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# On Rabbits, Foxes, Clouds and Rain

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Graham Feingold

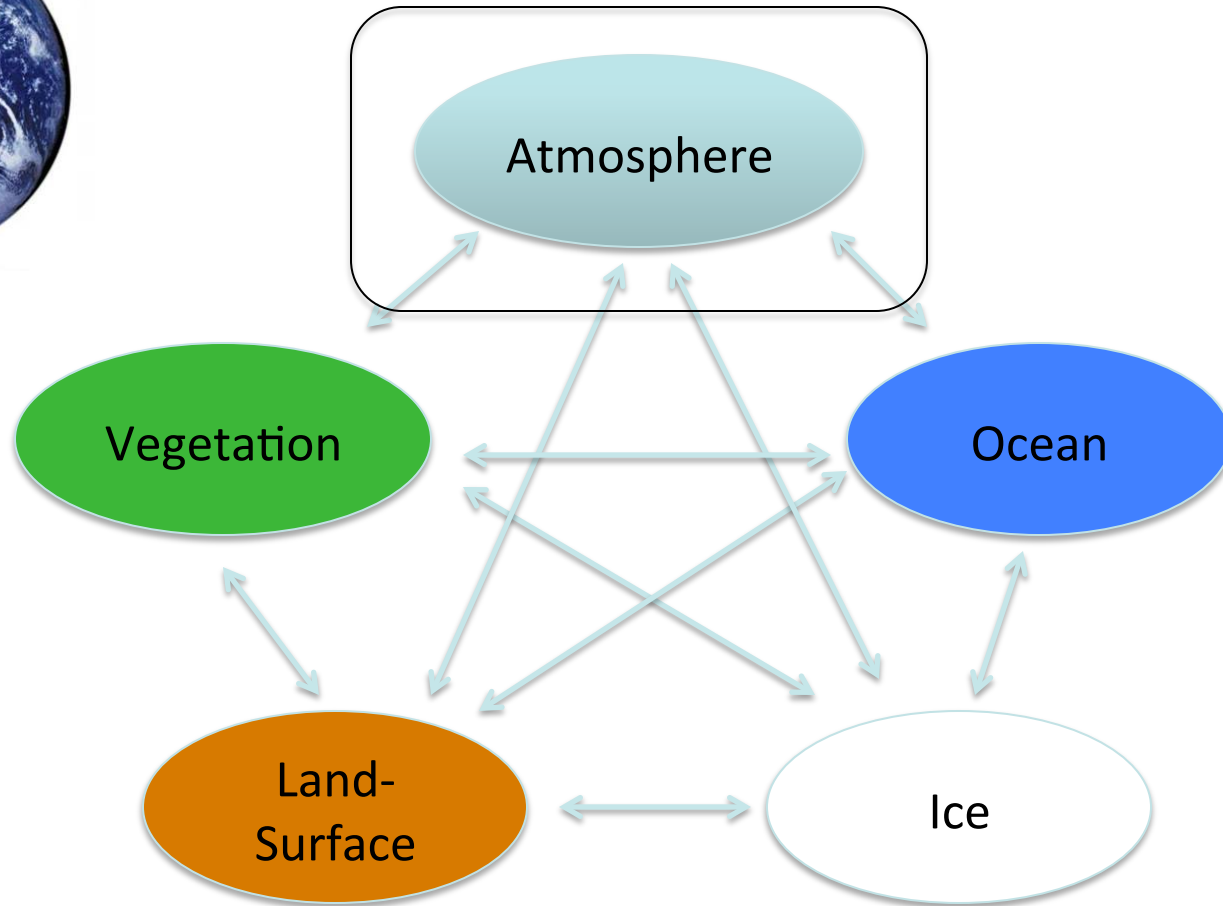
NOAA Earth System Research Laboratory, Boulder, Colorado

Acknowledgements: Ilan Koren, Weizmann Institute, Israel

University of Toronto, Physics Department  
March 1, 2012



# The Climate System



*A tightly coupled system with a number of important components*

# The Climate System



*Role of Clouds in Changing Planetary Albedo*

# Aerosol Enhances Cloud Brightness

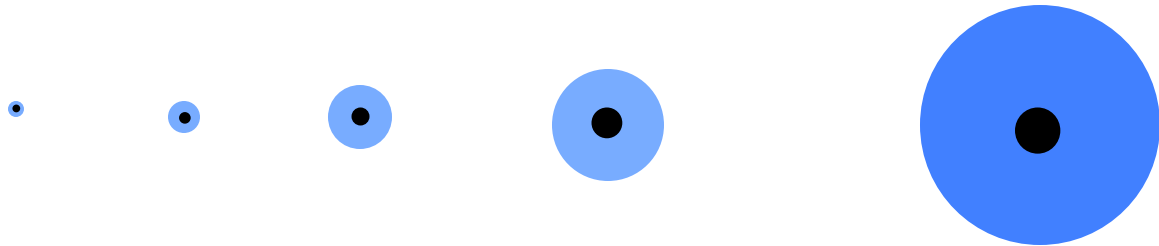


Ship track Images, Aqua, MODIS

Aerosol: A suspension of particles in a medium (air)

# A Very Short Course in Cloud Physics

# Drops form on tiny suspended particles



Larger drops form on larger aerosol particles  
Drops grow by vapour diffusion

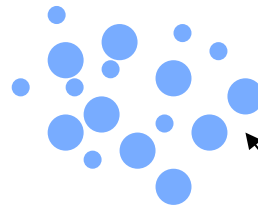
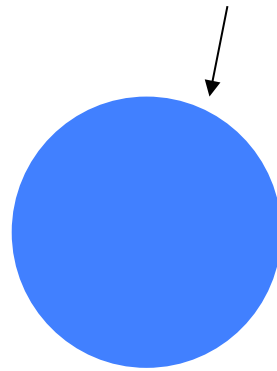
*More aerosol  $\rightarrow$  more, smaller drops (all else equal)*

Cloud Drops:  $\mu\text{m}$  to mm in size

Aerosol particles (sulfate, sea salt, etc.):  $< 1 \mu\text{m}$  in size

# Drop Coalescence generates Rain

Collector drop (larger fall velocity  $V_x$ )



Collected droplets  
(small fall velocity  $V_y$ )

*Coalescence efficiency is a strong function of drop size*

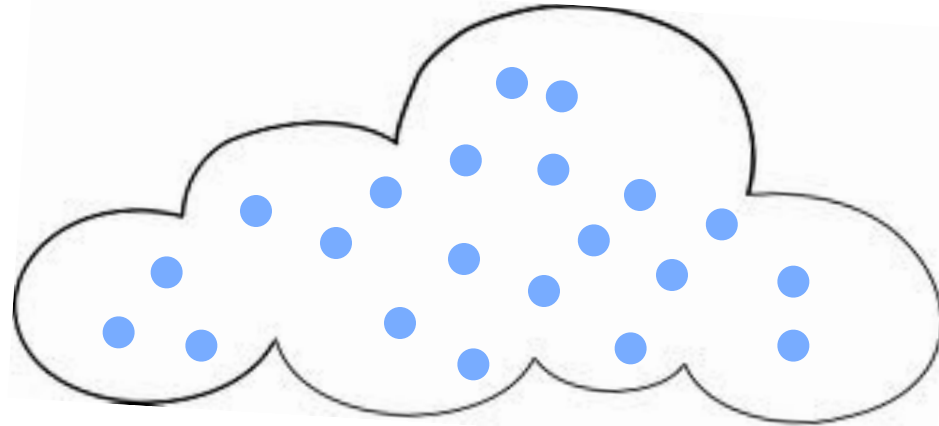


# Aerosol particles affect Precipitation

Few particles/large drops  
*Precipitation*



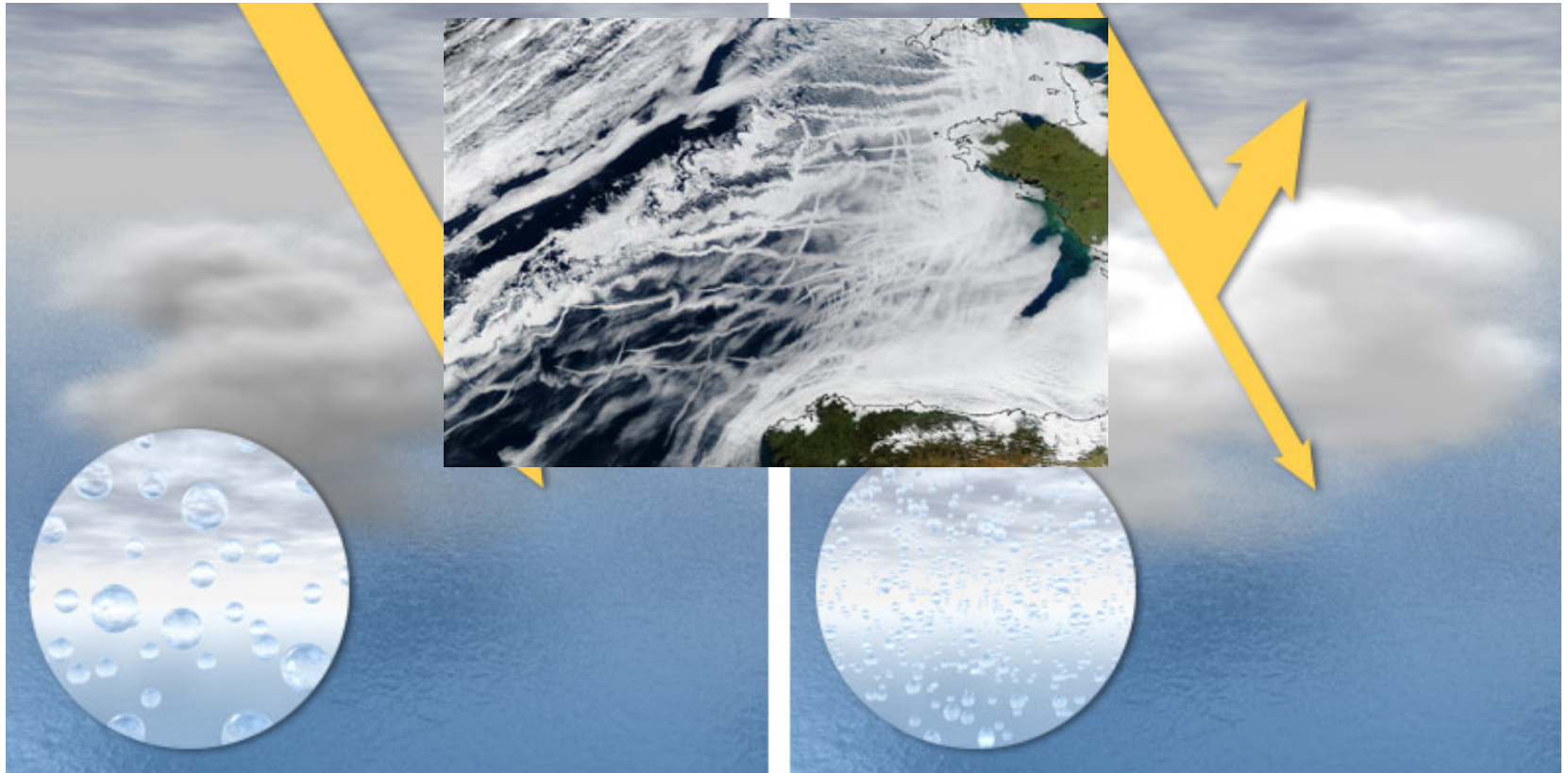
More particles/small drops  
*No precipitation*



# Aerosol particles enhance cloud brightness

Less reflective clouds (large drops)

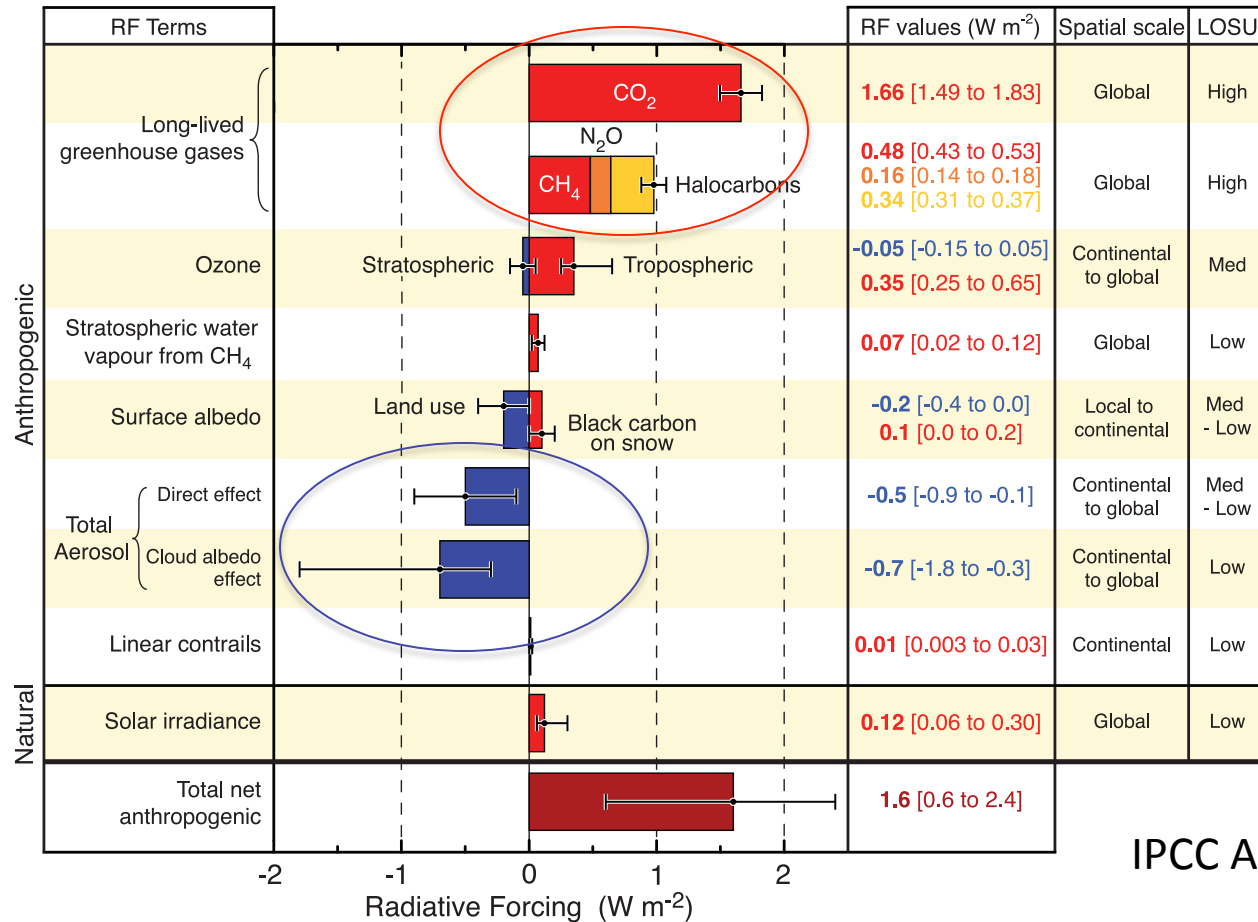
Reflective clouds (small drops)



Robert Simmon

# The Climate System

## RADIATIVE FORCING COMPONENTS



©IPCC 2007: WG1-AR4

IPCC AR4

*Role of Aerosol & Clouds in Changing Planetary Albedo*

# The Scale Problem

Aerosol-Cloud-Precipitation interactions and feedbacks must be represented at a range of spatiotemporal scales:

