The size of the proton from the Lamb shift in muonic hydrogen

for the CREMA collaboration

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Introduction:
Hydrogen, fundamental constants, QED tests and all that.
How large is the proton?

Muonic hydrogen:
(Finite) size does matter!

Experiment
- Principle
- Muon beam
- Laser system
- Data

Results
- muonic hydrogen #1 → proton charge radius
- muonic hydrogen #2 → proton’s Zemach (magnetic) radius
- muonic deuterium #1 → deuteron charge radius, polarizability
- muonic deuterium #2 + #3 → theory work ahead!
Hydrogen energy levels

\[ E = \frac{R_{\infty}}{n^2} \]

\[ V \sim \frac{1}{r} \]
Hydrogen energy levels

Energy

\begin{align*}
\text{n=3} & \quad 2P_{3/2} \\
\text{n=2} & \quad 2S_{1/2}, \ 2P_{1/2} \\
\text{n=1} & \quad 1S_{1/2}
\end{align*}

Shift: -43.5 GHz

Bohr

\[ E = \frac{R_{\infty}}{n^2} \]
\[ V \sim \frac{1}{r} \]

Dirac

e^{-} \text{ spin relativity}
Hydrogen energy levels

\[ E = \frac{R_{\infty}}{n^2} \]

\[ V \sim \frac{1}{r} \]

Shift:
-43.5 GHz 8.2 GHz

Bohr
\[ 1S_{1/2} \]

Dirac
\[ e^- \text{ spin} \]
relativity

Lamb
QED
Hydrogen energy levels

Energy levels:
- n=1
  - 1S_{1/2}
  - Shift: -43.5 GHz
  - 1.4 GHz hfs-splitting
    - proton-spin: \( H^{hfs} \sim \vec{\mu}_p \cdot \vec{\mu}_e \)
- n=2
  - 2S_{1/2}, 2P_{1/2}
  - Shift: 8.2 GHz
- n=3
  - 2P_{3/2}
  - Shift: 2P_{3/2}

Bohr:
- \( E = R_\infty / n^2 \)
- \( V \sim 1/r \)

Dirac:
- e^- spin relativity

Lamb:
- QED

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Hydrogen energy levels

**Bohr**
- \( E = \frac{R_{\infty}}{n^2} \)
- \( V \sim \frac{1}{r} \)

**Dirac**
- \( e^- \) spin
- relativity

**Lamb**
- QED

**hfs-splitting**
- proton-spin
- \( H^{\text{hfs}} \sim p_p \cdot p_e \)
- proton size
- \( V \sim \frac{1}{r} \)

**Shifts**
- -43.5 GHz
- 8.2 GHz
- 1.4 GHz
- 1.2 MHz

**Energy Levels**
- \( n=3 \)
- \( n=2 \)
- \( n=1 \)
Increasing accuracy !!!

The Rydberg constant:

\[ R_{\infty} = 10\,973\,731.568\,525 \pm 0.000\,073 \text{ m}^{-1} \] \( (u_r = 6.6 \cdot 10^{-12}) \)

is the most accurately determined fundamental constant.
Increasing accuracy ?!

Test of bound-state QED (Lamb shift) in Hydrogen:

QED-test is limited by the uncertainty of the proton rms charge radius.
Increasing accuracy ?!

Test of bound-state QED (Lamb shift) in Hydrogen:

- Hydrogen spectroscopy to test QED

\[ \nu_{1S-2S} = \frac{3}{4} R_\infty + \Delta L(r_p, \text{QED}) \]

MPQ 2S-8S/D
LKB, Paris

\[ E_{\text{fin.size}} = \frac{2}{3} \left( \frac{m_r}{m_e} \right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left( \frac{2\pi Z\alpha r_p}{\lambda_C} \right)^2 \]

- test QED \arrow[<] best non-H \( r_p = (0.895 \pm 0.018) \text{ fm} \) (2\%) \arrow[<] e-p scattering

- trust QED \arrow[<] extract \( r_p = (0.8768 \pm 0.0069) \text{ fm} \) (CODATA)

