COLLEGE PHYSICS

Chapter 3: Two-Dimensional Kinematics

Lesson 7

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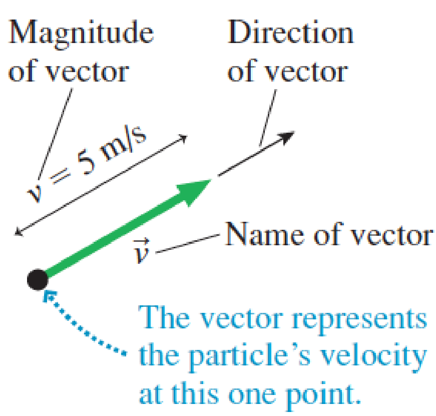
VECTORS

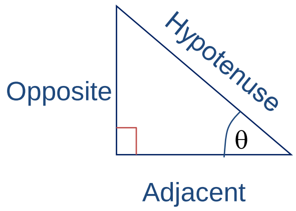
- A quantity having both a magnitude and a direction is called a **vector**
- The *geometric representation* of a vector is an arrow with the tail of the arrow placed at the point where the measurement is made



BUT FIRST … REVIEW!
RIGHT TRIANGLE TRIGONOMETRY

This is one of the most common things people are rusty with…

- $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$
- $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$
- $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$

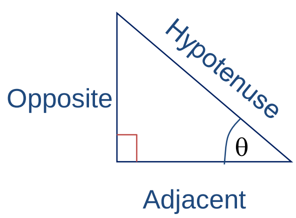


SOH CAH TOA

GIVE IT A TRY!

If the hypotenuse shown is 6m and the angle is 38°, what is the length of the adjacent side?

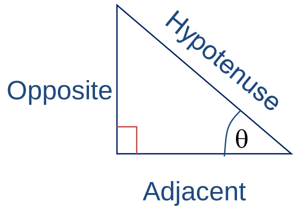
A. $(6\text{ m})\sin(38°)$
B. $(6\text{ m})\tan(38°)$
C. $(6\text{ m})\cos(52°)$
D. $(6\text{ m})\cos(38°)$
E. Not enough information

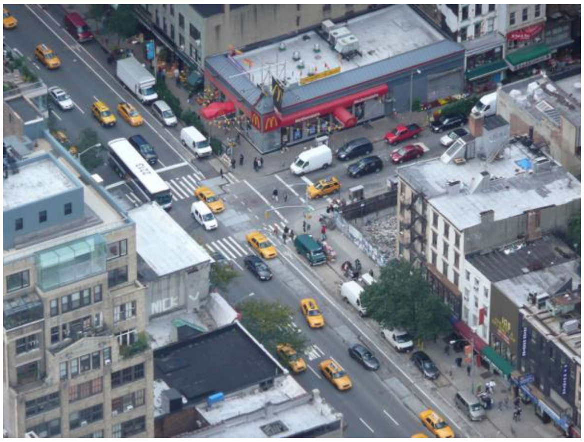


GIVE IT A TRY!

If the adjacent side below is 6 m and the opposite side is 4 m, what is the angle θ?

