The Department of Physics at the University of Toronto, March 15, 2012

The mirror symmetry in super-dense matter and chiral hydrodynamics

D. Kharzeev

Stony Brook University

BROOKHAVEN
NATIONAL LABORATORY

Collaborators

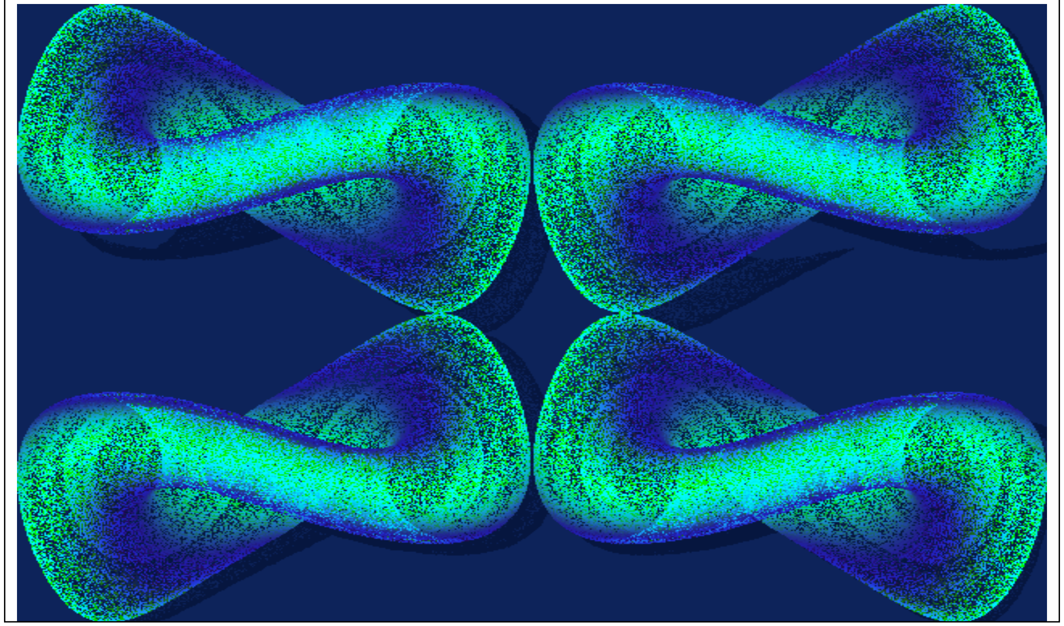
- G. Basar (Stony Brook)
- Y. Burnier (Stony Brook-Lausanne)
- G. Dunne (UConn)
- K. Fukushima (U Tokyo)
- J. Liao (U. Indiana)
- L. McLerran (BNL)
- D. Son (U. Washington)
- H. Warringa (Frankfurt)
- H.-U. Yee (Stony Brook)
- A. Zhitnitsky (U British Columbia)

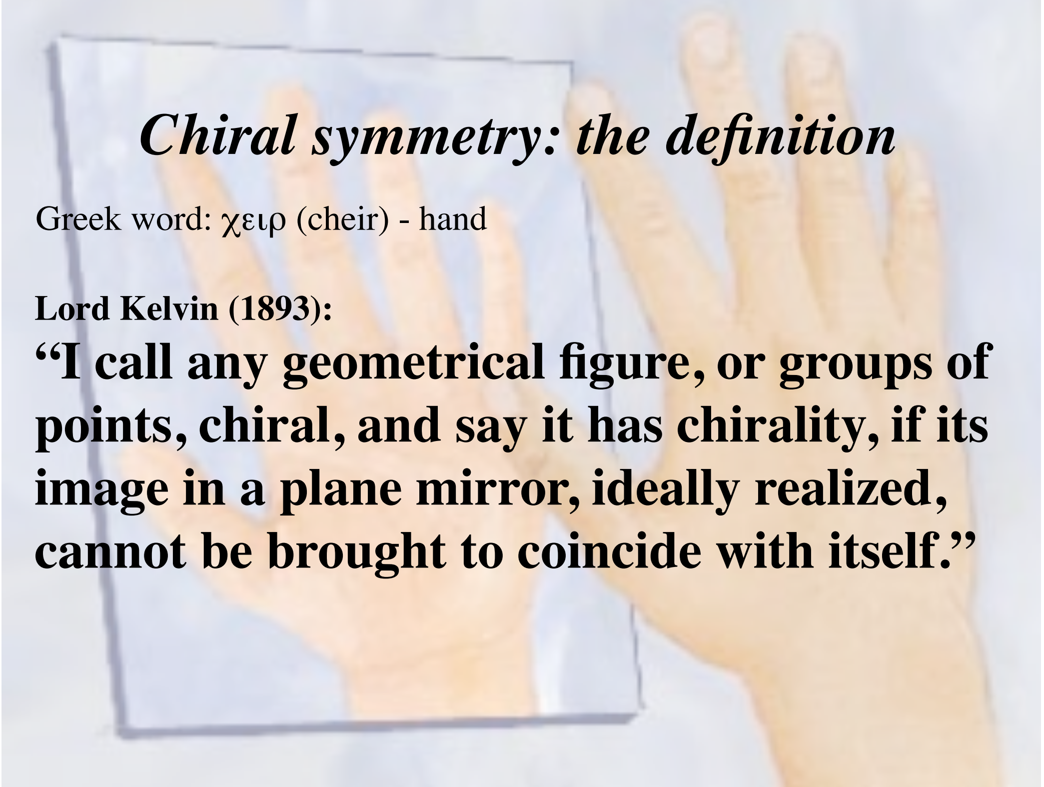
Work supported by the US Department of Energy

Outline

- Chiral symmetry and parity invariance
- Chern-Simons theory
- Chiral magnetic effect and <u>local</u> P and CP violation in hot quark-gluon matter at RHIC and LHC
- Chiral hydrodynamics: how quantum anomalies affect the macroscopic collective behavior at femto-, nano-, and parsec scales

What is chiral symmetry?





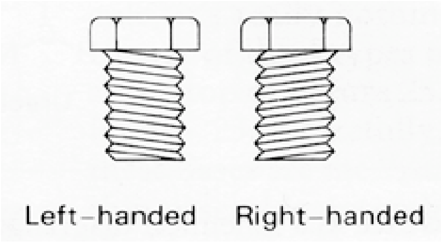
Chiral symmetry: the definition

Greek word: χειρ (cheir) - hand

Lord Kelvin (1893):

“I call any geometrical figure, or groups of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself.”

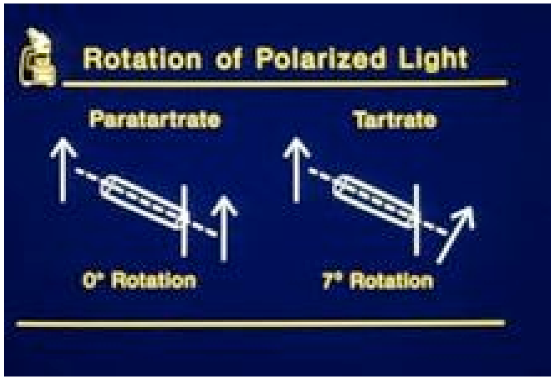
Examples



Not chiral:
nails, spin-0 bosons, …

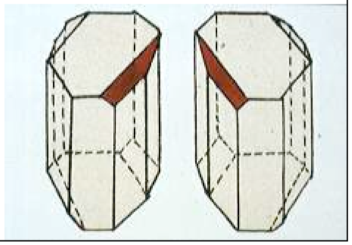
Chiral:
screws, spin-1/2 fermions,…

Polarized life



Louis Pasteur
1822-1895

Rotation of light polarization in tartaric acid - absent in synthesized one, but present in the one derived from wine lees ↔ different mixtures of left and right crystals



Truth at the bottom of the glass:

wine lees Sediment or deposit that forms in the bottom of wine casks during the fermentation process; used as a source of tartaric acid and tartrates.

Parity invariance:
left vs right

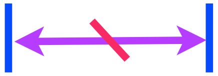
In 3+1 dimensions, Parity transformation is

$$\boxed{\vec{x} \longrightarrow -\vec{x}}$$

NB: different in the even
number of spatial dimensions!

Parity-even:

mass
energy
angular momentum
magnetic field
Maxwell field strength tensor



Parity-odd:

particle position
momentum
electric current
electric field
electromagnetic vector potential

Parity in gauge theories: Classical electrodynamics and Maxwell's equations

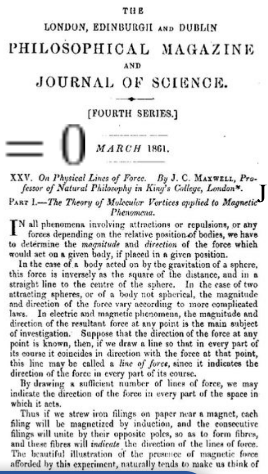
EM fields in the aether:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday-Maxwell induction

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$



THE
LONDON, EDINBURGH AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FOURTH SERIES.]

MARCH 1861.

XXV. *On Physical Lines of Force. By* J. C. MAXWELL, *Professor of Natural Philosophy in King's College, London**.

PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

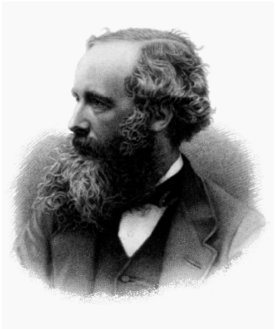
IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

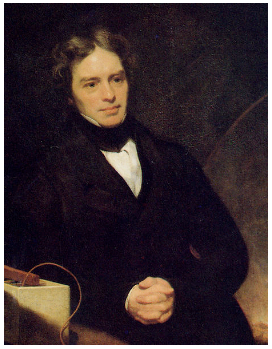
By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

Thus if we strew iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will *indicate* the direction of the lines of force. The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of

Maxwell electrodynamics is P and CP even

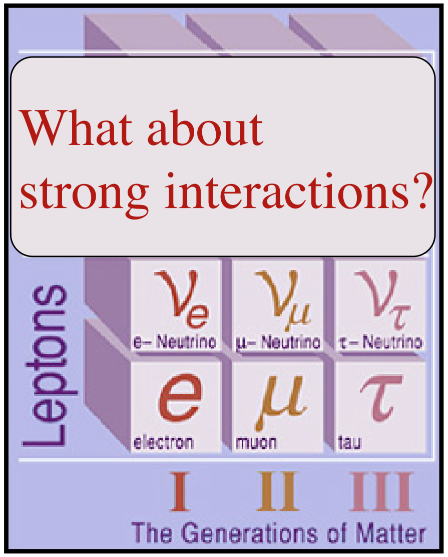


James C. Maxwell, 1831-1879

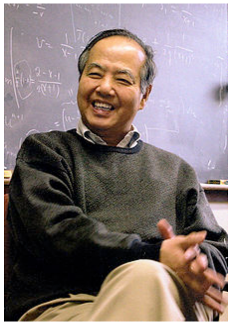


Michael Faraday, 1791-1867

P and CP invariances are violated by weak interactions



What about strong interactions?



T.D.Lee

C.N.Yang

1957

CP violation J.W.Cronin, V.L.Fitch

1980

Complex CKM mass matrix

Y. Nambu, M. Kobayashi, T. Maskawa

2008

Very strict experimental limits exist on the amount of global violation of P and CP invariances in strong interactions (mostly from electric dipole moments)

But: P and CP conservation in QCD is by no means a trivial issue…

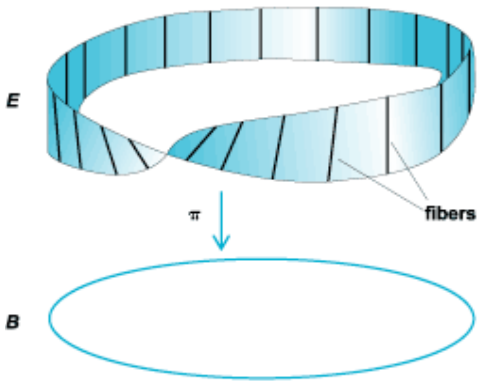
Can a local P and CP violation occur in QCD matter?