$\mu\text{m}$  to 1000s km

seconds to days

*The number of degrees of freedom of the system is staggering!*

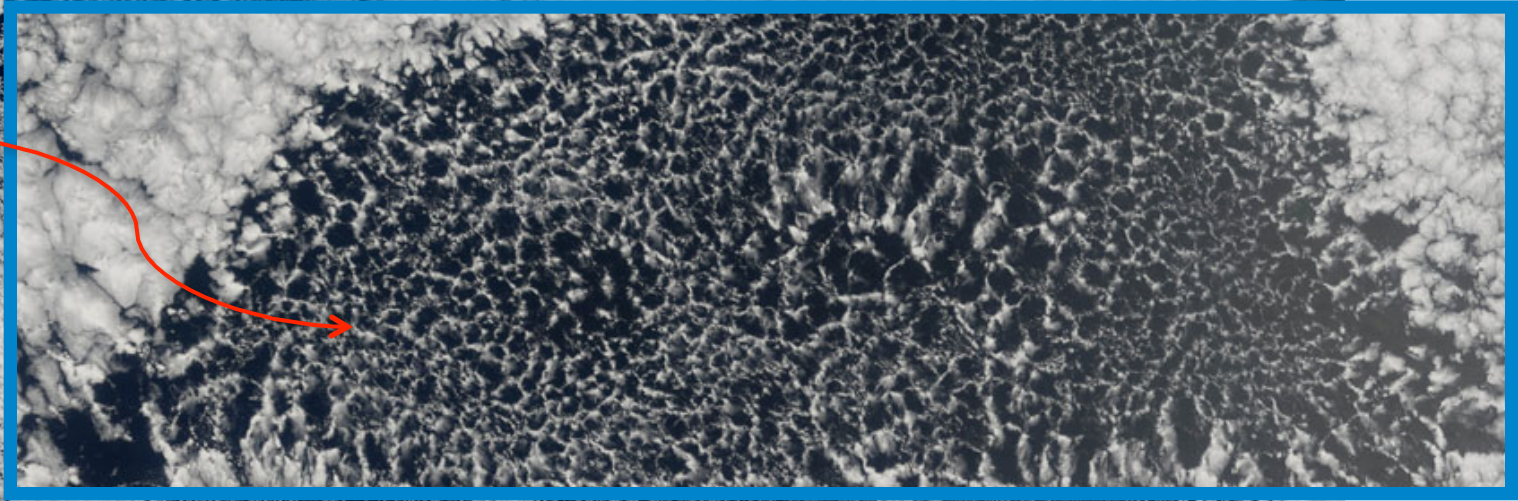
# A System Characterized by Complexity

- Dynamics
- Microphysics
- Radiation
- Aerosol

*A host of feedbacks between the components*

# Patterns: Mesoscale Cellular Convection in Stratocumulus

Open  
cellular  
convection



500 km

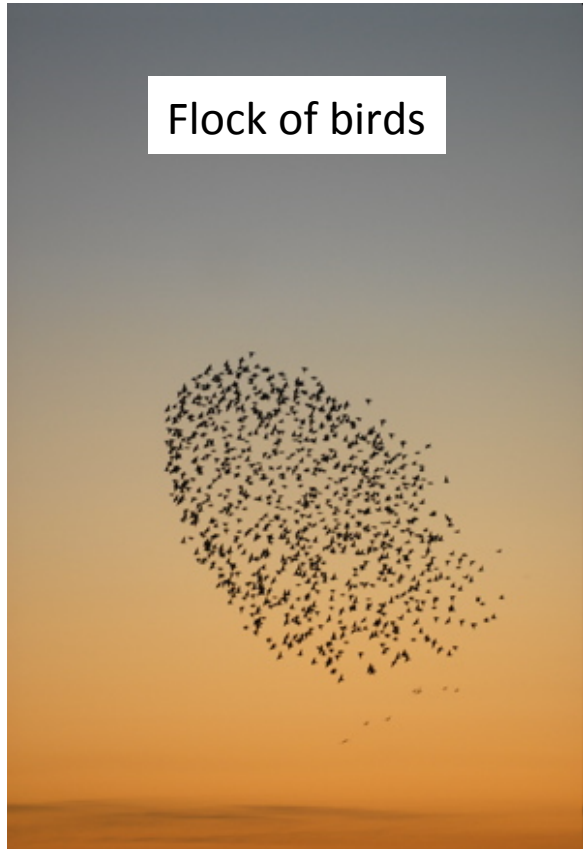


- *Patterns and emergence in atmospheric systems*

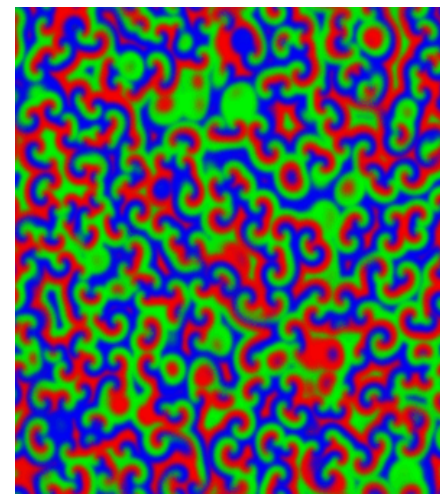
Closed  
cellular  
convection

# Other examples

Flock behaviour

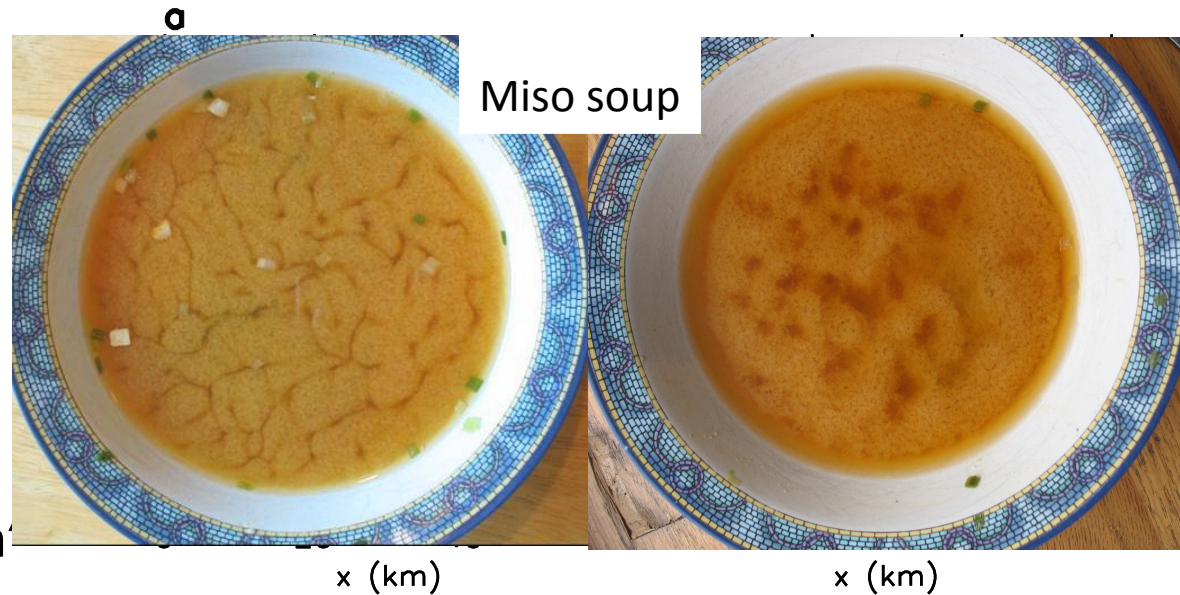


Numerical simulation of  
"Rayleigh-Bénard Convection"



Computer simulation  
of BZ reaction

Oscillatory behaviour in  
Belousov-Zhabotinskii chemical  
reactions



## *“Emergence”*

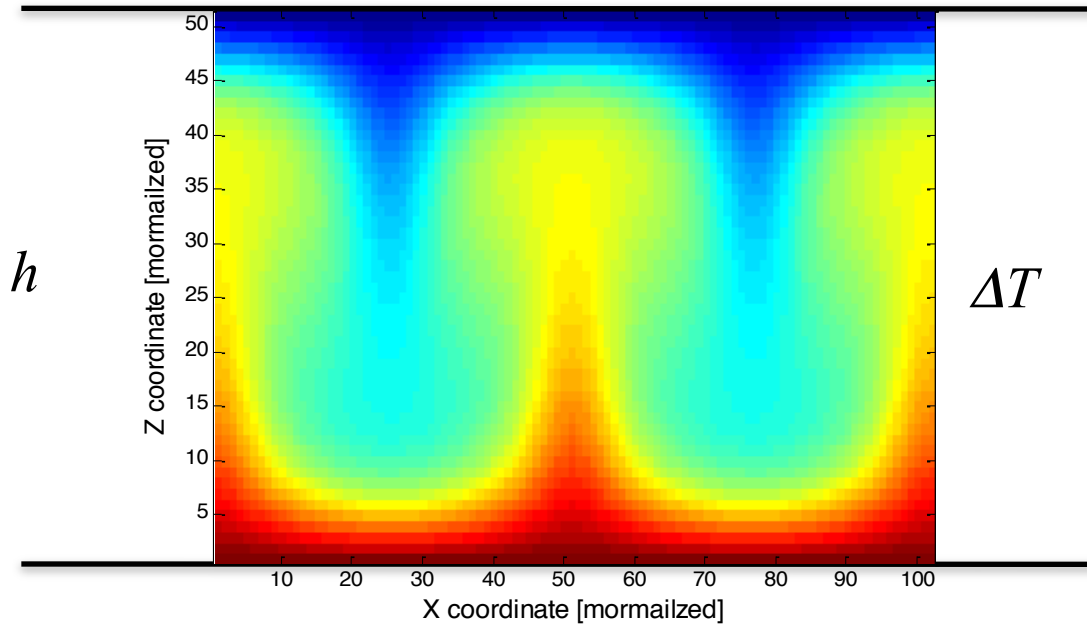
*System-wide patterns emerge from local interactions between elements that make up the system*

*Implication: Complex problems with huge number of degrees of freedom may be amenable to solution with much more simple set of equations*



# Rayleigh-Bénard Convection

*T (cold)*



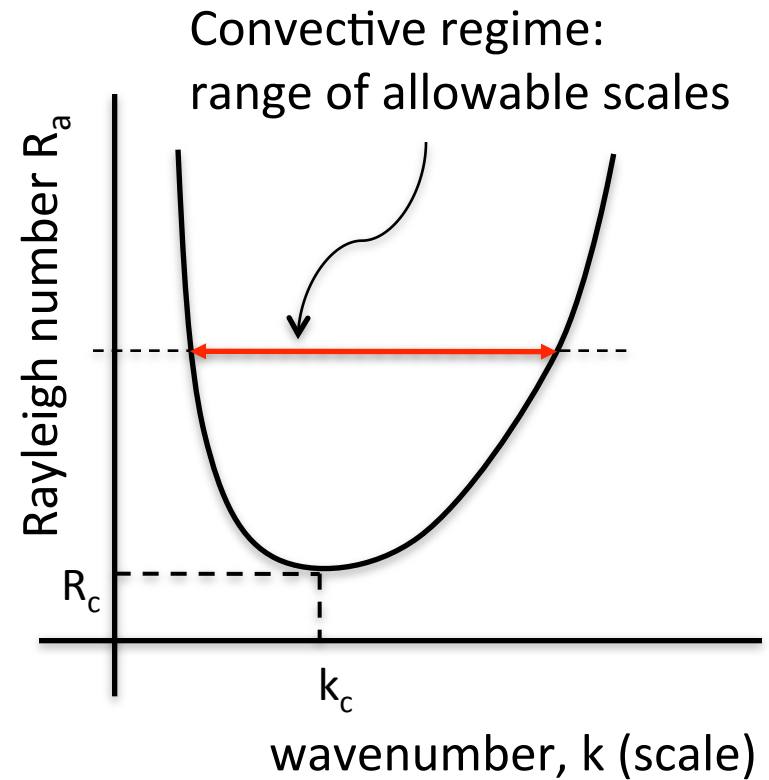
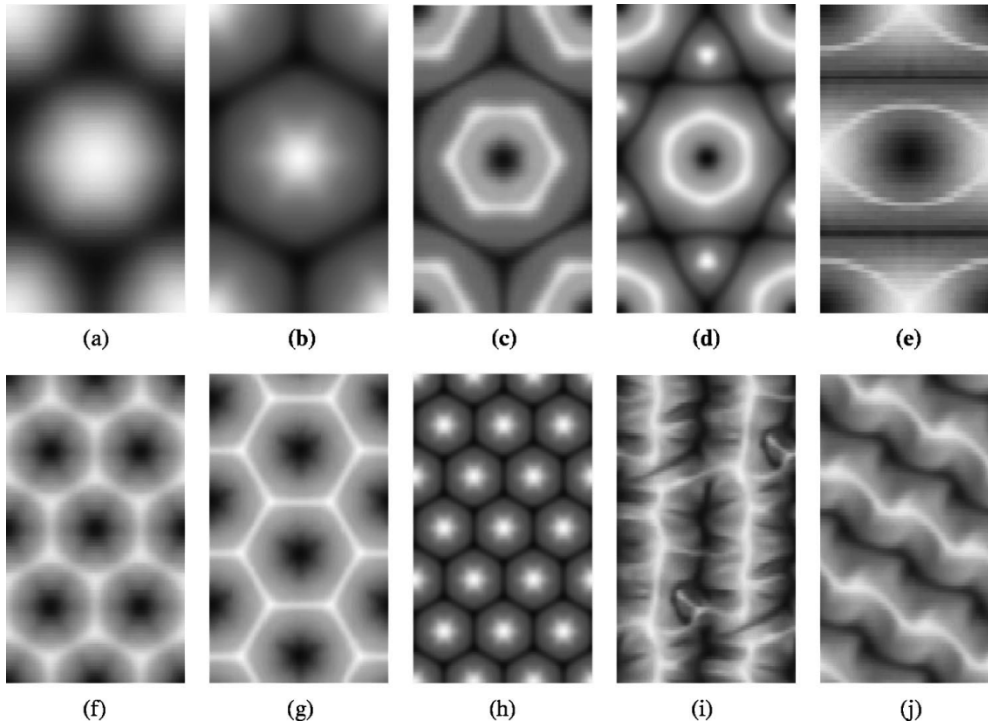
$\Delta T$  = temperature difference  
between surfaces  
 $\alpha$  = thermal expansion coefficient  
 $g$  = gravitational acceleration  
 $h$  = separation between the surfaces  
 $\nu$  = kinematic viscosity  
 $\chi$  = thermal diffusivity

*T (warm)*

$$R_a = \alpha g \Delta T h^3 / (\nu \chi)$$

*Transition from conduction to convection occurs when the Rayleigh number exceeds a critical value  $R_c$*

# Rayleigh-Bénard Convection



*Controlled lab experiments with different scales:  
Patterns and preferred modes*

