Torque

Why is a door knob located as far as possible from the door hinge? When you want to push open a door, you apply a force. <u>Where</u> you apply the force and <u>in what direction</u> you push are also important. If you apply the same force at two points, one twice as far from the point of rotation as the other, the force that is twice as far away has twice the torque and produces twice the amount of angular acceleration.
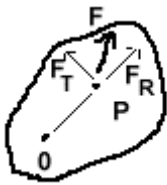
Torque
in circular motion, the force applied at a radial distance that changes the direction of motion of a rotation; torque can stop, start, or change the direction of circular motion; it is the "unbalanced force" of circular motion

$\tau = F \, d$

where τ is the torque in Newton meters (or N m), F is the perpendicular component of the applied force, and d is the radial distance (note: you may also think of d as being r, for radius)

Tangential and radial components of force A force *F* acts at at an angle at a point *P* on a rigid body that is free to rotate about an axis through *O* a distance *r* from the axis of rotation. Only the tangential component of *F* or F_T can have any effect on the rigid body. F_R (the radial component) passes through the axis of rotation and cannot cause the object to rotate.

The angle *F* makes with the angle of rotation is called ϕ

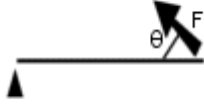**Torque (τ)** Torque is the "turning agent," or what causes the rotation about an axis

$\tau = r \, F \sin\phi$

Moment Arm ($r_{perpendicular}$) Perpendicular distance of the line of action of the force from the axis of rotation.

$\tau = F \, r_{perpendicular}$

moment arm.

Or, you may find the component of the force causing the torque.

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Resolve the force into its x and y components. Only the component that is perpendicular to the lever arm, or moment arm, causes a torque. In the image below, the vertical component F sin θ is perpendicular to the moment arm and thus causes a torque. The horizontal component F cos θ is parallel to the moment arm and does not cause a torque.
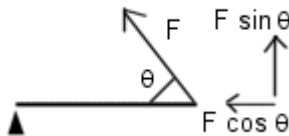
When a torque is applied, rotation occurs around a **pivot point,** or **fulcrum**. When more than one torque acts on a body, the acceleration produced is proportional to the net torque

Center of gravity
the point at which the entire weight of an object seems to act

<u>See-saw Torque</u>

<u>Motion of a rigid body about its center of mass</u>

Uniform
if an object is considered uniform, its center of gravity is at its geometric center

Rotational equilibrium
an object is said to be in rotational equilibrium when all the torques acting on it are balanced (or, $\Sigma \tau = 0$). A torque can cause a counterclockwise (cc) or a clockwise rotation (cw).
$\Sigma\tau_{cw} = \Sigma\tau_{cc}$

where Σ stands for "sum of"

Center of Mass (CM) The point where all the mass of an object seems to act. For example, if you look at the motion of a high jumper, there is one special spot that moves in a parabolic path. It would be the same point where you could balance that person. An object is in equilibrium as long as its CM is over its base level. An object is considered uniform when the CM is its geometric center. The CM position is given by:

$$x_{CM} = \frac{1}{M} \Sigma m_i x_i$$

where M is the total mass, m_i is the mass of the particle and x_i is the distance from the origin

Static Equilibrium

An analysis of static equilibrium is very important in engineering. The design engineer must identify and isolate all external forces and torques acting on the structure. With a good design and a correct choice of materials, structures can support loads. Landing gear of aircraft survive the shock of rough landings and bridges don't collapse under traffic loads and the wind.

Translational Equilibrium An object is in translational equilibrium (its momentum is constant) if the sum of the forces acting on it is zero.

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$

Rotational Equilibrium An object is in rotational equilibirum (its angular momentum is constant) if the sum of the torques acting on it is zero.

An object will be in equilibrium if it is suspended from its center of gravity or its center of gravity is below the suspension point.

Elasticity The branch of physics which deals with how objects deform when forces are applied to them.

Elastic Limit The point where the material being deformed suffers permanent deformation and will not return to its original shape.

There are three ways an object can change its dimensions when forces act on it:

1. An object can be deformed by **shearing** forces. It will behave like the pages of a book when under shear. An example: the movement of layers of rock in an earthquake.
2. An object can be deformed by **stretching** or **compressing** forces. An example: tensile forces that stretch the length of a string until it breaks. An example: Piling weights on a cylinder until it breaks.
3. An fluid can be deformed by **bulk** forces. An example: A fluid under high pressure can be compressed on all sides resuliting in a volume change.

A **stress** due to forces produces a **strain** or a deformation. Stress is proportional to strain and that proportionality cosntant is called its **modulus**. Stress is the product of modulus and strain. Stress is the ratio of the force exerted on the object to the cross-sectional area over which the force is exerted. Strain is the resulting deformation, whether it is the ratio of change in length to original length, change in height to original height, or change in volume to original volume.

$$\frac{F}{A} = E \frac{\Delta L}{L}$$

where F is the force, A is the cross-sectional area in m^2, E is Young's modulus, L is the original length, and ΔL is the change in length.

$$\frac{F}{A} = G \frac{\Delta h}{h}$$

where F is the force, A is the cross-sectional area in m^2, G is shear modulus, h is the original height, and Δ h is the change in height.

$$\frac{F}{A} = B \frac{\Delta V}{V}.$$

where F is the force, A is the cross-sectional area in m^2, B is bulk modulus, V is the original volume, and Δ V is the change in volume.
Remember, the ratio of F to A is the pressure (P) of the fluid.

