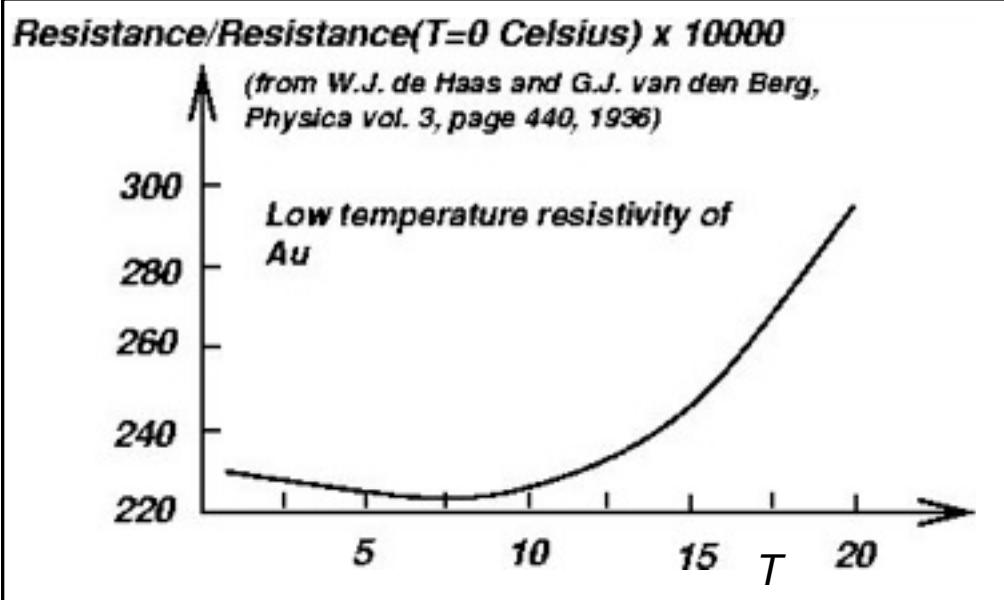
(a digression)



“Kondo”

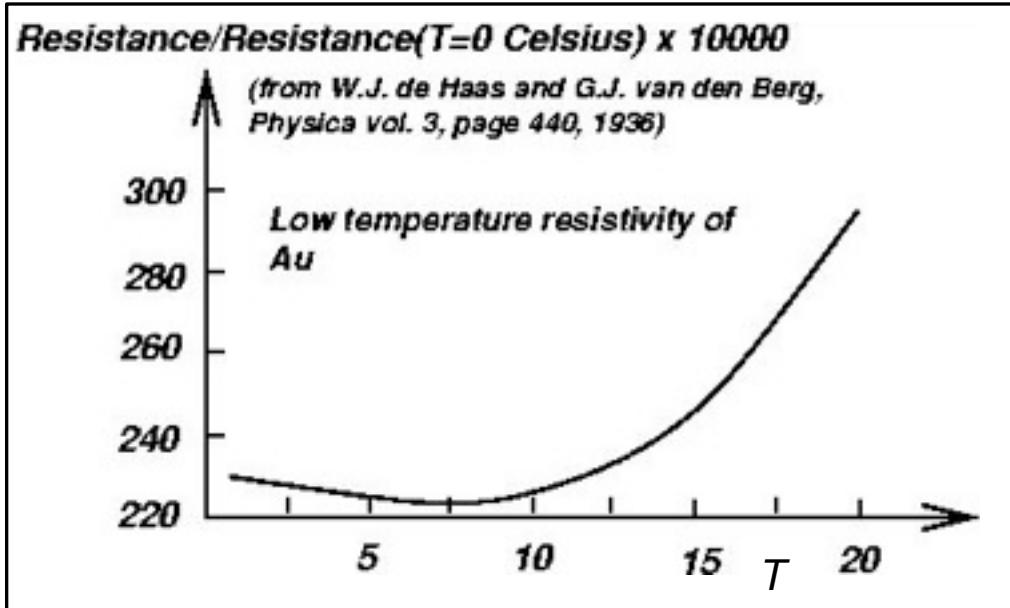
Kondo KHR-2 HV, the robot that plays soccer, fights with other bots and dances salsa

# Kondo effect (a digression)



# Kondo effect

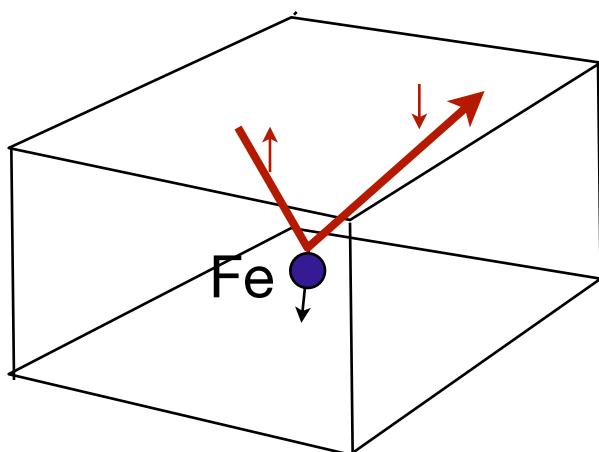
(a digression)



“A 75 year odyssey”

# Kondo effect

(a digression)



$$H = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J(\psi^\dagger \vec{\sigma} \psi) \cdot \vec{S}$$

Kondo (1962)

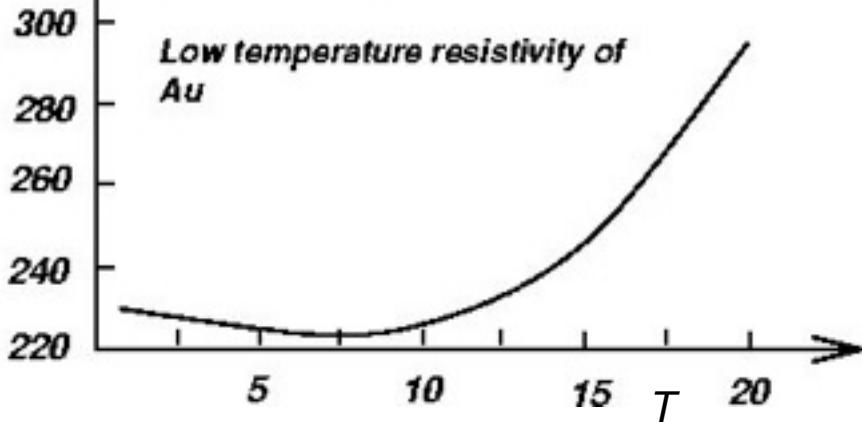


$$T_K = D\sqrt{J\rho} \exp\left[-\frac{1}{2J\rho}\right]$$

“Kondo Temperature”

Resistance/Resistance( $T=0$  Celsius)  $\times 10000$

(from W.J. de Haas and G.J. van den Berg,  
*Physica* vol. 3, page 440, 1936)

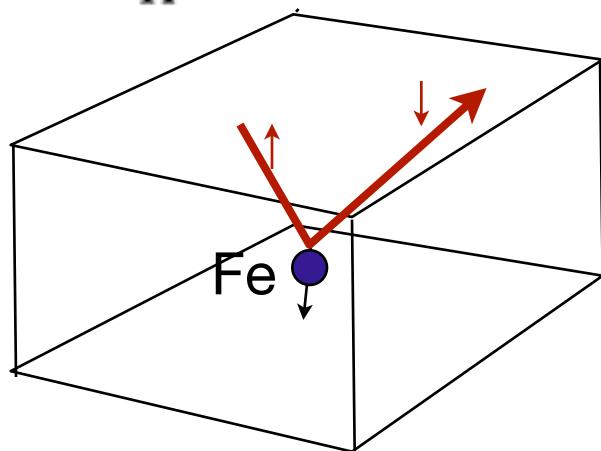


“A 75 year odyssey”

# Kondo effect

(a digression)

$T \gg T_K$



Spins asymptotically free

$$H = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J(\psi^\dagger \vec{\sigma} \psi) \cdot \vec{S}$$

Kondo (1962)

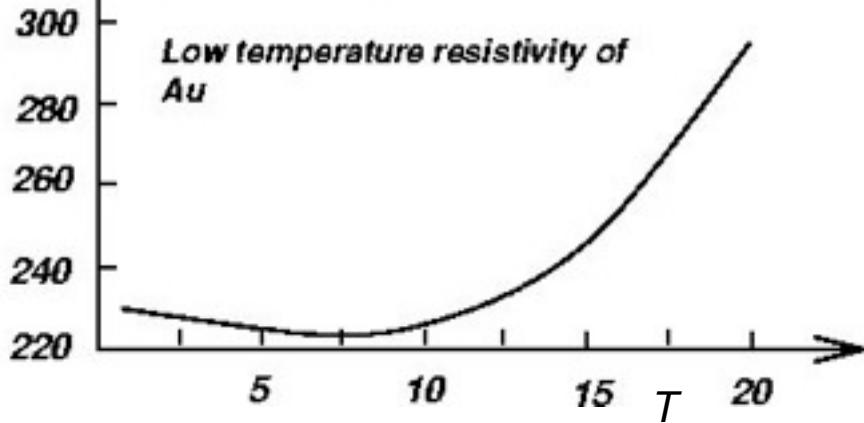


$$T_K = D\sqrt{J\rho} \exp\left[-\frac{1}{2J\rho}\right]$$

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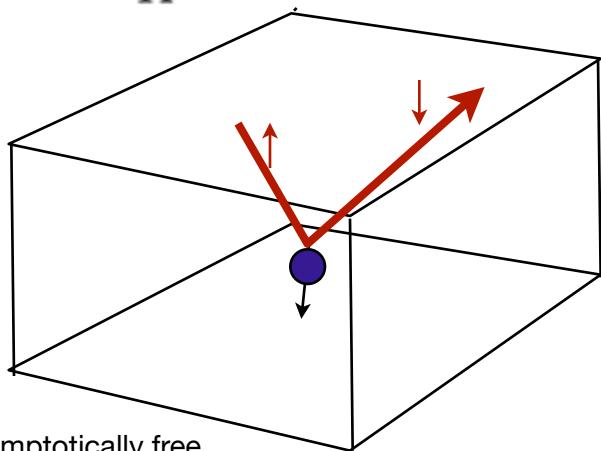
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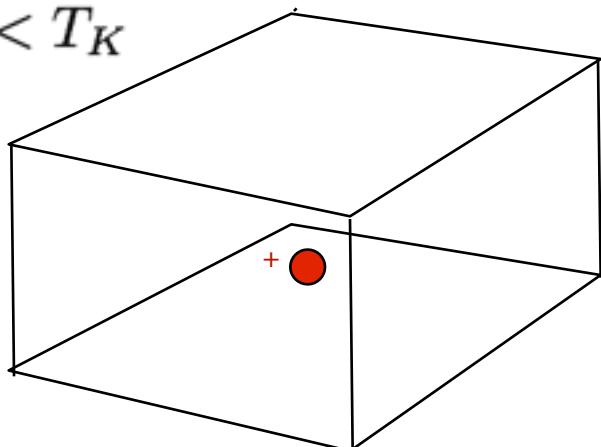


+

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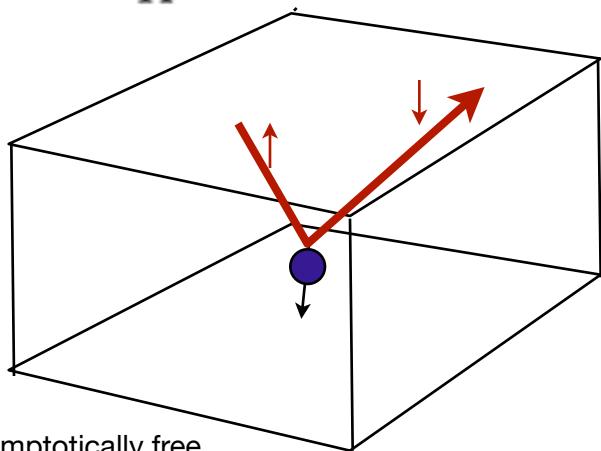
“Kondo Temperature”

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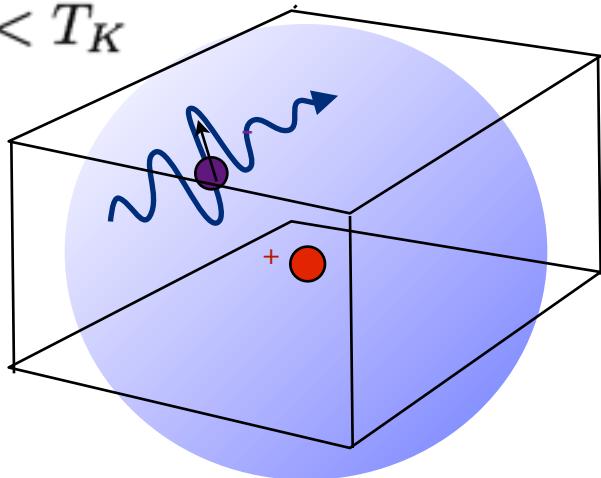


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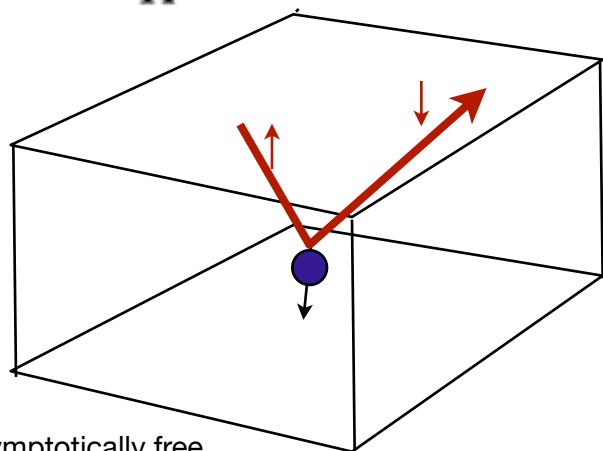
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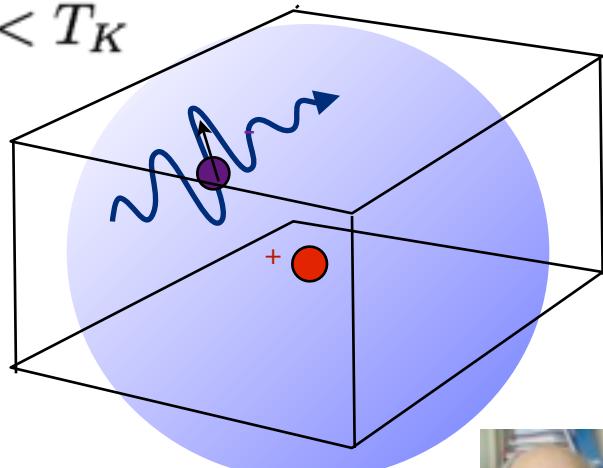


+

$$T_K = D\sqrt{J\rho} \exp\left[-\frac{1}{2J\rho}\right]$$

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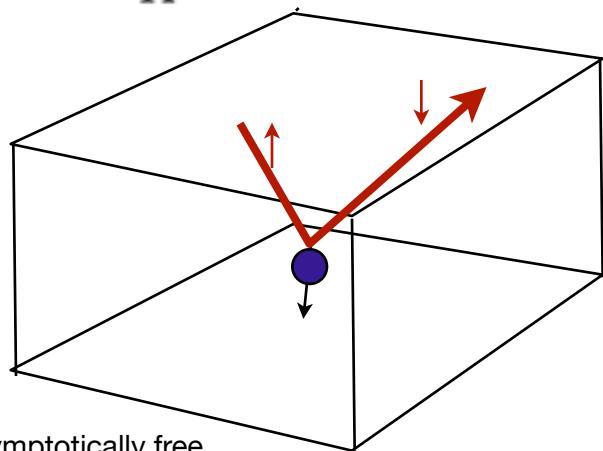


“Nozieres Local Fermi liquid”  
(Nozieres 76)



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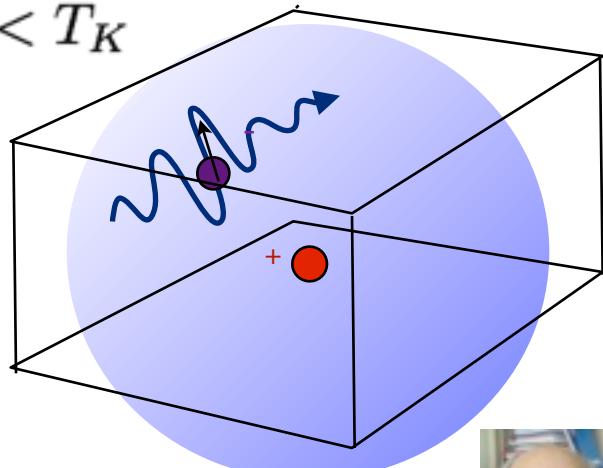


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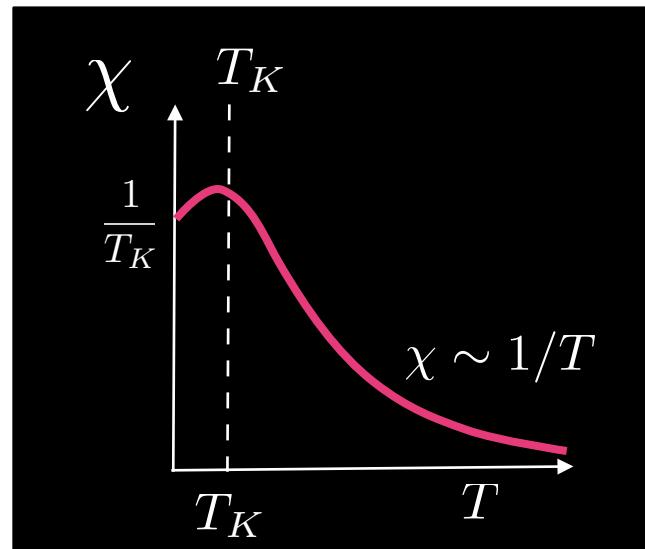
"Kondo Temperature"

$T \ll T_K$



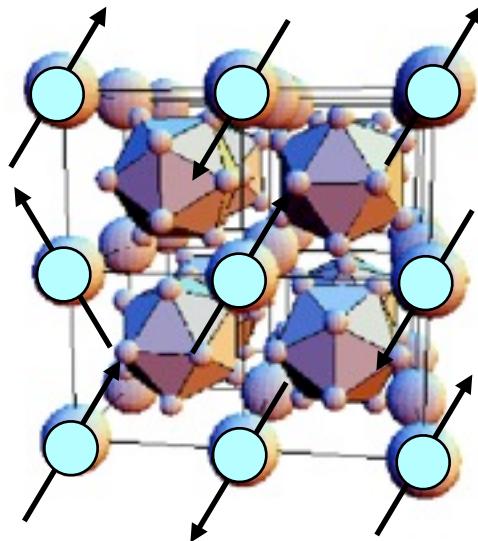
Spins absorbed into singlet ground-state

"Nozieres Local Fermi liquid"  
(Nozieres 76)



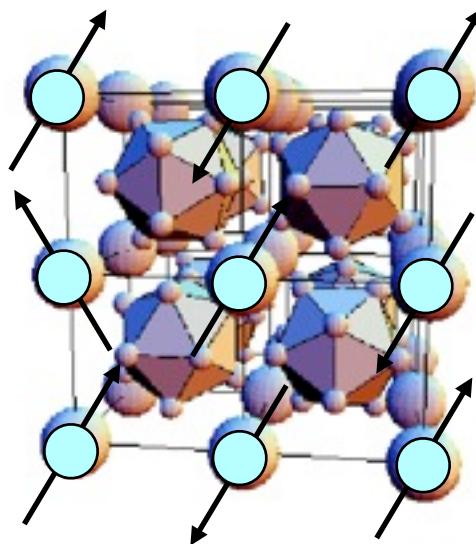
# Heavy Fermion Metals

[Review: cond-mat/0612006](https://arxiv.org/abs/cond-mat/0612006)

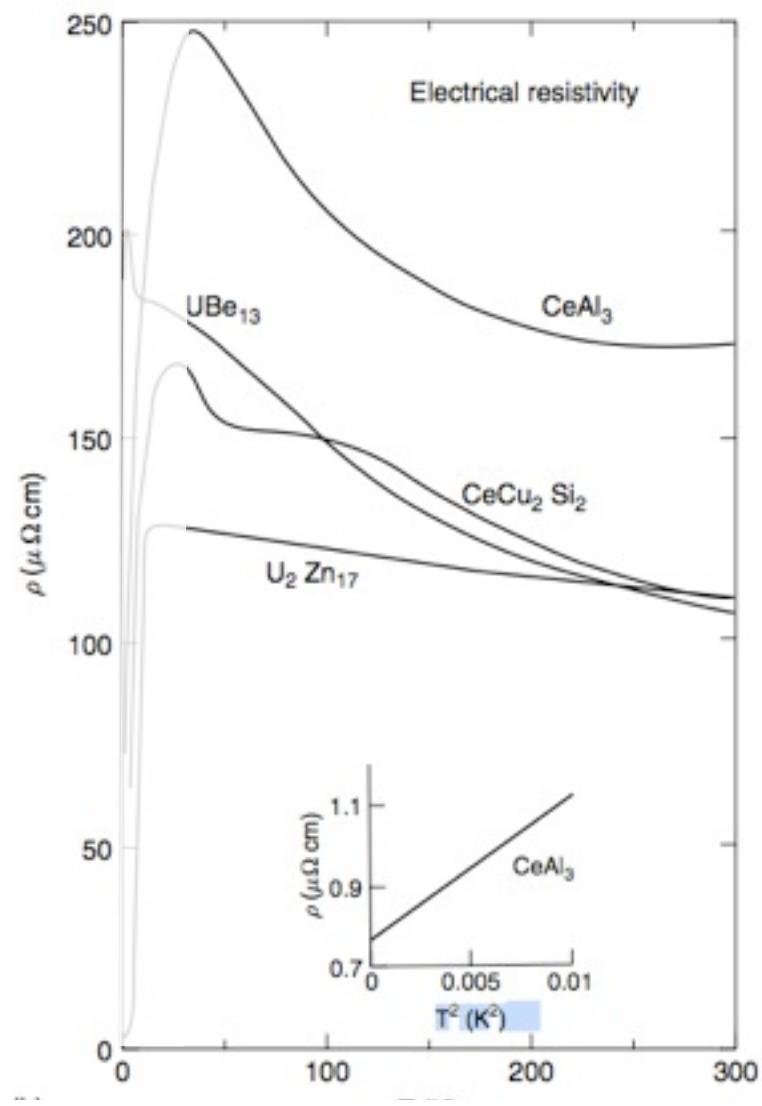


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[Review: cond-mat/0612006](https://arxiv.org/abs/cond-mat/0612006)

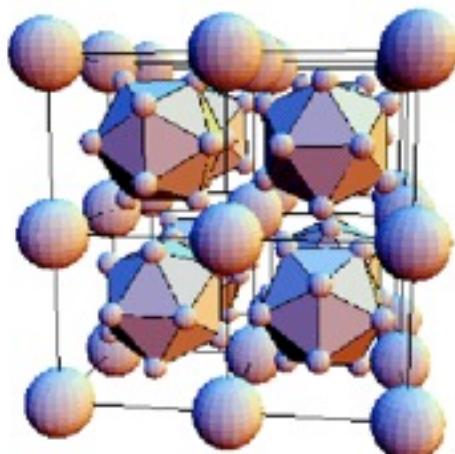


UBe<sub>13</sub>

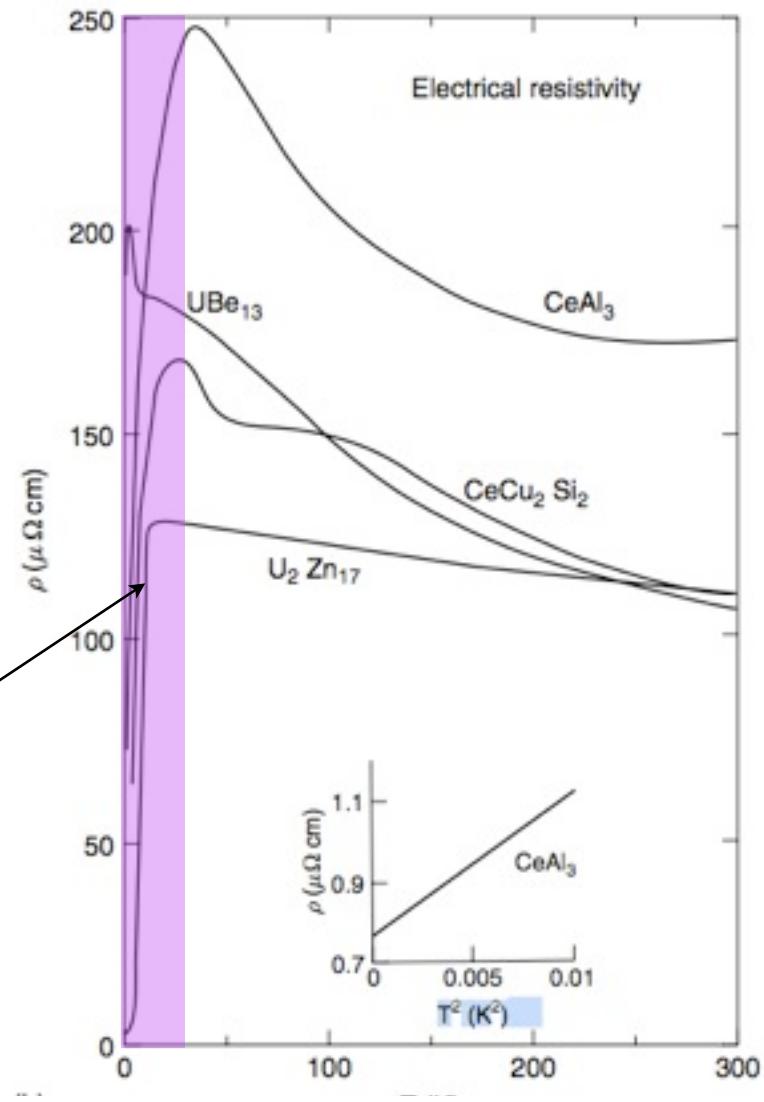


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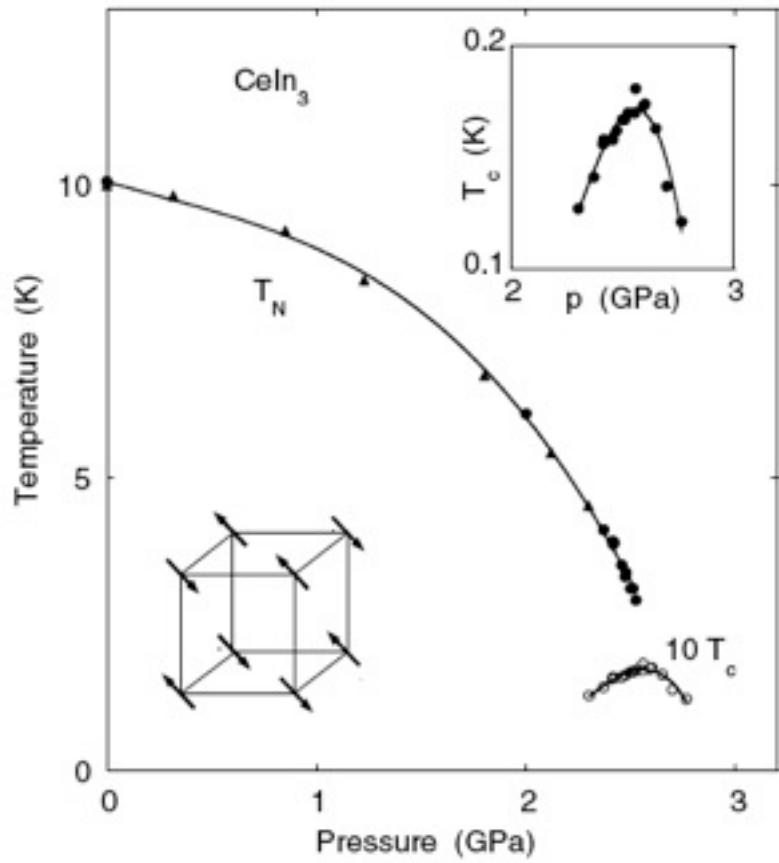
Coherent  
Heavy Fermi Liquid



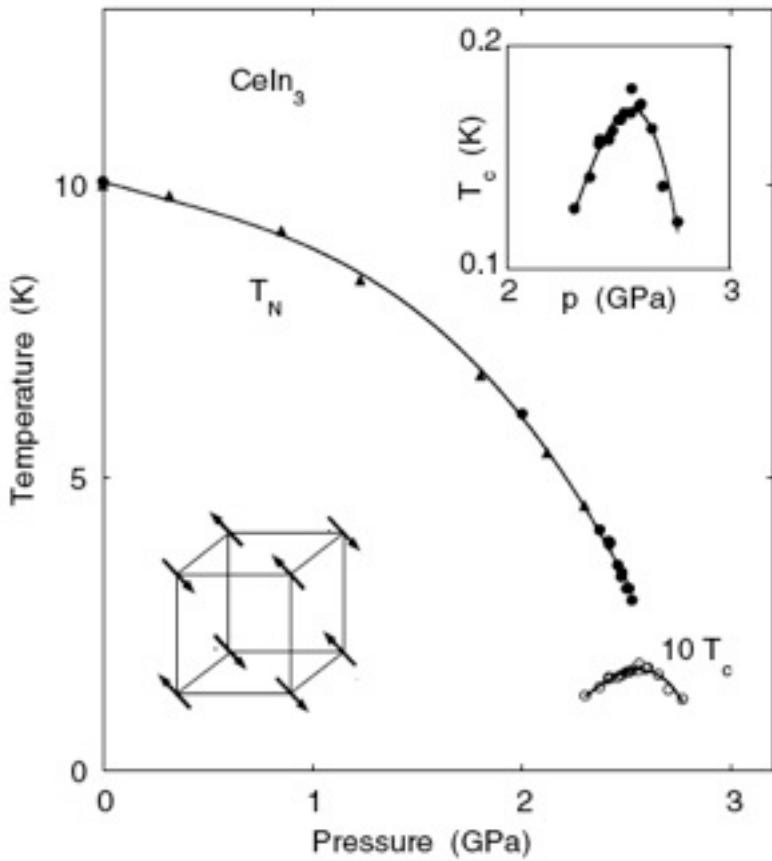
*Heavy Electron Quantum Criticality.*

# Experiments

# “Avoided Criticality”

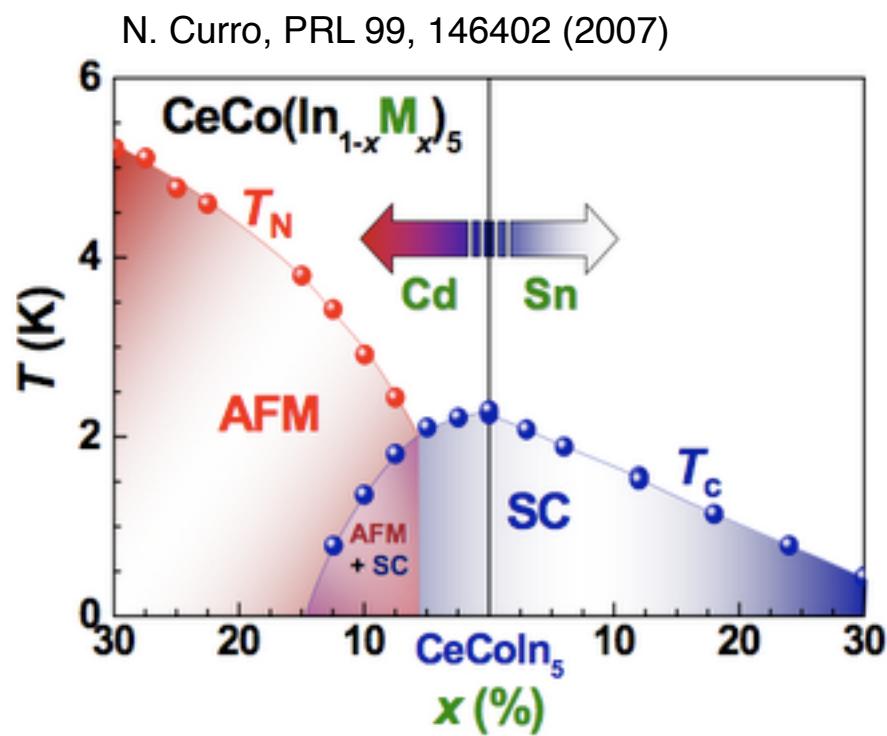


Mathur et al, Nature 394, 39 (1998)



