When you see a title like that, you need to worry: "Uh-oh, sounds philosophical."
Well, I just wanted to tell you two concrete stories about cases when my colleagues and I managed to do something useful by virtue of knowing something about inference. The ideas we needed were things I didn't know a few years ago, so I thought you might be interested too.

## Inference in biological physics

## Phil Nelson <br> University of Pennsylvania

## For these slides see:

WWW. physics.upenn.edu/~pcn

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There's more, of course, but t "Of course that's what we do -- everybody knows that." -- No. They don't.

## Part I:

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mortal

In classical logic it's fairly easy to spot errors of inference.

## An everyday question in clinical practice

To diagnose colorectal cancer, the hemoccult test-among others-is conducted to detect occult blood in the stool. This test is used from a particular age on, but also in routine screening for early detection of colorectal cancer. Imagine you conduct a screening using the hemoccult test in a certain region. For symptom-free people over 50 years old who participate in screening using the hemoccult test, the following information is available for this region:
The probability that one of these people has colorectal cancer is 0.3 percent. If a person has colorectal cancer, the probability is 50 percent that he will have a positive hemoccult test. If a person does not have colorectal cancer, the probability is 3 percent that he will still have a positive hemoccult test. Imagine a person (over age 50, no symptoms) who has a positive hemoccult test in your screening. What is the probability that this person actually has colorectal cancer? $\qquad$ percent

## G. Gigerenzer, Calculated risks

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Estimates (\%)

Here are the replies of 24 practicing physicians, who had an average of 14 years of professional experience:

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[^0]
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$P($ sick $\mid+)=P(+\mid$ sick $) \times \frac{P(\text { sick })}{P(+)}$

Posterior Likelihood Prior
estimate (given) estimate
(desired) (given)

## Finish working it out



C=Healthy, -

D=Healthy, +

## Finish working it out

$P($ sick $\mid+)=P(+\mid$ sick $) \times \frac{P(\text { sick })}{P(+)}$


$$
\begin{aligned}
P(+) & =B+D \\
& =\frac{B}{A+B}(A+B)+\frac{D}{C+D}(C+D) \\
& =P(+\mid \text { sick }) P(\text { sick })+P(+\mid \text { healthy }) P(\text { healthy }) \\
& =(0.5)(0.003)+(0.03)(0.997) \approx 0.03
\end{aligned}
$$

## Finish working it out

$$
\begin{array}{ll}
P(\text { sick } \mid+)=P(+\mid \text { sick }) \times \frac{P(\text { sick })}{P(+)} & \mathrm{A}=\text { Sick, }- \\
\text { Is that last factor a big deal? } & \mathrm{B}=\text { Sick, }+
\end{array}
$$



$$
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$\frac{P(\text { sick })}{P(+)} \approx \frac{0.003}{0.03} \approx 0.1$
It's huge: a positive test result means only a $5 \%$ chance you're sick. Not $97 \%$.

# Part II: Changepoint analysis in singlemolecule TIRF 

JF Beausang, Yale Goldman, PN

* Sometimes our model is not obviously connected with what we can actually measure experimentally, but and we need to makes a connection.
* Sometimes the model that interests us involves the behavior of actors that we can only see indirectly in our data; theory may be needed to separate them out from each other, and from noise.


Many thanks to Haw Yang. See also Lucas P. Watkins and Haw Yang J. Phys. Chem. B 2005

## Myosin V Processivity

We'd like to know things like: How does it walk? What are the steps in the kinetic pathway? What is the geometry of each state?
One classic approach is to monitor the position in space of a marker (e.g. a bead) attached to the motor. But this does not address the geometry of each state.


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The approach I'll discuss involves attaching a bifunctional fluorescent label to one lever arm. The label has a dipole moment whose orientation in space reflects that of the arm.

