When you see a title like that, you need to worry: "Uh-oh, sounds philosophical." *Well*, I just wanted to tell you two concrete stories about cases when my colleagues and I managed to do something useful by virtue of knowing something about inference. The ideas we needed were things I didn't know a few years ago, so I thought you might be interested too.

Inference in biological physics

Phil Nelson University of Pennsylvania

For these slides see:



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There's more, of course, but t

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In classical logic it's fairly easy to spot errors of inference.

An everyday question in clinical practice

To diagnose colorectal cancer, the hemoccult test—among others—is conducted to detect occult blood in the stool. This test is used from a particular age on, but also in routine screening for early detection of colorectal cancer. Imagine you conduct a screening using the hemoccult test in a certain region. For symptom-free people over 50 years old who participate in screening using the hemoccult test, the following information is available for this region:

The probability that one of these people has colorectal cancer is 0.3 percent. If a person has colorectal cancer, the probability is 50 percent that he will have a positive hemoccult test. If a person does not have colorectal cancer, the probability is 3 percent that he will still have a positive hemoccult test. Imagine a person (over age 50, no symptoms) who has a positive hemoccult test in your screening. What is the probability that this person actually has colorectal cancer? _____ percent

G. Gigerenzer, Calculated risks

Thursday, March 15, 2012

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A=Sick, –	C=Healthy, –
B=Sick, +	D=Healthy, +

$$P(\text{sick}|+) = P(+|\text{sick}) \times \frac{P(\text{sick})}{P(+)} \xrightarrow{A=\text{Sick},-} C=\text{Healthy},-$$
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Still need this

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Thursday, March 15, 2012

Finish working it out

$$P(\operatorname{sick}|+) = P(+|\operatorname{sick}) \times \underbrace{\frac{P(\operatorname{sick})}{P(+)}}_{P(+)} \quad A=\operatorname{Sick}, -$$
Is that last factor a big deal?
$$P(\operatorname{sick}) \text{ was given, but we need:} \qquad B=\operatorname{Sick}, +$$

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Finish working it out

$$P(\operatorname{sick}|+) = P(+|\operatorname{sick}) \times$$

$$\frac{P(\text{sick})}{P(+)} \quad \text{A=Sick, -}$$

B=Sick, +

Is that last factor a big deal? P(sick) was given, but we need: P(+) = B + D

D=Healthy, +

C=Healthy, -

 $= \frac{B}{A+B}(A+B) + \frac{D}{C+D}(C+D)$ = P(+|sick)P(sick) + P(+|healthy)P(healthy)= $(0.5)(0.003) + (0.03)(0.997) \approx 0.03$

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 $\frac{P(\text{sick})}{P(+)} \approx \frac{0.003}{0.03} \approx 0.1$ It's huge: a positive test result means only a 5% chance you're sick. *Not* 97%.

Part II: Changepoint analysis in singlemolecule TIRF

JF Beausang, Yale Goldman, PN

* Sometimes our model is not obviously connected with what we can actually measure experimentally, but and we need to makes a connection.

*Sometimes the model that interests us involves the behavior of actors that we can only see indirectly in our data; theory may be needed to separate them out from each other, and from noise.



Many thanks to Haw Yang. See also Lucas P. Watkins and Haw Yang *J. Phys. Chem. B* **2005**

We'd like to know things like: How does it walk? What are the steps in the kinetic pathway? What is the geometry of each state? One classic approach is to monitor the position in space of a marker (e.g. a bead) attached to the motor. But this does not address the geometry of each state.



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The approach I'll discuss involves attaching a bifunctional fluorescent label to one lever arm. The label has a dipole moment whose *orientation* in space reflects that of the arm.







Thursday, March 15, 2012



Polarized total internal reflection fluorescence microscopy



Fluorescence illumination by the evanescent wave eliminates a lot of noise, and importantly, maintains the polarization of the incident light. To tickle the fluorophore with every possible polarization, we need the incoming light to have at least two different beam directions.

pol-TIRF setup



pol-TIRF setup



8 polarized illuminations x 2 detectors = 16 fluorescent intensities per cycle

Thursday, March 15, 2012
Current state of the art



It's a bit more meaningful to convert from lab-frame angles θ , ϕ to actin-frame angles α , β . Even then, however, state of the art calculations give pretty noisy determinations, with pretty poor time resolution.