Mathematics: in search for "the most harmonious and the most beautiful"

" I always regarded mathematics as the method of obtaining the best shapes and dimensions of things; and this meant not only the most useful and economical, but chiefly the most harmonious and the most beautiful."

from a letter by Maxwell
to Galton

Gauge fields and topology

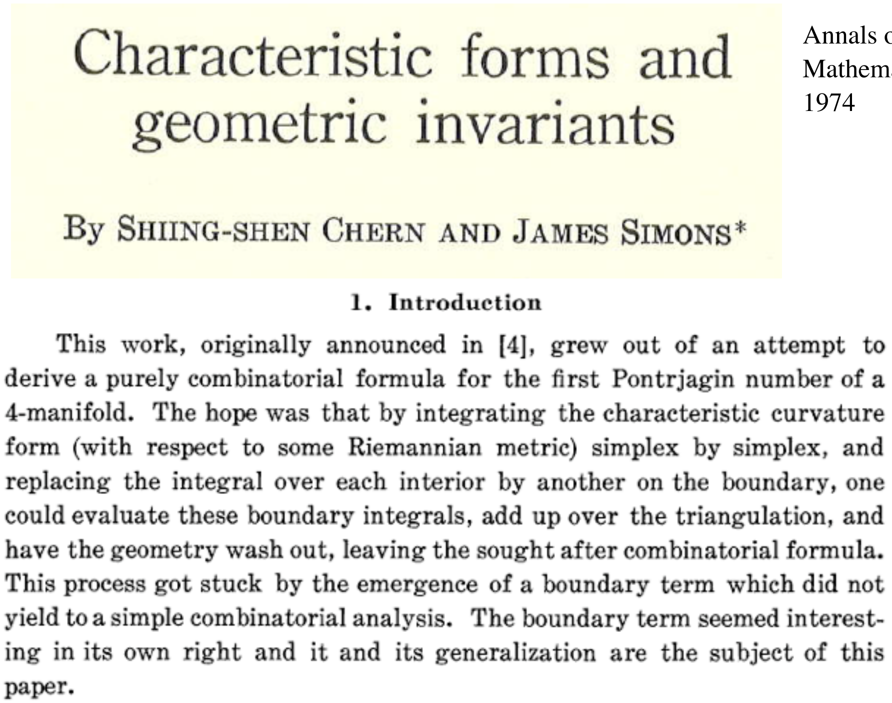


NB: Maxwell
electrodynamics
as a curvature
of a line bundle

Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories "live" in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Annals of
Mathematics,
1974



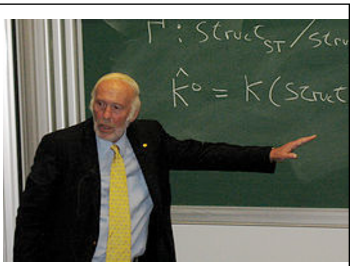
Characteristic forms and geometric invariants

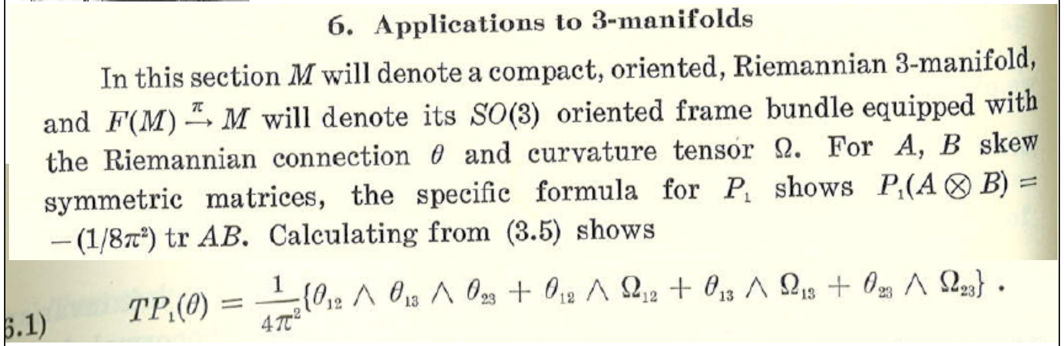
By Shiing-shen Chern and James Simons*

1. Introduction

This work, originally announced in [4], grew out of an attempt to derive a purely combinatorial formula for the first Pontrjagin number of a 4-manifold. The hope was that by integrating the characteristic curvature form (with respect to some Riemannian metric) simplex by simplex, and replacing the integral over each interior by another on the boundary, one could evaluate these boundary integrals, add up over the triangulation, and have the geometry wash out, leaving the sought after combinatorial formula. This process got stuck by the emergence of a boundary term which did not yield to a simple combinatorial analysis. The boundary term seemed interesting in its own right and it and its generalization are the subject of this paper.

Chern-Simons forms





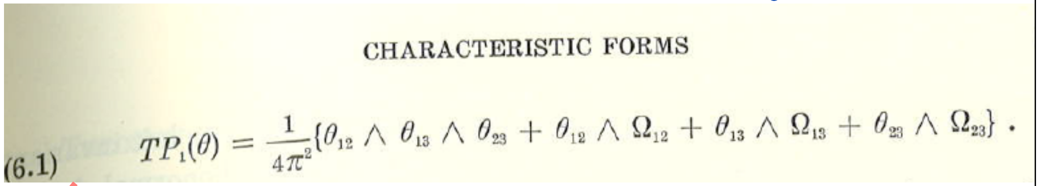
6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω. For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2)\ \mathrm{tr}\ AB$. Calculating from (3.5) shows

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\}\,. \tag{6.1}$$

What does it mean for a gauge theory?

Chern-Simons theory



CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12}\wedge\theta_{13}\wedge\theta_{23} + \theta_{12}\wedge\Omega_{12} + \theta_{13}\wedge\Omega_{13} + \theta_{23}\wedge\Omega_{23}\}\,. \tag{6.1}$$

What does it mean for a gauge theory?

Geometry ↔ **Physics**

Riemannian connection ↔ Gauge field

Curvature tensor ↔ Field strength tensor

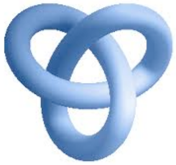
$$\boxed{S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)}$$

Abelian non-Abelian

Chern-Simons theory

$$S_{CS} = \frac{k}{8\pi} \int_M d^3x \; \epsilon^{ijk} \left(A_i F_{jk} + \frac{2}{3} A_i [A_j, A_k] \right)$$

Remarkable novel properties:



- gauge invariant, up to a boundary term
- topological - does not depend on the metric, knows only about the topology of space-time **M**
- when added to Maxwell action, induces a mass for the gauge boson - different from the Higgs mechanism!
- **breaks Parity invariance**

Chern-Simons theory and the vacuum of Quantum Chromodynamics

Equation:

$$D^\mu F^a_{\mu\nu} = 0$$

Belavin, Polyakov,
Tyupkin, Schwartz;
‘t Hooft; ...

Solution:

$$A^a_\mu(x) = \frac{2\boxed{\eta_{a\mu\nu}x_\nu}}{x^2+\rho^2} \longleftarrow$$

Coupling of
space-time
and color:

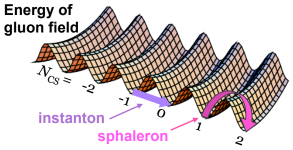
Integer $Q = \int d\sigma_\mu K_\mu$

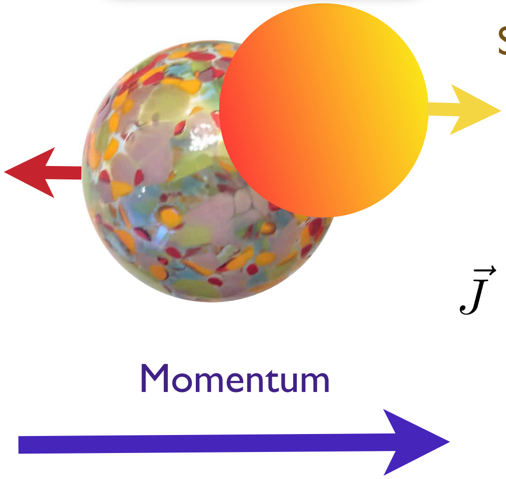
$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu,\, \nu = 1,\, 2,\, 3, \\ \delta_{a\mu} & \nu = 4, \\ -\delta_{a\nu} & \mu = 4. \end{cases}$$

$$K_\mu = \frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^a_\alpha\partial_\beta A^a_\gamma + \frac{1}{3}f^{abc}A^a_\alpha A^b_\beta A^c_\gamma\right)$$ Chern-Simons current

Topology-induced change of chirality

Right ↔ Left





Spin

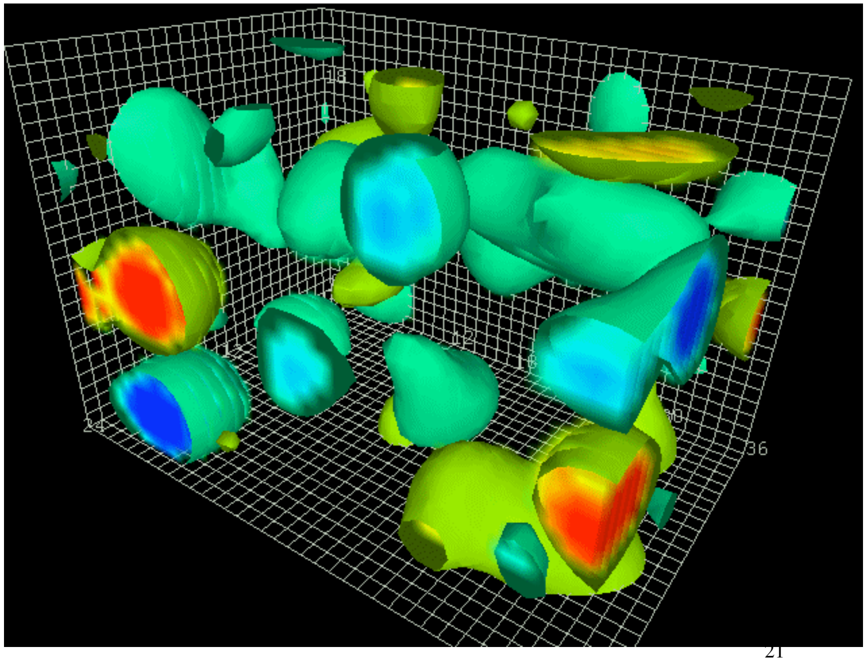
Color
SU(2) spin

$$\vec{J} = \vec{T} + \vec{S}$$

Momentum

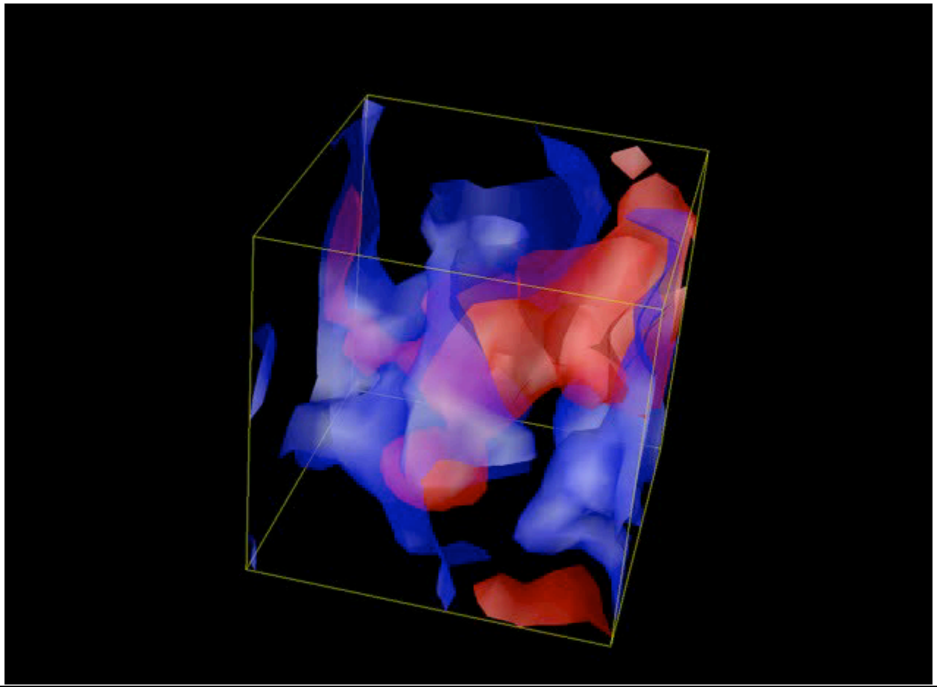
Topological number fluctuations in QCD vacuum