The unit for modulus is N/m^2 or Pascals (Pa).

Rotational Motion

Rotational motion is the motion of an object around an axis. Up to this point, we have only studied motion in a straight line (translational motion). Now we will study motion about an axis, or rotational motion. Objects can move translationally or rotationally or both. They can be in translational equilibrium (the sum of all the forces acting on the object is zero), but not in rotational equilibrium (the sum of all the torques acting on the object is zero), and vice versa. Or, they may be in both rotational and translational equilibrium.

Rotational Motion The translational motion of a rigid body is analyzed by describing the motion of its center of mass, as well as rotational motion about its center of mass. Each particle of a rotating rigid body has, at any moment, a linear velocity *v* and a linear acceleration *a*. The angular velocity is the same for <u>every</u> point in the rotating body at any instant, but the linear velocity is greater for points further from the axis of rotation.

Children on a merry-go-round all have different <u>linear</u> speeds (measured in m/s) depending on how far they are from the axis of rotation. They all have the same rotational speed (in rev/sec or rad/sec) no matter where they are located.

In the image below, a body is rotating about a fixed axis through its center. An object placed on the rotating object at point A that rotates to point B rotates through the same angle as an object placed at point a that rotates to point b. Both traveled the same angular distance θ. They did not travel the same tangential distance. One traveled the arc length AB in time t while the other traveled the arc length ab in time t.
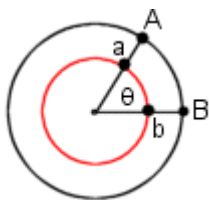
Angular displacement,θ
    is the angle about the axis through which the object turns. It is measured in degrees, revolutions, or the SI unit of radians.

1 revolution = 360° = 2 π radians
$\theta = d/r$
where d is the tangential distance and r is the radius.

Note: in rotational motion, it is easy to use the radius to convert back and forth between rotational and translational quantities. It is also easy to remember what to do. Think of the units! If you have a distance in meters, what would you do with the radius (also in meters) to convert it into radians? You would <u>divide</u> distance in meters by the radius in meters. Meters cancel leaving radians. A radian is a unit that serves as a place holder.

Angular Positon The object has rotated through some angle θ when it travels the distance *l* measured along the circumference of its circular path.

Radian One radian (rad) is defined as the angle subtended by an arc whose length is equal to the radius. In other words, if *l* = *r*, then θ is exactly equal to one radian.
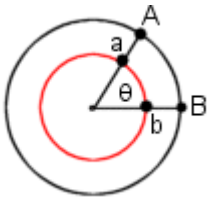
Angular speed (or velocity), ω

the rate at which an object rotates. The SI unit is rad/sec. In the image above, the object rotates through angle θ in time t. The angular speed or velocity is given by (remember, in translational motion v = d/t)

$\omega = \theta/t$

Angular speed (velocity) can be converted to the analogous translational speed (velocity) using the radius.

$\omega = v/r$
where v is the tangential velocity

Angular velocity (speed) and its relationship to frequency When an object rotates from some initial position θ_i to some final position θ_f, then its angular velocity (or speed) ω is equal to the change in the angular position $\Delta\theta = \theta_f - \theta_i$ divided by the change in time, or $\omega = \Delta\theta / t$ The angular velocity can be related to the frequency of rotation, *f*, where frequency is the number of complete revolutions per second. Since one revolution per second corresponds to an angle of 2π radians per second, $f = \omega / 2\pi$

Angular acceleration, α

the rate at which a rotating object changes angular speed. The SI unit is rad/s^2. The angular acceleration is the change in angular speed divided by time (remember, in translational motion, a = $(v_f - v_i)/t$)

$\alpha = (\omega_f - \omega_i)/t$

Angular acceleration can be converted to the analogous translational acceleration using the radius.

$\alpha = a/r$
where a is the tangential acceleration and r is the radius

Anglular Acceleration Angular acceleration is the change in angular velocity divided by the time to make this change. The average angular acceleration, $\alpha = \Delta \omega / t$

Radial Component of the linear acceleration The total linear acceleration a is the vector sum of the radial component of the acceleration and the tangential component of the acceleration. The radial component of the linear acceleration (or centripetal acceleration) can be written as $a_R = \omega^2 r$. Thus, the centripetal acceleration increases the farther you are from the axis of rotation. The children farthest out on a merry-go-round experience the greatest acceleration.

