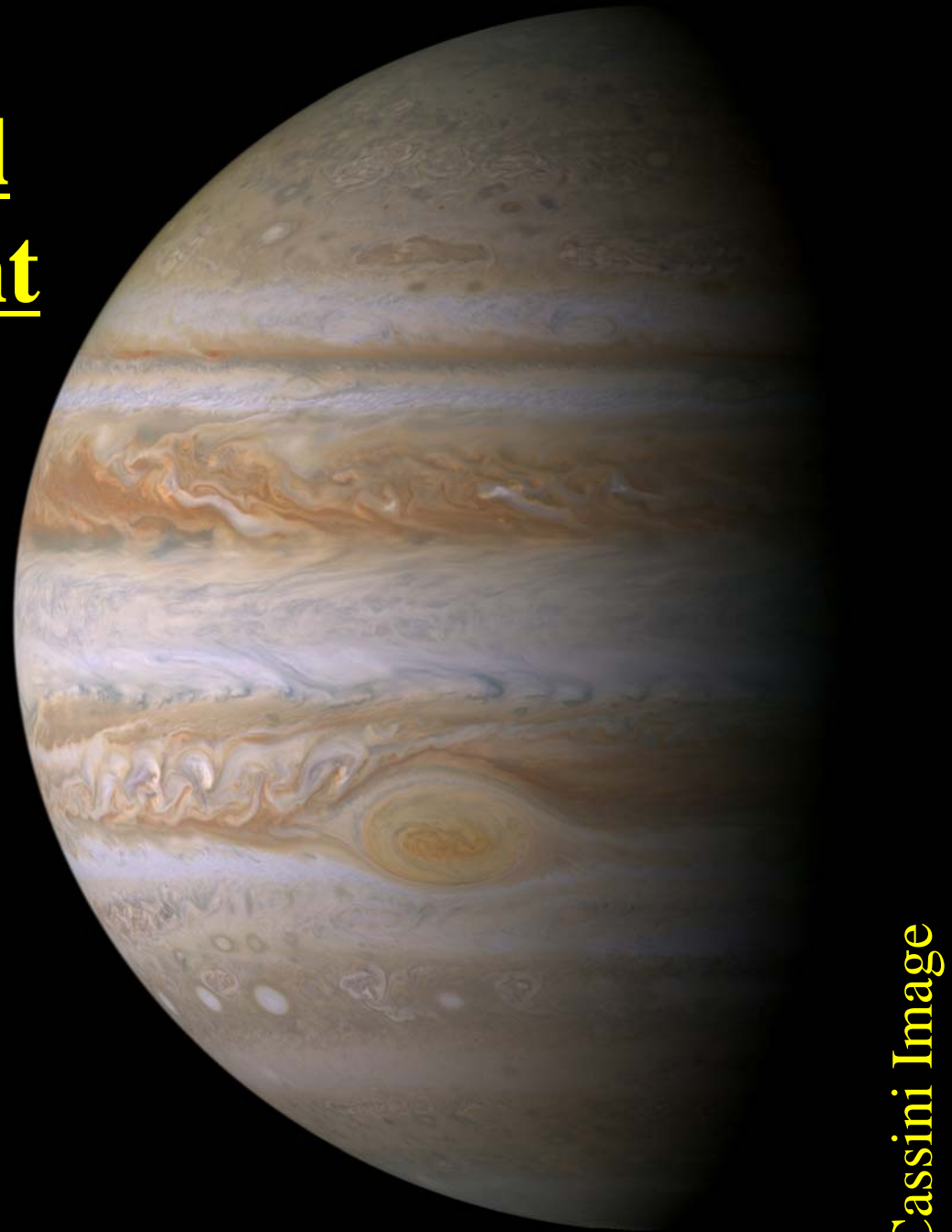


# Modeling Deep Convective Zonal Flows on the Giant Planets

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Cassini Image

# Acknowledgements

- Moritz Heimpel
  - University of Alberta
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- Johannes Wicht
  - Max Plank Institute for Aeronomy
  - [wicht@linmpi.mpg.de](mailto:wicht@linmpi.mpg.de)



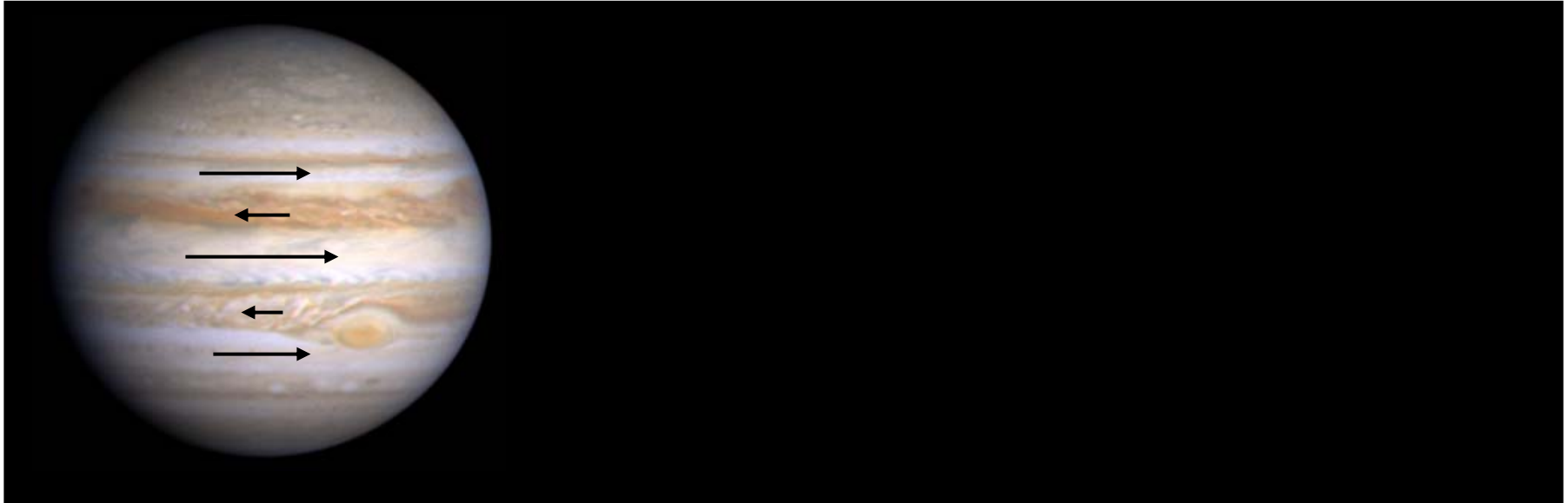
# Today's Seminar

- A new model of zonal flow dynamics on the Giant Planets
  1. Observations
  2. Shallow Layer Models
  3. Deep Convection Models
  4. Yano's Shallow Model
  5. Deep Jovian Convection Model
  6. Deep Convection Model for the Ice Giants

# 1. Observations of The Giant Planets

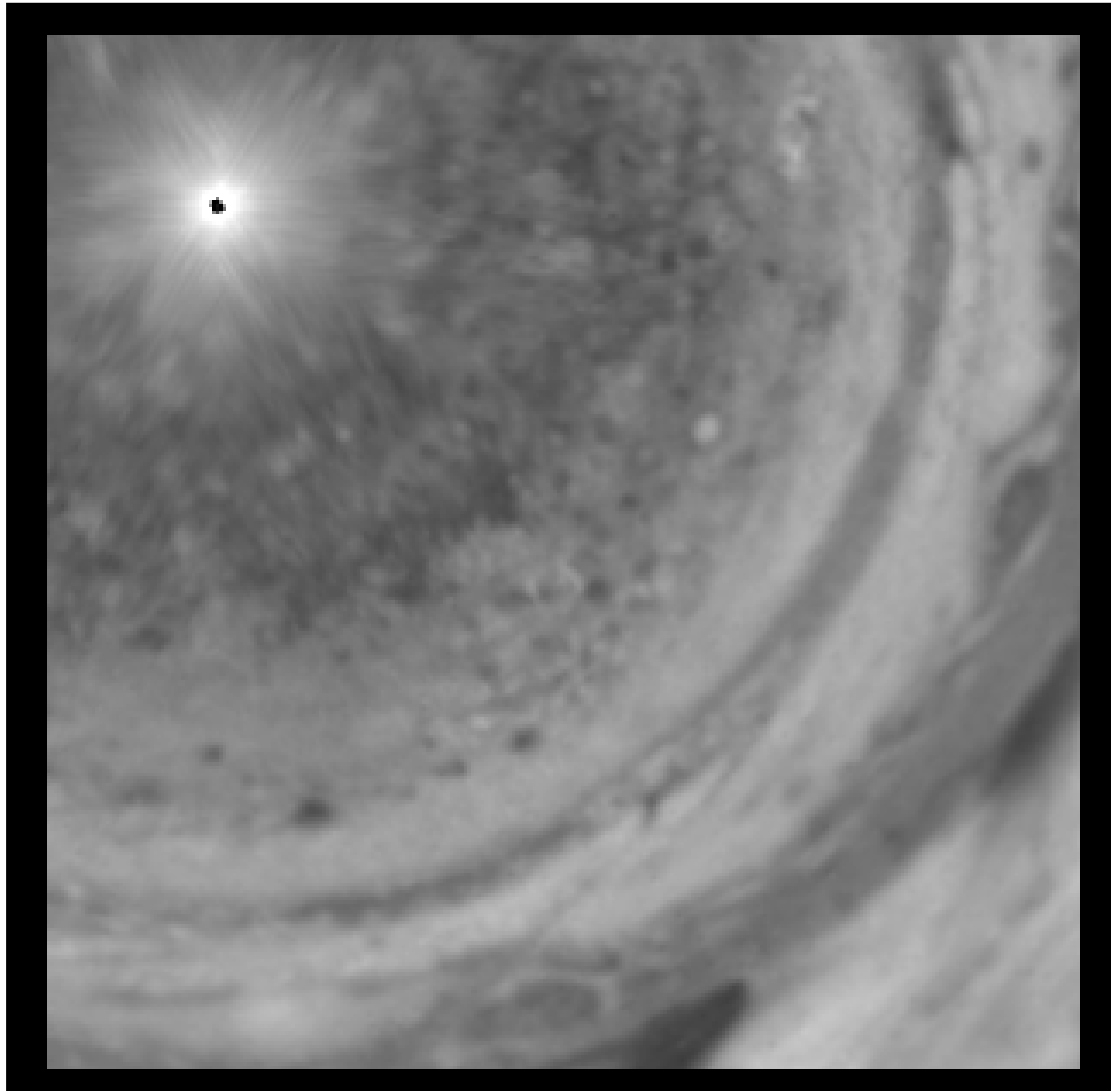
# Jovian Zonal Wind Observations

- Jovian winds movie from Cassini:

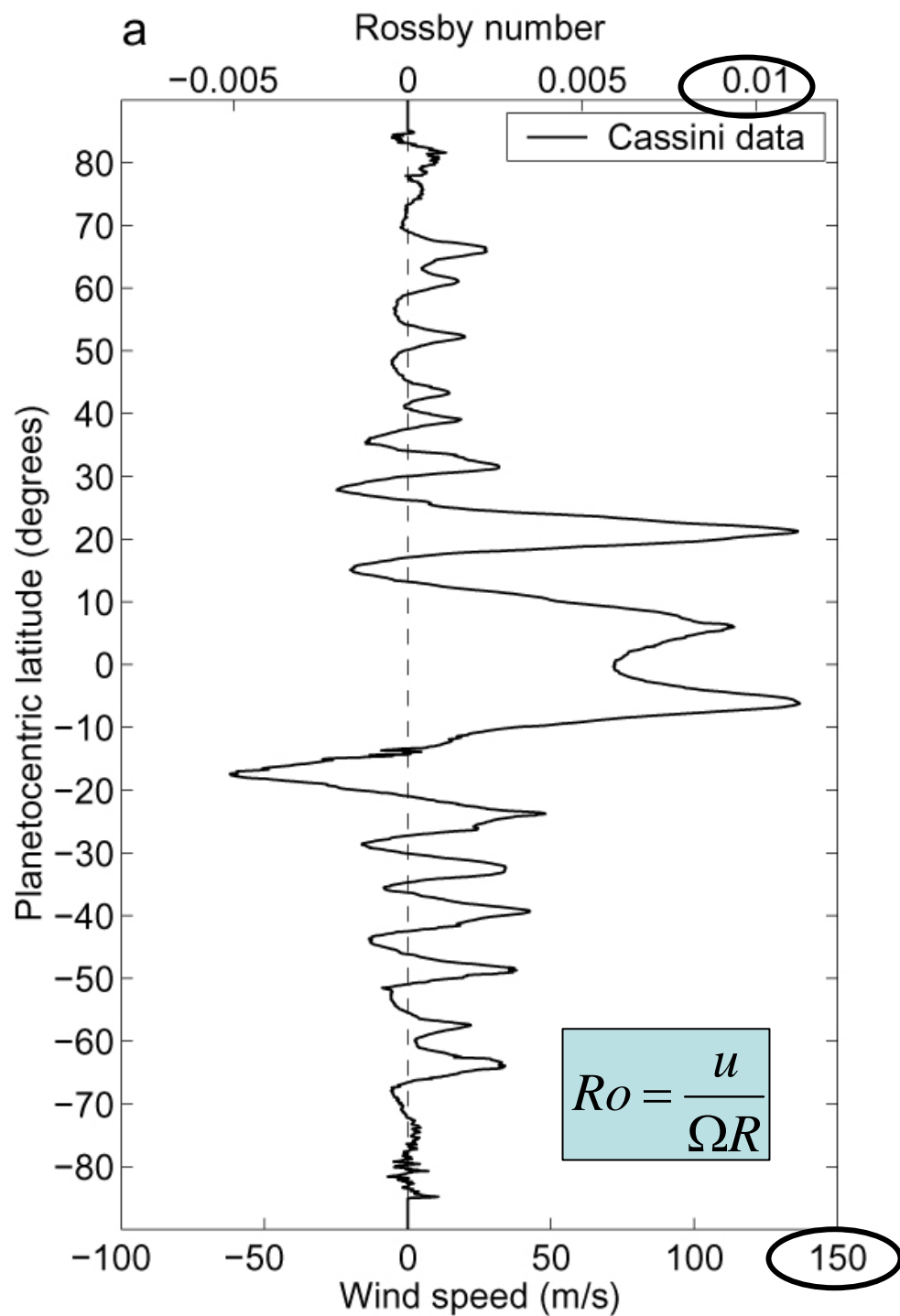


# Jovian Zonal Wind Observations

- Polar winds movie from Cassini:

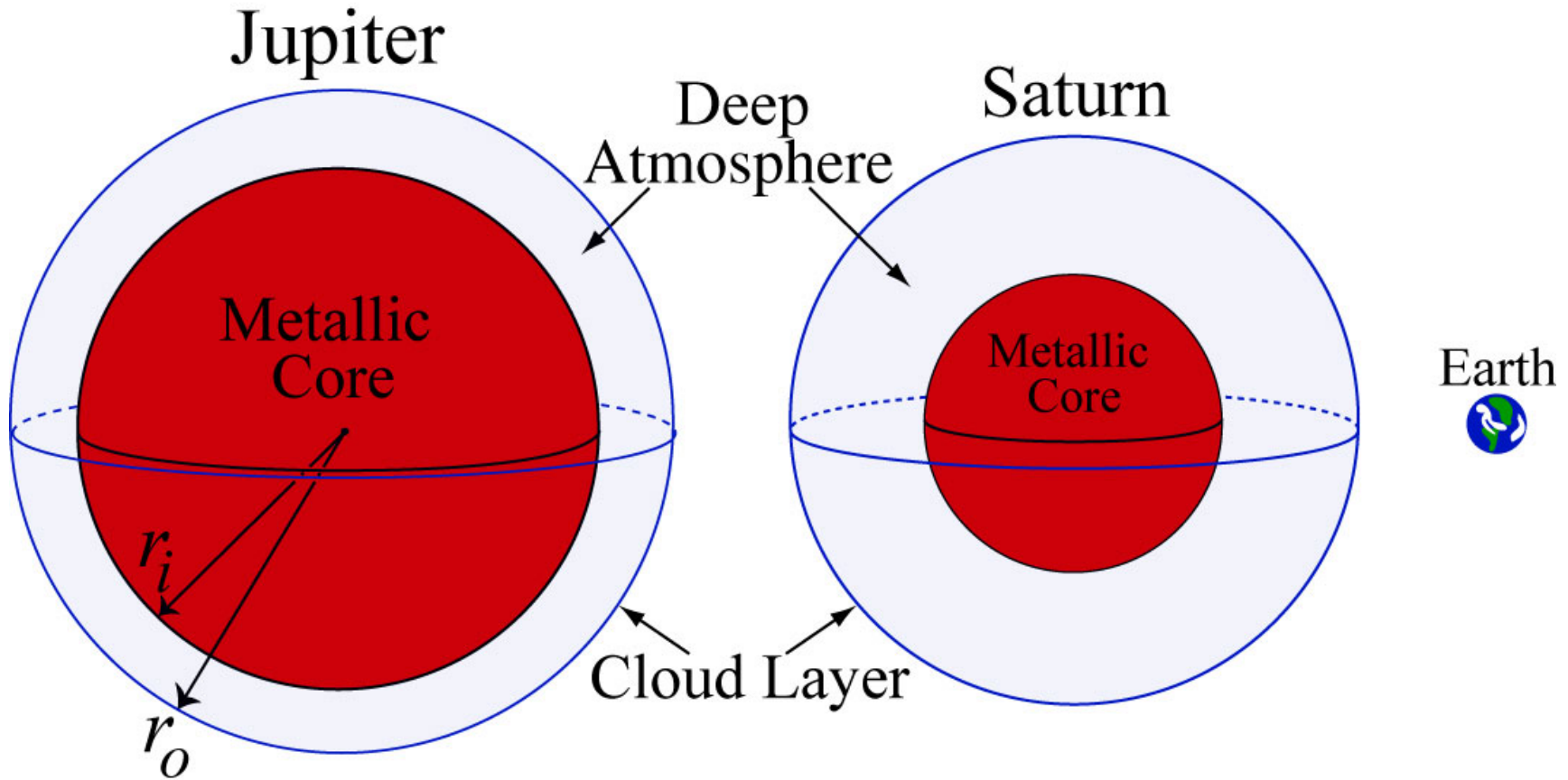


# Jovian Wind Observations



- East-west velocities w/r/t  $\mathbf{B}$ -field frame
- Powerful prograde equatorial jet
- Smaller wavelength higher latitude jets
- Fine scale jets near poles
- Net prograde winds

