COLLEGE PHYSICS

Chapter 3: Two-Dimensional Kinematics

Lesson 8

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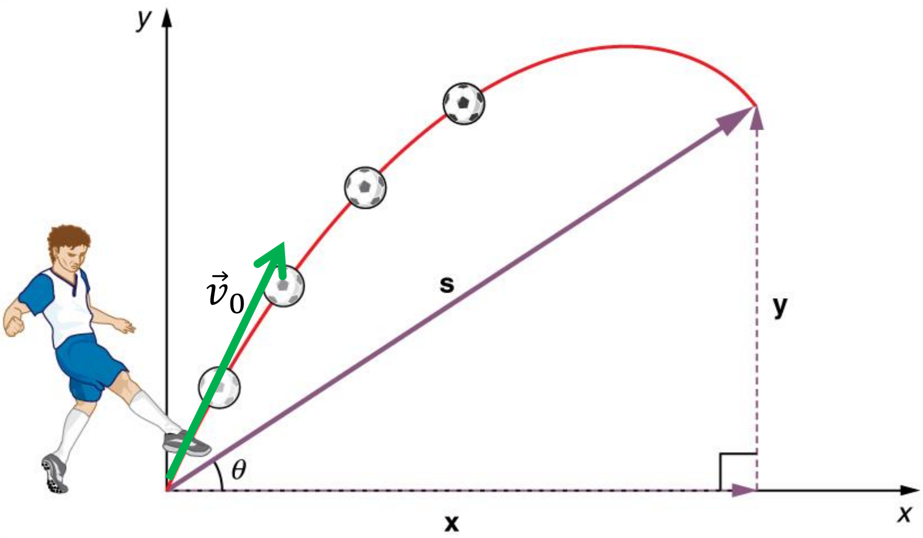
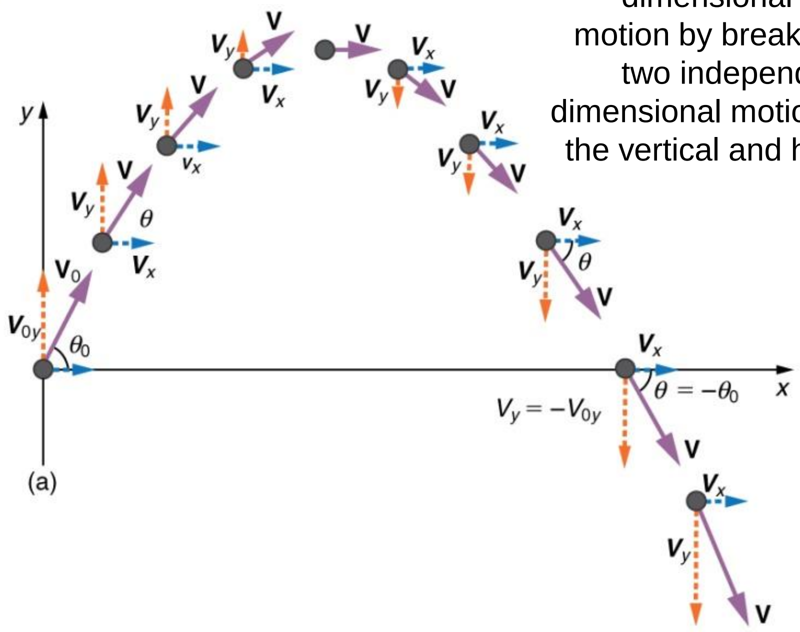
PROJECTILE MOTION

- Baseballs, tennis balls, Olympic divers, etc, all exhibit *projectile motion*
- A **projectile** is an object that moves in two dimensions under the influence of *only* gravity
- Projectile motion extends the idea of free-fall motion to include a **horizontal component** of velocity
- Air resistance is neglected

