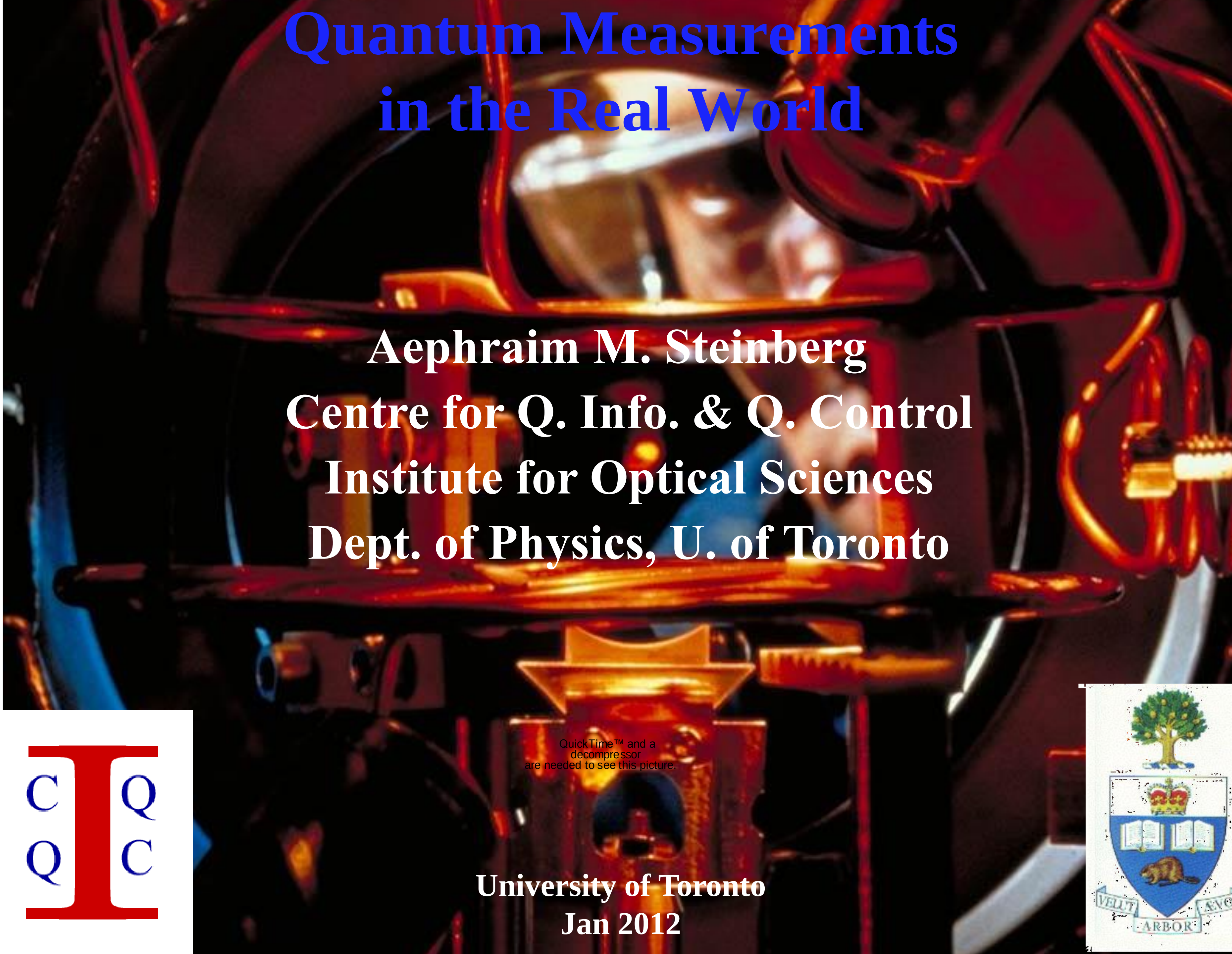
Quantum Measurements in the Real World



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University of Toronto

Jan 2012

The morals of the story

1 Quantum Measurement is much richer than the textbooks acknowledge

2 Different sorts of Q.Msmt's prove useful for different real-world tasks

Main topics:

- Optimal state discrimination

  (what's the best way to tell which of two quantum states you have?)

  - Generalized quantum measurements

- Weak measurement
  - Can we talk about the past in quantum mechanics?
  - ("Interaction-free" measurement, Hardy's Paradox, trajectories in two-slit interferometers, & more)

- Perhaps a few words on applications of measurement (eg quant. info)
  - Post-selection as an effective nonlinearity (logic gate)
  - Entangled states for phase super-resolution & their tomography

DRAMATIS PERSONÆ

CQIQC

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Max Touzel, Julian Schmidt, Xiaoxian Liu,
Timur Rvachov,Luciano Cruz, Morgan Mitchell,...

Some helpful theorists:

Daniel James, Pete Turner, Robin Blume-Kohout, Chris Fuchs, Howard Wiseman, János Bergou, John Sipe, Paul Brumer, Michael Spanner...

NSERC CRSNG

25 years cifar
Canadian Institute for Advanced Research

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QuantumWorks

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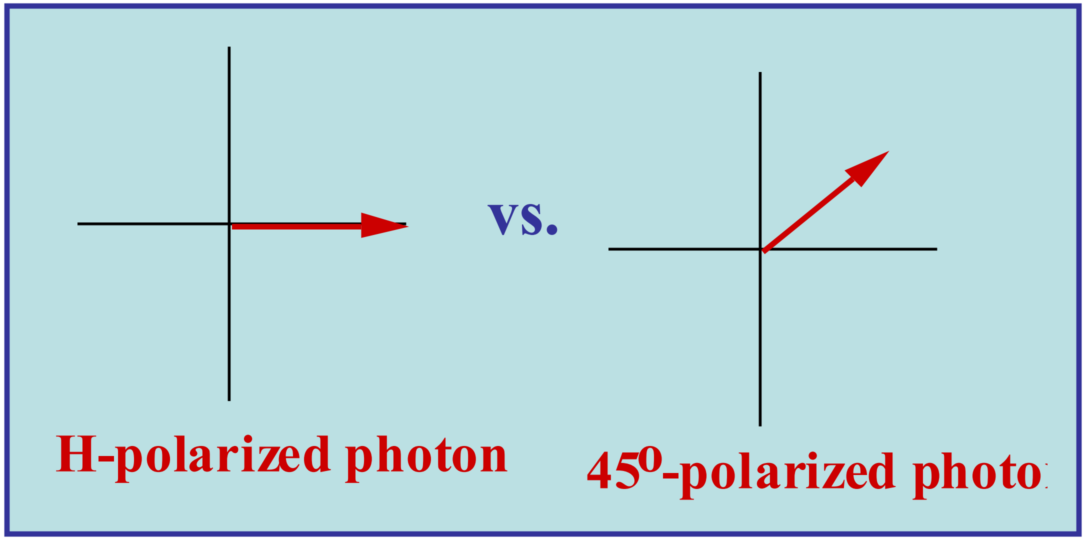
Distinguishing the indistinguishable...

How to distinguish non-orthogonal states optimally?



The view from the laboratory:

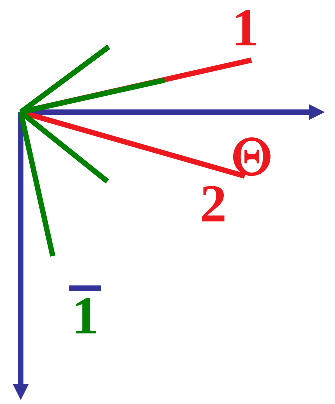
A measurement of a two-state system can only yield two possible results.

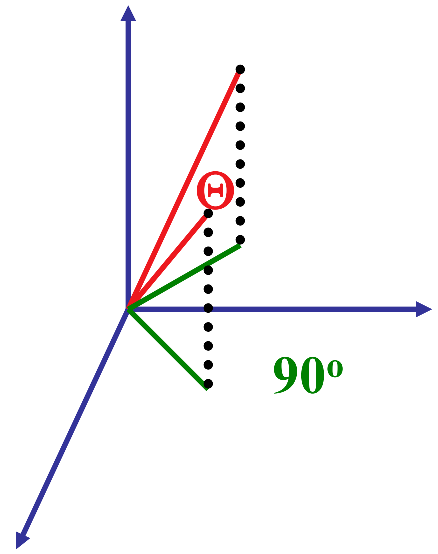
If the measurement isn't guaranteed to succeed, there are three possible results: (1), (2), and ("I don't know").

Therefore, to discriminate between two non-orth. states, we need to use an expanded (3D or more) system. To distinguish 3 states, we need 4D or more.

Use generalized (POVM) quantum measurements. [see, e.g., Y. Sun, J. Bergou, and M. Hillery, Phys. Rev. A 66, 032315 (2002).]

The geometric picture





Two non-orthogonal vectors
No projective measurement can tell with certainty which we have; if one basis vector is orth. to 1, the other *cannot* be orth. to 2.

If these two (red) vectors are embedded in a *3-dimensional* space, it becomes possible to find a pair of (green) orthogonal vectors each of which excludes one of the options!
(But now there is a third axis:
the z-axis is "inconclusive")

A test case

Consider these three non-orthogonal states:

$$|\psi_1\rangle_{in} = \begin{pmatrix} \sqrt{2/3} \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \;;\; |\psi_2\rangle_{in} = \begin{pmatrix} 0 \\ 1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix} \;;\; |\psi_3\rangle_{in} = \begin{pmatrix} 0 \\ -1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix}$$

Projective measurements can distinguish these states with *certainty* no more than 1/3 of the time.

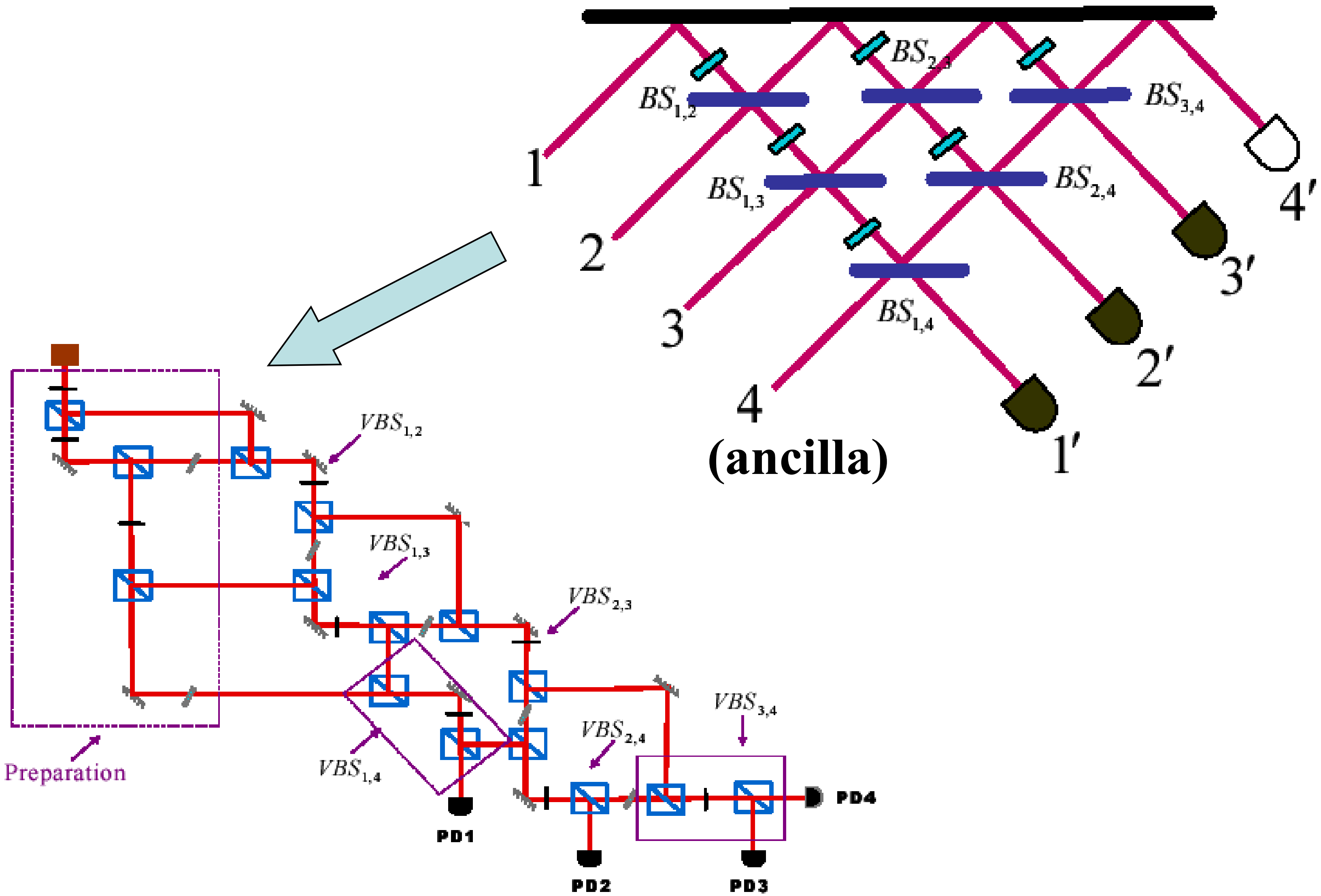
(No more than one member of an orthonormal basis is orthogonal to *two* of the above states, so only one pair may be ruled out.)