You could easily miss a short-lived state -- e.g. the elusive diffusive-search step (if it exists). Can't we do better? *IN Forkey et al. Nature 2003*

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Can we evade the cruel logic of photon statistics?

It turns out that *binning the data destroyed some information.* Something magical happens if instead of binning, we just we plot photon arrival time versus photon sequence number. Despite some ripples from Poisson statistics, it's obvious that each trace has a sharp changepoint, and moreover that the two changepoints found independently in this way are simultaneous. (A similar approach in the context of FRET was pioneered by Haw Yang.) *IF Beausang, YE Goldman, and*

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- Why did that trick work? How did we get such great time resolution from such cruddy data?
- *How well does it work?* If we have even fewer photons, for example because a state is short-lived, how can we quantify our confidence that any changepoint occurred at all?
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$$P \approx N \log(\Delta t) + n \log R + n' \log R' - \left(\frac{t_*}{\Delta t} - n\right) \left(R\Delta t\right) - \left(\frac{T - t_*}{\Delta T} - 1 - (N - n)\right) \left(R'\Delta t\right)$$
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$$R = n/t_*$$
, $R' = n'/(T - t_*)$

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More interestingly, we can substitute these optimal rates into the formula for *P* to find the likelihood as a function of putative changepoint:

Here's some very fake data; the photons arrive uniformly, not at random.

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Here are two lines corresponding to non-optimal choices of the changepoint. We'd like to see the likelihood function and how it selects the "right" changepoint, which for fake data is known.

Left: Some more realistic (Poisson-arrival) simulated data, shown in traditional binned form and in the improved version.

Right: Likelihood function for placement of the changepoint. Dashed line, maximum-likelihood point. Black triangle: Actual changepoint used in the simulation.

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Oh, yes -- the method also works on multiple-channel data. *Left:* one channel (red) starts with rare photons, then jumps to higher intensity. Another channel (blue) does the opposite. The sum of the intensities (black) doesn't change much at all.

Middle: "kink" representations of the same data. *Right:* both channels contribute to a likelihood function with a robust peak, even though there were only a total of just 200 photons in the entire dataset.

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Payoff

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Payoff

Oh, yes--it also works on real experimental data.

Now we can get back to the original motivation. Previously, people would take data from multiple polarizations, bin it, and pipe the inferred intensities into a maximum-likelihood estimator of the orientation of the fluorophore. That procedure leads to the rather noisy dots shown here.

One problem is that if a transition happens in the middle of a time bin, then the inferred orientation in that time bin can be crazy.

Here the solid lines are the inferred orientations of the probe molecule during successive states defined by changepoint analysis. We see a nice alternating stride in ϕ .

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Summary Part II

*When you only get a million photons, you'd better make every photon count.

*A simple maximum-likelihood analysis accomplishes this.

*In the context of TIRF it can dramatically improve the tradeoff between time resolution and accuracy.

Part III: Parallel recordings from dozens of individual neurons

- *Sometimes suggests a new kind of measurement that tests a model more stringently, or distinguishes two different models more completely, than previous measurements.
- *Sometimes our model is not obviously connected with what we can actually measure experimentally, and we need to make a connection.
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- *Sometimes suggests a new kind of measurement that tests a model more stringently, or distinguishes two different models more completely, than previous measurements.
- *Sometimes our model is not obviously connected with what we can actually measure experimentally, and we need to make a connection.
- ★Sometimes the model that interests us involves the behavior of actors that we can only see indirectly in our data; theory may be needed to separate them out from each other, and from noise.

Sources of energy

Experiments done in the lab of Vijay Balasubramanian (Penn).

Sources of energy

Experiments done in the lab of Vijay Balasubramanian (Penn).

Jason Prentice, Penn Physics

Jan Homann, Penn Physics

(plus Gasper Tkacik.)