QED-test is limited by the uncertainty of the proton rms charge radius.
The proton rms charge radius is not the most accurate quantity in the universe.

\[ r_p = 0.895(18) \text{ fm} \quad (u_r = 2\%) \]

CODATA: \[ r_p = 0.8768(69) \text{ fm} \quad (u_r = 0.8\%) \]
The proton rms charge radius is not the most accurate quantity in the universe.

e-p scattering: $r_p = 0.895(18)$ fm ($u_r = 2\%$)

CODATA: $r_p = 0.8768(69)$ fm ($u_r = 0.8\%$) 20x improvement

muonic hydrogen goal: $u_r = 0.1\%$
Proton charge radius and muonic hydrogen

muonic hydrogen = \( \mu^- \, p \)  
mass \( m_\mu = 207 \, m_e \)

\[
\Delta E_{\text{finite size}}(nl) \sim r_p^2 |\Psi(r=0)|^2
\]

\[
\langle r_{\text{orbit}} \rangle \sim \frac{\hbar}{Z \alpha m_r c} n^2
\]

\[
\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4 c^4}{3\hbar^2 n^3} m_r^3 r_p^2 \delta_{l0}
\]

Lamb shift in \( \mu p \): \( \Delta E(2P^{F=2}_{3/2} - 2S^{F=1}_{1/2}) = \)

\[
209.9779(49) - 5.2262 \, r_p^2 + 0.0347 \, r_p^3 \, [\text{meV}]
\]

finite size contribution is 2% of the \( \mu p \) Lamb shift measure \( \Delta E(2S-2P) \) to 30 ppm = 1.5 GHz

\[
\Rightarrow r_p \, \text{to} \, 10^{-3}
\]

\[
\Gamma_{2P} = 18.6 \, \text{GHz} \quad (\Gamma_{\text{rad.}})
\]
**μp Lamb shift experiment: Principle**

**“prompt”** \((t \sim 0)\)

- \(\mu^-\) stop in \(H_2\) gas
  - \(\Rightarrow \mu p^*\) atoms formed \((n \sim 14)\)
- 99%: cascade to \(\mu p(1S)\), emitting prompt \(K_\alpha, K_\beta\) ...
- 1%: long-lived \(\mu p(2S)\) atoms
  - lifetime \(\tau_{2S} \approx 1 \mu s\) at 1 mbar \(H_2\)

**“delayed”** \((t \sim 1 \mu s)\)

- Fire laser \((\lambda \approx 6 \mu m, \Delta E \approx 0.2 eV)\)
  - \(\Rightarrow\) induce \(\mu p(2S) \rightarrow \mu p(2P)\)
  - \(\Rightarrow\) observe delayed \(K_\alpha\) x-rays
  - \(\Rightarrow\) normalize delayed \(K_\alpha\) x-rays by prompt \(K_\alpha\) x-rays

μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays  (∼ 13 hours of data)
µp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

“prompt” ($t \sim 0$)

- 1 S
- 2 S
- 2 P
- n~14

99 %
1 %

10^4
10^3
10^2
10^1
1

events in 25 ns

time [µs]

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µp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

“prompt” \((t \sim 0)\)

\begin{itemize}
  \item 1 S
  \item 2 S
  \item 2 P
  \item \(n \sim 14\)
  \item 99 \%
  \item 1 \%
\end{itemize}

“delayed” \((t \sim 1 \mu s)\)

\begin{itemize}
  \item Laser
  \item 2 keV \(\gamma\)
  \item 2 P
  \item 1 S
\end{itemize}
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

“prompt” \((t \sim 0)\)

\[
\begin{align*}
1 S & \rightarrow 2 S \\
2 S & \rightarrow 2 P \\
99\% & \rightarrow 2 P
\end{align*}
\]