But a unitary transformation in a 4D space produces:

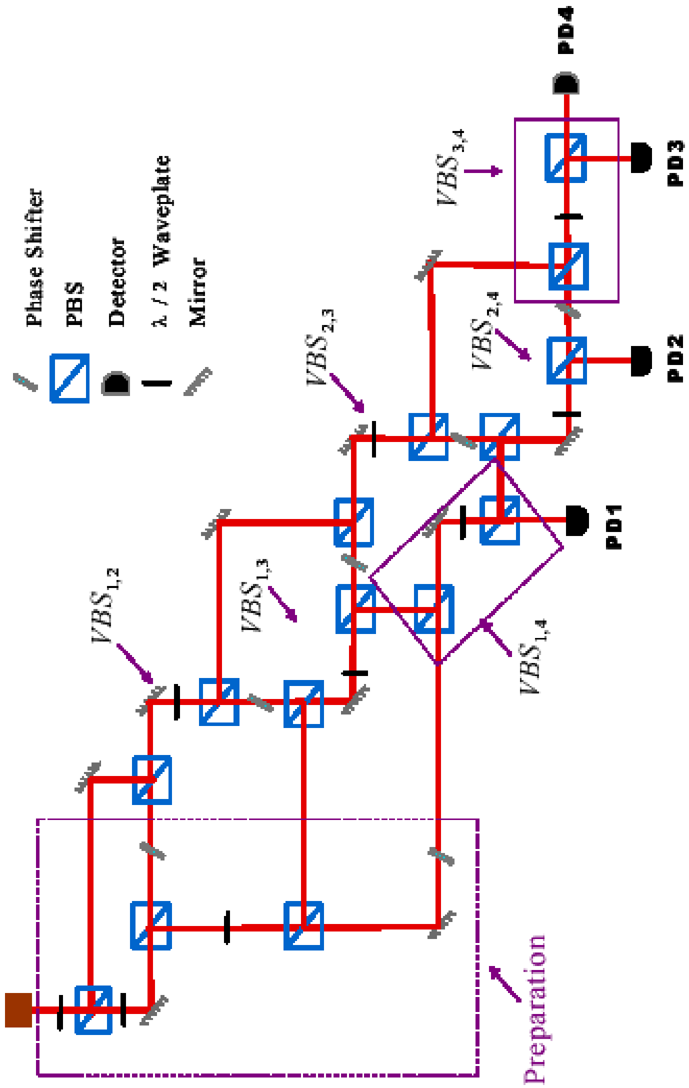
$$|\psi_1\rangle_{out} = \begin{pmatrix} 1/\sqrt{3} \\ 0 \\ 0 \\ \sqrt{2/3} \end{pmatrix} \quad |\psi_2\rangle_{out} = \begin{pmatrix} 0 \\ 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \quad |\psi_3\rangle_{out} = \begin{pmatrix} 0 \\ -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

…and these states can be distinguished with certainty up to 55% of the time

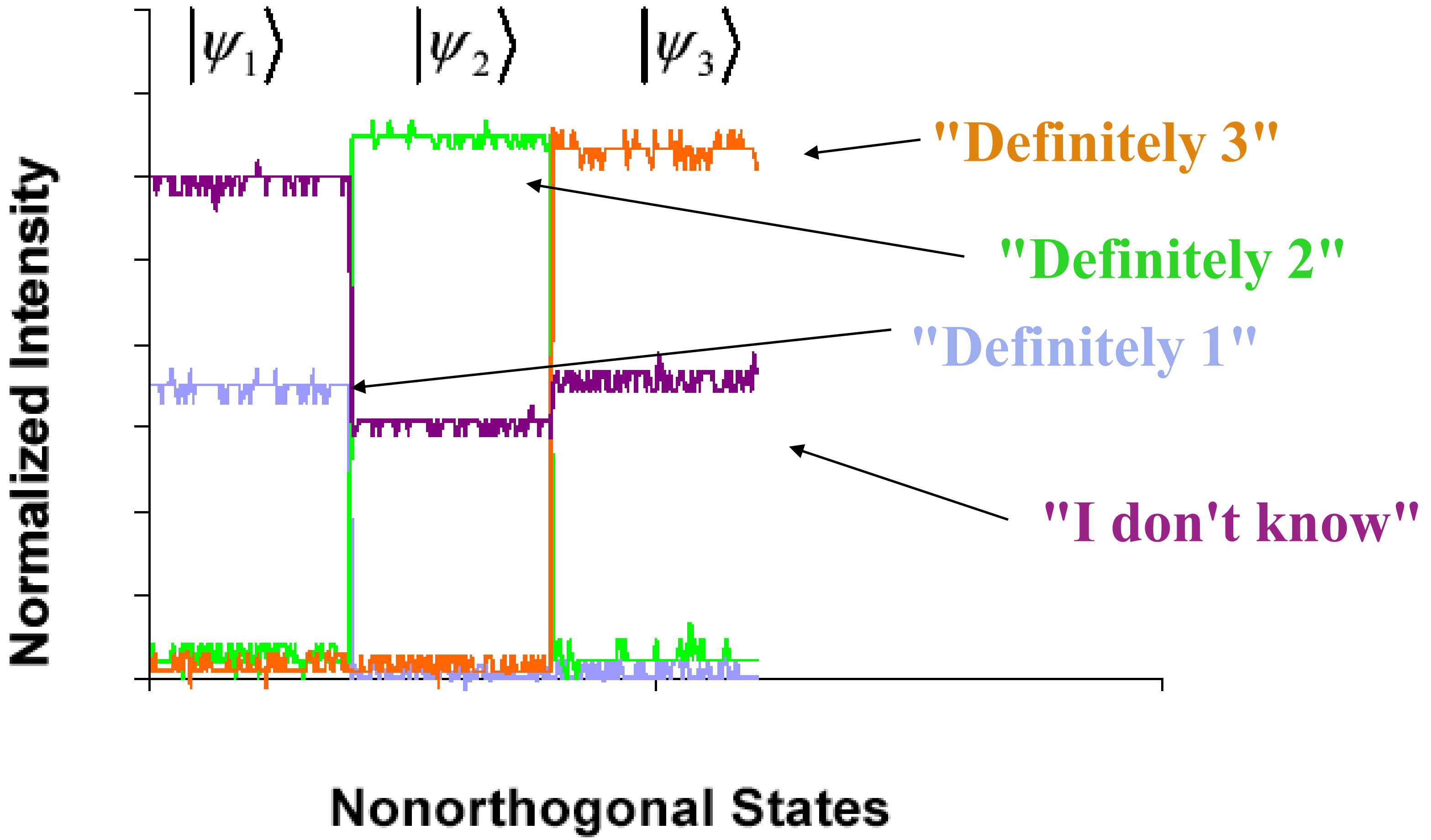
Experimental schematic



A 14-path interferometer for arbitrary 2-qubit unitaries...



Success!



The correct state was identified 55% of the time--
Much better than the 33% maximum for standard measurements.

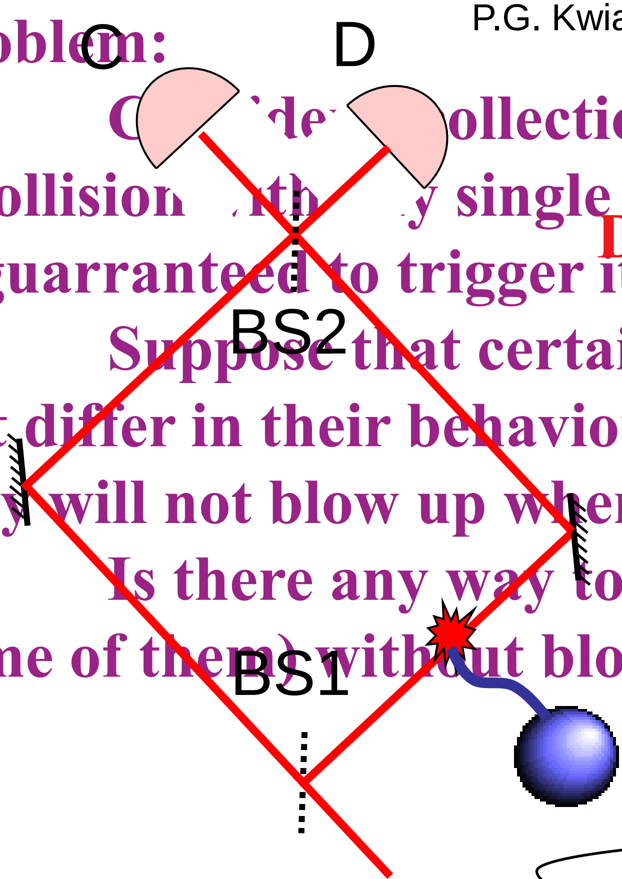
M. Mohseni, A.M. Steinberg, and J. Bergou, Phys. Rev. Lett. 93, 200403 (2004)

“Quantum Seeing in the Dark”

" Quantum seeing in the dark "

(AKA: "Interaction-free" measurement, aka "Vaidman's bomb")

A. Elitzur and L. Vaidman, Found. Phys. **23**, 987 (1993)
P.G. Kwiat, H. Weinfurter, and A. Zeilinger, Sci. Am. (Nov., 1996)



Problem:

Consider a collection of bombs so sensitive that a collision with any single particle (photon, electron, etc.) is guarranteed to trigger it.

Suppose that certain of the bombs are defective, but differ in their behaviour in *no way* other than that they will not blow up when triggered.

Is there any way to identify the working bombs (or some of them) without blowing them up?

Detector absent/ineffectual:

Only detector C fires

Detector working:

"boom!"	1/2
C	1/4
D	1/4

The bomb must be there... yet my photon never interacted with it.

Fanciful musing about this

Many feel that QM implies a tree falling in an empty forest makes no sounds.