A. $\tan^{-1}(6/4)$
B. $\cos^{-1}(6/4)$
C. $\cos^{-1}(4/6)$
D. $\tan^{-1}(4/6)$
E. $\sin^{-1}(4/6)$



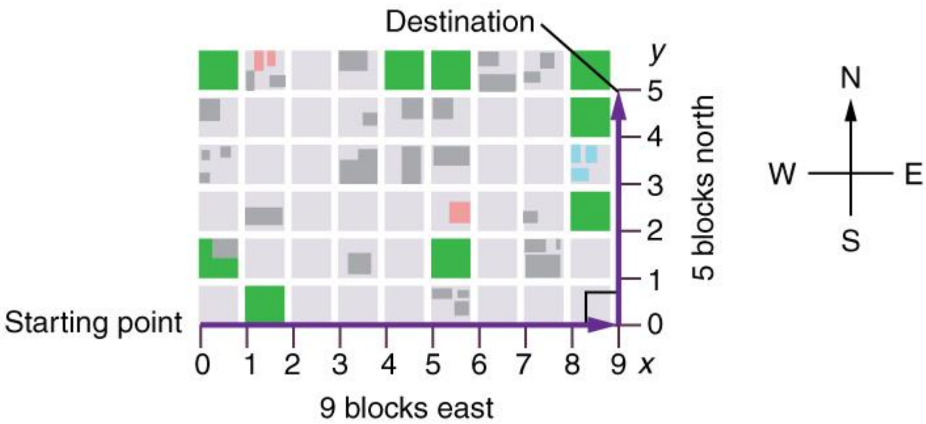


Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations.

Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths.

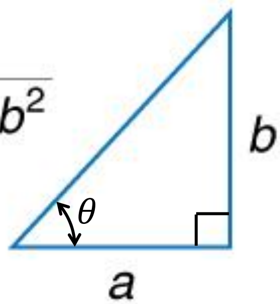
PROPERTIES OF VECTORS

- Suppose a pedestrian walks 9 blocks east, then turns left and walks 5 blocks north.
- In this city, every block is a 100 m x 100 m square



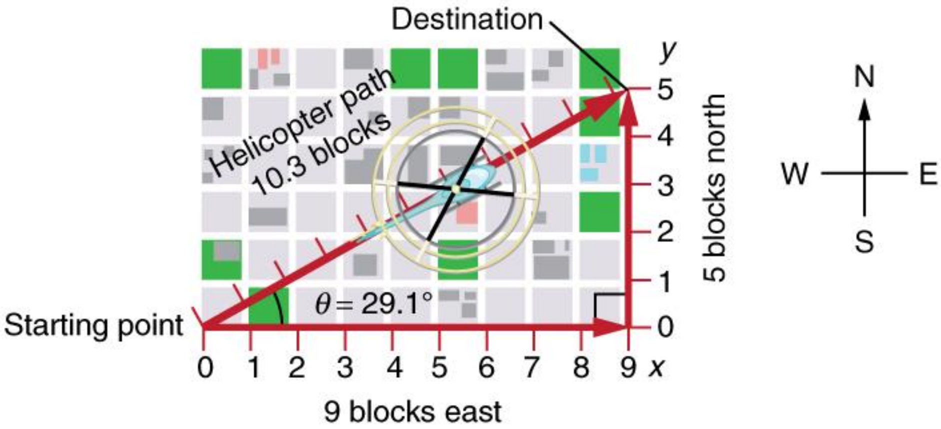
$$c = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$



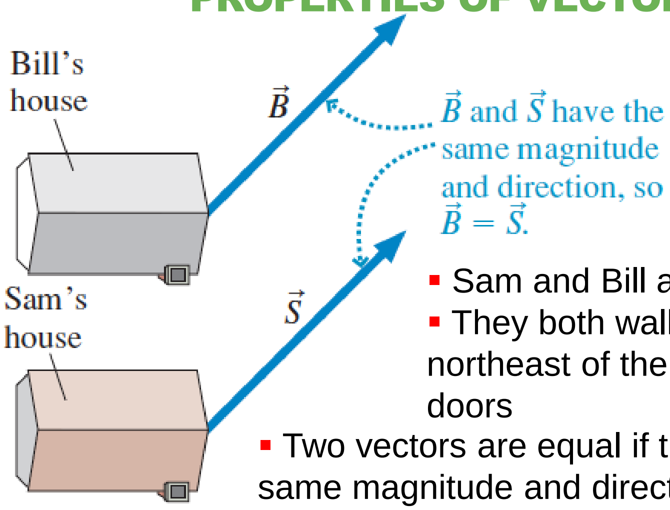
- The Pythagorean theorem relates the length of the legs of a right triangle, labeled *a* and *b* , with the hypotenuse, labeled *c* .
- The relationship is given by: $a^2 + b^2 = c^2$.
- The angle of ***c*** is given by: $\tan\theta = \frac{b}{a}$

DISPLACEMENT



The displacement is 10.3 blocks (or 1030 m) at an angle 29.1º north of east.