## Myosin V Processivity



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# Polarized total internal reflection fluorescence microscopy 



Fluorescence illumination by the evanescent wave eliminates a lot of noise, and importantly, maintains the polarization of the incident light.
To tickle the fluorophore with every possible polarization, we need the incoming light to have at least two different beam directions.

## pol-TIRF setup



## pol-TIRF setup



8 polarized illuminations $\times 2$ detectors $=16$ fluorescent intensities per cycle

## Current state of the art



It's a bit more meaningful to convert from lab-frame angles $\theta, \phi$ to actin-frame angles $\alpha, \beta$. Even then, however, state of the art calculations give pretty noisy determinations, with pretty poor time resolution.
You could easily miss a short-lived state -- e.g. the elusive diffusive-search step (if it exists). Can't we do better?

JN Forkey et al. Nature 2003

Unfortunately, the total photon counts from a fluorescent probe may not be very informative. Here we divided a time period of interest into 20 bins. There is some Poiss Horizontal axis is time. Vertical axis is binned photon count, PFI =polarized fluorescence intensity


JF Beausang, YE Goldman, and PCN, Meth. Enzymol. 487:431 (2011).
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If we classify the photons by polarization and bin them separately, that reveals a definite changepoint. But when exactly did it occur? Probably not at the dashed line shown, but how can we be more precise?

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It turns out that binning the data destroyed some information. Something magical happens if instead of binning, we just we plot photon arrival time versus photon sequence number. Despite some ripples from Poisson statistics, it's obvious that each trace has a sharp changepoint, and moreover that the two changepoints found independently in this way are simultaneous.
(A similar approach in the context of FRET was pioneered
 by Haw Yang.)

JF Beausang, YE Goldman, and PCN, Meth. Enzymol. 487:431 (2011).

## Now that I have your attention

- Why did that trick work? How did we get such great time resolution from such cruddy data?
- How well does it work? If we have even fewer photons, for example because a state is shortlived, how can we quantify our confidence that any changepoint occurred at all?
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Now: Divide the $N$ photons into $n$ that arrived before the putative changepoint, and $n^{\prime}=N-n$ that arrived after.
Take the limit $\Delta t \rightarrow 0$ :

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$P \approx N \log (\Delta t)+n \log R+n^{\prime} \log R^{\prime}-\left(\frac{t_{*}}{\Delta t}-n\right)(R \Delta t)-\left(\frac{T-t_{*}}{\Delta T}-1-(N-n)\right)\left(R^{\prime} \Delta t\right)$
$\approx$ const $+n \log R+n^{\prime} \log R^{\prime}-R t_{*}-R^{\prime}\left(T-t_{*}\right)$

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Maximize this first over $R$ and $R^{\prime}$ :

$$
R=n / t_{*}, \quad R^{\prime}=n^{\prime} /\left(T-t_{*}\right)
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Hypothesis is that photons are arriving in a Poisson process with rate $R$ from time 0 to time $t_{*}$, and thereafter arrive in another Poisson process with rate $R^{\prime}$.

$$
\begin{aligned}
& \log P\left(t_{1}, \ldots, t_{N} \mid R, R^{\prime}, t_{*}\right)=\sum_{k=1}^{t_{*} / \Delta t} \log \begin{cases}R \Delta t & \text { if a photon in this slice } \\
(1-R \Delta t) & \text { otherwise }\end{cases} \\
&+\sum_{k^{\prime}=t_{*} / \Delta t+1}^{T / \Delta t} \log \begin{cases}R^{\prime} \Delta t & \text { if a photon in this slice } \\
\left(1-R^{\prime} \Delta t\right) & \text { otherwise }\end{cases}
\end{aligned}
$$