“delayed” \((t \sim 1 \mu s)\)

\[
\begin{align*}
1 S & \rightarrow 2 S \\
2 S & \rightarrow 2 P
\end{align*}
\]

normalize \(\frac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha} \Rightarrow \text{Resonance}\)

\[
\text{CARTOON}
\]
πE5 area at PSI
Muon beam line

π⁻
10⁸ s⁻¹
p=100 MeV/c

B=4 T
B=2 T
B=4 T

HV= −19 kV
200 nm foil
for muon extraction

Solenoid
B=5 T

Hydrogen target

ExB

Muon detector
&
Frictional cooling

B=0.1 T

TOF
Velocity filter

Momentum filter

MEC
Muon beam line
Muon beam: inside 5 T solenoid

- $\mu^-$ enters the solenoid
- $e^-$ and $e^-$ produced
- $\mathbf{E} \times \mathbf{B}$ effect
- 10 cm separation
- Collimator
- Gas Target
- Laser pulse
- PM$_1$ to PM$_3$
The laser system

Main components:

- Thin-disk laser
  - fast response to detected $\mu^-$
- Frequency doubling
- TiSa laser:
  - frequency stabilized cw laser
  - injection seeded oscillator
  - multipass amplifier
- Raman cell
  - 3 Stokes: 708 nm $\rightarrow$ 6 $\mu$m
  - $\lambda$ calibration @ 6 $\mu$m
- Target cavity

The laser system

Yb:YAG thin-disk laser
- Oscillator 1030 nm
- Amplifier 708 nm, 400 mW
- SHG

CW TiSa laser
- Oscillator 1030 nm
- Amplifier 500 W
- SHG

Wave meter
- Verdi 5 W

Raman cell
- I$_2$/Cs

SHG

Thin-disk laser
- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single $\mu^{-}$ triggers the laser system
- $2S$ lifetime $\approx 1 \mu$s $\rightarrow$ short laser delay

The laser system

MOPA TiSa laser:
- Cw frequency stabilized laser
- referenced to a stable FP cavity
- FP cavity calibrated with \( I_2, \) Rb, Cs lines

\[
\nu_{FP} = N \cdot FRS
\]

\[
FRS = 1497.344(6) \text{ MHz}, \quad N \approx 2 \times 10^5.
\]

\[
\nu_{\text{TiSa}}^{\text{cw}} \text{ absolutely known with } \sigma = 30 \text{ MHz}
\]

\[
\Gamma_{2P-2S} = 18.6 \text{ GHz}
\]

Seeded oscillator

\[
\rightarrow \nu_{\text{TiSa}}^{\text{pulsed}} = \nu_{\text{TiSa}}^{\text{cw}}
\]

(frequency chirp \( \leq 100 \text{ MHz} \))

Multipass amplifier (2f- configuration)

gain=10
The laser system

Yb:YAG thin-disk laser
- Oscillator 1030 nm, 9 mJ
- Amplifier, 200 W
- SHG, 43 mJ
- TiSa Amp., 708 nm, 15 mJ

CW TiSa laser
- Wave meter
- I$_2$ / Cs
- Verdi, 500 W
- SHG, 23 mJ 515 nm 23 mJ
- TiSa Osc., 708 nm, 15 mJ

Raman cell:
- 708 nm $\rightarrow$ H$_2$, 6.02 μm
- 1$^{st}$ Stokes 2$^{nd}$ Stokes 3$^{rd}$ Stokes
- $\nu_{708\text{ nm}} - 3 \cdot \hbar \nu_{\text{vib}}$
tunable $\omega_{\text{vib}}(p, T) = \text{const}$

6 μm monitoring
- H$_2$ O
- Ge-filter
- 6 μm cavity
The laser system

Yb:YAG thin-disk laser

- Oscillator 1030 nm
  - 9 mJ 200 W
- Amplifier
  - 500 W
  - 9 mJ
  - 43 mJ
  - SHG
  - 23 mJ 515 nm 23 mJ

SHG

- TiSa Amp.
- 708 nm, 15 mJ

SHG

- 6 µm monitoring
- H₂O
- 20 m 0.25 mJ

Raman cell

- Ge-filter
- 6 µm cavity

SHG

- 190 mm
- 25 mm
- 12 mm
- 2 mm
- 3 mm
- Laser pulse
- Horiz. plane
- Vert. plane