# Aerosol/drizzle selects the state

Albedo



Closed-cell  
Albedo  $\sim 0.6$   
(non-precipitating)

high aerosol

*Onset of drizzle*

*results in a transition to open-cell convection*

WRF Model  
+ 2-moment

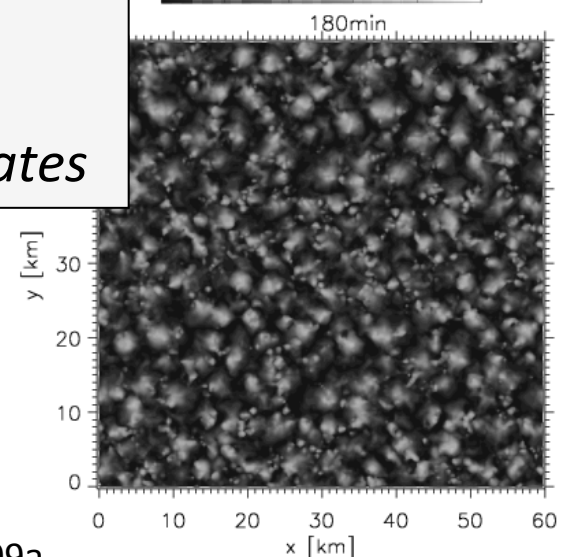
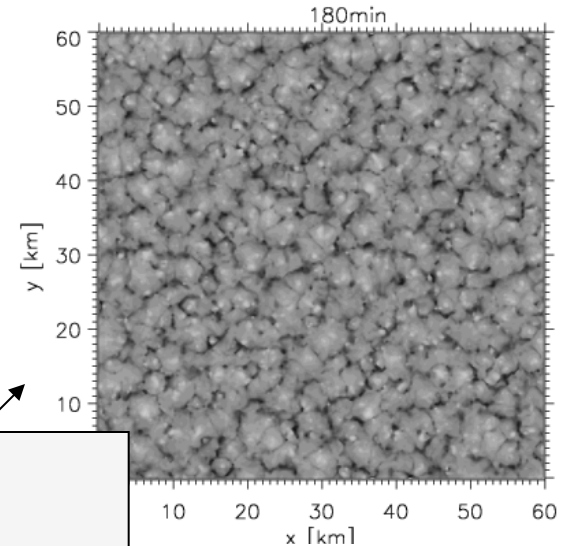
(i) Aerosol “selects” the state of the system (same meteorology)

(ii) The stable state oscillates

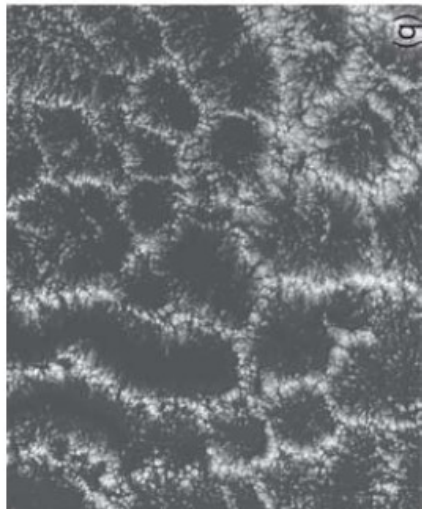
Open-cell  
Albedo  $\sim 0.2$   
(precipitating)

low aerosol

Albedo

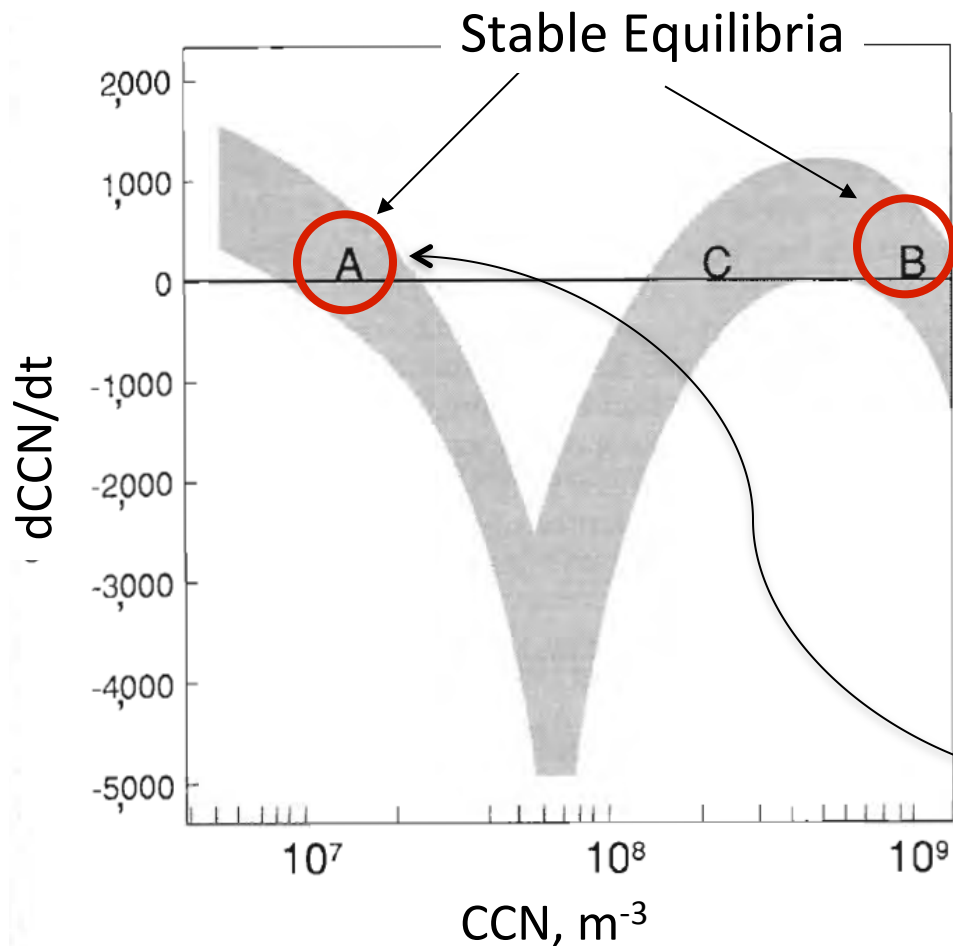


Satellite images

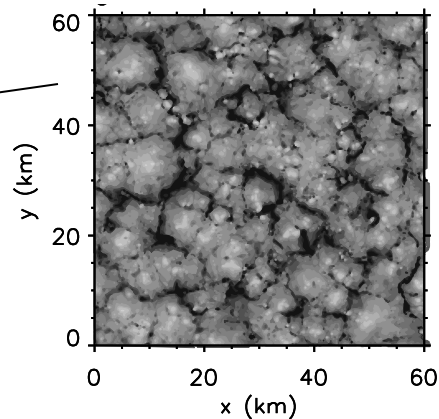


# System Equilibria

*Atmospheric systems prefer certain modes*

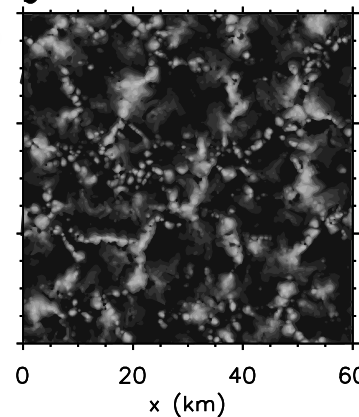


Closed cell



Non-drizzling,  
closed-cell  
mode

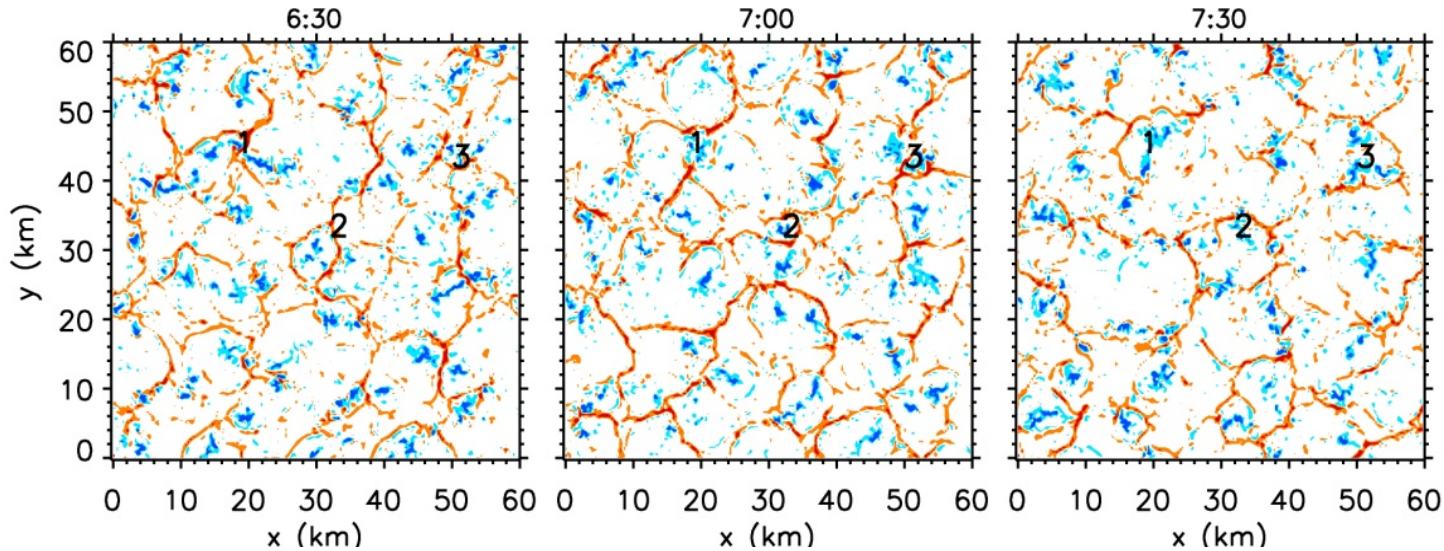
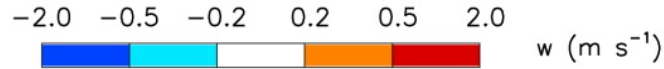
Open cell



Drizzling,  
open-cell mode

Baker and Charlson, 1990  
Mixed-layer model

# Rearrangement of Open Cells



**Red:** Updrafts

**Blue:** Downdrafts/precipitation

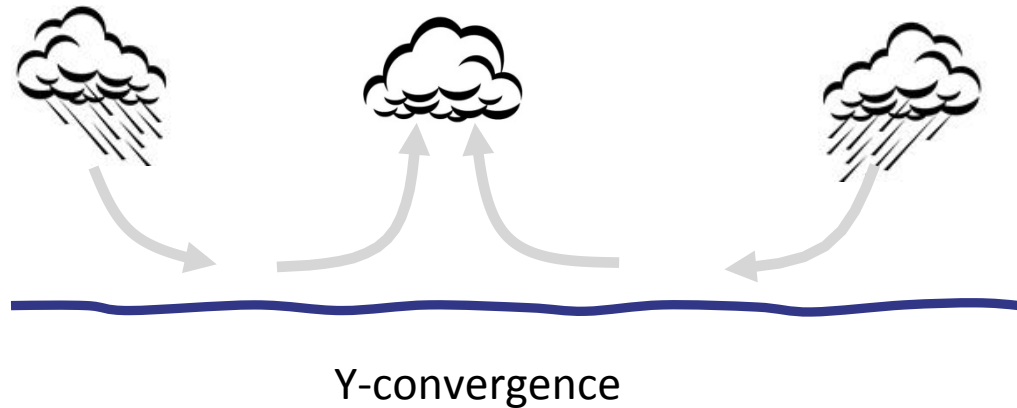
**Y**-shaped surface convergence zone  
is region favoured for new convection

↓  
Precipitation is initiated

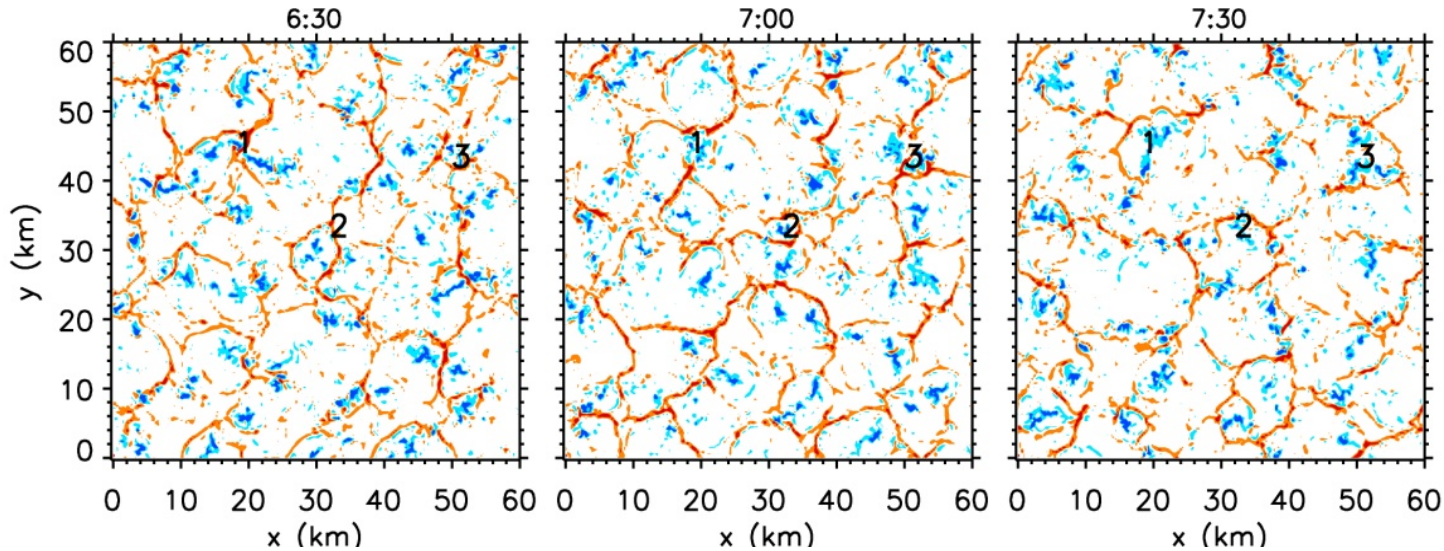
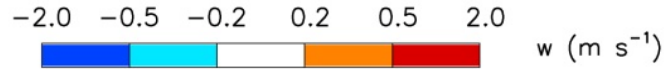
↓  
Downdrafts, opening of cell

↓  
Surface divergence

*Feingold, Koren, Wang, Xue, Brewer (2010)*



# Rearrangement of Open Cells



**Red:** Updrafts

**Blue:** Downdrafts/precipitation

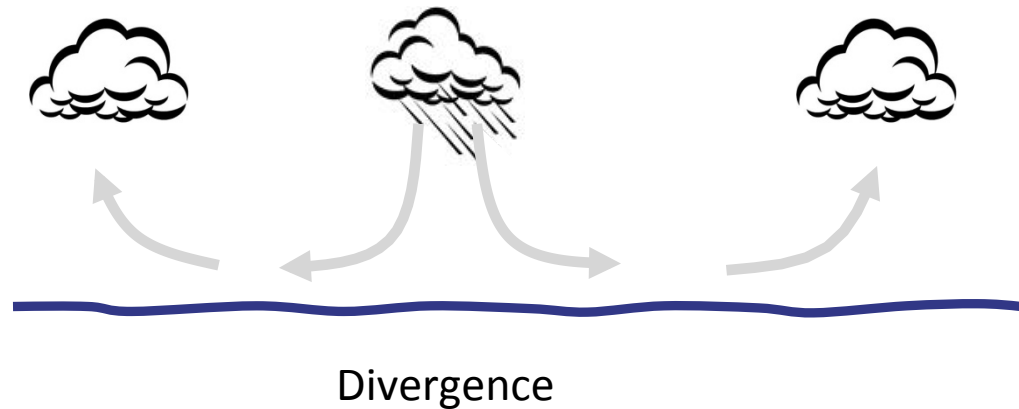
**Y**-shaped surface convergence zone  
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Precipitation is initiated