("cooled" configurations)

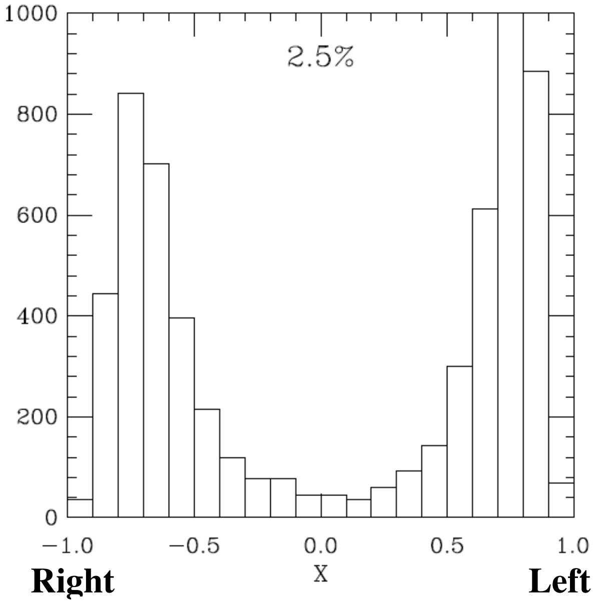


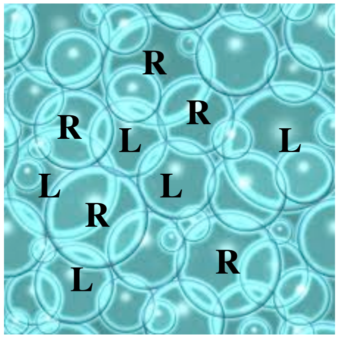
Topological number fluctuations in QCD vacuum

ITEP Lattice Group



The chiral nature of the QCD “aether”



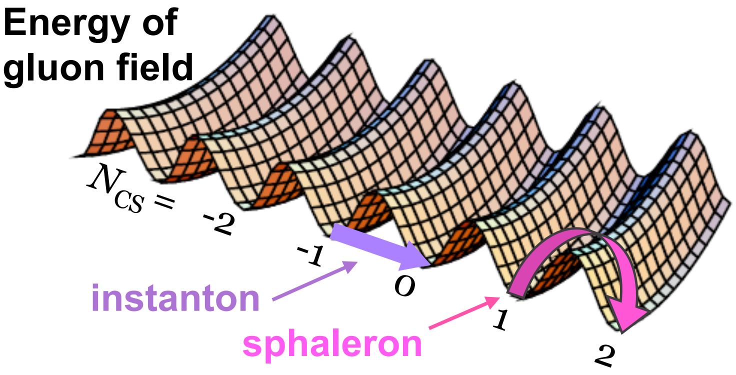


T. DeGrand, A. Hasenfratz,
Phys.Rev.D65:014503,2002

$$\tan(\frac{\pi}{4}(1+X(x))) = \frac{|\psi_L(x)|}{|\psi_R(x)|} = (\frac{\psi_L^\dagger(x)\psi_L(x)}{\psi_R^\dagger(x)\psi_R(x)})^{1/2}$$

Sphaleron transitions at finite energy or temperature

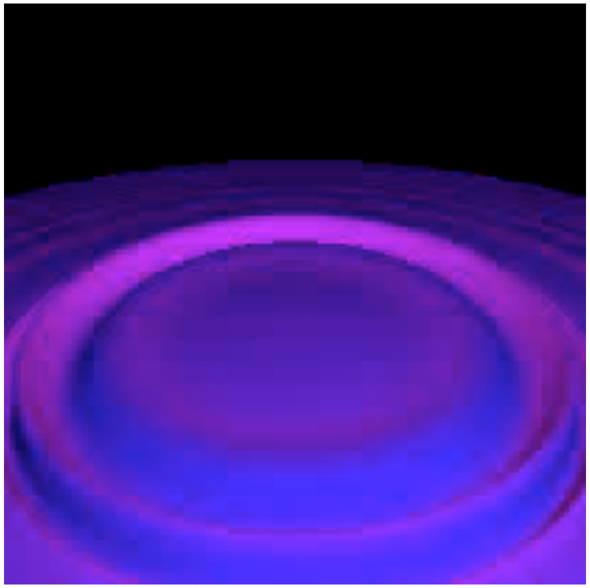
$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int_0^t \langle (q(x)q(0) + q(0)q(x)) \rangle d^4x$$



Sphalerons:
random walk of
topological charge at finite T:

$$\langle Q^2 \rangle = 2\Gamma V t, \quad t \to \infty$$

Sphaleron transitions at finite energy or temperature



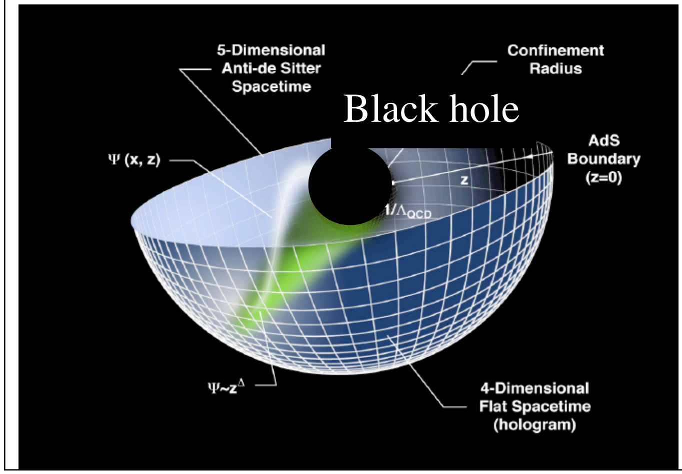
C. Rebbi, http://scv.bu.edu/visualization/gallery

Topological number diffusion at strong coupling

Chern-Simons number
diffusion rate
at strong coupling

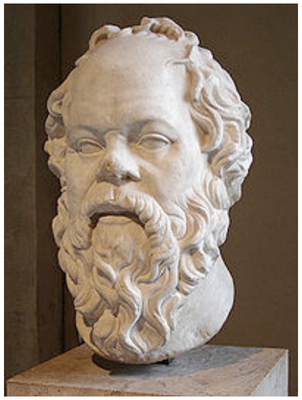
$$\Gamma = \frac{(g_{\rm YM}^2 N)^2}{256\pi^3} T^4$$

D.Son,
A.Starinets
hep-th/
020505



NB: This calculation is completely analogous to the calculation of shear viscosity that led to the “perfect liquid”

The metaphor of the cave, 380 B.C.



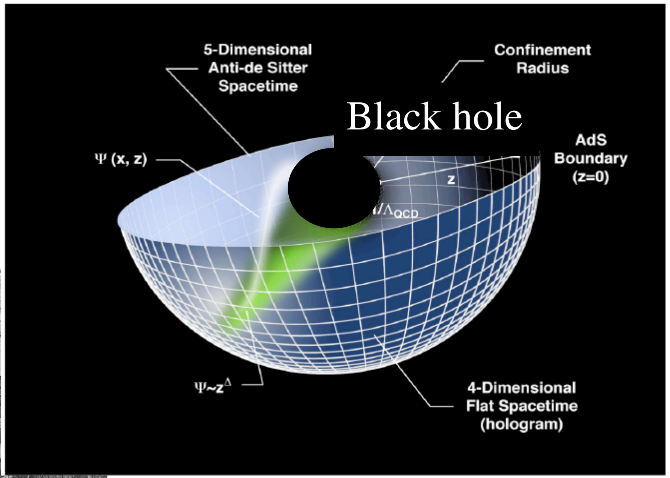
Socrates (Σωκράτης)
469 - 399 B.C.

“Physical objects and physical events are only "shadows" of their ideal or perfect forms, and exist only to the extent that they instantiate the perfect versions of themselves”

Socrates, in Plato’s “Republic”

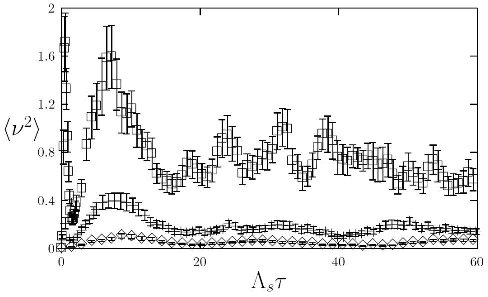
“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

The metaphor of the cave, 2012 A.D.

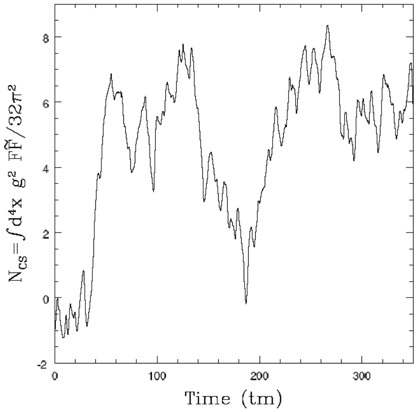


“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

Diffusion of Chern-Simons number in QCD: real time lattice simulations

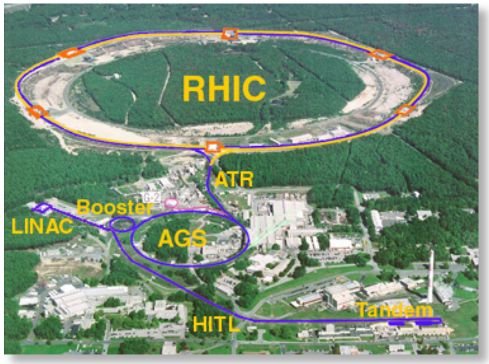


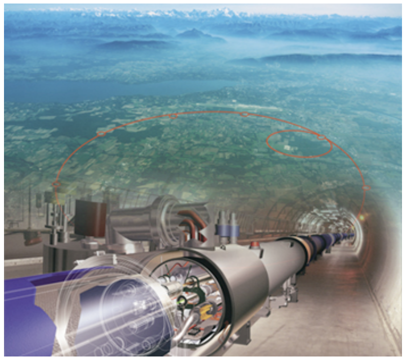
DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002



P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

Experimental test of Chern-Simons dynamics in hot QCD: Heavy ion collisions

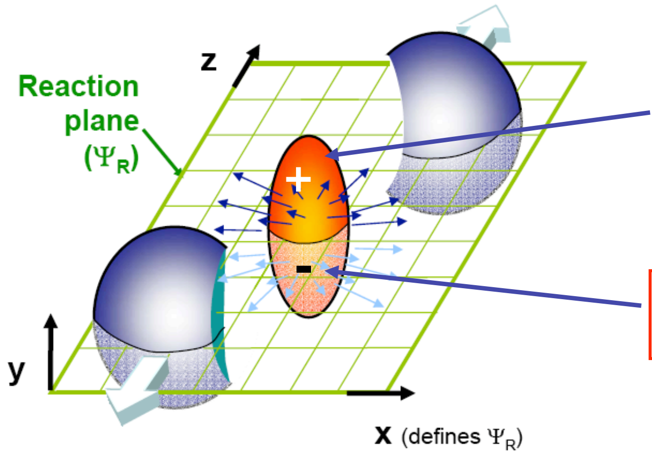




LHC

NICA,
JINR

Charge asymmetry w.r.t. reaction plane as a signature of strong P violation



excess of positive charge

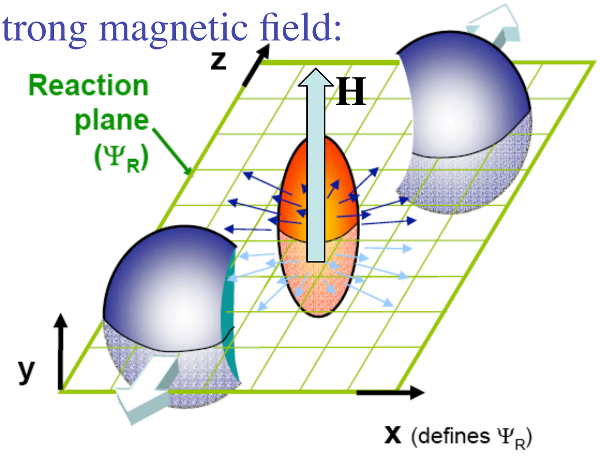
excess of negative charge

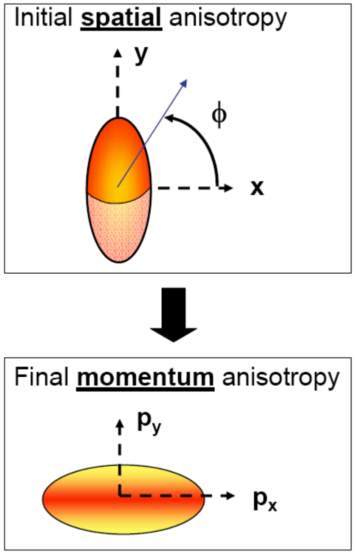
Electric dipole moment of QCD matter!