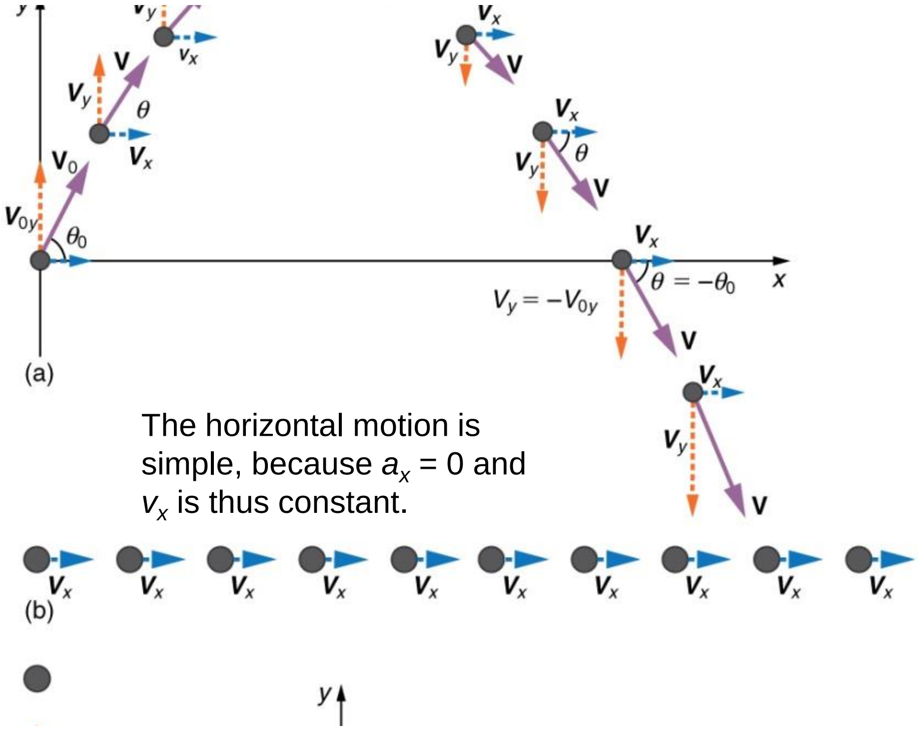
- Projectiles in two dimensions follow a *parabolic trajectory* as shown in the photo

PROJECTILE MOTION

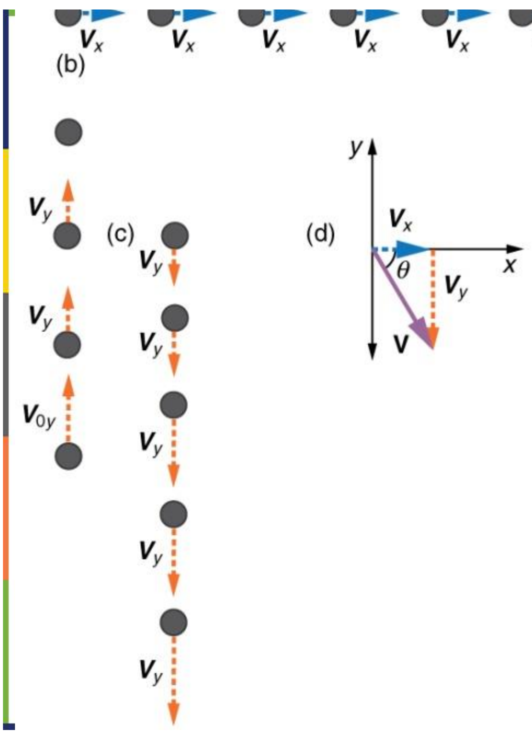




We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.

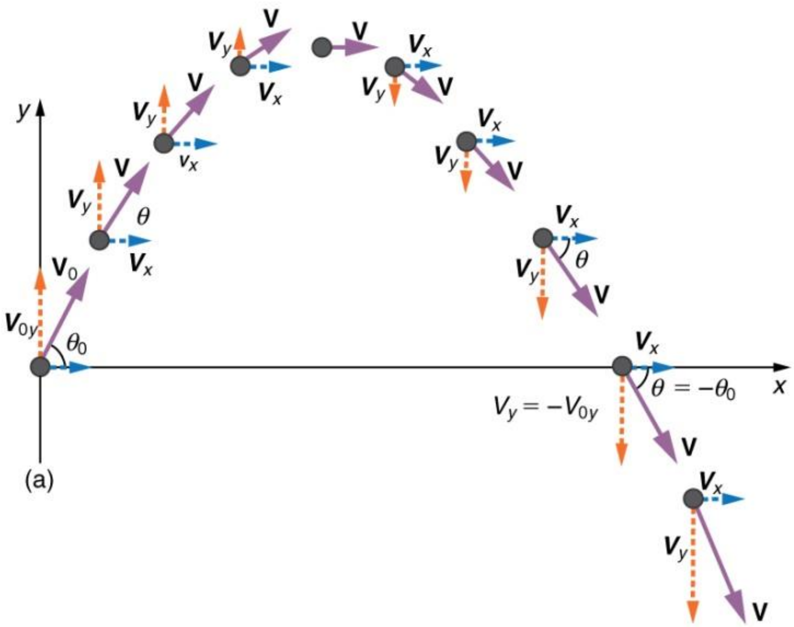


The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant.