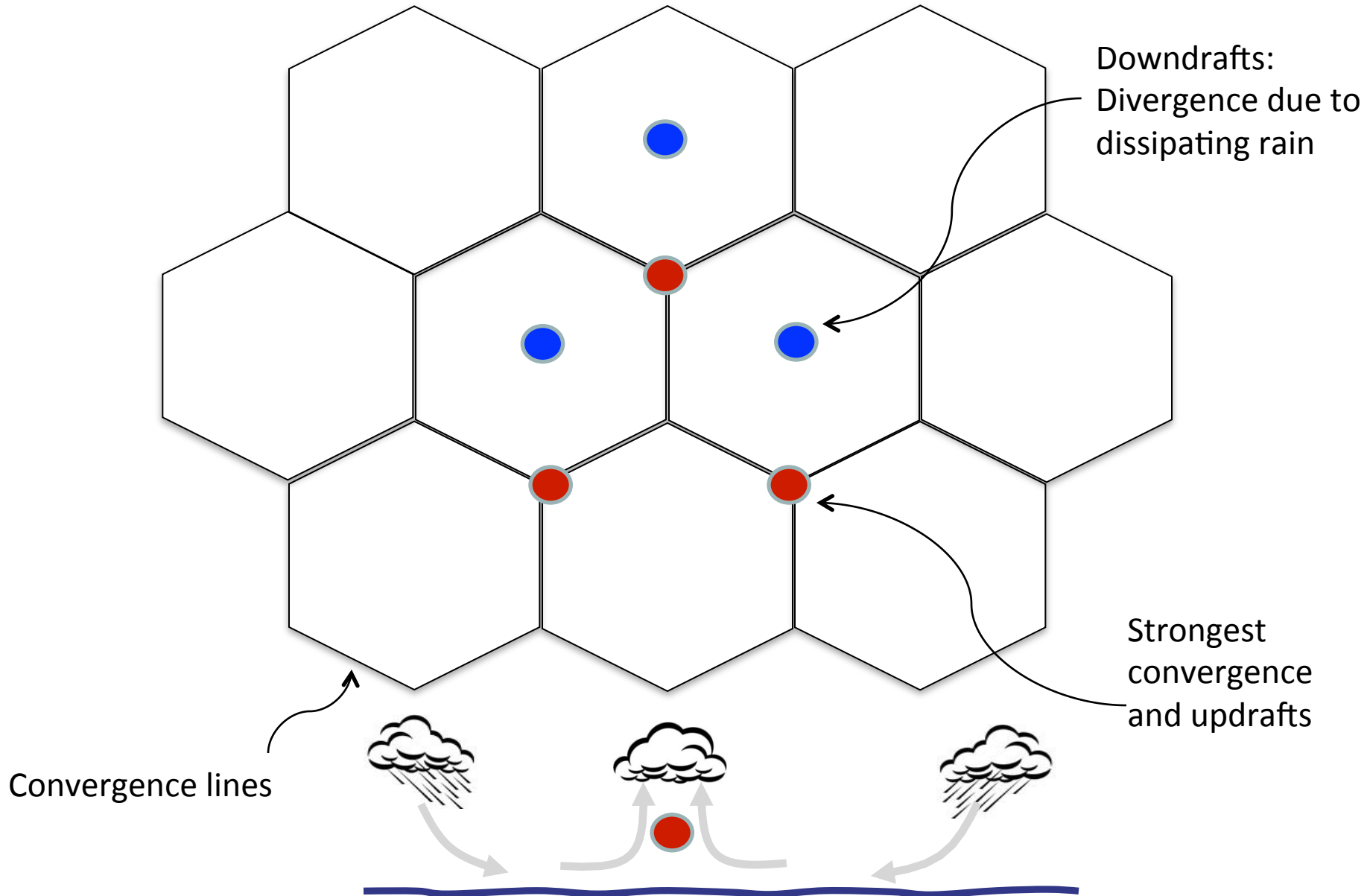
↓  
Downdrafts, opening of cell

↓  
Surface divergence

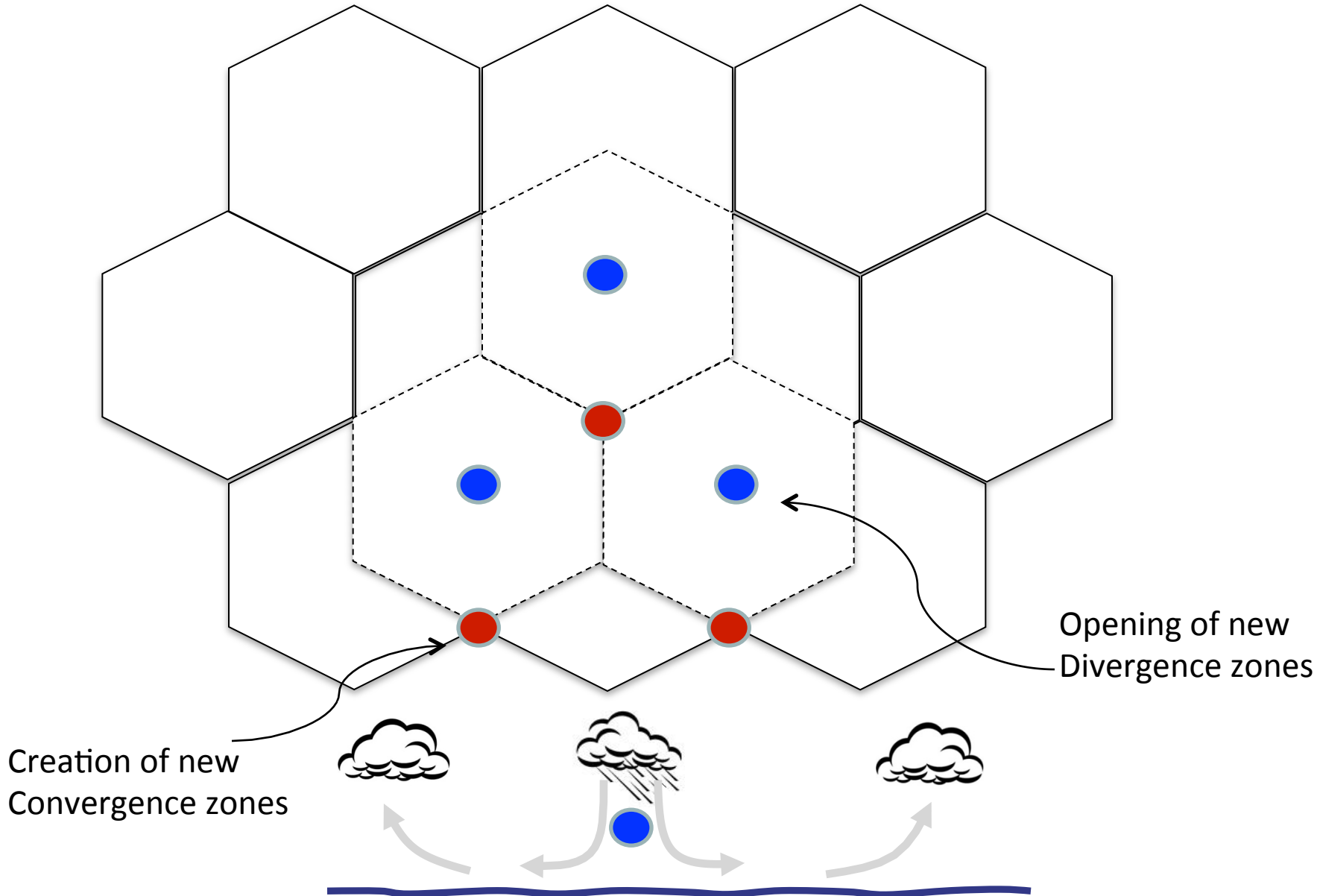
*Feingold, Koren, Wang, Xue, Brewer (2010)*