DK, Phys.Lett.B633(2006)260 [hep-ph/0406125]

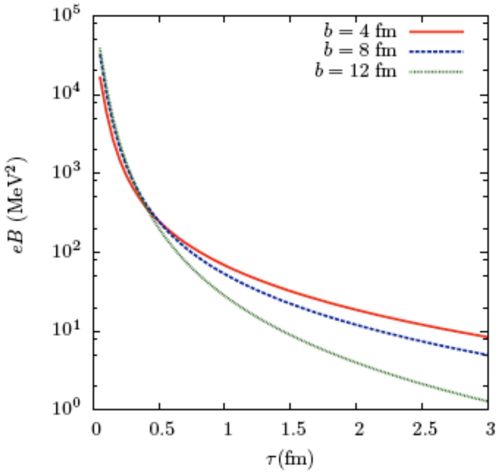
Is there a way to observe topological charge fluctuations in experiment?

Relativistic ions create a strong magnetic field:





Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory



In a conducting plasma, Faraday induction can make the field long-lived:

K.Tuchin, arXiv:1006.3051

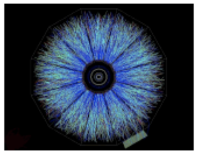
DK, McLerran, Warringa, Nucl Phys A803(2008)227

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

Comparison of magnetic fields

The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5 x 10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

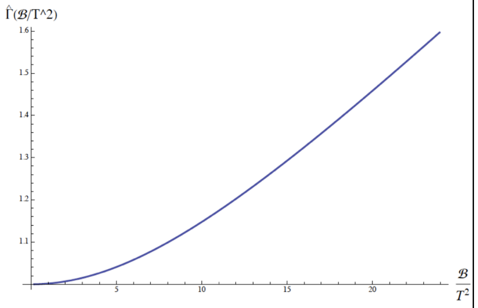
The Chern-Simons diffusion rate in an external magnetic field

strongly coupled N=4 SYM plasma in an external $U(1)_R$ magnetic field through holography

G. Basar, DK, arXiv:1202.2161

weak field:

$$\Gamma_{CS} = \frac{(g^2N)^2}{256\pi^3}T^4\left(1+\frac{1}{6\pi^4}\frac{\mathcal{B}^2}{T^4}+\mathcal{O}(\frac{\mathcal{B}^4}{T^8})\right)$$



strong field increases the rate:

$$\Gamma(\mathcal{B},T) = \frac{(g^2N)^2}{384\sqrt{3}\pi^5}\mathcal{B}T^2$$

dimensional reduction

Chiral Magnetic Effect in a chirally imbalanced plasma

Fukushima, DK, Warringa, PRD‘08

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of B, vector e.m. current is not conserved:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through

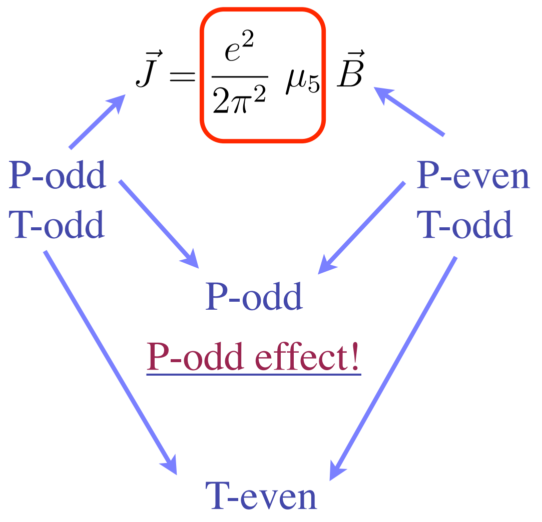
$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\boxed{\vec{J} = \frac{e^2}{2\pi^2}\, \mu_5\, \vec{B}}$$

Coefficient is fixed by the axial anomaly, no corrections

Chiral magnetic conductivity: discrete symmetries



$$\vec{J} = \boxed{\frac{e^2}{2\pi^2}\,\mu_5}\,\vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

P-odd effect!

T-even

Non-dissipative current!

(quantum computing etc)

cf Ohmic conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd,
dissipative

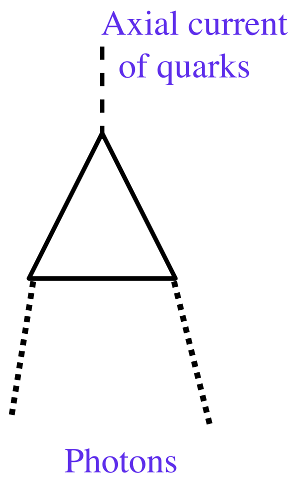
From QCD to electrodynamics: Maxwell-Chern-Simons theory

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4} P_\mu J^\mu_{CS}$$

$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu \theta = (\dot{\theta}, \vec{P})$$

$$\boxed{\begin{gathered}
\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + \underline{c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)}, \\
\vec{\nabla} \cdot \vec{E} = \rho + \underline{c\vec{P} \cdot \vec{B}}, \\
\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \\
\vec{\nabla} \cdot \vec{B} = 0,
\end{gathered}}$$

EM fields in QCD “aether”



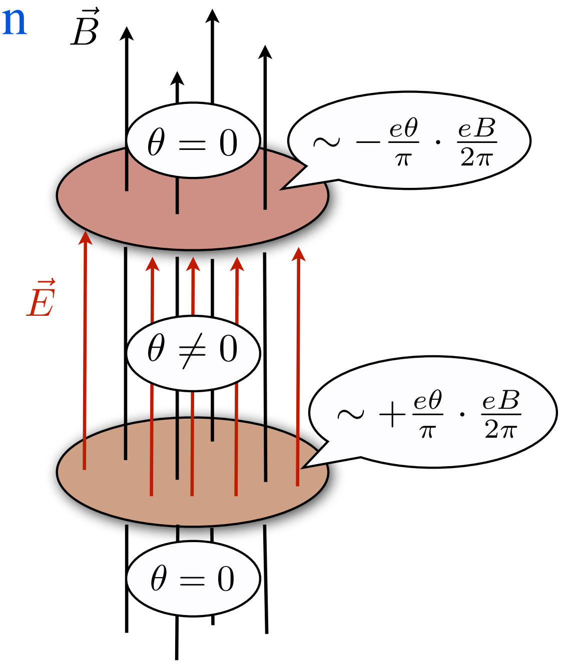
Annals Phys. 325 (2010) 205-218

The Chiral Magnetic Effect I:
Charge separation

$$\vec{\nabla}\cdot\vec{E} = \rho + c\vec{P}\cdot\vec{B}$$

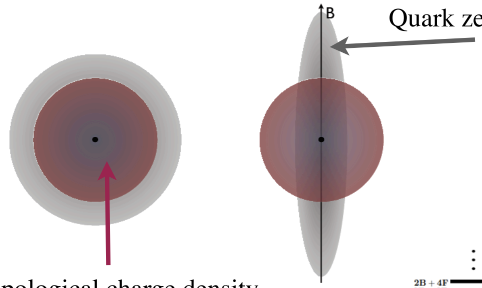
$$\vec{P} \equiv \vec{\nabla}\theta$$

$$\boxed{d_e = \sum_f q_f^2 \left(e\,\frac{\theta}{\pi}\right)\left(\frac{eB\cdot S}{2\pi}\right) L}$$



DK, Annals Phys. 325 (2010) 205-218
e-Print: **arXiv:0911.3715**

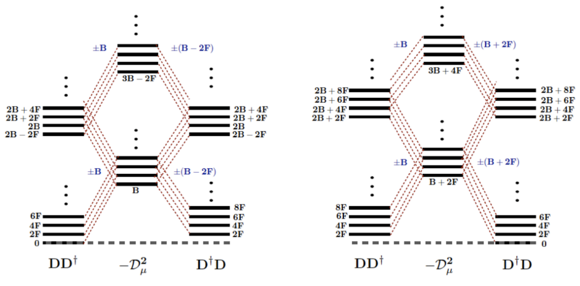
Electric dipole moment of QCD instanton in an external magnetic field



Quark zero mode density

Topological charge density

G. Basar, G. Dunne, DK,
arXiv:1112.0532 [hep-th]



$\tau_3 = +1$ $\tau_3 = -1$

Asymmetry between left and right modes induces the e.d.m. in an external B

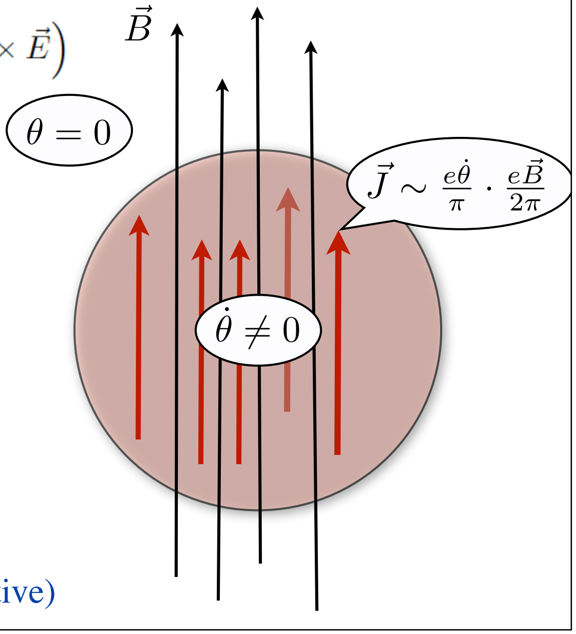
$$\sigma_3^E = -\bar{\psi}_L \sigma_3 \psi_L + \bar{\psi}_R \sigma_3 \psi_R$$

The chiral magnetic effect II: chiral induction

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)$$

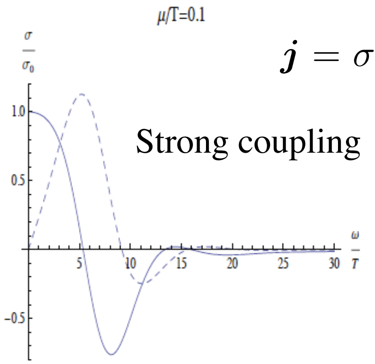
$$\boxed{\vec{J} = -\frac{e^2}{2\pi^2}\dot{\theta}\vec{B}}$$

T-even
(reversible,
non-dissipative)



Holographic chiral magnetic effect: the strong coupling regime (AdS/CFT)

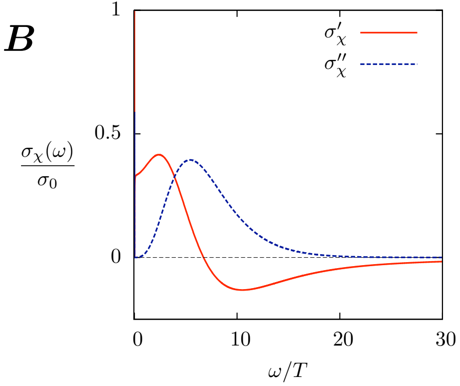
$$\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$$



Strong coupling

H.-U. Yee, arXiv:0908.4189,
JHEP 0911:085, 2009;
V. Rubakov, arXiv:1005.1888, ...