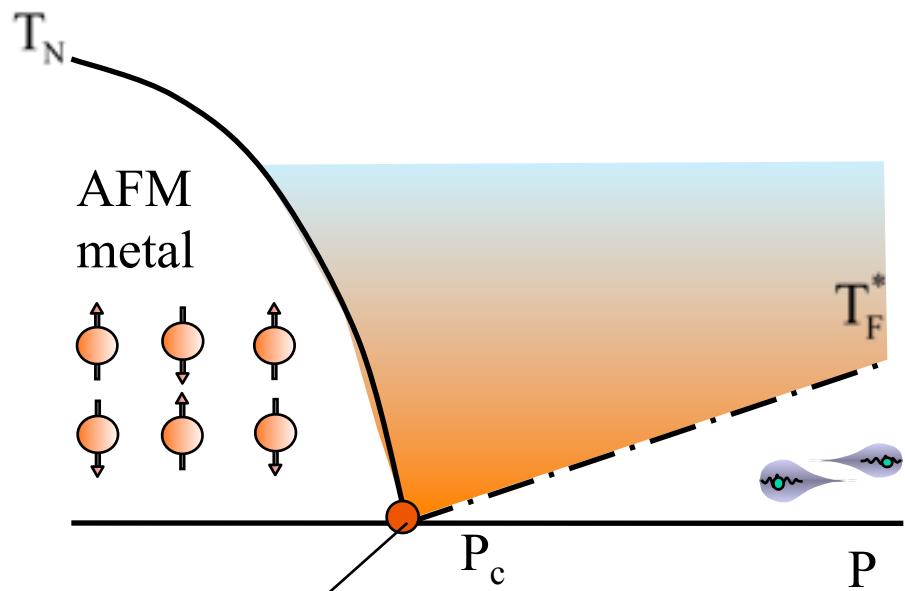
Mathur et al, Nature 394, 39 (1998)

# “Avoided Criticality”

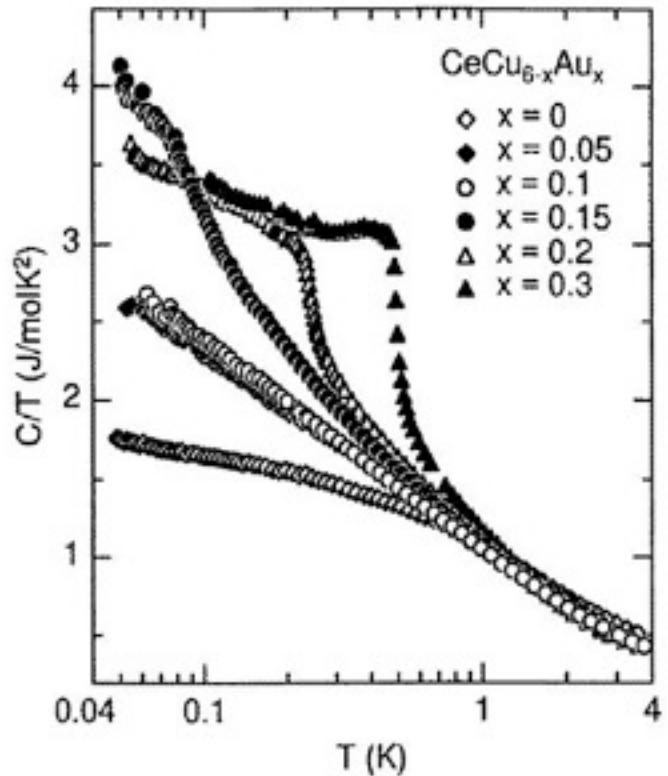


# Quantum Criticality: divergent specific heat capacity

Heavy Fermion  
Materials

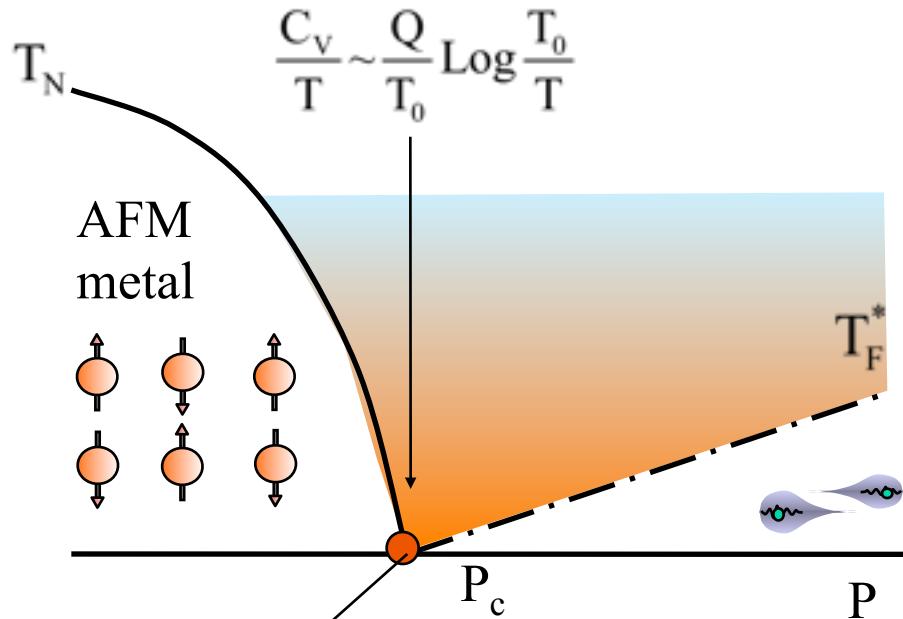


H. Von Lohneysen (1996)



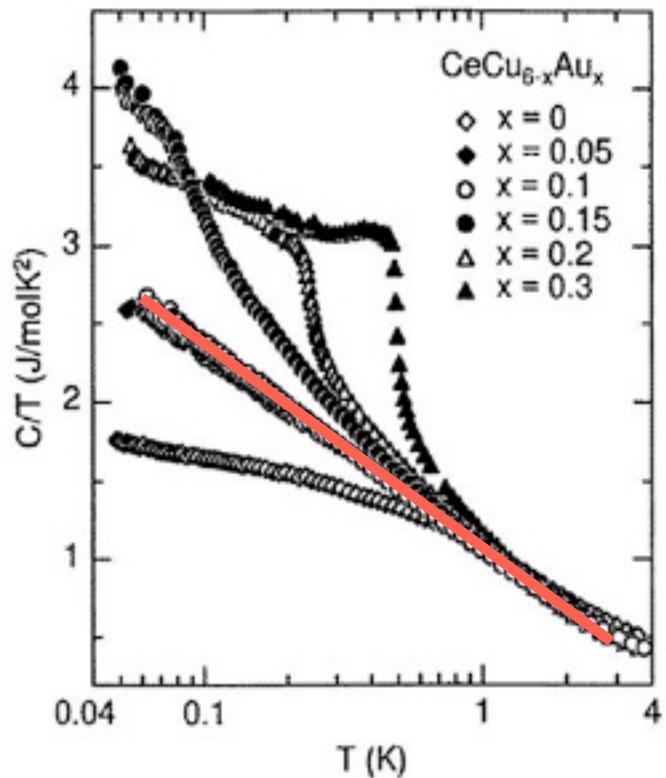
# Quantum Criticality: divergent specific heat capacity

Heavy Fermion  
Materials



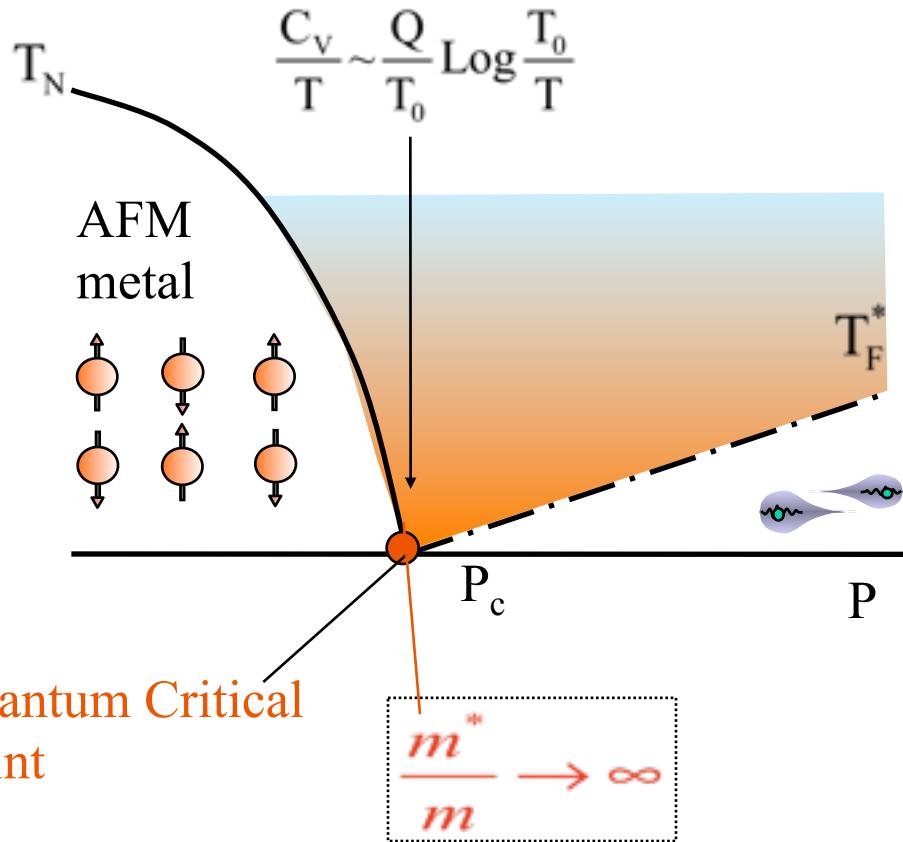
Quantum Critical  
Point

H. Von Lohneysen (1996)

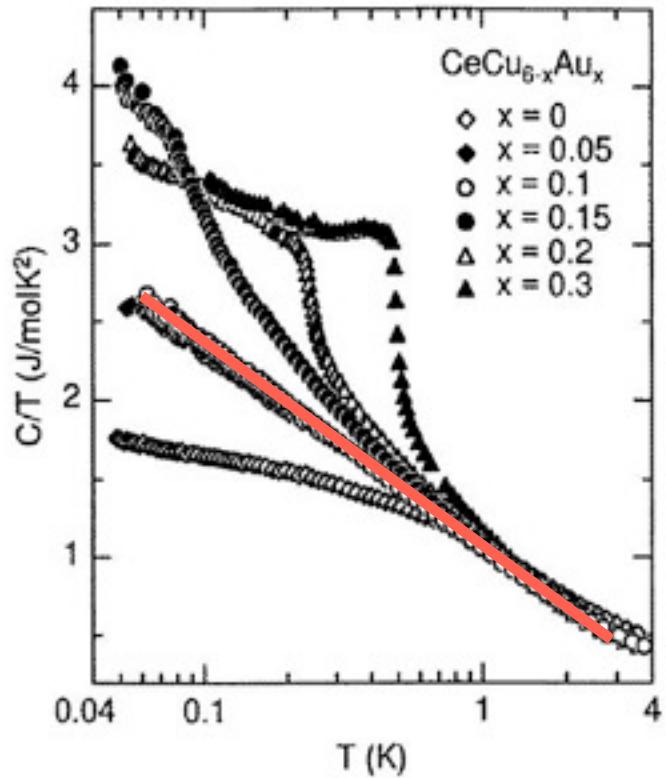


# Quantum Criticality: divergent specific heat capacity

Heavy Fermion  
Materials

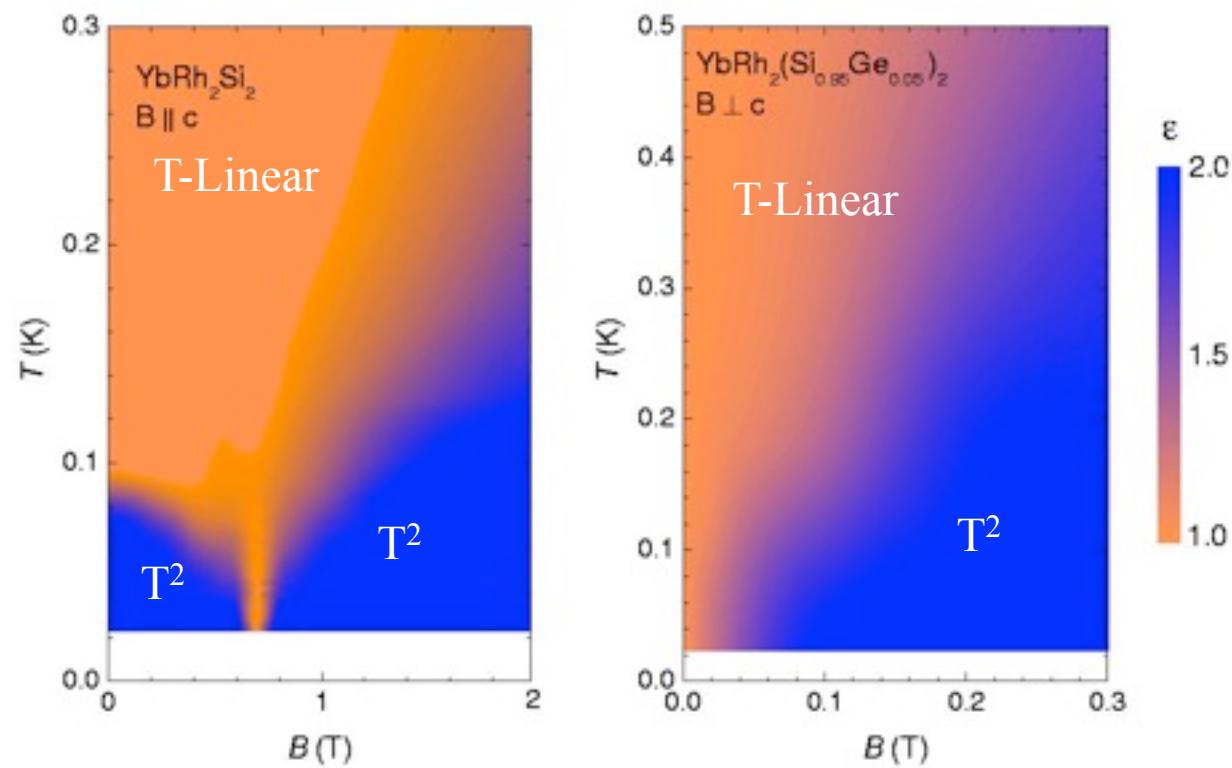
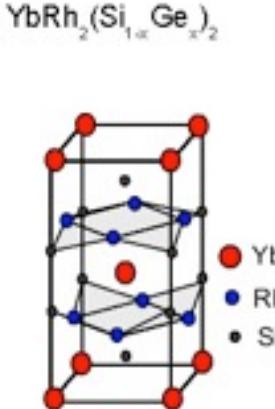


H. Von Lohneysen (1996)

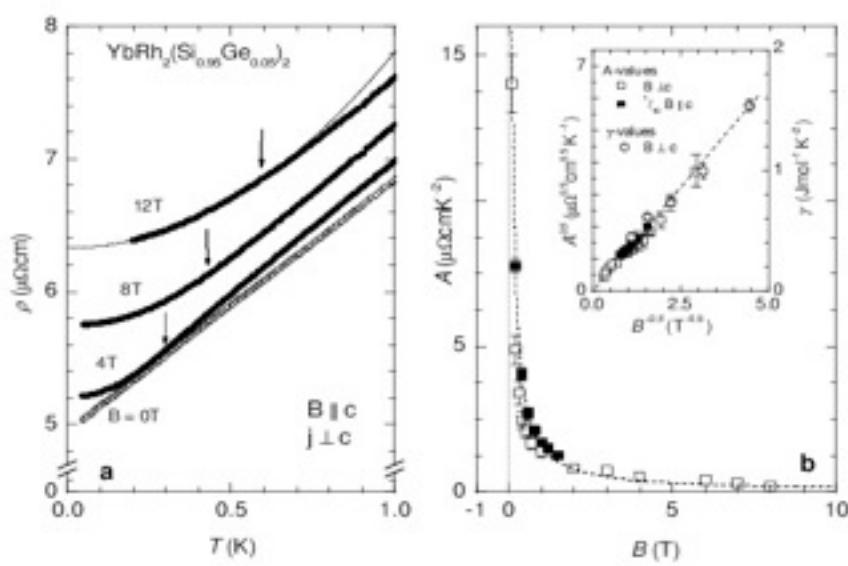


# Divergence of Interaction and Effective Mass

Gegenwart et al (2002)



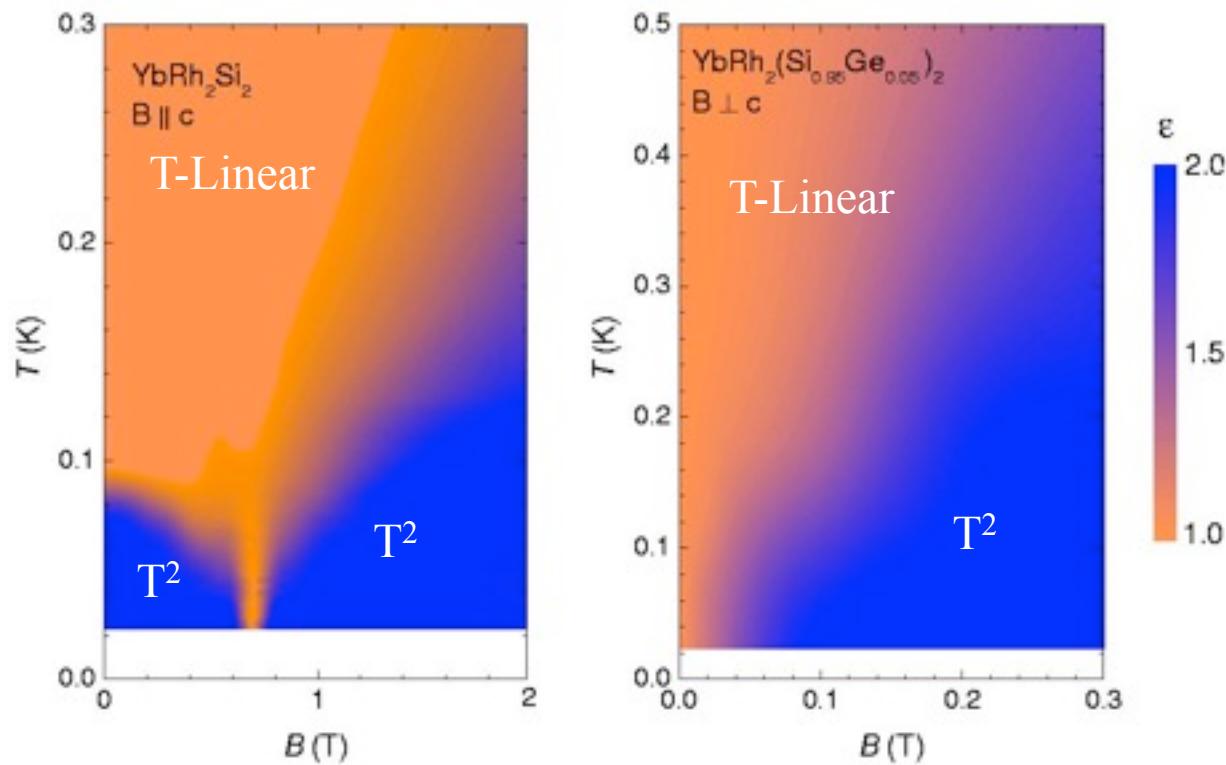
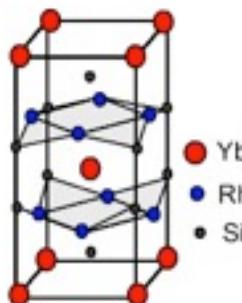
# Divergence of Interaction and Effective Mass



$$\rho = AT^2 + \rho_0$$

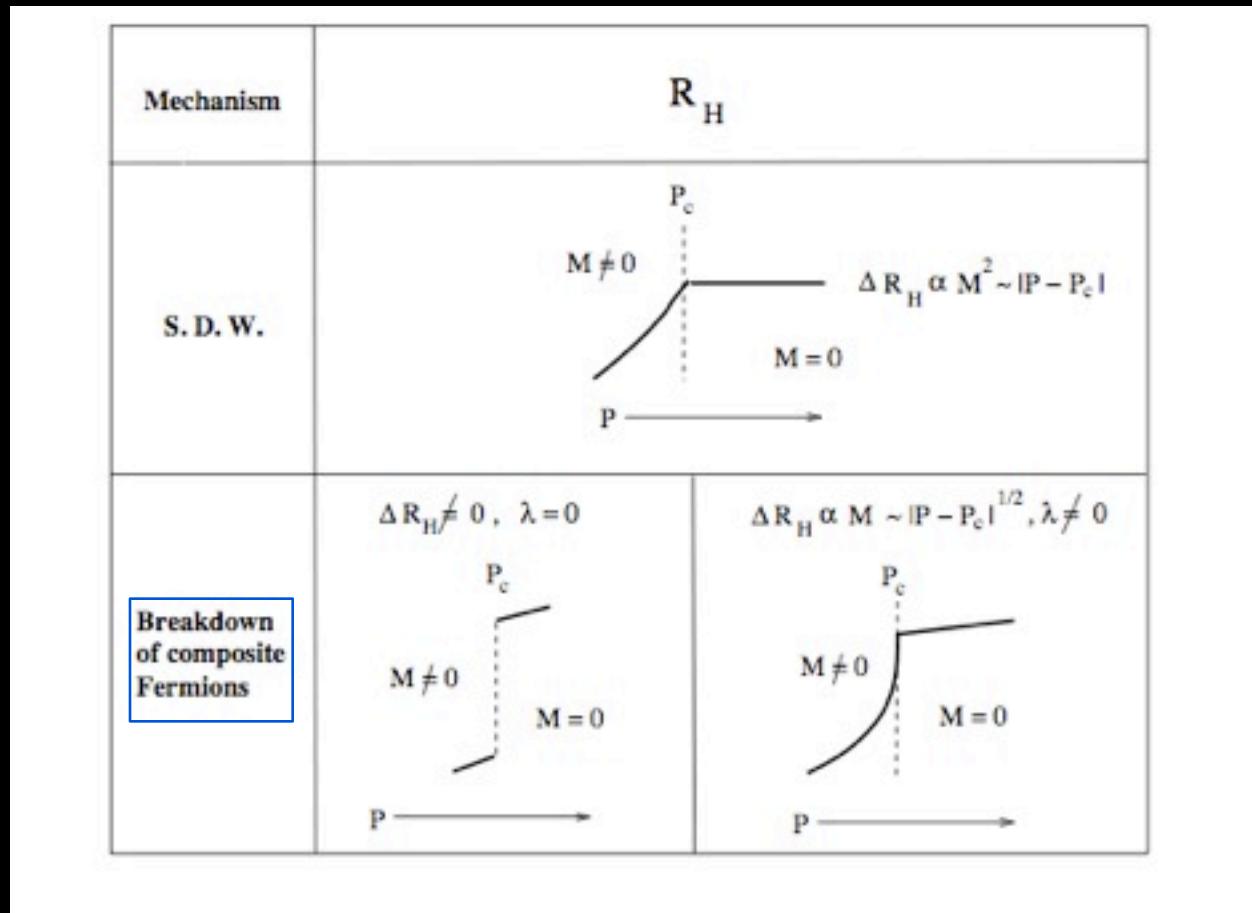
$$A \propto \frac{1}{T_F^2} \propto \frac{1}{B - B_c}$$

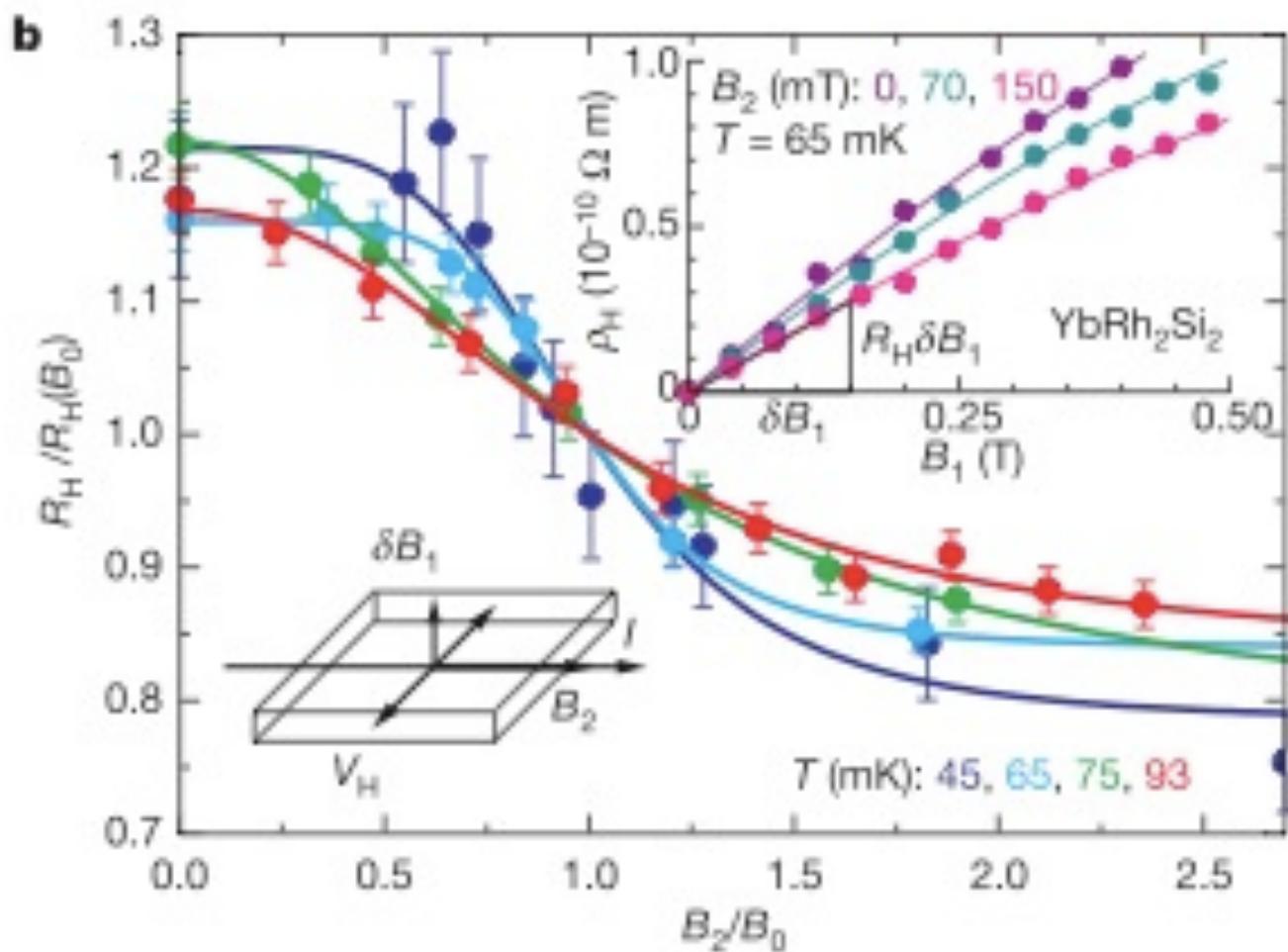
Gegenwart et al (2002)

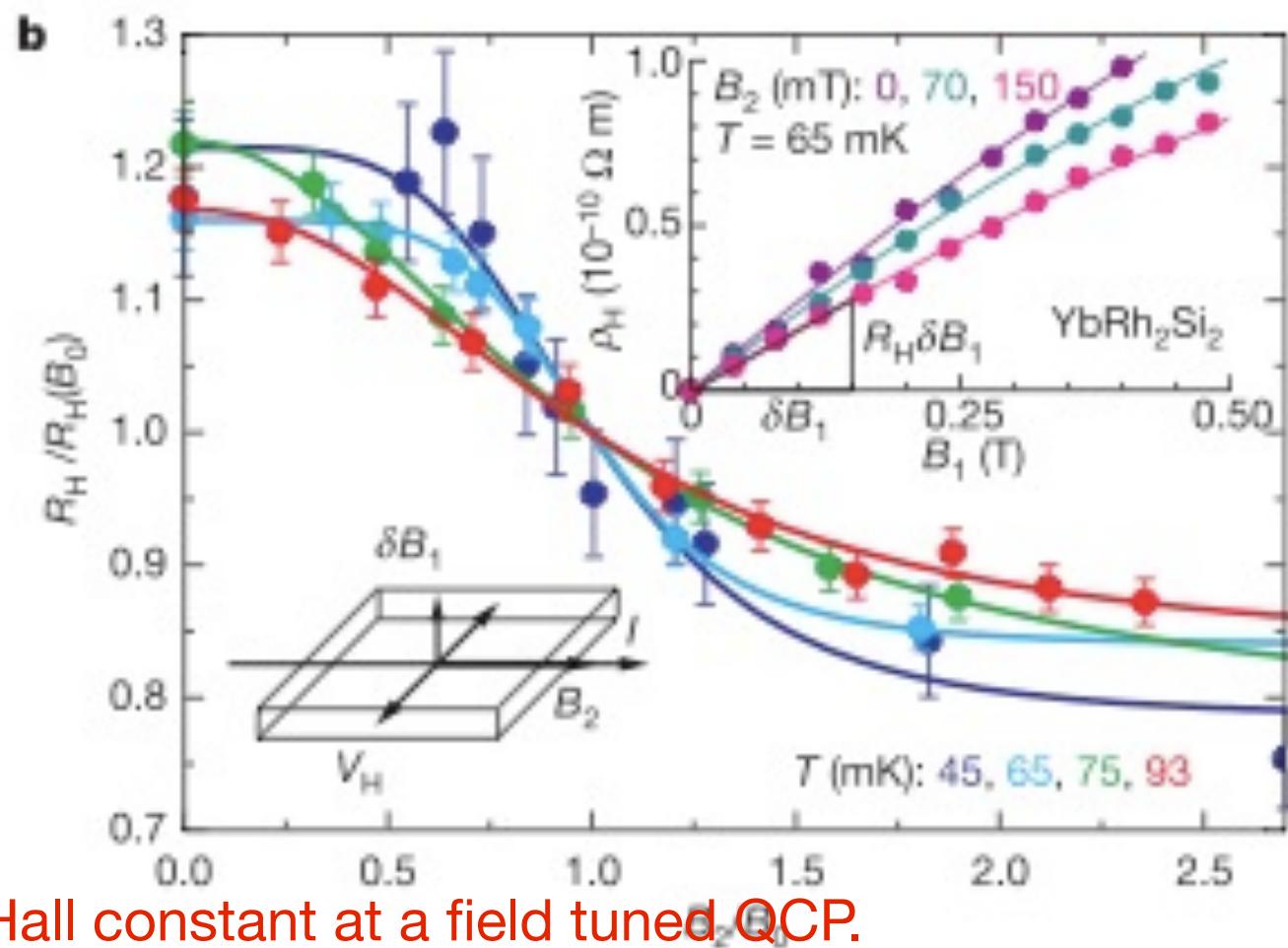


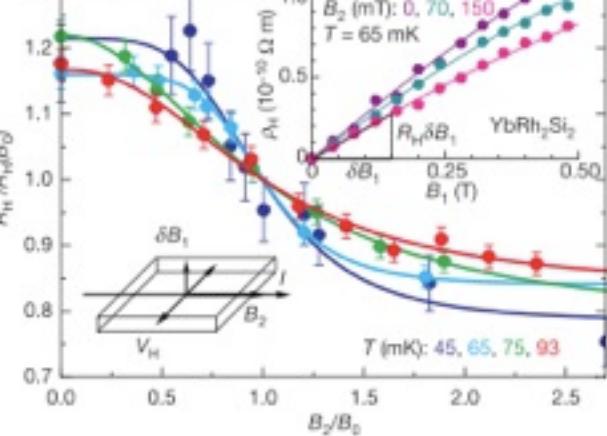
“How do fermions get heavy and die?” PC, Pepin, Si and Ramazashvili, J. Cond Matt. ,13}, R723 (2001).

anticipated an abrupt change in FS when a composite heavy electron undergoes a Kondo “breakdown”.