(Many thanks to Michael Berry and Olivier Marre, Princeton; Bart Borghuis, Janelia Farms; Michael Freed and others at Penn Retina Lab; Joerg Sander, U Alberta; Ronen Segev, BGU, Chris Wiggins, Columbia.)

Really big picture

Retina is also an approachable, yet still complex, part of the brain. It's a 2D carpet consisting of "only" three layers of neurons.

Optics Visual scene in Visual Visual Scene in Visual V

Really big picture

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Optics Retinal Visual scene in Retinal illumination pattern Retinal ganglion cell spike trains
It matters



It matters



Summary, Part III





Cf Meister, Pine, and Baylor 1994. Incredibly, one can keep a mammalian retina alive in a dish for over 6 hours while presenting it stimuli and recording its activity.

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What's in the dish



Michael Berry, Princeton



67 ms of data, viewed as a movie. [data have been smoothed]

Classic: Gerstein+Clark 1964; Abeles+Goldstein 1977; Schmidt 1984.



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Some spikes move across the array:



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Mostly we are hearing retinal ganglion cells, as desired, because they're the ones that spike.

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Some spikes move across the array:

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The spike-sorting problem is: Given raw data like these, convert to a list of discrete events (which cells fired at what times).

Classic: Gerstein+Clark 1964; Abeles+Goldstein 1977; Schmidt 1984.



Not-so-simple events

Unfortunately many events are complex, with multiple overlapping spikes in many locations. And of course these may be the most interesting ones!

It really matters because "Failure in identification of overlapping spikes from multiple neuron activity causes artificial correlations" [Bar-Gad '01]. Moreover, when we graduate to bigger arrays, nearly all events will involve overlaps in time!!

Many authors say **bursts** are a big problem, but here is a nice fit that we obtained with no special effort. See later.

We even handle overlapping spikes, which some algorithms do not attempt. See later.



JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).



[Sorry, no time to discuss our method for this step.]

Superposing 50 traces chosen from 284 in this cluster shows that they really do all resemble each other.

Occasional events in which this event collides with another don't affect the "archetype waveform" (template) (next slide).

Although the *shape* of each instance of the archetype is quite constant, still its *amplitude* has significant variation.



JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).





Noise covariance

Vanilla least-squares fitting is not appropriate for time series, because it assumes that every sample is independent of all others--whereas actually, successive samples are correlated.

Here is the covariance of channel #13 with all other channels (after an initial spatial filter, also obtained from data). For reference, each channel has a single blue curve showing an exponential function.

We see that #13 is correlated only with itself, and it has a simple covariance matrix that is easy to invert. The inverse covariance thus obtained defines our correlated Gaussian model of the noise.

[Again: The covariance is **not** a delta function, contrary to what is assumed in naive least-squares fitting.]



On inference

Suppose we measure some experimental data, and wish to make an inference about some situation that we cannot directly observe. That is, we imagine a variety of worlds with different values of *X*, and ask which is most probable given the observed data.

See M. Denny and S. Gaines, Chance in Biology.

On inference

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If we know the probability that those data would have arisen in a world with a particular value of *X*, then Bayes's formula gives us what we actually want:

$$P(X|\text{observed data}) = P(\text{data}|X) \frac{P(X)}{P(\text{data})}$$

We can ignore the denominator, if all we want is to compare two hypotheses (e.g. maximize over *X*).

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For our application, we'd like $\mathbf{P}(\text{spikes | data})$, where "data" is an observed waveform and "spikes" refers to a collection of spike archetypes μ_1, \ldots occurring at times t_1, \ldots with amplitudes A_1, \ldots relative to the amplitude of the corresponding archetype (neuron). Bayes's formula gives what we want as

 $\mathbf{K} \times (\text{likelihood}) \times (\text{prior}) = \mathbf{KP} (\text{data } | \text{spikes})\mathbf{P} (\text{spikes})$

See M. Denny and S. Gaines, Chance in Biology.