# Giant Planet Atmospheres

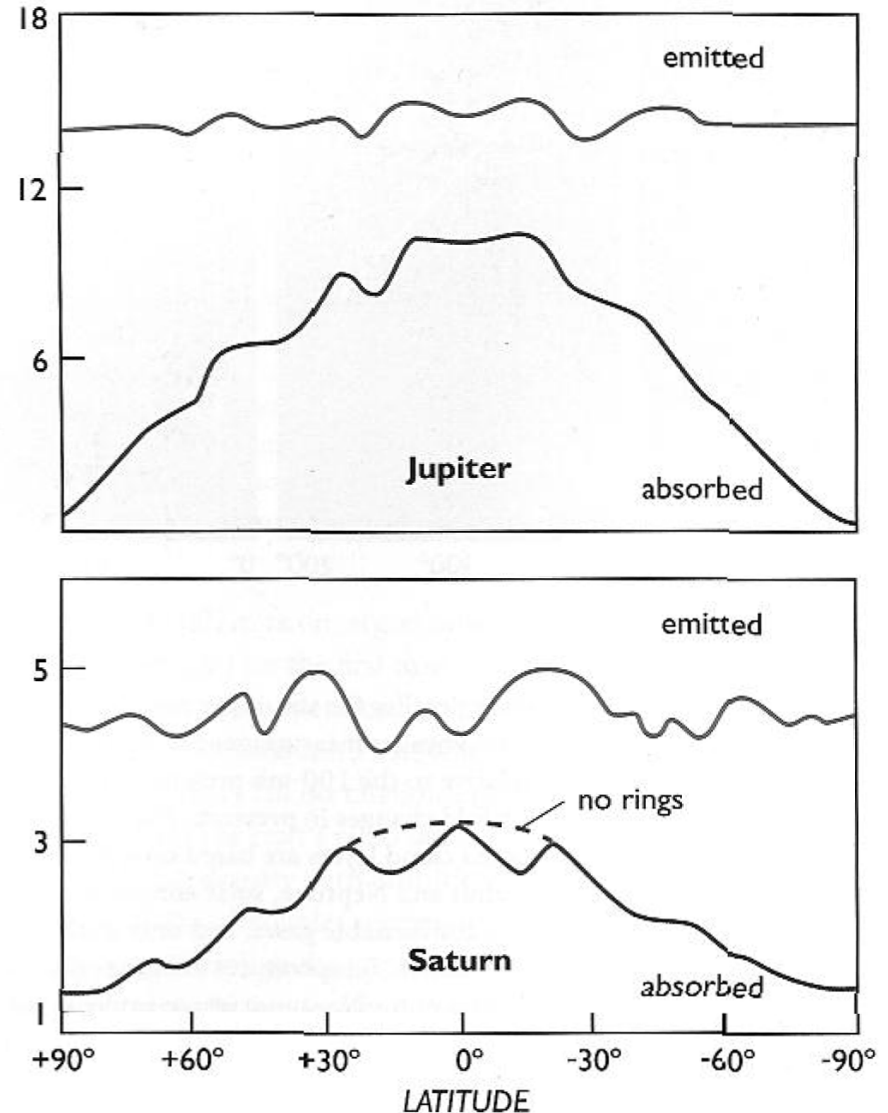


- Deep atmospheric shell aspect ratios ( $\chi = r_i/r_o$ )
  - Jupiter:  $\chi = 0.75 \sim 0.95$ , Saturn:  $\chi = 0.4 \sim 0.8$



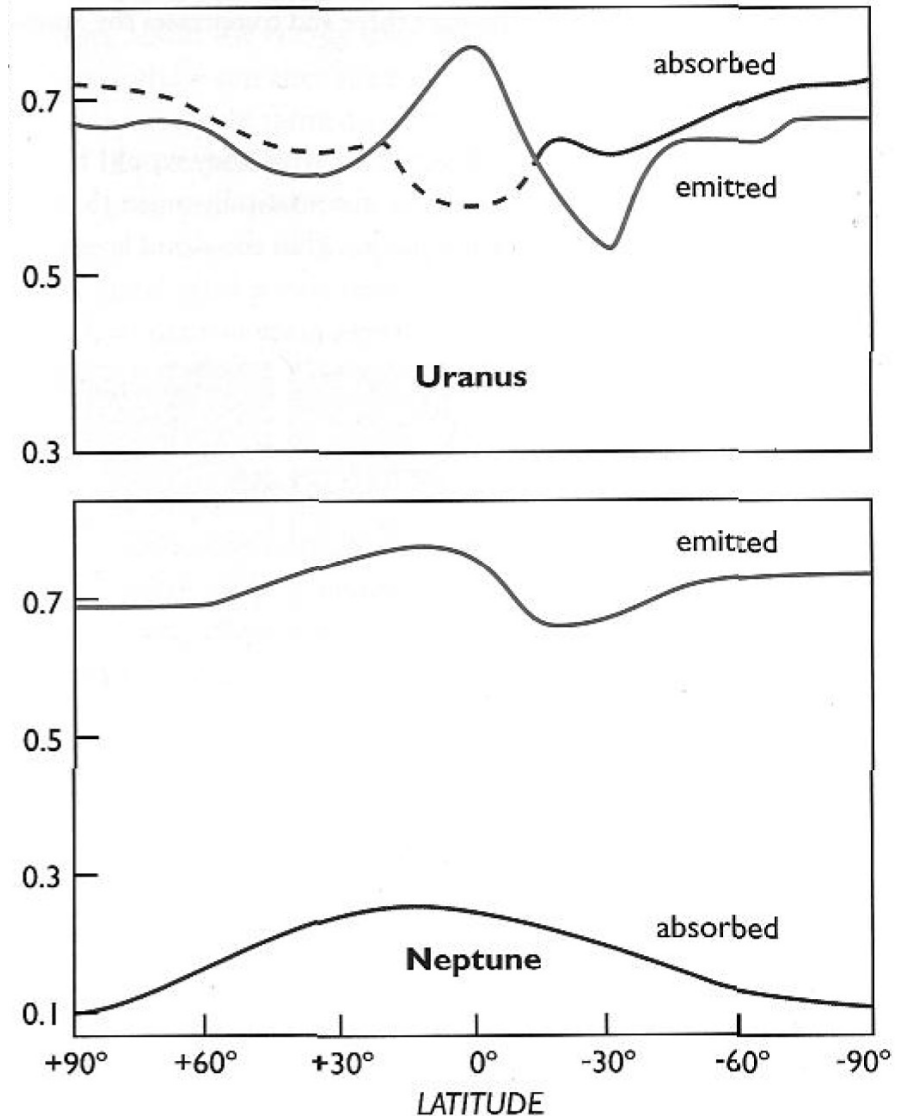
# Thermal Emission

- Jupiter and Saturn's thermal emission ~ twice solar insolation
- Neptune's ~ 3 times solar insolation
- Uranus is anomalous
  - Zonal winds could be due to either shallow or deep energy sources



# Thermal Emission

- Jupiter and Saturn's thermal emission ~ twice solar insolation
- Neptune's ~ 3 times solar insolation
- Uranus is anomalous
  - Zonal winds could be due to either shallow or deep energy sources



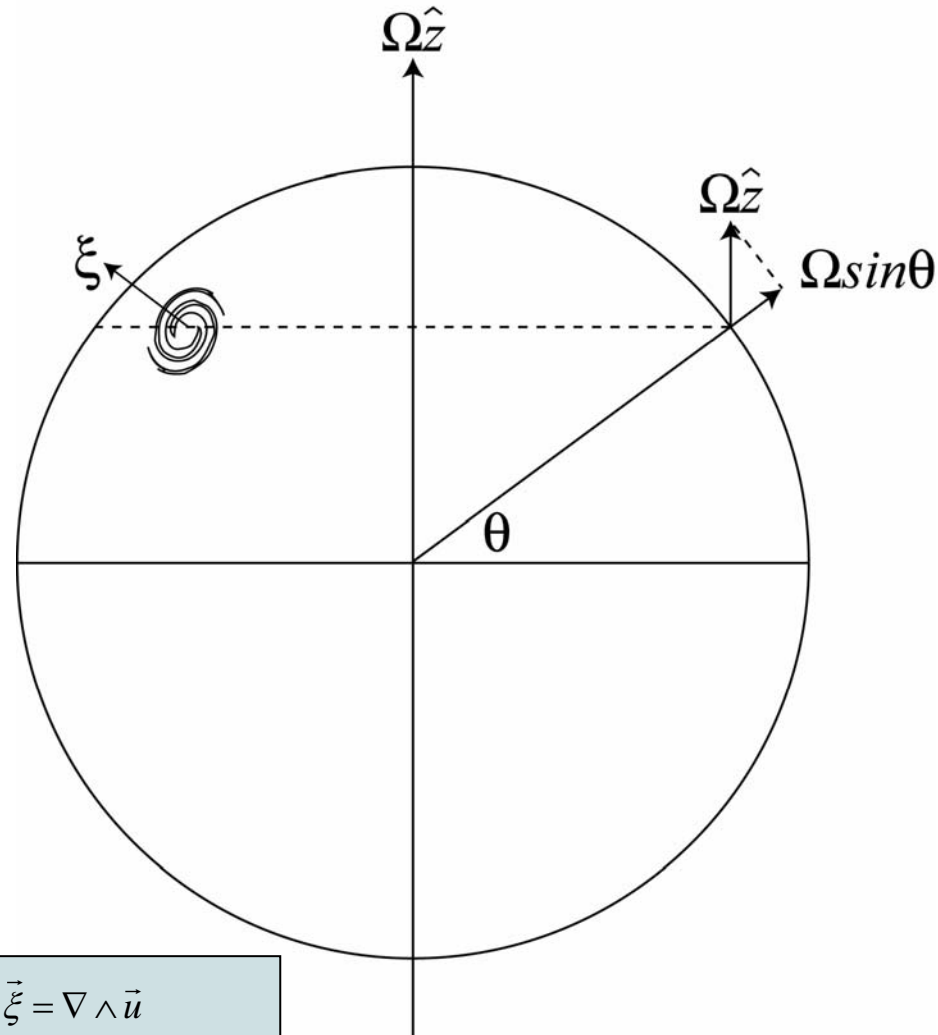
# Zonal Wind Models

- Shallow layer models
- Deep Convection models
  - Both models are able to generate aspects of the observed zonal winds

## 2. Shallow Layer Models

# Shallow Layer Models

- Turbulent flow on a rotating spherical surface of radius  $r_o$
- No convection
- Simulates dynamics of outer weather layer



local vorticity:  $\vec{\xi} = \nabla \wedge \vec{u}$

planetary vorticity:  $\vec{f} = 2\Omega \sin \theta \hat{z}$

fluid layer depth:  $h$

# Turbulence

- 3D Turbulence: Energy cascades from injection scale eddies down to dissipation scale eddies
- 2D Turbulence: “Inverse Cascade” of energy; eddies tend to grow to domain size

# Potential Vorticity Conservation

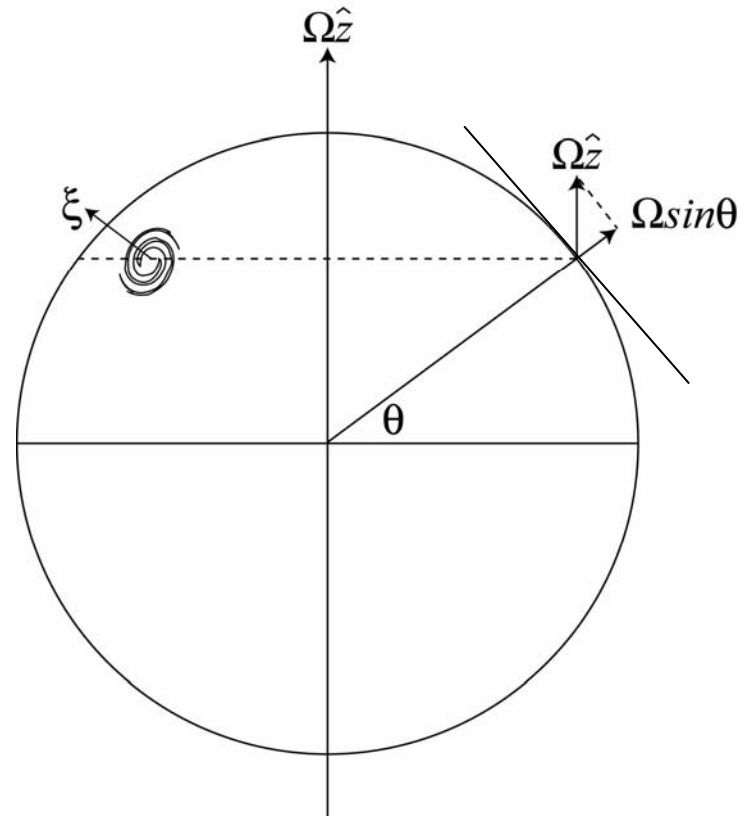
- In an inviscid shallow layer, potential vorticity (PV),  $q$ , is conserved.

$$\frac{dq}{dt} = \frac{d}{dt} \left( \frac{\xi + f}{h} \right) = 0$$

local vorticity  $\vec{\xi} = \nabla \wedge \vec{u}$

planetary vorticity  $\vec{f} = 2\Omega \sin\theta$

fluid layer depth  $h$



# $\beta$ - Plane Approximation

- Taylor expand planetary vorticity,  $f$ :

$$f(\theta) = 2\Omega \sin \theta$$

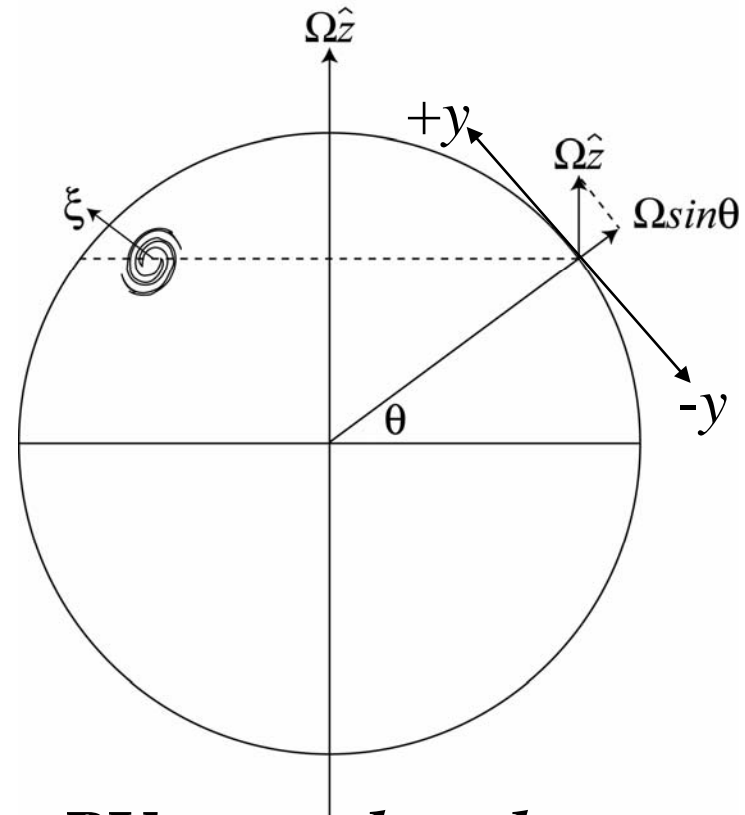
$$\approx f_o + (\nabla f) y$$

$$= f_o + \beta y$$

- On a spherical surface

$$\beta = 2\Omega \cos \theta / r_o$$

- Ambient PV gradient: Increasing PV towards poles





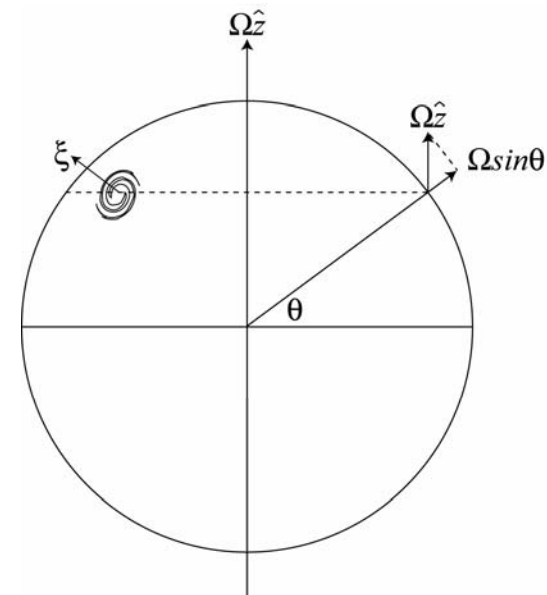
# Potential Vorticity Conservation

- For inviscid 2D flow on the  $\beta$ -plane:

$$\frac{d}{dt} \left( \frac{\xi + f}{h} \right) = \frac{d}{dt} \left( \frac{\xi + f_o + \beta y}{h} \right) = 0$$

$$(\xi + \beta y) = \text{const.}$$

$$(U/L + \beta L) \sim \text{const.}$$



- For sufficiently large  $L$  :