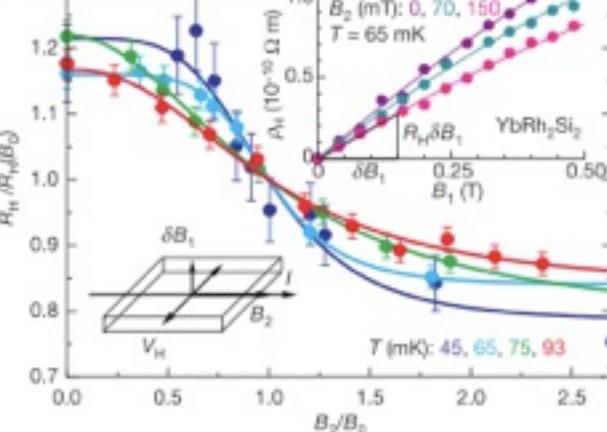
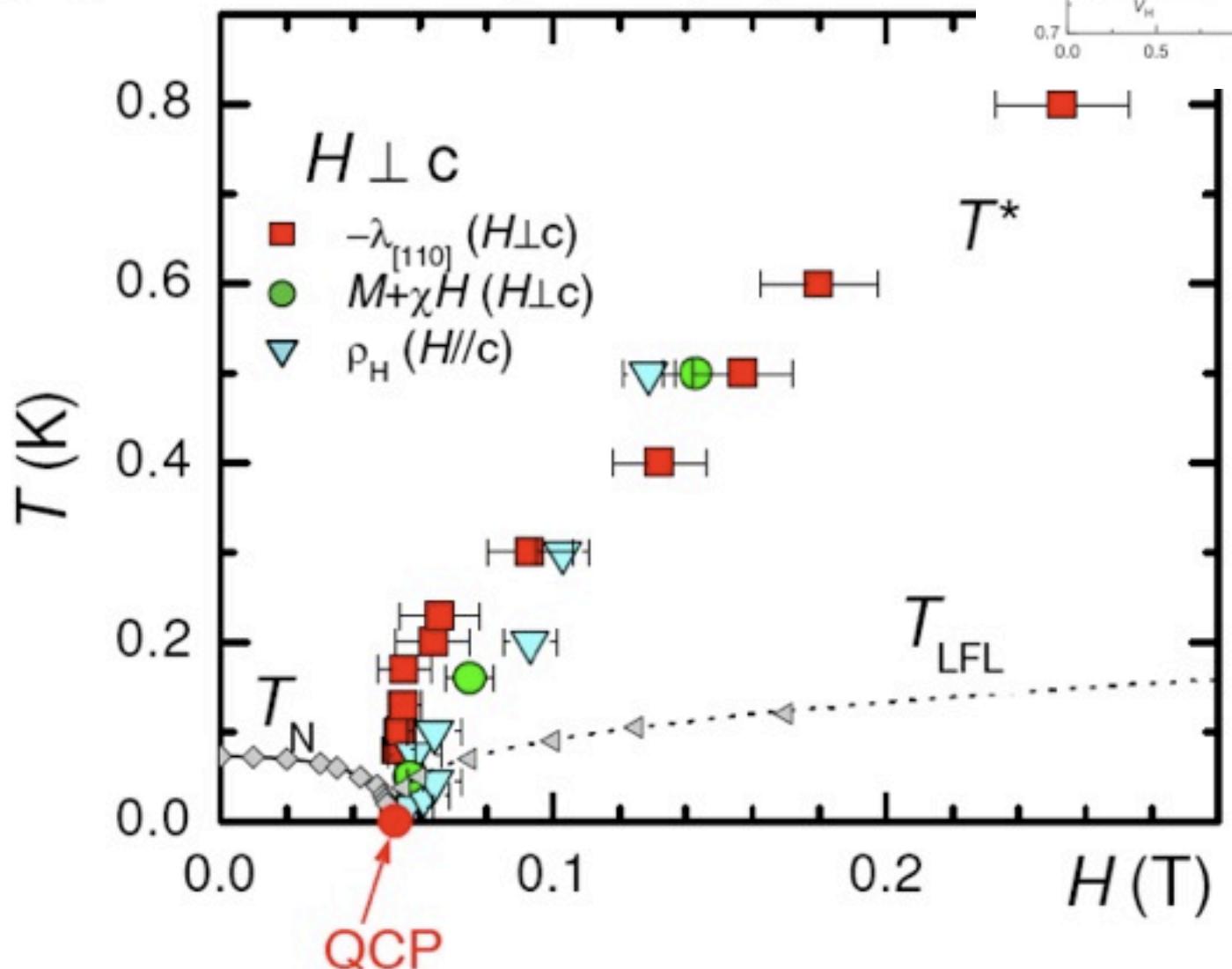




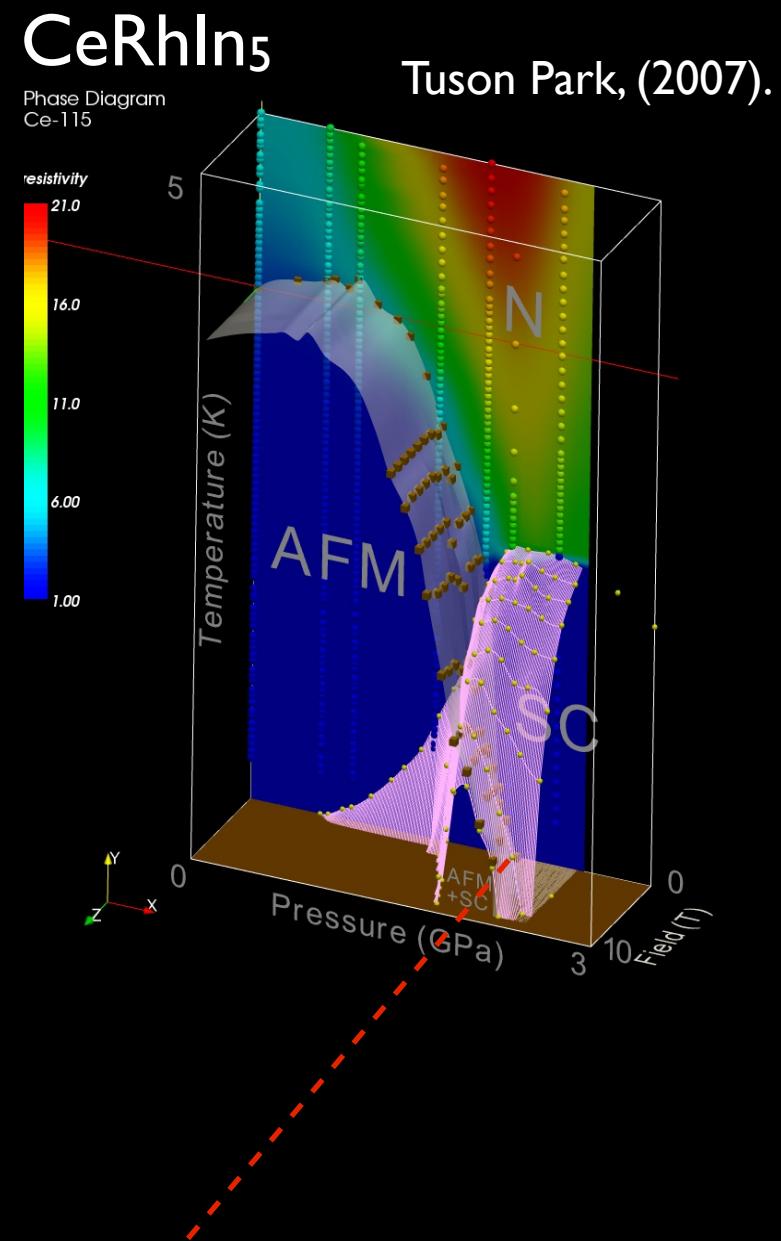




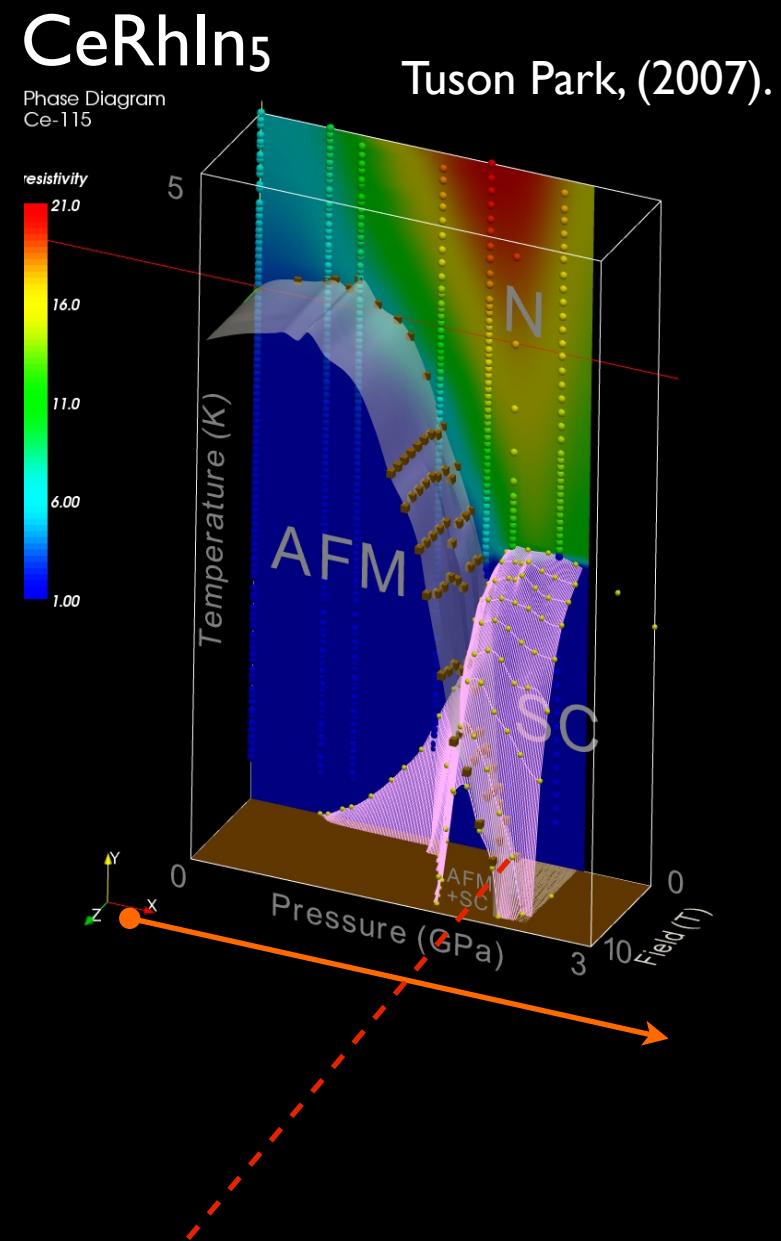
$T_{\text{Hall}}$  represents a new energy scale ( $T^*$ )



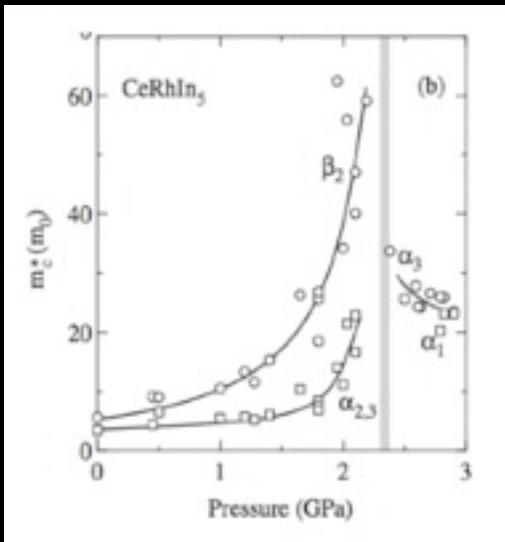
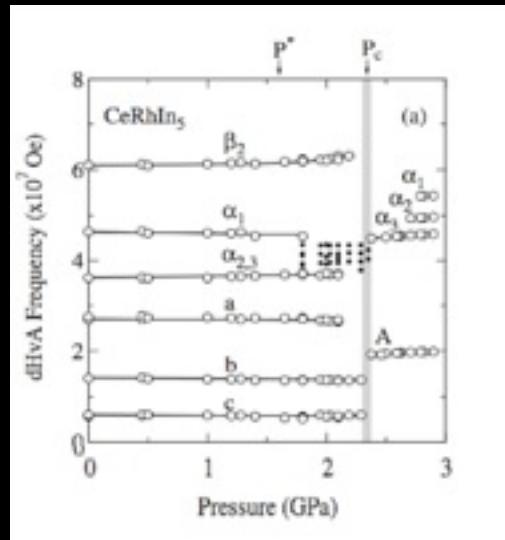
# Reconstruction of the Fermi Surface and mass divergence



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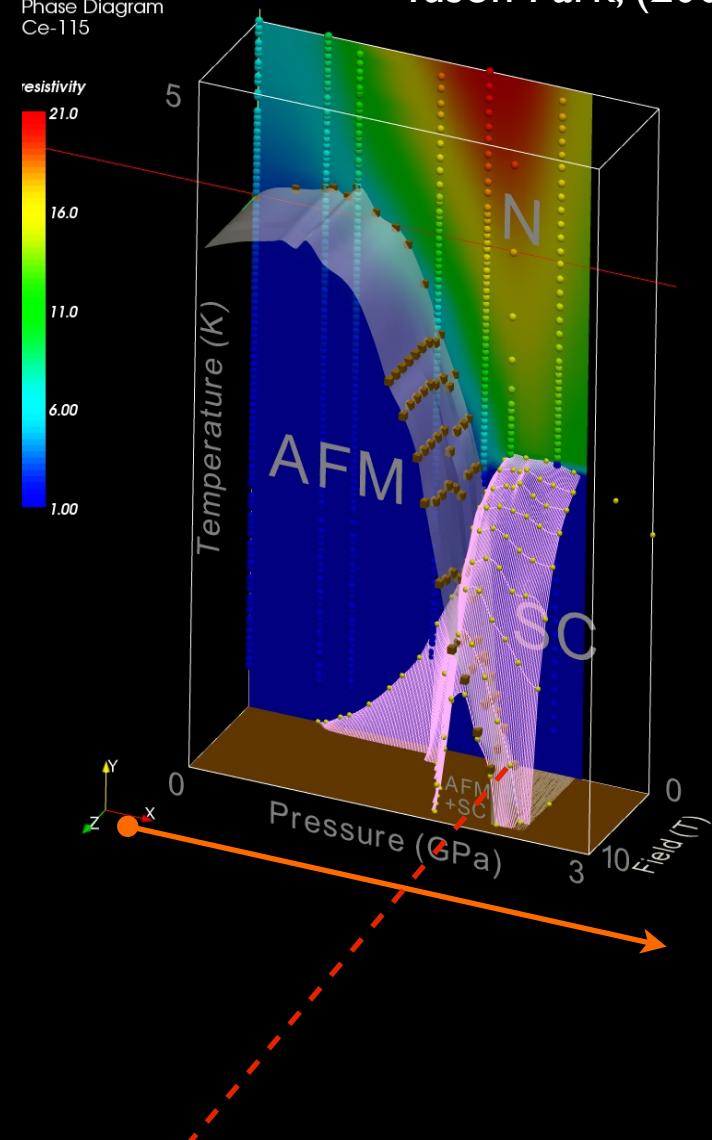


Shimuzu et al (2006)

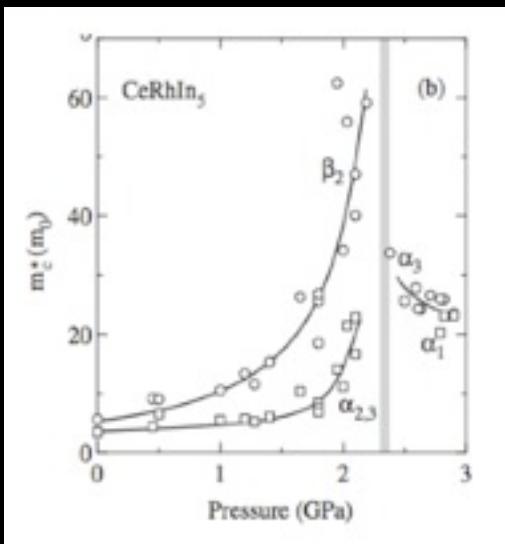
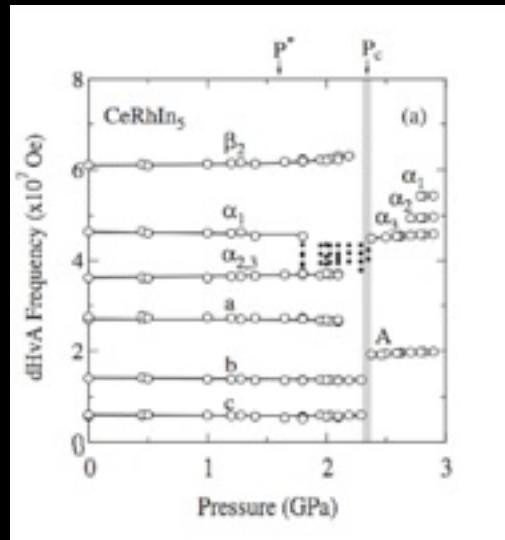
CeRhIn<sub>5</sub>

Phase Diagram  
Ce-115

Tuson Park, (2007).



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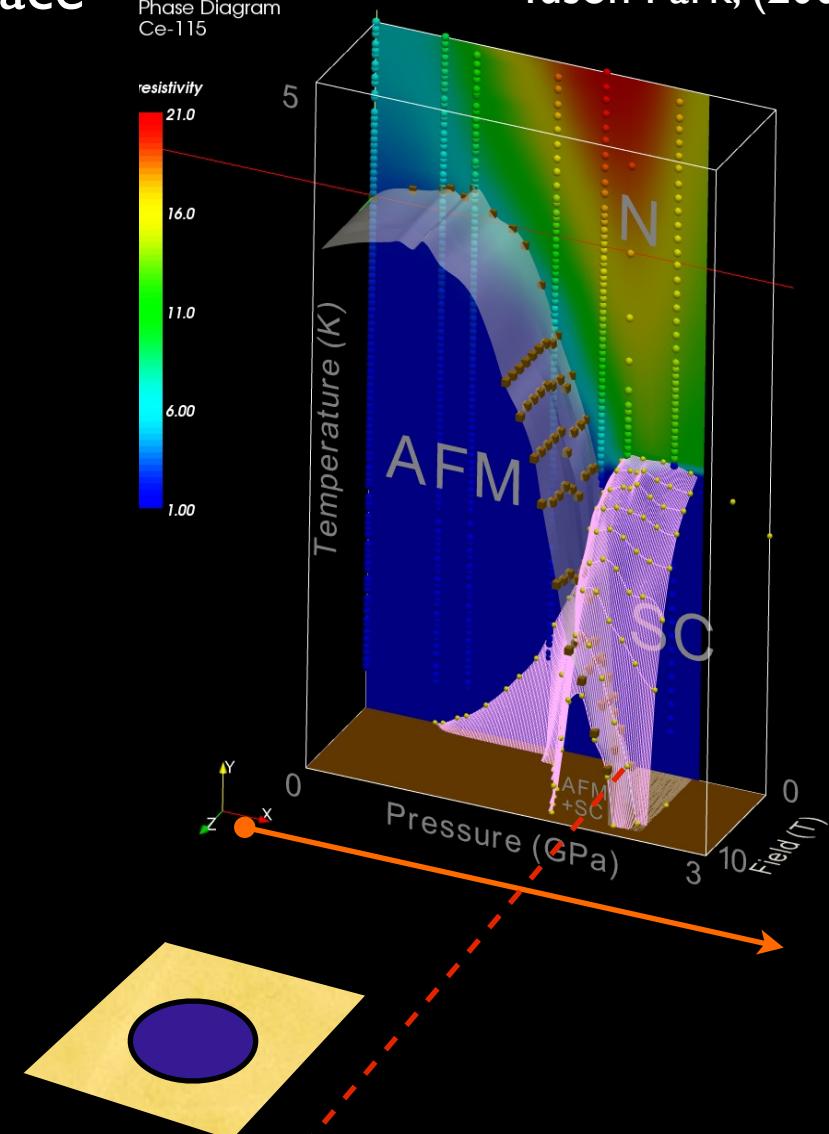


Shimuzu et al (2006)

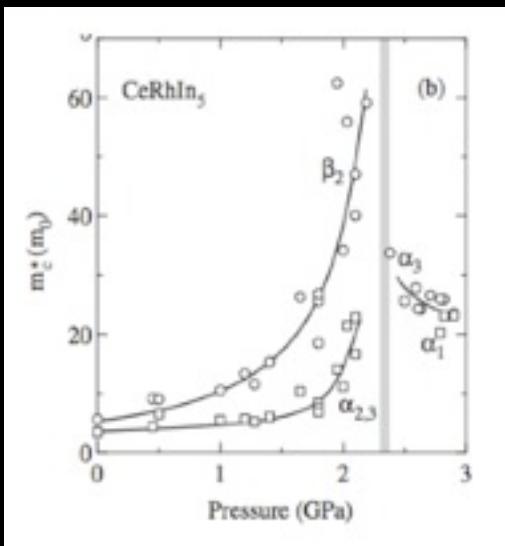
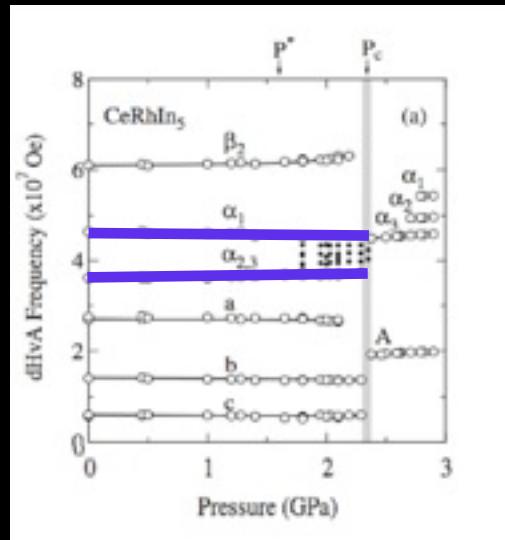
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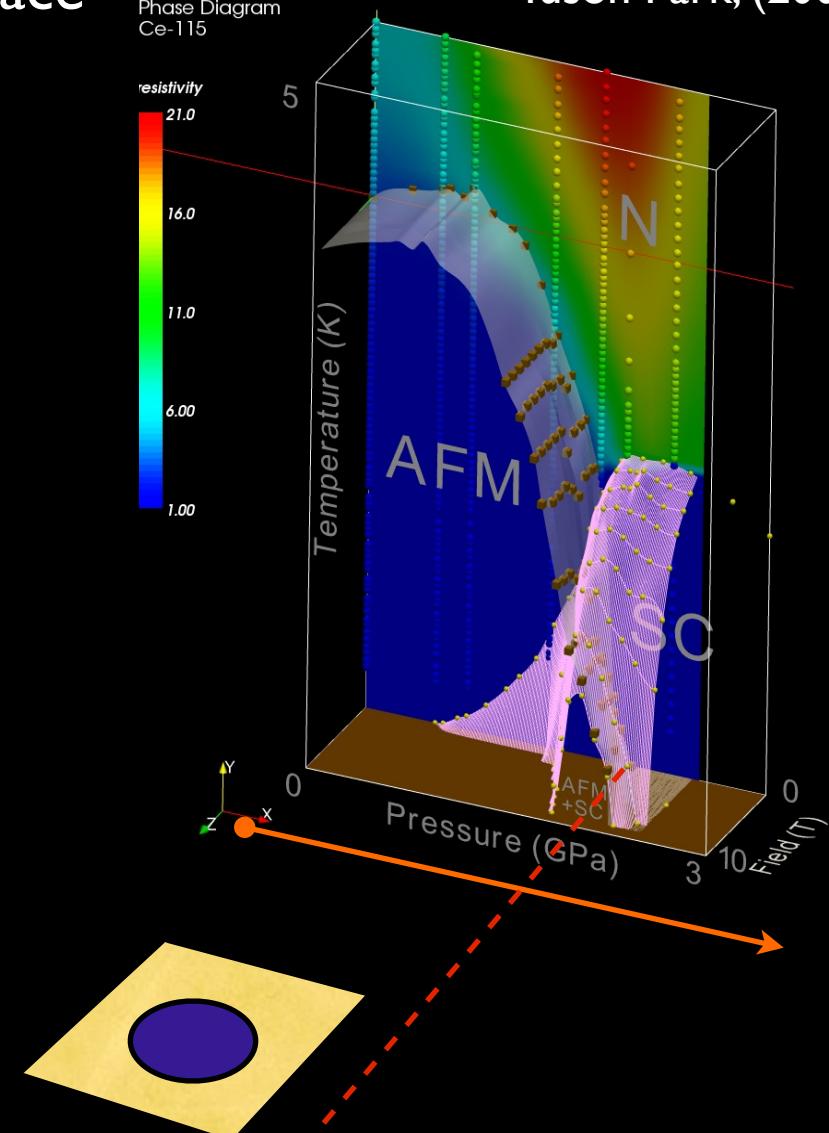