# Surface Convergence Patterns

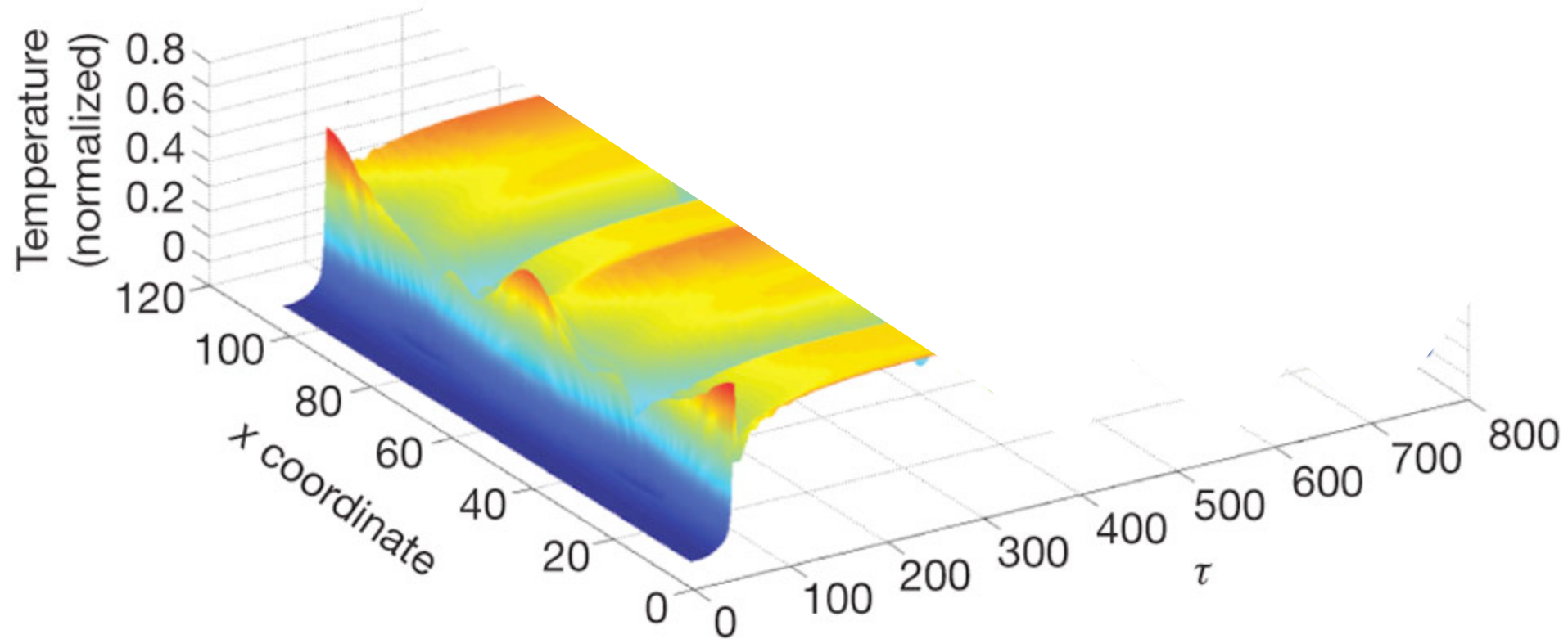


# Shifting of the Patterns

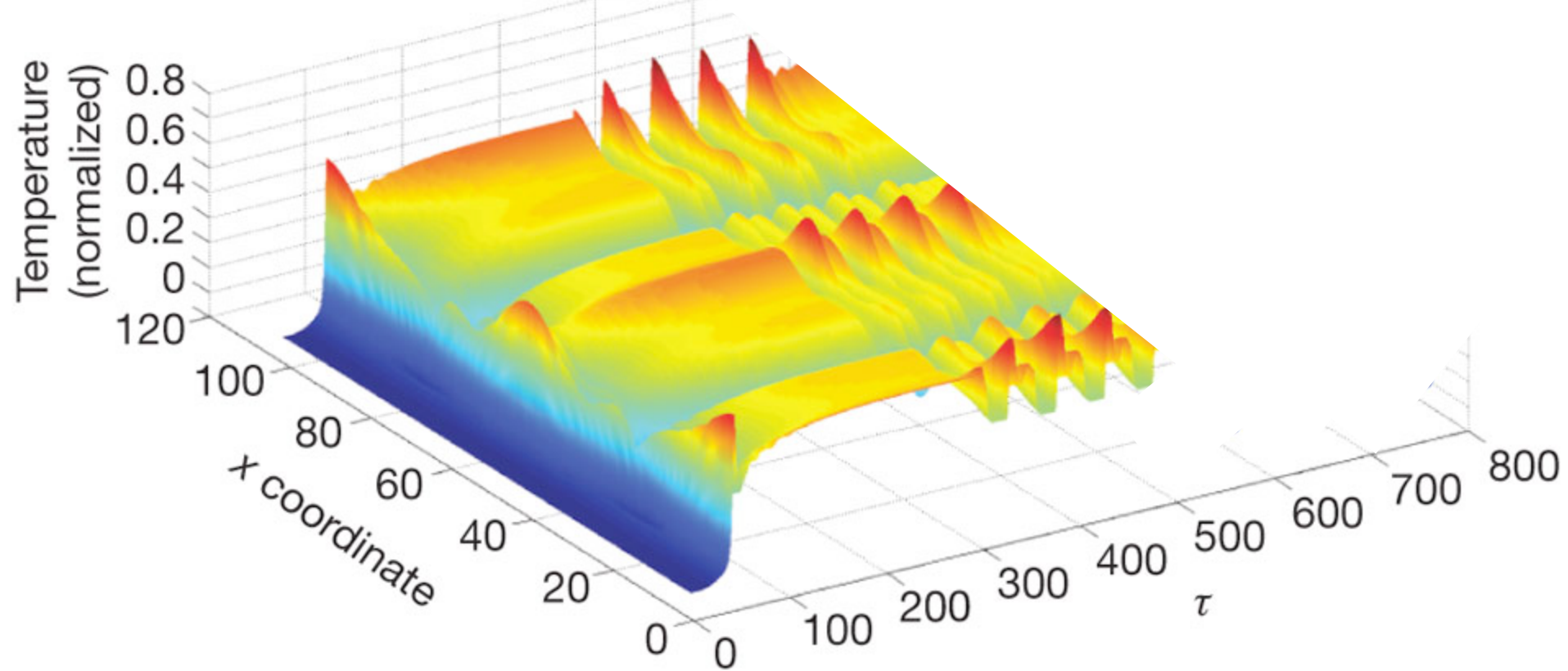


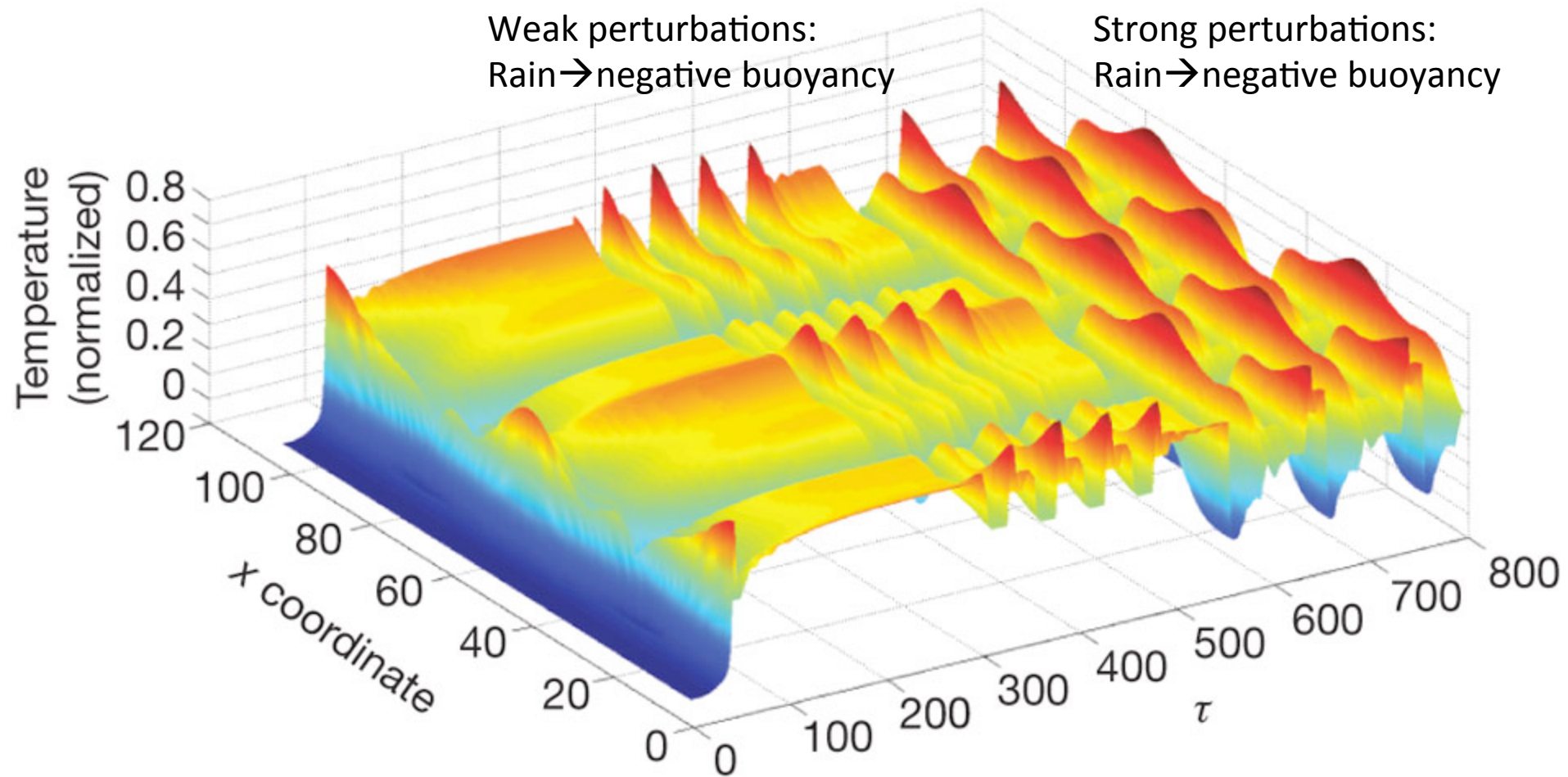


### Steady-state convection

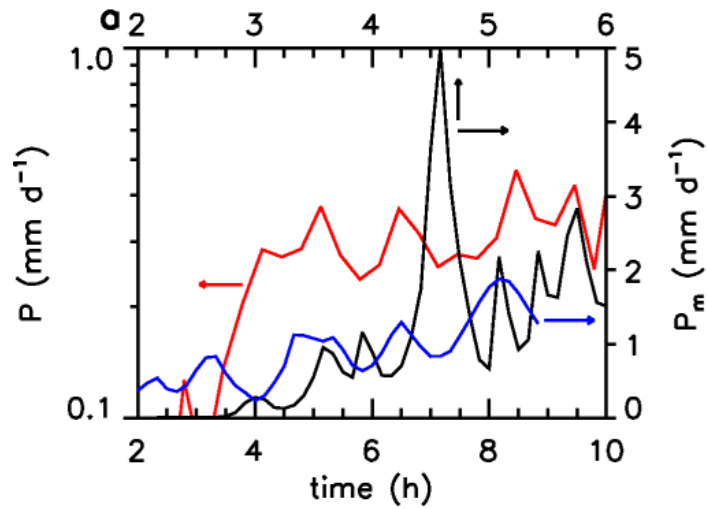


Weak perturbations:  
Rain  $\rightarrow$  negative buoyancy



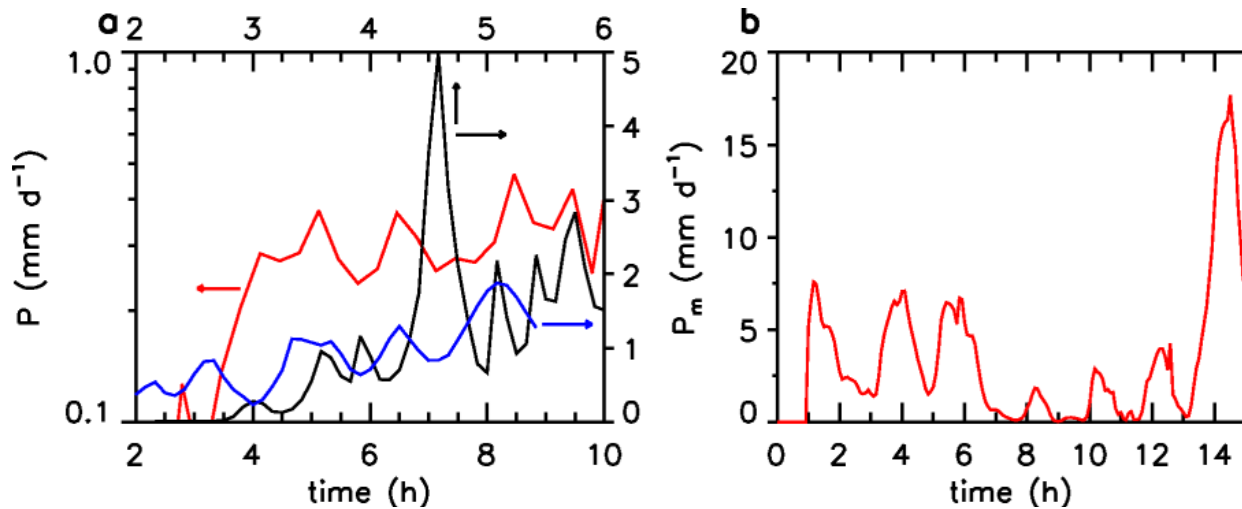


# Synchronization: Oscillations in Precipitation



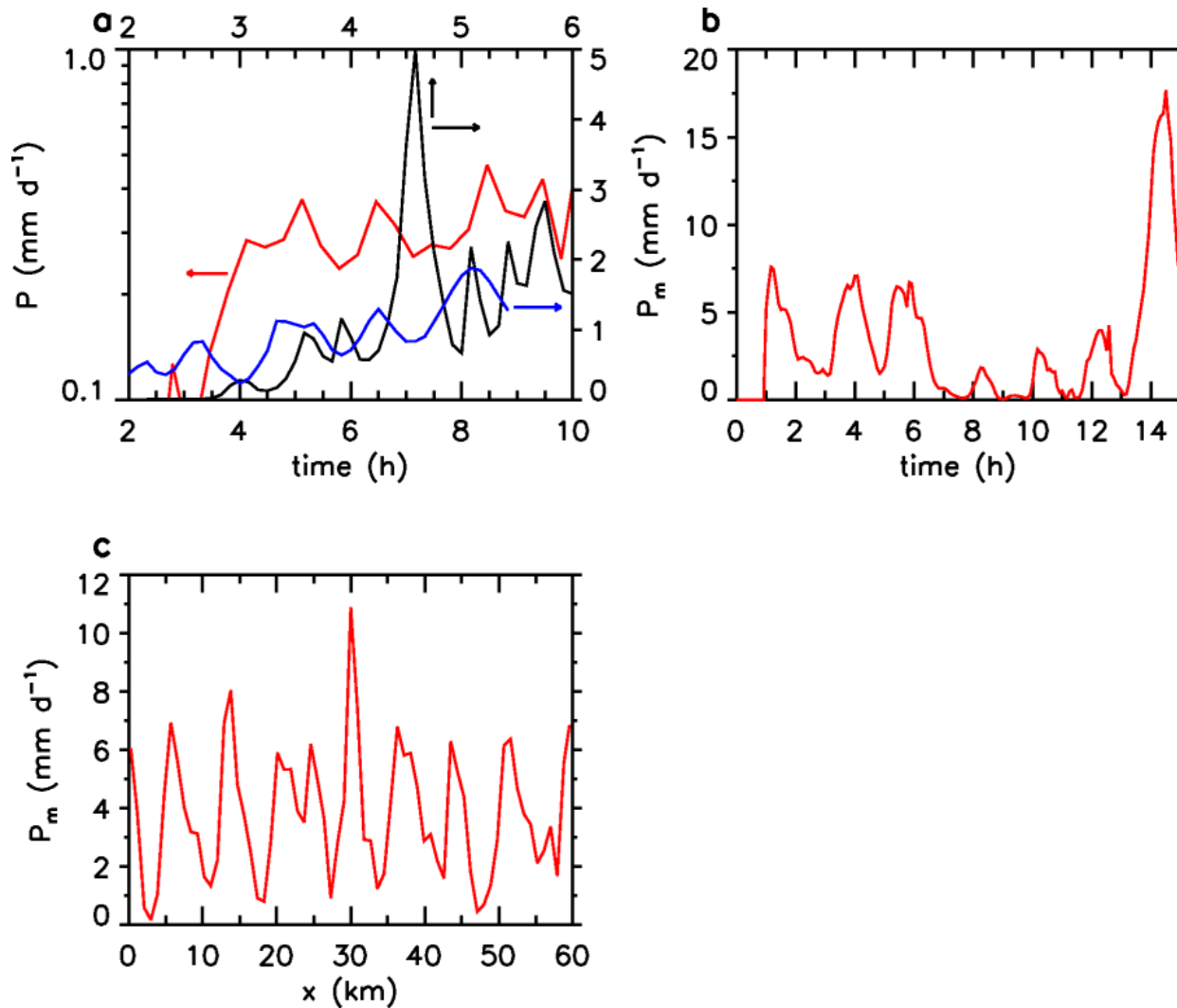
3 LES cases:  
**DYCOMS**  
ATEX  
**VOCALS**

# Synchronization: Oscillations in Precipitation



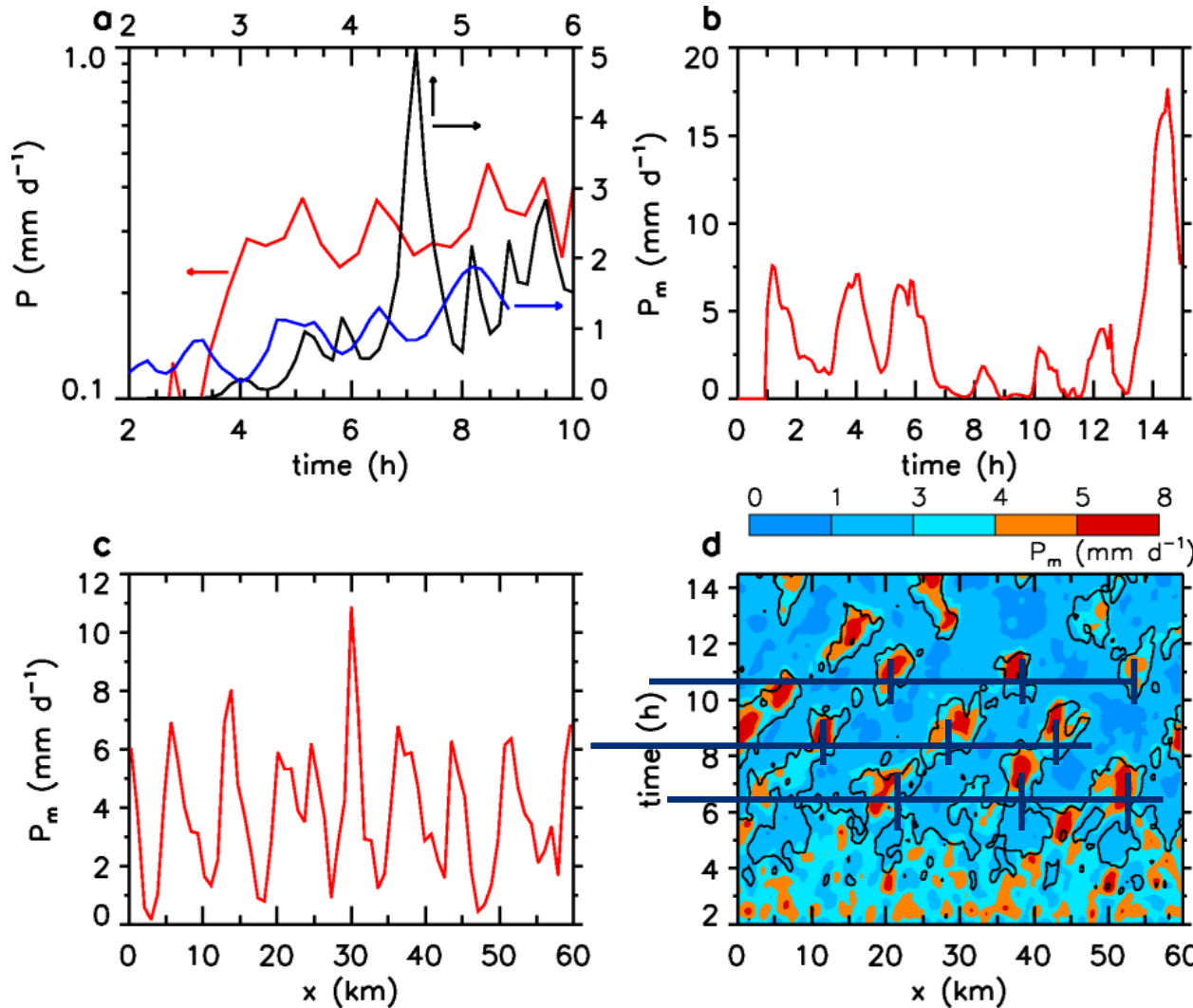
3 LES cases:  
**DYCOMS**  
ATEX  
**VOCALS**