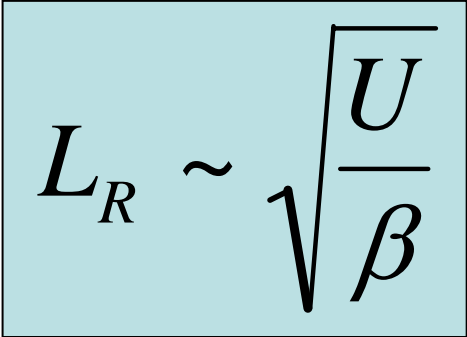
$$U/L \sim \beta L \Rightarrow L \sim \sqrt{\frac{U}{\beta}}$$

local vorticity  $\vec{\xi} = \nabla \wedge \vec{u}$

planetary vorticity  $\vec{f} = 2\Omega \sin\theta \hat{z}$

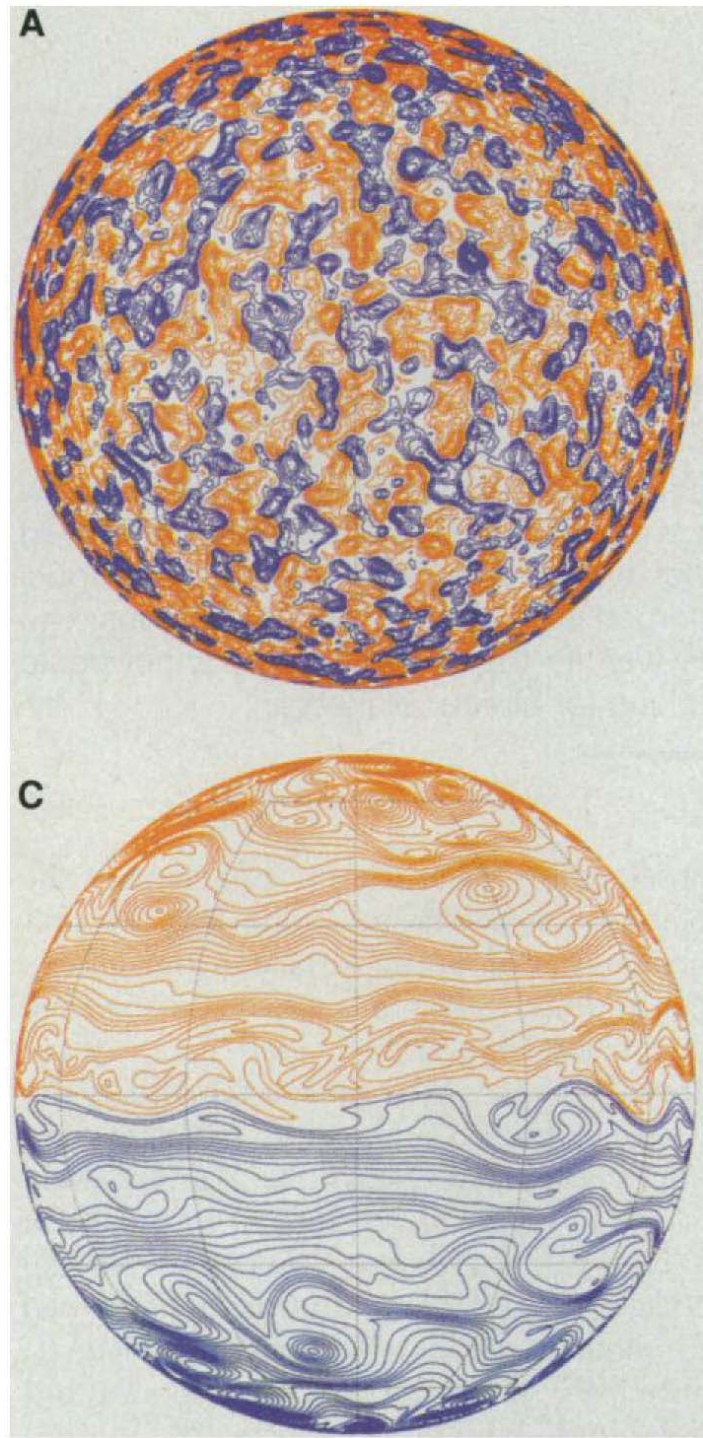
fluid layer depth  $h$

# Shallow Layer Rhines Length

- Rhines, *JFM*, 1975
- Rhines scale,  $L_R$ , where curvature effects truncate 2D inverse cascade:  

$$L_R \sim \sqrt{\frac{U}{\beta}}$$
- At  $L_R$ , turbulent eddies cease to grow
- Energy gets transferred into zonal motions
  - Scale of zonal jets

# Shallow Layer Models

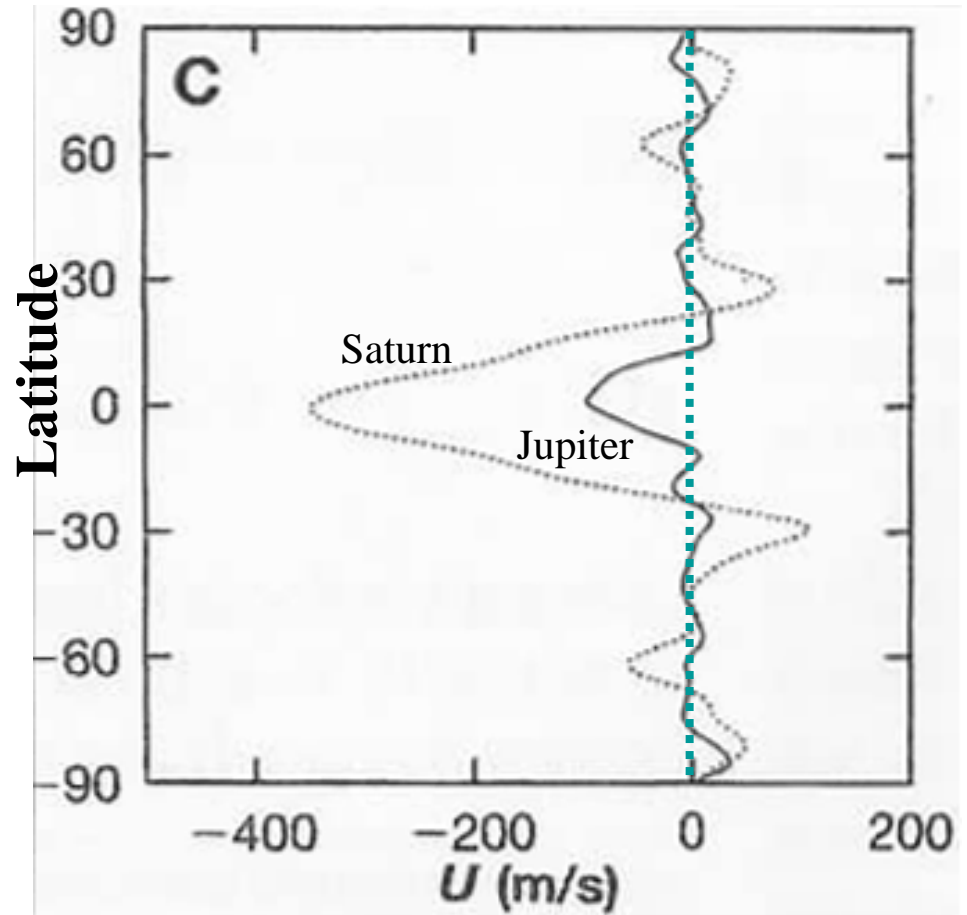
- Shallow layer turbulence evolves into:
  - **Large-scale zonal flows**
  - Alternating bands
  - Coherent vortices



# Shallow Layer Models

- Shallow layer turbulence evolves into:
  - **Large-scale zonal flows**
  - Alternating bands
  - Coherent vortices

Equatorial jets  
are westward



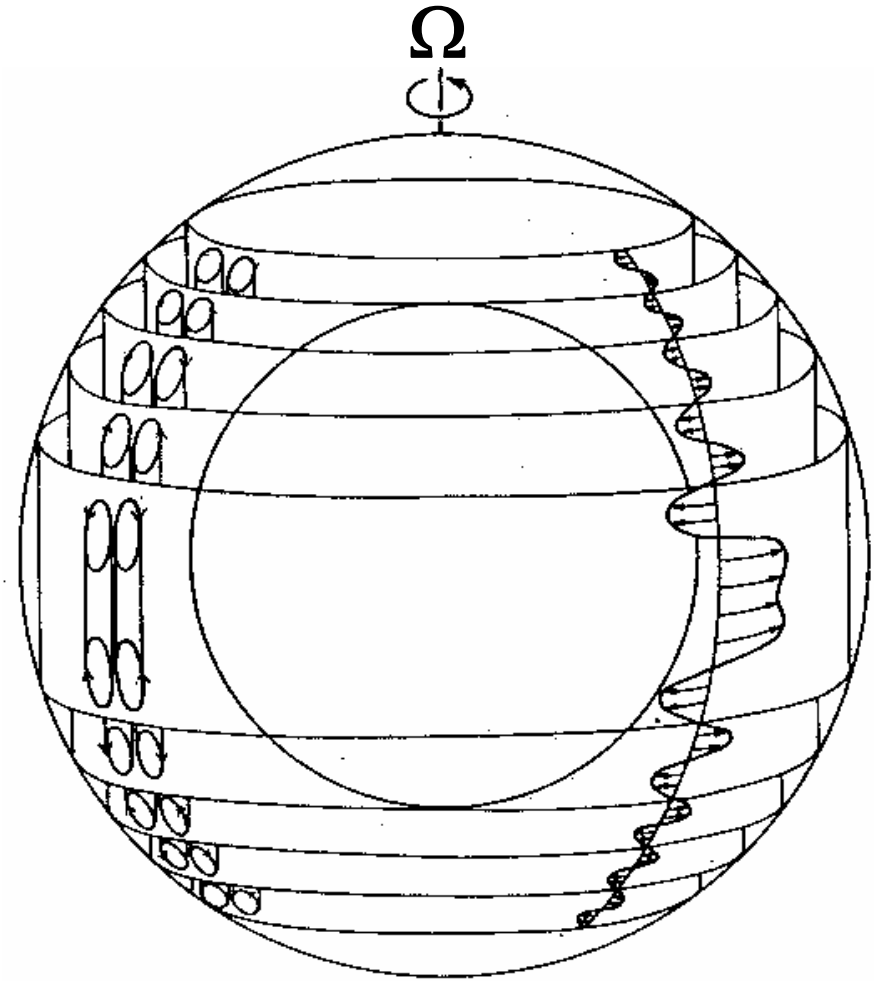
# Shallow Layer Models

- 2D turbulence evolves into Rhines scale zonal jets
- Shallow layer  $\beta$ -effect
  - Potential vorticity increases towards poles
  - Gives retrograde equatorial jets
    - Iacono et al., 1999; Vasavada & Showman, 2005

# 3. Deep Models

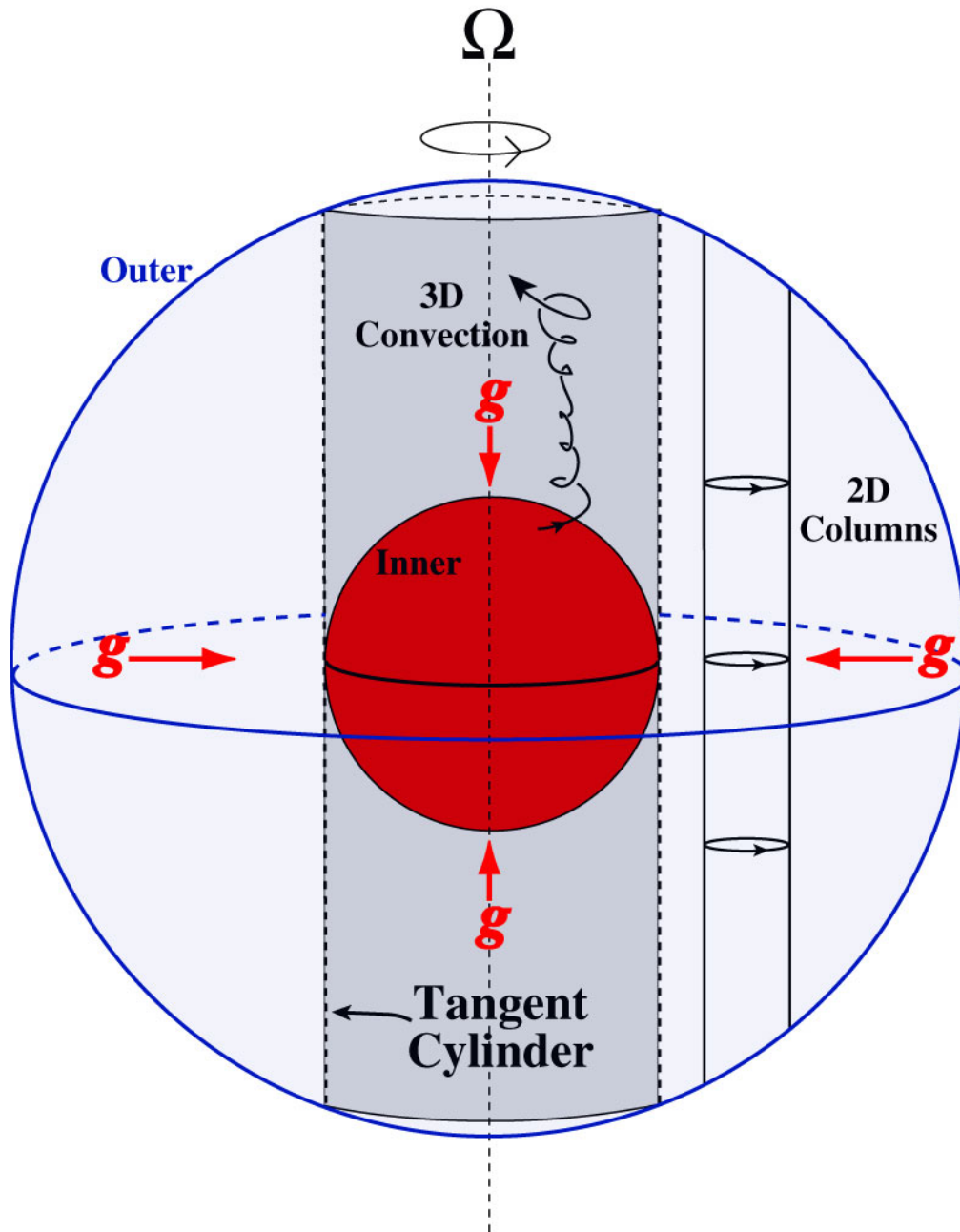
# Deep Layer Models

- Busse, 1976
  - Deep spherical shell dynamics
  - Multiple jets require nested cylinders of convection columns
    - And even then...