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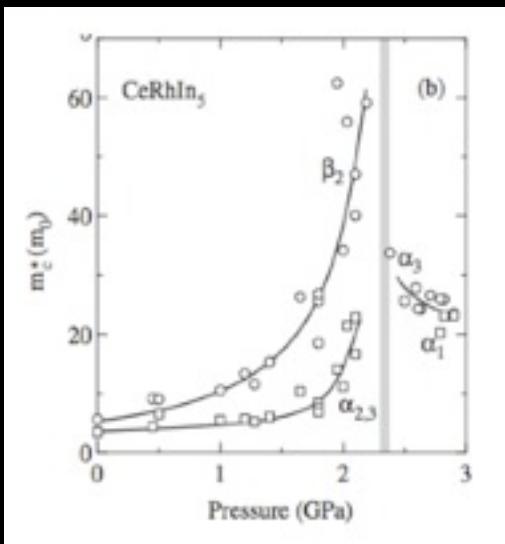
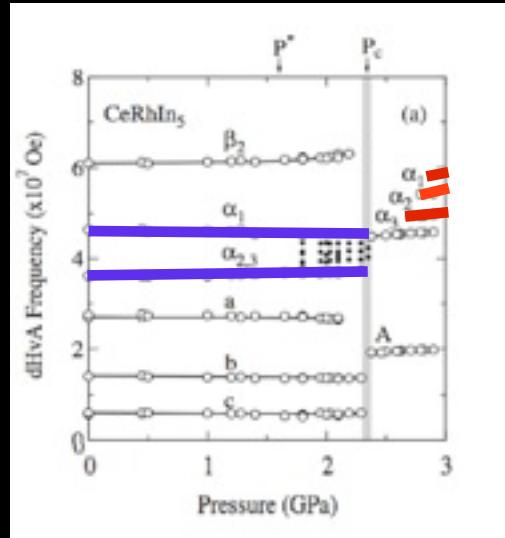
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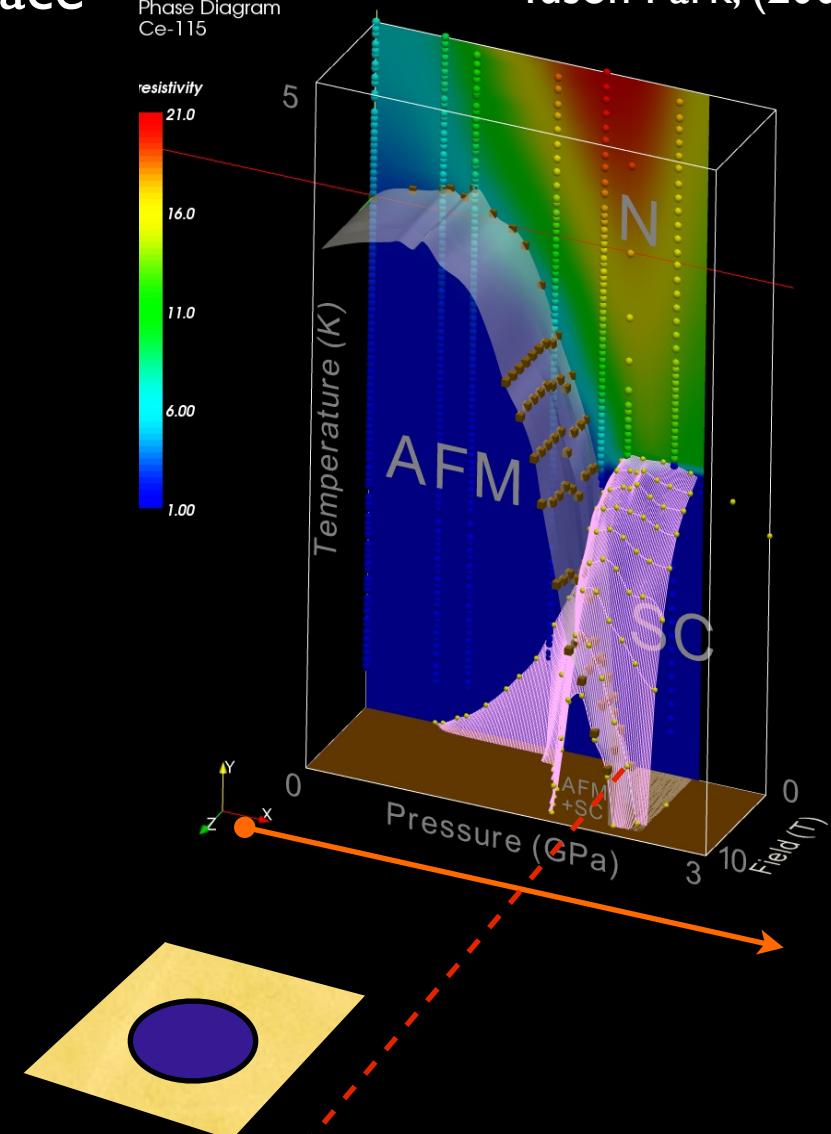


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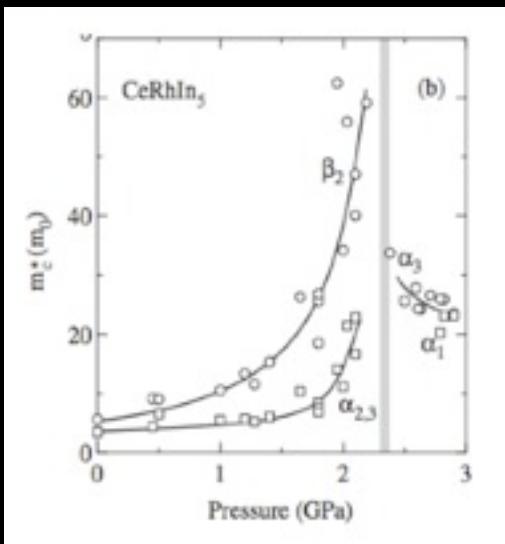
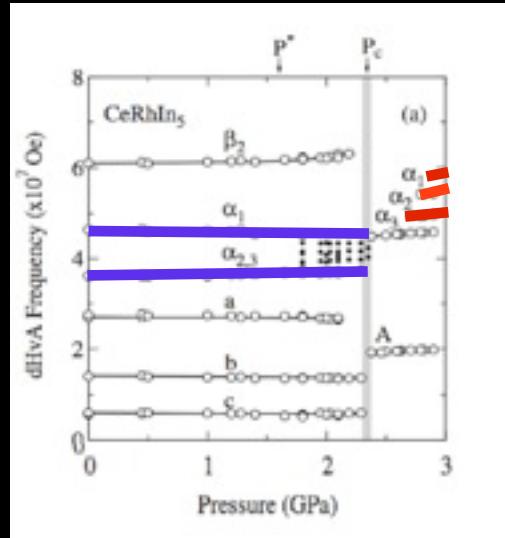
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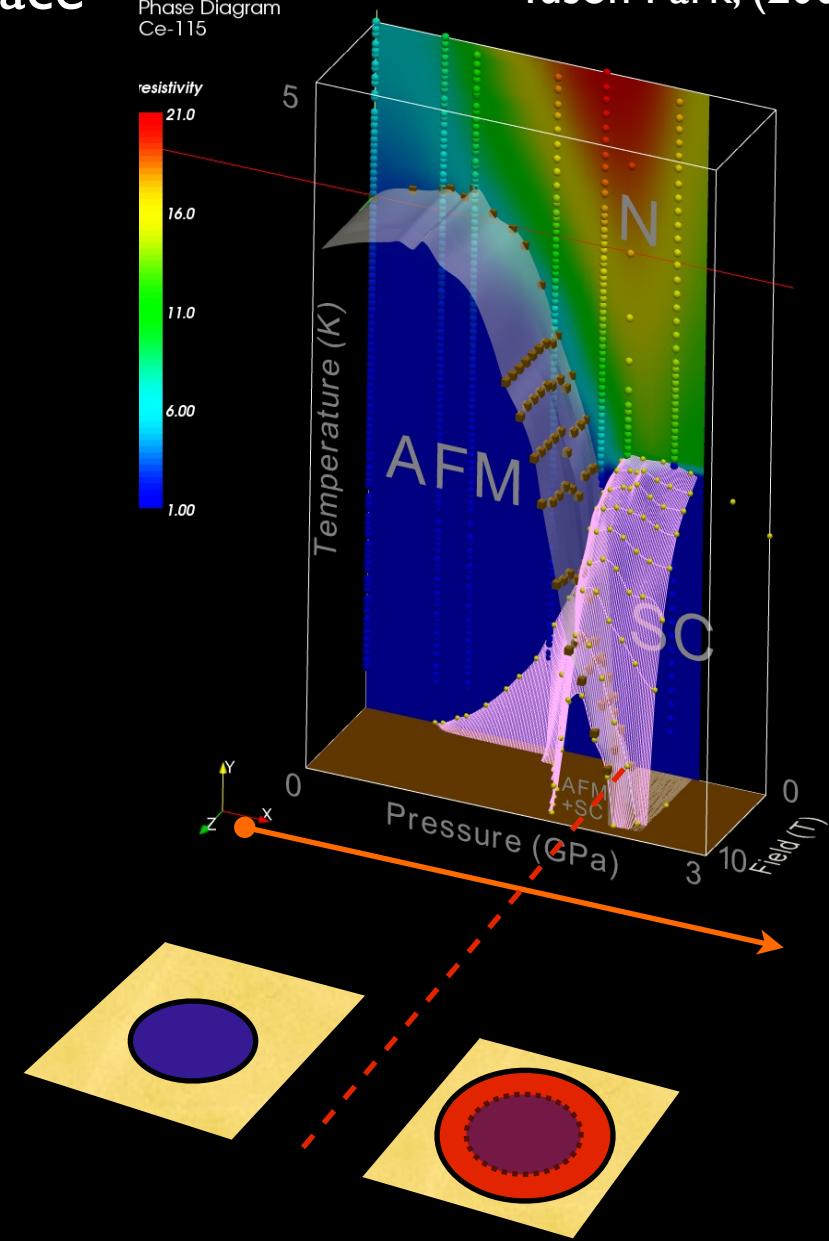


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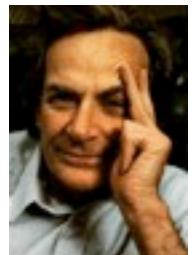
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Phase Diagram  
Ce-115

Tuson Park, (2007).



Black Hole in the Phase Diagram.

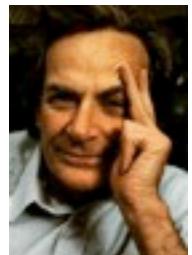


Feynman



Hertz

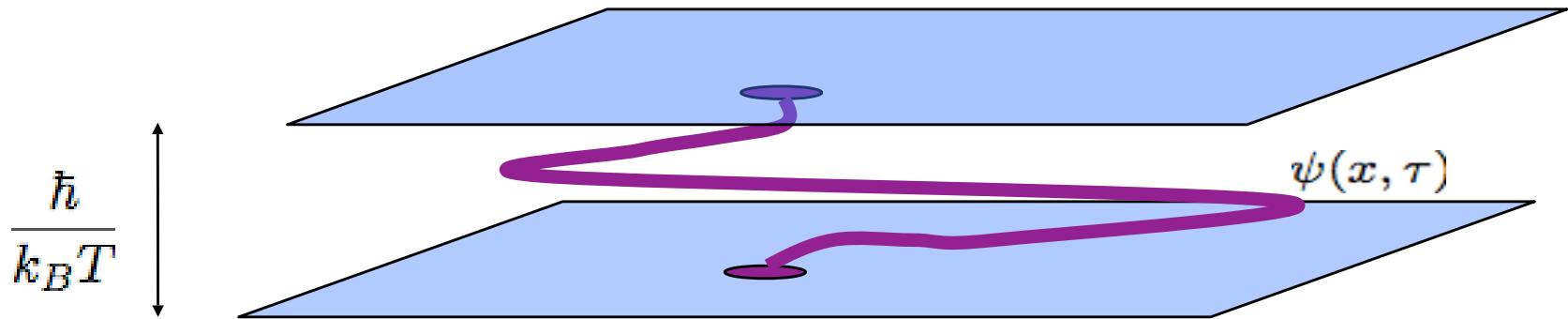
$$Z = \sum_{\text{Histories}} \exp \left[ - \int_0^{1/T} L[\psi(x, \tau)] d\tau \right]$$

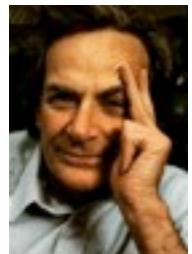


Feynman

Hertz

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Feynman

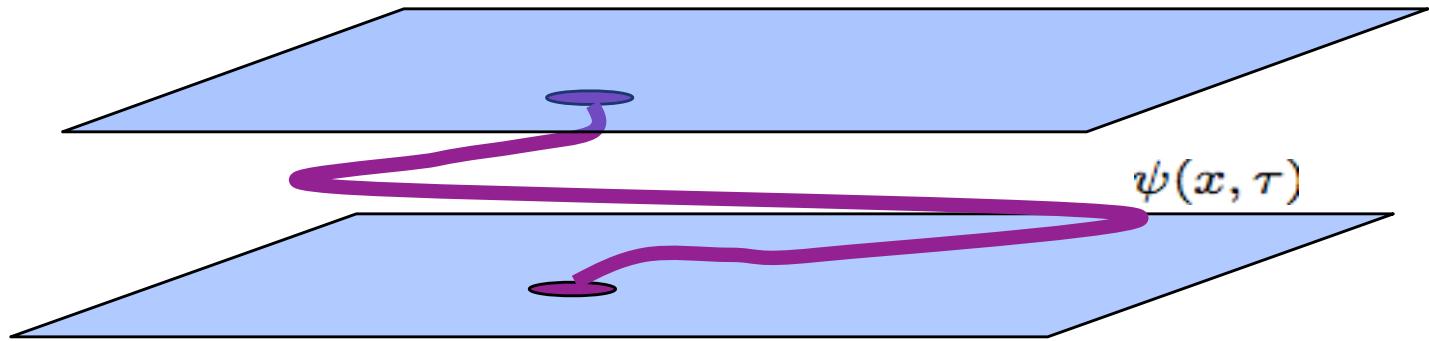
Hertz

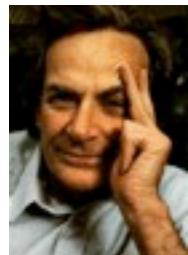
Temperature:  
Boundary condition  
in time.

$$Z = \sum_{\text{Histories}} \exp \left[ - \int_0^{1/T} L[\psi(x, \tau)] d\tau \right]$$

Sachdev (1999), Continentino (2001), Palova (2009).

$$\frac{\hbar}{k_B T}$$





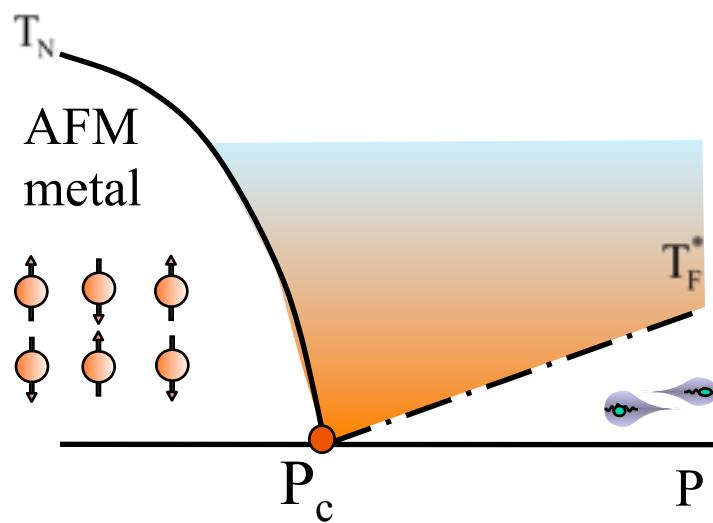
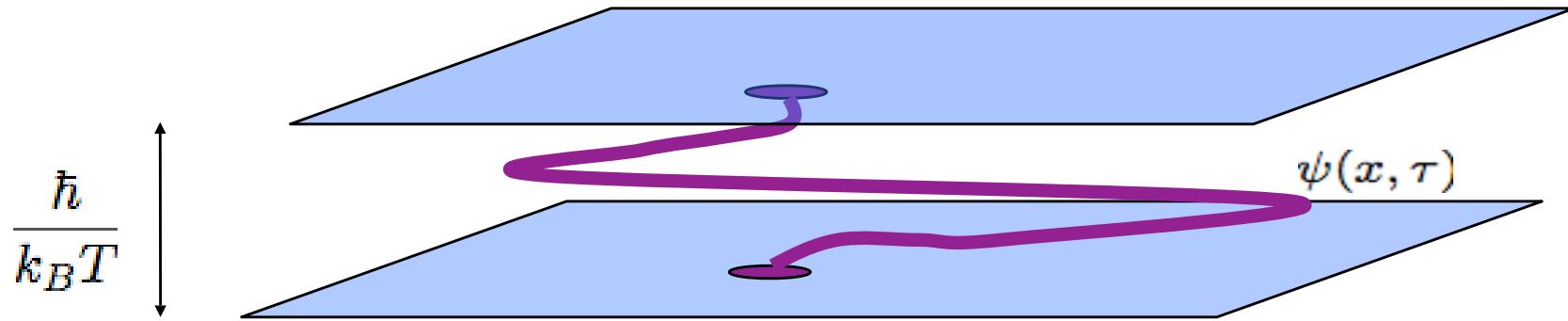
Feynman

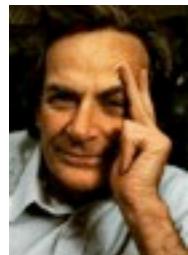
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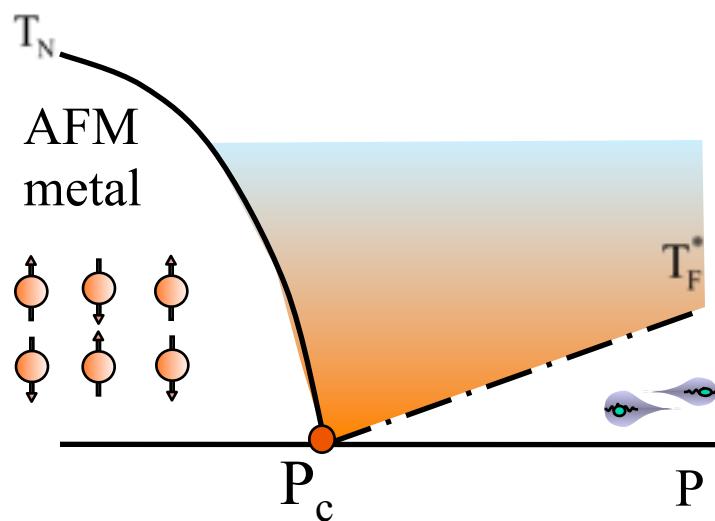
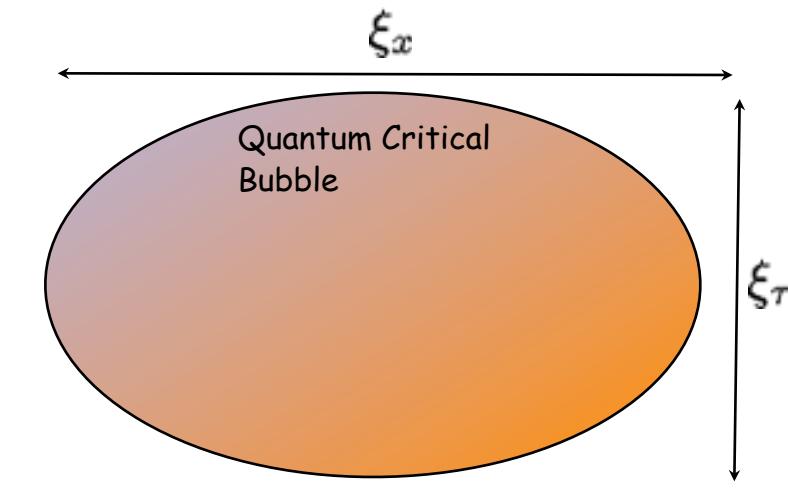


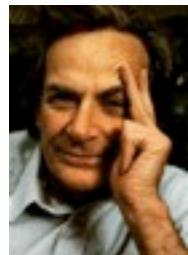
Feynman      Hertz

# Temperature: Boundary condition in time.

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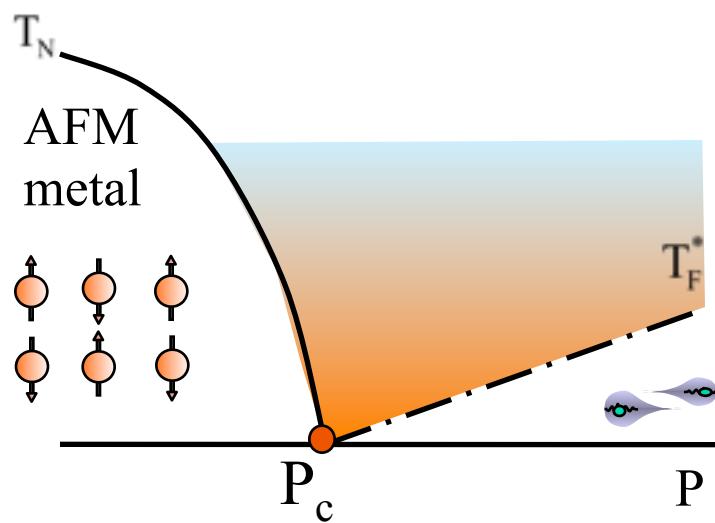
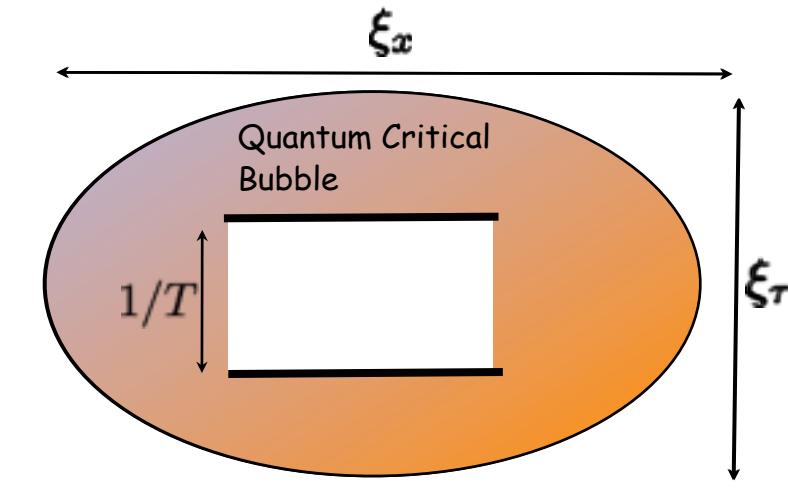


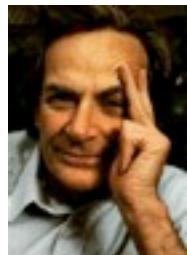
Feynman      Hertz

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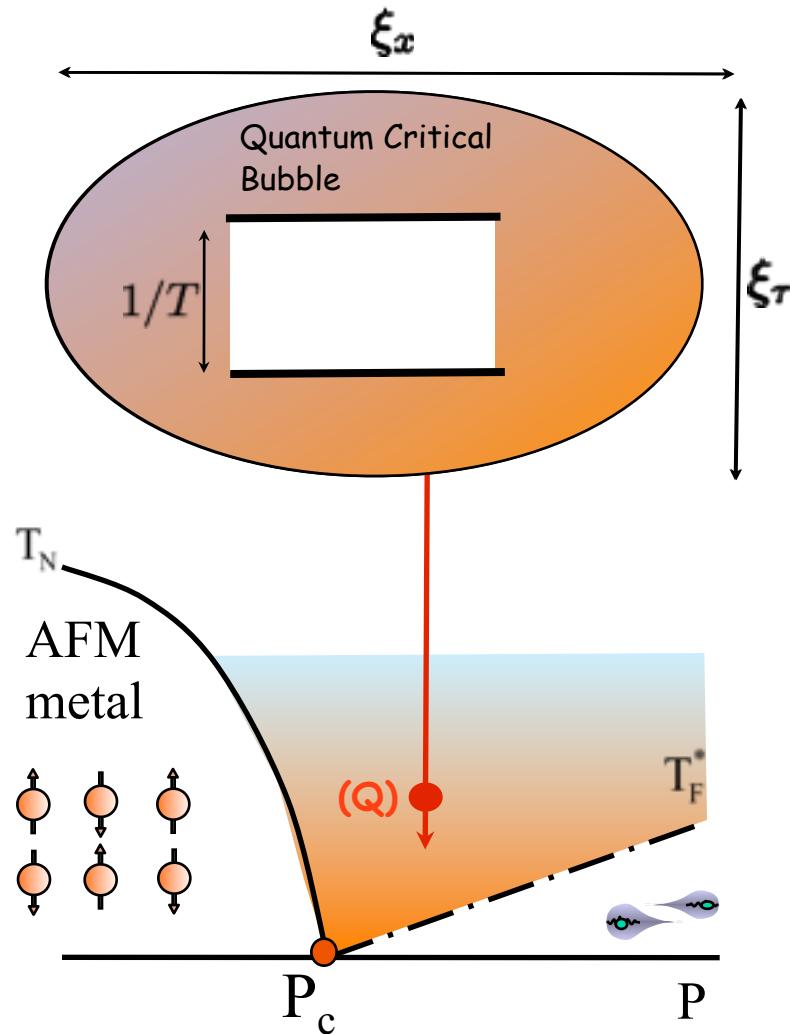


Feynman      Hertz

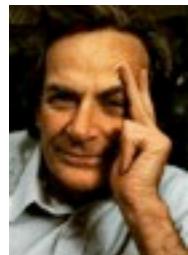
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**(Q)** Quantum critical region:  
interior of correlation bubble.

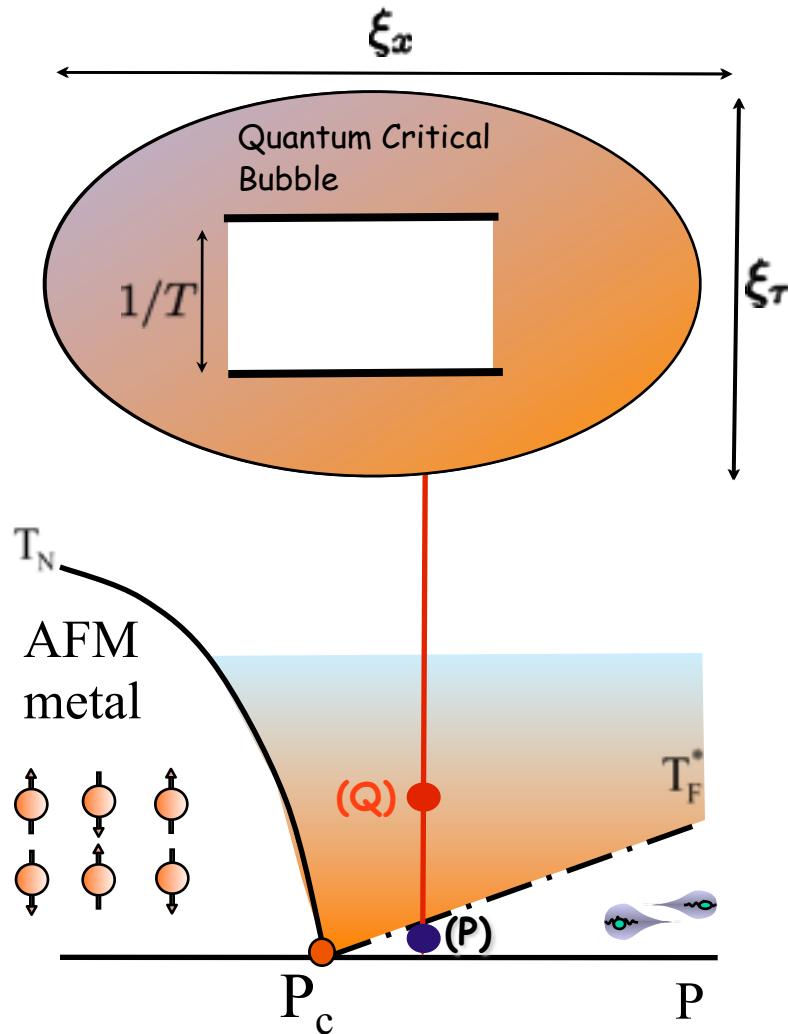


Feynman      Hertz

# Temperature: Boundary condition in time.

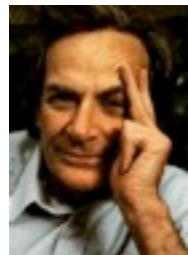
Sachdev (1999), Continentino (2001), Palova (2009).

$$Z = \sum_{\text{Histories}} \exp \left[ - \int_0^{1/T} L[\psi(x, \tau)] d\tau \right]$$



**(Q)** Quantum critical region:  
interior of correlation bubble.

**(P)** Paramagnet: probes  
exterior of correlation bubble



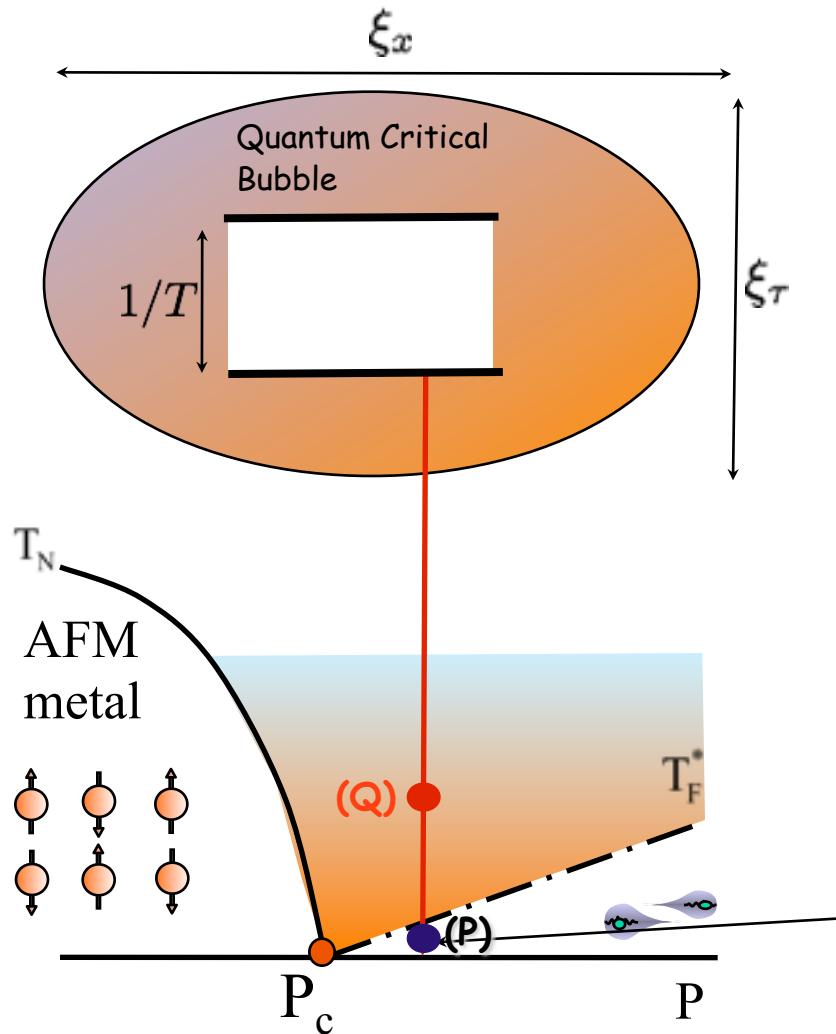
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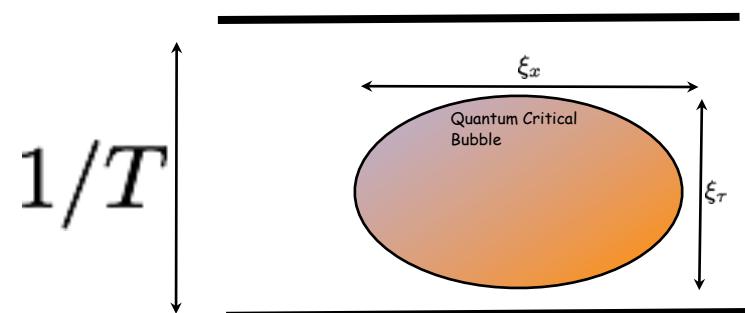
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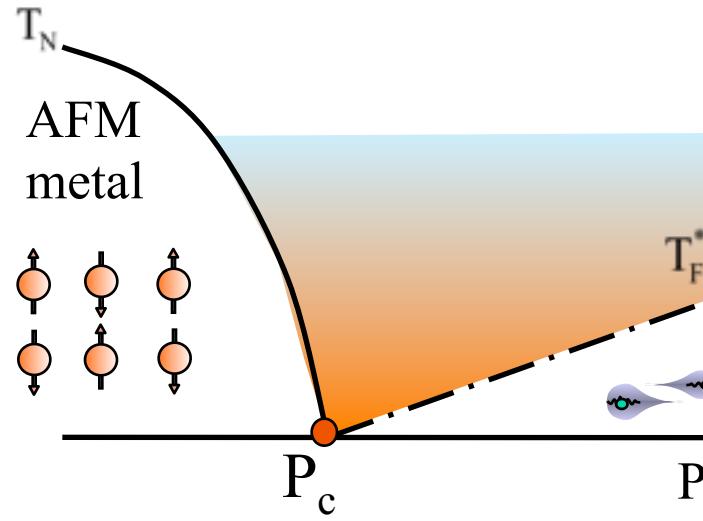