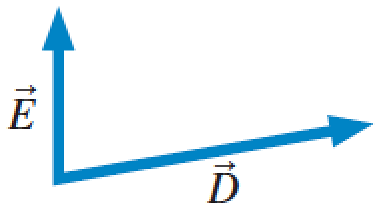
PROPERTIES OF VECTORS



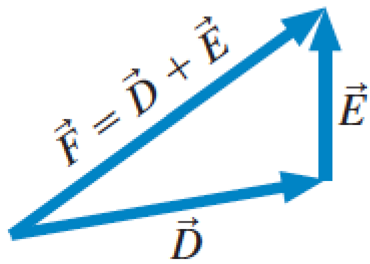
- Sam and Bill are neighbors
- They both walk 200 m to the northeast of their own front doors
- Two vectors are equal if they have the same magnitude and direction
- This is true regardless of the starting points of the vectors
- ***B*** = ***S***

VECTOR ADDITION
GEOMETRIC METHOD 1: TIP-TO-TAIL

- To evaluate ***F*** = ***D*** + ***E***, you could move ***E*** over and use the tip-to-tail rule



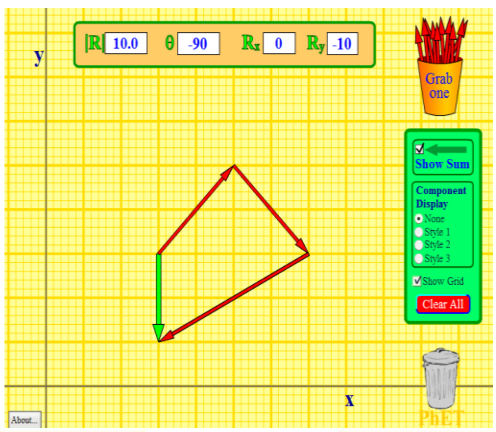
What is $\vec{D} + \vec{E}$?



Tip-to-tail rule:
Slide the tail of $\vec{E}$ to the tip of $\vec{D}$.

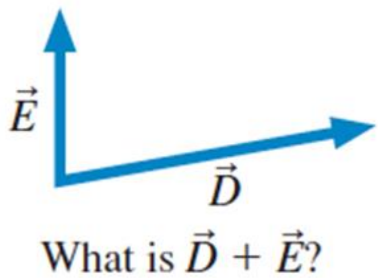
EXPLORE!
VECTOR ADDITION

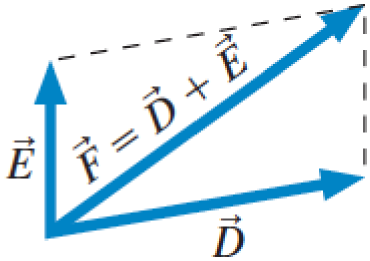
http://phet.colorado.edu/en/simulation/vector-addition



VECTOR ADDITION
GEOMETRIC METHOD 2: PARALLELOGRAM METHOD

- Alternatively, ***F*** = ***D*** + ***E*** can be found as the diagonal of the parallelogram defined by ***D*** and ***E***

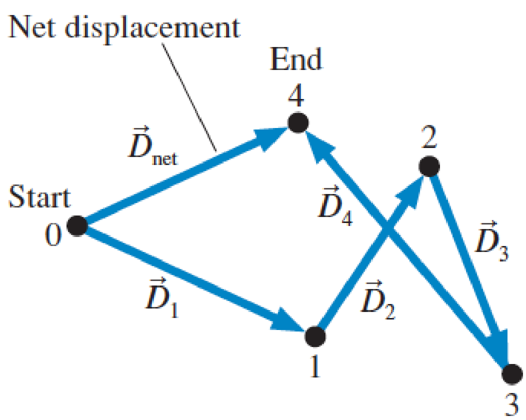




Parallelogram rule: Find the diagonal of the parallelogram formed by $\vec{D}$ and $\vec{E}$.