Busse, *Icarus*, 1976

# Rotating Spherical Shell Dynamics



- Geostrophic quasi-2D dynamics
  - Axial flow structures
- Tangent cylinder (TC) flow barrier



# The Topographic $\beta$ -effect

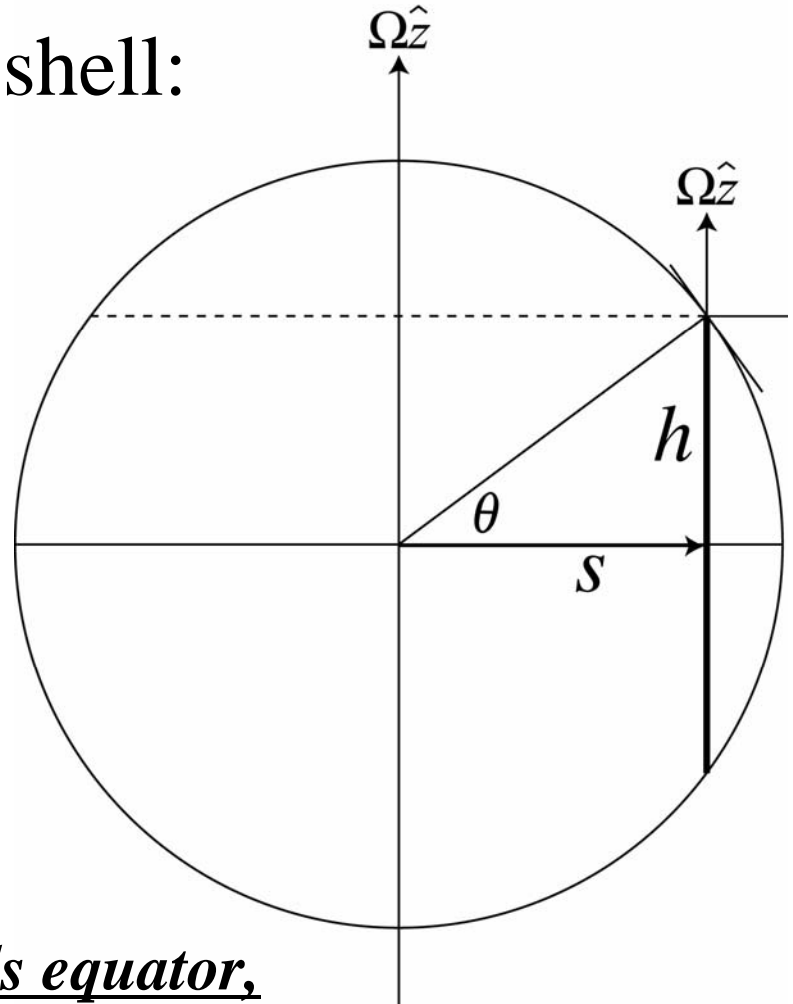
- PV Conservation in a deep shell:

$$\frac{d}{dt} \left( \frac{\xi + f_o}{h_o \left( 1 + \left( \frac{dh}{ds} \right) s \right)} \right) = 0$$

$$(\xi - \beta_t s) = \text{const.}$$

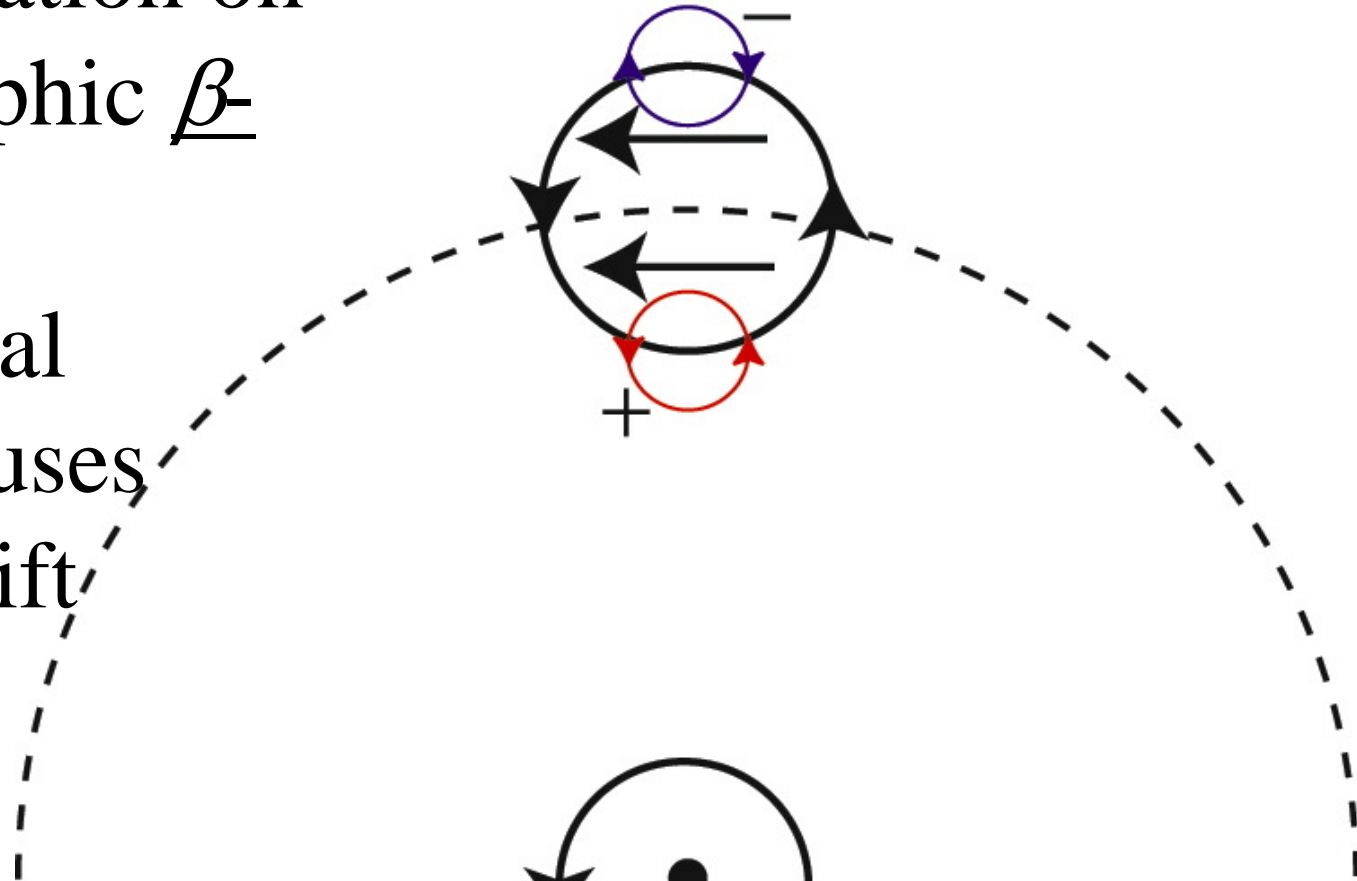
$$\text{where } \beta_t = \frac{f_o}{h_o} \frac{dh}{ds}$$

- PV gradient: Increasing PV towards equator, opposite the shallow layer case



# The Topographic $\beta$ -effect

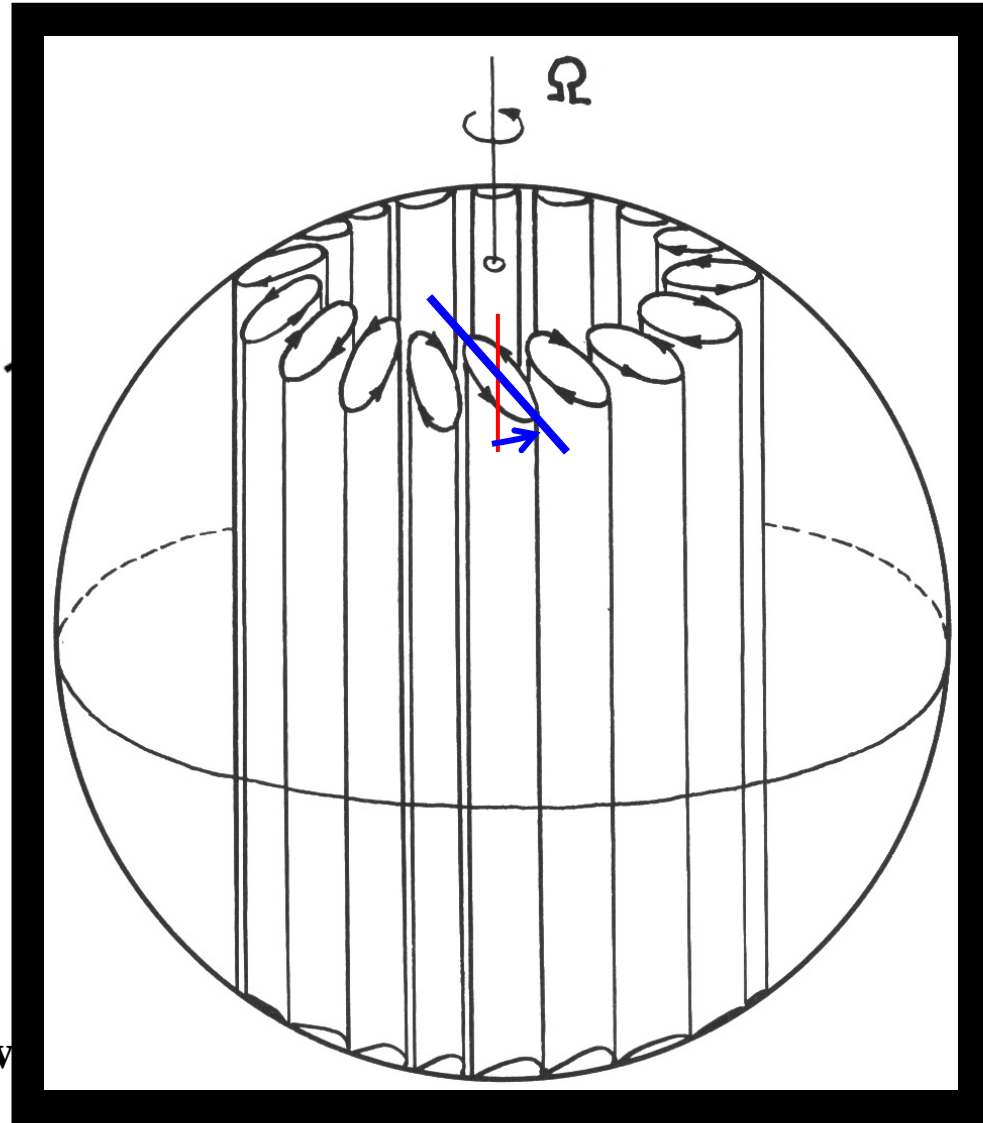
- PV conservation on the topographic  $\beta$ -plane
- Induced local vorticity causes eastward drift



Equatorial plane, viewed from above

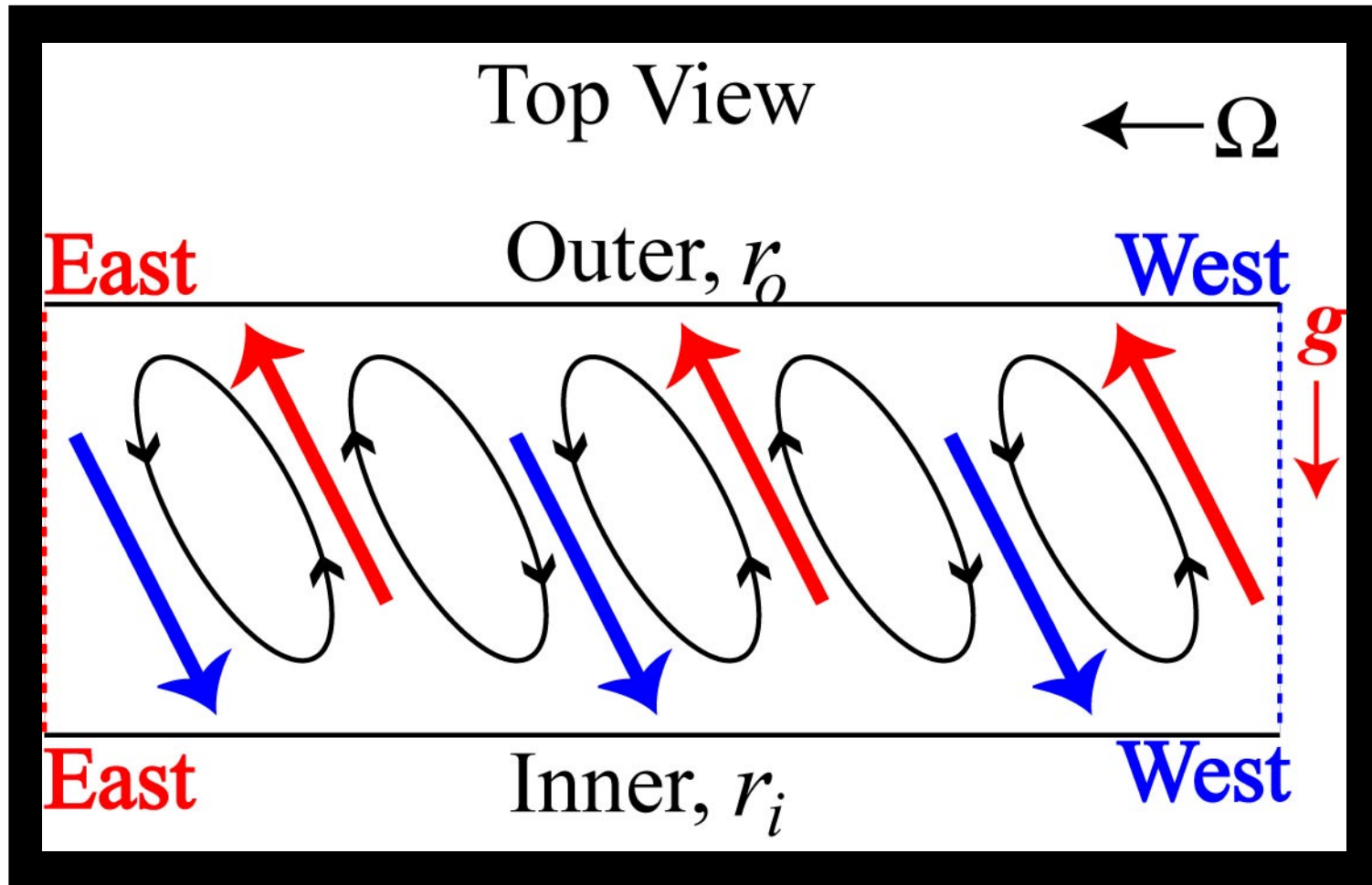
# The *Topographic* $\beta$ -effect

- PV conservation on *spherical boundary* causes latitudinally-varying drift rate & tilted columns



Equatorial plane, viewed from above

# Reynolds Stresses



Equatorial plane, viewed from above

# Quasi-laminar Deep Convection Models

**Tangent Cylinder**



QuickTime?and a  
YUV420 codec decompressor  
are needed to see this picture.

QuickTime?and a  
YUV420 codec decompressor  
are needed to see this picture.

North pole view of  
**equatorial plane**  
isotherms

Outer boundary heat  
flux from 20°N lat

# Shallow vs. Deep Model Comparison

- Shallow Layer:

- Imposed turbulence on a spherical shell
- Increasing PV with latitude

- *Retrograde equatorial jet*

- Higher latitude Rhines scale jets

- Deep Convection:

- Quasi-laminar, QG convection
- Decreasing PV with latitude

- Prograde equatorial jet

- *No higher latitude alternating jets*

# Yano's Shallow Layer Model

Yano et al., *Nature* (2003)

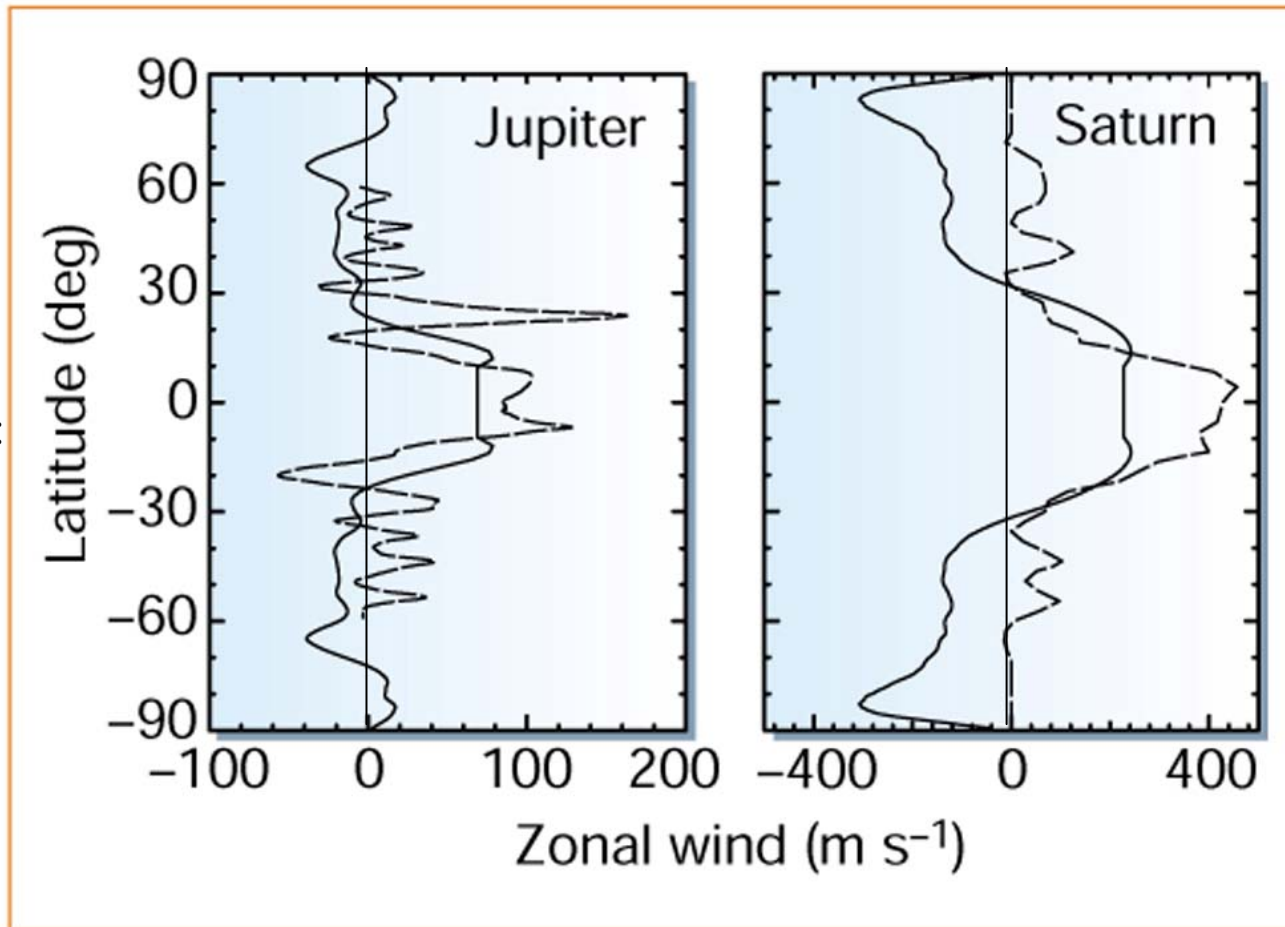
# Yano's Shallow Layer Model

- Yano et al., 2003
  - 2D shallow layer turbulence model
  - **Chooses a different function for  $\beta$** 
    - **Full sphere topographic  $\beta$  parameter**
      - Opposite sign as shallow layer  $\beta$
  - Simulates geostrophic turbulence in a sphere using the 2D shallow layer equations