Design: insensitive to misalignment

Transverse illumination
Large volume

Dielectric coating with $R \geq 99.9\%$ (at 6 µm)

→ Light makes 1000 reflections
→ Light is confined for $\tau=50$ ns
→ 0.15 mJ saturates the $2S - 2P$ transition
The laser system

Yb:YAG thin-disk laser
- Oscillator 1030 nm
  - 9 mJ
- Amplifier
  - 43 mJ
- SHG
  - 23 mJ

cw TiSa laser
- Oscillator 1030 nm
  - 9 mJ
- Amplifier
  - 43 mJ
- SHG
  - 23 mJ
- Verdi
  - 5 W
- I$_2$ / Cs
- cw TiSa 708 nm
  - 400 mW

SHG
- 708 nm, 15 mJ
- TiSa Osc.
- TiSa Amp.

Water absorption
- 6 µm monitoring
- H$_2$O
  - 6 µm
  - 0.25 mJ
- Ge-filter
- μ
- 6 µm cavity

- Vacuum tube for 6 µm beam transport.
- Direct frequency calibration at 6 µm.

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6 $\mu$m wavelength calibration

- 6 $\mu$m light calibration: $\text{H}_2\text{O}$ vapor absorption measurement in air / cell

$\text{H}_2\text{O}$ absorption lines known to a few MHz (HITRAN)

$\Rightarrow \delta \nu \approx 300 \text{ MHz uncertainty}$ (6 ppm of $\Delta E_{2S-2P}$) due to our calibration accuracy over the whole wavelength range $\lambda = 5.5 \ldots 6.1 \mu$m

- Laser frequency detuning is measured in number of Fabry-Perot cavity fringes
  - grid spacing of our measurement: $\text{FSR}(\text{FP}) = 1497.344(6)$ MHz
  - all measured resonances are within $\pm 70$ FP fringes of a $\text{H}_2\text{O}$ line
TiSa lasers
Target, cavity and detectors
The situation June 28, 2009, 18:00

Laser frequency [1 unit = 1497.332(3) MHz, arb. offset]
The situation June 28, 2009, 18:00

data 2003

signal [delayed/promt events]

laser frequency [1 unit = 1497.332(3) MHz, arb. offset]

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The situation June 28, 2009, 18:00

laser frequency [1 unit = 1497.332(3) MHz, arb. offset]
The situation June 28, 2009, 18:00

Movie of The Search

Signal [delayed/prompt events]

Laser frequency [1 unit = 1497.332(3) MHz, arb. offset]
\[ \mu \text{p} \left( 2S_{1/2}(F=1) \rightarrow 2P_{3/2}(F=2) \right) \]

\begin{itemize}
  \item Stat.: 700 MHz (14 ppm)
  \item but 75 GHz away from prediction \((\Gamma = 18.6 \text{ GHz})\)
\end{itemize}
The time spectra

Laser ON resonance

Laser OFF resonance

1.32 \times 10^6 events

1.02 \times 10^6 events

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The resonance: discrepancy, sys., stat.

Water-line/laser wavelength: 300 MHz uncertainty

Δν water-line to resonance: 200 kHz uncertainty

Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy:
5.0 σ ↔ 75 GHz ↔ δν/ν = 1.5 × 10^{-3}
The resonance: discrepancy, sys., stat.

Water-line/laser wavelength: 300 MHz uncertainty

Δν water-line to resonance: 200 kHz uncertainty

550 events measured on resonance where 155 bgr events are expected

fit Lorentz + flat bgr ⇒ $\chi^2$/dof = 28.1/28
width agrees with expectation
bgr agrees with laser OFF data

$\chi^2$/dof = 283/31 for flat line → 16σ

Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy: 5.0 σ ↔ 75 GHz ↔ $\delta \nu/\nu = 1.5 \times 10^{-3}$
Uncertainty budget and sensitivity

- **Statistics**
  - Center position uncertainty ($\sim 4\%$ of $\Gamma$) 700 MHz