ADDITION OF MORE THAN TWO VECTORS

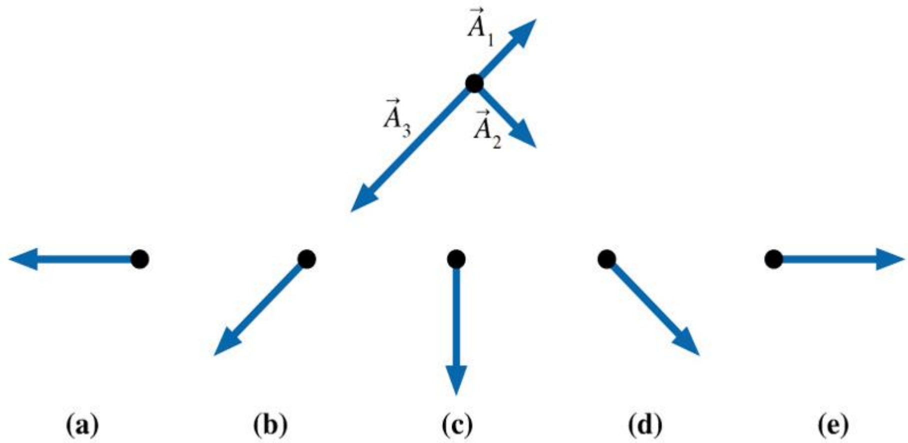
- The figure shows the path of a hiker moving from initial position 0 to position 1, then 2, 3, and finally arriving at position 4



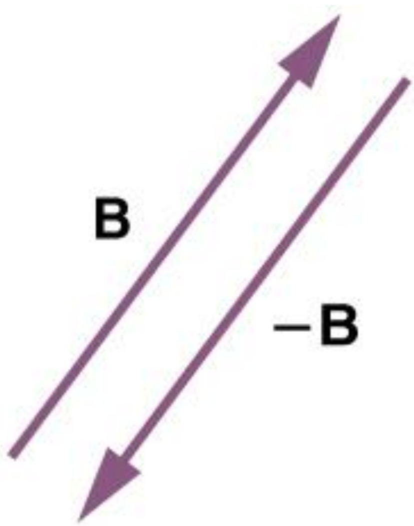
- The hiker's net displacement, an arrow from position 0 to 4, is: $\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$

GIVE IT A TRY!

Which figure shows the vector sum $\mathbf{A_1} + \mathbf{A_2} + \mathbf{A_3}$?