# Yano's Shallow Layer Model

- Prograde equatorial jet
- Broad Rhines scale high latitude jets



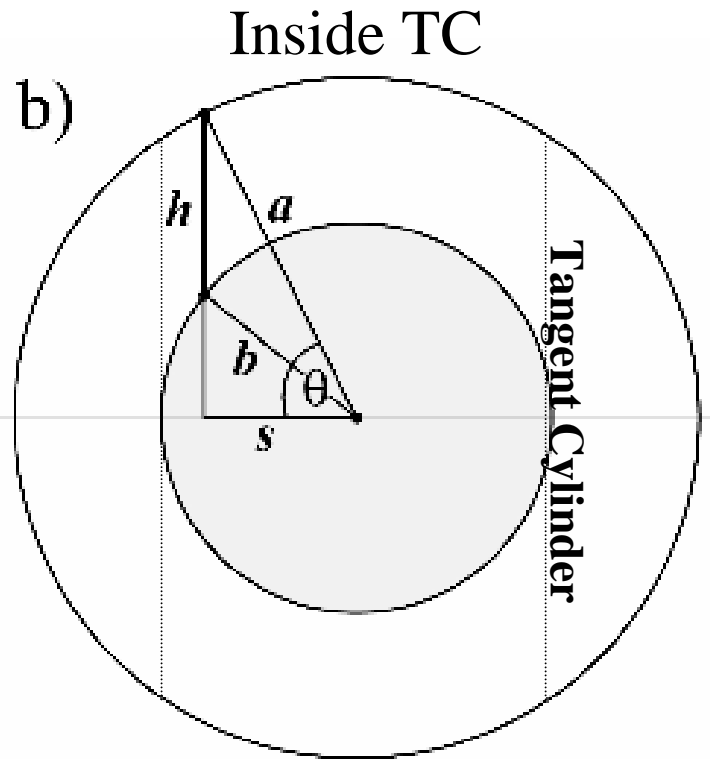
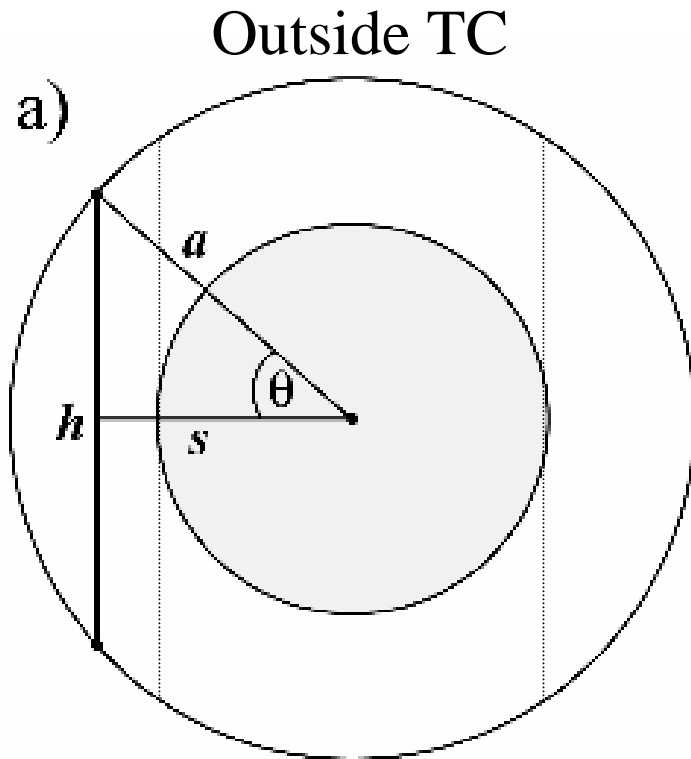
# Yano's Shallow Layer Model

- Sign of  $\beta$  controls direction of equatorial jet
- Broad high latitude jets do not resemble Jovian observations
- Suggests that Rhines scale jets can form from quasigeostrophic ( $\sim 2D$ ) convective turbulence in a deep 3D model

# 4. New Deep Convection Model

Heimpel, Aurnou & Wicht, *Nature*,  
**438**: 193-196 (2005)

# Spherical Shell Rhines Scale



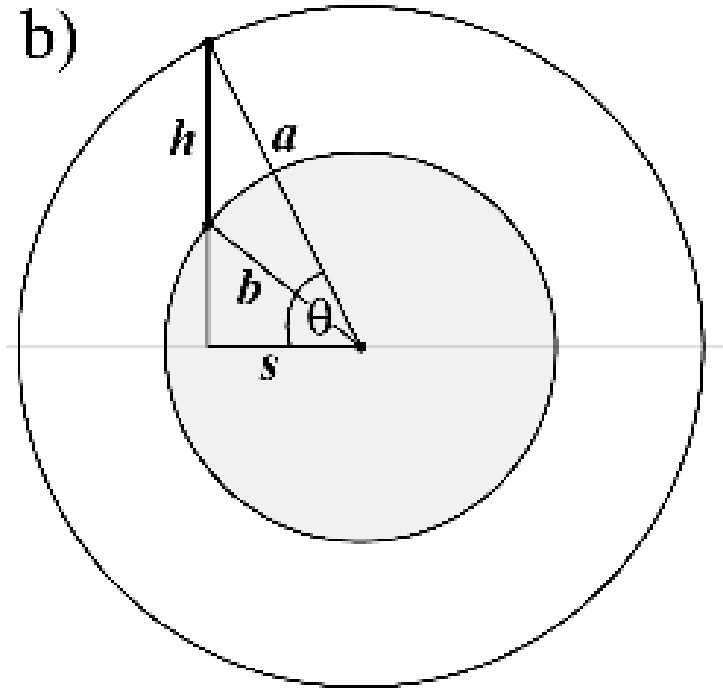
$$\frac{1}{h} \left| \frac{dh}{ds} \right| = \frac{1}{r_o} \frac{\cos \theta}{\sin^2 \theta}$$

$$\frac{1}{h} \left| \frac{dh}{ds} \right| = \frac{1}{r_o} \frac{\cot \theta}{\sqrt{\chi^2 - \cos^2 \theta}}$$

(Scaling discontinuity across TC)

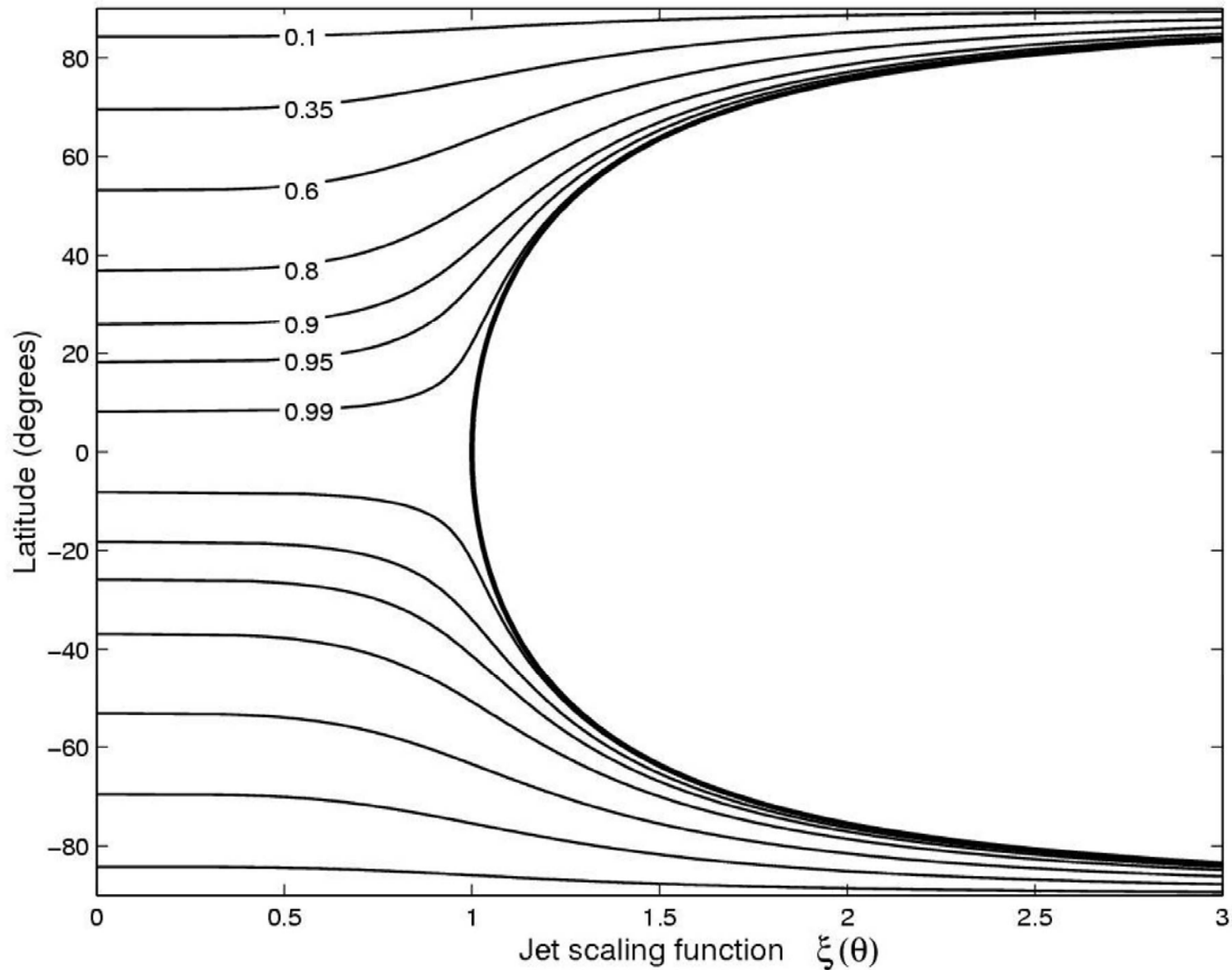
# Spherical Shell Rhines Scale

- The Rhines scale is **discontinuous** across the tangent cylinder
- In thin shells:
  - The  $\beta_t$  parameter becomes larger inside the tangent cylinder
  - $L_R$  becomes smaller