Now: Divide the $N$ photons into $n$ that arrived before the putative changepoint, and $n^{\prime}=N-n$ that arrived after.
Take the limit $\Delta t \rightarrow 0$ :

$$
\begin{aligned}
P & \approx N \log (\Delta t)+n \log R+n^{\prime} \log R^{\prime}-\left(\frac{t_{*}}{\Delta t}-n\right)(R \Delta t)-\left(\frac{T-t_{*}}{\Delta T}-1-(N-n)\right)\left(R^{\prime} \Delta t\right) \\
& \approx \text { const }+n \log R+n^{\prime} \log R^{\prime}-R t_{*}-R^{\prime}\left(T-t_{*}\right)
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$$

Maximize this first over $R$ and $R^{\prime}$ :

$$
R=n / t_{*}, \quad R^{\prime}=n^{\prime} /\left(T-t_{*}\right)
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OK, duh, that was no surprise! But it does explain why we can just lay a ruler along the cumulative plot to get our best estimate of the before and after rates.

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OK, duh, that was no surprise! But it does explain why we can just lay a ruler along the cumulative plot to get our best estimate of the before and after rates.
More interestingly, we can substitute these optimal rates into the formula for $P$ to find the likelihood as a function of putative changepoint:

## Application

## Application



Here's some very fake data; the photons arrive uniformly, not at random.

## Application



Here are two lines corresponding to non-optimal choices of the changepoint. We'd like to see the likelihood function and how it selects the "right" changepoint, which for fake data is known.

## Application



Here is our log-likelihood function as a function of putative changepoint time.

Here's some very fake data; the photons arrive uniformly, not at random.



Left: Some more realistic (Poisson-arrival) simulated data, shown in traditional binned form and in the improved version.

Right: Likelihood function for placement of the changepoint. Dashed line, maximum-likelihood point. Black triangle: Actual changepoint used in the simulation.



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Right: Likelihood function for placement of the changepoint. Dashed line, maximum-likelihood point. Black triangle: Actual changepoint used in the simulation.


Oh, yes -- the method also works on multiple-channel data. Left: one channel (red) starts with rare photons, then jumps to higher intensity. Another channel (blue) does the opposite. The sum of the intensities (black) doesn't change much at all.
Middle: "kink" representations of the same data. Right: both channels contribute to a likelihood function with a robust peak, even though there were only a total of just 200 photons in the entire dataset.

## 




JF Beausang, YE Goldman, and PCN, Meth. Enzymol. 487:431 (2011).

## Payoff



JF Beausang, YE Goldman, and PCN, Meth. Enzymol. 487:431 (2011).

## Payoff



Oh, yes-it also works on real experimental data.
Now we can get back to the original motivation. Previously, people would take data from multiple polarizations, bin it, and pipe the inferred intensities into a maximum-likelihood estimator of the orientation of the fluorophore. That procedure leads to the rather noisy dots shown here.
One problem is that if a transition happens in the middle of a time bin, then the inferred orientation in that time bin can be crazy.

Here the solid lines are the inferred orientations of the probe molecule during successive states defined by changepoint analysis. We see a nice alternating stride in $\phi$.


JF Beausang, YE Goldman, and PCN, Meth. Enzymol. 487:431 (2011).

## Summary Part II


*When you only get a million photons, you'd better make every photon count.

* A simple maximum-likelihood analysis accomplishes this.
* In the context of TIRF it can dramatically improve the tradeoff between time resolution and accuracy.


## Part III: Parallel recordings from dozens of individual neurons

* Sometimes suggests a new kind of measurement that tests a model more stringently, or distinguishes two different models more completely, than previous measurements.
* Sometimes our model is not obviously connected with what we can actually measure experimentally, and we need to make a connection.
*Sometimes the model that interests us involves the behavior of actors that we can only see indirectly in our data; theory may be needed to separate them out from each other, and from noise. individual neurons

*Sometimes the model that interests us involves the behavior of actors that we can only see indirectly in our data; theory may be needed to separate them out from each other, and from noise.


## Sources of energy

Experiments done in the lab of Vijay Balasubramanian (Penn).

## Sources of energy

Experiments done in the lab of Vijay Balasubramanian (Penn).