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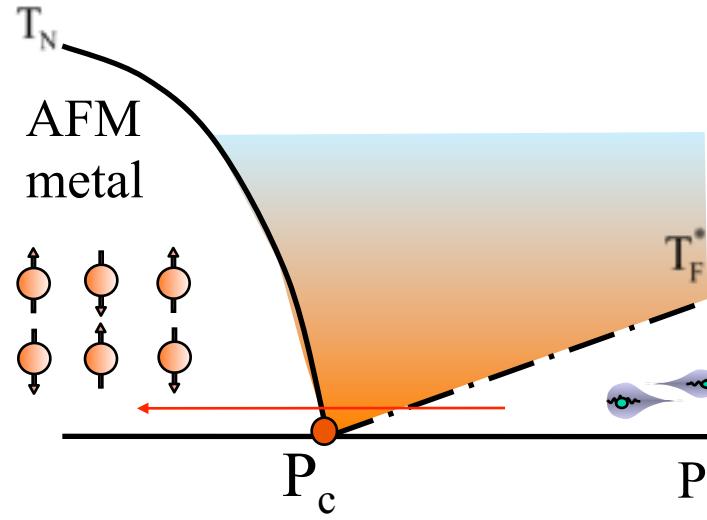
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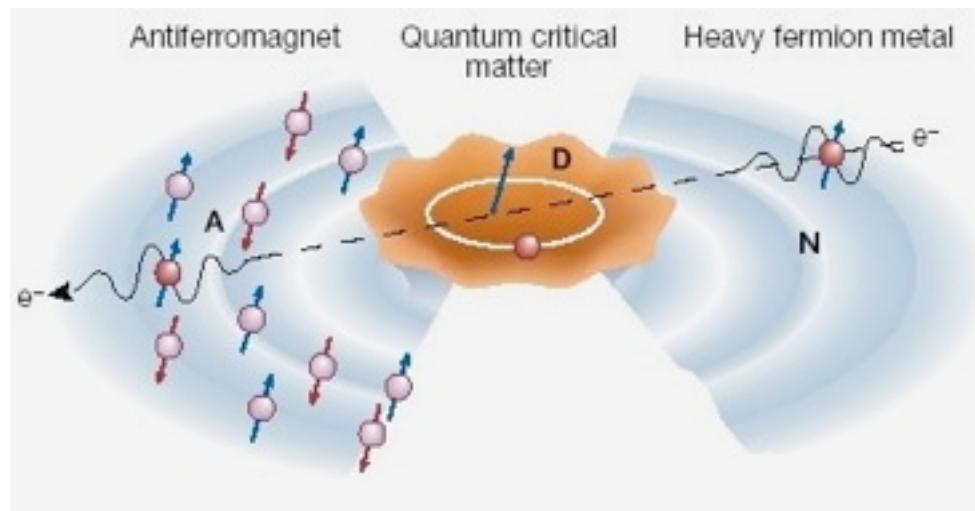
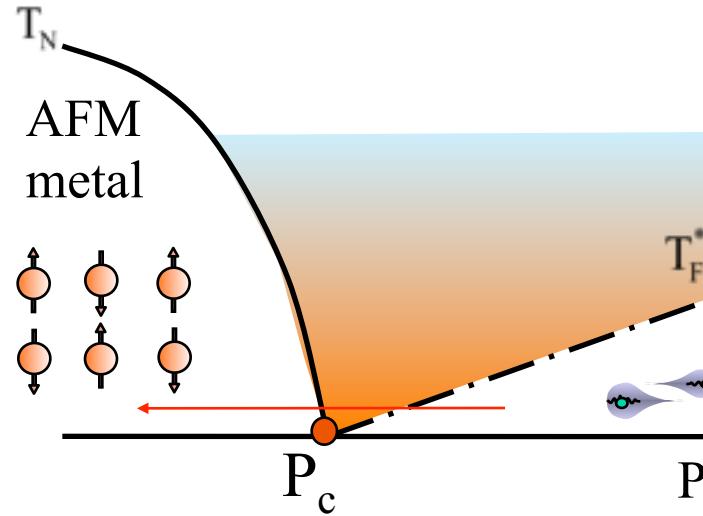
# "Black Hole in the Phase Diagram".



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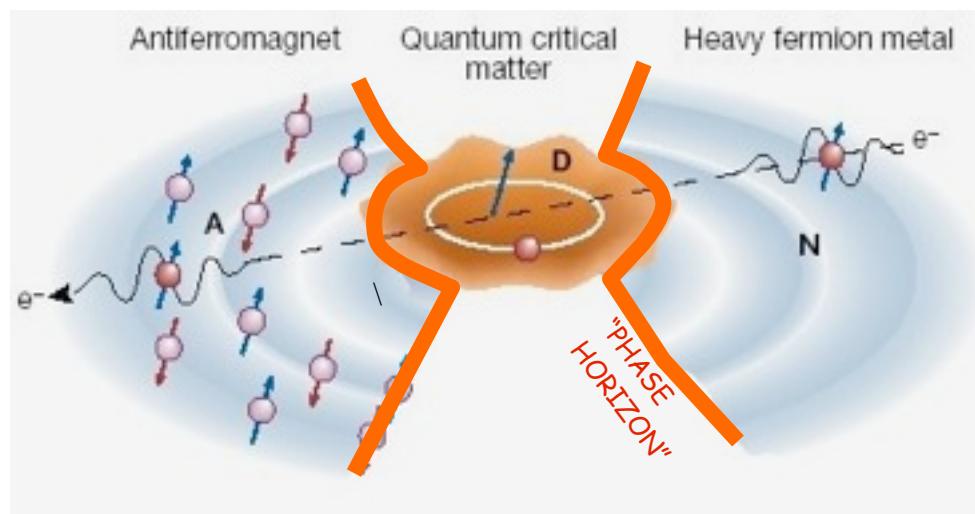
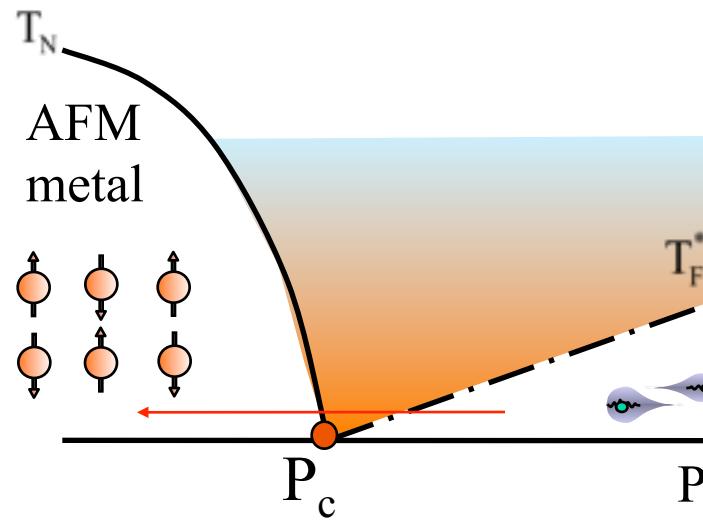


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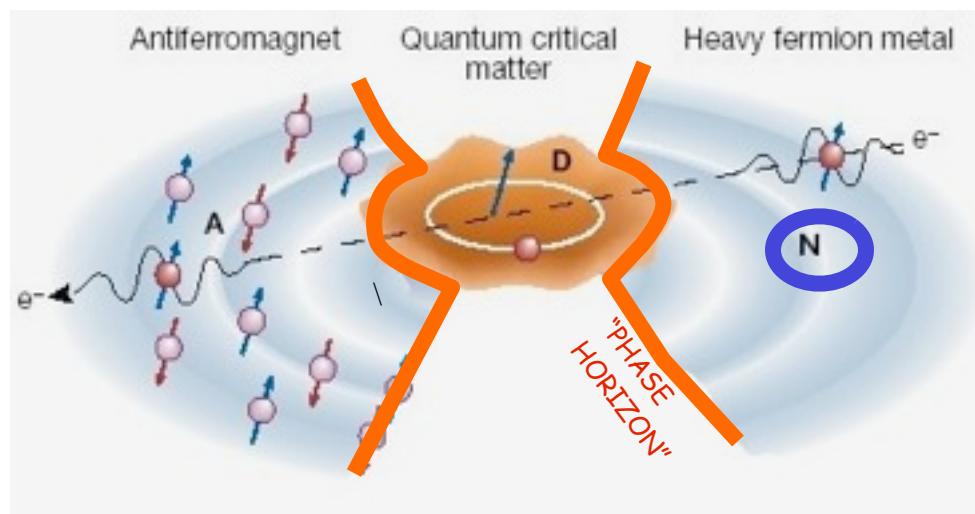
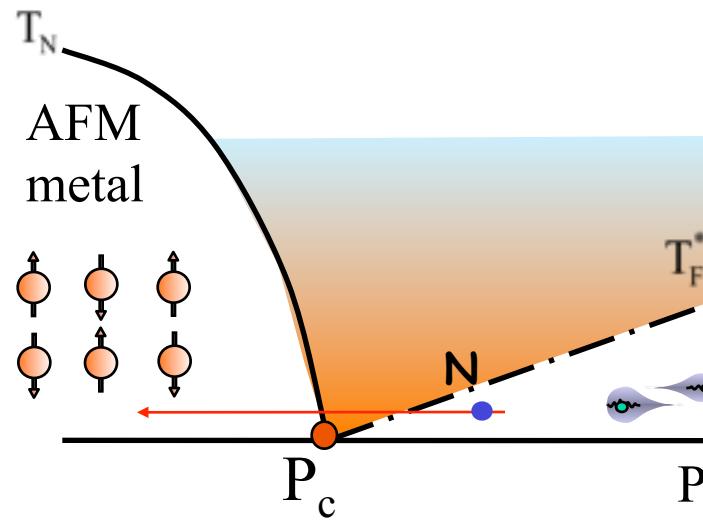
P.C and A. Schofield, Nature (2005)

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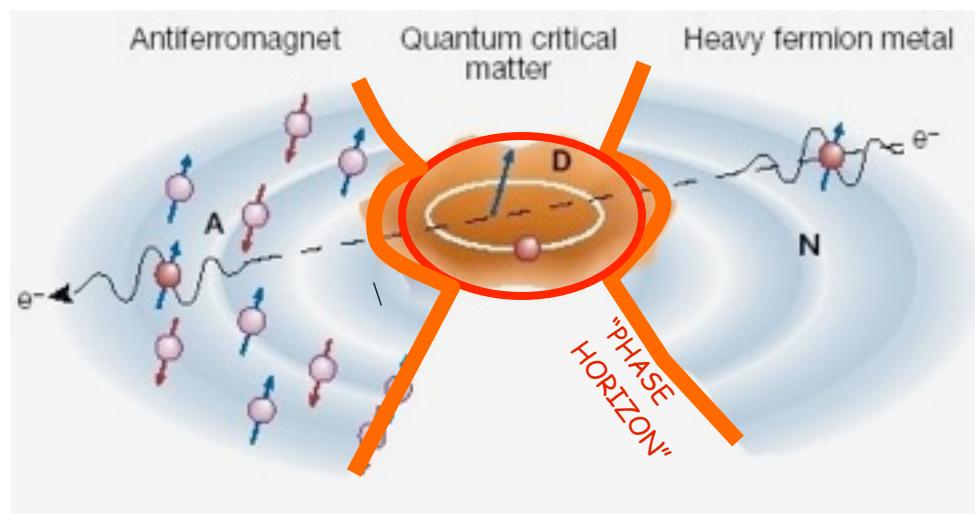
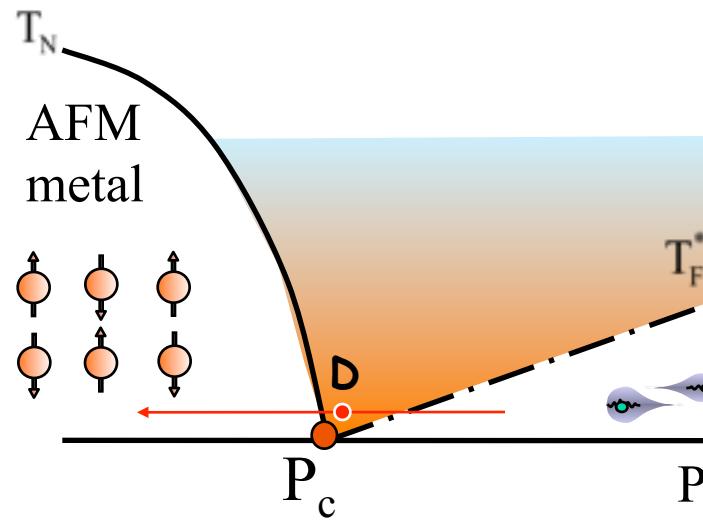
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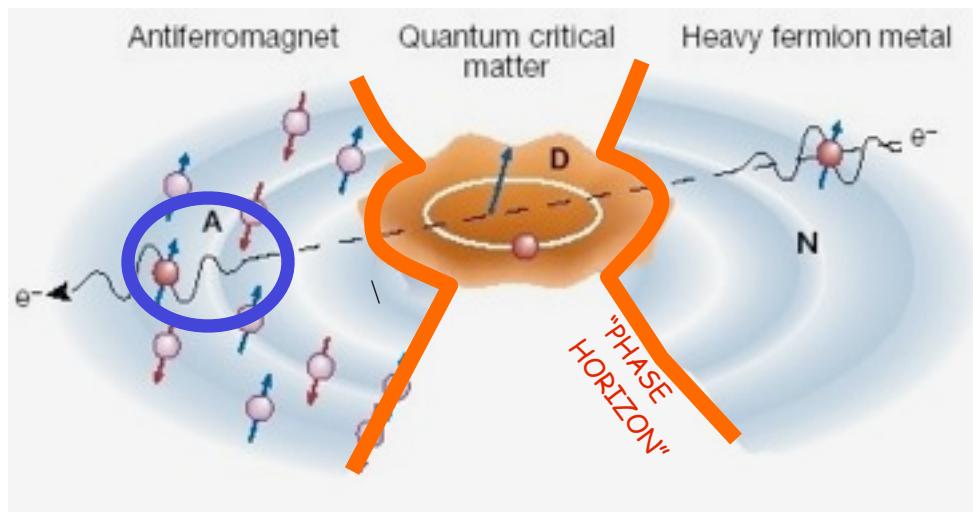
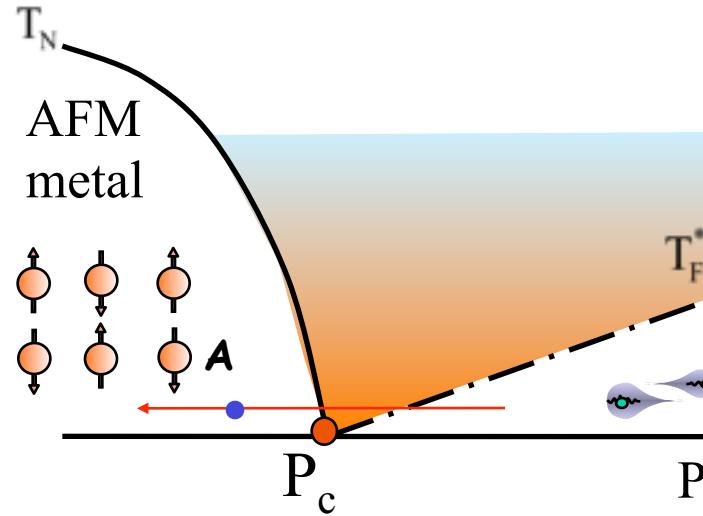
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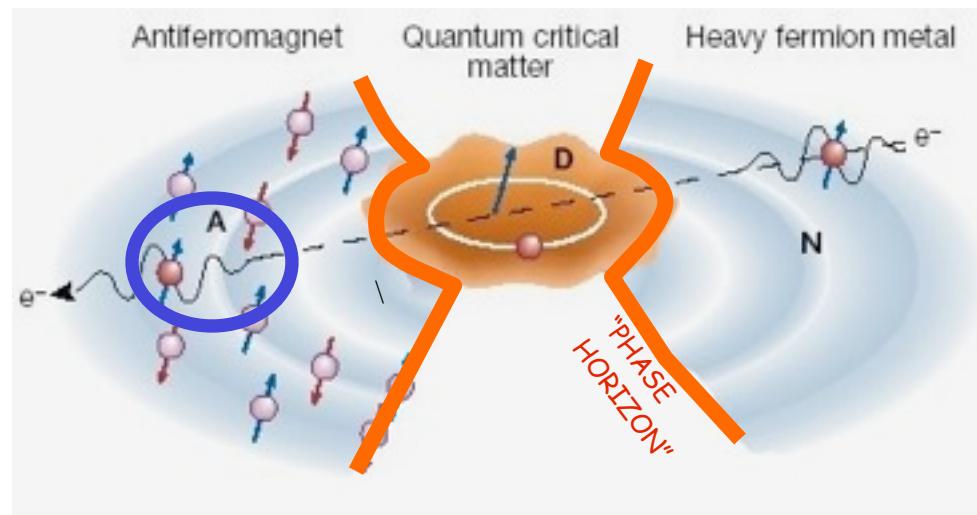
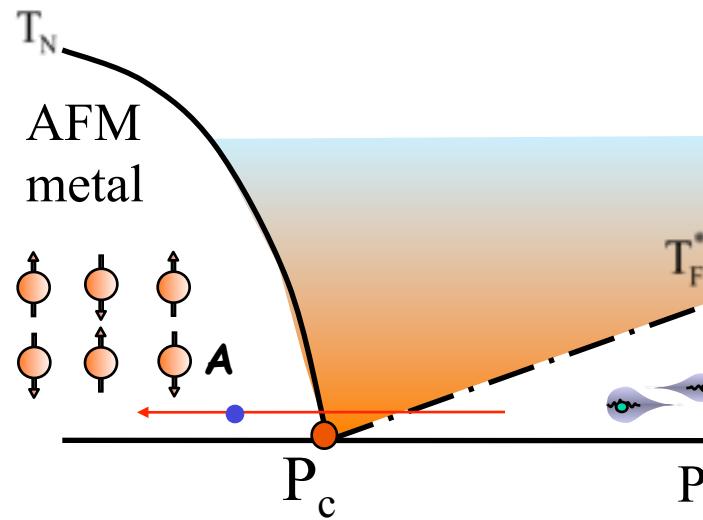
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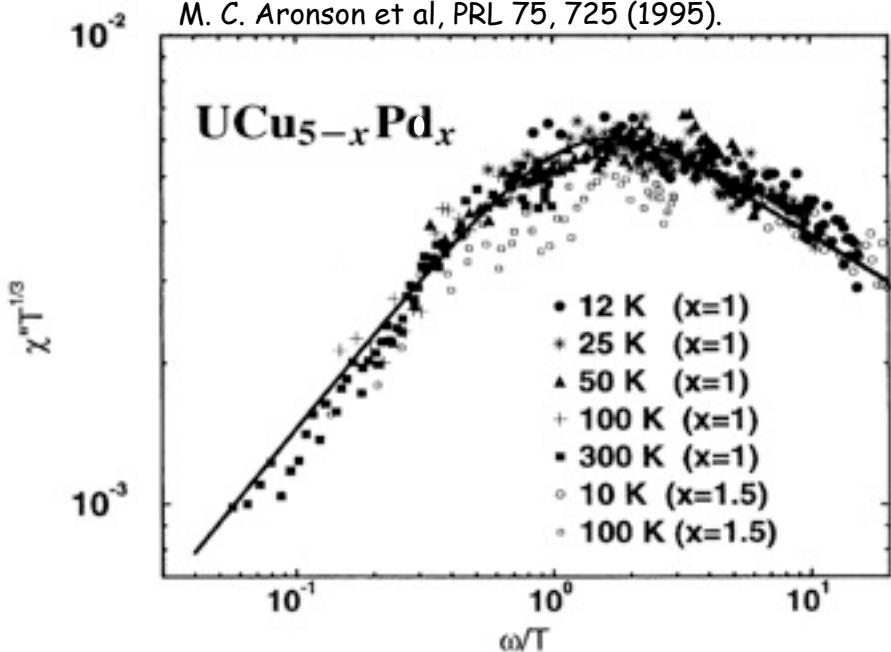
John Hertz: Critical droplet is **Quantum** if  $\hbar\omega(q)|_{q=\xi^{-1}} \gg k_B T$

# E/T Scaling:



Meigan Aronson

M. C. Aronson et al, PRL 75, 725 (1995).

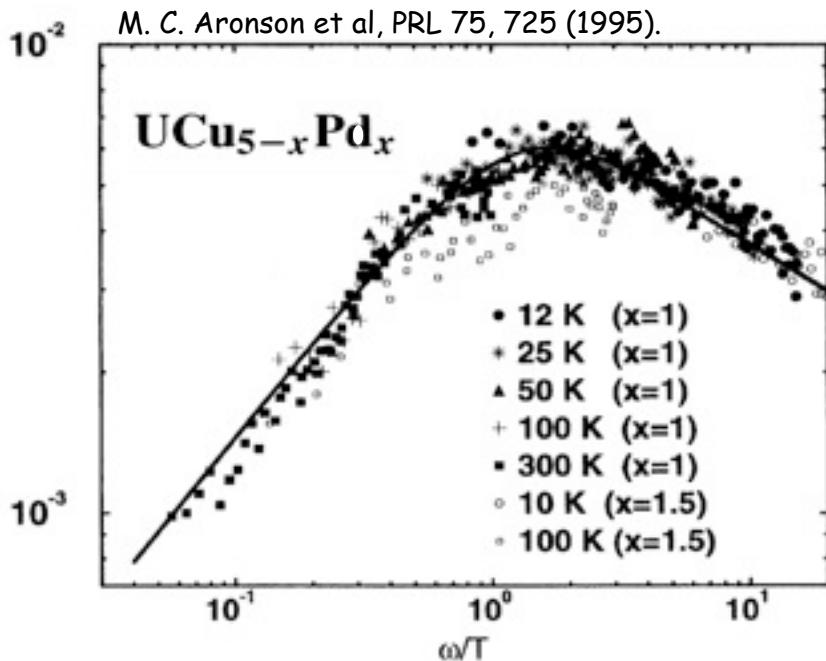


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$$\chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right)$$



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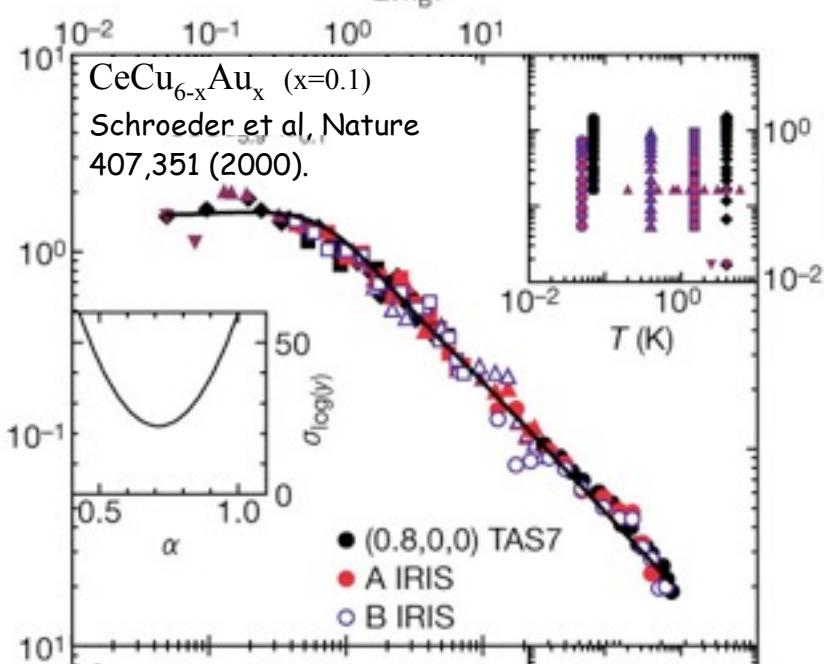
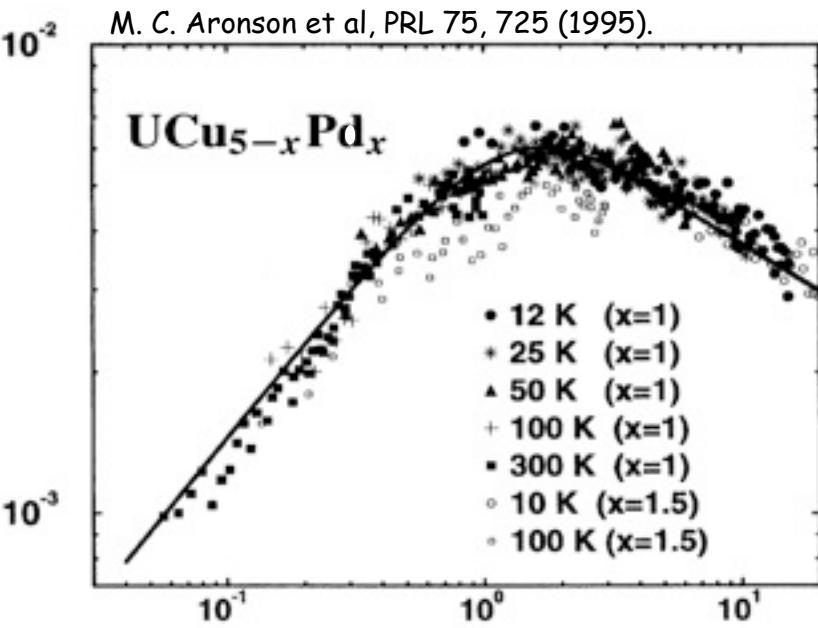


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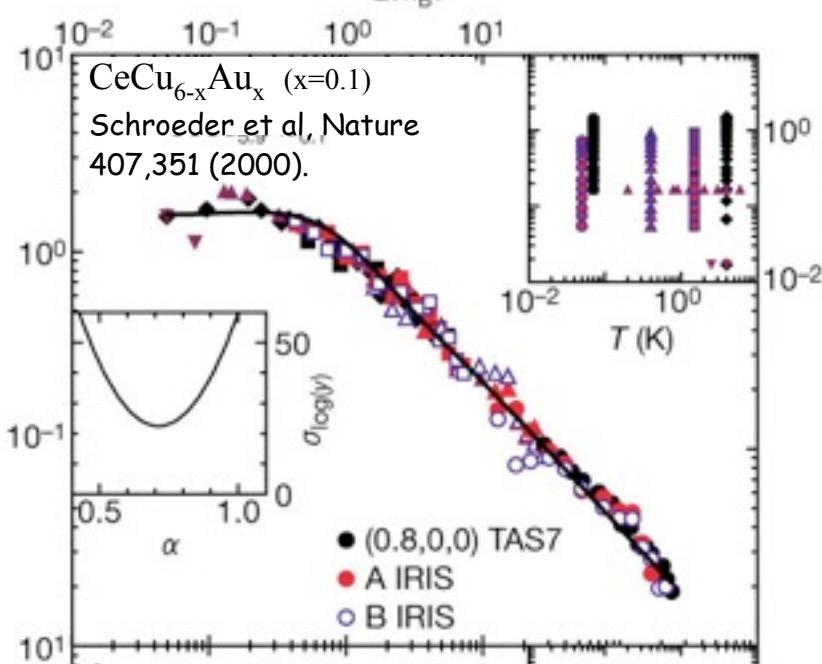
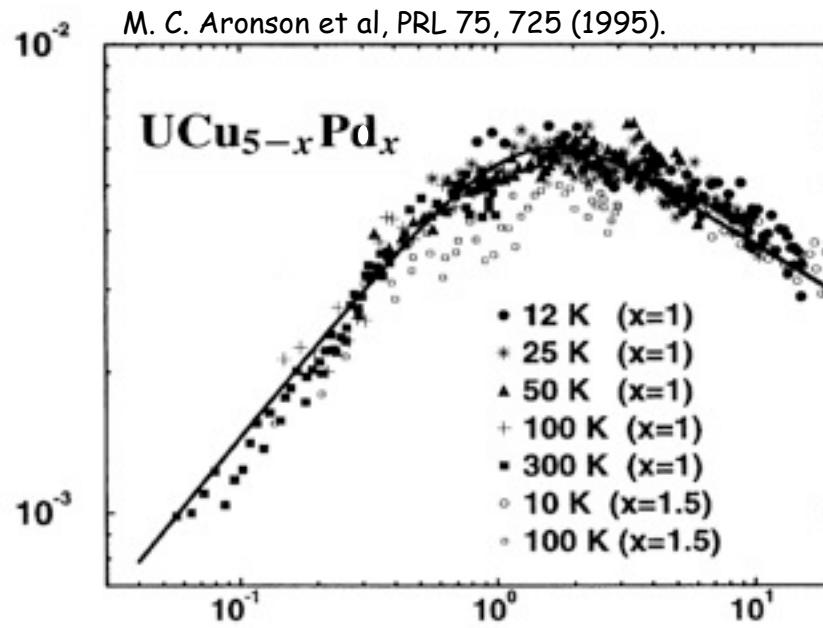
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Physics Below the upper  
Critical Dimension.