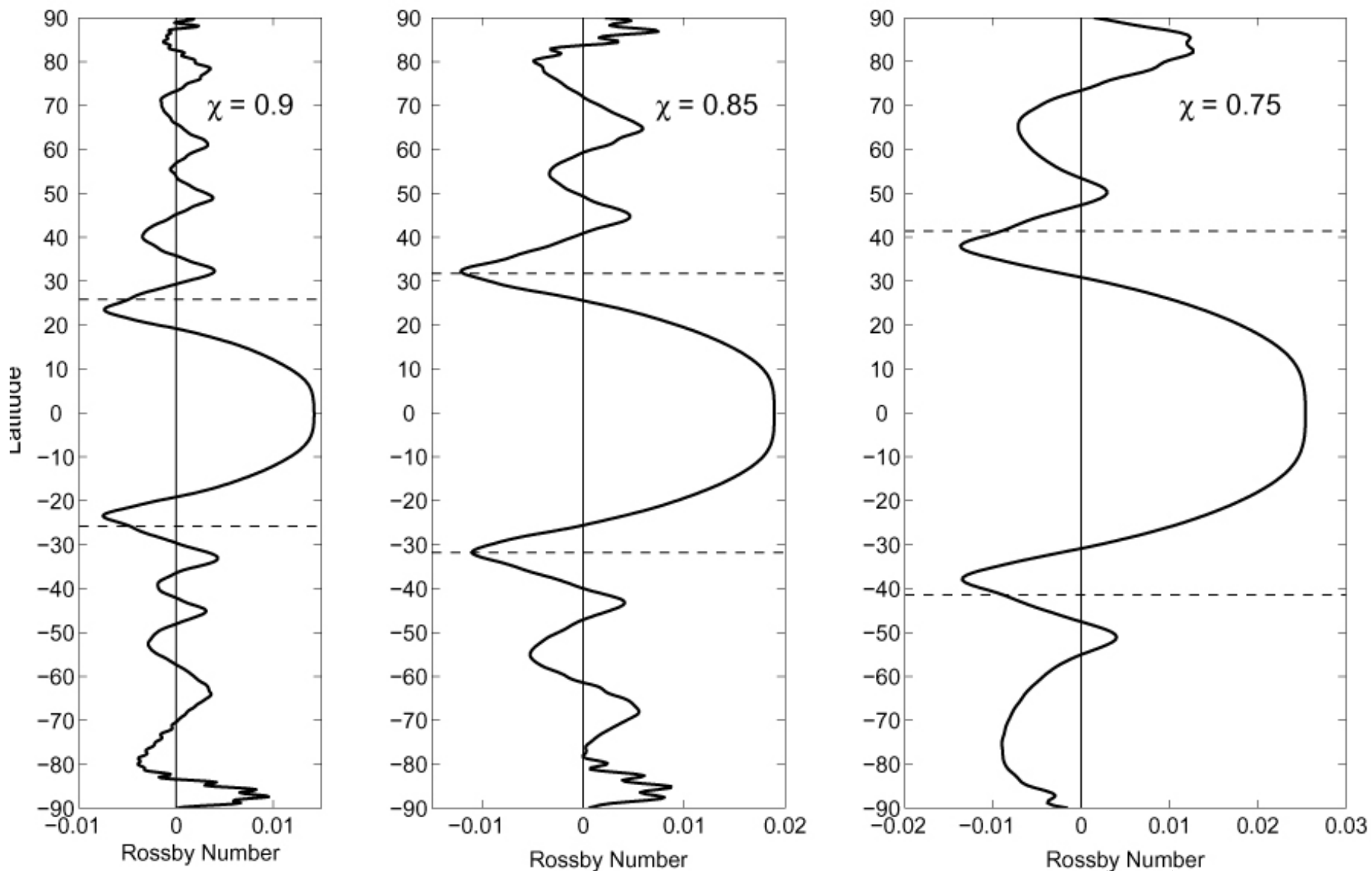


$$L_R \sim \sqrt{\frac{U}{\beta}}$$

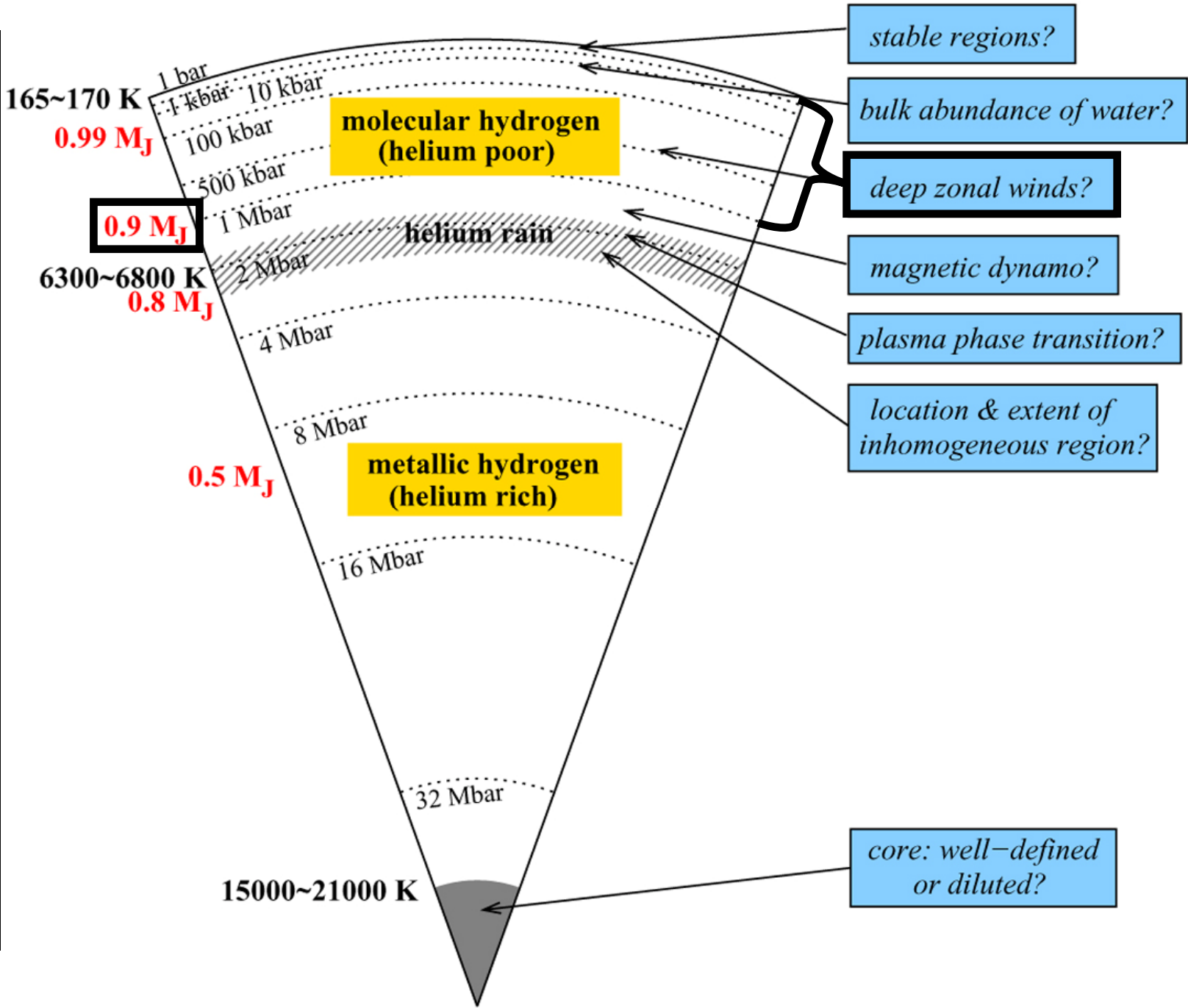
# Geometric Scaling Function



# Zonal Winds vs. Radius Ratio Tests



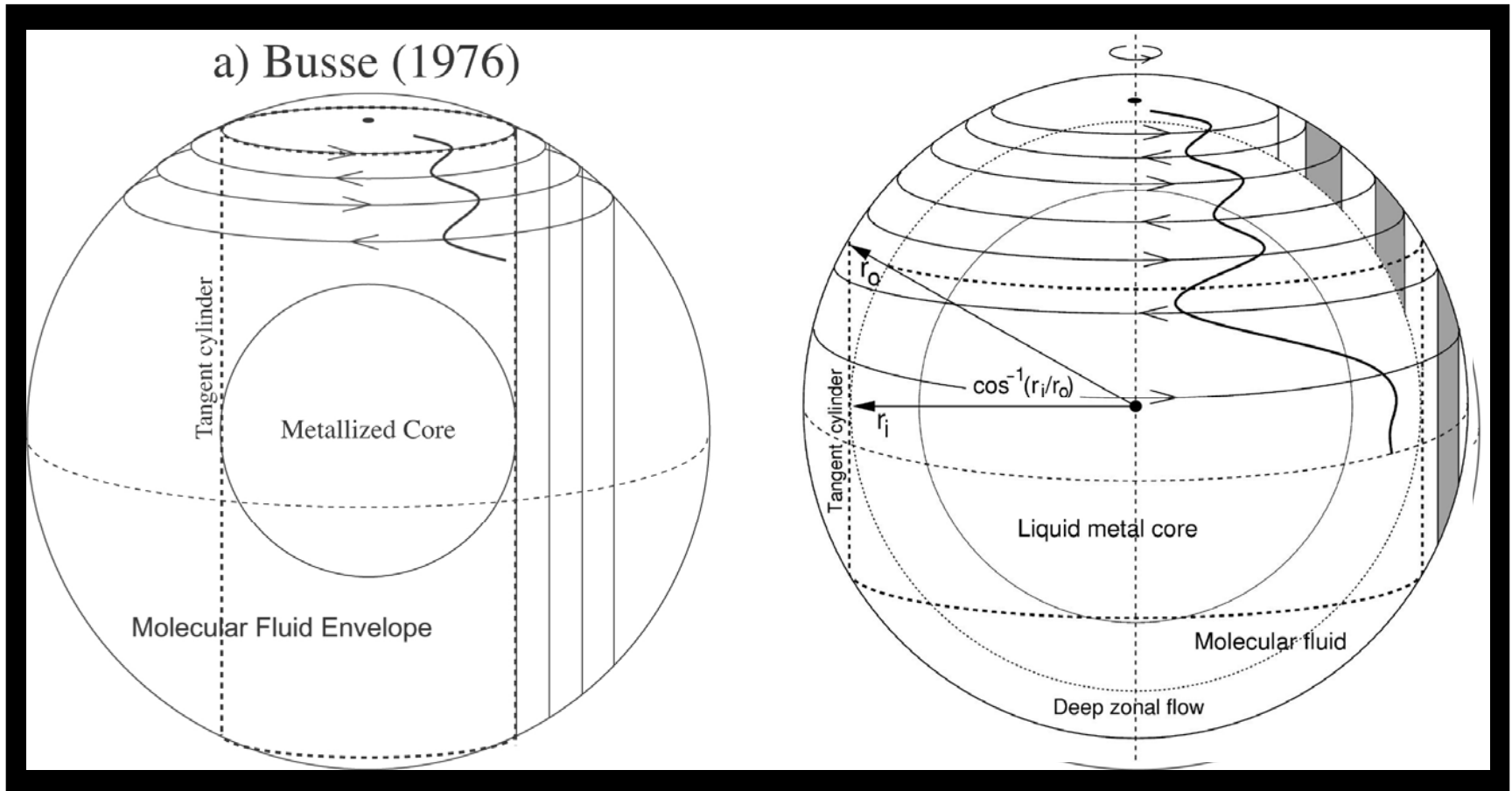
# Recent Jovian Interior Models





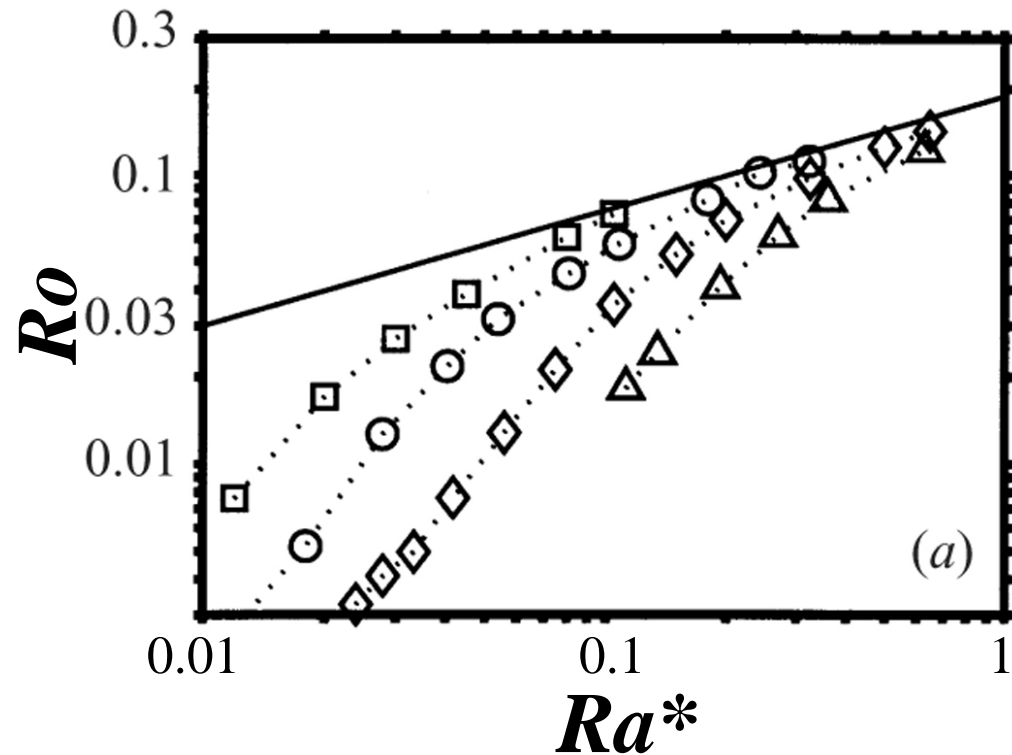
# New Deep Convection Model

- Rapidly-rotating, turbulent convection
- Thin shell geometry ( $\chi = 0.90$ ; *7000 km deep*)



# $\chi=0.35$ Rotating Convection Studies

- Christensen, 2002
  - Zonal flow from fully-developed, quasigeostrophic turbulence
  - Asymptotic regime



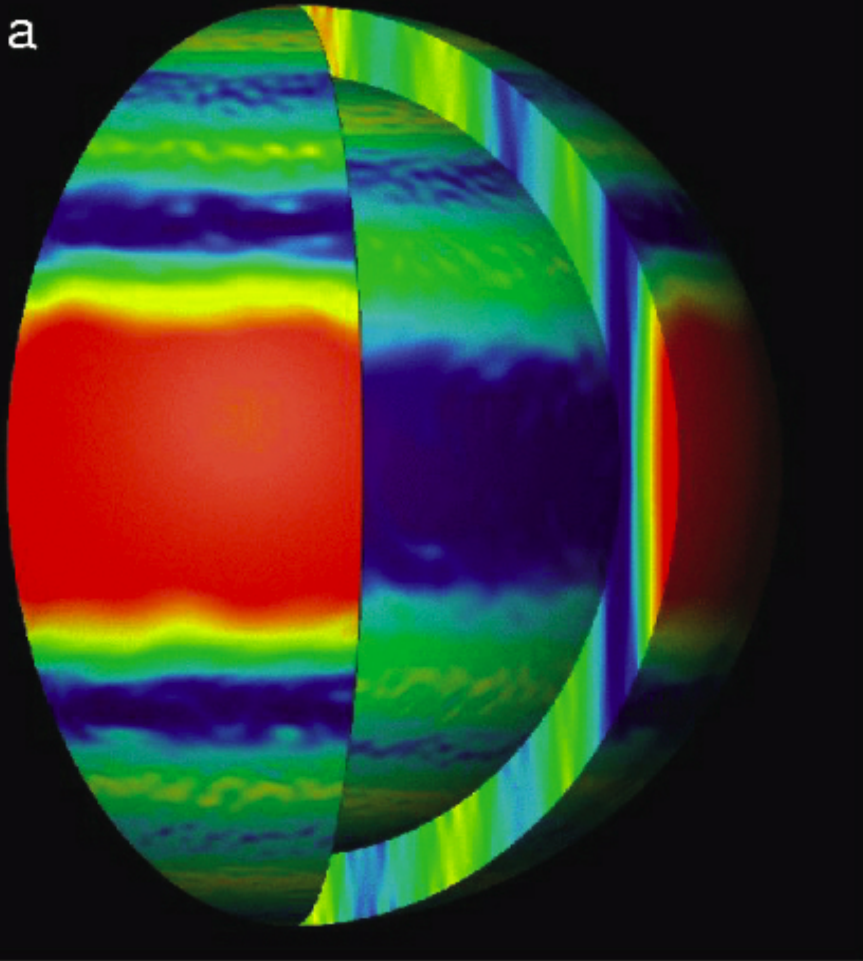
# New Deep Convection Model

- Solve Navier-Stokes equation
- Energy equation
- Boussinesq (*incompressible*) fluid
  
- Wicht's MagIC2 spectral transform code
  - Spherical harmonics in lat. & long.
    - Difficulties in resolving flows near poles
  - Chebyshev in radius
  - Courant time-step using grid representation

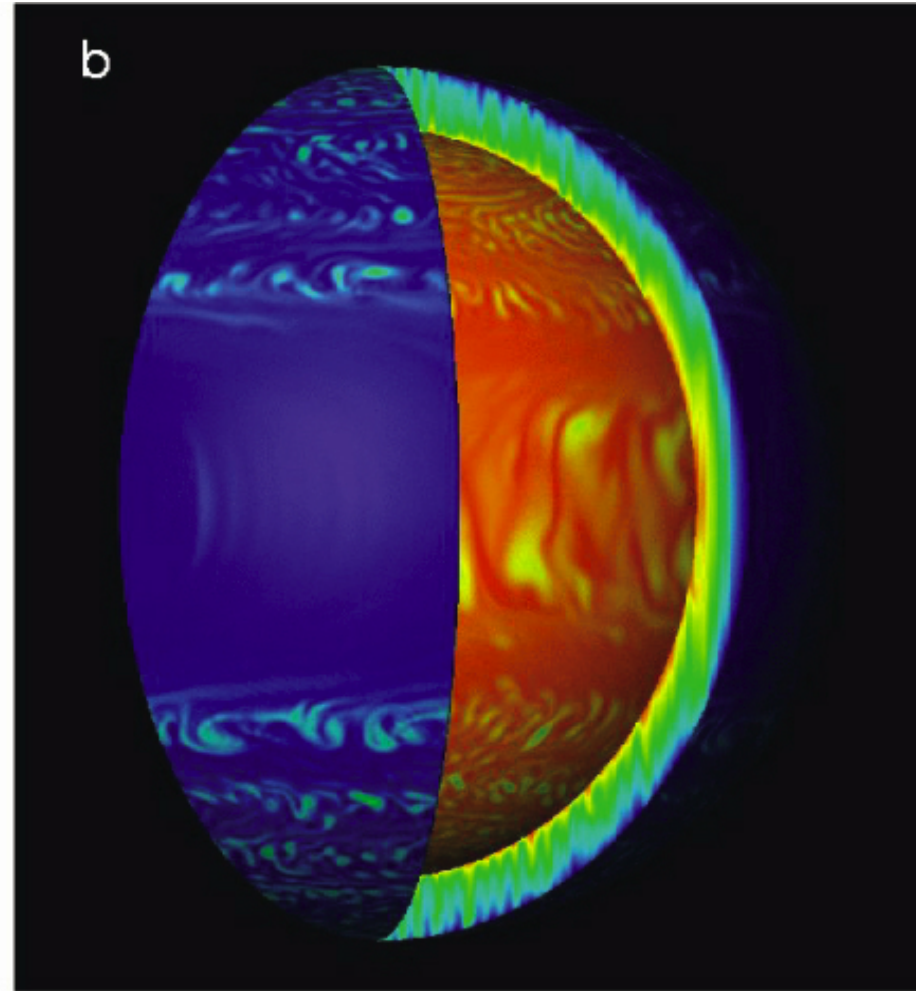
# New Deep Convection Model

- Input Parameters:
  - $Ra = 1.67 \times 10^{10}$
  - $E = 3 \times 10^{-6}$
  - $Pr = 0.1$ 
    - $Ra^* = RaE^2/Pr = 0.15$
  - $r_i/r_o = \chi = 0.90$
- Resolution
  - $L_{\max} = 512$ , grid:  $\phi$  768,  $\theta$  192,  $r$  65
  - Hyperdiffusion and 8-fold  $\phi$ -symmetry
- Output Parameters:
  - $Re \sim 5 \times 10^4$
  - $Ro \sim 0.01$