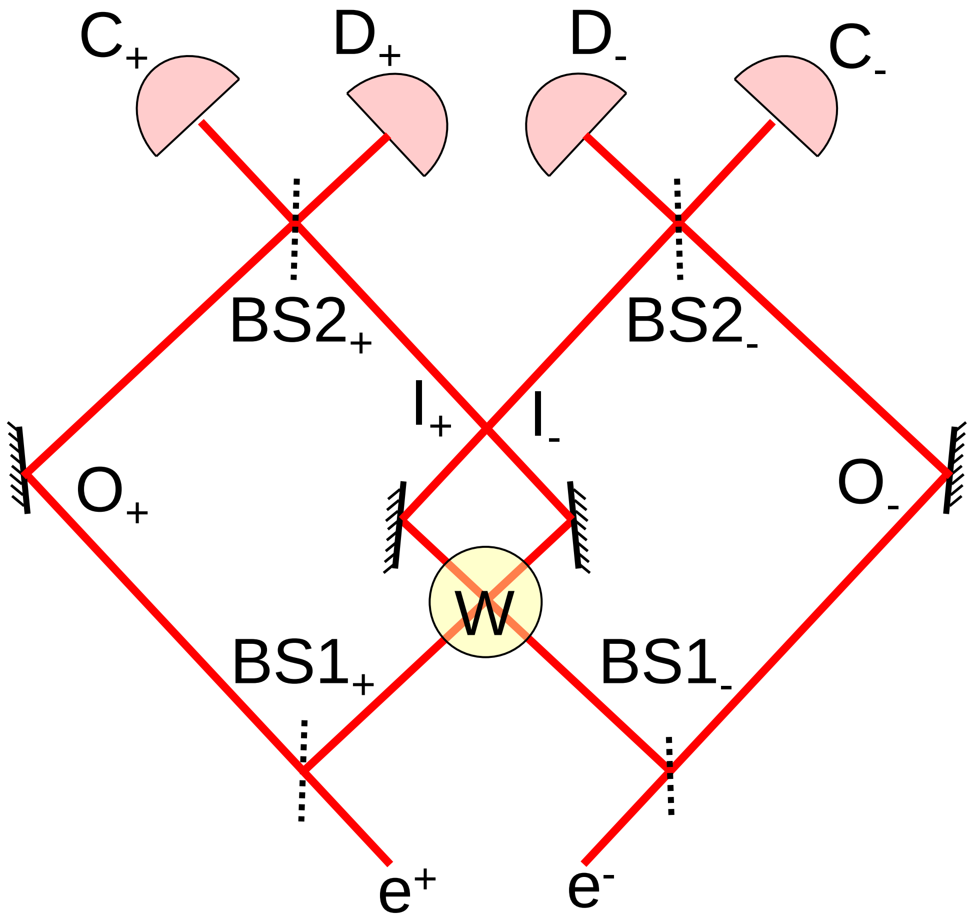
Not only is this an inappropriate conclusion, but:

- **QM says you can tell that a tree *would have made a sound had it fallen*, even if it doesn't fall!**

- **QM is not a theory of what happens, but of all the possible things which could happen.**

Hardy's Paradox

(for Elitzur-Vaidman "interaction-free measurements")



Outcome	Prob
D_+ and C_-	1/16
D_- and C_+	1/16
C_+ and C_-	9/16
D_+ and D_-	1/16
Explosion	4/16

D+ e- was in

D- e+ was in

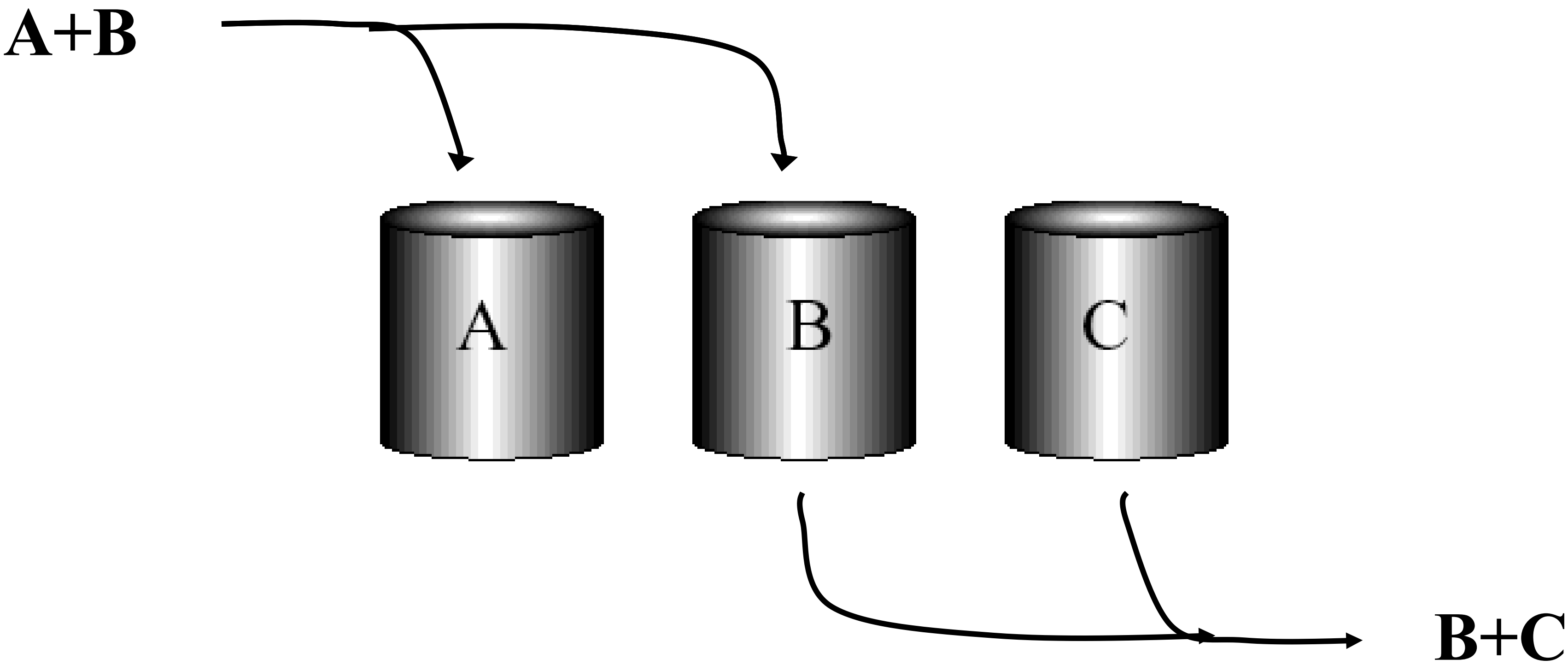
D+D- ?

But… if they were both in, they should have annihilated!

Can we talk about what goes on behind closed doors?

**("Postselection" is the big new buzzword in QIP...
but how should one describe post-selected states?)**

Predicting the past...



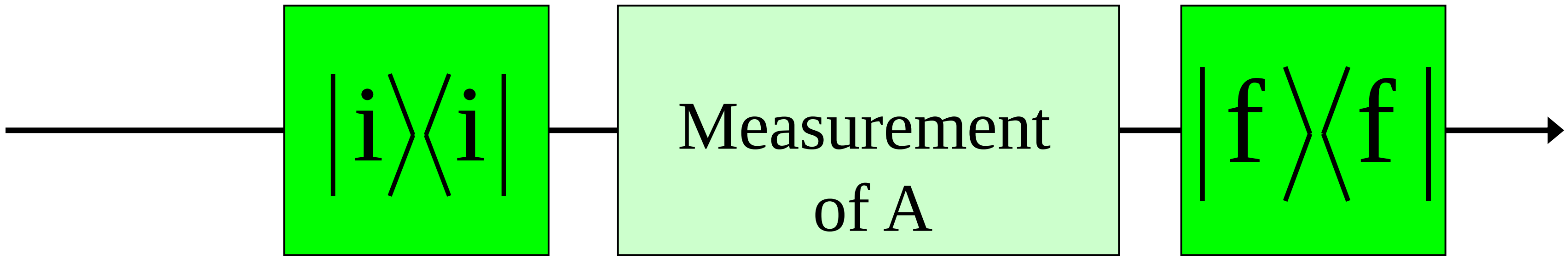
What are the odds that the particle was in a given box (e.g., box B)?

It had to be in B, with 100% certainty.

Conditional measurements (Aharonov, Albert, and Vaidman)

AAV, PRL 60, 1351 ('88)

Prepare a particle in |i> …try to "measure" some observable A…
***postselect* the particle to be in |f>**

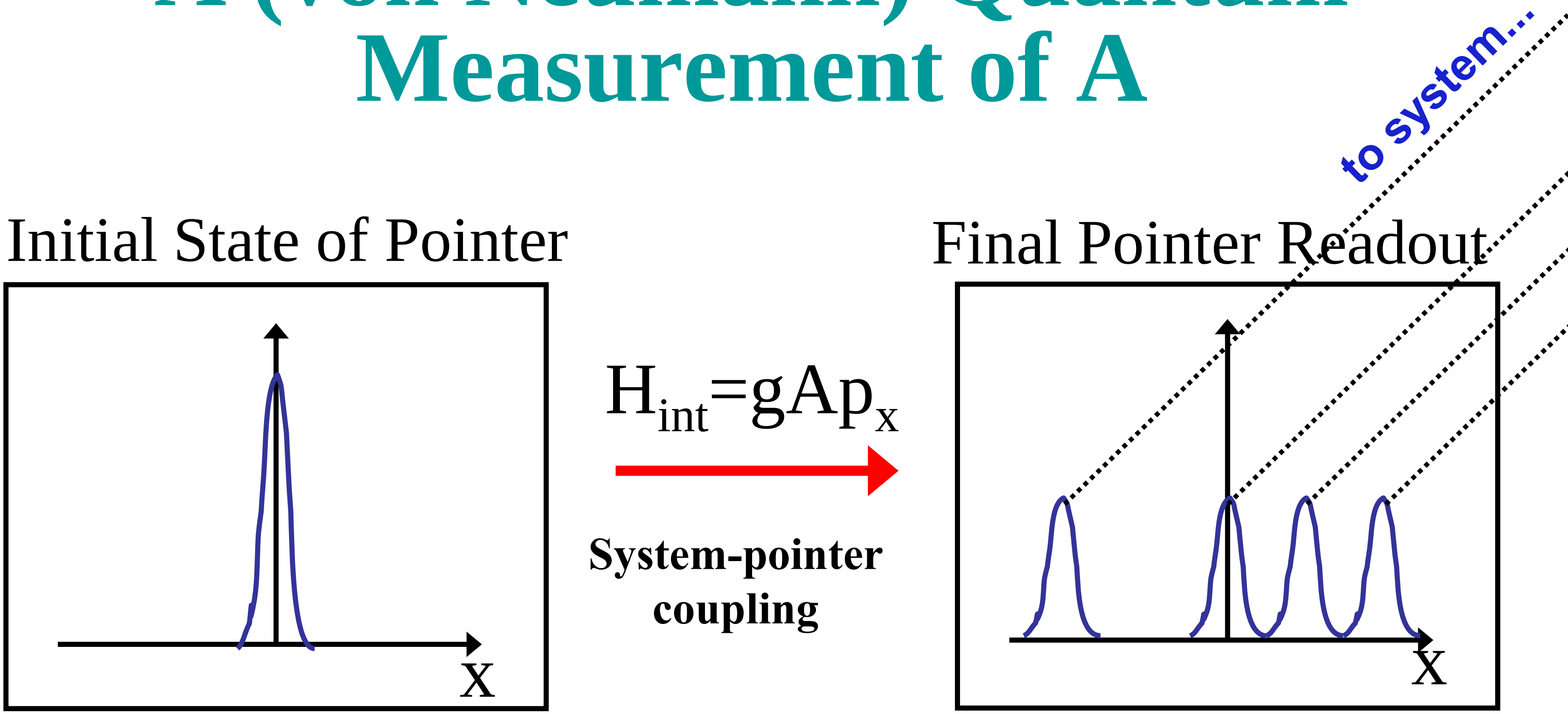


Does <A> depend more on i or f, or equally on both?
Clever answer: both, as Schrödinger time-reversible.
Conventional answer: i, because of collapse.

Reconciliation: measure A "weakly."
Poor resolution, but little disturbance.

⇒ **the "weak value"**
(but how to determine?)

