COLLEGE PHYSICS

Chapter 2 INTRODUCTION: Kinematics in One Dimension

Lesson 5

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EQUATIONS FOR CONSTANT ACCELERATION:

1. $x = x_0 + \left(\frac{v_0 + v}{2}\right)t$

2. $v = v_0 + at$

3. $x = x_0 + v_0 t + \frac{1}{2}at^2$

4. $v^2 = {v_0}^2 + 2a(x - x_0)$

Strategy: When a = constant, you can use one of these equations to solve for an unknown.

Equations for Constant Acceleration

Idea: An object moves along the x-axis with constant acceleration, a.

Initial conditions:

At time $t=0$, $x=x_0$, $v=v_0$

Finally:

At some time, t, the object has position x and velocity v.

Elapsed time: $\Delta t = t$

Displacement: $\Delta x = x - x_0$

Change in velocity: $\Delta v = v - v_0$

First Equation:

Use definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$$

$$\Rightarrow \bar{v} t = x - x_0$$

$$\Rightarrow x = x_0 + \bar{v} t$$

Note that when $a =$ constant, $\bar{v}$ is exactly half-way between v_0 and v:

$$\bar{v} = \frac{v_0 + v}{2}$$

$$\boxed{x = x_0 + \left(\frac{v_0 + v}{2}\right) t} \qquad (1)$$

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Example.

A Ducati motorcycle can go from 0 to 60 mph in 3 seconds. Assuming constant acceleration, how far does it travel in this time?

[Note: 1 mile = 1600 m]

First, let's convert to SI units:

$$v = 60\ \frac{\cancel{\text{miles}}}{\cancel{\text{hr}}}\left(\frac{1600\ \text{m}}{1\ \cancel{\text{miles}}}\right)\left(\frac{1\ \cancel{\text{hr}}}{3600\ \text{s}}\right) = 26.667\ \frac{\text{m}}{\text{s}}$$

$v_0 = 0 \qquad t = 3\ \text{s}$

Set $x_0 = 0$, solve for x

Use (1): $$x = 0 + \left(\frac{0 + 26.667}{2}\right)3$$

$$\boxed{x = 40\ \text{m}}$$

Equations for Constant Acceleration

Second Equation

Use definition of average acceleration:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$$

Note that when $a =$ constant, $a = \bar{a}$.

$$\Rightarrow \quad a = \frac{v - v_0}{t}$$

$$\Rightarrow \quad at = v - v_0$$

$$\Rightarrow \quad \boxed{v = v_0 + at} \qquad (2)$$

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Example.

The maximum acceleration of your car is a "quarter-gee": $a = 9.8\ \text{m/s}^2 \div 4 = 2.45\ \text{m/s}^2$. If you are driving at 50 km/hr, what is the minimum time required to accelerate to 120 km/hr?

First, let's convert to SI units:

$$v_0 = 50\ \frac{\text{k}\cancel{\text{m}}}{\cancel{\text{hr}}}\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{hr}}}{3600\ \text{s}}\right) = 13.889\ \frac{\text{m}}{\text{s}}$$

$$v_f = 120\ \frac{\text{km}}{\text{hr}}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = 33.333\ \frac{\text{m}}{\text{s}}$$

$\Rightarrow$ Minimum time to change velocity will be when your acceleration is maximum:

$$a = 2.45\ \text{m/s}^2$$

Use eq. (2): $v = v_0 + at$, solve for t:

$$at = v - v_0$$

$$t = \frac{v - v_0}{a} = \left(\frac{33.333 - 13.889}{2.45}\right)$$

$$\boxed{t = 7.9\ \text{s}}$$

Equations for Constant Acceleration

Third Equation

Recall:

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t \qquad (1)$$

$$\boxed{v = v_0 + at} \qquad (2)$$

Let's eliminate v by plugging (2) into (1).