# Synchronization: Oscillations in Precipitation



3 LES cases:  
**DYCOMS**  
**ATEX**  
**VOCALS**

# Synchronization: Oscillations in Precipitation



3 LES cases:  
**DYCOMS**  
**ATEX**  
**VOCALS**

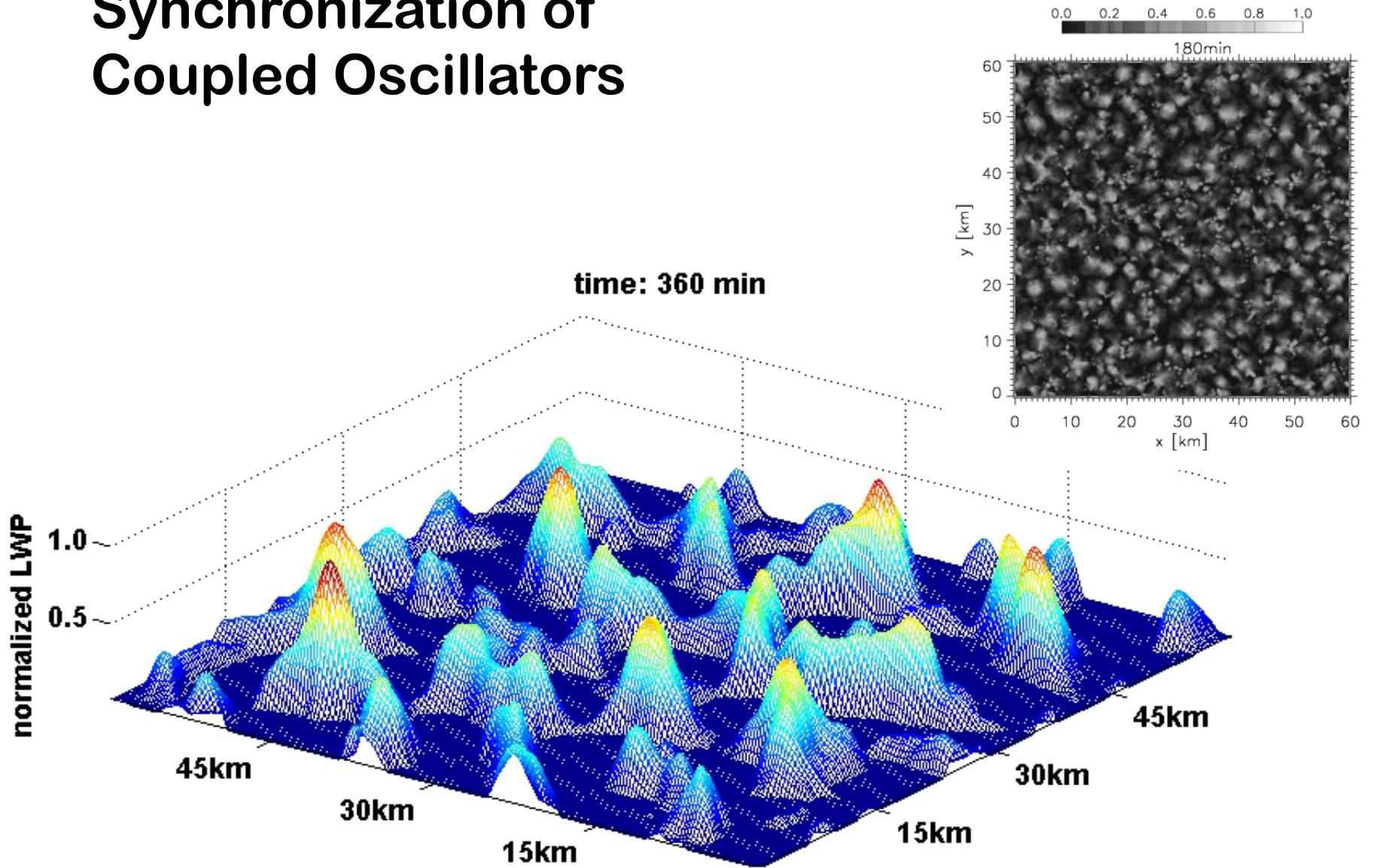
Hovmuller diagram

*Shift in rain "grid"*

Colored contours: rain

Contours: updraft

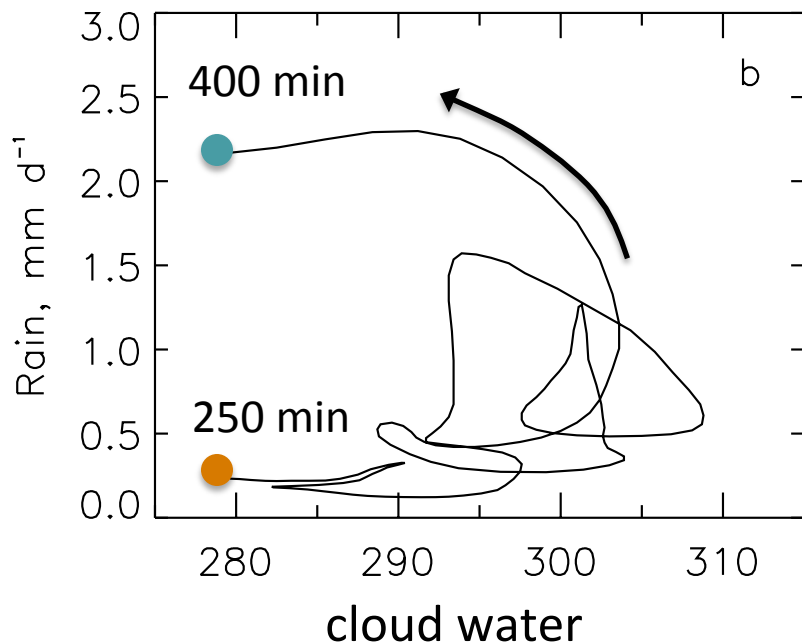
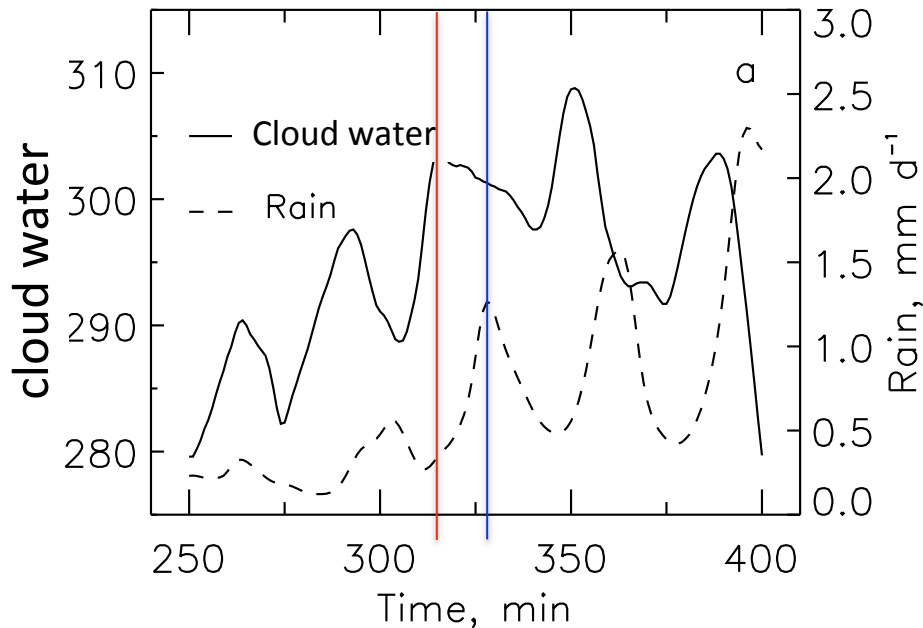
# Synchronization of Coupled Oscillators



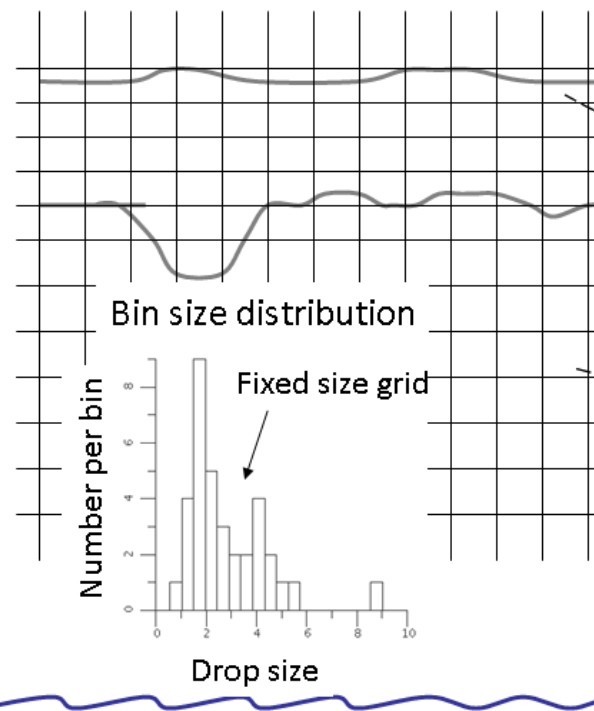


**Are Oscillating Patterns Common?**

# Large Eddy Simulation of Aerosol-Cloud-Precipitation



Large Eddy Simulation:  
Solution to Navier-Stokes Eqns on  
3-D grid ( $\sim 200 \times 200 \times 200$ )



Anticlockwise loops in  $R$ ; Cloud phase space

# Predator-Prey Model

Lotka-Volterra Equations  
(circa 1926)



$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = -y(\gamma - \delta x)$$

$x$  = prey

$y$  = predator

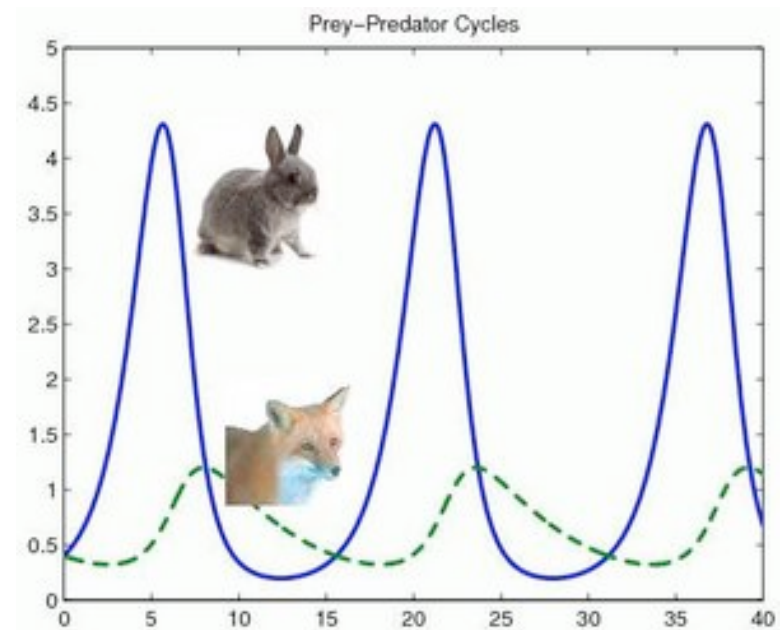
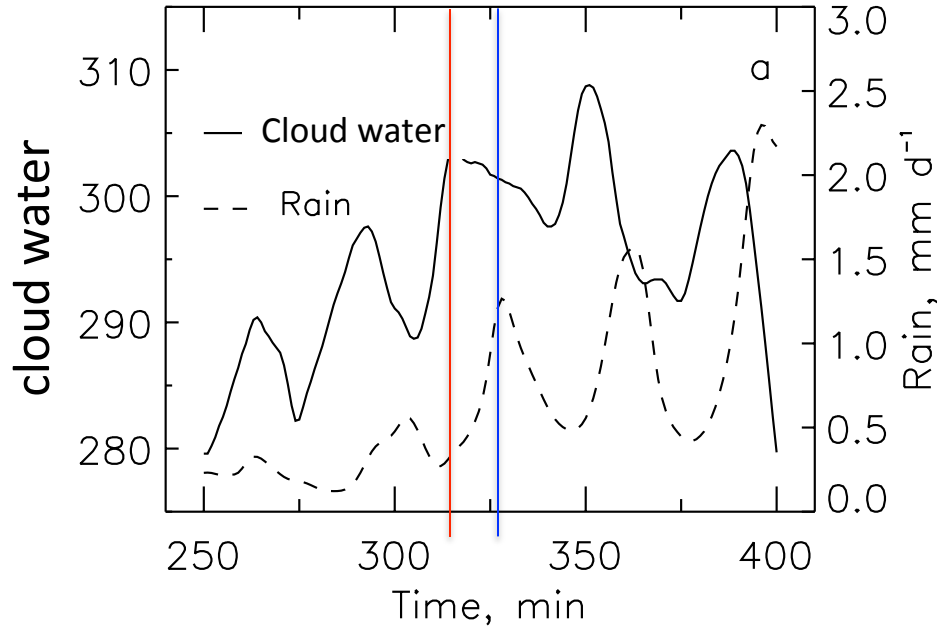


Image courtesy of Wikipedia

4 parameters:  
 $\alpha, \beta, \gamma, \delta$

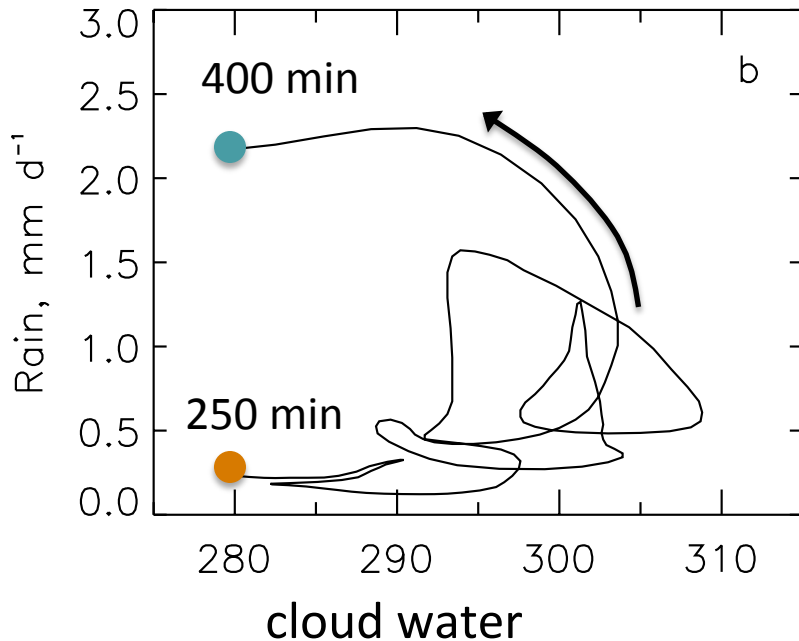
# Predator-Prey Model



Clouds=Rabbits; Rain=Foxes

- Cloud builds up
- Rain follows some time behind
- Rain destroys cloud
- Cloud regenerates (meteorological forcing, colliding outflows, etc)

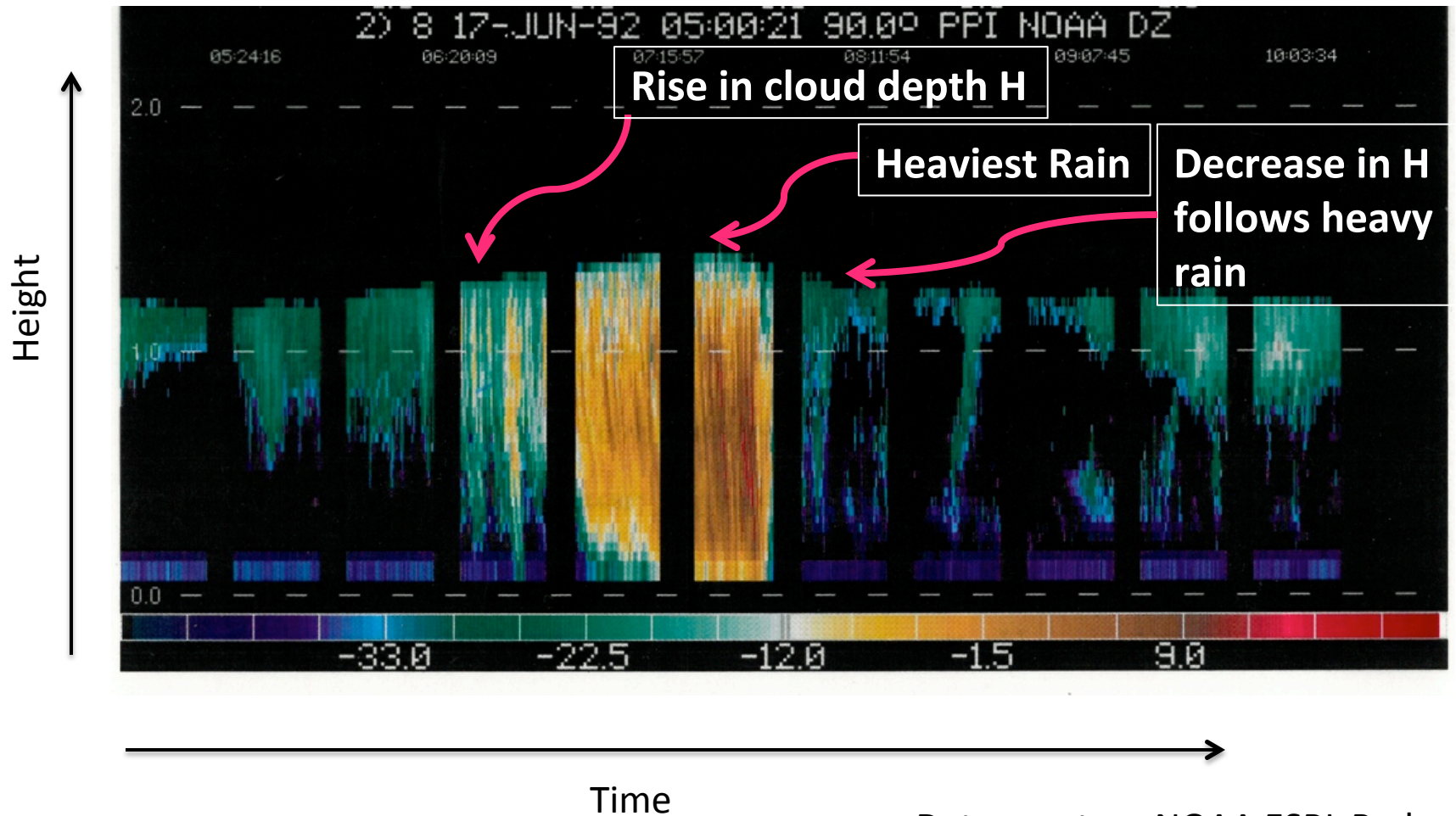
and so on...