# The Standard Model

# Standard Model: Quantum SDW?



Doniach



Schrieffer



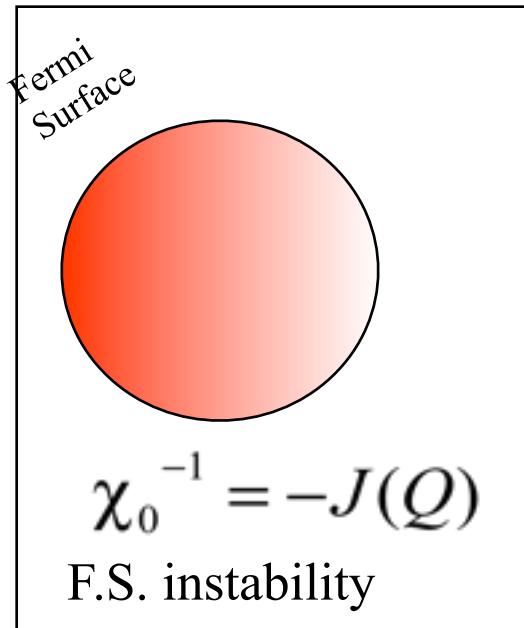
Hertz



Millis

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

$$d_{eff} = d + \textcolor{red}{z}$$



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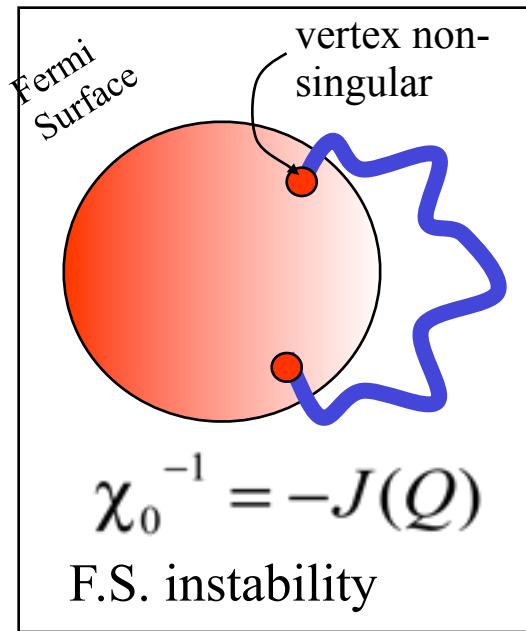
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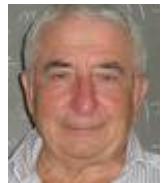
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$$\chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega/\Gamma)$$

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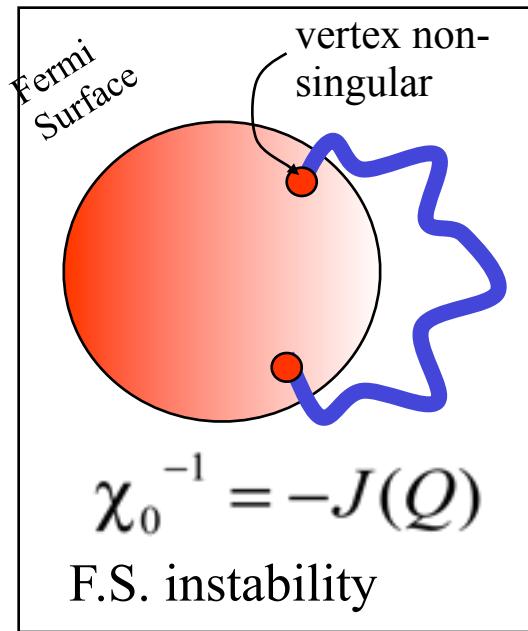
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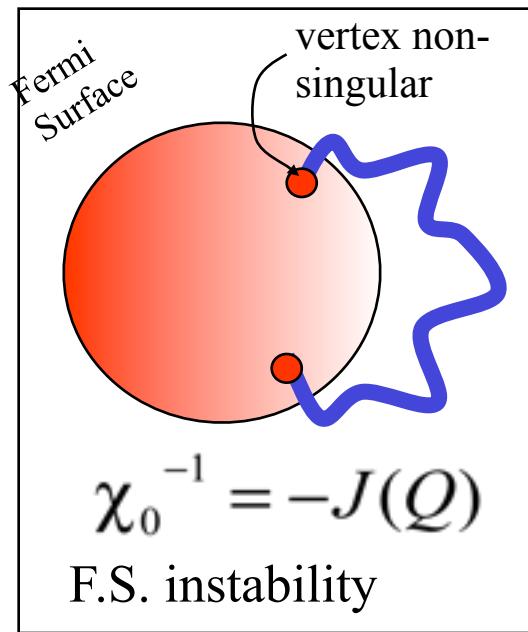
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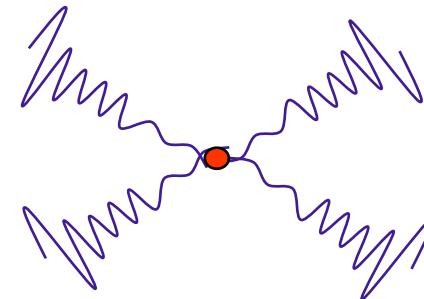
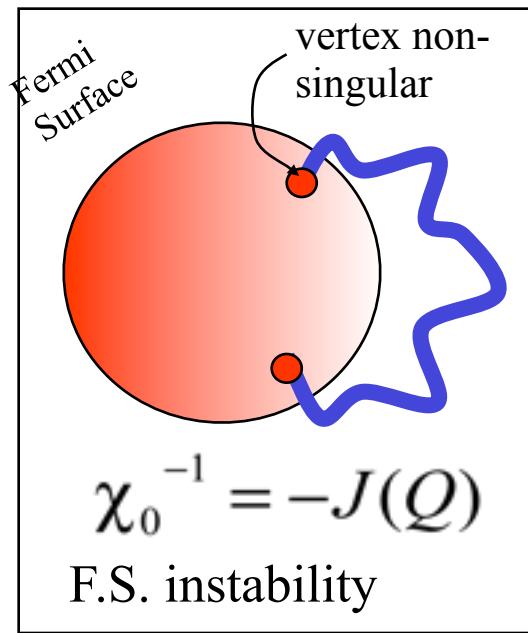
Time counts as  $z=2$  scaling dimensions

# Standard Model: Quantum SDW?



- Moriya, Doniach, Schrieffer (60s)
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- Millis (93)

$$d_{eff} = d + z$$



If  $d + z = d + 2 > 4$  :  
 $\phi^4$  terms “irrelevant”  
Critical modes are Gaussian.  
T is not the only energy scale.

$$\chi^{-1}(q, \omega) \propto (\xi^{-2} + (q - Q)^2 - i\omega/\Gamma)$$

$$\tau^{-1} \propto \xi^{-2}$$

Time counts as  $z=2$  scaling dimensions

New Ideas:

Break up of the electron.

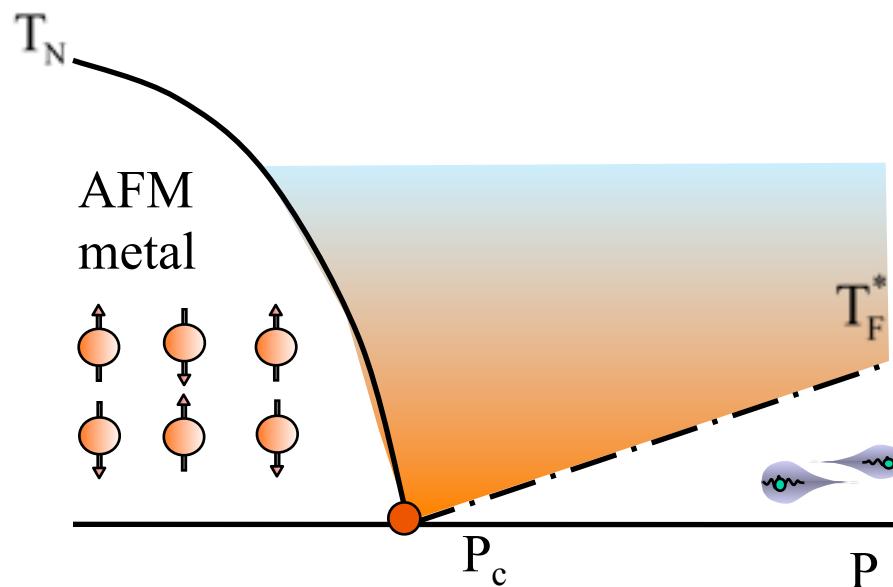
$$H = \sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_j (\psi^\dagger_j \vec{\sigma} \psi_j) \cdot \vec{S}_j$$

"THE BATTLEGROUND"  
Kondo Lattice Model  
(Kasuya, 1951)

$$H = \sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_j (\Psi^\dagger_j \vec{\sigma} \Psi_j) \cdot \vec{S}_j$$

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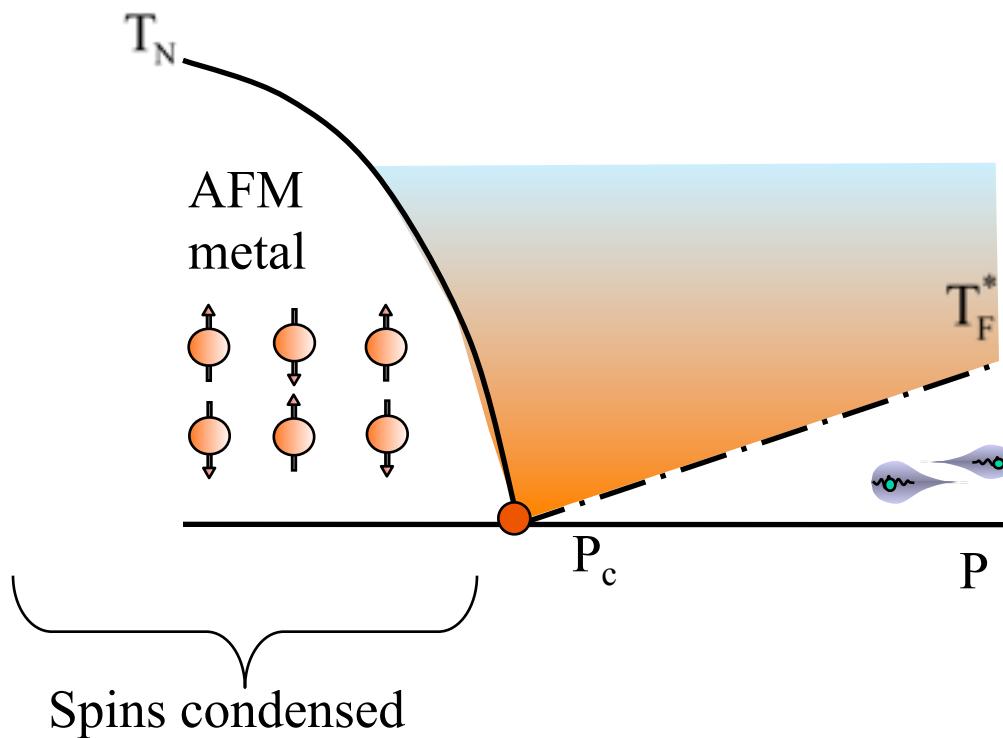
### Heavy Fermion Materials



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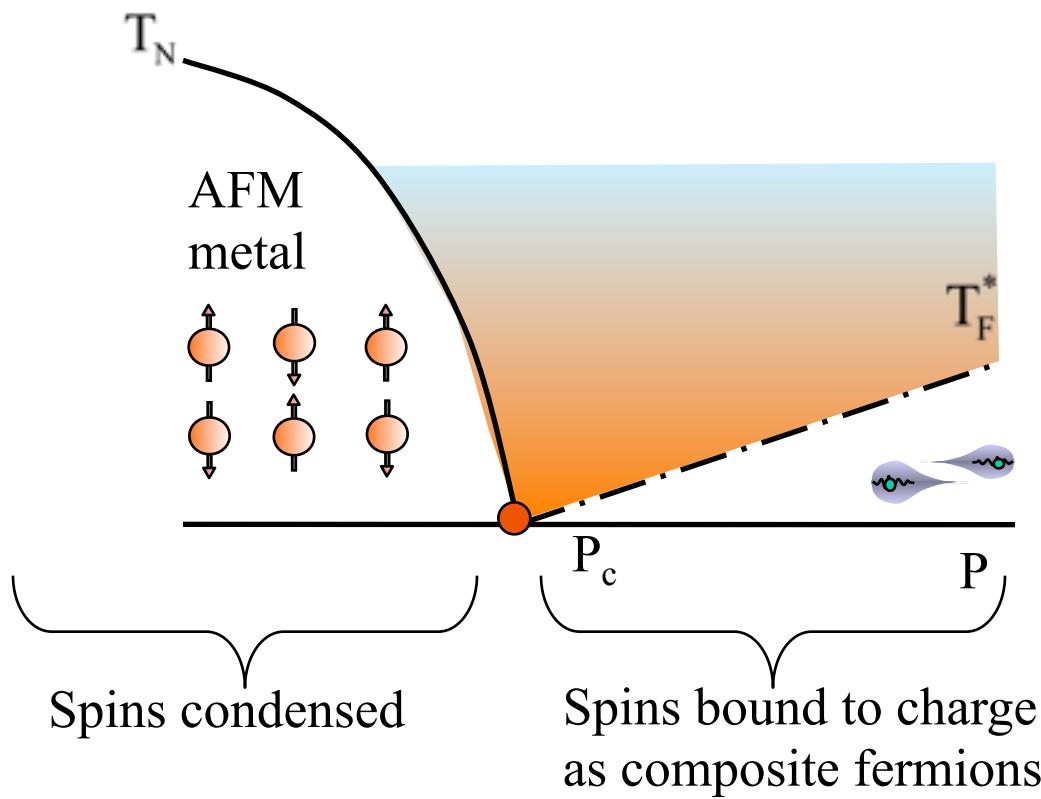
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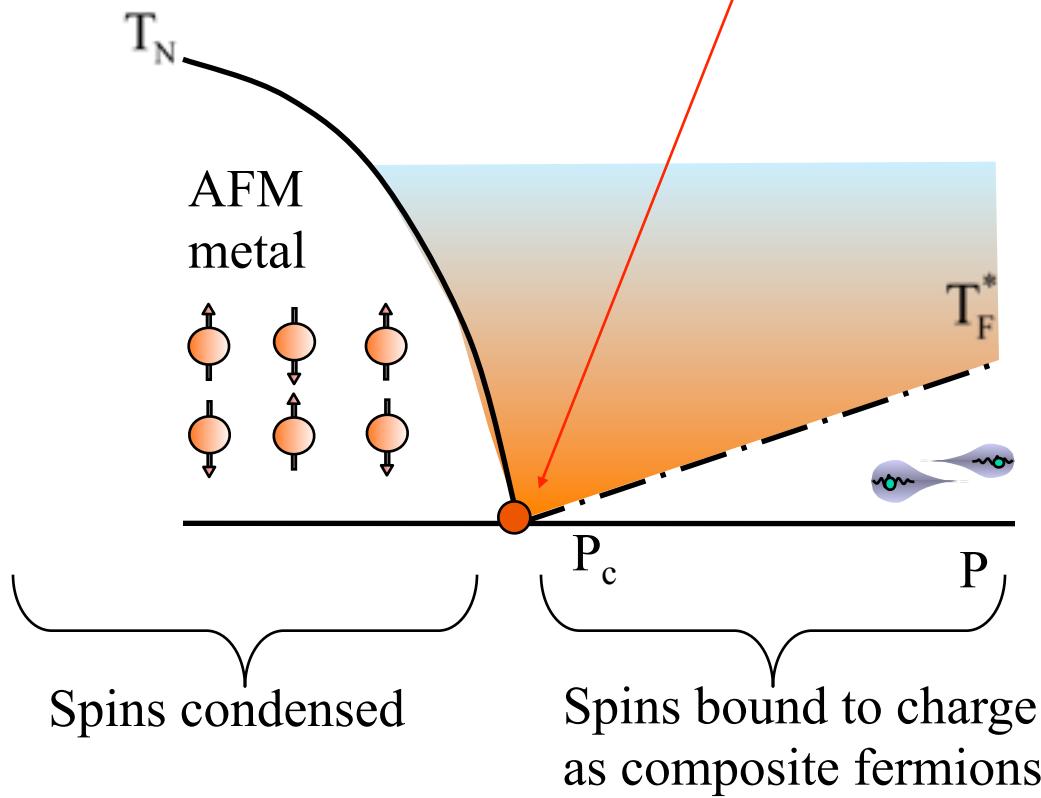


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"THE BATTLEGROUND"  
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Heavy Fermion  
Materials

Deconfinement of spin:  
fundamentally new kind of  
zero mode.

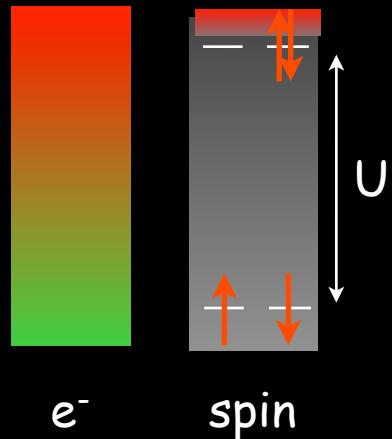


# New Methods

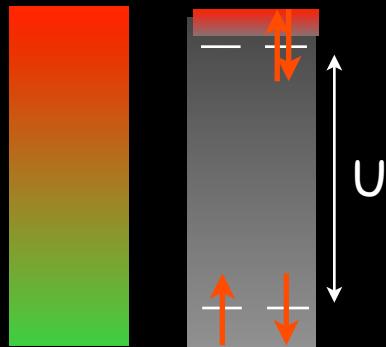


$e^-$

## New Methods



# New Methods



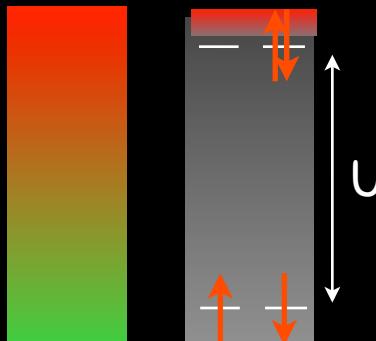
$e^-$       spin

$\Phi$        $W$

Elimination of States  
implies Gauge Fields.

(Read Newns, PC, Millis Lee... 80's)

# New Methods



A diagram showing a vertical rectangle divided into two horizontal sections. The top section is red and contains a pair of orange arrows pointing up and down. The bottom section is grey and contains a pair of orange arrows pointing up and down. A double-headed vertical arrow between the two sections is labeled 'U'.

$$\vec{S} = b^\dagger_\alpha \left( \frac{\vec{\sigma}}{2} \right)_{\alpha\beta} b_\beta$$

J. Schwinger '55

$e^-$       spin       $U(1) :$        $b_j \rightarrow e^{i\theta_j} b_j$

$\Phi$       W

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# New Methods



$e^-$  spin

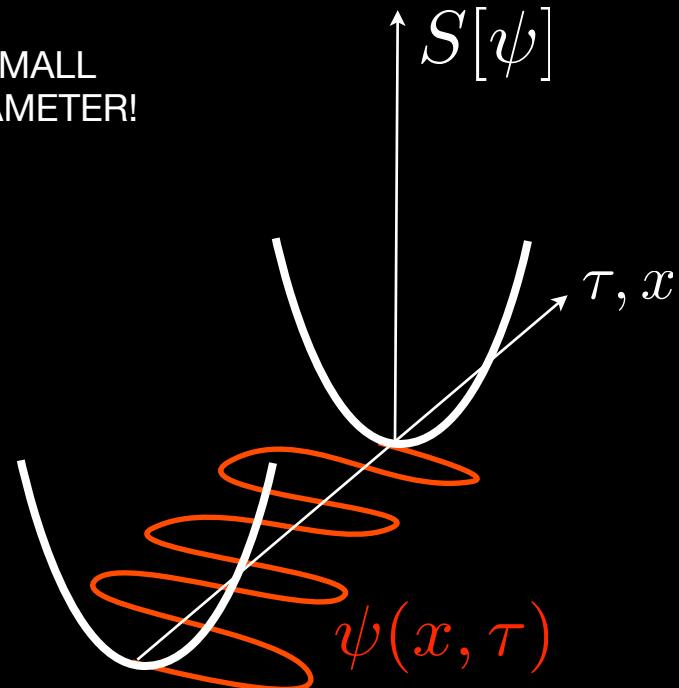
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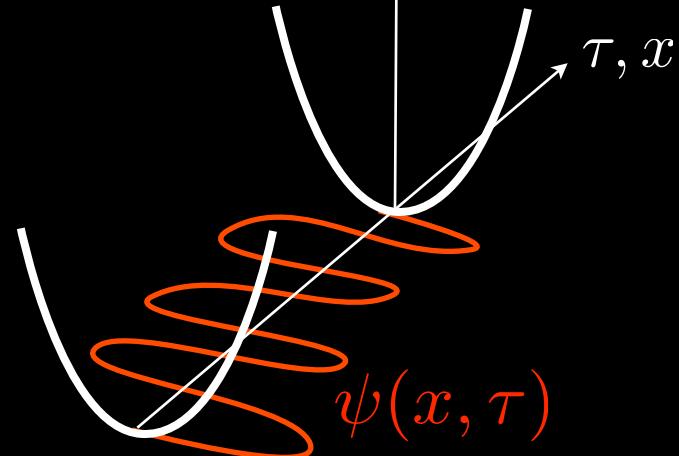
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NO SMALL  
PARAMETER!

$$S[\psi]$$

$\tau, x$



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Large N : family of models with "N" spin components, which retain the key physics and can be solved in the large N limit.

# New Methods



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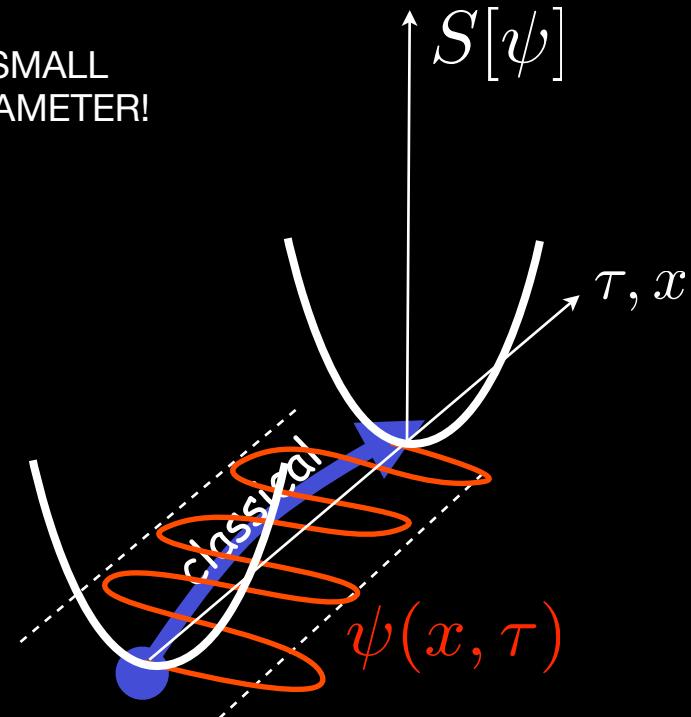
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Elimination of States  
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$$\frac{1}{N} \sim \hbar_{eff}$$

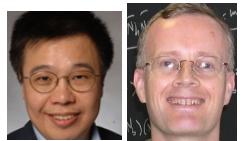
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# New Ideas

# New Ideas

Si, Ingersent



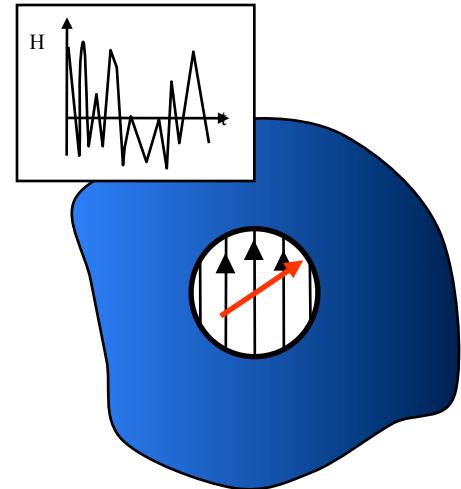
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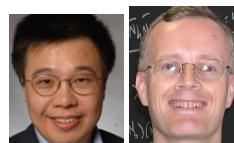
Fluctuations critical in time.

Requires a two dimensional spin fluid



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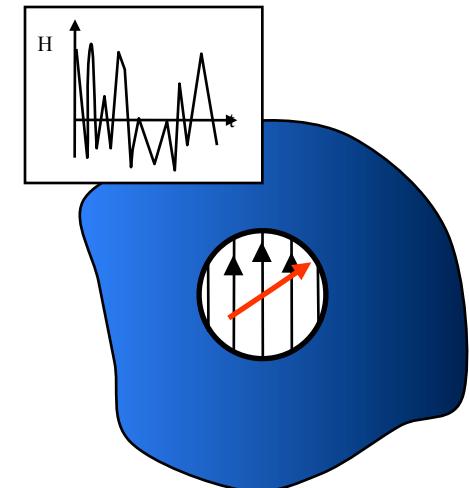


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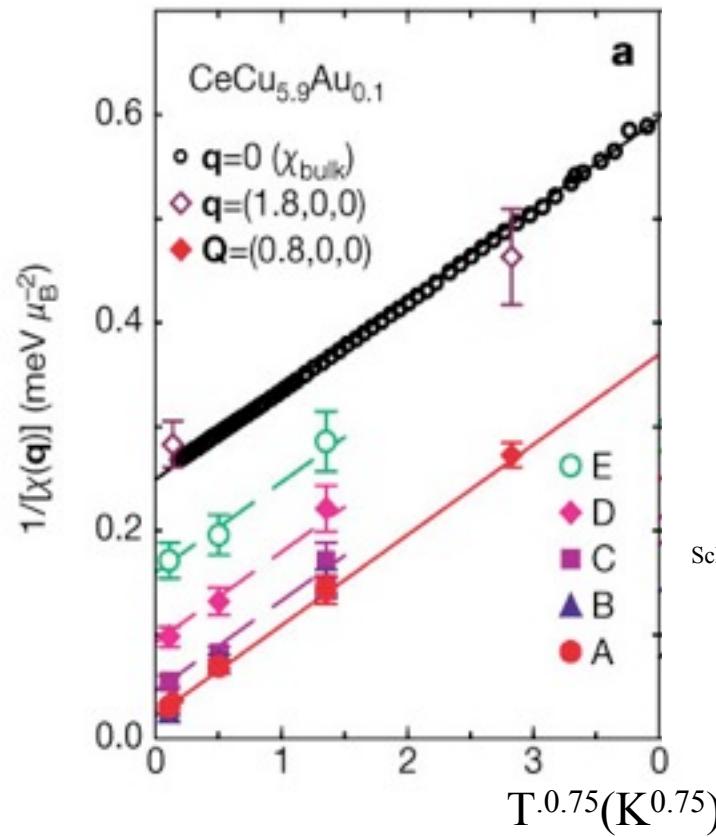
Fluctuations critical in time.



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## Locality of critical fluctuations

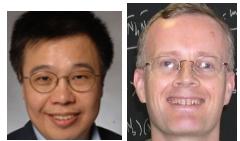
$$\chi^{-1} = \chi_0^{-1} + AT^\alpha$$



Schroeder et al, Nature 407, 351 (2000).

# New Ideas

Si, Ingersent



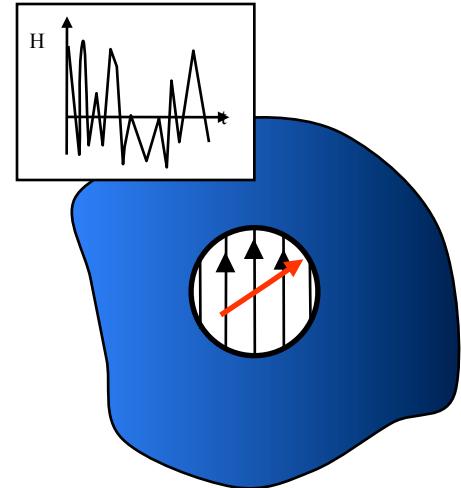
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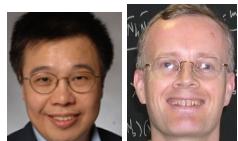
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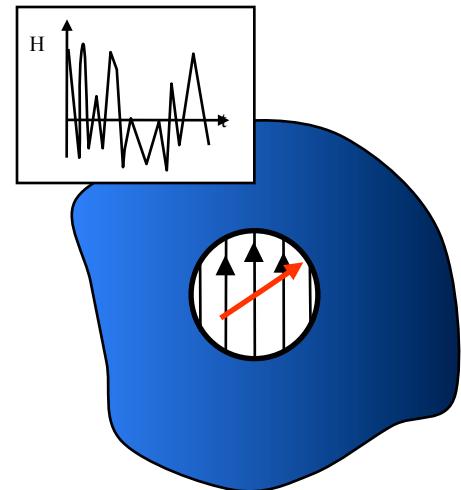
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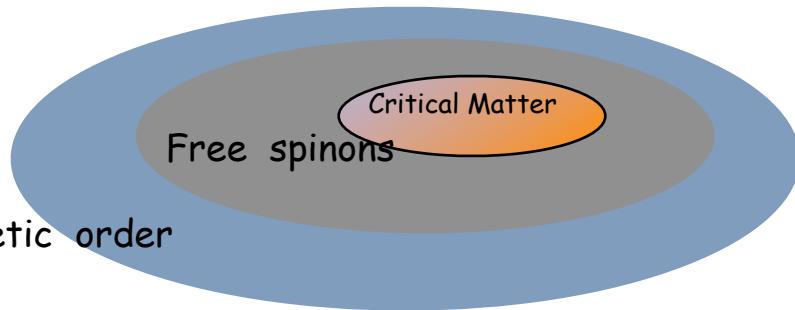


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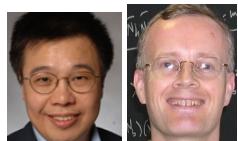


Senthil      Sachdev      Vishwanath



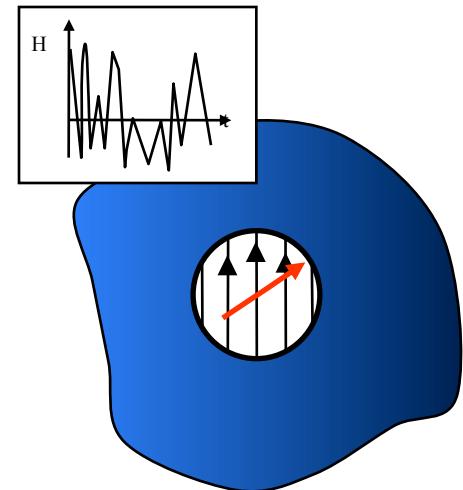
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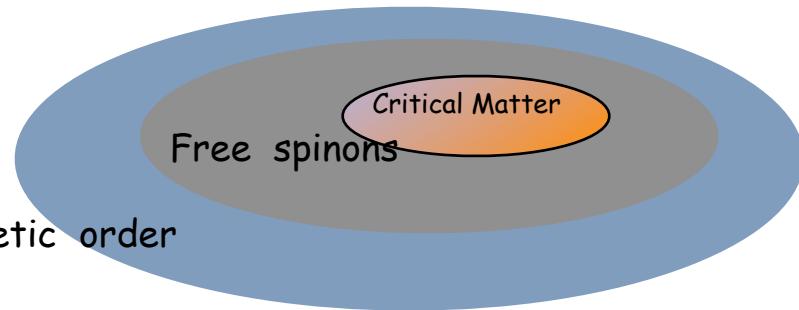


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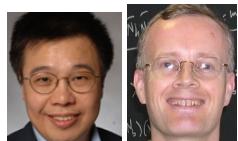


- Search for a new mean-field theory.