- **Systematics**
  - Laser frequency ($\text{H}_2\text{O}$ calibration) 300 MHz
  - AC and DC stark shift < 1 MHz
  - Zeeman shift (5 Tesla) < 30 MHz
  - Doppler shift < 1 MHz
  - Collisional shift 2 MHz

- **Total uncertainty of the line determination** 760 MHz

- **Discrepancy with prediction** 75 300 MHz

Systematic effects are small since they scale like $1/m$

Finite size effect scales like $m^3$
Proton radius

\( \nu(2S_{1/2}^F=1 \rightarrow 2P_{3/2}^F=2) = 49881.88(76) \text{ GHz} \).

\[ \tilde{L}^{\text{exp.}} = 206.2949(32) \text{ meV} \]

\[ \tilde{L}^{\text{th.}} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV} \]

\[ \Rightarrow r_p = 0.84184(36)(56) \text{ fm} \]

\[ u_{\text{exp}} = 4.3 \times 10^{-4} \]

\[ u_{\text{theo}} = 6.7 \times 10^{-4} \]

\[ r_p = 0.84184(67) \text{ fm} \quad (u_r = 8 \times 10^{-4}) \]

CODATA 2006: \( r_p = (0.8768 \pm 0.0069) \text{ fm} \), from H e-p scattering: \( r_p = (0.895 \pm 0.018) \text{ fm} \)

3.1\( \sigma \) from e-p scatt.
5.0\( \sigma \) from CODATA
\( r_p \) is 4\% smaller
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) theory wrong?  \( \mu p \) experiment wrong?

H theory wrong?

H experiments wrong?  \( \rightarrow R_\infty \) wrong?
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} \text{75 GHz} \\ \text{.31 meV} \\ \text{0.15 \%} \end{cases} \]

\( \mu_p \) theory wrong?  
\( \mu_p \) experiment wrong?  
H theory wrong?  
H experiments wrong? \( \rightarrow R_\infty \) wrong?

\( \mu_p \) theory wrong?

Discrepancy = 0.31 meV  
Theory uncert. = 0.005 meV  
\( \Rightarrow 64\delta(\text{theory}) \) deviation
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\
0.31 \text{ meV} \\
0.15 \% \end{cases} \]

\( \mu_p \) theory wrong?  
\( \mu_p \) experiment wrong?  
H theory wrong?  
H experiments wrong?  \( \rightarrow R_\infty \) wrong?

\( \mu_p \) experiment wrong?

Frequency mistake by 75 GHz (\( \leftrightarrow \) 0.15%)?

That is 100 \( \sigma \) !  
\( \sigma_{\text{tot}} = 760 \) MHz,  
[ 700 MHz\text{stat}, \ 300 \text{MHz}_{\text{syst}} ]

2\text{nd line in } \mu_p \text{ agrees with this 1\text{st line!}} (\rightarrow \text{next slides})
What may be wrong?

\[
\tilde{L}^{\text{theo.}}_{\mu p}(r_p^{\text{CODATA}}) - \tilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 
75 \text{ GHz} \\
0.31 \text{ meV} \\
0.15 \% 
\end{cases}
\]

- \( \mu p \) theory wrong?
- \( \mu p \) experiment wrong?
- H theory wrong?
- H experiments wrong? \( \rightarrow R_\infty \) wrong?

H experiments wrong?
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ .31 \text{ meV} \end{cases} \]

\[ R_\infty \text{ with our } r_p \]

- \( \mu_p \) theory wrong?
- \( H \) theory wrong?
- \( H \) experiments wrong?

F. Biraben, spring 2010
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) theory wrong?  
\( \mu p \) experiment wrong?  
H theory wrong?  
H experiments wrong? \( \rightarrow R_{\infty} \) wrong?

H experiments wrong?

H(1S-2S): 60 kHz \( \rightarrow 1700\sigma \)

all H(2S-\( n\ell \)) where \( n\ell = 2P, 4, 6, 8S/D, 12D \)  
in the same direction by 1...3\( \sigma \)
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ .31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) theory wrong?  \( \mu p \) experiment wrong?