$$x = x_0 + \left[\frac{v_0 + (v_0 + at)}{2}\right]t$$

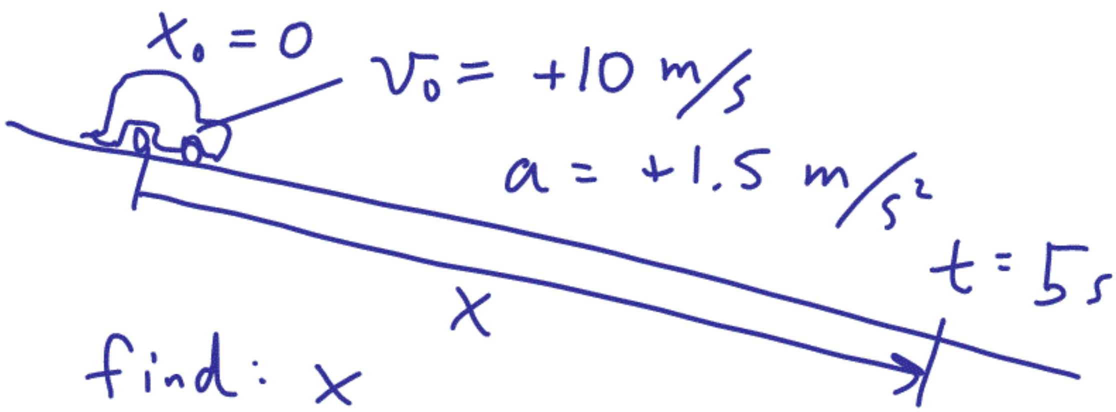
$$= x_0 + (2v_0 + at)\frac{t}{2}$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \qquad (3)$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2} \qquad (3)$$

Example.

A car rolls down a hill with an initial velocity of v = +10 m/s, and a constant acceleration of a = 1.5 m/s^2. How far does it roll in 5 seconds?



find: x

Use (3):

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 0 + (10)(5) + \frac{1}{2}(1.5)(5)^2 \\ &= 50 + 18.75 \end{aligned}$$

$$\boxed{x = 69\text{m}}$$

Equations for Constant Acceleration

Fourth Equation

Recall:

$$x = x_0 + \left(\frac{v_0 + v}{2}\right) t \qquad (1)$$

$$v = v_0 + at \qquad (2)$$

First, let's solve (2) for t:

$$v - v_0 = at$$

$$\Rightarrow \boxed{t = \frac{v - v_0}{a}}$$

Next, let's eliminate t by plugging this into eq. (1).

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right)$$

$$x = x_0 + \frac{1}{2a}(v + v_0)(v - v_0)$$

$$= x_0 + \frac{1}{2a}(v^2 - v_0^2)$$

Solve for v^2:

$$x - x_0 = \frac{1}{2a}\left(v^2 - v_0^2\right)$$

$$2a(x - x_0) = v^2 - v_0^2$$

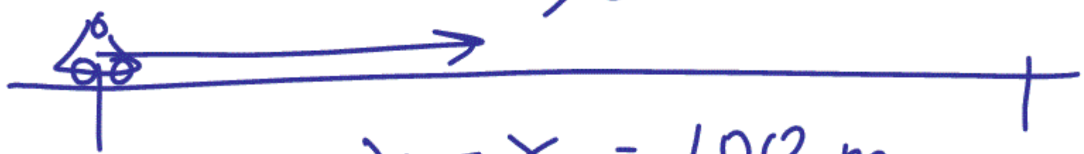
$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \qquad (4)$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \qquad (4)$$

Example.

A Ducati motorcycle starts from rest, and accelerates at $a = 8.9$ m/s^2. How fast is it going when it has traveled 100 m?

$v_0 = 0$, $a = 8.9\ m/s^2$



$x - x_0 = 100$ m.

$v = ?$

Use (4): $v^2 = v_0^2 + 2a(x - x_0)$

$= 0 + 2(8.9)(100)$

$v^2 = 1780$

$v = \sqrt{1780} = 42.190$ m/s

$$\boxed{v = 42\ m/s}$$

<u>Assess</u>: What is this in km/hr?

$$v = 42.19\ \frac{\cancel{m}}{\cancel{s}}\left(\frac{1\ km}{1000\ \cancel{m}}\right)\left(\frac{3600\ \cancel{s}}{1\ hr}\right)$$

$v = 152$ km/hr .... That's fast!

EQUATIONS FOR CONSTANT ACCELERATION:

1. $x = x_0 + \left(\frac{v_0 + v}{2}\right) t$ Does not contain a ! (but you know it's constant)

2. $v = v_0 + at$ Does not contain position!

3. $x = x_0 + v_0 t + \frac{1}{2} a t^2$ Does not contain v_f !

4. $v^2 = {v_0}^2 + 2a(x - x_0)$ Does not contain t !

Strategy: Figure out which variable you don't know and don't care about, and use the equation which doesn't contain it.

PROBLEM-SOLVING STEPS

1. Draw a simple sketch. Decide what direction is positive, and note it on your sketch.
2. Make a list of what is given: "knowns". Remember, "stopped" means velocity is zero.
3. Identify exactly what needs to be determined in the problem: "unknowns".
4. Find an equation or set of equations that can help you solve the problem.
5. Convert to S.I. units, plug the knowns into the appropriate equations, solve the problem.
6. Check to see if the answer is reasonable: does it make sense?