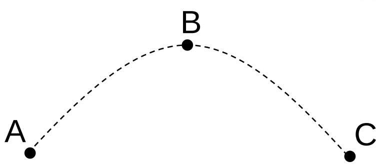


The velocity in the vertical direction decreases as the object rises, is zero at the object's highest point, and increases as the object falls towards the Earth again.

The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.



GIVE IT A TRY!



A tennis ball is launched at an angle, and flies through the air in a parabolic path, as shown, A→ B → C.

At point B

A. the velocity is zero.

B. the velocity is horizontal.

C. the velocity is vertical.

D. the velocity is up and to the right.

E. the velocity is down and to the right.

EQUATIONS FOR PROJECTILE MOTION

Horizontal $a_x = 0$	Vertical $a_y = -g$
$x = x_0 + v_x t$	$y = y_0 + \left(\frac{v_{0y} + v_y}{2}\right) t$
$v_x = v_{0x} =$ constant	$v_y = v_{0y} - gt$
	$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$
	${v_y}^2 = {v_{0y}}^2 - 2g(y - y_0)$

Note: The only common variables between the horizontal and vertical motions is time *t*.

GIVE IT A TRY!

A bullet is fired horizontally at the same moment that another bullet is dropped from rest. Both bullets begin at the same height above the ground. Which bullet hits the ground first?

A. The bullet which was fired horizontally.

B. The bullet which was dropped from rest

C. They both hit the ground at the same time

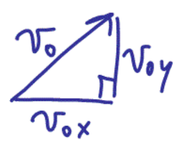
WORKED EXAMPLE: RANGE

- A cannonball is fired with an initial speed v_0, at an angle θ_0 above horizontal.
- How far does the cannonball travel horizontally before it returns back to the same height?

Set $x_0 = 0$, $y_0 = 0$

Final position: $x = R$, $y = y_0 = 0$

Initially: $\sin\theta_0 = \frac{v_{0y}}{v_0}$



$\Rightarrow v_{0y} = v_0 \sin\theta_0$

$v_{0x} = v_0 \cos\theta_0$

<u>y-component</u> known: y_0, y, $a_y = -g$

<u>important</u>: t, because it is the same for the x-component.

Not important: v_f, so use $y_f = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$$(y_f - y_0) = v_{0y} t - \frac{1}{2} g t^2$$

$(y_f - y_0) = 0$, $v_{0y} = v_0 \sin\theta_0$

$$v_0 \sin\theta_0 t = \frac{1}{2} g t^2$$

assume $t \neq 0$, divide both sides by t:

$$v_0 \sin\theta_0 = \frac{1}{2} g t$$

, solve for t:

$$\boxed{t = \frac{2 v_0 \sin\theta_0}{g}}$$

<u>x-component</u>: use $x_f = x_0 + v_x t$

$$v_x = v_{x0} = v_0 \cos\theta_0$$

$x_f = R$, $x_0 = 0$

$$\Rightarrow R = v_0 \cos\theta_0 t$$

$$R = v_0 \cos\theta_0 \left[\frac{2 v_0 \sin\theta_0}{g}\right]$$

$$R = \frac{2 v_0^2 \cos\theta_0 \sin\theta_0}{g}$$

We can use the double-angle formula from trigonometry: $\sin 2\theta = 2 \sin\theta \cos\theta$

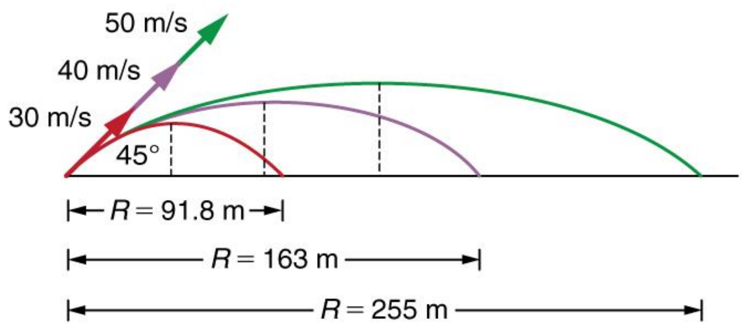
$$R = \frac{v_0^2}{g} \left[2 \cos\theta_0 \sin\theta_0\right]$$