Weak coupling



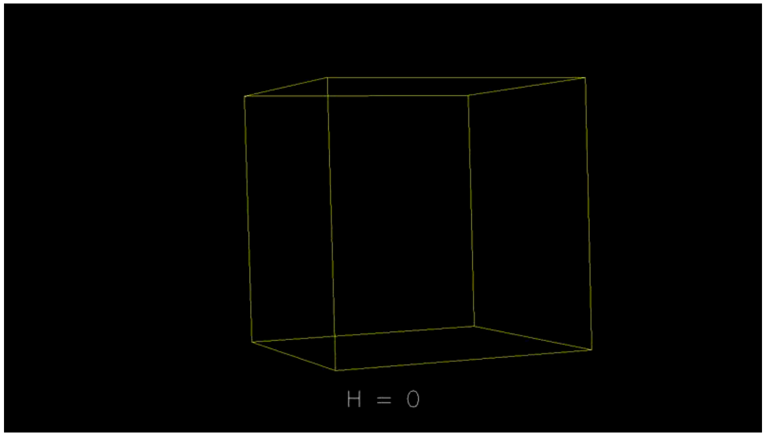
D.K., H. Warringa
Phys Rev D80 (2009) 034028

A. Rebhan et al, JHEP 0905, 084 (2009), G.Lifshytz, M.Lippert, arXiv:0904.4772;...
E. D' Hoker and P. Krauss, arXiv:0911.4518; A. Gorsky, P. Kopnin, A. Zayakin, arXiv:1003.2293,

CME persists at strong coupling - **hydrodynamical formulation?**

“Numerical evidence for chiral magnetic effect in lattice gauge theory”,

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov, ArXiv 0907.0494; PRD

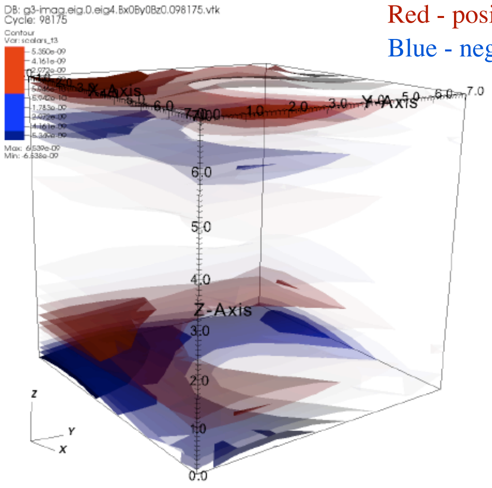


Red - positive charge
Blue - negative charge

SU(2) quenched, Q = 3; Electric charge density (H) - Electric charge density (H=0)

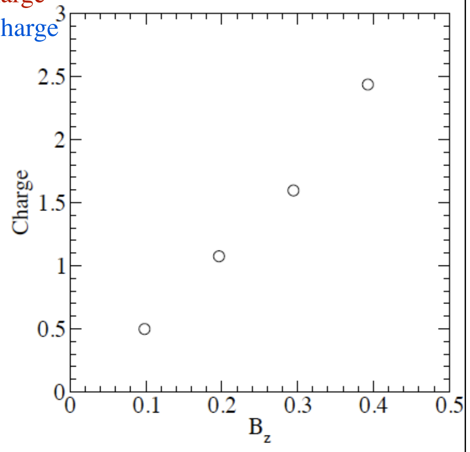
“Chiral magnetic effect in 2+1 flavor QCD+QED”,

M. Abramczyk, T. Blum, G. Petropoulos, R. Zhou, ArXiv 0911.1348;



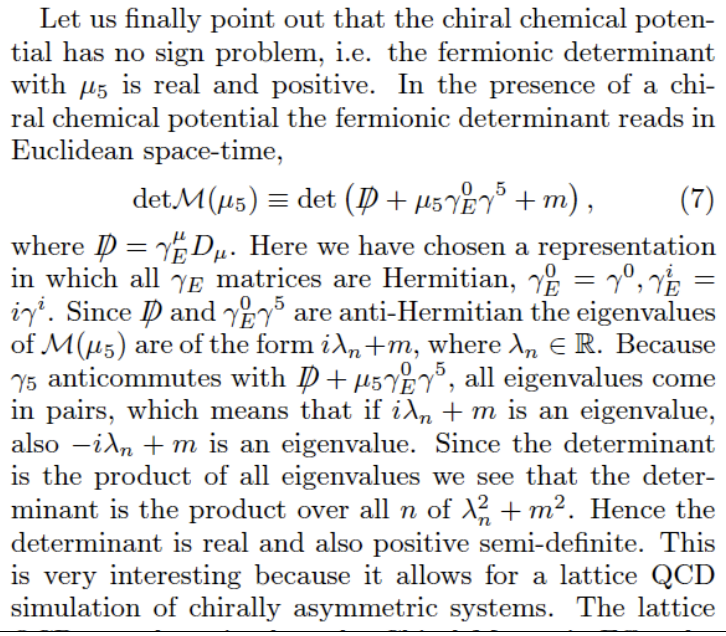
Red - positive charge

Blue - negative charge



2+1 flavor Domain Wall Fermions, fixed topological sectors, 16^3 x 8 lattice

No sign problem for the chiral chemical potential - direct lattice studies are possible



Let us finally point out that the chiral chemical potential has no sign problem, i.e. the fermionic determinant with μ_5 is real and positive. In the presence of a chiral chemical potential the fermionic determinant reads in Euclidean space-time,

$$\det\mathcal{M}(\mu_5) \equiv \det\left(\not{D} + \mu_5\gamma_E^0\gamma^5 + m\right), \qquad (7)$$

where $\not{D} = \gamma_E^\mu D_\mu$. Here we have chosen a representation in which all γ_E matrices are Hermitian, $\gamma_E^0 = \gamma^0, \gamma_E^i = i\gamma^i$. Since $\not{D}$ and $\gamma_E^0\gamma^5$ are anti-Hermitian the eigenvalues of $\mathcal{M}(\mu_5)$ are of the form $i\lambda_n + m$, where $\lambda_n \in \mathbb{R}$. Because γ_5 anticommutes with $\not{D} + \mu_5\gamma_E^0\gamma^5$, all eigenvalues come in pairs, which means that if $i\lambda_n + m$ is an eigenvalue, also $-i\lambda_n + m$ is an eigenvalue. Since the determinant is the product of all eigenvalues we see that the determinant is the product over all n of $\lambda_n^2 + m^2$. Hence the determinant is real and also positive semi-definite. This is very interesting because it allows for a lattice QCD simulation of chirally asymmetric systems. The lattice

Fukushima, DK,
Warringa, PRD‘08

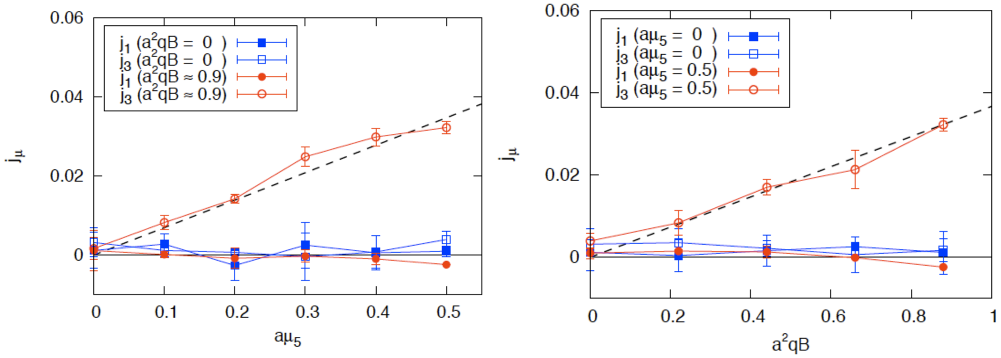
arXiv:1105.0385, PRL

Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

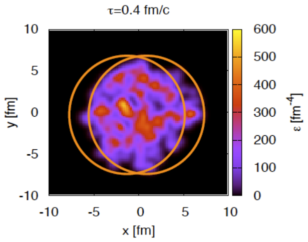
(Dated: May 3, 2011)

We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.

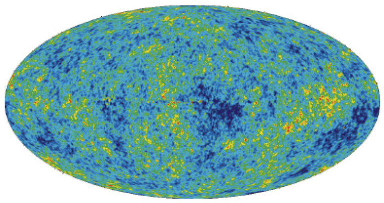


Hydrodynamics:
an effective low-energy
Theory **Of E**verything (**TOE**)

- Hydrodynamics states that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)



Little Bang



WMAP, Astrophys.J.Suppl.170:288,2007

Big Bang

The rise, fall and rebirth of hydrodynamical approach to hadronic matter

- Early applications (Fermi, Landau, Hagedorn, ...) were motivated by the idea of strong interactions among the constituents in dense matter; (positive beta functions (screening) in all known theories);

- Asymptotic freedom (anti-screening), Bjorken scaling, partons

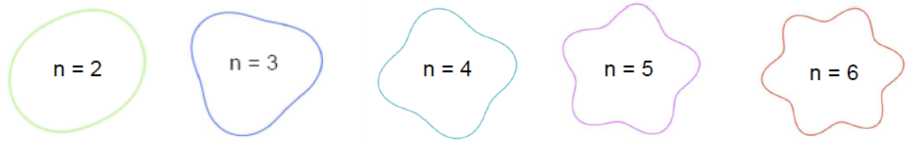
↓

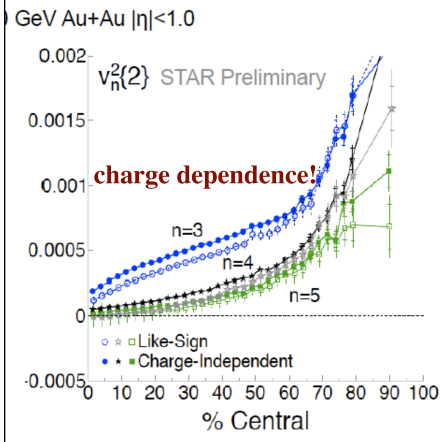
quasi-ideal quark-gluon plasma, applicability of hydrodynamics questioned

- RHIC, LHC: **strongly coupled plasma**; new theoretical tool: holography

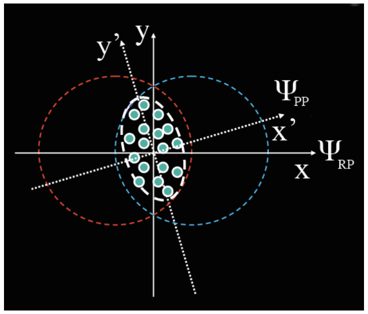
Hydrodynamics

$$N_{pairs} \propto 1 + 2v_1^2 \cos\Delta\varphi + 2v_2^2 \cos 2\Delta\varphi + 2v_3^2 \cos 3\Delta\varphi + 2v_4^2 \cos 4\Delta\varphi + \ldots$$



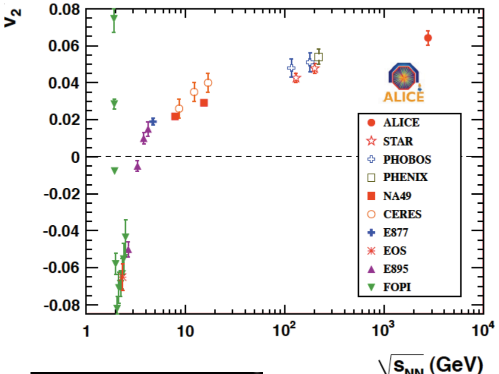


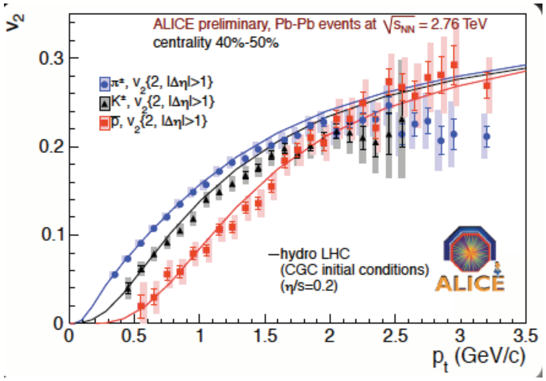
charge dependence!