A (von Neumann) Quantum Measurement of A



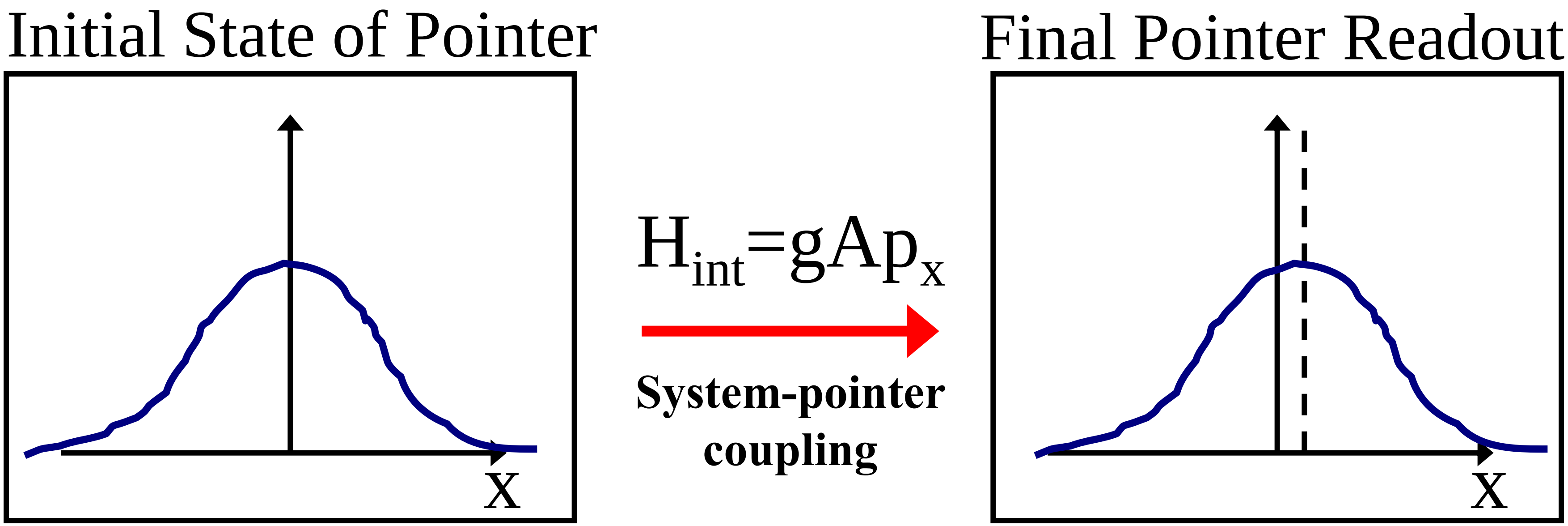
Well-resolved states

System and pointer become entangled

Decoherence / "collapse"

Large back-action

A Weak Measurement of A



Poor resolution on each shot.

Negligible back-action (system & pointer separable)

Strong: $|\Psi\rangle_s \phi_p(x) \rightarrow \sum_i c_i |\psi_i\rangle_s \phi_p(x - g a_i)$

Weak: $|\Psi\rangle_s \phi_p(x) \rightarrow |\Psi\rangle_s \phi_p(x - g\langle A_s\rangle)$

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$$

Back to Hardy's Paradox...

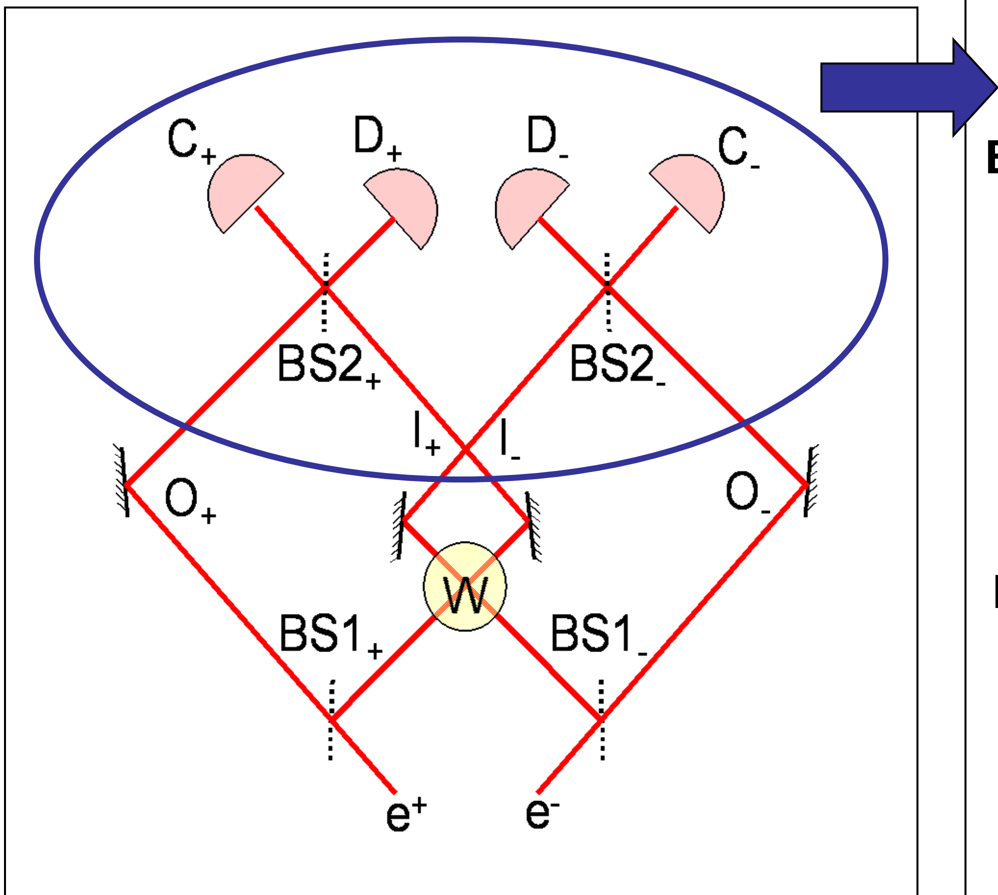
Interpretational digression

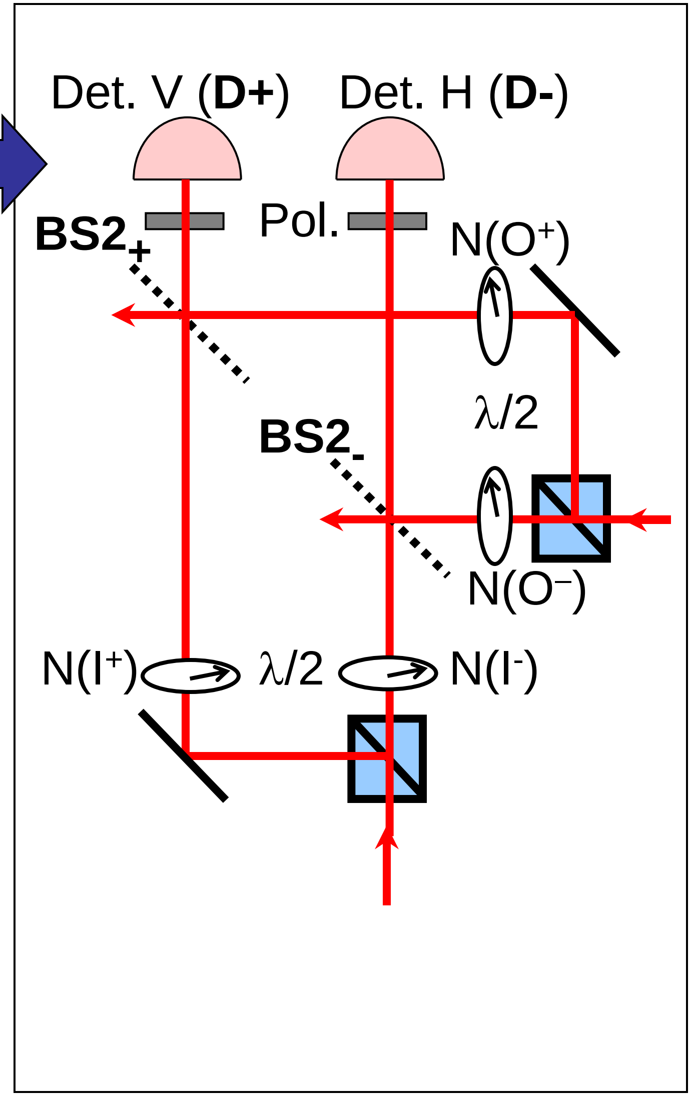
Note: Hardy's reading of his paradox (filtered through me) is that it's simply not fair to ascribe real values to *potential* measurements, without knowing which sets of measurements are really going to be done -- quantum mechanics is known to be *contextual.*

Weak measurements, on the other hand, are *non*-contextual, and allow one to ask what properties a system had before post-selection.

What questions is one really allowed to ask?

Weak Measurements in Hardy's Paradox





But what can we say about where the particles were or weren't, once D+ & D– fire?

Probabilities	e- in	e- out	
e+ in	**0**	1	1
e+ out	1	**–1**	0
	1	0	

AND THESE ARE IN FACT THE PREDICTIONS FOR THE WEAK VALUES!

Y. Aharonov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen, PLA 301, 130 (2002); quant-ph/0104062

How to measure (weak) *joint* probabilities?

Resch &Steinberg, PRL 92,130402 ('04)

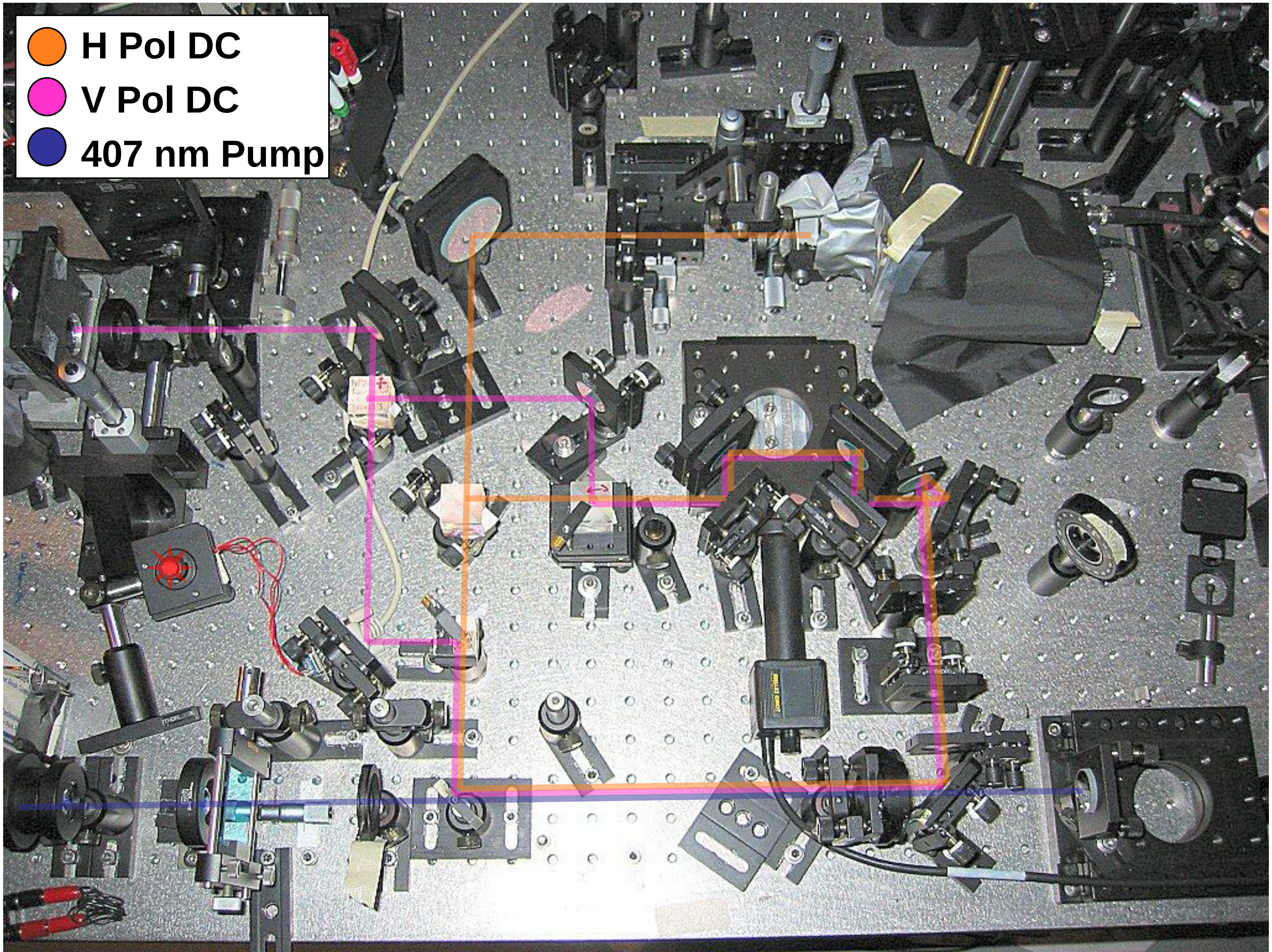
Use *two* pointers and couple individually to the two observables of interest ("A" and "B");
then use their *correlations* to draw conclusions about P_{AB}.

$$H_{int} = g_A |A\rangle\langle A| p_x + g_B |B\rangle\langle B| p_y$$

We have shown that the real part of P_{ABW} can be extracted from such correlation measurements:

$$\mathrm{Re}(P_{ABW}) = \frac{2\langle xy\rangle}{g_A g_B t^2} - \mathrm{Re}(P^*_{AW} B_{BW})$$

Using a “photon switch” to implement Hardy’s Paradox



Weak Measurements in Hardy's Paradox

Ideal Weak Values

	$N(I^-)$	$N(O^-)$	
$N(I^+)$	0	1	1
$N(O^+)$	1	–1	0
	1	0	

Experimental Weak Values ("Probabilities"?)