Rain preys on clouds



# Vertical Profile of Radar reflectivity (a proxy for Rainrate) from N. Atlantic (Porto Santo, 1992; ASTEX)



# Balance Equations: average system state

Cloud Depth  $H$

$$\frac{dH}{dt} = \frac{H_0 - H}{\tau_1} + \dot{H}_r(t - T)$$

Loss term due to rain

Rainrate  $R$

$$R = \alpha H^3 N_d^{-1}$$

Empirically and theoretically based

$$R(t) = \frac{\alpha H^3 (t - T)}{N_d (t - T)}$$

Delay function  
(time for rain to develop)

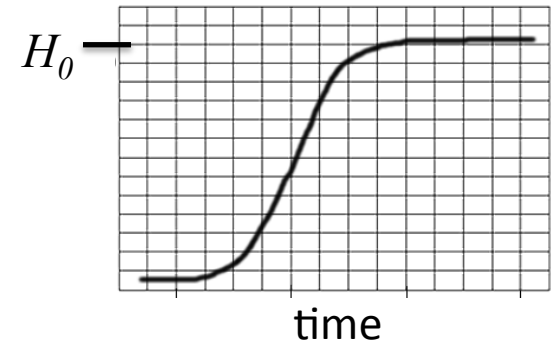
Drop concentration  $N_d$

$$\frac{dN_d}{dt} = \frac{N_0 - N_d}{\tau_2} + \dot{N}_d(t - T)$$

Loss term due to rain

Notes:

Source terms represent a range of forcings that result in exponential rise to  $H_0$  or  $N_0$  within a few  $\tau$



$N_d$  (or aerosol) modulates  $H$ - $R$  interaction

# Loss terms

Cloud Depth  $H$  loss term

$$\dot{H}_r = \frac{dH}{dt} = \frac{dH}{dLWP} \frac{dLWP}{dt} \quad ; \quad \frac{dLWP}{dt} = -R$$

$$\dot{H}_r = -\frac{1}{c_1 H} R = -\frac{\alpha H^2}{c_1 N_d}$$

Drop concentration  $N_d$  loss term (collision-coalescence)

$$\dot{N}_d = -c_2 N_d R$$

Wood 2006

# Balance Equations

Cloud Depth  $H$

$$\frac{dH}{dt} = \frac{H_0 - H}{\tau_1} + \dot{H}_r(t - T)$$

Rainrate  $R$

$$R(t) = \frac{\alpha H^3(t - T)}{N_d(t - T)}$$

Drop concentration  $N_d$

$$\frac{dN_d}{dt} = \frac{N_0 - N_d}{\tau_2} + \dot{N}_d(t - T)$$

Notes:

Five parameters:

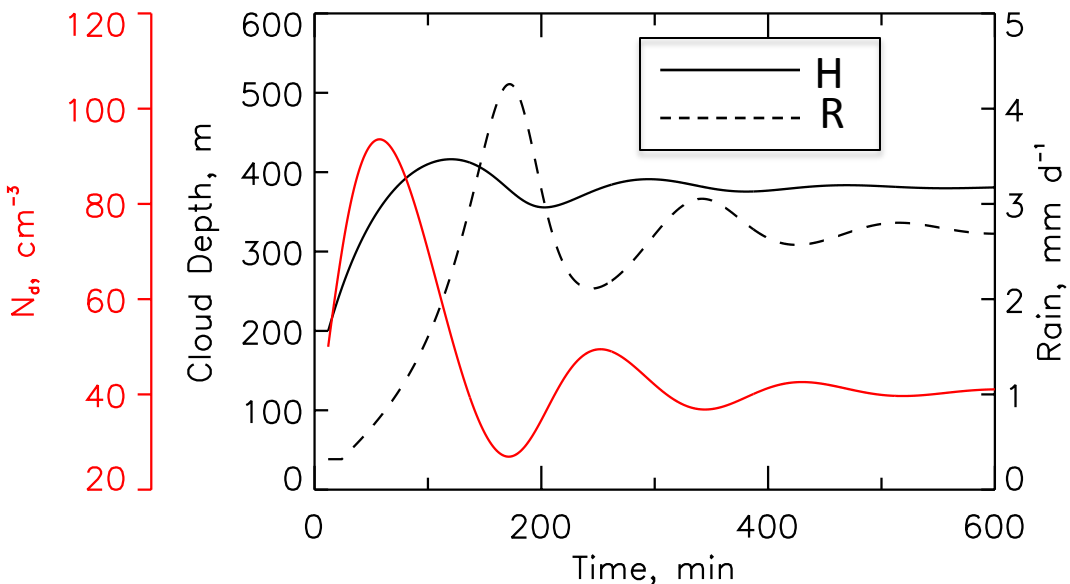
Carrying Capacity:  $H_0, N_0$

Time constants:  $\tau_1, \tau_2$

Delay time:  $T$



# Oscillating Solutions: Steady State



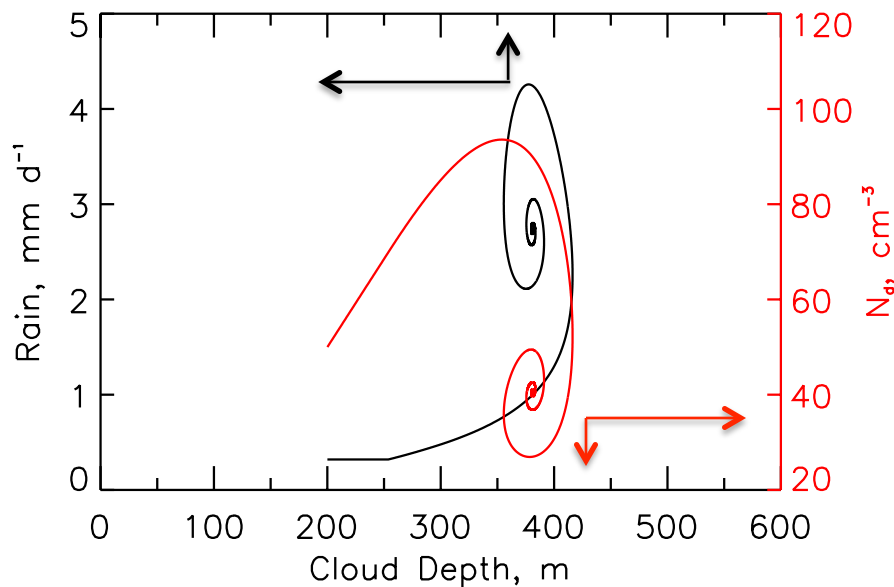
*At steady state:  
Aerosol sources are sufficient  
to maintain balance between  
sources and rainfall removal*

$$H_0 = 530 \text{ m}$$

$$N_0 = 180 \text{ cm}^{-3}$$

$$\tau_1 = \tau_2 = 60 \text{ min}$$

$$T = 10 \text{ min}$$

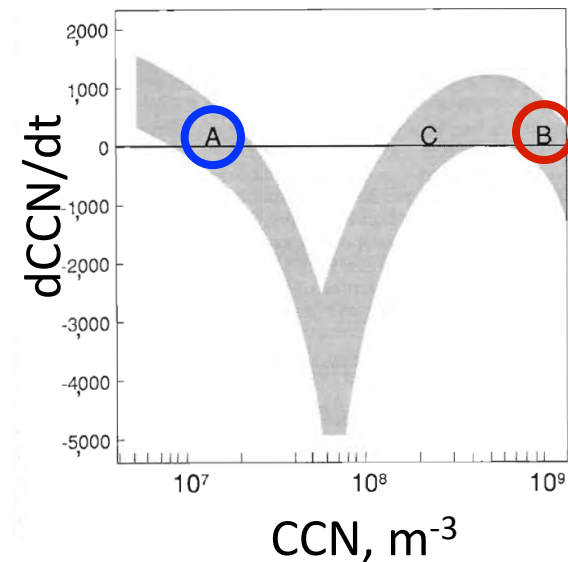
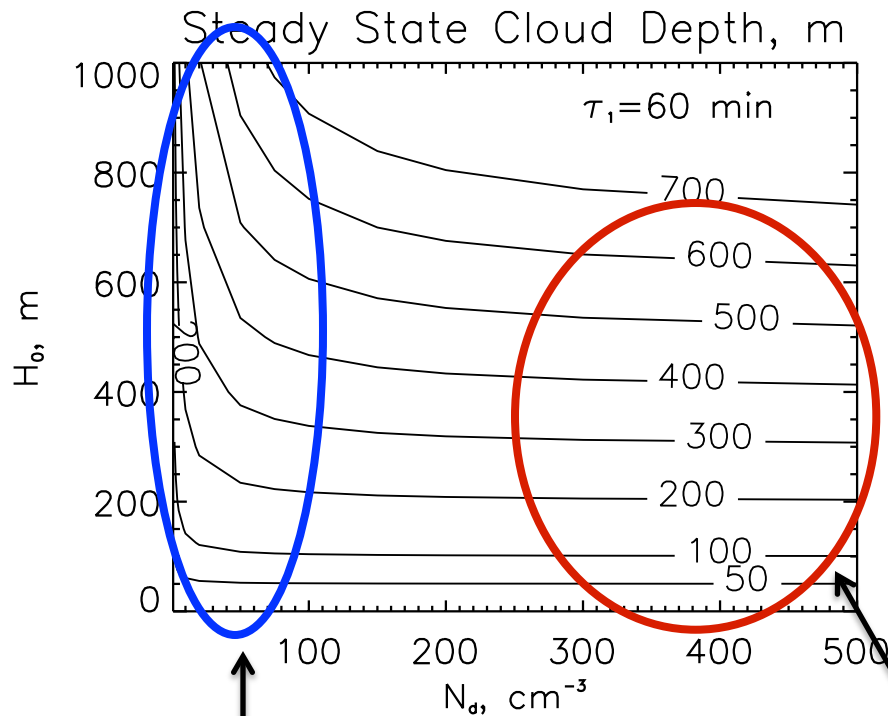


—  $N$ ;  $N$   
—  $H$ ;  $R$

7 day simulation

# Steady State Solution to Cloud Depth H

$$\frac{dH}{dt} = \frac{H_0 - H}{\tau_1} + \dot{H}_r(t - T) = 0 \quad H = \frac{(N_d^2 + 4\gamma\tau_1 N_d H_0)^{\frac{1}{2}} - N_d}{2\gamma\tau_1}$$

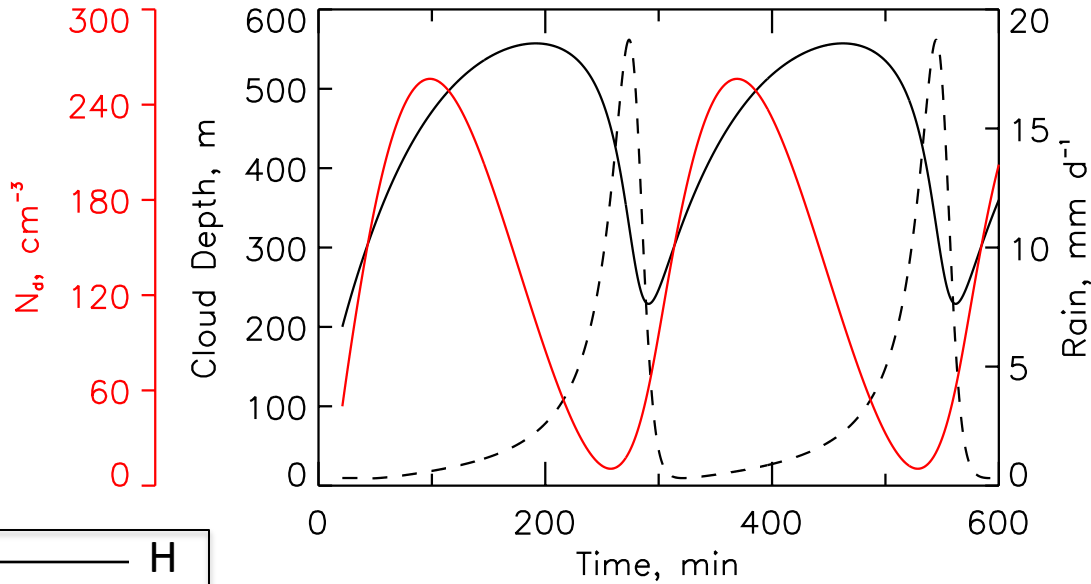


Baker and Charlson, 1990

Cloud Depth determined by  
drop concentration  $N_d$

Cloud Depth determined by  $H_0$

# Oscillating Solutions: No Steady State

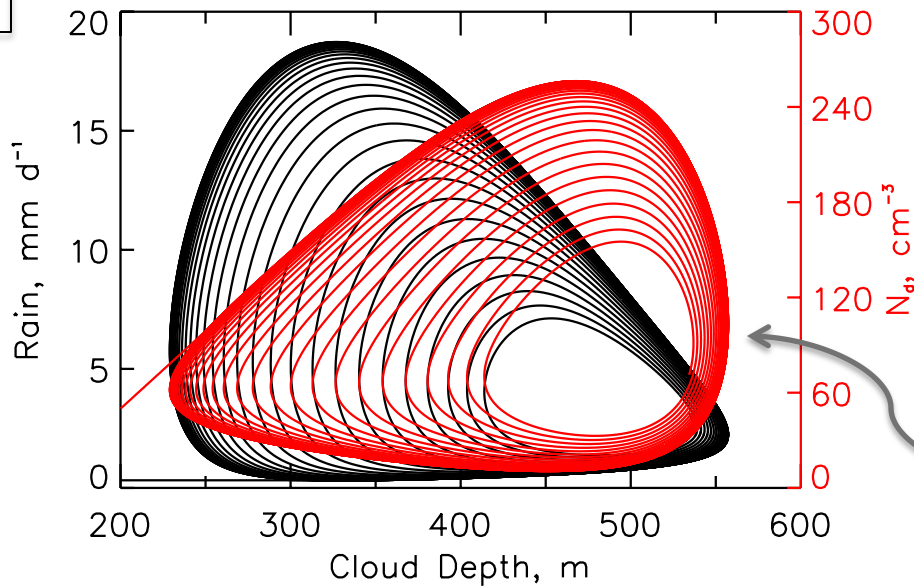


$$\frac{dH}{dt} = \frac{H_0 - H}{\tau_1} + \dot{H}_r(t - T)$$

$$R(t) = \frac{\alpha H^3(t - T)}{N_d(t - T)}$$

$$\frac{dN_d}{dt} = \frac{N_0 - N_d}{\tau_2} + \dot{N}_d(t - T)$$

$H_0 = 670 \text{ m}$   
 $N_0 = 515 \text{ cm}^{-3}$   
 $\tau_1 = \tau_2 = 84 \text{ min}$   
 $T = 21.5 \text{ min}$