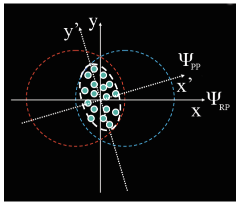


P. Sorensen
[STAR]
QM 2011

The remarkable success of hydrodynamics at RHIC and LHC





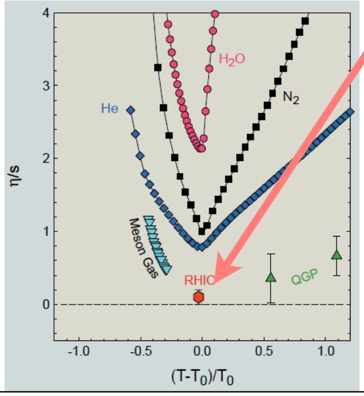


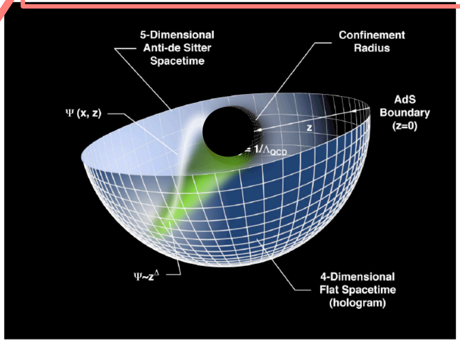
R. Snellings [ALICE Coll.] Talk at QM2011

Quantifying the transport properties of QCD matter

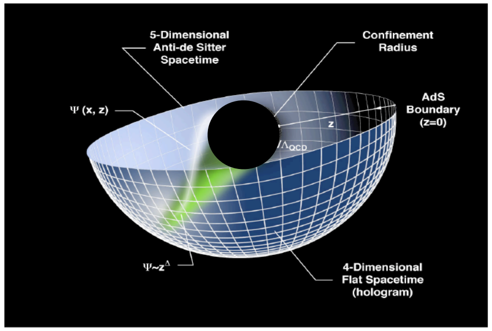
- Hydrodynamics:
  an effective low-energy theory, expansion in the ratio of thermal length 1/T to the typical variation scale L, $\epsilon \equiv \frac{1}{LT}$
- Each term in this derivative expansion is multiplied by an appropriate transport coefficient

very small shear viscosity - “perfect liquid”; strong coupling





Low-energy effective ToE: hydrodynamics



Holographic view:

Particle contents of supergravity: **gravitons, dilatons, axions**

= fields on the boundary

Caveman's view:	AdS$_5$ "Reality":
Shear viscosity	Graviton propagation
Bulk viscosity Deviation from conformal symmetry	Dilaton propagation
Rate of topological transitions	Axion propagation

Hydrodynamics and anomalies

- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

Chiral MagnetoHydroDynamics (CMHD) - relativistic hydrodynamics with triangle anomalies and external electromagnetic fields

First order (in the derivative expansion) formulation:
D. Son and P. Surowka, arXiv:0906.5044

Constraining the new anomalous transport coefficients:
positivity of the entropy production rate, $\partial_\mu s^\mu \geq 0$

$$\nu^\mu = -\sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi\omega^\mu + \xi_B B^\mu,$$

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu,$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right), \quad \xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right).$$

CME
(for chirally
imbalanced
matter)

Chiral MagnetoHydroDynamics (CMHD) - relativistic hydrodynamics with triangle anomalies and external electromagnetic fields

First order hydrodynamics has problems with causality and is numerically unstable, so second order formulation is necessary;

Complete second order formulation of CMHD:
DK and H.-U. Yee, 1105.6360; Phys Rev D

Many new transport coefficients - use conformal/Weyl invariance;
still 18 independent transport coefficients related to the anomaly.
15 that are specific to 2nd order:

new

$$\begin{aligned}
&\sigma^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \omega^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\sigma_{\nu\alpha}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\omega_{\nu\alpha}\ ,\ \sigma^{\mu\nu}\omega_\nu\ ,\\
&\sigma^{\mu\nu}E_\nu\ ,\ \sigma^{\mu\nu}B_\nu\ ,\ \omega^{\mu\nu}E_\nu\ ,\ \omega^{\mu\nu}B_\nu\ ,u^\nu\mathcal{D}_\nu E^\mu\ , \qquad (2.60)\\
&\epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha B_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha E_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha B_\beta\ .
\end{aligned}$$

Many new anomaly-induced phenomena!

Chiral MagnetoHydroDynamics (CMHD) - relativistic hydrodynamics with triangle anomalies and external electromagnetic fields

DK and H.-U. Yee, 1105.6360

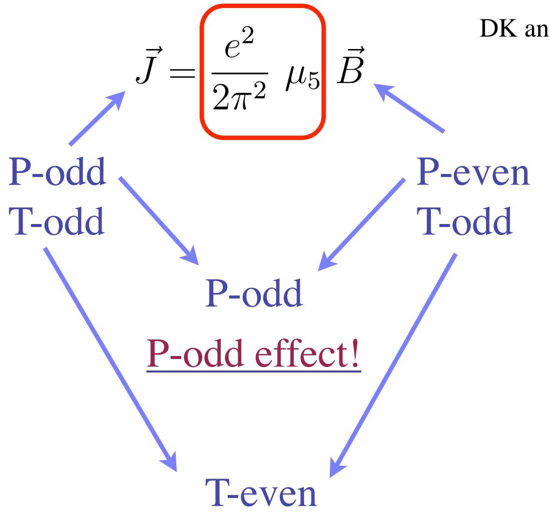
Positivity of entropy production - still too many unconstrained transport coefficients...

$$\begin{aligned} T\mathcal{D}_\mu s^\mu &= 2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \sigma\left(T\Delta^{\mu\nu}\mathcal{D}_\nu\bar{\mu} - E^\mu\right)\left(T\Delta_{\mu\alpha}\mathcal{D}^\alpha\bar{\mu} - E_\mu\right) \\ &+ \left(-T\xi\mathcal{D}_\mu\bar{\mu} + T\mathcal{D}_\mu D + \left(\frac{\xi}{n} - \frac{2TD_B}{n}\right)\mathcal{D}_\mu p\right)\omega^\mu \\ &+ \left(-T\xi_B\mathcal{D}_\mu\bar{\mu} + T\mathcal{D}_\mu D_B + \left(\frac{\xi_B}{n} - \kappa\frac{\mu}{n}\right)\mathcal{D}_\mu p\right)B^\mu \\ &- \sigma_{\mu\nu}\tau^{\mu\nu}_{(2)} - \left(T\mathcal{D}_\mu\bar{\mu} - E_\mu\right)\nu^\mu_{(2)} + T\mathcal{D}_\mu s^\mu_{(2)} + \cdots \\ &+ \frac{1}{n}\left(-F^{\mu\alpha}\nu_{\alpha(1)} + \mathcal{D}_\alpha\tau^{\alpha\mu}_{(1)}\right)\left((\xi - 2TD_B)\omega_\mu + (\xi_B - \kappa\mu)B_\mu\right) \quad . \end{aligned}$$

Is there another guiding principle?

No entropy production from T-even anomalous terms

DK and H.-U. Yee, 1105.6360



$$\vec{J} = \boxed{\frac{e^2}{2\pi^2}\ \mu_5}\ \vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

P-odd effect!

T-even

Non-dissipative current!

(time-reversible - no arrow of time, no entropy production)

cf Ohmic conductivity:

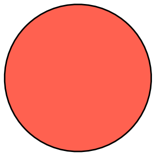
$$\vec{J} = \sigma \vec{E}$$

T-odd, dissipative

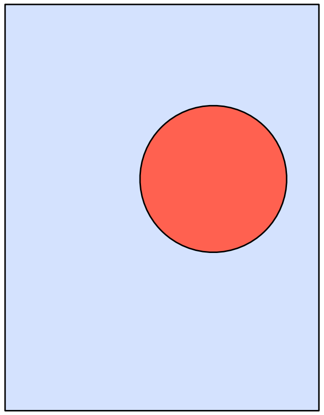
No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360

Entropy grows



$$\partial_\mu s^\mu \geq 0$$



Mirror reflection: entropy decreases ?

$$\partial_\mu s^\mu \leq 0$$

Decrease is ruled out by 2nd law of thermodynamics

↓

$$\partial_\mu s^\mu = 0$$

No entropy production from T-even anomalous terms

1st order hydro: Son-Surowka results are reproduced

2nd order hydro: 13 out of 18 transport coefficients
are computed;
but is the “guiding principle” correct?

DK and H.-U. Yee, 1105.6360

Can we check the resulting relations between the transport coefficients?

e.g.

$$\begin{aligned} \bar{\lambda}_1 &= \frac{2\bar{\eta}}{\bar{n}}\left(\bar{\xi} - 2\bar{D}_B\right) \quad , \\ \bar{\lambda}_2 + \bar{\xi}_1 &= \left(\frac{2\bar{\eta}}{\bar{n}}\left(\bar{\xi} - 2\bar{D}_B\right)\right)' + \left(\frac{\bar{\eta}}{\bar{p}} - \frac{2\bar{\eta}'}{\bar{n}}\right)\left(\bar{\xi} - 2\bar{D}_B\right) \end{aligned}$$

The fluid/gravity correspondence

Long history:

Hawking, Bekenstein, Unruh;
Damour ’78;
Thorne, Price, MacDonald ’86 (membrane paradigm)

Recent developments motivated by AdS/CFT:

Policastro, Kovtun, Son, Starinets ’01 (quantum bound)
Bhattacharya, Hubeny, Minwalla, Rangamani ’08
(fluid/gravity correspondence)

Some of the transport coefficients of 2nd order hydro computed;
enough to check some of our relations, e.g.

J. Erdmenger et al, 0809.2488;
N. Banerjee et al, 0809.2596

$$\bar\lambda_1 = \frac{2\bar\eta}{\bar n}\left(\bar\xi - 2\bar D_B\right) \quad ,$$
$$\bar\lambda_2 + \bar\xi_1 = \left(\frac{2\bar\eta}{\bar n}\left(\bar\xi - 2\bar D_B\right)\right)' + \left(\frac{\bar\eta}{\bar p} - \frac{2\bar\eta'}{\bar n}\right)\left(\bar\xi - 2\bar D_B\right)$$

It works

Other holographic
checks work as well:

DK and H.-U. Yee, 1105.6360

The CME in relativistic hydrodynamics: The Chiral Magnetic Wave

DK, H.-U. Yee, arXiv:1012.6026 [hep-th]; PRD

$$\vec{j}_V = \frac{N_c\, e}{2\pi^2}\mu_A \vec{B}; \quad \vec{j}_A = \frac{N_c\, e}{2\pi^2}\mu_V \vec{B},$$

CME Chiral separation

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c\, e\vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry is restored)

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2}\partial_1 - D_L \partial_1^2\right) j^0_{L,R} = 0$$

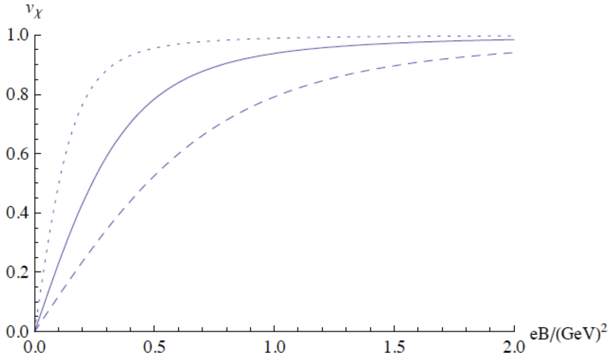
Gapless collective mode is the carrier of CME current in MHD:

$$\omega = \mp v_\chi k - i D_L k^2 + \cdots$$

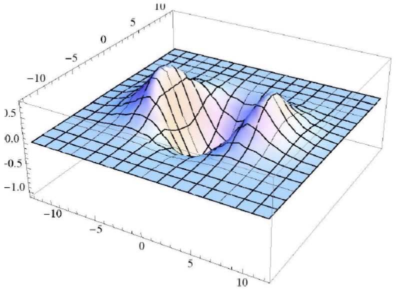
The Chiral Magnetic Wave

The velocity of CMW computed in Sakai-Sugimoto model (holographic QCD)

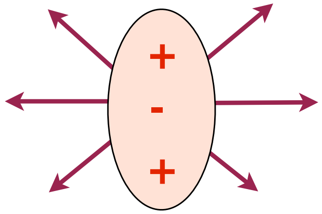
In strong magnetic field, CMW propagates with the speed of light!