Kristy Simmons, Penn Neuroscience

(plus Gasper Tkacik.)
(Many thanks to Michael Berry and Olivier Marre, Princeton; Bart Borghuis, Janelia Farms; Michael Freed and others at Penn Retina Lab; Joerg Sander, U Alberta; Ronen Segev, BGU, Chris Wiggins, Columbia.)

## Really big picture

Retina is also an approachable, yet still complex, part of the brain. It's a 2D carpet consisting of "only" three layers of neurons.

Retinal
illumination pattern

Retinal
ganglion cell
spike trains


# Really big picture 

Retina is also an approachable, yet still complex, part of the brain. It's a 2D carpet consisting of "only" three layers of neurons.


Retinal
illumination pattern

## Retina

## It matters



Your source for the latest research news


## Artificial Retina More Capable of Restoring Normal Vision; Animal Study Shows Including Retina's Neural 'Code' Improved Prosthetic

ScienceDaily (Nov. 16, 2010) - Researchers have developed an artificial retina that has the capacity to reproduce normal vision in mice. While other prosthetic strategies mainly increase the number of electrodes in an eye to capture more information, this study concentrated on incorporating the eye's neural "code" that converts pictures into signals the brain can understand.

The research was presented at

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$$

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| :--- |
| Reddit $\%$ Slashdot F Fark ShareThis |

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Emai
给 Reddit \% Slashdot F Fark ShareThis

## Summary, Part III


2. Clustering
3. Fitting
4. Performance

## Get data

Cluster

Fit
Interpret

Cf Meister, Pine, and Baylor 1994. Incredibly, one can keep a mammalian retina alive in a dish for over 6 hours while presenting it stimuli and recording its activity.
94.


Cf Meister, Pine, and Baylor 1994. Incredibly, one can keep a mammalian retina alive in a dish for over 6 hours while presenting it stimuli and recording its activity.


## What's in the dish



Michael Berry, Princeton

## Simple events



67 ms of data, viewed as a movie.<br>[data have been smoothed]

## Simple events



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Some spikes move across the array:


## Simple events



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Some spikes move across the array:


The spike-sorting problem is: Given raw data like these, convert to a list of discrete events (which cells fired at what times).

## Not-so-simple events

which electrode, $x$


Unfortunately many events are complex, with multiple overlapping spikes in many locations. And of course these may be the most interesting ones!

It really matters because "Failure in identification of overlapping spikes from multiple neuron activity causes artificial correlations" [Bar-Gad '01]. Moreover, when we graduate to bigger arrays, nearly all events will involve overlaps in time!!

Many authors say bursts are a big problem, but here is a nice fit that we obtained with no special effort. See later.

We even handle overlapping spikes, which some algorithms do not attempt. See later.



















JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).
2. Clustering
3. Fitting
4. Performance

[Sorry, no time to discuss our method for this step.]

Superposing 50 traces chosen from 284 in this cluster shows that they really do all resemble each other.

Occasional events in which this event collides with another don't affect the









 "archetype waveform" (template) (next slide).

Although the shape of




 each instance of the archetype is quite constant, still its amplitude has significant variation.





JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).

## Resulting archetype

We scaled each instance of each archetype to get best agreement with the others, then took the median at each time point to find our best estimate of the consensus waveform (blue). As a check, the pointwise mean waveform looks the same (red).







Fit
Interpret

## Noise covariance

Vanilla least-squares fitting is not appropriate for time series, because it assumes that every sample is independent of all others--whereas actually, successive samples are correlated.
Here is the covariance of channel \#13 with all other channels (after an initial spatial filter, also obtained from data). For reference, each channel has a single blue curve showing an exponential function.

We see that \#13 is correlated only with itself, and it has a simple covariance matrix that is easy to invert. The inverse covariance thus obtained defines our correlated Gaussian model of the noise.
[Again: The covariance is not a delta function, contrary to what is assumed in naive leastsquares fitting.]