# $\chi=0.90$ Model Results

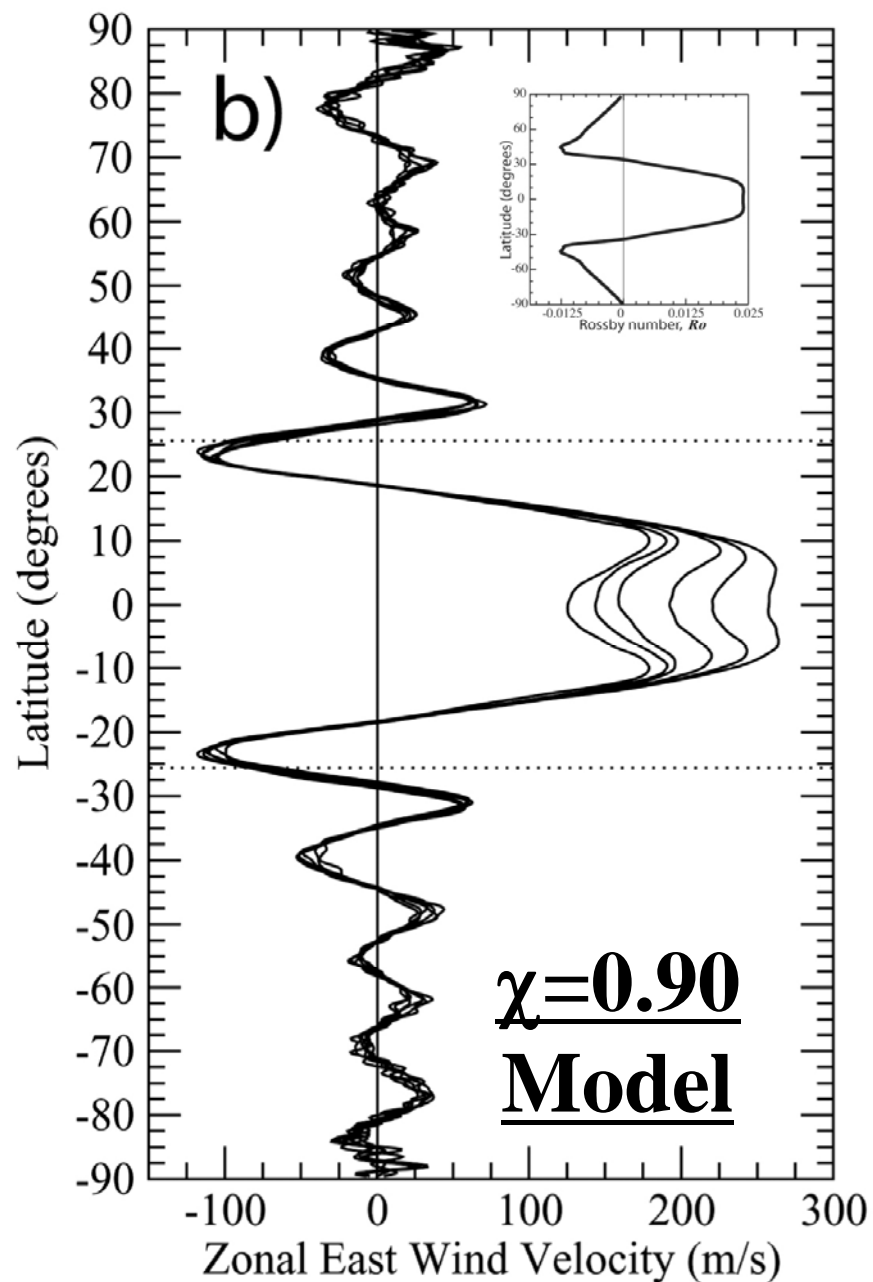
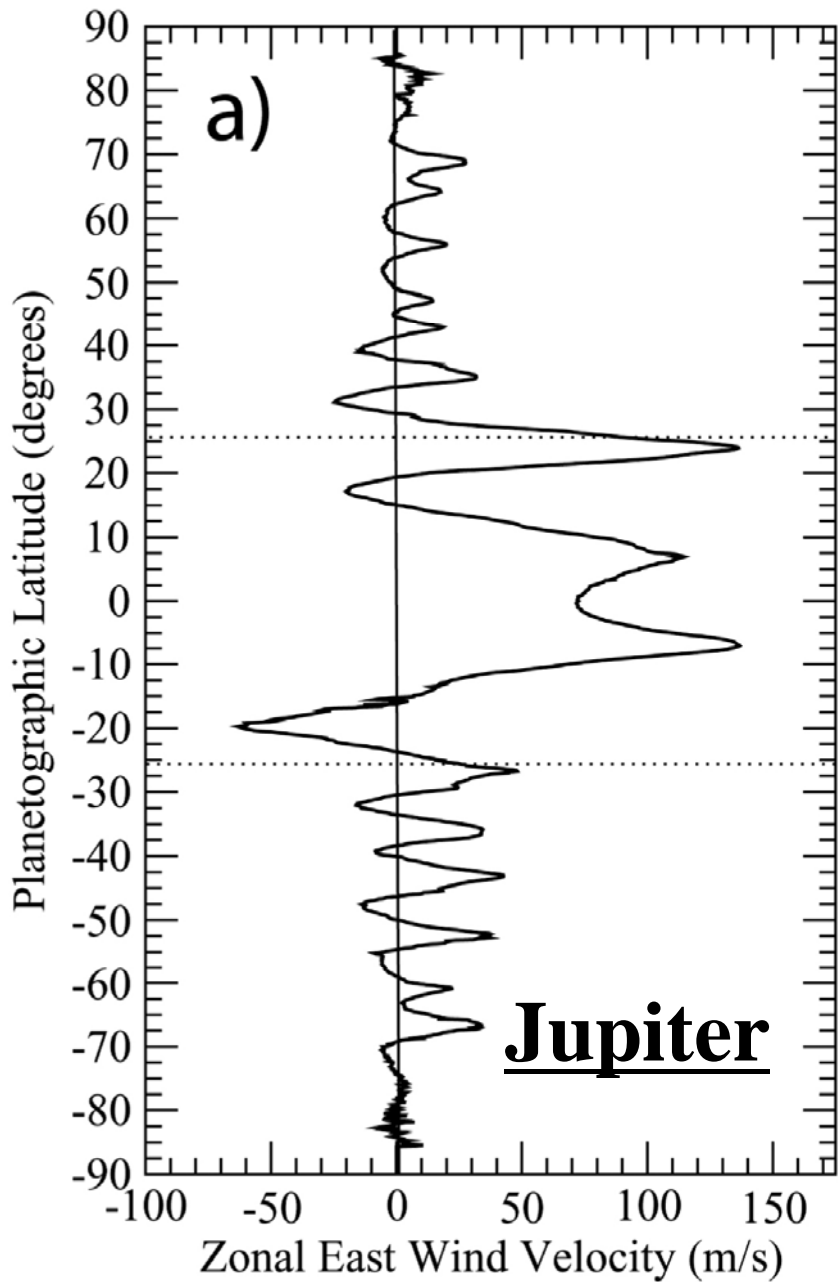


Azimuthal Velocity



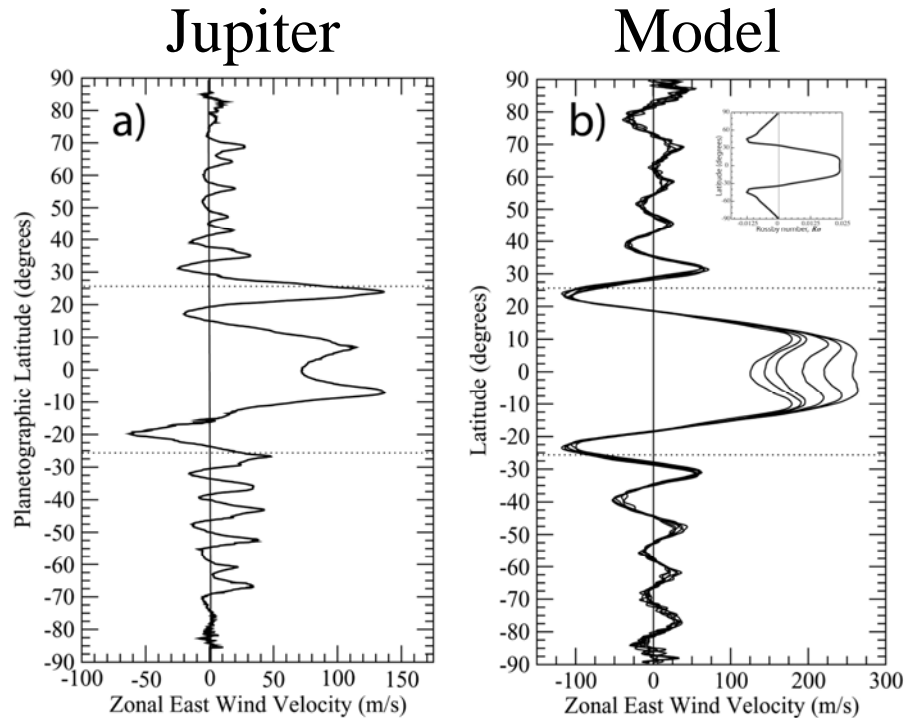
Temperature

# Zonal Wind Profiles



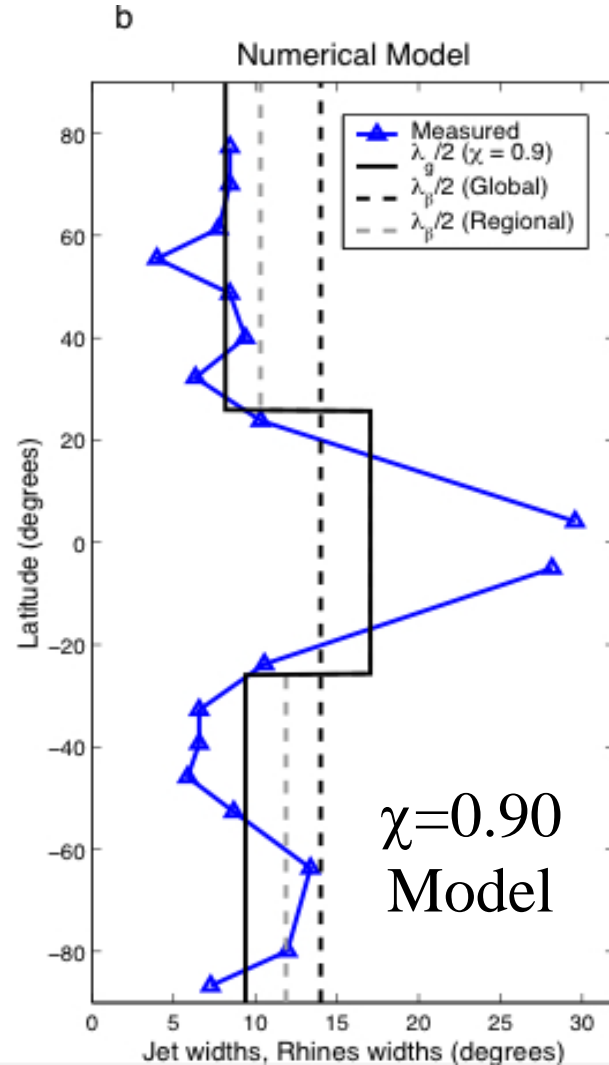
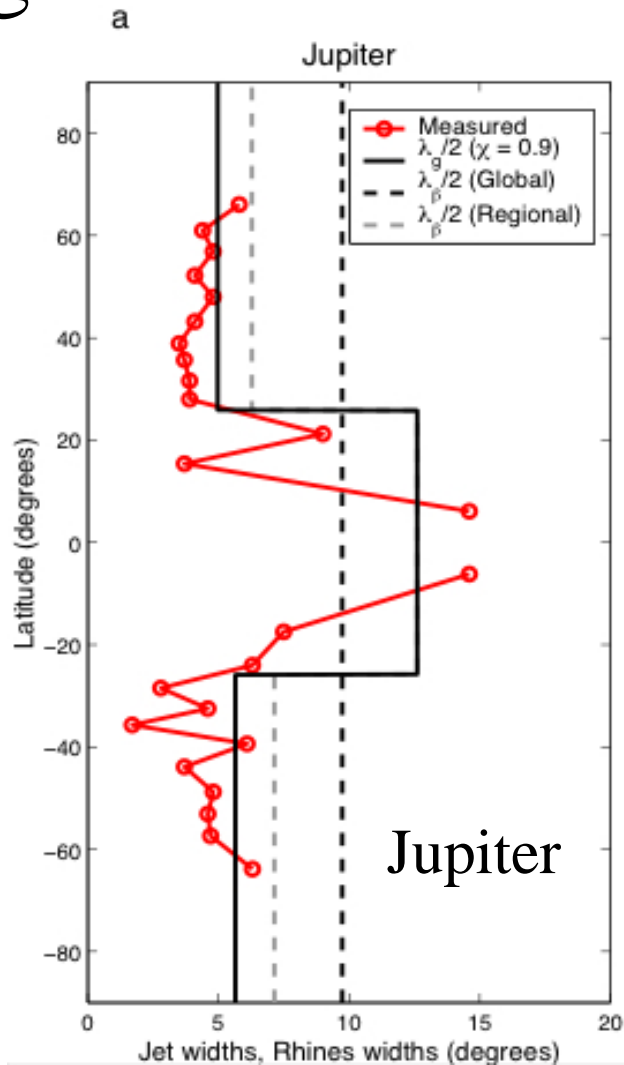
# The “Tuning” Question

- We have chosen
  - the radius ratio
  - The lowest Ekman number feasible
    - i.e., Fastest rotation rate
  - A Rayleigh number such that our *peak*  $Ro$  is similar to Jupiter’s
    - Close to asymptotic regime of Christensen, 2002



# 3D Spherical Shell Rhines Scale

- “Regional” 3D shell Rhines scale





# New Deep Convection Model

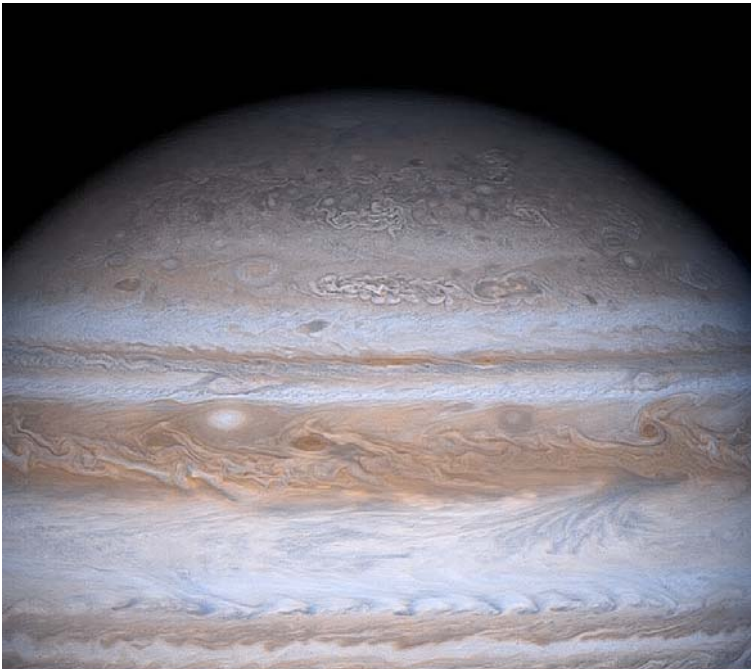
- Quasigeostrophic convective turbulence in a relatively thin spherical shell
- Generates **coexisting** prograde equatorial jets & alternating higher latitude jets
- Model (& Jovian observations) follows Rhines scaling for a 3D shell
  - Scaling **discontinuity** across the tangent cylinder
  - Smaller-scale jets inside tangent cylinder

# Following Work 1

- Why do we need “regional” Rhines fits?
  - Expensive to test with full 3D models
    - 2D quasigeostrophic mechanical models
    - 2D quasigeostrophic convection models

# Following Work 2

- How do we model high latitude fine scale jets?
  - Due to odd “plane layer” mode (Roberts, 1968)?
  - Local models or non-SH global models



QuickTime?and a  
DV - NTSC decompressor  
are needed to see this picture.

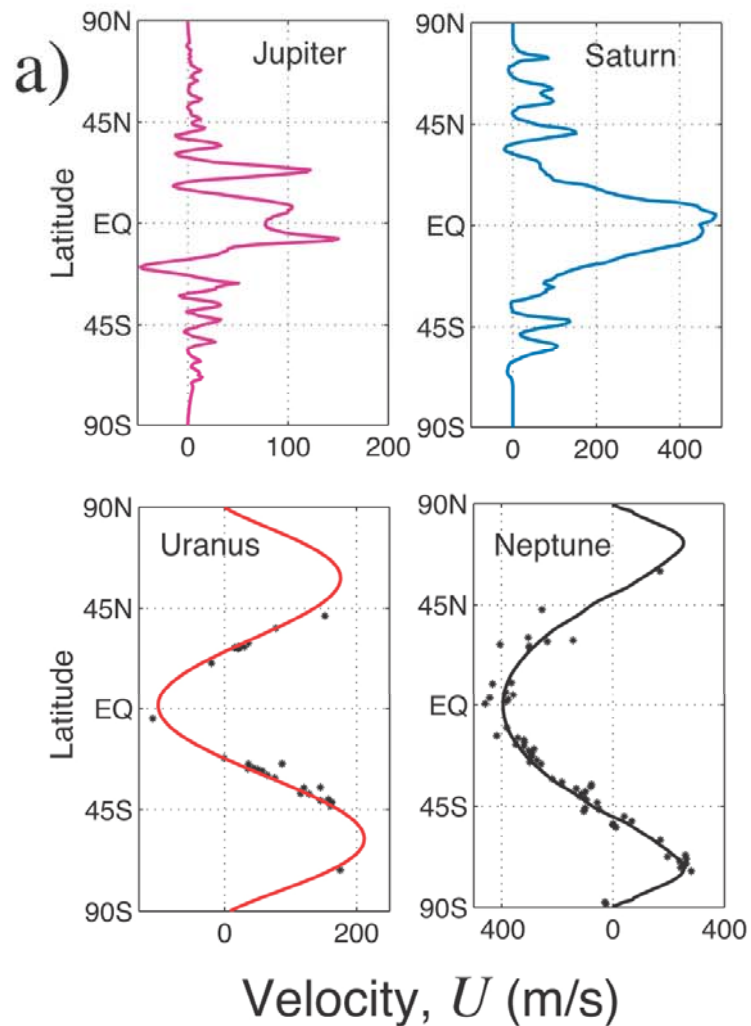
# Following Work 3

- Assumed zonal flow truncation at depth due to electromagnetic braking
  - Simple 2D EM braking model
- Assumed Boussinesq fluid
  - How does compressibility change Boussinesq Rhines scaling?