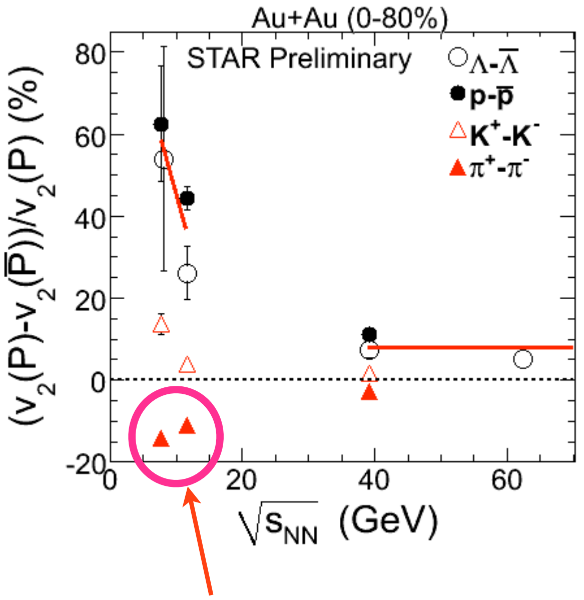
DK, H.-U. Yee,
arXiv:1012.6026 [hep-th]



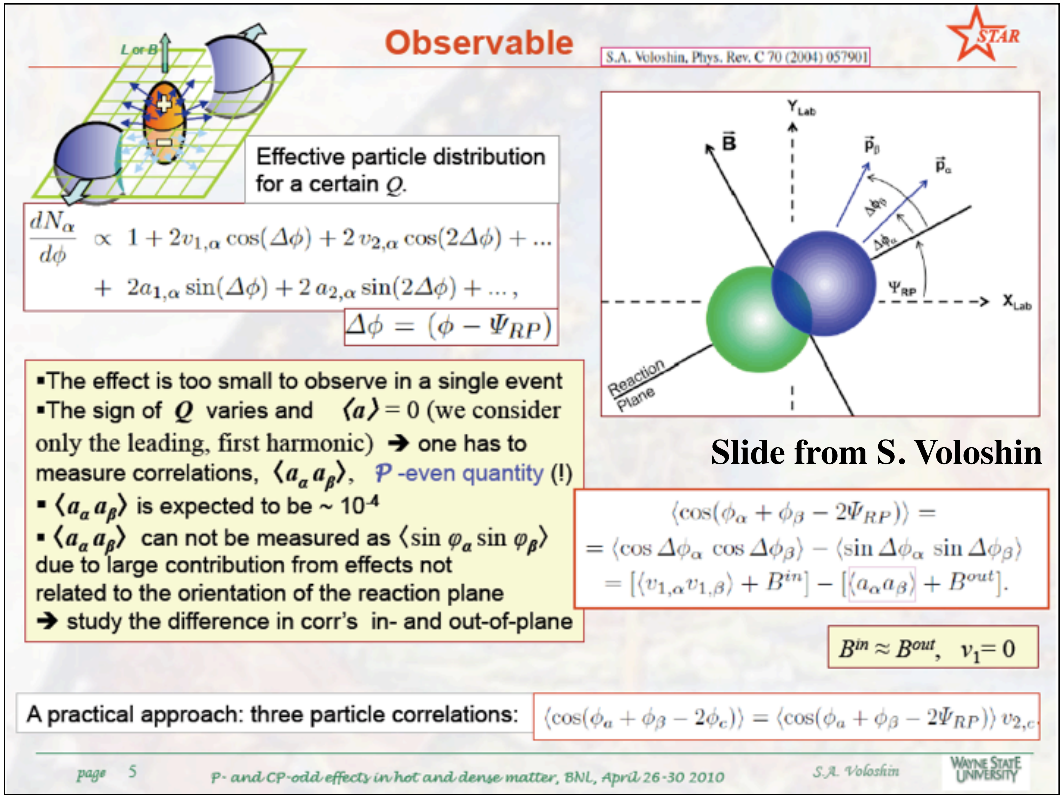
Y.Burnier, DK, J. Liao, H.-U.Yee,
arXiv:1103.1307 - PRL



Anomaly-induced
quadrupole moment
at finite baryon
density



Chiral magnetic wave or
a mundane effect (Coulomb, resonances)?



Observable

S.A. Voloshin, Phys. Rev. C 70 (2004) 057901

Effective particle distribution for a certain Q.

$$\frac{dN_\alpha}{d\phi} \propto 1 + 2v_{1,\alpha}\cos(\Delta\phi) + 2\,v_{2,\alpha}\cos(2\Delta\phi) + \ldots$$
$$+\ 2a_{1,\alpha}\sin(\Delta\phi) + 2\,a_{2,\alpha}\sin(2\Delta\phi) + \ldots,$$

$$\Delta\phi = (\phi - \Psi_{RP})$$

- The effect is too small to observe in a single event
- The sign of Q varies and $\langle a \rangle = 0$ (we consider only the leading, first harmonic) ➔ one has to measure correlations, $\langle a_\alpha a_\beta \rangle$, $\mathcal{P}$ -even quantity (!)
- $\langle a_\alpha a_\beta \rangle$ is expected to be ~ 10^{-4}
- $\langle a_\alpha a_\beta \rangle$ can not be measured as $\langle \sin\varphi_\alpha \sin\varphi_\beta \rangle$ due to large contribution from effects not related to the orientation of the reaction plane

➔ study the difference in corr's in- and out-of-plane

Slide from S. Voloshin

$$\begin{aligned}&\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP})\rangle = \\ &= \langle\cos\Delta\phi_\alpha\ \cos\Delta\phi_\beta\rangle - \langle\sin\Delta\phi_\alpha\ \sin\Delta\phi_\beta\rangle \\ &= [\langle v_{1,\alpha}v_{1,\beta}\rangle + B^{in}] - [\boxed{\langle a_\alpha a_\beta\rangle} + B^{out}].\end{aligned}$$

$$B^{in} \approx B^{out}, \quad v_1 = 0$$

A practical approach: three particle correlations: $\langle\cos(\phi_a + \phi_\beta - 2\phi_c)\rangle = \langle\cos(\phi_a + \phi_\beta - 2\Psi_{RP})\rangle\, v_{2,c}$

page 5 P- and CP-odd effects in hot and dense matter, BNL, April 26-30 2010 S.A. Voloshin

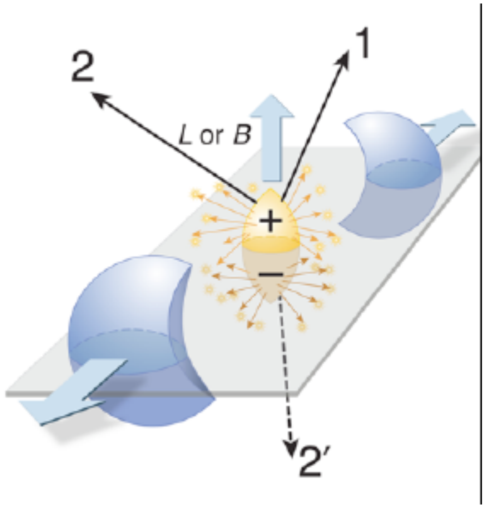
WAYNE STATE UNIVERSITY

PRL **103,** 251601 (2009)

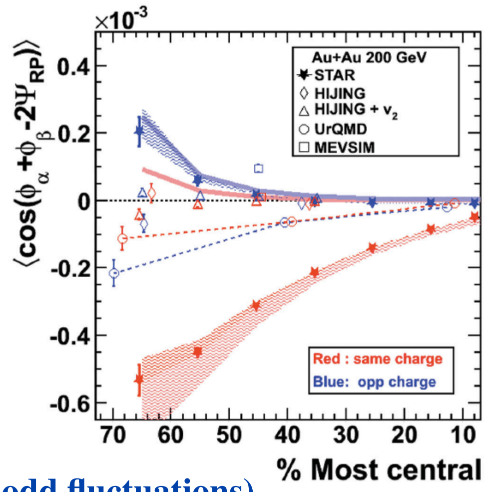
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

week ending
18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

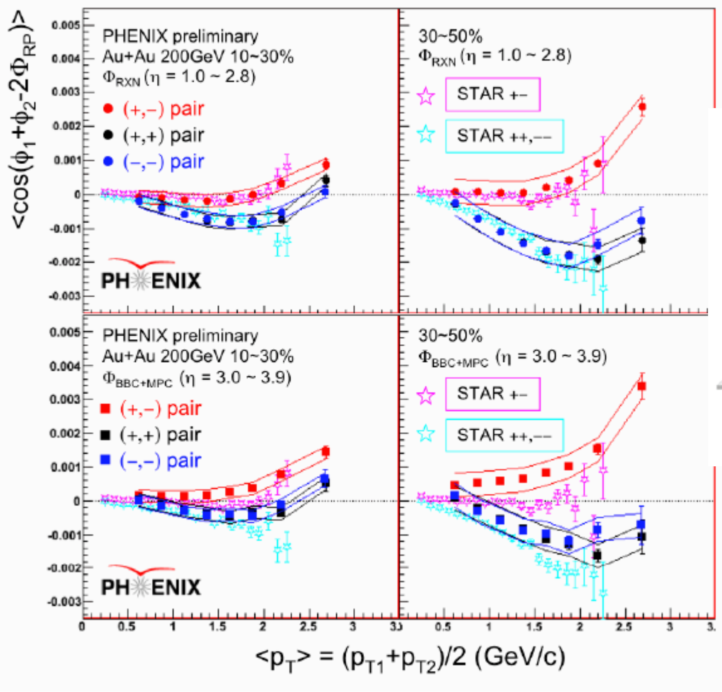


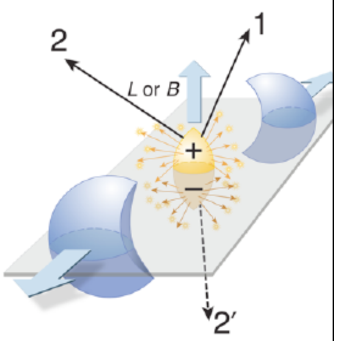
(STAR Collaboration)



NB: P-even quantity (strength of P-odd fluctuations)

S.Esumi et al
[PHENIX Coll]
April 2010





Relatively good agreement between PHENIX & STAR

The New York Times

In Brookhaven Collider, Scientists Briefly Break a Law of Nature

By DENNIS OVERBYE
Published: February 15, 2010

Physicists said Monday that they had whacked a tiny region of space with enough energy to briefly distort the laws of physics providing the first laboratory demonstration of the kind of process that scientists suspect has shaped cosmic history.

Newsweek

Quark Soup

Physicists create conditions not seen since the big bang.

Feb 16, 2010

Sharon Begley

NewScientist

Atom smasher shows vacuum of space in a twist

17:27 15 February 2010 by Rachel Courtland

USA TODAY

Scientists re-create high temperatures from Big Bang

REUTERS

Hottest Temperature Ever Heads Science to Big Bang

Are the observed fluctuations of charge asymmetries a convincing evidence for the local parity violation?