## On inference

Suppose we measure some experimental data, and wish to make an inference about some situation that we cannot directly observe. That is, we imagine a variety of worlds with different values of $X$, and ask which is most probable given the observed data.

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If we know the probability that those data would have arisen in a world with a particular value of $X$, then Bayes's formula gives us what we actually want:

$$
P(X \mid \text { observed data })=P(\text { data } \mid X) \frac{P(X)}{P(\text { data })}
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We can ignore the denominator, if all we want is to compare two hypotheses (e.g. maximize over $X$ ).

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For our application, we'd like (spikes I data), where "data" is an observed waveform and "spikes" refers to a collection of spike archetypes $\mu_{1}, \ldots$ occurring at times $t_{1}, \ldots$ with amplitudes $A_{1,}$. relative to the anplitude of the corresponding archetype (neuron). Bayes's formula gives what we want as
$\mathbf{K} \times($ likelihood $) \times($ prior $)=\mathbf{K P}($ data I spikes $) \mathbf{P}($ spikes $)$

See M. Denny and S. Gaines, Chance in Biology.

## Bayesian idea

Previous slide expressed $\mathbf{P}$ (spikes I data) as:

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\mathbf{K} \times(\text { likelihood }) \times(\text { prior })=\mathbf{K} \mathbf{P}(\text { data } \mid \text { spikes }) \mathbf{P}(\text { spikes })
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To get the prior, $\mathbf{P}$ (spikes), assume that for a single spike it has the form

$$
\underline{P}^{\text {cell }}(\mu) \underline{P^{\mathrm{time}}(t)} \underline{P}^{\mathrm{ampl}}(A \mid \mu)
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The three factors are respectively the popularity of this neuron, uniform in time, and a Gaussian reflecting its typical amplitude and amplitude variability. We get these priors from the data subset used in clustering.

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The three factors are respectively the popularity of this neuron, uniform in time, and a Gaussian reflecting its typical amplitude and amplitude variability. We get these priors from the data subset used in clustering.

To get the likelihood function $\mathbf{P}$ (data I spikes), suppose that the data consist of one archetype, plus noise. And suppose that the noise is some Gaussian, independent of which spikes fired. We know all about this Gaussian from our measurement of noise covariance.

Then the likelihood is that distribution, evaluated at the difference between the actual waveform and the idealized one. [Pouzat et. al. 2002]

## Bayesian idea, II

We start with an experimental trace ("data").
We find its peak (absolute minimum), and start looking for a spike there.
We ask for the likelihood ratio between the hypotheses of no spike versus one spike of given type, at given time, with given amplitude.
$\star$ To compute the likelihood of no spike, evaluate the noise distribution on the trace.
\# To compute the probability of one spike, choose a spike archetype and a value of $\mathbf{t}$, the spike time. Holding the "data" fixed, the probability is now a Gaussian function in the remaining parameter $\mathbf{A}$, so it's fast and easy to marginalize over A.

## [Nuts and Bolts]

Let $V_{\alpha}(t)$ be measured voltage, electrode $\alpha$ and $F_{\mu \alpha}(t)$ be archetype waveform of type $\mu$. Define the deviation

$$
[\delta \mathbf{V}]_{\alpha t}=V_{\alpha}(t)-A F_{\mu \alpha}\left(t-t_{1}\right)
$$

Then the probability that one spike, of type $\mu$, is present is
$P($ spikes $\mid$ data $)=K_{\mu} \exp \left[-\frac{\left(A-\gamma_{\mu}\right)^{2}}{2 \sigma_{\mu}^{2}}-\frac{1}{2}(\delta \mathbf{V})^{\mathrm{t}} \mathrm{C}^{-1}(\delta \mathbf{V})\right]$
which is a Gaussian in A. So it's easy to marginalize over A: just complete the square! [Here $K_{\mu}=P^{\text {cell }}(\mu) P^{\text {time }}\left(t_{1}\right)\left(2 \pi \sigma_{\mu}^{2}\right)^{-1 / 2}$ doesn't depend on A.]