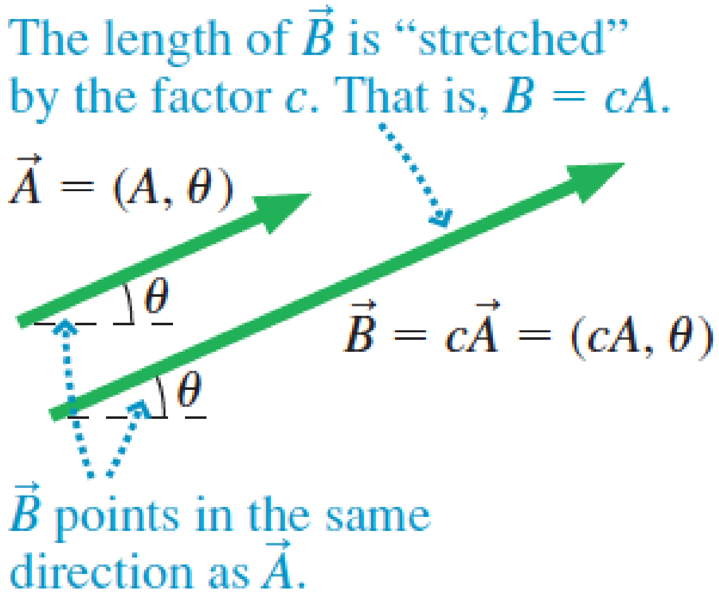


MORE VECTOR MATHEMATICS

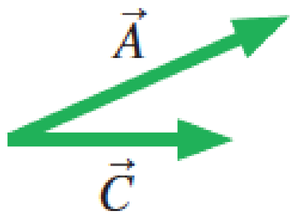


- The negative of a vector is just another vector of the same magnitude, but pointing in the opposite direction.
- $\mathbf{B} + (-\mathbf{B}) = \mathbf{0}$, the zero-vector

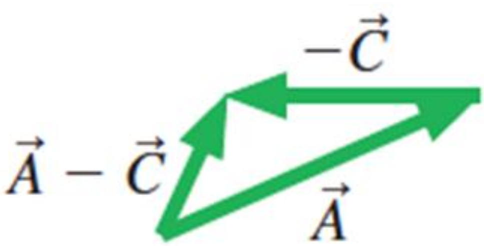
MULTIPLYING A VECTOR BY A SCALAR



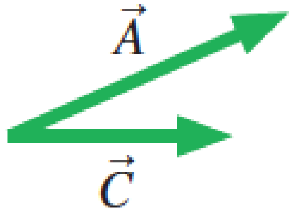
VECTOR SUBTRACTION



Vector subtraction: What is $\vec{A} - \vec{C}$?
Write it as $\vec{A} + (-\vec{C})$ and add!



VECTOR SUBTRACTION



Vector subtraction: What is $\vec{A} - \vec{C}$?
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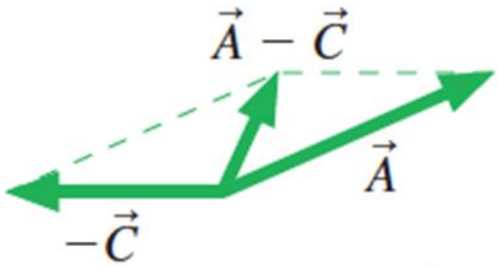
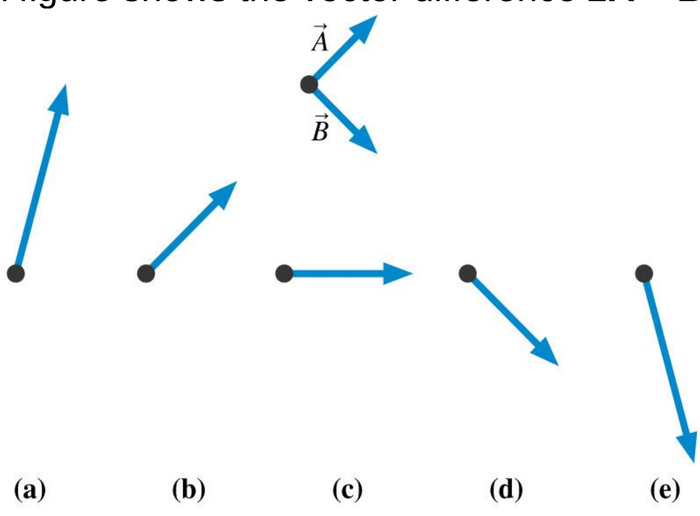


GIVE IT A TRY!

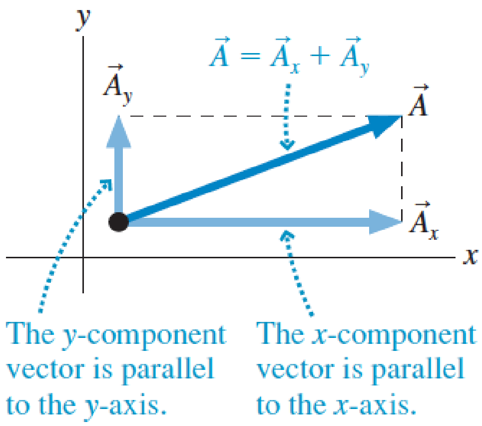
Which figure shows the vector difference 2**A** – **B**?