H theory wrong?  H experiments wrong?  \( \rightarrow R_{\infty} \) wrong?

H theory wrong!

New Physics! :-)

And what about e-p scattering ?!?!?
Discussions...

1005.4879 Karshenboim et al: previously missing QED term.
1005.4880 Karshenboim et al: previously missing QED term.
1007.1419 Krutov, Martynenko: Ground-state HFS of $e\mu^3He$
1007.5076 Bernauer et al: New Mainz electron scattering value
1008.3536 Jaeckel, Roy: “Spectroscopy as a test of Coulomb’s law”
1008.3861 De Rujula: “QED is not endangered by the proton’s size”
1008.4225 Vanderhaegen, Walcher: “Long range structure of the nucleon”
1008.4345 Cloet, Miller: “Third Zemach moment of the proton”
1008.4384 Garcia et al: “Hyperfine splitting in hydrogen with form factors”
1008.4546 De Rujula: “Comment on Third Zemach moment of the proton”
1008.4619 Hill, Paz: “Model independent extraction of the proton charge radius from electron scattering”
Discussions...

1008.3536 Jaeckel, Roy: "Spectroscopy as a test of Coulomb’s law"
extra hidden photons, minicharged particles
cause deviations from Coulomb’s law.

$\mu p$ transition can NOT be explained this.
(contradicts Lamb shift measurements in ordinary hydrogen)
De Rujula: “QED is not endangered by the proton’s size”

A large third Zemach moment

\[ \langle r_p^3 \rangle_2 = \int d^3 r_1 \, \int d^3 r_2 \, \rho(r_1) \, \rho(r_2) \, |r_1 - r_2|^3 \]

of the proton can explain all three measurements: \( \mu_p \), H, e-p

\( \rho(r) \) is not a simple Dipole, but has “core” and “tail”

Sick’s CF fit to e-p data has \( \chi^2 / \text{dof} = 1.65 \) for \( \text{dof} = 310 \)

\[ \implies p = 3.9 \times 10^{-12} \]

“This casts doubt even on the corresponding extracted value
of the mean square radius ...”

Hill, Paz: “Model independent extraction of the proton charge radius from
electron scattering”
Discussions...

1008.4345 Cloet, Miller: “Third Zemach moment of the proton”
(is a comment on De Rujula)

Such a large third Zemach moment is impossible.

\[ \langle r_p^3 \rangle_{(2)}^{(DeRujula)} = 36.6 \pm 6.9 \text{ fm}^3 \]
\[ \langle r_p^3 \rangle_{(2)}^{(Sick)} = 2.71 \pm 0.13 \text{ fm}^3 \]
1005.4879 Karshenboim et al: previously missing QED term

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so-called “model-independent” fits are in fact model-dependent uncertainties may have been underestimated
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<td>866$^{+52}_{-56}$</td>
<td>959$^{+85}_{-93}$</td>
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<td>$\chi^2$</td>
<td>32.81</td>
<td>32.51</td>
<td>-</td>
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<td>$z$ expansion (no bound)</td>
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<tr>
<td>$k_{\text{max}}$</td>
<td>918$^{+9}_{-9}$</td>
<td>868$^{+28}_{-29}$</td>
<td>879$^{+64}_{-69}$</td>
<td>1022$^{102}_{114}$</td>
<td>1193$^{152}_{174}$</td>
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<td>$\chi^2$</td>
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<tr>
<td>$k_{\text{max}}$</td>
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<td>868$^{+28}_{-29}$</td>
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<td>880$^{+39}_{-61}$</td>
<td>880$^{+39}_{-62}$</td>
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<tr>
<td>$\chi^2$</td>
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<td>32.52</td>
<td>32.48</td>
<td>32.46</td>
<td>32.45</td>
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</table>

Table 1: Proton charge radius extracted from data of Table 1 of [18] ($Q^2 \lesssim 0.04\text{GeV}^2$) in units of $10^{-18}$ m, using different functional behaviors of the form factor. Dashes denote fits that do not constrain the slope to be positive.
Discussions...