(PC et al JCM, 2001, Rech et al 2005, Lebanon et al 2006, Pepin 2006, 2008, Paul

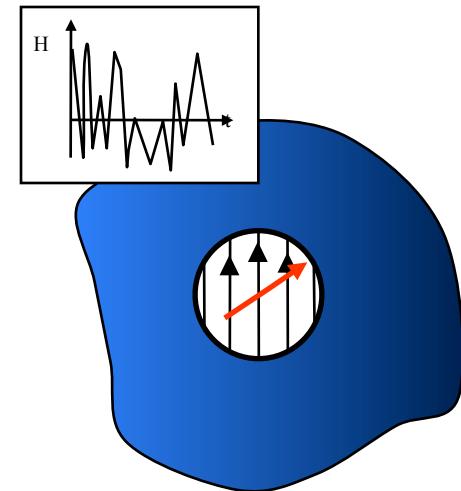
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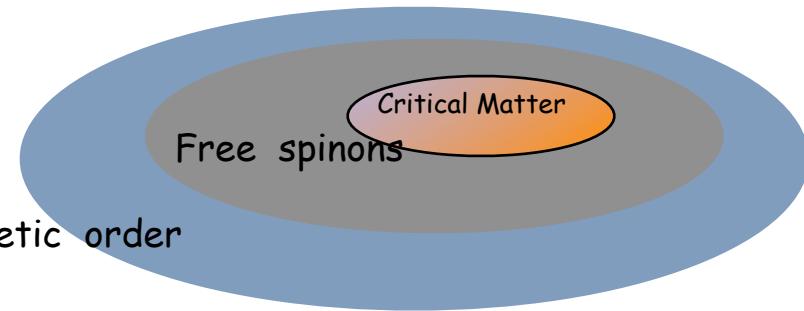


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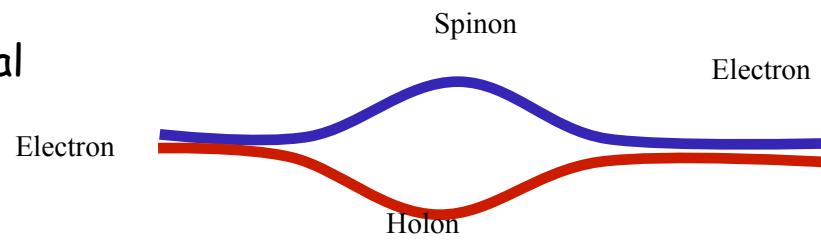
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Pepin



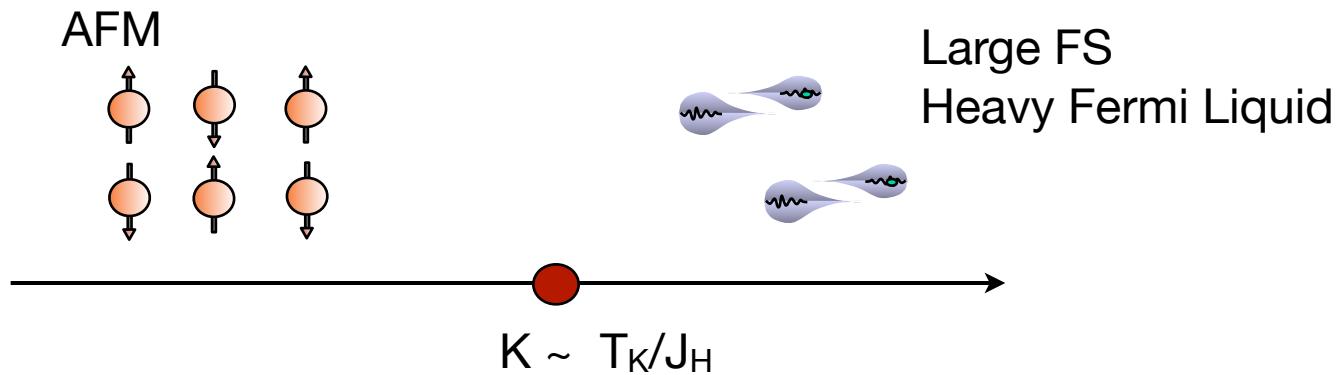
# Q-Frustration

# Kondo meets frustration

- Frustration and Kondo have different effects.

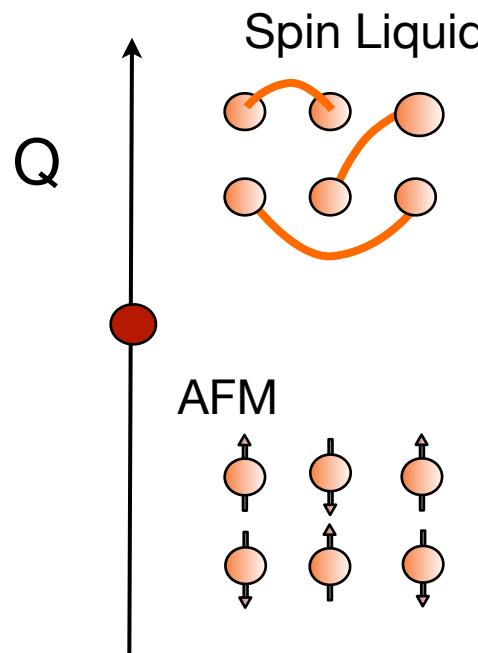
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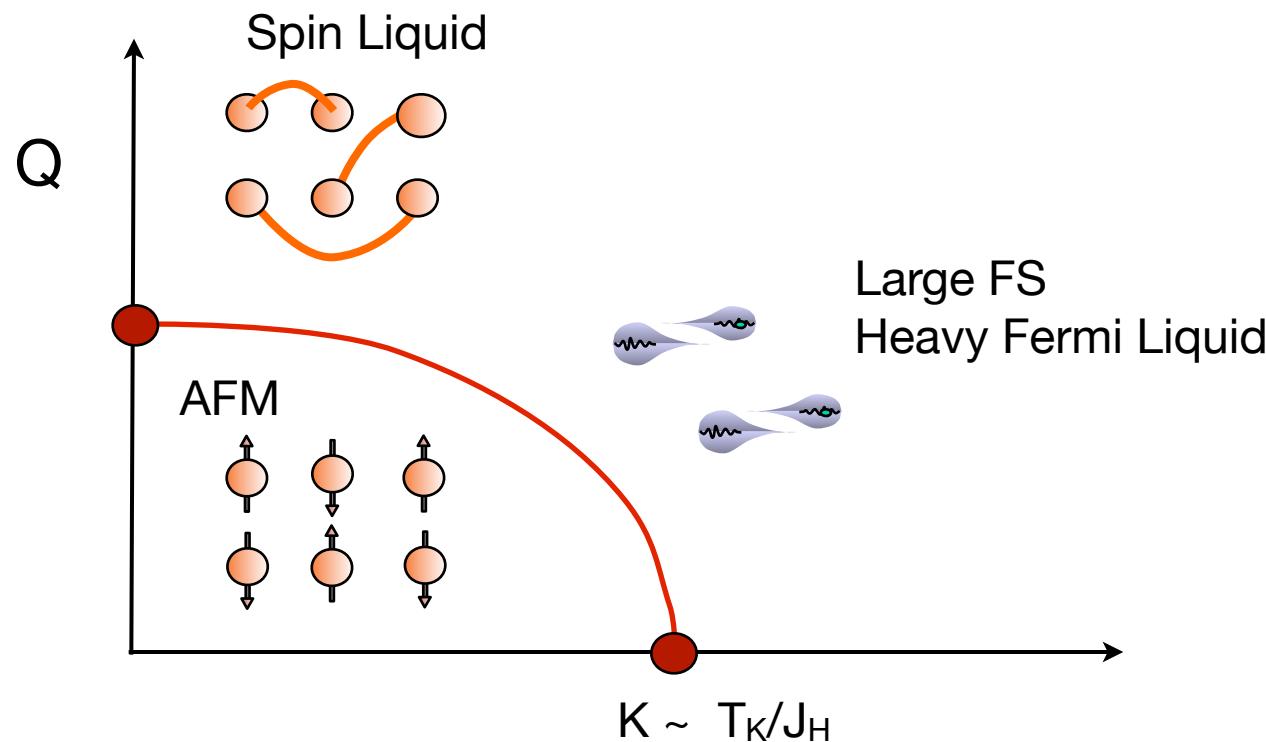
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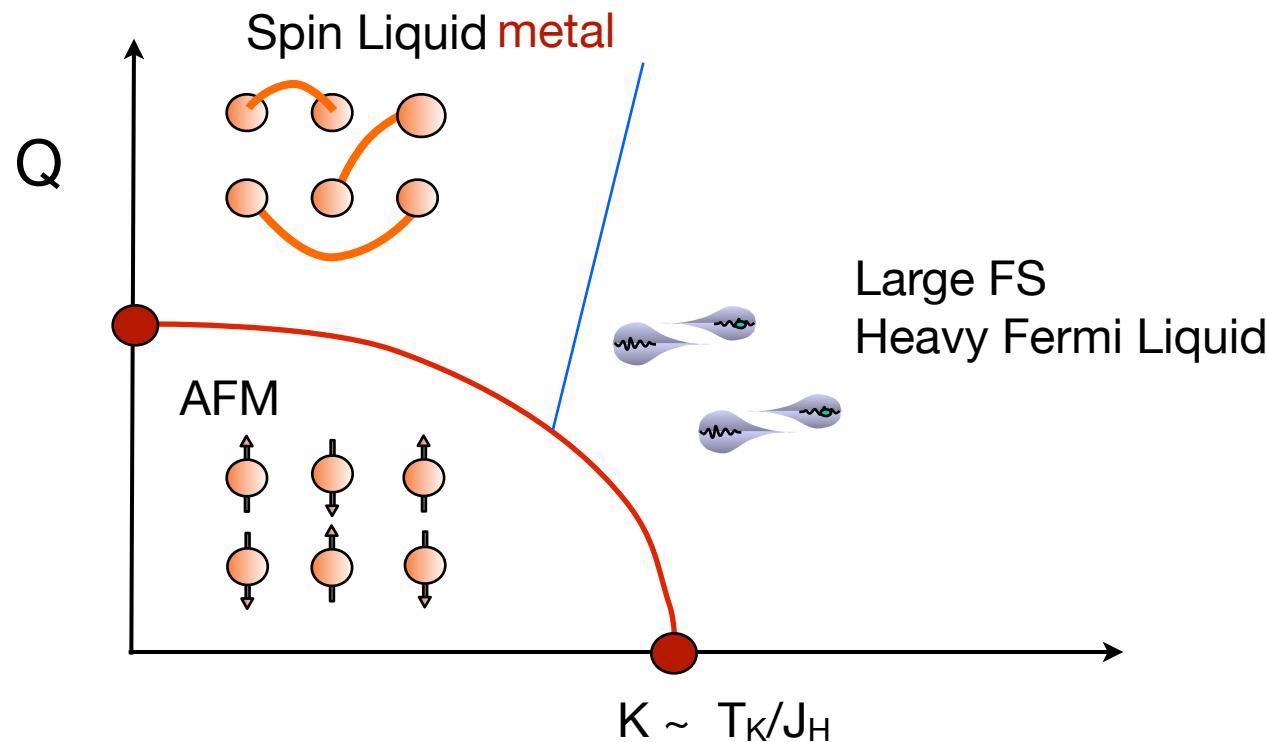
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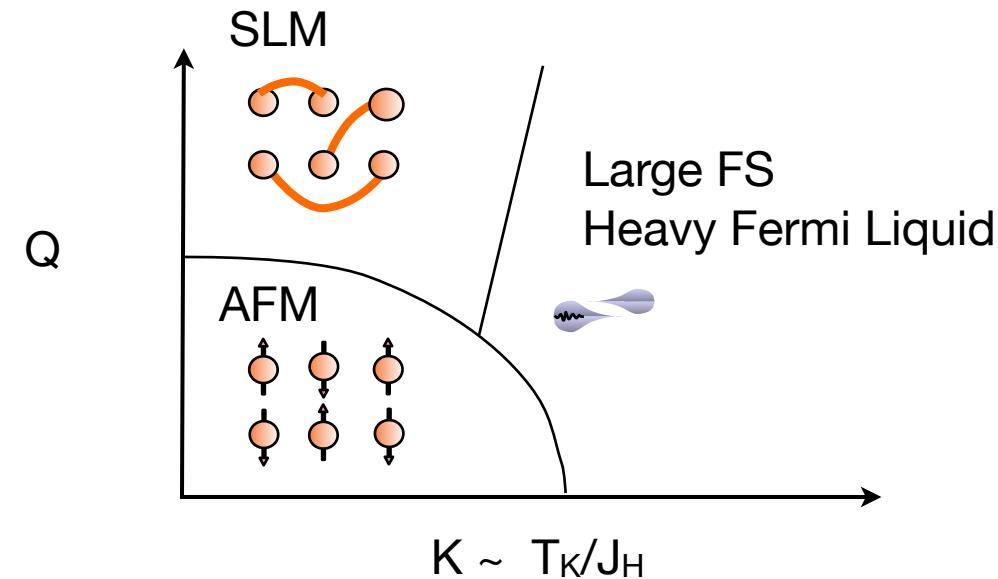


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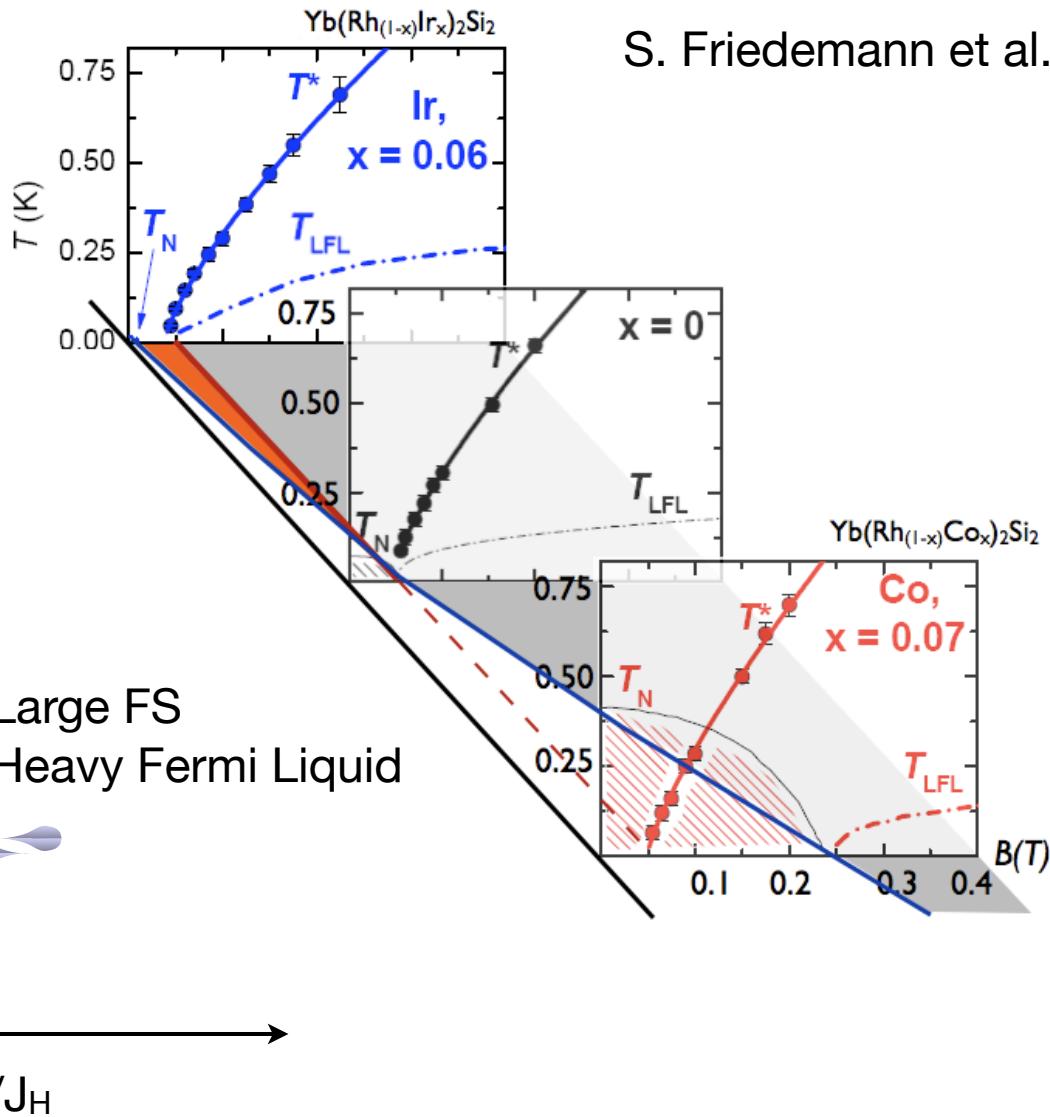
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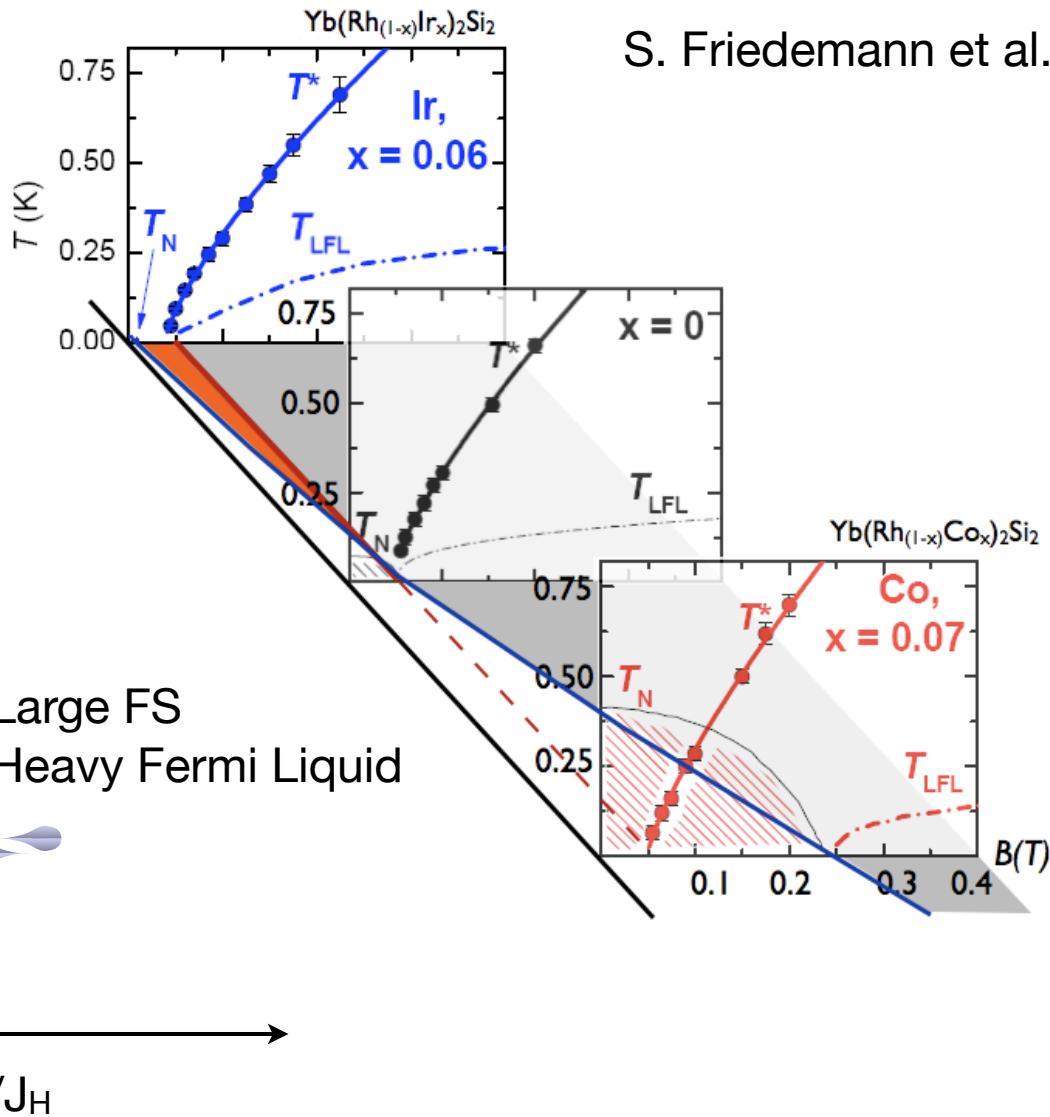


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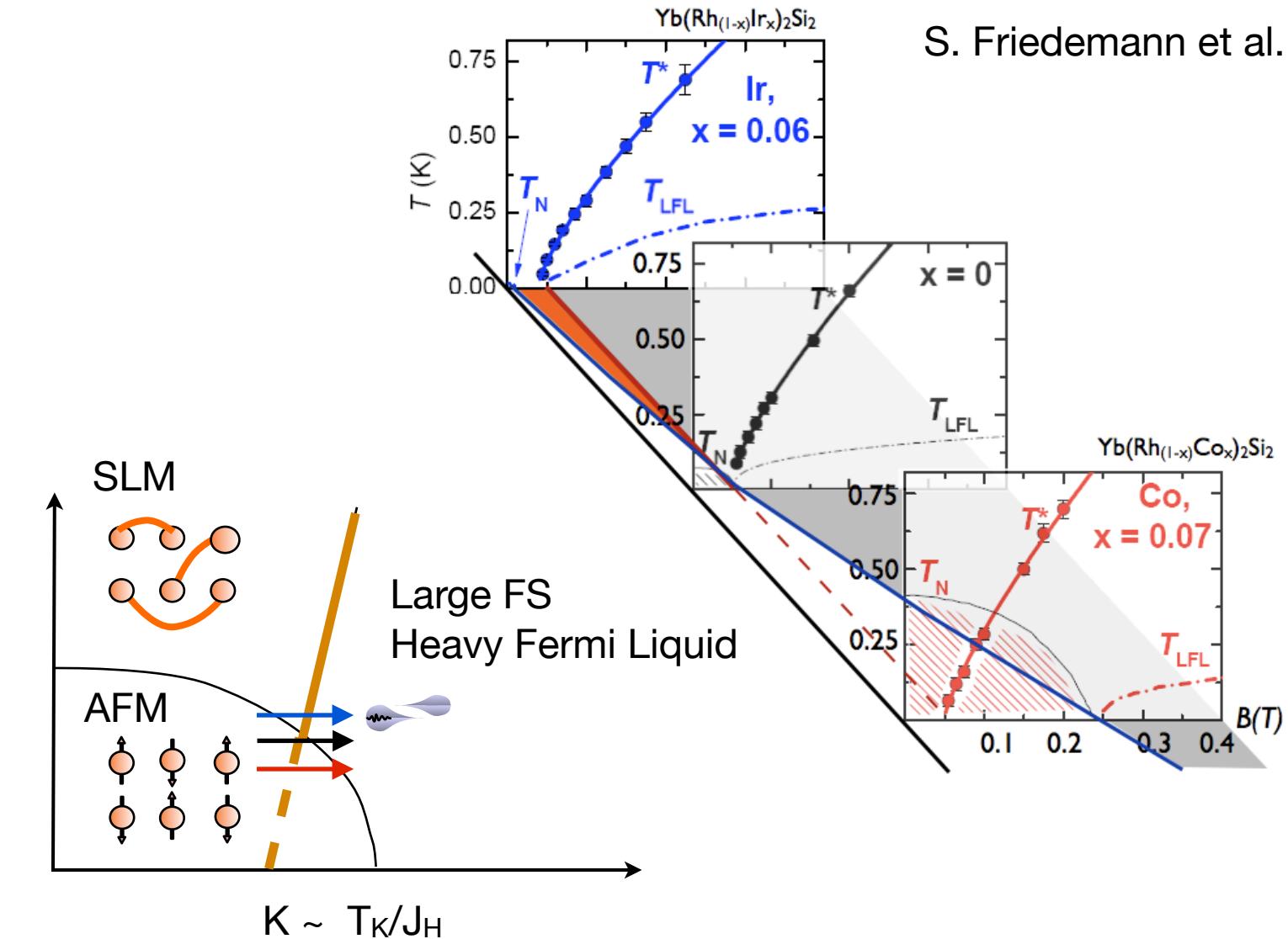


S. Friedemann et al. (2009).

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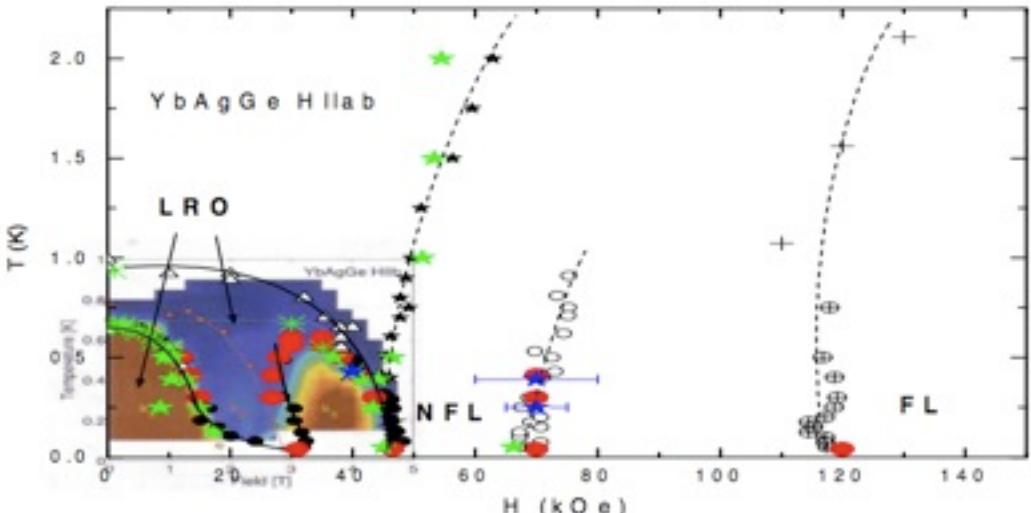


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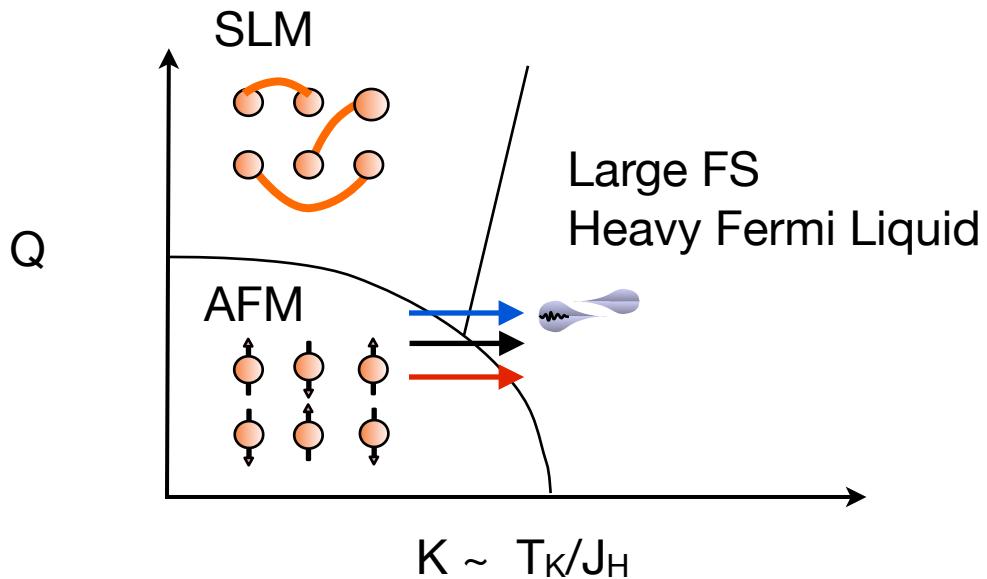


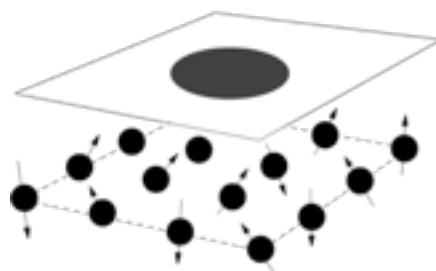
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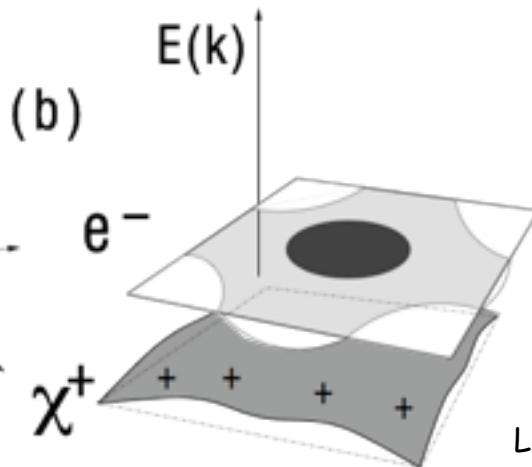


Canfield et al (unpublished)



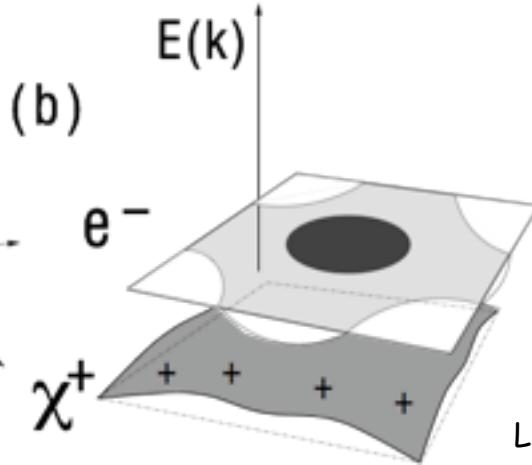
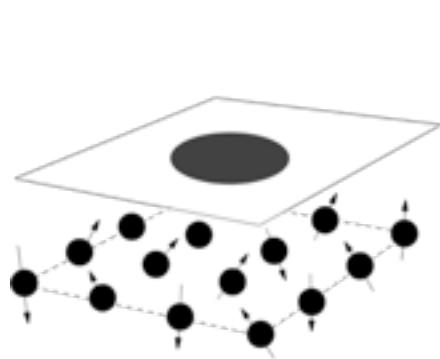