# 5. Modeling Zonal Flows on the Ice Giants

Aurnou, Heimpel & Wicht, submitted to *Icarus*  
(2006)

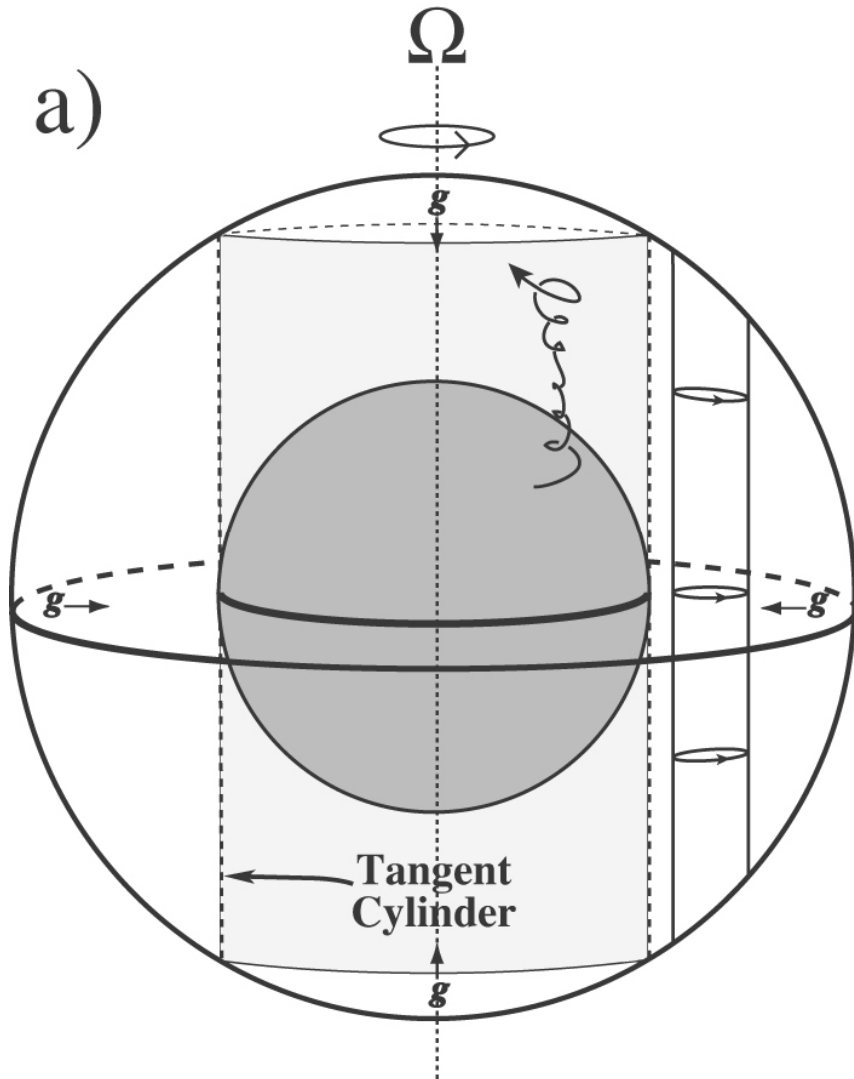
# Giant Planet Zonal Wind Profiles



Adapted from Sukoriansky et al., 2002

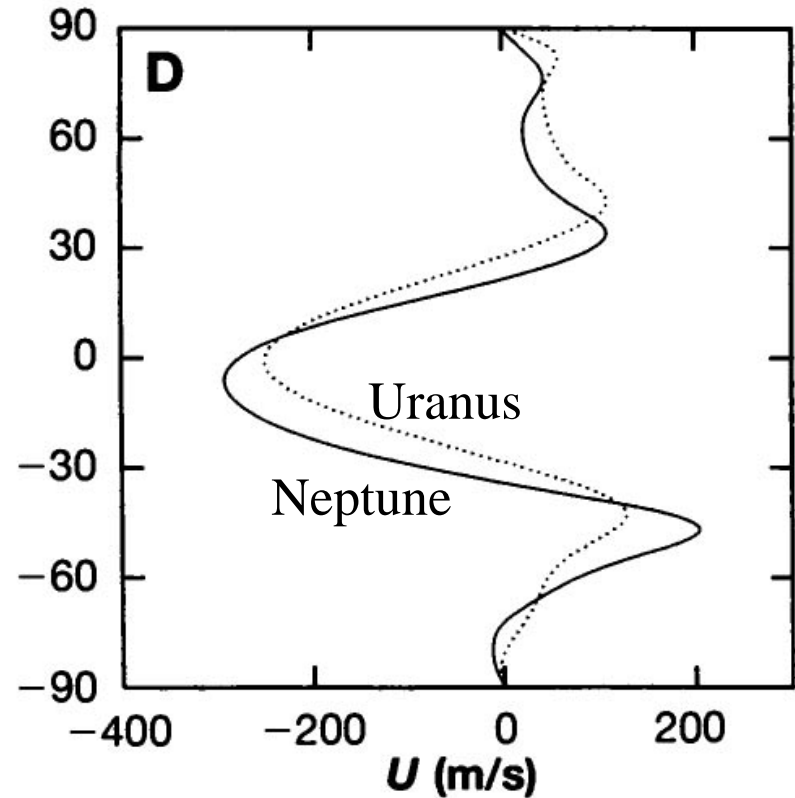
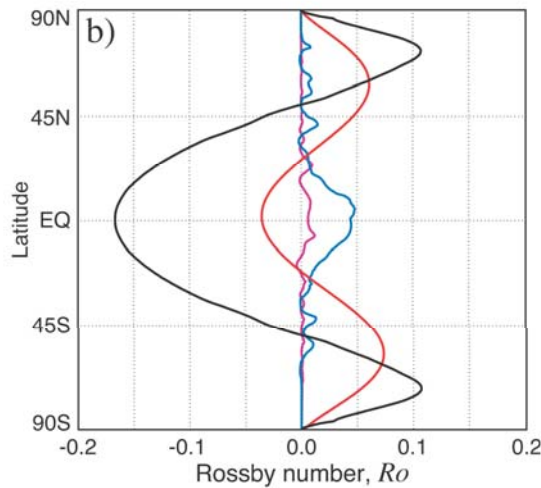
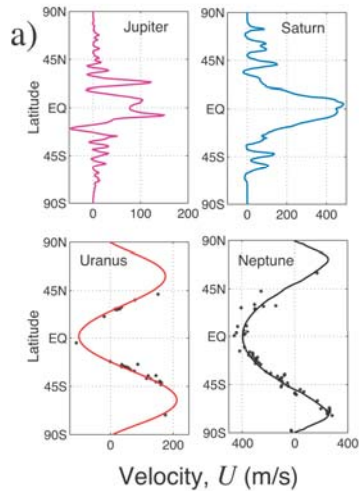
# Angular Momentum in Deep Shells

a)



**Rotation  $\gg$  Buoyancy**

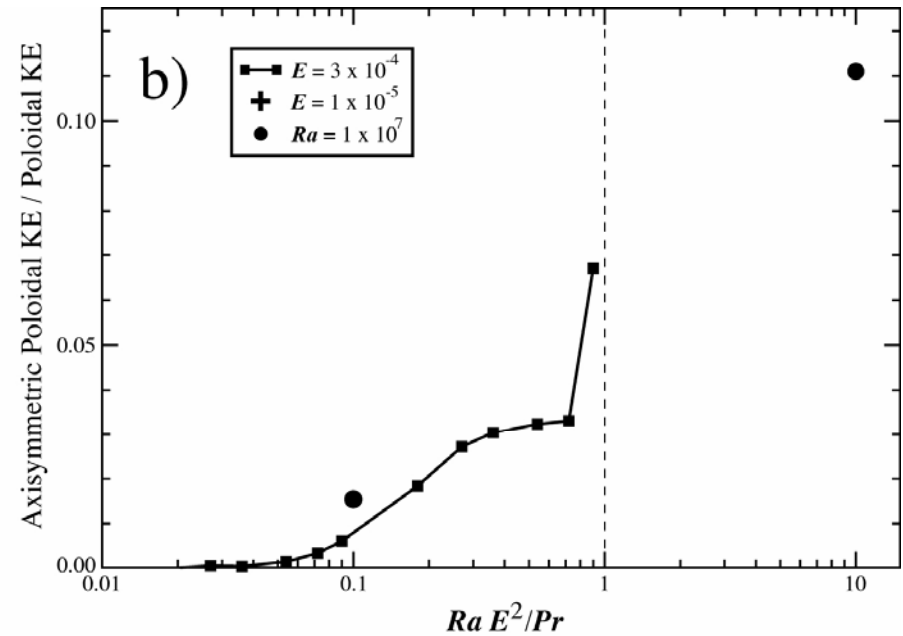
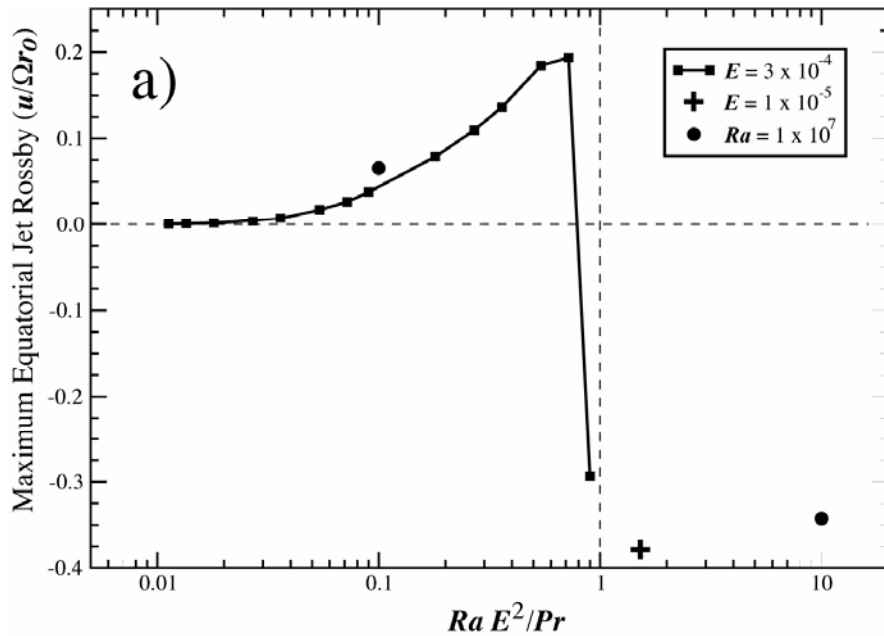
# Shallow Layer Models



- Shallow layer turbulence naturally evolve to:
  - **Large-scale zonal flows**
  - **Retrograde equatorial jets**

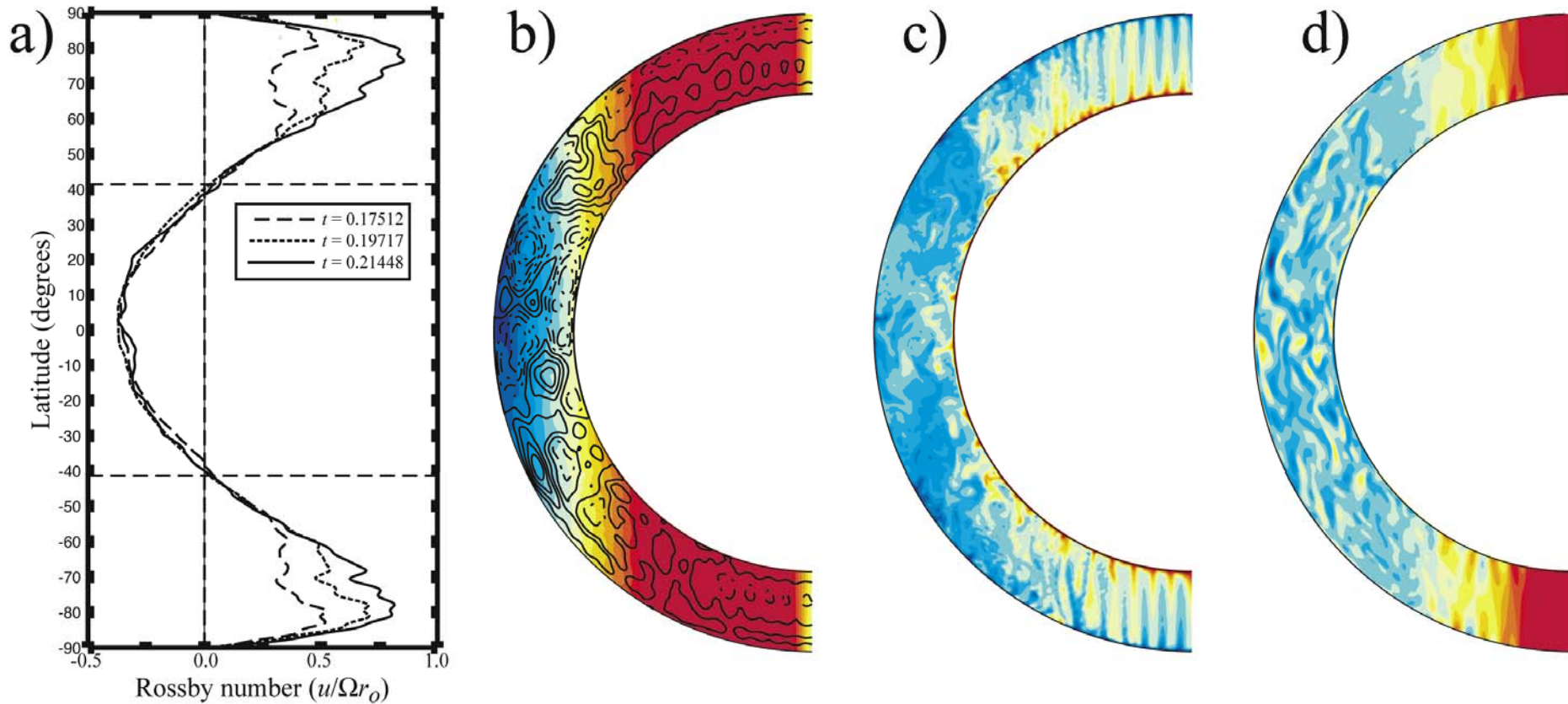


# Deep Convection Model ( $\chi = 0.75$ )



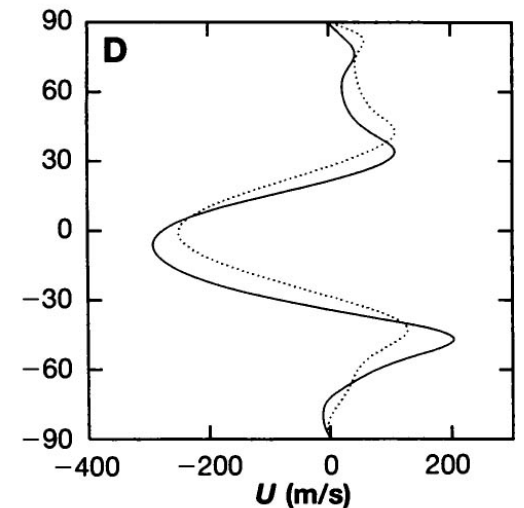
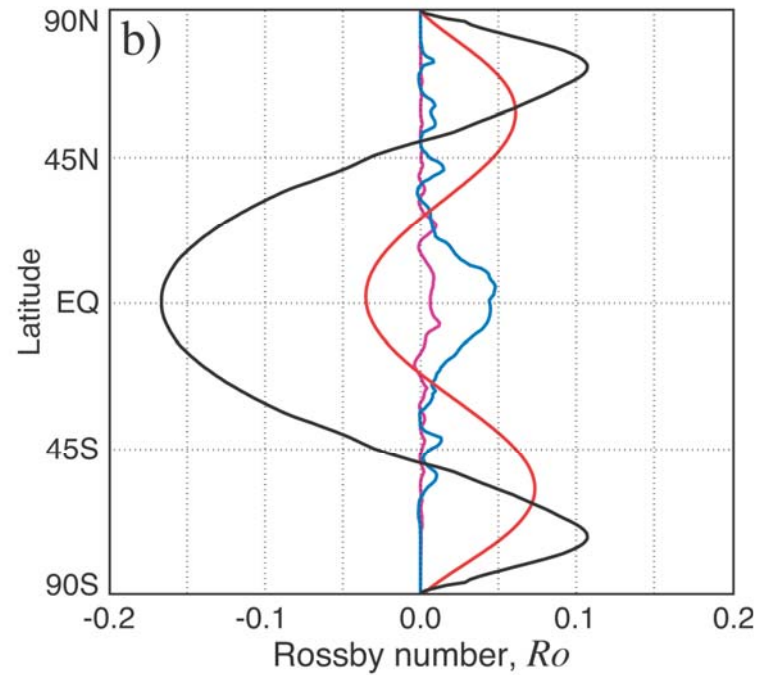
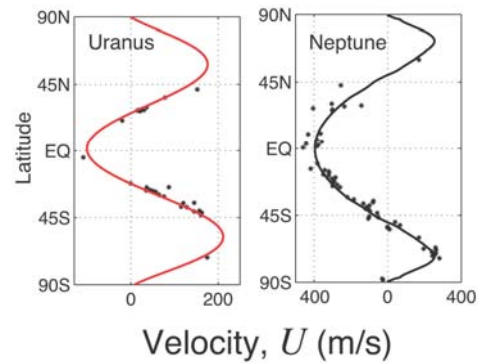
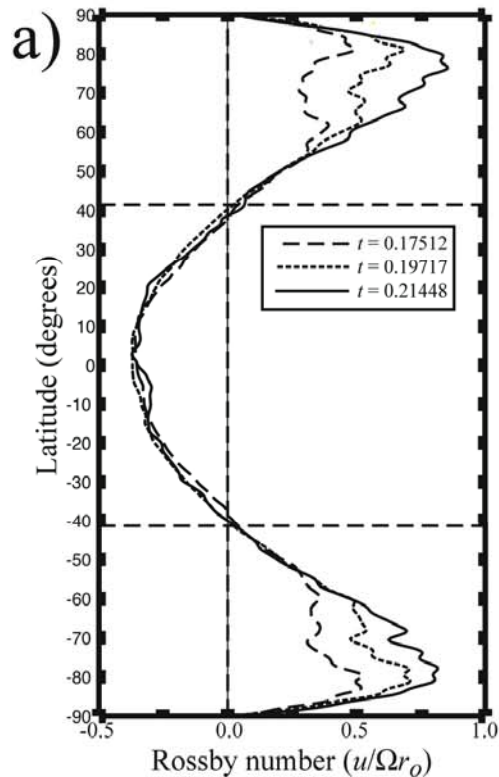
- a) Equatorial Jet vs.  $Ra^*$ 
  - $Ra^* = \text{Buoyancy} / \text{Coriolis}$
- b) Axisym. poloidal KE fraction vs.  $Ra^*$

# Deep Convection Model ( $\chi = 0.75$ )



- Retrograde jet case with  $Ra^* = 1.5$ 
  - $Ra = 1.5 \times 10^9$ ;  $E = 10^{-5}$ ;  $Pr = 0.1$

# Deep Convection Model ( $\chi = 0.75$ )



# Ice Giants Work

- Deep convection model for the Ice Giants
- Can deep models explain the thermal emissions of U vs. N?
  - Modeling
  - Subaru Observations

QuickTime?and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.

Questions?