COMPONENT VECTORS

- The figure shows a vector *A* and an *xy*-coordinate system that we've chosen
- We can define two new vectors *parallel to the axes* that we call the **component vectors** of *A*, such that:

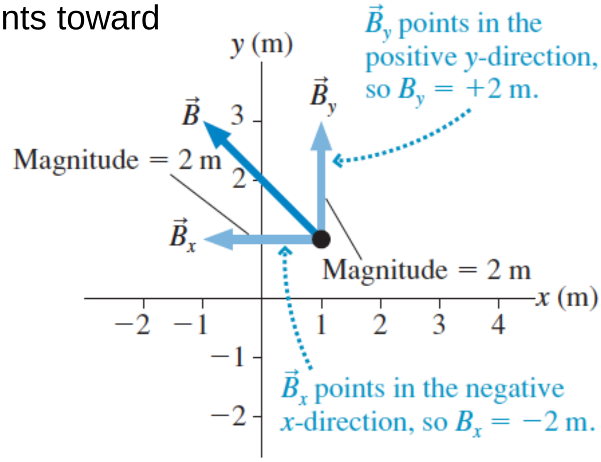
$$\vec{A} = \vec{A}_x + \vec{A}_y$$



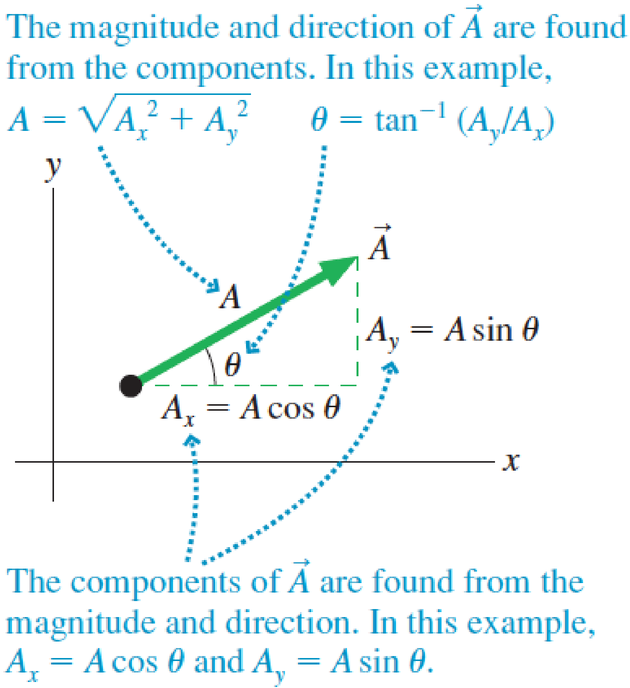
- We have broken *A* into two perpendicular vectors that are parallel to the coordinate axes
- This is called the **decomposition** of *A* into its component vectors