In the image below, the radial component of the acceleration, a_R, represents the centripetal acceleration. The tangential component of the acceleration, a_{tan}, represents the acceleration that is measured tangent to the circle. The total linear acceleration of a rotating object is the vector sum of the two components
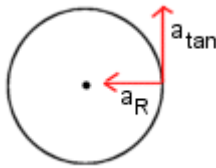
The equations for linear (tangential or translational) motion can be transformed into analogous rotational forms:

$d = v\,t$	$\theta = \omega\,t$
$d = d_o + v_i\,t + \frac{1}{2}\,at^2$	$\theta = \theta_o + \omega_i\,t + \frac{1}{2}\,\alpha\,t^2$
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha\,t$
$v_f^2 = v_i^2 + 2\,ad$	$\omega_f^2 = \omega_i^2 + 2\,\alpha\theta$

Just as an unbalanced force is required to change the motion of an object in linear (translational) motion, a torque is required to change the motion of an object in rotation motional.

$\tau = F\,r$
where r is the radius

Newton's 2nd law can be transformed into its analogous rotational form:

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F = ma	$\tau = I \alpha$

Rolling motion of a wheel or sphere Rolling *without slipping* involves both rotation and translation. Remember the relationship between angular velocity of the rotating object and the linear speed of the axel, or $\omega = v/r$. At any moment when the rotating wheel is in contact with the ground, at that point of contact the wheel is momentarily at rest. The velcity of the axel equals *v*; the velocity at the top of the wheel is 2*v*.

moment of inertia, I

the rotational inertia of a rotating body. It is analogous to mass in translational motion. The rotational inertia not only depends upon the rotating body's mass, but also the distribution of that mass.

Advanced look at moment of inertia: You can think of a rotating rigid body as conisting of many particles located at different radial distances from the axis of rotation. The moment of inertia of the rotating body is simply the sum of the masses of each particle multiplied by the square of the distance of that particle from the axis of rotation.

$I = \Sigma m r^2 = m_1 r^2_1 + m_2 r^2_2 + m_3 r^2_3 + m_n r^2_n$

Until this point, our study of physics has been concerned with translational motion, or motion in the *xy* plane. The standard English alphabet supplies the variables for this motion. We use the Greek alphabet for variables to distinguish rotational motion from translational motion. The following table lists variables for translational motion and the analogous rotational variable with their SI variables.

distance/displacement	d in m	θ in rad
speed/velocity	v in m/s	ω in rad/s
acceleration	a in m/s^2	α in rad/s^2
force	F in Newtons	τ in N m
mass	m in kg	I in kg m^2

Rotational Dynamics

A force is required to make an object start rotating about an axis. The direction of the force and where it is applied is important. A torque produces angular acceleration. A torque is required to start the rotation of a body. A rotating rigid body can be considered as consisting of many particles located at various distances from the axis of rotation. The sum of the torques due to each of these particles is just the total torque. The moment of inertia, *I*, tells how the mass of the body is distributed about the axis of rotation. In rotational motion, $\tau = I \alpha$

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Some rotational moments of inertia for solid bodies:

Thin hoop (bicycle wheel or ring) of radius r	$I = mr^2$
Solid disk (solid cylinder, record, or pulley wheel) of radius r	$I = 1/2\ mr^2$
Uniform sphere (star) of radius r	$I = 2/5\ mr^2$
Long, uniform rod of length L with its axis of rotation through its center	$I = 1/12\ mr^2$
Long, uniform rod of length L with its axis of rotation through one end	$I = 1/3\ mr^2$

Rotational Kinetic Energy

A body rotating about an axis is said to have rotational kinetic energy. It is analogous to translational kinetic energy. Its SI unit is Joules. An object that rotates about its center of mass undergoes translational motion that has both translational and rotational KE, if the axis is fixed. For an object that rolls without slipping down an incline, the original potential energy is equal to the sum of the translational kinetic energy and the rotational kinetic energy.

$1/2\ m\ v^2$	$1/2\ I\ \omega^2$

An object that rotates while its center of mass (CM) translates will have both rotational and translational kinetic energies. The total kinetic energy of such an object is given by:

$$KE = 1/2\ mv^2_{CM} + 1/2\ I_{CM}\omega^2$$

An object sliding down an incline (without rolling) transforms all of its potential energy into translational kinetic energy. An object that rolls down the inlcine transforms part of its potential energy to translational kinetic energy and part to rotational kinetic energy. Consider several objects at the top of an incline: a box which slides down the incline and a hoop, a solid cylinder, and a sphere which roll down the incline. Which will reach the bottom first? The sliding box which converts <u>all</u> its potential energy into translational kinetic energy. The hoop will be last because it converts the greatest amount of its potential energy into rotational kinetic energy. The more translational kinetic energy an object has, the faster it reaches the bottom of the incline.

Angular Momentum and its Conservation

Linear momentum has its analogous quantity, angular momentum, *L*.

$p = m\ v$	$L = I\ \omega$

Angular momentum is an important concept because it remains constant if no external torques are acting. The law of conservation of angular momentum is one of the conservation laws of physics. If zero net torque is acting on a body and the body is rotating about a fixed axis, the law of conservation of angular momentum states

$I \omega$ = constant.

Many things can be understood when one analyzes their motion using conservation of angular momentum. An ice skater spinning on the ice with her arms extended has a large I and a small ω; an ice skater spinning on the ice with her arms pulled close to her body has a small I and a large ω. Since angular momentum must be conserved, when her I decreased by bringing her arms close to her body, her angular speed must increase.