	$N(I^-)$	$N(O^-)$	
$N(I^+)$	0.243±0.068	0.663±0.083	0.882±0.015
$N(O^+)$	0.721±0.074	–0.758±0.083	0.087±0.021
	0.925±0.024	–0.039±0.023	

J.S. Lundeen and A.M. Steinberg, *Phys. Rev. Lett.* **102**, 020404 (2009);
also Yokota *et al.*, *New. J. Phys.* **11**, 033011 (2009).

Can we understand what is really happening physically?

$$\mathrm{Re}\,\langle \hat{\sigma}^{-}_{zP}\hat{\sigma}^{-}_{zE}\rangle = \frac{R_{\nearrow\nearrow} + R_{\nwarrow\nwarrow} - R_{\nwarrow\nearrow} - R_{\nearrow\nwarrow}}{R_{\nearrow\nearrow} + R_{\nwarrow\nwarrow} + R_{\nwarrow\nearrow} + R_{\nearrow\nwarrow}}$$

$$\langle \hat{N}(M_P)\hat{N}(M_E)\rangle_W = g^{-2}\mathrm{Re}\langle \hat{\sigma}^{-}_{zP}\hat{\sigma}^{-}_{zE}\rangle, \qquad = \frac{R_{\circlearrowleft\circlearrowleft} + R_{\circlearrowright\circlearrowright} - R_{\circlearrowright\circlearrowleft} - R_{\circlearrowleft\circlearrowright}}{R_{\circlearrowleft\circlearrowleft} + R_{\circlearrowright\circlearrowright} + R_{\circlearrowright\circlearrowleft} + R_{\circlearrowleft\circlearrowright}},$$

TABLE I. The measured coincidence rates needed to determine the weak values.

E	P	$R_{\nearrow\nearrow}$	$R_{\nearrow\nwarrow}$	$R_{\nwarrow\nwarrow}$	$R_{\nwarrow\nearrow}$	$R_{\circlearrowleft\circlearrowleft}$	$R_{\circlearrowright\circlearrowright}$	$R_{\circlearrowleft\circlearrowright}$	$R_{\circlearrowright\circlearrowleft}$	g_E	g_P
O	O	556	834	583	730	750	543	666	571	0.674	0.541
I	I	2261	772	115	746	1030	762	913	729	0.635	0.570
I	O	1152	1079	351	179	484	655	452	654	0.635	0.541
O	I	1051	260	329	769	715	609	388	825	0.674	0.570

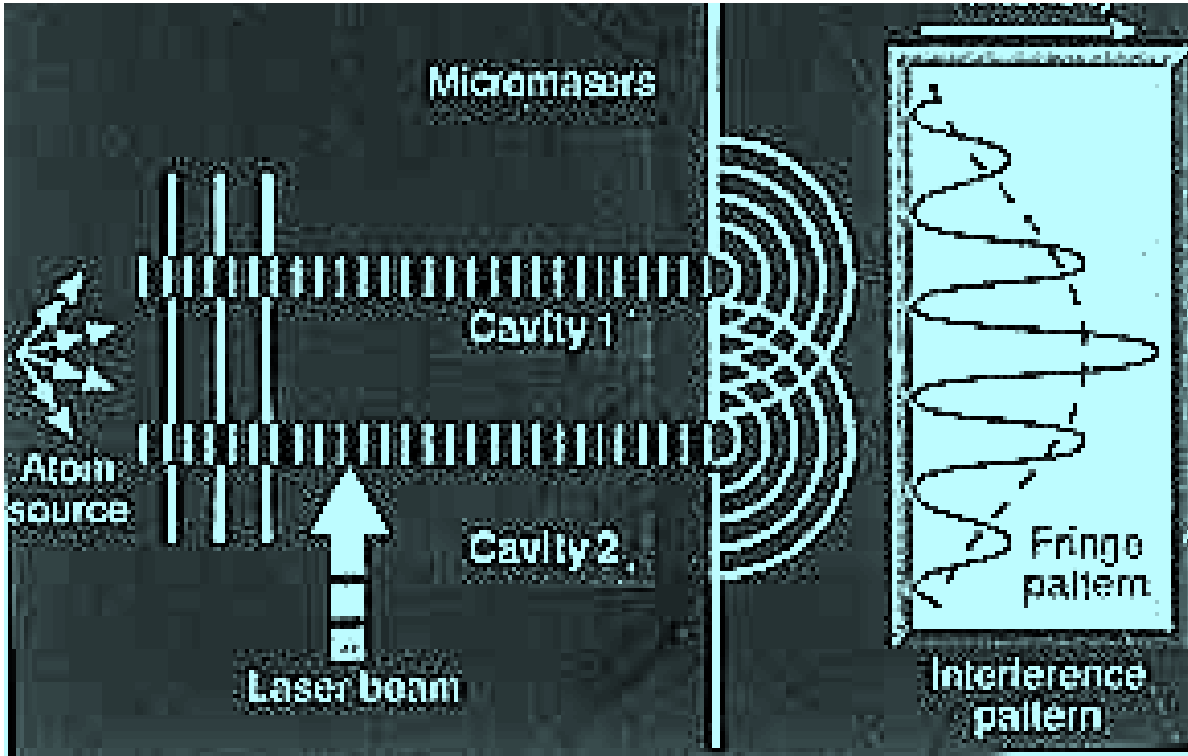
What is the meaning of the negative joint occupation? Recall that the joint values are extracted by studying the polarization rotation of both photons in coincidence. Consider a situation in which both photons always simultaneously passed through two particular arms. When a polarization rotator is placed in each of these arms, it would tend to cause their polarizations to rotate in a correlated fashion; when P was found to have 45° polarization, E would also be more likely to be found at 45° than −45°. Experimentally, we find the reverse—when P is found to have 45° polarization, E is preferentially found at −45° (and vice versa), as though it had rotated in the direction opposite to the one induced by the physical wave plate. As in all weak-measurement experiments, a negative

Some other experiments using weak measurement to study foundations...

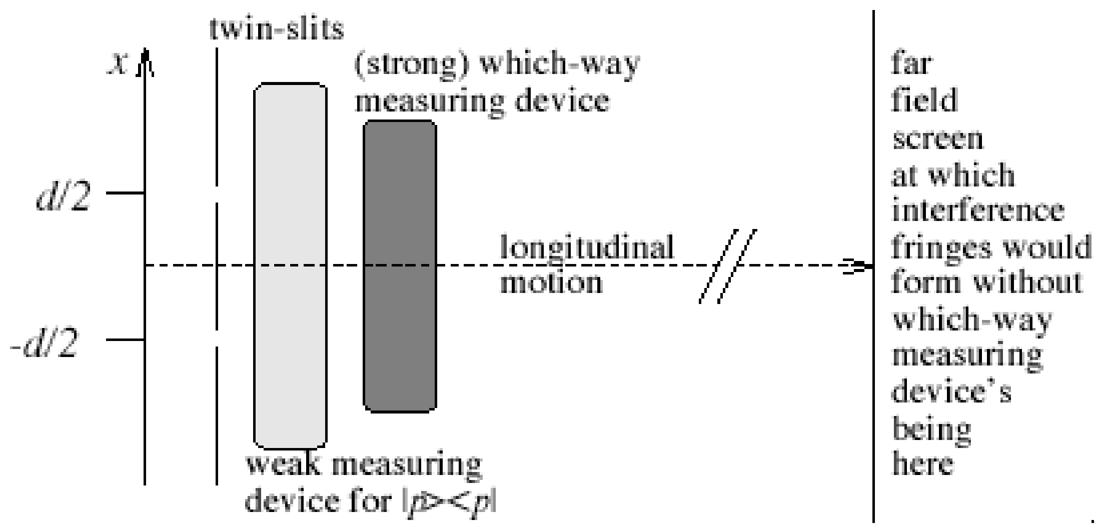
Which-path controversy (Scully, Englert, Walther vs the world?)

[Reza Mir *et al.*, New. J. Phys. **9**, 287 (2007)]

Which-path measurements destroy interference.
This is usually explained via measurement backaction & HUP.
Suppose we use a *microscopic* pointer.
Is this really irreversible, as Bohr would have all measurements?
Need it disturb momentum?
Which is «more fundamental» – uncertainty or complementarity?



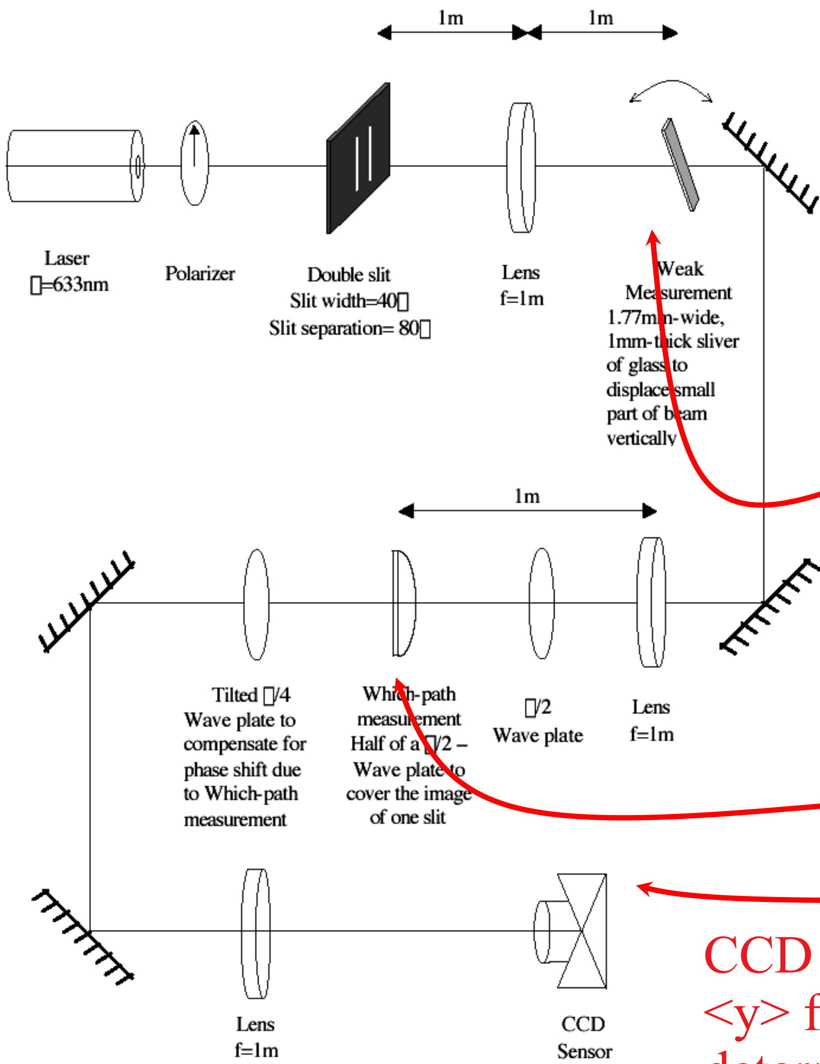
Weak measurements to the rescue!