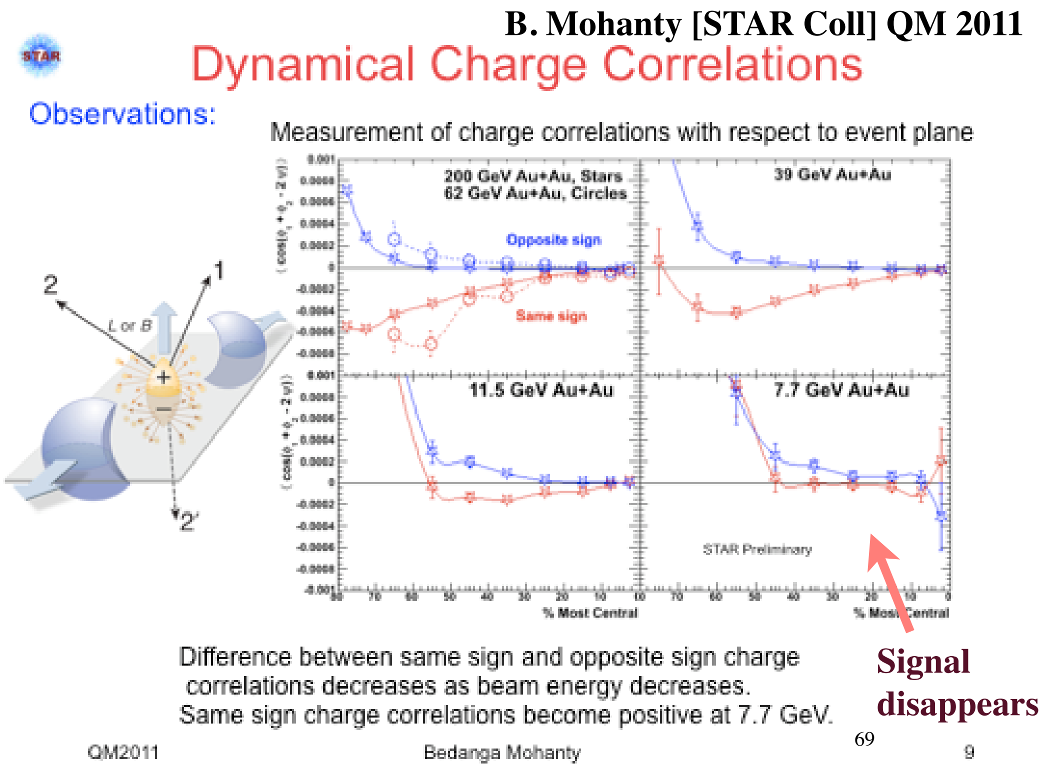
A number of open questions that still have to be clarified:

in-plane vs out-of-plane, new observables?	e.g. A. Bzdak, V. Koch, J. Liao, arXiv:0912.5050; 1005.5380; ...
physics “backgrounds”	e.g. M. Asakawa, A. Majumder, B. Muller, arXiv:1003.2436 S. Pratt and S. Schlichting, arXiv:1005.5341 F. Wang, arXiv: 0911.1482; ...

Fortunately, a number of analytical and numerical (lattice)
tools are available to theorists,
and the new data (low energy, PID asymmetries, U-U)
will hopefully come - **this question can be answered!**

B. Mohanty [STAR Coll] QM 2011

Dynamical Charge Correlations



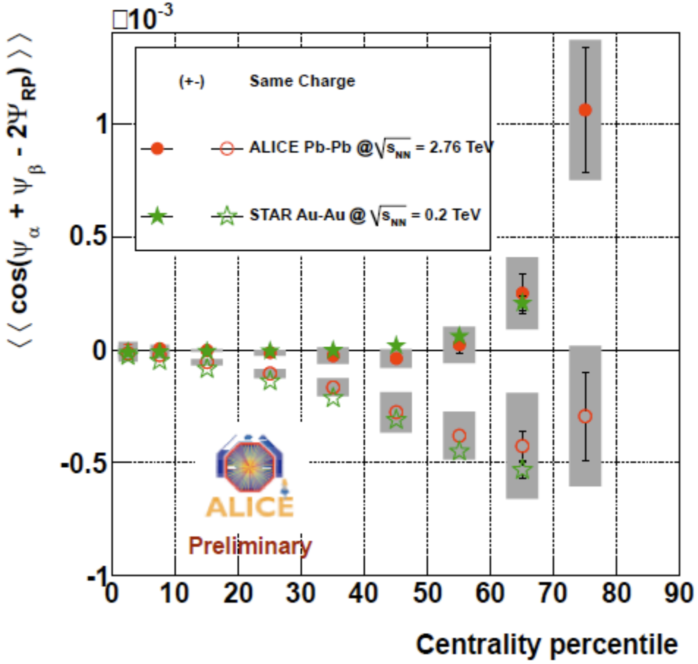
Observations:

Measurement of charge correlations with respect to event plane

Difference between same sign and opposite sign charge correlations decreases as beam energy decreases.
Same sign charge correlations become positive at 7.7 GeV.

Signal disappears

CME studies at the LHC



P. Cristakoglou, J. Schukraft [ALICE Coll] Talks at QM 2011

A new test: baryon asymmetry

DK, D.T.Son
arXiv:1010.0038; PRL

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2}\left[\mathrm{tr}(VAQ)\vec{B} + \mathrm{tr}(VAB)2\mu\vec{\omega}\right]$$

CME

Vorticity-induced
“Chiral Vortical Effect”

$$J_E^{CME} \sim \frac{2}{3} \;\; (N_f = 3) \;\; \text{or} \;\; \frac{5}{9} \;\; (N_f = 2)$$

$$J_B^{CME} = 0 \;\; (N_f = 3) \;\; \text{or} \;\; \sim \frac{1}{9} \;\; (N_f = 2).$$

$$J_E^{CVE} = 0 \;\; (N_f = 3) \;\; \text{or} \;\; \sim \frac{1}{3} \;\; (N_f = 2);$$

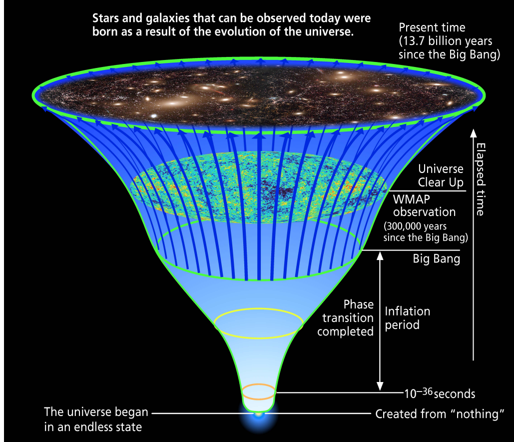
CME:
(almost) only
electric charge

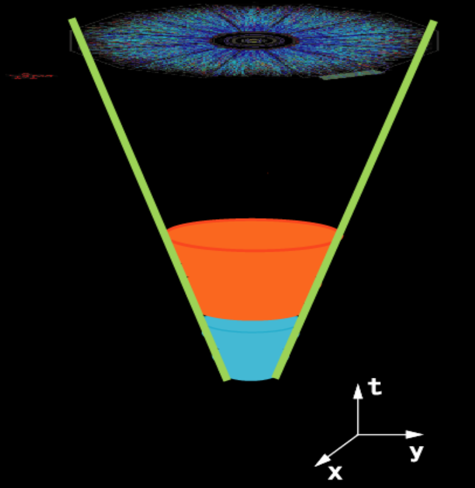
CVE:
(almost) only
baryon charge

$$J_B^{CVE} \sim 1 \;\; (N_f = 3) \;\; \text{or} \;\; \sim \frac{2}{3} \;\; (N_f = 2).$$

There has to be a positive correlation between
electric charge and baryon number! mixed correlators - e.g. $\Lambda \;\; \pi^+$

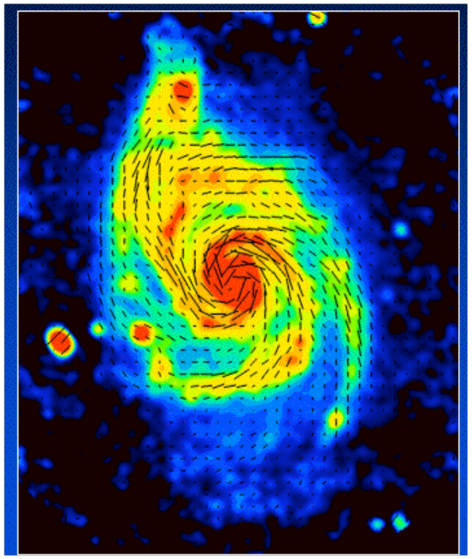
What are the implications for the Early Universe?





T.Hatsuda

What is the origin of cosmic magnetic fields?



Magnetic fields are abundant in the Universe at large scales:

3 μG field in Milky Way;

1-40 μG fields in clusters of galaxies

Is the CMB polarized?

Magnetic field in M51:
Polarization of emission

Beck 2000

What is the origin of magnetic fields in the Universe?

Primordial magnetic field (E.Fermi, 1949)?

Dynamo in proto-galaxy? Stars? Galaxy?

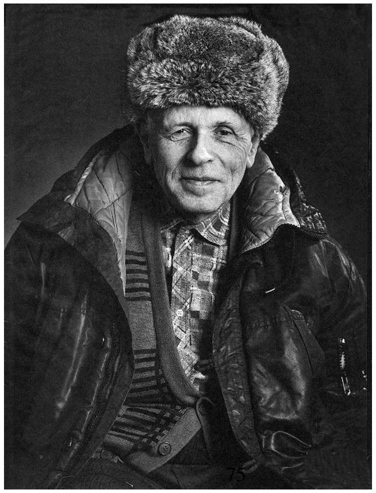
Coupling of chromo-magnetic and magnetic fields; axions; instability in Maxwell-Chern-Simons theory, ...=>

Primordial magnetic helicity generation at the QCD phase transition?

What is the origin of the matter-antimatter asymmetry in the Universe?

A.D. Sakharov,
JETP Lett. 5 (1967) 24

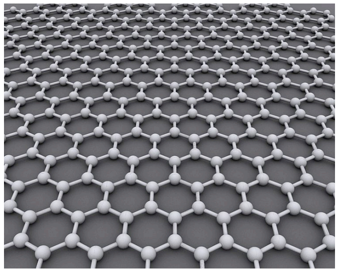
1. B violation
2. CP violation
3. Non-equilibrium dynamics



Can CP violation in the Big Bang
be a dynamical fluctuation, similar
to what happens in heavy ion collisons?

Chiral fermions and topology in condensed matter systems

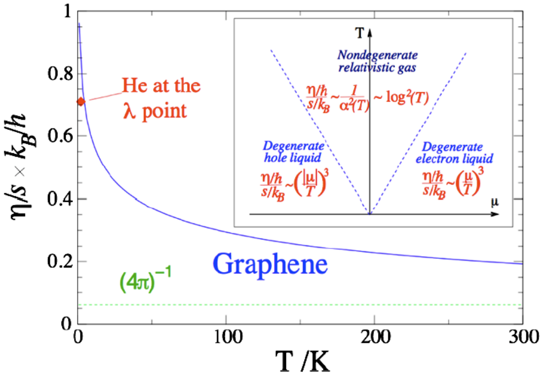
Novel appplication: graphene



Similar to QGP in
several ways:
strongly coupled, perfect liquid
behavior, chiral fermions, ..

Massless (2+1) fermions

M.Muller, J.Schmalian, L. Fritz,
PRL **103,** 025301 (2009)



Magnetized graphene: e.g. I.Aleiner, DK, A. Tsvelik, Phys Rev B '07;
M.Khodas, I.Zaliznyak, DK, Phys Rev B'09

Summary

Interplay of topology, anomalies and magnetic field
leads to the Chiral Magnetic Effect;
confirmed by lattice QCD x QED,
evidence from RHIC and LHC

CME and related anomaly-induced phenomena
are an integral part of relativistic hydrodynamics
(Chiral MagnetoHydroDynamics)