Bayesian idea

Previous slide expressed P(spikes | data) as:

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Priors con itself--an benefit o and can b

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To get the **prior**, $\mathbf{P}(\text{spikes})$, assume that for a single spike it has the form $P^{\text{cell}}(\mu)P^{\text{time}}(t)P^{\text{ampl}}(A|\mu)$

The three factors are respectively the popularity of this neuron, uniform in time, and a Gaussian reflecting its typical amplitude and amplitude variability. We get these priors from the data subset used in clustering.

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To get the **likelihood function P**(data I spikes), suppose that the data consist of one archetype, plus noise. And suppose that the noise is some **Gaussian**, independent of which spikes fired. We know all about this Gaussian from our measurement of noise covariance.

Then the likelihood is that distribution, evaluated at the difference between the actual waveform and the idealized one. [Pouzat et. al. 2002]

Bayesian idea, II

We start with an experimental trace ("data"). We find its peak (absolute minimum), and start looking for a spike there.

We ask for the **likelihood ratio** between the hypotheses of no spike versus one spike of given type, at given time, with given amplitude.

- ★To compute the likelihood of no spike, evaluate the noise distribution on the trace.
- ★To compute the probability of one spike, choose a spike archetype and a value of t, the spike time. Holding the "data" fixed, the probability is now a Gaussian function in the remaining parameter A, so it's fast and easy to marginalize over A.

Let $V_{\alpha}(t)$ be measured voltage, electrode α and $F_{\mu\alpha}(t)$ be archetype waveform of type μ . Define the deviation $[\delta \mathbf{V}]_{\alpha t} = V_{\alpha}(t) - AF_{\mu\alpha}(t - t_1)$



which is a Gaussian in A. So it's easy to marginalize over A: just complete the square! [Here $K_{\mu} = P^{\text{cell}}(\mu)P^{\text{time}}(t_1) \left(2\pi\sigma_{\mu}^2\right)^{-1/2}$ doesn't depend on A.]

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Next, we sweep over a range of t to find the best value of likelihood ratio for this spike type. [We only check t values close to the peak of the event.]

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Then we choose the winner among spike types.

If the winner's likelihood ratio is good enough (bigger than about 1), we say there's a spike here. **That's the absolute criterion I promised earlier**.

JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).



est our assumptions

Can we really assume that the spikes from a particular cell differ only in overall amplitude? We took many events that contained a single spike of each type. Point by point in time, we subtracted the scaled shifted archetype and found the residual (on each channel).

Green: the archetype itself. Red: mean deviation from archetype. Blue: std deviation from archetype. We really do subtract spikes pretty completely.



Successfully fit overlaps



Successfully fit overlaps

Successfully fit overlaps

Top: Closeup of four channels, showing three fit archetypes found by the algorithm. Bottom: sum of those fits (color) versus actual data (black).







Each cell has a receptive field...

... and they tile the whole visual field. MEA recording is **high throughput**: We got dozens of cells all at once. Here are cells from just one functional group, "on cells." Each putative receptive field is a single connected region of image space.



KD Simmons, JS Prentice, G Tkacik, J Homann, PCN, V Balasubramanian, submitted.
Receptive fields

Once you've got the spike trains, you can find receptive fields etc. Here's a typical spike-triggered average.

How interesting--guinea pig retina has a lot of these highly anisotropic receptive fields. The "surround" doesn't surround the "center"!

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Takehome Part III



I described how we identify the individual ganglion cell signals from a hash of noise and overlapping real signals:

Takehome Part III



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There is something weirdly -- *unreasonably* -- effective about approaching biological systems with a physical model. I don't understand why. I don't *need* to understand why.

Thursday, March 15, 2012

Often, when we want to justify theory, we scratch our heads and say, "Well Hodgkin and Huxley was a big deal."

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I'd just like to suggest that this attitude, though common, misses out on some of what theory can do for you. Particularly, a *physical model* can give a lot of dividends.

We like to teach famous success stories in science, but we don't always remember to present them as showcases of the utility of physical modeling.

Thanks



University of Pennsylvania

For these slides see: www.physics.upenn.edu/~pcn

Thursday, March 15, 2012

Thanks



University of Pennsylvania



NSF DMR, BIO, IBN



NSF NSEC



National Eye Institute Training grant ONAL WSHUTES

Computational Neuroscience Training grant

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