To find the probability of a given momentum transfer, measure the *weak probability* of each possible initial momentum, conditioned on the final momentum observed at the screen...

Wiseman, PLA 311, 285 (2003)

Convoluted implementation...



Glass plate in focal plane measures $P(p_i)$ weakly (shifting photons along y)

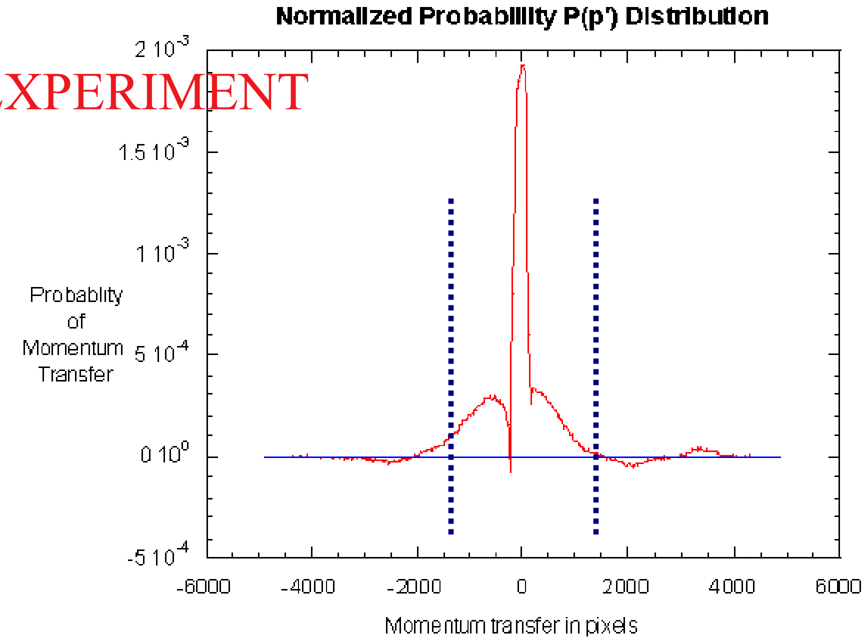
Half-half-waveplate in image plane measures path strongly

CCD in Fourier plane measures $\langle y \rangle$ for each position x; this determines $\langle P(p_i) \rangle_{wk}$ for each final momentum p_f.

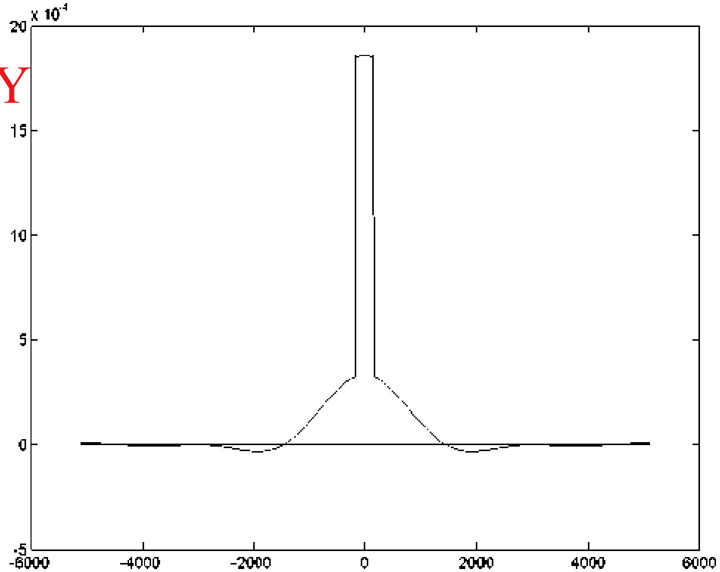
The distribution of the integrated momentum *transfer*

R. Mir *et al.*, New. J. Phys. 9, 287 (2007)

EXPERIMENT



THEORY



Note: the distribution extends well beyond h/d.

On the other hand, all its moments are (at least in theory, so far) 0.

The former fact agrees with Walls *et al*; the latter with Scully *et al*.

For weak distributions, they may be reconciled *because the distributions may take negative values* in weak measurement.

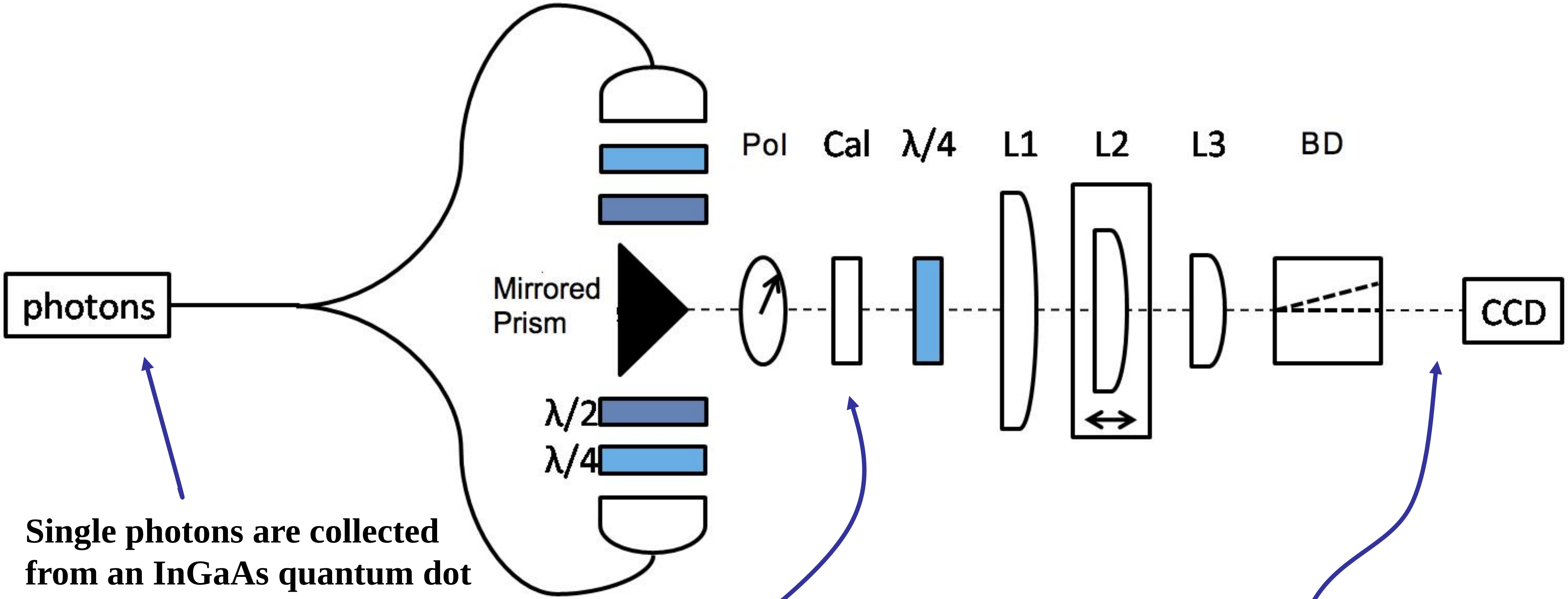
Can we follow trajectories in the interferometer too?

Bohmian trajectories:

QuickTime™ and a
decompressor
are needed to see this picture.

Prediction: Weak measurement will reveal these trajectories [Wiseman New. J. Phys. 9, 165 (2007)]

Weakly measuring photon trajectories



Single photons are collected from an InGaAs quantum dot provided by NIST.

A birefringent calcite crystal weakly measures propagation direction by creating a small k-dependent polarisation rotation.

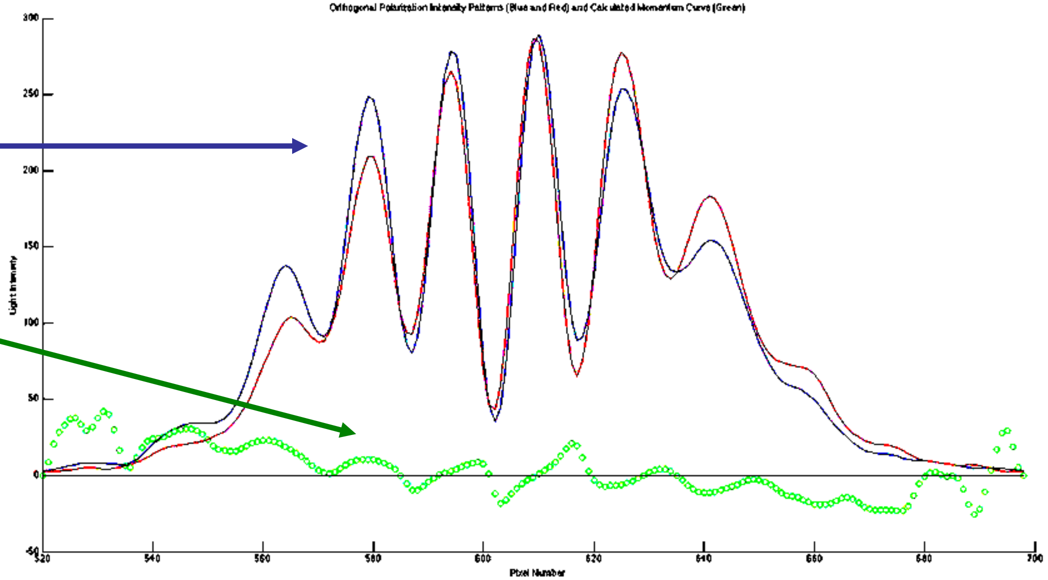
Imaging a chosen plane on a CCD camera allows us to postselect on position. The pol. rotation at each x is measured by subtracting two copies of the fringe pattern, one for H and one for V.

Some early data

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

Raw data

Local momentum extracted from subtraction



Reconstructed trajectories

QuickTime™ and a decompressor are needed to see this picture.

S. Kocsis *et al.*,
Science 332, 1170 (2011)

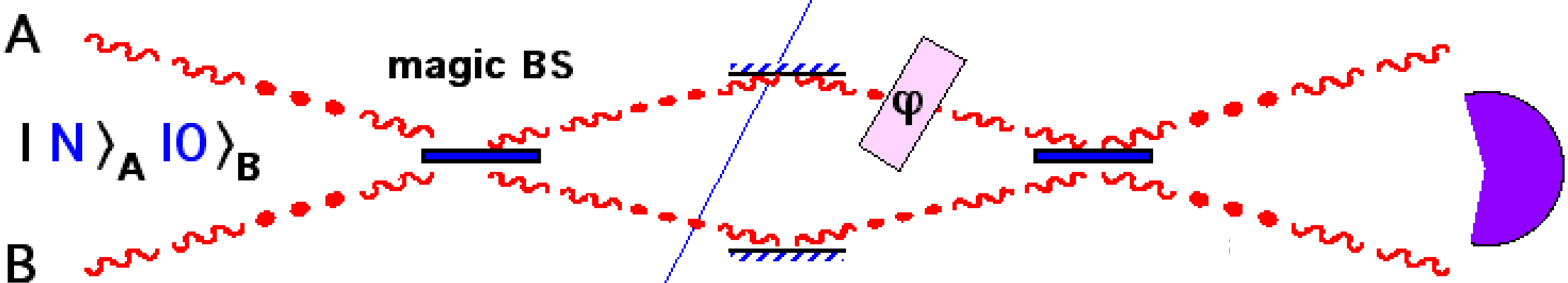
Using measurement to make entanglement; *and* using entanglement to make better measurements...