1005.4879 Karchenboim et al: previously missing QED term.

1008.4619 Hill, Paz: “Model independent extraction of the proton charge radius from electron scattering”

![Graph showing variation of fitted proton charge radius as a function of maximum $Q^2$.]
New $r_p$ and $R_\infty$

Proton radius (fm)

- Orsay, 1962
- Stanford, 1963
- Saskatoon, 1974
- Mainz, 1980
- Mainz, free norm.
- dispersion fit
- Paris, 1996
- Garching, 1997
- Paris, 1999
- Rosenfelder, 2000
- Eides, 2001
- Sick, 2003
- Pachucki Jentschura, 03
- CODATA 2006

New $r_p$ and $R_\infty$

New $r_p$ and $R_\infty$

accuracy of the Rydberg constant

- single measurements
- least-square adjustments
- our value from muonic hydrogen

More measurements
\[ \mu p \left( 2S_{1/2}(F=0) \rightarrow 2P_{3/2}(F=1) \right) \text{ at } \lambda = 5.5 \mu m \]

- \( \sigma_{\text{position}} = 1.1 \text{ GHz } \leftrightarrow 25 \text{ ppm } (\Gamma = 18.6 \text{ GHz}) 
- \text{Position fits perfectly with theory using new } r_p 

Extract HFS and \( r_{\text{Zemach}} \)
\[
\mu d \left( 2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2) \right)
\]

(still preliminary)

281 events
76 bgr.

- \[\sigma_{\text{position}} = 880 \ \text{MHz} \iff 17 \ \text{ppm} \ (\Gamma = 18.6 \ \text{GHz})\]
- Position does not fit with prediction: \(3.5\sigma\) deviation

DEUTERIUM

Hyperfine structure of muonic deuterium

Prediction (with new \(r_p\))

Extract \(r_d\) and d. pol.

Randolf Pohl
U of T
Sept. 30, 2010
p. 28
\( \mu d \ (2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2 \text{ and } 1/2)) \)

(still preliminary)

- \( \sigma_{\text{position}} = 2.2 \text{ GHz} \leftrightarrow 43 \text{ ppm} \) \( (\Gamma = 18.6 \text{ GHz}) \)
- Relative pos. fit to each others but not with the first \( \mu d \) line
- Background well known from previous \( \mu d \) line

\[ \begin{align*}
2P_{3/2} & \quad F=5/2 \quad 0.737 \text{ meV} \quad 0.723 \text{ meV} \\
2P_{3/2} & \quad F=3/2 \quad 0.8051 \text{ meV} \quad 0.8041 \text{ meV} \\
2P_{1/2} & \quad F=3/2 \quad 0.6104 \text{ meV} \\
2P_{1/2} & \quad F=1/2 \quad 1.4015 \text{ meV} \\
2S_{1/2} & \quad F=3/2 \quad 2.0103 \text{ meV} \\
2S_{1/2} & \quad F=1/2 \quad 4.3287 \text{ meV} \\
2S_{1/2} & \quad F=0 \quad 6.058 \text{ meV}
\end{align*} \]
Summary

- measured $\mu p \ (2S_{1/2}(F=1) \rightarrow 2P_{3/2}(F=2))$ to 15 ppm (stat. + syst.)
  $\rightarrow r_p$ to $8 \times 10^{-4}$ (experimental precision $4 \times 10^{-4}$)

- $r_p = 0.84184 \pm 0.00067$ fm is 5σ away from CODATA-2006

  The proton is 4% smaller, and the Rydberg constant $R_\infty$ is 4.9 sigma off

- measured $\mu p \ (2S_{1/2}(F=0) \rightarrow 2P_{3/2}(F=1))$ to 25 ppm (stat., online)
  exactly at the position deduced with our new $r_p$
  $\rightarrow$ Zemach radius to a few % (radius of the magnetic moment distribution)

- measured $\mu d \ (2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2))$ to 20 ppm (stat., online)
  not exactly where we expected it from $r_p$ and H-D isotope shift
  $\rightarrow$ deuteron polarizability