—  $H; N$   
—  $H; R$

Oscillation around a steady state

7 day simulation

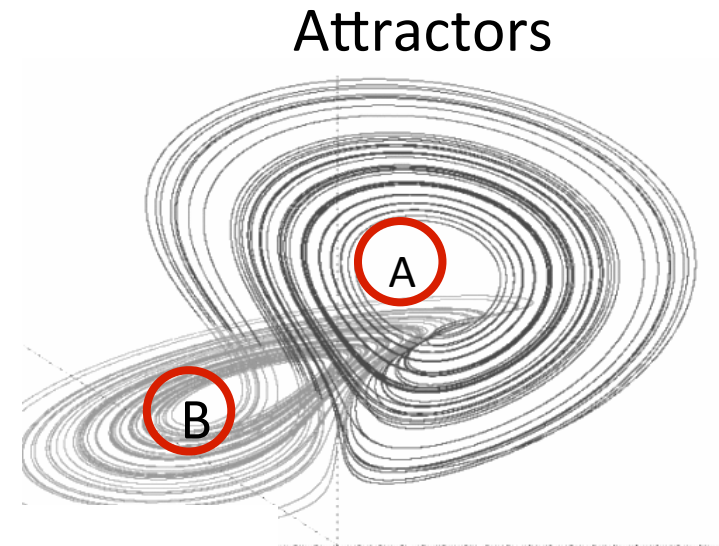
# Stability

*How stable are the stable states?*

*How readily does the system transition from one state to another?*

*States A and B are stable  
and self-sustaining*

*Small perturbations strengthen  
the resilience of the state*

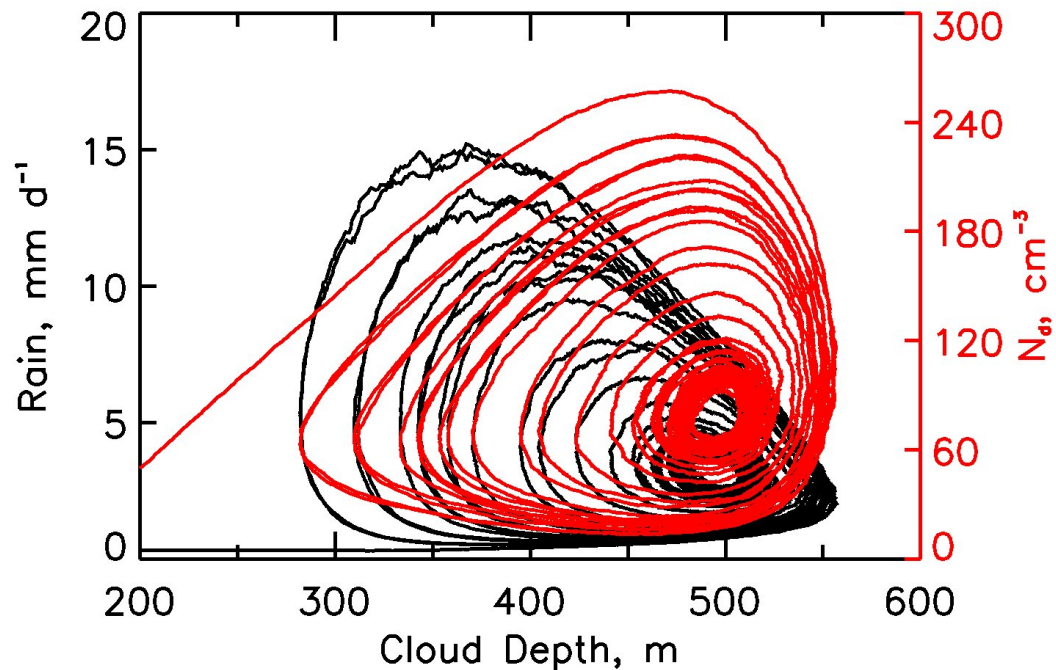
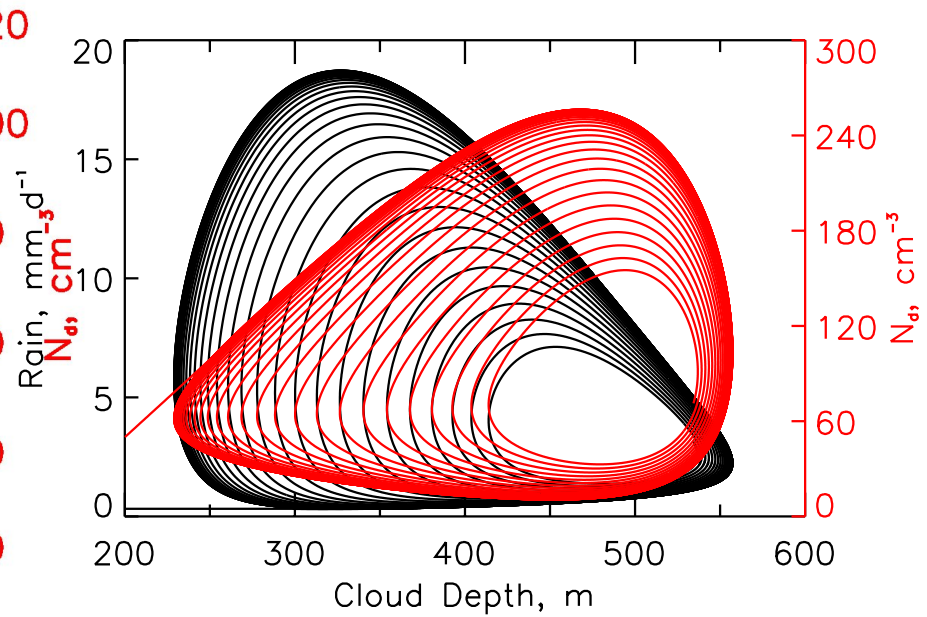
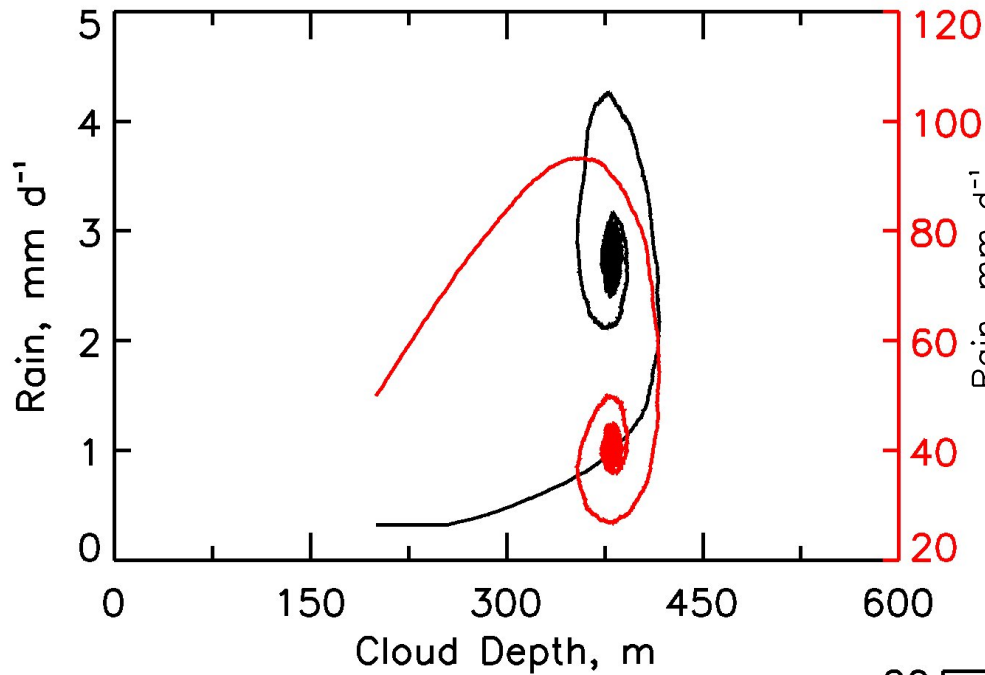


Lorenz, 1954

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

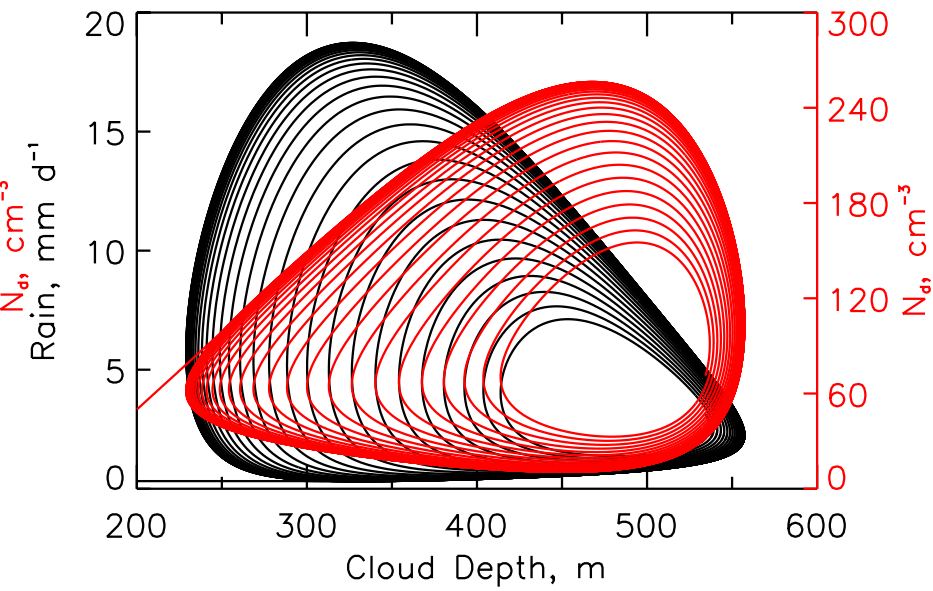
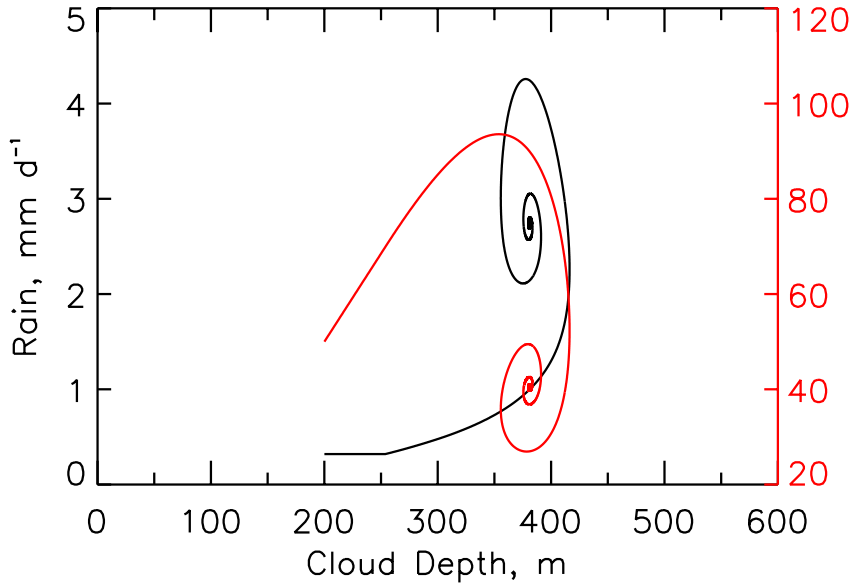
$$\frac{dz}{dt} = xy - \beta z$$



*± 50% perturbations to  $H_0$  and  $N_0$  every second: Solutions are robust*

*Small perturbations strengthen the resilience of the state;*

*Large enough perturbations will lead to collapse*



$H_0 = 530$  m  
 $N_0 = 180$  cm<sup>-3</sup>  
 $\tau_1 = \tau_2 = 60$  min  
 $T = 10$  min

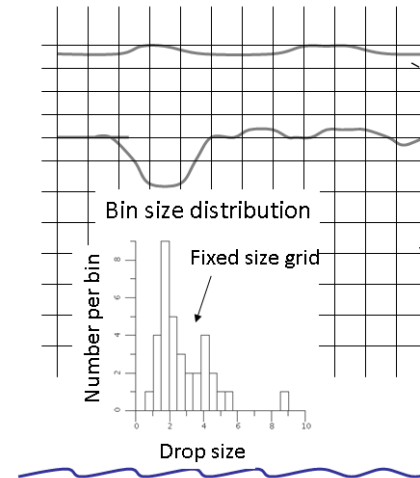
$H_0 = 670$  m  
 $N_0 = 515$  cm<sup>-3</sup> ← Stabilizing agent  
 $\tau_1 = \tau_2 = 84$  min  
 $T = 21.5$  min

*Rosenzweig Paradox (1971): the more "food" (larger  $H_0$ ) you give a predator-prey system, the more unstable it will be*

# The Way Forward?

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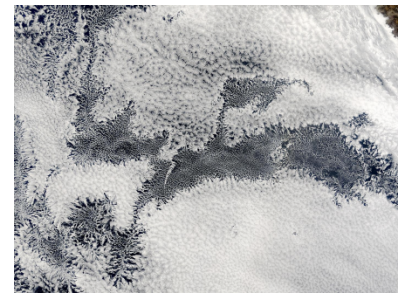
1) Process level understanding  
“Reductionist” or “Newtonian”



2) Systems approach  
“Darwinian”



Synthesis of “Newtonian” and “Darwinian” \*



\* John Harte, Physics Today October 2002

# Summary

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- *Emergence: coherent patterns emerge from local interactions*
  - *Open/Closed cells*
  - *Flock behaviour*
  - *Oscillating chemical reactions*
- *Can we exploit emergence to represent complex systems?*
- *Open cellular state: Coupled system that constantly rearranges itself*
  - *Local interactions: convergence of precipitating outflows*
  - *synchronized rain, oscillations in open-cell state*
- *Aerosol-cloud-precipitation system contains elements of the predator-prey problem*
  - *Coupled oscillations in Cloud-Rain “Populations”*
  - *Bifurcation*



# References

- Feingold, Koren, Wang, Xue, Brewer, 2010: Precipitation generated oscillations in open-cellular cloud fields, *Nature August 2010*
- Koren and Feingold, 2011: Aerosol-cloud-precipitation system as a predator-prey problem, *PNAS, July 2011*.