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JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).

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Next, we sweep over a range of $t$ to find the best value of likelihood ratio for this spike type. [We only check $\mathbf{t}$ values close to the peak of the event.]

Then we choose the winner among spike types.
If the winner's likelihood ratio is good enough (bigger than about 1), we say there's a spike here. That's the absolute criterion I promised earlier.

JS Prentice, J Homann, KD Simmons, G Tkacik, V Balasubramanian, PCN, PLoS ONE 6(7): e19884 (2011).

## Test our assumptions

Can we really assume that the spikes from a particular cell differ only in overall amplitude? We took many events that contained a single spike of each type. Point by point in time, we subtracted the scaled shifted archetype and found the residual (on each channel).

Green: the archetype itself. Red: mean deviation from archetype.
Blue: std deviation from archetype.
We really do subtract spikes pretty completely.











## Successfully fit overlaps



## Successfully fit overlaps

## Successfully fit overlaps

Top: Closeup of four channels, showing three fit archetypes found by the algorithm.
Bottom: sum of those fits (color) versus actual data (black).



Interpret

## Each cell has a receptive field...

... and they tile the whole visual field. MEA recording is high throughput: We got dozens of cells all at once. Here are cells from just one functional group, "on cells." Each putative receptive field is a single connected region of image space.


Region of retina responded to by ganglion cell \#1, etc.

KD Simmons, JS Prentice, G Tkacik, J Homann, PCN, V Balasubramanian, submitted.

## Receptive fields

Once you've got the spike trains, you can find receptive fields etc. Here's a typical spike-triggered average.

How interesting--guinea pig retina has a lot of these highly anisotropic receptive fields. The "surround" doesn't surround the "center"!

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## Takehome Part III



I described how we identify the individual ganglion cell signals from a hash of noise and overlapping real signals:

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## Full circle

\author{
Medical tests <br> ```
tests

```
}

OK, I was a scatterbrain and gave you three talks. But wait -- if I can fill in the spaces

Changepoint
Analysis

\title{
Full circle
}

OK, I was a scatterbrain and gave you three talks. But wait -- if I can fill in the spaces
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\title{
Full circle
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Medical tests


Theory can cut across apparently different kinds of experiment, offering useful methods to one domain from another without having to reinvent everything.

\section*{Wait, there's more}


\section*{Wait, there's more}


\section*{Wait, there's more}


\section*{Wait, there's more}


There is something weirdly -- unreasonably -- effective about approaching biological systems with a physical model. I don't understand why. I don't need to understand why.

\section*{Go long}

\section*{Go long}

Often, when we want to justify theory, we scratch our heads and say, "Well Hodgkin and Huxley was a big deal."

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My point so far is that theory is needed every day. It's our microscope; our Geiger counter; it helps us to see the invisible.

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Another context in which theory enters laboratory discussions is, " H theory to get this thing published. Go do some theory, run some ANOVA, whatever."
I'd just like to suggest that this attitude, though common, misses out on some of what theory can do for you. Particularly, a physical model can give a lot of dividends.

\section*{Go long}

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Indeed. But that sort of cherry-picking approach can leave the impression that theory is something that happens every 50 years or so. It's also too reverent.

My point so far is that theory is needed every day. It's our microscope; our Geiger counter; it helps us to see the invisible.
To emphasize that, I didn't select famous examples; instead I have told you about the two things I happen to be working on right now (a random choice, you'll agree).

Another context in which theory enters laboratory discussions is, " H theory to get this thing published. Go do some theory, run some ANOVA, whatever."
I'd just like to suggest that this attitude, though common, misses out on some of what theory can do for you. Particularly, a physical model can give a lot of dividends.

We like to teach famous success stories in science, but we don't always remember to present them as showcases of the utility of physical modeling.

\section*{Thanks}


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