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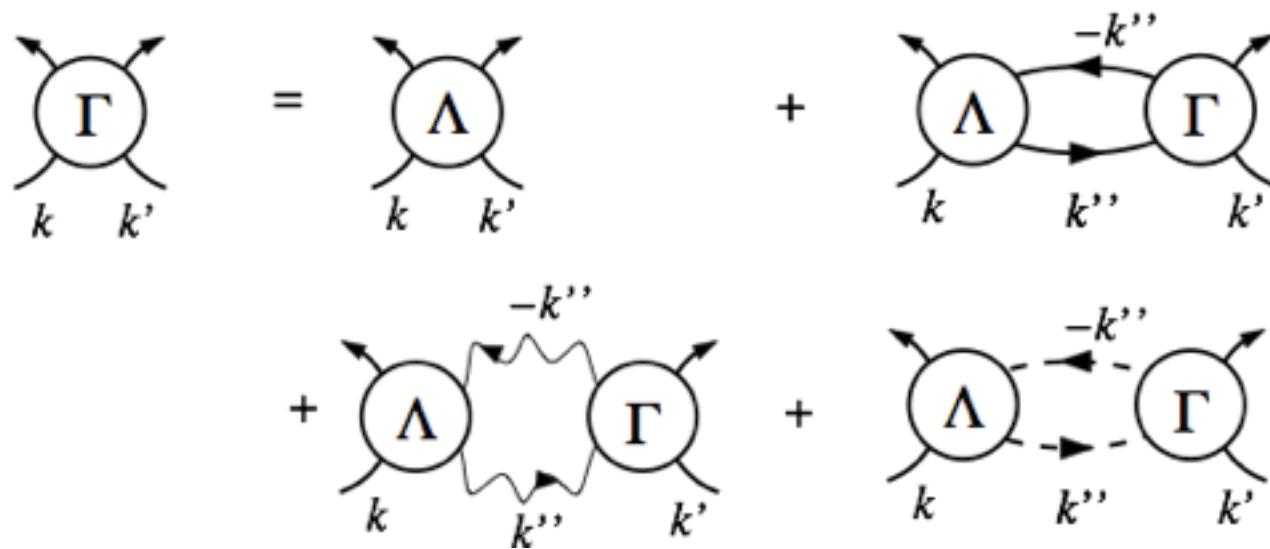
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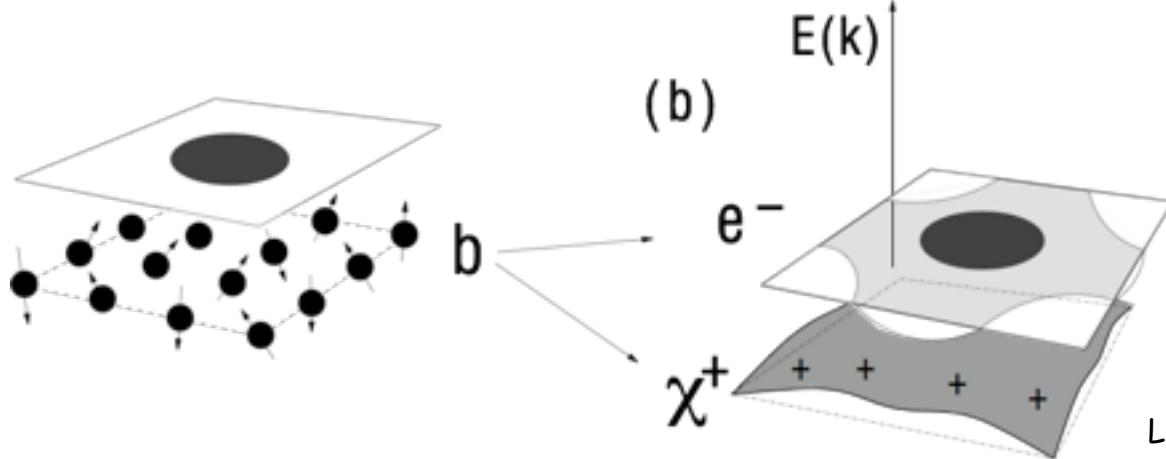
Luttinger sum rule for Kondo Lattice (Oshikawa, 2000) P.C, I.Paul, J. Rech (05)



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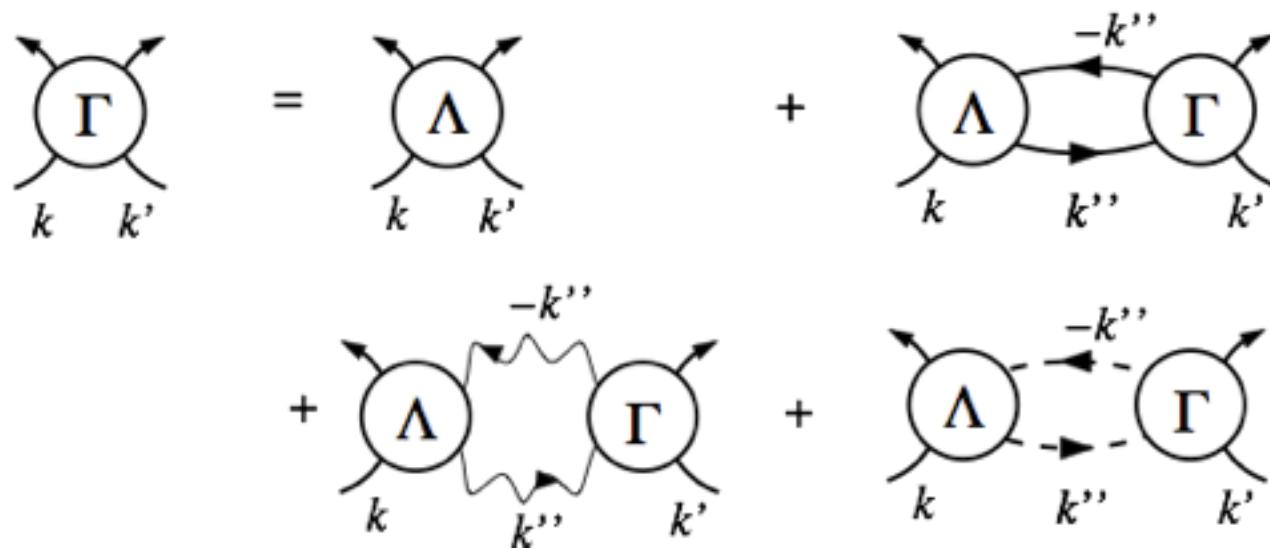
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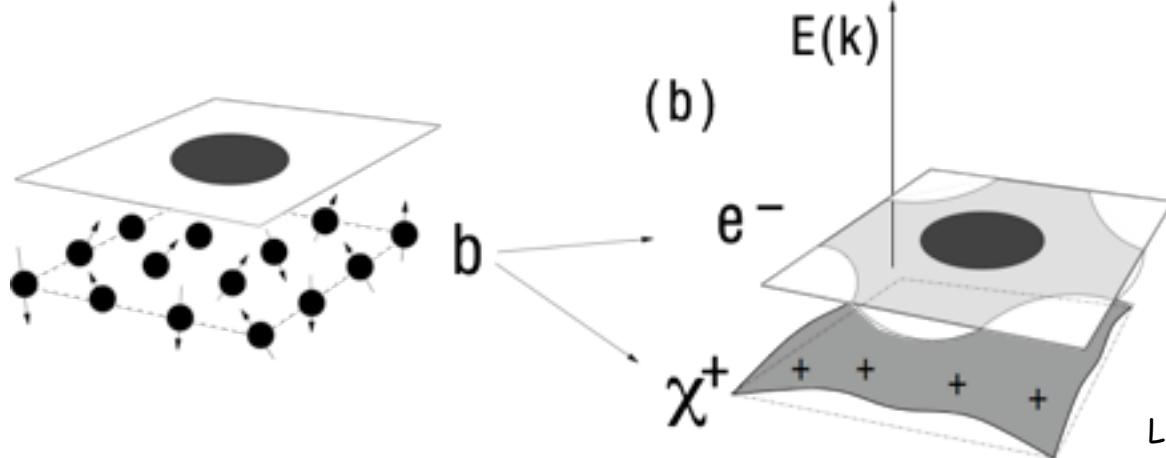


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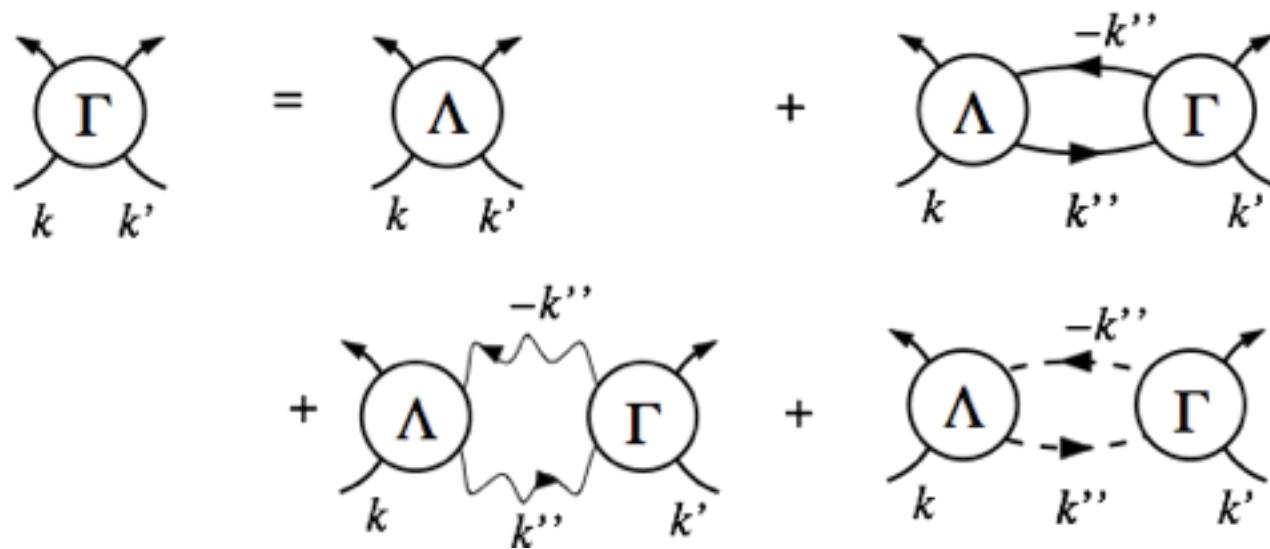


Virtual spinons.



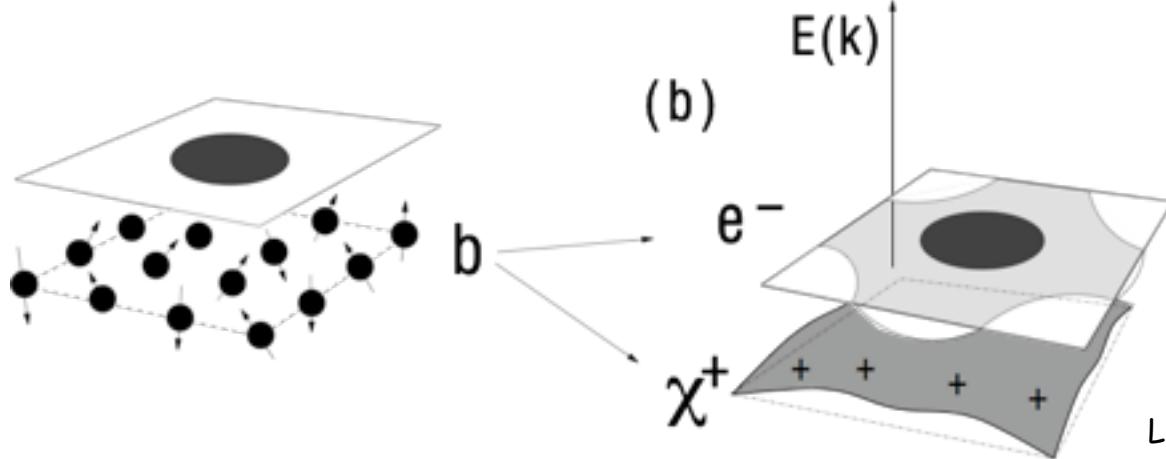
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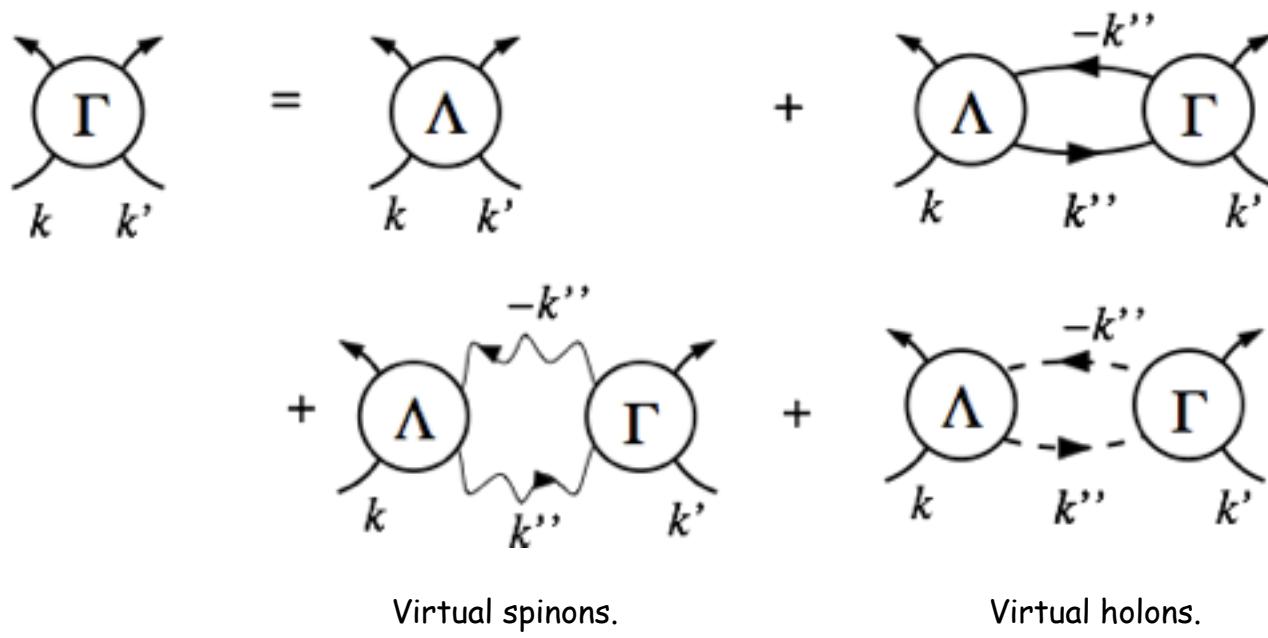
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Virtual holons.



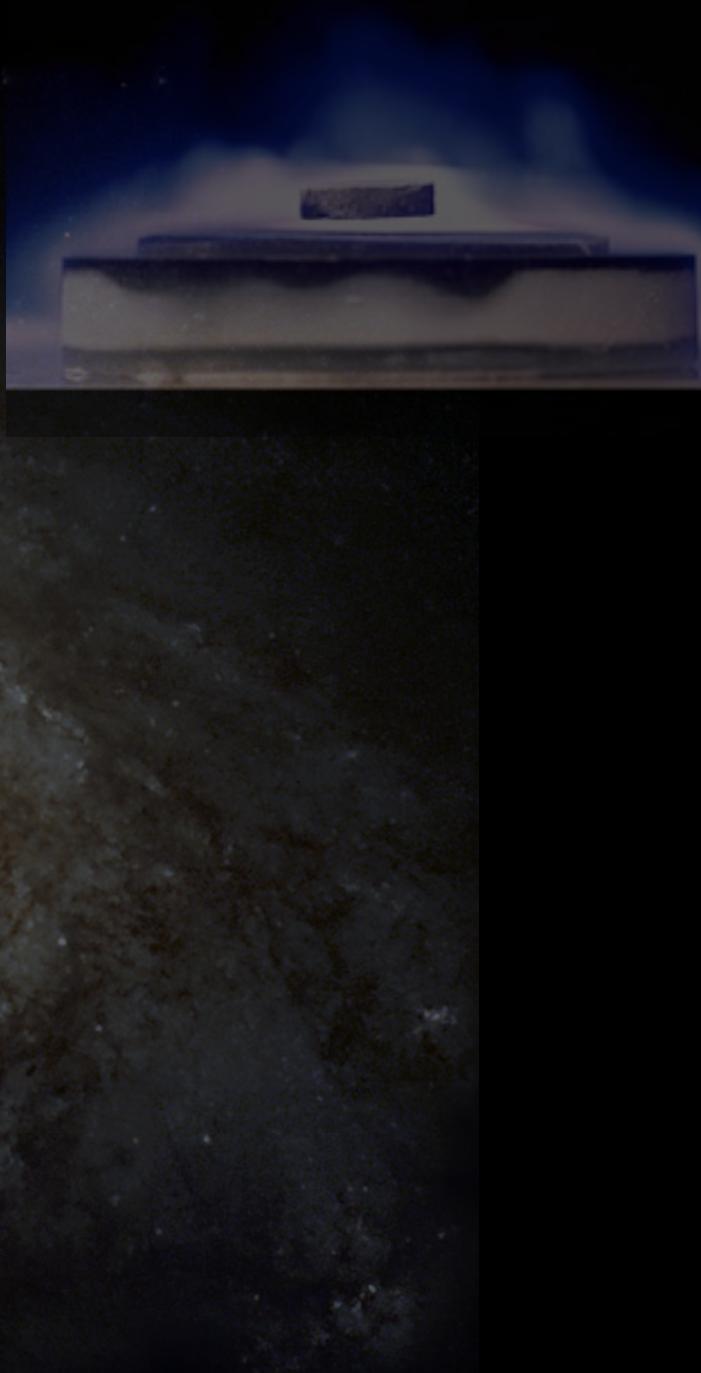
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Fermi Liquid scattering parameters determined primarily by excitation of low-lying spinon and holon states..

Towards a new Conjunction of ideas.



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# Collaborators



Flint



Nev.



Dzero



Rech



Lebanon

Rebecca Flint

Rutgers

Andriy Nevidomskyy

Rutgers

Maxim Dzero

Columbia/Rutgers

Jerome Rech,

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Eran Lebanon,

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Indranil Paul,

CNRS, Grenoble

Lucia Palova

Rutgers

Premi Chandra

Rutgers

Gergely Zarand

Budapest

Olivier Parcollet

SpHT Paris.

Andy Schofield

Birmingham

Qimiao Si

Rice, Houston

Catherine Pepin

SpHT Paris.

Almut Schroeder

Kent State

Gabriel Aeppli

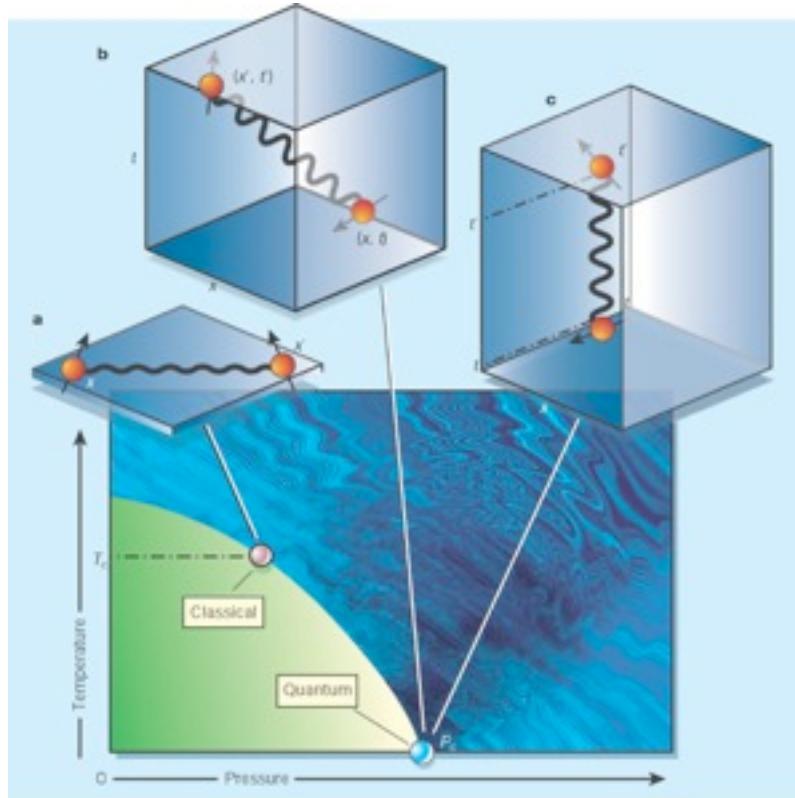
LCN

Hilbert v. Lohneysen

Karlsruhe

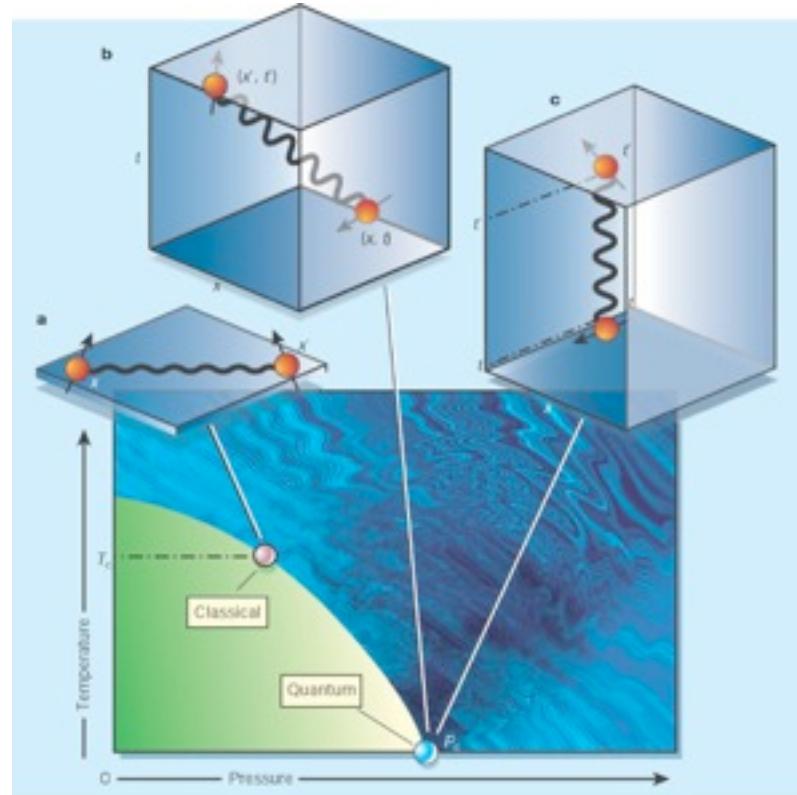


# Conclusions



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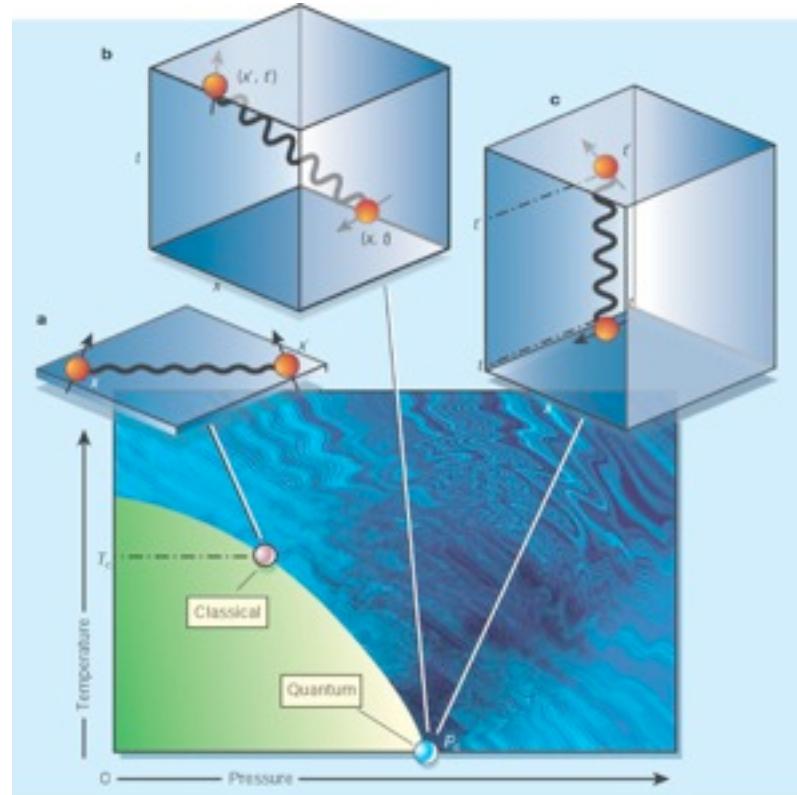
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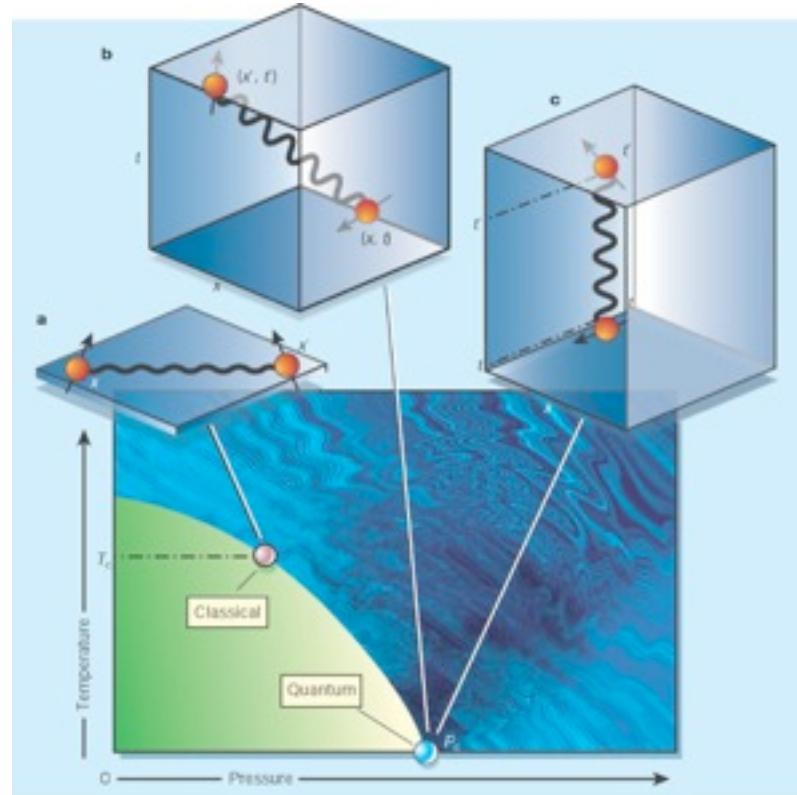


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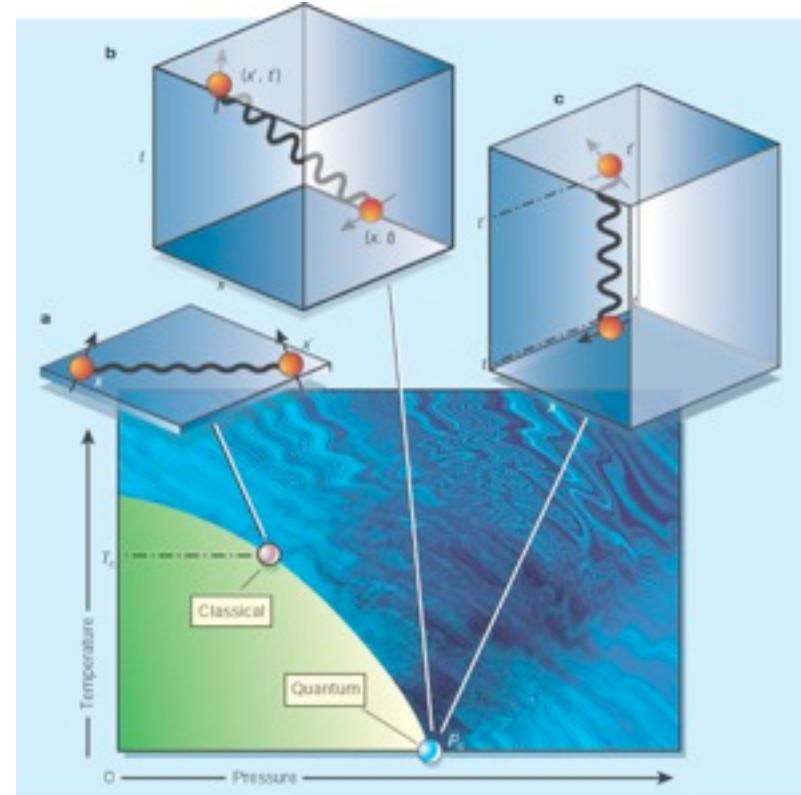


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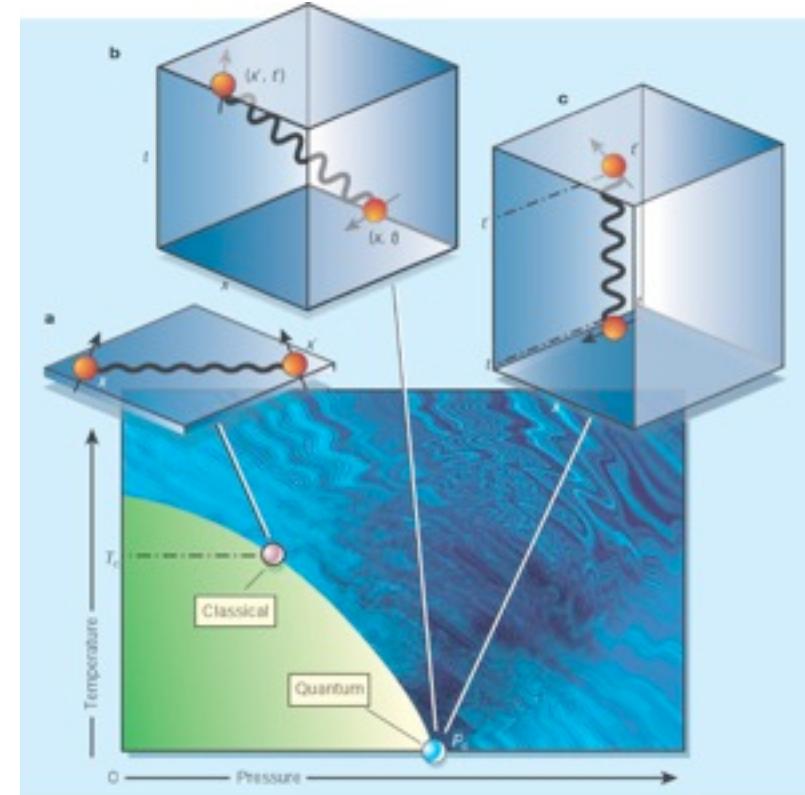
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New rule in material physics: avoided criticality.  
New phases develop in order to avoid the  
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