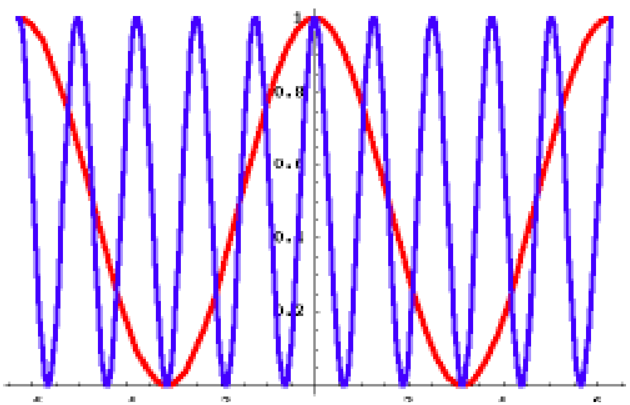
Jon Dowling's Slide of Magic BS

N Photons

$$|N\rangle_A|0\rangle_B + e^{iN\varphi}|0\rangle_A|N\rangle_B$$

N-Photon Detector





Oscillates N times as fast

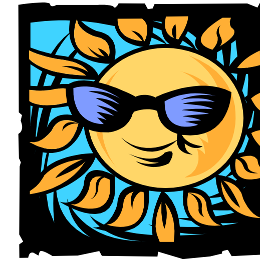
$$\frac{1+\cos\varphi}{2}$$ uncorrelated

$$\frac{1+\cos N\varphi}{2}$$ correlated

φ = kx

Δφ: $1/\sqrt{N} \rightarrow 1/N$

Highly number-entangled states ("3003" experiment).



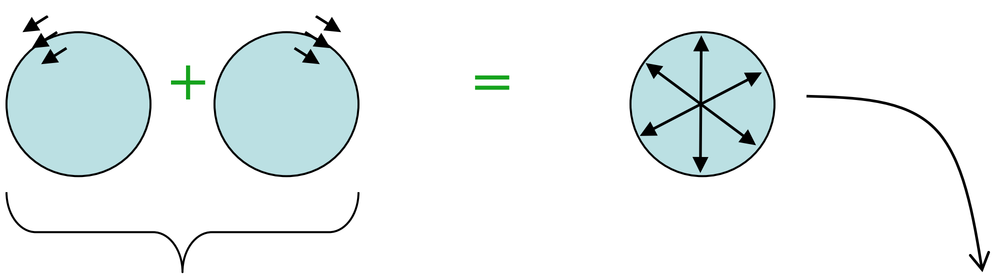
M.W. Mitchell *et al.*, Nature **429**, 161 (2004)

States such as |N,0> + |0,N> ("N00N" states) could revolutionize metrology (from atomic clocks to optical-interferometric sensing), and have been proposed for lithography as well.

But how to make them?

Important factorisation:

$$\left(a^{\dagger 3} + b^{\dagger 3}\right) = \left(a^{\dagger} + b^{\dagger}\right)\left(a^{\dagger} + e^{2\pi i/3} b^{\dagger}\right)\left(a^{\dagger} + e^{-2\pi i/3} b^{\dagger}\right)$$

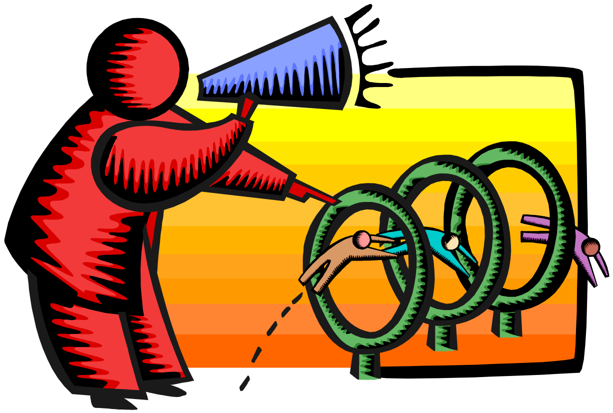


A "noon" state

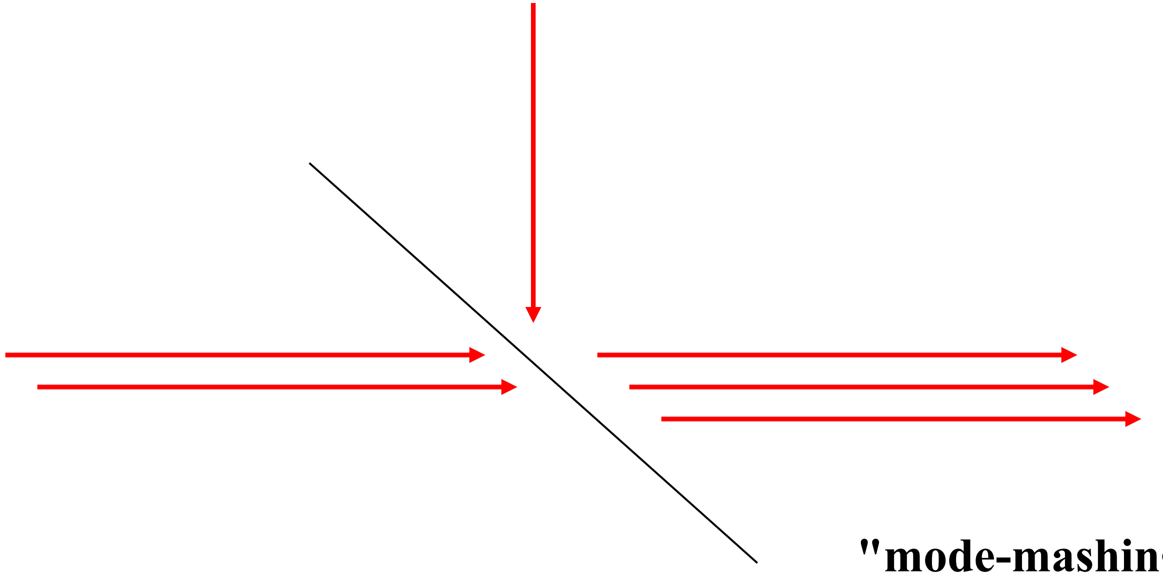
A really odd beast: one 0° photon, one 120° photon, and one 240° photon... but of course, you can't tell them apart, let alone combine them into one mode!

Theory: H. Lee *et al.*, Phys. Rev. A 65, 030101 (2002); J. Fiurásek, Phys. Rev. A 65, 053818 (2002)

Trick #1



How to combine three non-orthogonal photons into one spatial mode?



"mode-mashing"

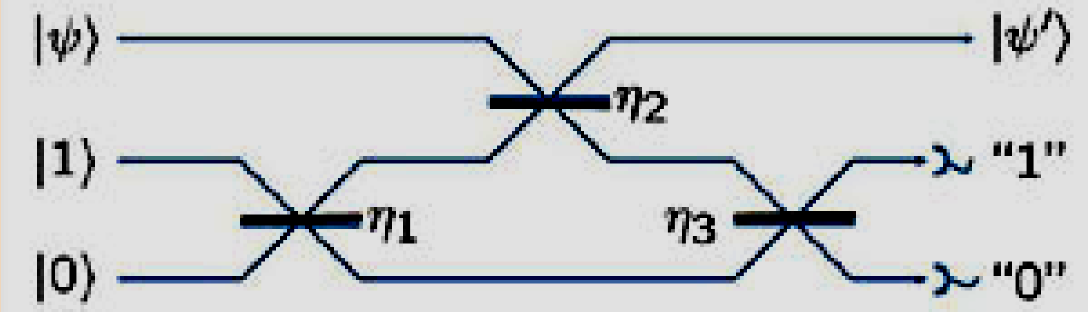
Yes, it's that easy! If you see three photons out one port, then they all went out that port.

⇒ Post-selective nonlinearity

Cf. “KLM”: measurement itself can act as an entangling logic gate!

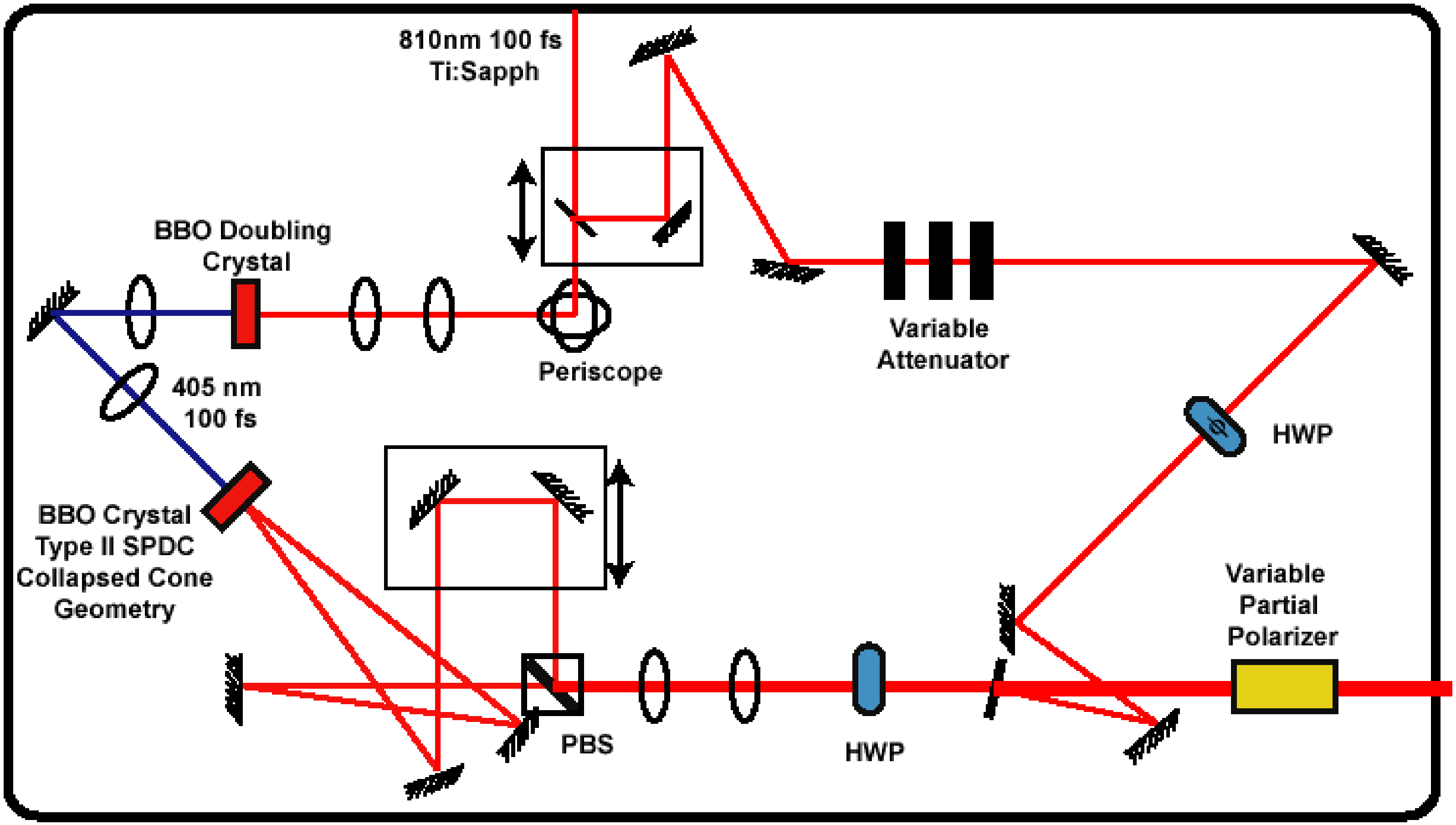
INPUT STATE a|0> + b|1> + c|2>

OUTPUT STATE a|0> + b|1> – c|2>

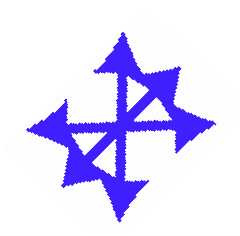


Knill, Laflamme, Milburn Nature 409, 46, (2001), and others after; cf. also Raussendorf & Briegel, Phys Rev Lett 86, 5188 (2001).

Making triphoton states...



E.g.,



$HV(H+V) \approx R^3 + R^2L + RL^2 + L^3$

In HV basis, $H^2V + HV^2$ looks "number-squeezed"; in RL basis, phase-squeezed.