$$\boxed{R = \frac{v_0^2 \sin 2\theta_0}{g}}$$

RANGE

$$R = \frac{{v_0}^2 \sin 2\theta_0}{g}$$

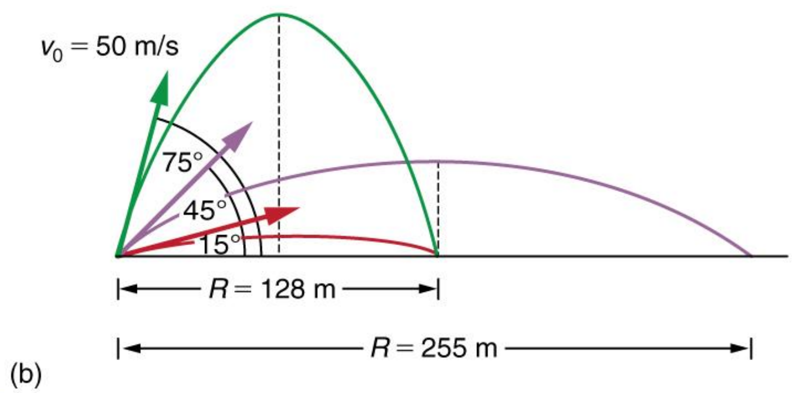
- As you increase the initial speed, v_0, while keeping the angle the same, the range increases.



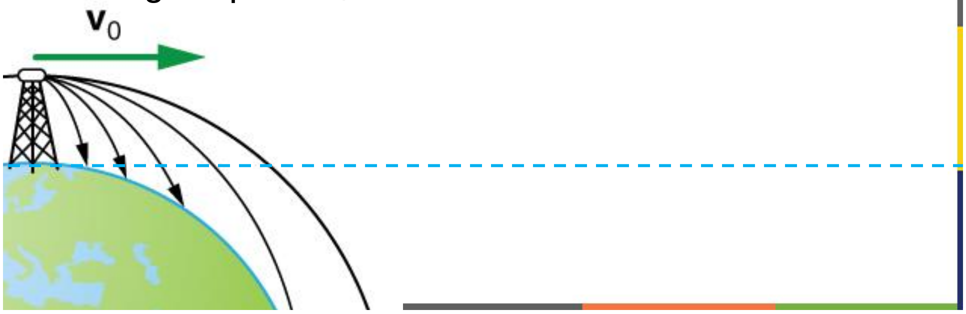
RANGE

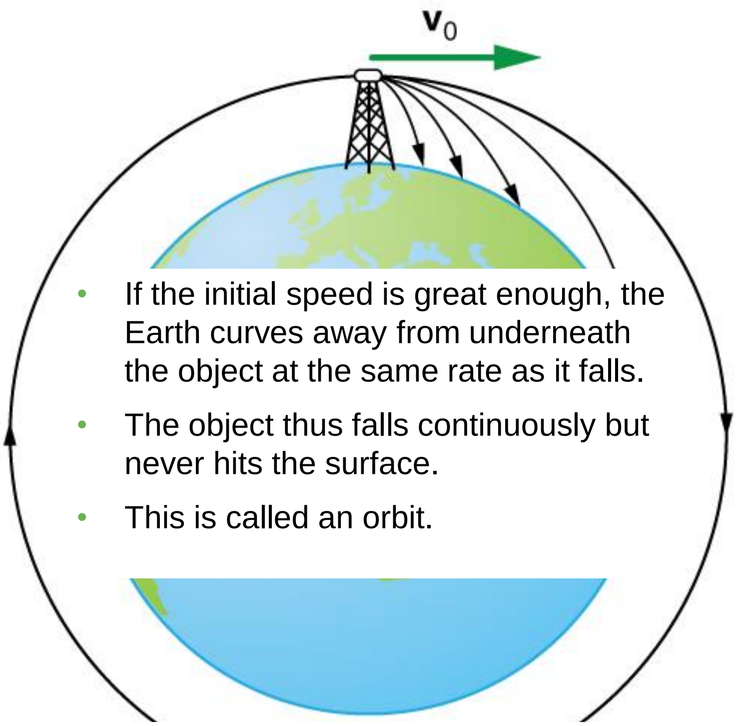
$$R = \frac{{v_0}^2 \sin 2\theta_0}{g}$$

- As you increase the angle of the initial velocity, θ_0, the range increases as θ_0 goes from 0° to 45°, then decreases as θ_0 goes from 45° to 90°.



- When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth.
- If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path.
- The actual range is larger than predicted by the range equation, due to the curvature of the Earth.





- If the initial speed is great enough, the Earth curves away from underneath the object at the same rate as it falls.
- The object thus falls continuously but never hits the surface.
- This is called an orbit.