- measured $\mu d \ (2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2))$
  observed $\mu d \ (2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=1/2))$
  $\rightarrow$ check calculations in $\mu d$

http://muhy.web.psi.ch
Yeah!
Outlook: Lamb shift in muonic helium

- **CREMA** collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure $\Delta E(2S-2P)$ in $\mu^4\text{He}$, $\mu^3\text{He}$
- $\Rightarrow$ alpha particle and helion charge radius to $3 \times 10^{-4}$ (0.0005 fm)
Outlook: Lamb shift in muonic helium

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- Goal: Measure $\Delta E(2S-2P)$ in $\mu^4$He, $\mu^3$He
- $\Rightarrow$ alpha particle and helion charge radius to $3 \times 10^{-4}$ (0.0005 fm)
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  - Help to solve the proton size puzzle
  - Absolute charge radii of helion, alpha
  - Low-energy effective nuclear models: $^1\text{H}$, $^2\text{D}$, $^3\text{He}$, $^4\text{He}$
  - Better bound-state QED test together with $\text{He}^+(1S-2S)$ [Udem @ MPQ]
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- Identical muon beam
- Similar laser, no Raman cell ($\rightarrow$ more pulse energy)
- Similar, maybe better x-ray detectors (8.2 keV)
- Event rate: 16-48 events per hour (not 6 per hour, $\mu^p$)
- Line with 300 GHz (1 nm!)
µp Lamb shift collaboration in 2009

F. KOTTMANN
A. ANTOGNINI, T.W. HÄNSCH, T. NEBEL,
R. POHL
D. TAQUU

E.-O. Le BIGOT, F. BIRABEN, P. INDELICATO,
L. JULIEN, F. NEZ

F.D. AMARO, J.M.R. CARDOSO, D.S. COVITA,
L.M.P. FERNANDES, J.A.M. LOPEZ, C.M.B. MONTEIRO,
J.M.F. DOS SANTOS, J.F.C.A. VELOSO

A. GIESEN, K. SCHUHMANN
T. GRAF

C.-Y. KAO, Y.-W. LIU

P. RABINOWITZ

A. DAX, P. KNOWLES, L. LUDHOVA,
F. MULHAUSER, L. SCHALLER

ETH Zürich, Switzerland
MPQ, Garching, Germany
PSI, Switzerland
Laboratoire Kastler Brossel, Paris, France
Department of Physics, Coimbra, Portugal
Dausinger + Giesen, Stuttgart, Germany
Institut für Strahlwerkzeuge, Stuttgart, Germany
National Tsing Hua University, Hsinchu, Taiwan
Department of Chemistry, Princeton, USA
former members, spent holidays at run 2009
Proton Size Investigators thank you for your attention
## Contributions to the $\mu p$ Lamb shift

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<thead>
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<th>#</th>
<th>Contribution</th>
<th>Value</th>
<th>Unc.</th>
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<td>Relativistic one loop VP</td>
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<td>4</td>
<td>NR two-loop electron VP</td>
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<td>5</td>
<td>Polarization insertion in two Coulomb lines</td>
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<td>NR three-loop electron VP</td>
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<td>Three-loop VP (total, uncorrected)</td>
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<td>Recoil finite size</td>
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<td>Sum</td>
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Contributions to the $\mu p$ Lamb shift

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<th>Pachucki</th>
<th>Borie</th>
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<td>Leading nuclear size contribution</td>
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</tbody>
</table>

P. Indelicato, 2010
Contributions to the $\mu p$ Lamb shift

Lamb shift: $\Delta E_{LS} = 206.0573(45) - 5.2262r^2_p + 0.0347r^3_p$ meV

$u = 0.0045$ meV dominated by proton polarizability

2S Hyperfine structure: $\Delta E_{2S}^{HFS} = 22.8148 (78)$ meV

using $R_Z = 1.022$ fm and scatter.

Fine structure: $\Delta E_{FS} = 8.352082$ meV

2P$_{3/2}$ Hyperfine structure: $\Delta E_{HFS}^{2P_{3/2}} = 3.392588$ meV