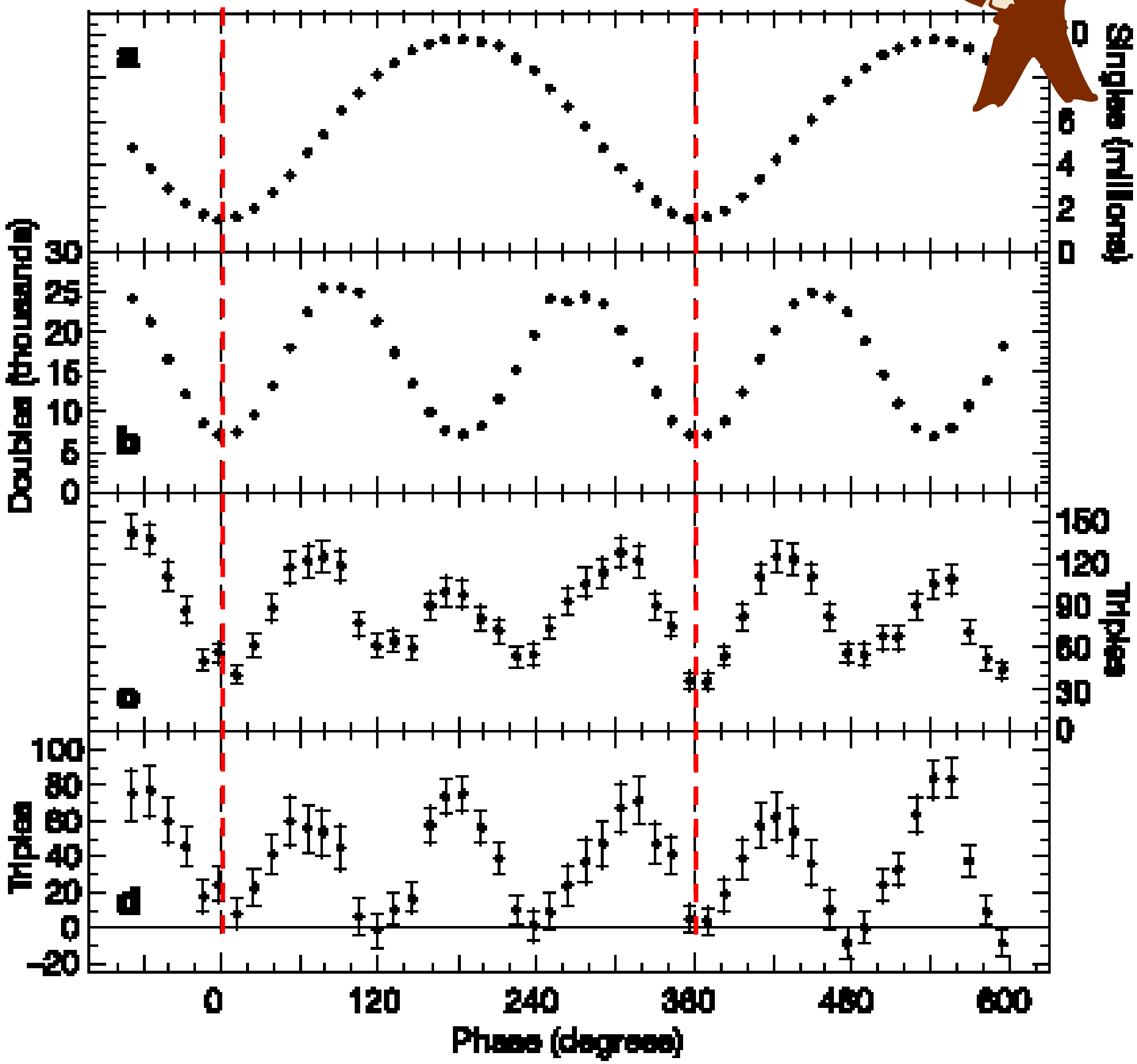
It works!

Singles:

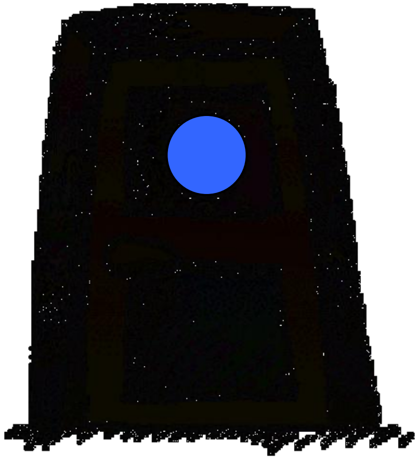
Coincidences:

Triple coincidences:

Triples (bg subtracted):



M.W. Mitchell, J.S. Lundeen, and A.M. Steinberg, Nature 429, 161 (2004)



A glimpse at a few other things in progress...

Is weak measurement good for anything *practical*?

$$A_w = \frac{\langle f|A|i\rangle}{\langle f|i\rangle}$$

may be very big if the postselection (<f|i>) is very unlikely...

QuickTime™ and a
decompressor
are needed to see this picture.

Can one photon act like many photons?

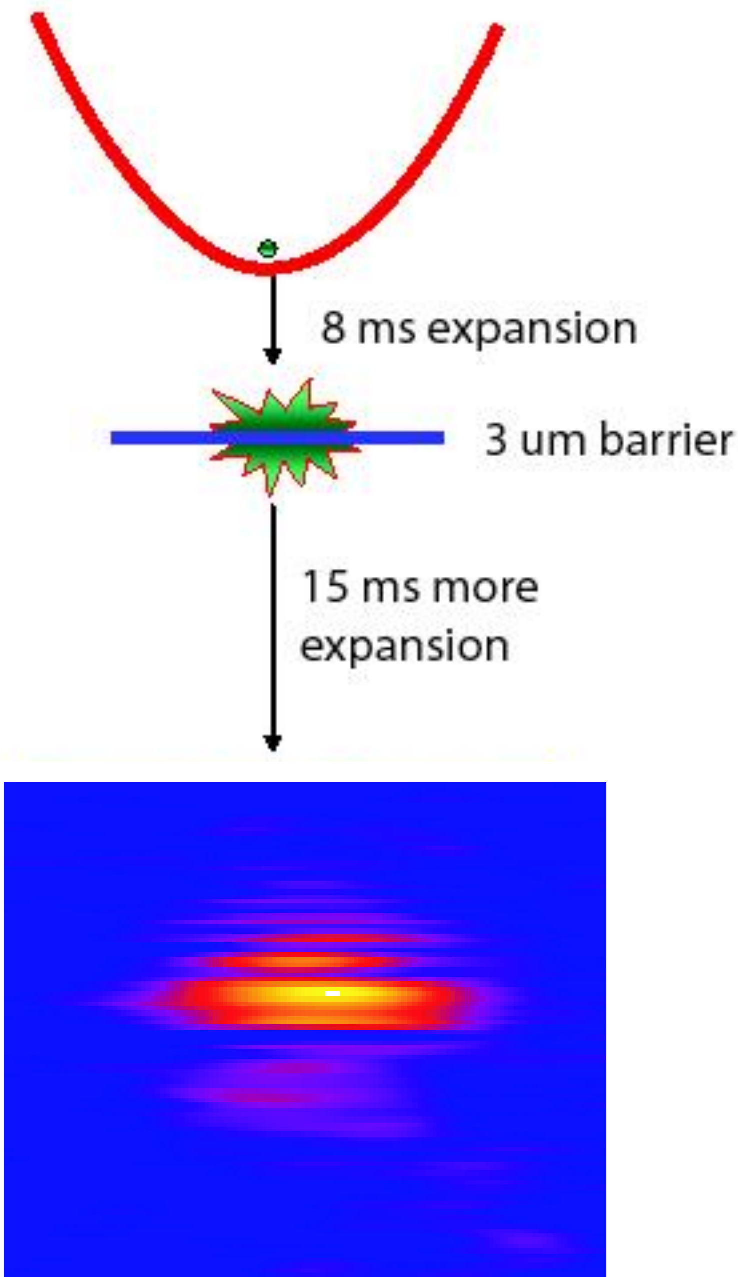
QuickTime™ and a
decompressor
are needed to see this picture.

$\langle n \rangle_w$ may be >> 1.

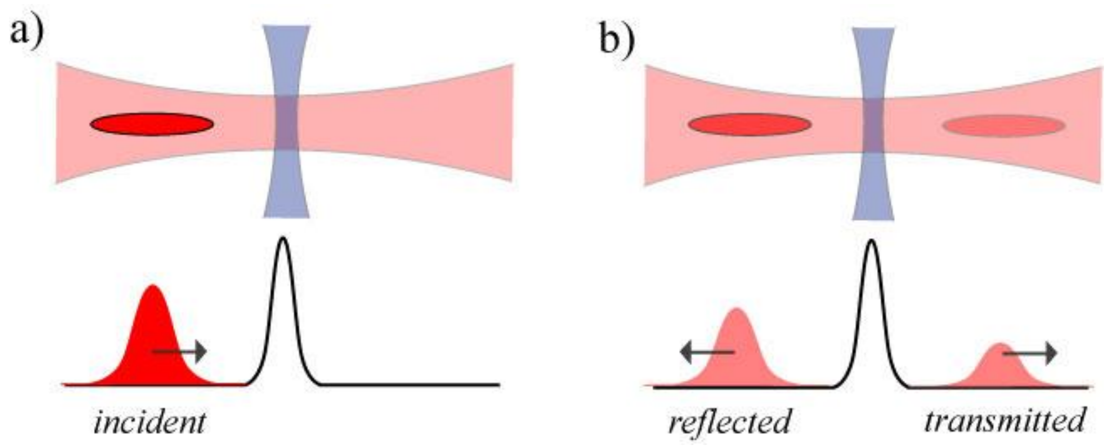
When the post-selection succeeds, the phase shift on the probe may be much larger than the phase shift due to a single photon -- *even though there only ever* is *at most one signal photon*!

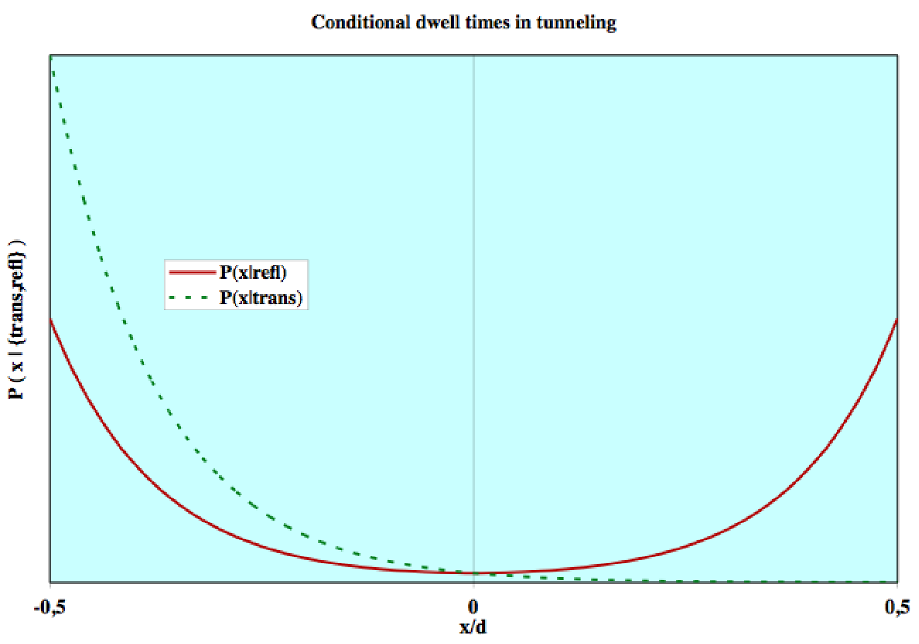
Weak Measurement Amplification of Single-Photon Nonlinearity,
Amir Feizpour, Xingxing Xing, and Aephraim M. Steinberg
Phys Rev Lett 107, 133603 (2011)

Measuring the phase of an atom



Measuring the tunneling time?





The morals of the story, again

1\. **There are many *different* "quantum measurements"!
And they are *good* for something.**

1\. **Post-selected systems often exhibit surprising behaviour which can be probed using weak measurements.**

2\. **These weak measurements may "resolve" various paradoxes... admittedly while creating new ones (negative probability)!**

4\. **All of the claims in Hardy's "paradox" are borne out by weak measurement.
Retrodiction (and "intradiction," to mangle some jargon) is alive and well in quantum mechanics.**

5\. **A postselected particle can be certain to have been in each of two places at the same time, yet can never be in both at the same time.**

6\. **A series of tunneling-time experiments is still under preparation at U of T.**