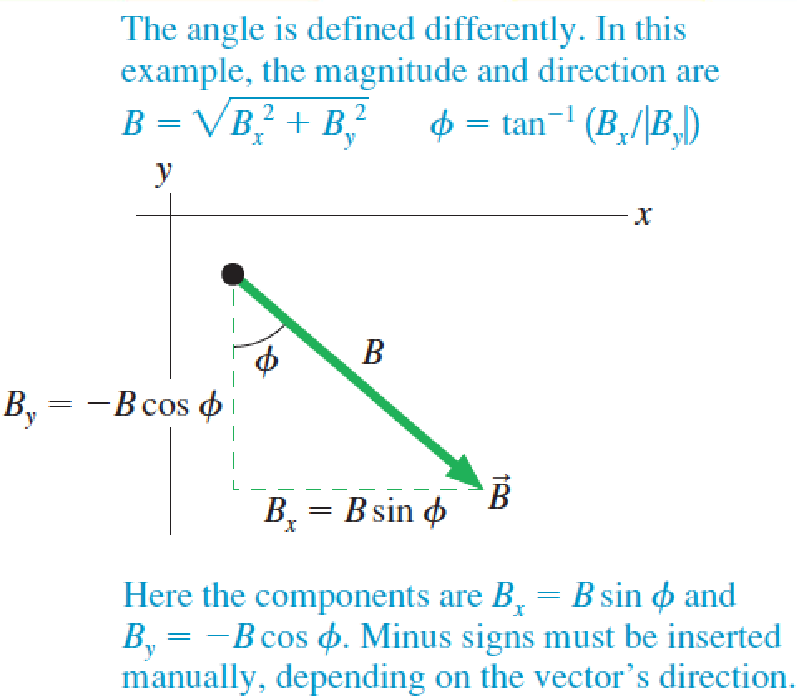
COMPONENTS

- We can describe each component vector with a single number called the component
- The component tells us how big the component vector is, and, with its sign, which ends of the axis the component vector points toward



Switching between (magnitude, direction) notation and (*x*-component, *y*-component) notation:





VECTOR ADDITION AND SUBTRACTION: ANALYTICAL METHOD

- We can perform vector addition by adding the *x*- and *y*-components separately
- For example, if ***D*** = ***A*** + ***B*** + ***C***, then:

$$D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

- Similarly, to find ***R*** = ***P*** – ***Q*** we would compute:

$$R_x = P_x - Q_x$$

$$R_y = P_y - Q_y$$

- To find ***T*** = *c**S***, where *c* is a scalar, we would compute:

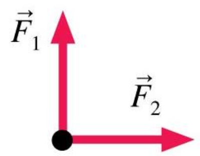
$$T_x = cS_x$$

$$T_y = cS_y$$

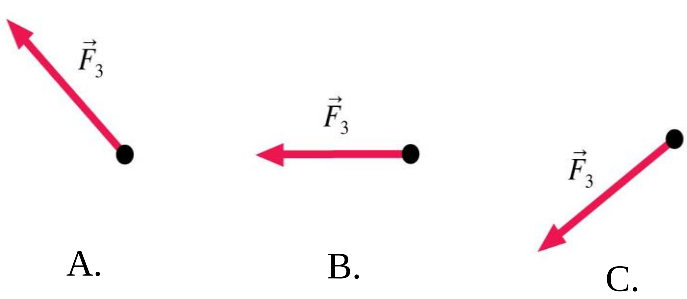
GIVE IT A TRY!

The sum $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ points to the left.

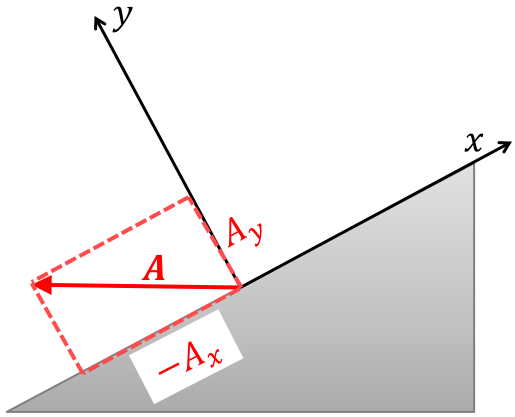
Two of three forces are shown. Which is the missing third force?



Two of the three forces exerted on an object



TILTED AXES AND ARBITRARY DIRECTIONS



- For some problems it is convenient to tilt the axes of the coordinate system
- The axes are still perpendicular to each other, but there is no requirement that the *x*-axis has to be horizontal
- Tilted axes are useful if you need to determine component vectors "parallel to" and "perpendicular to" an arbitrary line or surface